Division of Work and Fragmented Information: An Explanation for the Diminishing Marginal Product of Labor

Harashima, Taiji

Kanazawa Seiryo University

30 May 2014

Division of Work and Fragmented Information:
An Explanation for the Diminishing Marginal Product of Labor

Taiji HARASHIMA*

May 2014

Abstract

In this paper, an explanation for the diminishing marginal product of labor is demonstrated in a model that incorporates the concept of entropy from information theory. First, I introduce the concept of "division of work" and argue that the division of work (i.e., the allocation of tasks in the production process) and not the division of labor (i.e., worker specialization) is the source of the diminishing marginal product of labor. Division of work results in a fragmentation of the information that workers can access, and inefficiencies other than the commonly assumed factors of redundancy and congestion in labor inputs are generated by this fragmentation of information. The introduced inefficiency is modeled using the concept of entropy from information theory and the experience curve effect theory. The mechanism of the diminishing marginal product of labor is well explained by the model, and the model is similarly used to explore the diminishing marginal product of capital.

JEL Classification code: D24, D80, E23, J24, M54
Keywords: Diminishing marginal product; Division of work; Division of labor; The experience curve effect; The quantity of information; Entropy

---

*Correspondence: Taiji HARASHIMA, Kanazawa Seiryo University, 10-1 Goshomachi-Ushi, Kanazawa-shi, Ishikawa, 920-8620, Japan.
Email: harashim@seiryo-u.ac.jp or t-harashima@mve.biglobe.ne.jp.

1 INTRODUCTION

Traditionally, a diminishing marginal product of labor has been regarded as a law in economics (e.g., Barro and Sala-i-Martin, 2004), even in production functions at the macro level. This property has also predominantly been empirically supported. The cause of the diminishing marginal product of labor is usually attributed to an increase in the amount of redundancy and congestion as labor inputs increase. As a result, the marginal product diminishes. This reasoning has generally been regarded to be too natural to question, particularly for production in small factories, and almost no recent studies have explored the reasons underlying labor's diminishing marginal product. The law of the diminishing marginal product of labor has been applied widely, including in production functions at the macro level.

There is a factor, however, that may eliminate redundancy and congestion in labor inputs at the macro level. If the division of labor is well managed in an economy, redundancy and congestion may disappear. With an appropriate arrangement of division of labor, therefore, the marginal product of labor may not diminish at the macro level. Adam Smith (1776) viewed the division of labor as the specialization of an individual worker's or organization's skills through education, training, and practice that increases productivity. Smith argued that increases in productivity are derived from (1) the dexterity of each worker, (2) time saved doing a more specialized task relative to switching among different activities, and (3) facilitation of production through the use of machines. These three features, however, are basically unrelated to redundancy and congestion. Features (1) and (3) are a function of a worker's increased skill as a result of specialization, not of mitigated redundancy or congestion. Feature (2) is also unrelated because time spent moving to and from various activities becomes unnecessary as a result of a worker's specialization regardless of the existence of redundancy and congestion. As a whole, Smith's concept of the division of labor is basically indifferent to redundancy and congestion. Therefore, the division of labor will not eliminate redundancy and congestion.

However, there is another factor that may eliminate redundancy and congestion. It is superficially similar to the division of labor. In this paper, I call it the "division of work". Division of work does not indicate the level of specialization of workers skills but rather the distribution of differentiated work tasks. I propose that it is not division of labor but division of work that influences redundancy and congestion. If the division of work is well arranged, redundancy and congestion at the macro level will be nearly eliminated.

If well-managed division of work can eliminate redundancy and congestion, why has the marginal product of labor been observed to diminish? The observed diminishing marginal product implies that there may be another inefficiency that even the division of work cannot eliminate. This inefficiency may be the true factor that causes the marginal product of labor to diminish. This paper first examines the nature of division of work in detail and then the relationship between it and the diminishing marginal product of labor. An important feature of the division of work is fragmentation of information among workers. Harashima (2009, 2011, 2012) showed that fragmented information will result in a diminishing marginal product. In this paper, the mechanism shown in the models in Harashima (2009, 2011, 2012) is examined in more detail in the context of entropy as presented in information theory (e.g., Shannon, 1948; Theil, 1967, 1972) and the experience curve theory (e.g., Wright, 1936; Hirsch, 1952; Boston Consulting Group [BCG], 1972). The concept of the quantity of information as a modification of the concept of entropy is introduced in the model presented in this paper. The model shows that marginal product of labor is closely related to the quantity of information that a worker possesses, that the quantity of information is inversely proportionate to the number of workers, and that the quantity of information does not decrease at the same rate that the quantity of labor increases. The quantity of information is shown to be the key factor in the diminishing marginal product of labor.

The paper is organized as follows. In Section 2, the differences between divisions of labor and work are examined and the importance of the fragmentation of information resulting from the division of work is shown. In Section 3, the effect of fragmentation of information is modeled based on the concept of entropy as presented in information theory, and the close relation between the quantity of information and the experience curve effect is shown. The reasons for the diminishing marginal product of labor are explained in Section 4 based on the mechanism developed in Section 3. In Section 5, the diminishing marginal product of capital is examined and concluding remarks are offered in Section 6.

2 DIVISIONS OF LABOR AND WORK

2.1 Differences between divisions of labor and work

The concept of division of labor has a long history dating back to Smith (1776). On the other hand, the idea of division of work has attracted little attention (e.g., Rizavi and Sofer, 2009). Division of work may be confused with division of labor, and it may be viewed as merely expressing the division of labor concept in different words. They are, however, completely different concepts as described below.

2.1.1 Division of labor

Based on Smith (1776), the division of labor is defined here as "the specialization of a worker's skills," where this specialization will continue throughout the worker's career. Each worker is assigned a specialized skill. Hence, division of labor has the three features discussed in the Introduction. These advantages will always prevail in an economy. As noted in the Introduction, they are basically indifferent to redundancy and congestion, which means that specialization is optimized depending on the levels of technology (A) and capital inputs (K) regardless of the quantity of labor inputs (L). As a result, division of labor will have the following features:

N1-1: For each given set of A and K, there exists a best division of labor.
N1-2: Division of labor is kept best in any given period in an economy by market arbitrage.

N1-1 reflects Smith's view of division of labor and indicates that the best division of labor does not depend on L. The degree of specialization is determined by A and K. For example, medical doctors are divided into many specialized fields at present, but formerly a medical doctor dealt with various diseases because existing medical knowledge and capital were relatively primitive as compared with the present. Even if the number of doctors increased in those earlier periods, the degree of specialization would not necessarily change because it was constrained by the existing level of medical knowledge and capital.

2.1.2 Division of work

Division of work is defined here as "the assignment of differentiated tasks to each worker", where (1) workers who possess the same specialized skill may be assigned to different tasks, (2) tasks are divided for each new project or contract, and (3) a divided task seldom encompasses a worker's entire career.

Division of work does not imply a distribution of specialized skills but rather a distribution of differentiated tasks. Even if some workers have the same specialized skill, they may be assigned to different tasks, and conversely, a worker with a specialized skill may be assigned to a variety of tasks that are not necessarily related to that skill. In this paper, a unit of tasks is called "divided work". Because division of work is the assignment of differentiated divided works, division of work has the following features:

N2-1: For each given set of A, K, and L, there exists a best division of work.

N2-2: Division of work is kept best in any period in an economy by market arbitrage.

N2-1 indicates that the best division of work depends on not only A and K but also on L. The key difference from N1-1 is the inclusion of L. This difference means that division of work is influenced by redundancy and congestion in labor inputs. If L changes, the division of work has to be adjusted to best allocate the division of work for the changed set of A, K, and L according to N2-2. For example, if L increases for a given set of A and K, divided work will be adjusted so as to completely eliminate redundancy and congestion. This change will be implemented by adjusting production processes and personnel allocations.

Given N1-1, N1-2, N2-1, and N2-2, divisions of labor and work move differently when L changes. If L changes while A and K are unchanged, the division of labor is unchanged but the division of work may change because by N1-1, the best division of labor is unchanged even if L changes, and by N1-2, division of labor does not change, but on the other hand, by N2-1, the best division of work may be different before and after L changes; thus, by N2-2, the division of work may change to make it optimal if L changes. If the best division of work is different when L is different for any given set of A and K, then if L changes while A and K are unchanged, the best division of work must also change.

2.2. The advantages and disadvantages of division of labor

The advantages of division of labor have previously been discussed, but Smith (1776) also noted some disadvantages: division of labor can result in a mental stagnation in workers, and because workers are confined to a single repetitive task, they can become ignorant of the world. These disadvantages, however, are not studied in theoretical economics because Pareto efficiency is indifferent to them. Economic outcomes may be indirectly affected negatively by the disadvantages to some extent (e.g., workers' abilities may deteriorate), but these negative aspects of the division of labor will not change with a change in L for the same reason the advantages will not.

2.3 The advantages and disadvantages of division of work

2.3.1 The advantage: counterbalance

As N2-1 and N2-2 indicate, redundancy and congestion in labor inputs can be eliminated by division of work. Appropriate division of work maximizes the production for a given set of A, K, and L. N2-1 and N2-2 indicate that any negative effect of an increase in L can be counterbalanced by reorganizing the division of work.

2.3.2 The disadvantage: fragmented information

The well-established concept of the diminishing marginal product of labor suggests that division of work nevertheless cannot remove all inefficiencies with regard to labor inputs, most likely because there are inefficiencies other than redundancy and congestion. Another possible source of inefficiency is the fragmentation of information resulting from the division of work. Each worker experiences only a fraction of the entire production process, and these divided and isolated workers can access only a fraction of the information on the entire production process. It is difficult for a worker to know information held by workers at different production sites. This problem is not a result of division of labor because the specialization of skills does not necessarily imply that workers' access to information is limited. Workers with specialized skills will usually be assigned to specific divided works, and thus the information they have access to may be incomplete, but this limited access is a result of division of work, not division of labor.

Because all of the labor inputs are correlated in production processes owing to the division of work (i.e., production cannot be complete if any part of the divided work is not completed), the problem of fragmented information is especially problematic when workers engage in intellectual activities. Intellectual activities such as decision-making and innovation will be enhanced if the amount of relevant information available to the worker increases. This correlation among all of the labor inputs indicates that all pieces of information about the entire production process need to be completely known by each worker to enable completely correct decision-making. However, only a portion of the information is available to each worker. Workers will inevitably face unexpected problems because of fragmented and incomplete information. When an unexpected problem occurs, workers with fragmented or incomplete information will make different, usually worse, decisions than those with complete information. As a result, overall productivity decreases.

For example, a CEO of a large company may know the overall production plan but not the local and minor individual incidents that happen at each production site each day. In contrast, each worker at each production site may know little of the overall plan but a great deal about local and minor incidents that occur for each specific divided task that each worker engages in at each production site. To be most efficient, even if many unexpected incidents occur, all of the workers and the CEO need to know all about the entire process because all of the labor inputs are correlated owing to the division of work. Practically speaking, however, it is nearly impossible for each worker to access all of the experiences of every other worker.

Division of work cannot simultaneously solve inefficiencies caused by redundancy and congestion and those caused by fragmented and incomplete information. Although a greater level of division of work eliminates the former, it generates the latter. Work is actually divided greatly in most workplaces. This implies that inefficiency resulting from redundancy and congestion is much more serious than that caused by information fragmentation, so work is divided greatly despite the fact that information fragmentation harms productivity.

2.3.3 Solving unexpected problems with fragmented information

Because of fragmentation of information, ordinary workers inevitably will face many unexpected problems as discussed above, and the level of efficiency decreases. If ordinary workers are able to solve at least some of the unexpected problems, inefficiency will be mitigated. Usually, however, ordinary workers are implicitly assumed to do only what they are ordered to do and nothing else, but to solve an unexpected problem, they are required to do more. They need to grasp a situation, speculate on the detailed structure of the entire production process, prioritize actions, and create innovations. They have to engage in intellectual activities to discover unknown mechanisms or rules; that is, ordinary workers have to do more than just what they are ordered to do and are required to possess intelligence. Of course, ordinary workers are not robots who only follow orders. They are human beings who have intelligence and are thus fundamentally different from machines—only humans can fix unexpected problems by creating innovations, and they indeed can at least partly address unexpected problems even with incomplete information. Workers' ability to fix unexpected problems (i.e., use their intelligence) appears indispensable in production processes because many minor but unforeseeable incidents actually do occur on a regular basis.

It is not possible for ordinary workers to completely mitigate inefficiencies because each worker can access only fragmented and incomplete information when creating innovations to fix unexpected problems. As the level of division of work increases, workers are less able to correctly estimate the full structure of the entire production process and correctly prioritize actions to solve unexpected problems. Therefore, the amount of mitigated inefficiency resulting from division of work will depend on the level of information fragmentation.

3 THE QUANTITY OF INFORMATION AND EXPERIENCE

In this section, I examine how information is fragmented, how the degree of fragmentation can be indexed, and how this index is related to division of work.

3.1 Entropy

In information theory (e.g., Shannon, 1948; Theil, 1967, 1972), the information content of a random variable x_i ($\in X$) is defined as

$$-\ln(p_i) \ ,$$

where $X = \{x_1, x_2, x_3, \ldots, x_n\}$ and p_i is the probability of occurrence of x_i. The entropy of X is defined as

$$H(X) = -\sum_{i=1}^{n} p_i \ln(p_i) \ . \tag{1}$$

Entropy in information theory can be interpreted as indicating the value of information contained in a message and also as indicating the "quantity" of information. Therefore, this concept can be used to examine fragmented information, particularly for measuring the magnitude of fragmentation. Higher levels of fragmented information will be seen as smaller levels of entropy for each worker.

3.2 The quantity of information and division of work

Let a "sign" be a fragment of information. A worker will observe various signs that are related to a divided work. Suppose that a worker observes N_L signs in a given divided work. Even if some signs are qualitatively identical, they are accounted for as different signs if they are observed at different times or different places. If work was not divided, all signs could be observed by a worker, but work is divided, so a worker can only observe a portion of the total number of signs. Information that the unobserved signs conveys therefore remains unknown to the worker. As the division of work increases, the number of signs a worker can observe out of the total number of signs decreases. That is, division of work constrains the quantity of information a worker can know.

Signs are categorized by types, and signs within each type are qualitatively identical. Suppose that m types can be perceived in a divided work ($s_1, s_2, s_3, \ldots, s_m$). The probability of occurrence of a type per a sign in a period is assumed to be constant on average, and the occurrence probability of type s_i is p_i. The probability that type s_i is perceived for the first time after a worker observes j signs is

$$p_i(1-p_i)^{j-1}$$

Hence, the probability that type s_i is perceived at least once when a worker observes N_L signs is

$$p_i + p_i(1-p_i) + p_i(1-p_i)^2 + \cdots + p_i(1-p_i)^{N_L-1} = 1-(1-p_i)^{N_L} \ .$$

In this paper, by modifying the concept of entropy, the quantity of information is defined as follows. For the information content $-\ln(p_i)$, the quantity of information for N_L signs is

$$\Pi(N_L) = -\sum_{i=1}^{m} \left\{ \left[1-(1-p_i)^{N_L}\right] \ln(p_i) \right\} \ .$$

The weight p_i in $H(X)$ (equation [1]) is replaced with $1-(1-p_i)^{N_L}$ in $\Pi(N_L)$. This means that the value of information (the information content $-\ln(p_i)$) is same for any N_L, but the probability of obtaining this information increases as N_L increases from p_i to $1-(1-p_i)^{N_L}$. If $N_L = 1$, $\Pi(N_L)$ is equal to entropy, but as N_L increases, $\Pi(N_L)$ gradually diverges from entropy. $\Pi(N_L)$ indicates the quantity of information that a worker can possess by observing N_L signs. Suppose that $1 > p_i \geq p_{i+1} > 0$ for any i. In particular, suppose for simplicity that $p_i = i^{-\tau}$ where $\tau > 1$. Hence, $\Pi(N_L)$ is redefined as $\Pi(N_L, \tau)$ such that

$$\Pi(N_L, \tau) = -\sum_{i=1}^{m} \left\{ \left[1-(1-p_i)^{N_L}\right] \ln(p_i) \right\} = \sum_{i=1}^{m} \left\{ \left[1-(1-i^{-\tau})^{N_L}\right] i^{-\tau} \ln(i^{\tau}) \right\} \ . \qquad (2)$$

Because $1 > p_i > 0$ for any i, $\Pi(N_L, \tau)$ is a finite number.

3.3 Diminishing marginal increase in the quantity of information

The quantity of information increases as N_L increases (i.e., as the worker can observe more signs in the divided work), but the marginal increase in the quantity of information diminishes because

$$\begin{aligned} \frac{d\left[1-(1-i^{-\tau})^{N_L}\right]}{dN_L} &= -\frac{d \exp\left[N_L \ln(1-i^{-\tau})\right]}{d\left[N_L \ln(1-i^{-\tau})\right]} \frac{d\left[N_L \ln(1-i^{-\tau})\right]}{dN_L} \\ &= -(1-i^{-\tau})^{N_L} \ln(1-i^{-\tau}) \ . \end{aligned}$$

Because $(1-i^{-\tau})^{N_L} > 0$ and $\ln(1-i^{-\tau}) < 0$,

$$\frac{d\left[1-(1-i^{-\tau})^{N_L}\right]}{dN_L} > 0 \ ;$$

thus, by Eq. (2),

$$\frac{d\Pi(N_L, \tau)}{dN_L} > 0 \ .$$

In addition,

$$\frac{d^2\left[1-(1-i^{-\tau})^{N_L}\right]}{dN_L^2} = -(1-i^{-\tau})^{N_L} \left[\ln(1-i^{-\tau})\right]^2 \ .$$

Because $(1-i^{-\tau})^{N_L} > 0$ and $\ln(1-i^{-\tau}) < 0$,

$$\frac{d^2\left[1-\left(1-i^{-\tau}\right)^{N_L}\right]}{dN_L^2}<0 \quad ;$$

thus, by Eq. (2),

$$\frac{d^2\Pi\left(N_L,\tau\right)}{dN_L^2}<0 \quad .$$

Therefore, the quantity of information a worker possesses increases at a slower rate relative to the worker's increase in experiences (N_L).

The features of $\frac{d\Pi\left(N_L,\tau\right)}{dN_L}>0$ and $\frac{d^2\Pi\left(N_L,\tau\right)}{dN_L^2}<0$ indicate that, for many values of τ, $\Pi_{NL,\tau}$ can be approximated by exponential functions such that

$$\Pi\left(N_L,\tau\right)\approx \mu N_L^{\beta} \tag{3}$$

for $N_L < \overline{N}$, where μ, β, and $\overline{N}$ are positive constants and $0 < \beta < 1$.

3.4 The quantity of information and the experience curve

The functional form of Eq. (3) is similar to that for the "experience curve effect" that has long been studied in economics and business administration. This similarity implies that the mechanism of the experience curve effect can be explained based on the concept of the quantity of information.

3.4.1 The experience curve

The experience curve effect states that the cost of doing a task will be reduced the more often the task is performed. Workers who perform repetitive tasks exhibit an improvement in performance as the task is repeated a number of times. The primary idea of the experience curve effect (the "learning curve effect" in earlier literature) dates back to Wright (1936), Hirsch (1952), Alchian (1963), and Rapping (1965). The importance of the learning curve effect was emphasized by the Boston Consulting Group (BCG) in the late 1960s and early 1970s (e.g., BCG, 1972). The experience (or learning) curve effect has been applied in many fields, including business management, strategy, and organizational studies (e.g., on airplanes, Wright, 1936; Asher, 1956; Alchian, 1963; Womer and Patterson, 1983; in shipbuilding, Searle and Goody, 1945; on machine tools, Hirsch, 1952; in metal products, Dudley, 1972; in nuclear power plants, Joskow and Rozanski, 1979; Zimmerman, 1982; in chemical products, Lieberman, 1984; Argote et al., 1990; in food services, Reis, 1991). More recently, it has also been applied to technology and policy analysis, particularly energy technologies (e.g., Yelle 1979; Dutton and Thomas, 1984; Hall and Howell, 1985; Lieberman, 1987; Argote and Epple, 1990; Criqui et al., 2000; McDonald and Schrattenholzer, 2001; van der Zwaan and Rablc, 2003, 2004; Miketa and Schrattenholzer, 2004; Papineau, 2006). An empirical problem of the experience curve effect is to distinguish dynamic learning effects from static economies of scale. After surveying empirical studies, Lieberman (1984) concluded that, in general, static scale economies are statistically significant but small in magnitude relative to learning-based economies (see also Preston and Keachie, 1964; Stobaugh and Townsend, 1975; Sultan, 1976; Hollander, 2003).

The experience curve effect is usually expressed by the following functional form:

$$C_N = C_1 N^{-(1-\alpha)} \quad , \tag{4}$$

where C_1 is the cost of the first unit of output of a task, C_N is the cost of the nth unit of output, N is the cumulative amount of output and is interpreted as the experience of a worker engaging in the task, and α is a constant parameter ($0 < \alpha < 1$). $\frac{C_{2N}}{C_N}$ and $1 - \alpha$ are often called the progress ratio and learning rate, respectively. This log-linear functional form is most commonly used because of its simplicity and good fit to empirical data. Empirical studies have shown that α is usually between 0.6 and 0.9. Studies by BCG in the 1970s showed that the experience curve effects for various industries ranged from cost reductions of 10–25% for every doubling of output (i.e., $0.58 \leq \alpha \leq 0.85$) (e.g., BCG, 1972). Dutton and Thomas (1984) present the distribution of progress ratios obtained from a sample of 108 manufacturing firms. The ratios mostly range from 0.7 to 0.9 (i.e., $0.48 \leq \alpha \leq 0.85$) and average 0.82 (i.e., $\alpha = 0.71$). OECD/IEA (2000) argues that industry-level progress ratios have a similar distribution as the firm-level ones shown in Dutton and Thomas (1984; see also, e.g., Hirsch, 1956; Womer and Patterson, 1983; Womer, 1984; Ayres and Martinas, 1992; Williams and Terzian, 1993).

The magnitude of α (or equivalently the progress ratio or learning rate) may be affected by various factors (e.g., Hirsch, 1956; Adler and Clark, 1991; Pisano et al., 2001; Argote et al., 2003; Sorenson, 2003; Wiersma, 2007). Nevertheless, the average α is usually observed to be about 0.7 (i.e., a progress ratio of 0.8 and a learning rate of 0.3) as shown in BCG (1972), Dutton and Thomas (1984), and OECD/IEA (2000). It therefore seems reasonable to assume that α is 0.7 on average.

An important element that experience conveys is information. By accumulating experience by doing a task, a worker increases the amount of information known about the task and the information becomes more complete. In this sense, N, which indicates experience in Eq. (4), reflects the current amount of information a worker possesses about a task. Accumulated experiences will improve efficiency in implementing a task because the worker's amount of information on the task increases. However, if other factors remain the same, the magnitude of improvement will diminish as N accumulates because the information on the task will approach saturation.

Because the essence of experience is that it conveys information, the experience curve effect can be extended to a wide variety of tasks. The tasks need not be limited to a worker's repeated actions, that is, tasks whose experiences are divided by periods. For example, a human activity can be divided into many experiences, each of which is experienced by different workers. Each experience conveys a subset of information, and a part of the subset overlaps with subsets regarding other experiences. The experience curve effect will be applicable to this kind of task if N is interpreted as a subset all worker experiences, so a task in a period with experiences that are divided among workers is also applicable to the experience curve effect in the same way as a task performed by a worker whose experiences are divided by periods. By extension, this logic suggests that tasks applied to the experience curve effect should not be limited to ones with experiences divided only by periods or workers. As long as the task is a human intellectual activity and its experiences are divided by factors other than periods or workers, the task will also be applicable to the experience curve effect because it has the common nature that each divided experience conveys only a subset of all of the information that affects a worker's intellectual activities. Nevertheless, the concept of the experience curve effect should not be expanded infinitely. It can only be applied to the tasks of workers, the performances of which differ depending on the amount of information the worker has.

3.4.2 The quantity of information and the experience curve effect

As shown in Section 3.2, a worker observes N_L signs for assigned divided work. Section 3.4.1

shows that experience in Eq. (4) reflects the current amount of information a worker possesses about a task, and in addition, that the magnitude of improvement will diminish as N accumulates because the information on the task will approach saturation. This nature of the experience curve is very similar to that of the quantity of information, as shown in Sections 3.2 and 3.3. This similarity implies that N in Eq. (4) can be replaced with N_L.

Let all signs in an economy be $\tilde{N}$. N_L is thereby part of $\tilde{N}$. For simplicity, suppose that all workers are identical and the number of signs each worker observes is also identical. Thereby, the experience of a worker (N_L) is inversely proportionate to the number of workers. Hence,

$$N_L = L^{-1}\beta_L \quad ,$$

where β_L is a constant. $N_L L(=\beta_L)$ indicates the total signs in the economy; thus, $\beta_L = \tilde{N}$. Here, $\tilde{N}$ can be normalized such that $\tilde{N} = 1$ by adjusting the scale of L. Because $\tilde{N} = 1$, then

$$N_L = L^{-1} \quad . \tag{5}$$

Let C_{L,N_L} be the magnitude of inefficiency in a worker's divided work caused by fragmented information when each worker's experience is N_L. C_{L,N_L} does not indicate the inefficiency initially generated by fragmented information but the inefficiency that remains even after mitigation by a worker who has dealt with unexpected problems. Costs will increase proportionally with increases in inefficiency; thus, C_{L,N_L} also indicates costs. C_{L,N_L}^{-1} can be interpreted as the amount of productivity in a worker's divided work, which increases as the amount of mitigation by the worker increases.

In Eq. (4), N is interpreted as the experience of a worker engaging in divided work. In Eq. (5), N_L is also interpreted as the worker's experience for a specific divided work. Therefore, N in Eq. (4) can be replaced with N_L. Hence,

$$C_{L,N_L} = C_{L,1} N_L^{-(1-\alpha)} , \tag{6}$$

where $C_{L,1}$ is the inefficiency when $N_L = 1$ (i.e., $N_L = \tilde{N}$ and $L = 1$). Section 3.3 shows that the quantity of information can be approximated by

$$\Pi(N_L, \tau) = \mu N_L^{\beta} \quad . \tag{7}$$

Therefore, combining Eq. (6) and (7),

$$C_{L,N_L} = C_{L,1}\left[\frac{\Pi(N_L, \tau)}{\mu}\right]^{\frac{-(1-\alpha)}{\beta}} \quad . \tag{8}$$

If $1-\alpha = \beta$, then

$$C_{L,N_L} = \frac{C_{L,1}\mu}{\Pi(N_L, \tau)} \quad . \tag{9}$$

Eqs. (8) and (9) indicate that inefficiency C_{L,N_L} decreases as the quantity of information increases and imply that the quantity of information is the driving force behind the experience curve effect. The probable mechanism of the experience curve effect is that, as they amass larger quantities of information, workers can solve more unexpected problems by using their intelligence and the level of inefficiency thereby decreases. As discussed in Section 2, an important element that experience conveys is information. An increase in the quantity of information (i.e., an increase in experience) enables a worker to better understand the entire work scheme, create more innovations, and reduce the quantity of inefficiency caused by the division of work.

3.4.3 The average quantity of information over the various types of work

The value of τ will vary depending on the type of work being done. Eq. (7) therefore indicates that the value of α will also be different across various types of work if α and β are correlated such that $1-\alpha=\beta$. As shown in Section 3.4.1, the estimates of α in the experience curves of various industries are usually between 0.6 and 0.9. This distribution reflects different values of τ in various industries. Nevertheless, the average α of an economy has usually been observed to be about 0.7 across both economies and time periods. Hence, the average quantity of information across the various types of work should also be similar across economies and time period.

4 THE QUANTITY OF INFORMATION AND DIMINISHING MARGINAL PRODUCT OF LABOR

4.1 Diminishing marginal product of labor

4.1.1 Mitigation by intelligence and the quantity of information

Because $C_{L,N_L}^{-1}=C_{L,1}^{-1}L^{\alpha-1}$, the productivity C_{L,N_L}^{-1} is proportional to $L^{\alpha-1}$ not L^{-1}. That is, even if L increases (i.e., signs per L decrease), the decrease in productivity is partially mitigated by the term L^{α} because of workers' intelligence and innovations. From Eq. (9) and $C_{L,N_L}^{-1}=C_{L,1}^{-1}L^{\alpha-1}$,

$$L^{\alpha-1}=\mu\Pi(N_L,\tau) \quad . \tag{10}$$

Eqs. (8), (9), and (10) indicate that C_{L,N_L}^{-1} is proportional to $L^{\alpha-1}$ because the quantity of information $\Pi(N_L,\tau)$ decreases at a slower rate than L increases.

4.1.2 Effective labor input

Let W_L be the total amount of workers' provision of labor input that is supplemented by workers' innovations to mitigate the inefficiency resulting from fragmented and incomplete information. Hence, $L^{-1}W_L$ is the amount of a worker's provision of labor input. Because the amount of a worker's provision of labor input increases as productivity (C_{L,N_L}^{-1}) increases, then $L^{-1}W_L$ is directly proportional to C_{L,N_L}^{-1} such that

$$\frac{W_L}{L} = \frac{\gamma_L}{C_{L,N_L}} \quad , \tag{11}$$

where γ_L is a constant (i.e., γ_L indicates the output per worker in a period when $C_{L,N_L} = 1$). Substituting Eqs. (5) and (6) into Eq. (11) gives

$$W_L = \frac{\gamma_L}{C_{L,N_L}} L = \frac{\gamma_L}{C_{L,1} N_L^{-(1-\alpha)}} L = \frac{\gamma_L}{C_{L,1} L^{1-\alpha}} L = \frac{\gamma_L}{C_{L,1}} L^{\alpha} \quad . \tag{12}$$

Eq. (12) indicates that, instead of L, the labor input effectively provided by workers (L^{α}) is directly proportional to W_L; thus, the effective labor input $\tilde{L}$ is

$$\tilde{L} = \upsilon_L W_L = \omega_L L^{\alpha} \quad , \tag{13}$$

where υ_L and ω_L are positive constant parameters and $\omega_L = \frac{\upsilon_L \gamma_L}{C_{L,1}}$. Even in production functions at a macro level, the effective labor input should approximately have the functional form shown in Eq. (13), and the equation clearly indicates that the marginal product of labor is diminishing, even at the macro level.

4.2 The quantity of information and the rational expectation hypothesis

Suppose that the initial inefficiency (i.e., the level of inefficiency prior to mitigation by workers' innovations) when $N_L = 1$ is $\rho C_{L,1}$ where $\rho > 1$. This initial level of inefficiency is reduced to $C_{L,1}$ by ρ^{-1} as a result of the use of workers' intelligence to innovate. Eq. (6) indicates that, as N_L decreases (i.e., as L increases), the reduction of the initial level of inefficiency decreases; that is, $\rho C_{L,1}$ is reduced to $C_{L,N_L} = C_{L,1} N_L^{-(1-\alpha)}$ by $\rho^{-1} N_L^{-(1-\alpha)}$. For simplicity, suppose that $1 - \alpha = \beta$; thus, by Eq. (7), $\rho^{-1} N_L^{-(1-\alpha)} = \frac{\mu}{\rho \Pi(N_L, \tau)}$. When experience is N_L, the initial level of inefficiency is reduced by $\frac{\mu}{\rho \Pi(N_L, \tau)}$ as a result of the use of workers' intelligence. As the quantity of information increases, the initial level of inefficiency decreases at a higher rate. In other words, as the quantity of information increases, workers can more accurately find the true solutions to unexpected problems. This relationship indicates that the degree of inaccuracy of solutions to unexpected problems is inversely proportional to $\Pi(N_L, \tau)$.

The inaccuracy of a solution to an unexpected problem can be measured by the variance ($\boldsymbol{\sigma}^2$) of differences between actually observed values and predictions based on the solution. That is, the solution that a worker eventually chooses for an unexpected problem will not always be the best, but rather it diverges from the best solution with a variance of $\boldsymbol{\sigma}^2$. As shown above, the variance (i.e., the degree of inaccuracy) will be inversely proportional to $\Pi(N_L, \tau)$, most simply such that

$$\frac{1}{\Pi(N_L, \tau)} = \psi \sigma^2 , \tag{14}$$

where ψ is a positive constant.

Eq. (14) is consistent with the hypothesis of rational expectations, which states that agents' predictions of the future value of economically relevant variables are not systematically wrong, or more simply, the mean prediction of agents is usually correct. The inefficiency caused by the division of work is related not to the mean but to the variance of differences between actually observed values and predictions. If the agents are rational, the mean prediction will not be systematically wrong (i.e., the expectation is rational), but the variance of predictions will take on different values depending on the quantity of information workers possess.

4.3 The quantity of information and the central limit theorem

Suppose that when $\Pi(N_L, \tau) = 1$, the expected degree of inaccuracy of each solution is σ_1^2; that is, $1 = \psi\sigma_1^2$ by Eq. (14). Therefore,

$$\frac{\psi\sigma_1^2}{\Pi(N_L,\tau)} = \psi\sigma^2,$$

by Eq. (14); thus,

$$\sigma^2 = \frac{\sigma_1^2}{\Pi(N_L,\tau)} \quad . \tag{15}$$

The relation described by Eq. (15) is very similar to the central limit theorem. The central limit theorem indicates that the distribution of a random sample of size n with mean μ and variance $\tilde{\sigma}^2$ is close to the normal distribution with mean μ and variance $\frac{\tilde{\sigma}^2}{n}$. The quantity of information $\Pi(N_L, \tau)$ can be interpreted as the size of a random sample. In this random sample, $\Pi(N_L, \tau)$ units of information (i.e., $\Pi[N_L, \tau]$samples) imparts an inaccuracy of $\sigma^2 = \frac{\sigma_1^2}{\Pi(N_L,\tau)}$. As the quantity of information $\Pi(N_L, \tau)$ increases, the level of inaccuracy is reduced at the same rate. With increasing quantities of information, the probability that a worker chooses a solution close to the best solution increases because of the same mechanism shown in the central limit theorem.

5 THE QUANTITY OF INFORMATION AND DIMINISHING MARGINAL PRODUCT OF CAPITAL

As with labor input, capital inputs also have the property of a diminishing marginal product. This property can also be explained by the quantity of information in a similar manner as that discussed for labor.

5.1 The quantity of information

A worker encounters a unique combination of varieties of technology per unit capital ($K^{-1}A$). Let N_A be a worker's average encounter frequency with each variety of technology per unit capital in a period. Many (usually minor) unexpected incidents occur in the production process that are dealt with by ordinary workers. Suppose that an incident requires resolving a problem with a set of different varieties of technology. Hence, the incidents can be categorized by type, each of which implicitly indicates a set of different varieties of technology. The types are different from each other, but incidents within the same type are qualitatively identical.

Suppose that there are m types of incidents in a production process (i_1, i_2, i_3,…, i_m). The probability of the occurrence of a given type per a variety in a period in the production process (i.e., the probability that a type occurs when a worker is encountering a variety) is assumed to be constant on average, and the occurrence probability of type i_q is p_q. The probability that type i_q occurs at least once when a worker's average encounter frequency is N_A is

$$p_q + p_q(1-p_q) + p_q(1-p_q)^2 + \cdots + p_q(1-p_q)^{N_A-1} = \frac{\left[1-(1-p_q)^{N_A}\right]}{1-(1-p_q)} p_q = 1-(1-p_q)^{N_A} .$$

This probability can be interpreted as the probability of the experience of dealing with type i_q with an N_A encounter frequency. Hence, the quantity of information of the production process with an N_A encounter frequency is

$$\Pi(N_A) = \sum_{i=1}^{m} \left\{ \left[1-(1-p_i)^{N_A}\right] \ln\left(\frac{1}{p_i}\right) \right\} .$$

$\Pi(N_A)$ indicates the quantity of information that a worker can possess by encountering (experiencing) a variety N_A times. (N_A therefore indicates a worker's experience on a variety per capital.) As was the case with $\Pi(N_A)$, suppose that $1 > p_i \geq p_{i+1} > 0$ for any i, particularly $p_i = i^{-\tau}$ where $\tau > 1$. Hence, $\Pi(N_A)$ can be redefined as $\Pi(N_A, \tau)$ such that

$$\Pi(N_A, \tau) = \sum_{i=1}^{m} \left\{ \left[1-(1-p_i)^{N_A}\right] \ln\left(\frac{1}{p_i}\right) \right\} = \sum_{i=1}^{m} \left\{ \left[1-(1-i^{-\tau})^{N_A}\right] \ln(i^{\tau}) \right\}.$$

Because $1 > p_i > 0$ for any i, $\Pi(N_A, \tau)$ is a finite number.

The features of $\frac{d\Pi(N_A, \tau)}{dN_A} > 0$ and $\frac{d^2\Pi(N_A, \tau)}{dN_A^2} < 0$ indicate that for many values of τ, $\Pi(N_A, \tau)$ can be approximated by exponential functions such that

$$\Pi(N_A, \tau) = \nu N_A^{\gamma} \tag{16}$$

for $1 \leq N_L < \overline{N}$ where ν, γ, and $\overline{N}$ are positive constants and $0 < \gamma < 1$. In particular, if $\overline{N}$ is not large, $\Pi(N_A, \tau)$ will be approximated very well by Eq. (16).

5.2 Diminishing marginal product of capital

A worker encounters a unique combination of varieties of technologies ($K^{-1}A$) per unit capital. As $K^{-1}A$ increases, the number of varieties per unit capital increases; thus, N_A will decrease because the probability of encountering each of the varieties in $K^{-1}A$ in a period decreases. The amount of $K^{-1}A$ therefore will be inversely proportional to a worker's experience on a variety per capital (N_A) such that

$$N_A = \beta_A \left(\frac{A}{K} \right)^{-1} ,$$

where β_A is a positive constant. Normalizing the worker's average encounter frequency β_A to equal 1, then

$$N_A = \left(\frac{A}{K} \right)^{-1} . \tag{17}$$

Let C_{A,N_A} be the amount of inefficiency resulting from imperfect technology embodied in capital when a worker utilizes a variety of technologies in $K^{-1}A$ in a period. C_{A,N_A} does not indicate the level of inefficiency initially generated by imperfect technology but that remaining after mitigation by workers' innovations. Costs increase proportionally to increases in inefficiency; thus, C_{A,N_A} also indicates costs. Conversely, C_{A,N_A}^{-1} can be interpreted as a productivity in supplementing imperfect technology by creating innovations when a worker utilizes a variety of technologies in $K^{-1}A$ in a period and each worker's experience is N_A. The creation of innovations will increase as the frequency of a worker's encounters with a variety of technologies in $K^{-1}A$ increases (i.e., the productivity in supplementing imperfect technology by creating innovations will increase as the number of experiences increases). Hence, the inefficiency C_{A,N_A} will decrease as the encounter frequency increases. The experience curve effect indicates that inefficiency C_{A,N_A} declines (i.e., productivity C_{A,N_A}^{-1} increases) as a worker's experience on a variety per capital (N_A) increases (i.e., $K^{-1}A$ becomes smaller) such that

$$C_{A,N_A} = C_{A,1} N_A^{-(1-\alpha)} , \tag{18}$$

where $C_{A,1}$ is the level of inefficiency when $N_A = 1$. Note that α is the constant parameter ($0 < \alpha < 1$) used in Eq. (4).

Section 5.1 shows that the quantity of information $\Pi(N_A, \tau)$ can be approximated by Eq. (16). Therefore, combining Eq. (16) and (18),

$$C_{A,N_A} = C_{A,1} \left[\frac{\Pi(N_A, \tau)}{\nu} \right]^{\frac{-(1-\alpha)}{\gamma}} . \tag{19}$$

If $1 - \alpha = \gamma$, then

$$C_{A,N_A} = \frac{C_{A,1}\nu}{\Pi(N_A, \tau)} . \tag{20}$$

Eqs. (19) and (20) indicate that inefficiency C_{A,N_A} decreases as the quantity of information increases and imply that the quantity of information is the driving force behind the experience curve effect.

The amount of technology input per unit capital will increase as C_{A,N_A}^{-1} increases because the inefficiency is mitigated by an increased amount of workers' innovations. Thus, the amount of technology input per unit capital when a worker uses a variety of technologies in $K^{-1}A$ will be inversely proportional to C_{A,N_A} such that

$$W_A\left(\frac{A}{K}\right)^{-1} = \frac{\gamma_A}{C_{A,N_A}} \quad , \tag{21}$$

where W_A is the amount of technology input per unit capital when a worker utilizes a unique combination of varieties of technologies in $K^{-1}A$, and γ_A is a positive constant (i.e., γ_A indicates the amount of technology input per unit capital when a worker utilizes a unique combination of varieties of technologies $K^{-1}A$ in a period when $C_{A,N_A} = 1$). Substituting Eqs. (17) and (18) into Eq. (21) gives

$$W_A = \frac{\gamma_A}{C_{A,N_A}}\left(\frac{A}{K}\right) = \frac{\gamma_A}{C_{A,1}N_A^{-(1-\alpha)}}\left(\frac{A}{K}\right) = \frac{\gamma_A}{C_{A,1}\left(\frac{A}{K}\right)^{1-\alpha}}\left(\frac{A}{K}\right) = \frac{\gamma_A}{C_{A,1}}\left(\frac{A}{K}\right)^{\alpha} \quad . \tag{22}$$

The amount of technology embodied in a unit of capital is $K^{-1}A$. Because technology is imperfect, however, that level of technology input cannot be effectively realized. At the same time, the inefficiency resulting from these imperfections is mitigated by innovations created by ordinary workers even though it is not completely eliminated. Eq. (22) indicates that the magnitude of mitigation depends on $K^{-1}A$, and that, with the mitigation, technology input per unit capital is effectively not equal to $K^{-1}A$ but directly proportionate to $W_A = \frac{\gamma_A}{C_{A,1}}\left(\frac{A}{K}\right)^{\alpha}$. By Eq. (22), therefore, the effective technology input per unit capital ($\tilde{A}$) is

$$\tilde{A} = \upsilon_A W_A = \omega_A\left(\frac{A}{K}\right)^{\alpha} \quad , \tag{23}$$

where υ_A and ω_A are positive constant parameters and $\omega_A = \frac{\upsilon_A\gamma_A}{C_{A,1}}$. Eq. (23) clearly shows that marginal product of capital is diminishing.

6 CONCLUDING REMARKS

The reason for the diminishing marginal product of labor is usually explained by redundancy and congestion in labor inputs. However, there is a factor that may eliminate redundancy and congestion in labor inputs at the macro level: division of work. If division of work is well arranged, redundancy and congestion at the macro level will be almost removed.

If division of work almost eliminates redundancy and congestion, why has the marginal product of labor been empirically observed to be diminishing? The observed diminishing marginal product implies that there is another inefficiency that division of work does not remove. In this paper, I show that fragmentation of information among workers resulting from the division of work is the source of the diminishing marginal product of labor. The mechanism of fragmentation of information shown in Harashima (2009, 2011, 2012) is more deeply examined in the context of the concept of entropy in information theory (e.g., Shannon, 1948; Theil, 1967, 1972) and the theory of the experience curve effect (e.g., Wright, 1936; Hirsch, 1952; BCG, 1972). The model shows that the marginal product of labor is closely related to the quantity of information that a worker possesses, and the quantity of information is the key factor in the diminishing marginal product of labor.

Aspects of the concepts presented in this paper could be beneficial in the field of business management. Although I have assumed that the effects of division of work are identical across firms, they actually will be different among firms. The inefficiency caused by division of work inside a firm will be reduced by improving information sharing among workers in the firm. If a firm succeeds in improving information sharing, it will have an advantage over other competitive firms.

References

Adler, Paul S. and Kim B. Clark (1991) "Behind the Learning Curve: A Sketch of the Learning Process," *Management Science*, Vol. 37, No. 3, pp. 267–281.

Alchian, Armen A. (1963) "Reliability of Progress Curves in Airframe Production," *Econometrica*, Vol. 31, No. 4, pp. 679–693.

Argote, Linda, Sara L. Beckman and Dennis Epple (1990): "The Persistence and Transfer of Learning in Industrial Settings," *Management Science*, Vol. 36, No. 2, pp. 140–154.

Argote, Linda and Dennis Epple (1990) "Learning Curves in Manufacturing," *Science,* Vol. 247, No. 4945, pp. 920–924.

Argote, Linda, Bill McEvily and Ray Reagans (2003) "Managing Knowledge in Organizations: An Integrative Framework and Review of Emerging Themes," *Management Science*, Vol. 49, No. 4, pp. 571–582.

Asher, Harold (1956) *Cost-Quantity Relationships in the Airframe Industry*, Report 291, RAND Corporation, Santa Monica, CA.

Ayres, Robert U. and Katalin Martinas (1992), "Experience and Life Cycle: Some Analytical Implications", *Technovation*, Vol. 12, No. 7, pp. 465–486.

Barro, Robert J. and Sala-i-Martin, Xavier (2004) *Economic Growth*, 2nd edition, MIT Press, Cambridge, MA.

Boston Consulting Group (BCG) (1972) *Perspectives on Experience*, Boston, MA.

Criqui, Patrick, Jean-Marie Martin, Leo Schrattenholzer, Tom Kram, Luc Soete and Adriaan Van Zon (2000) "Energy Technology Dynamics," *International Journal of Global Energy Issues*, Vol. 14, No. 1-4, pp. 65–103.

Dudley, Leonard (1972) "Learning and Productivity Changes in Metal Products," *American Economic Review*, Vol. 62, No. 4, pp. 662–69.

Dutton, John M. and Annie Thomas (1984) "Treating Progress Functions as a Managerial Opportunity," *Academy of Management Review*, Vol. 9, No. 2, pp. 235–247.

Hall, Graham and Sydney Howell (1985) "The Experience Curve from the Economist's Perspective," *Strategic Management Journal*, Vol. 6, No. 3, pp. 197–212.

Harashima, Taiji (2009) "A Theory of Total Factor Productivity and the Convergence Hypothesis: Workers' Innovations as an Essential Element," *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 15508.

Harashima, Taiji (2011)"A Model of Total Factor Productivity Built on Hayek's View of Knowledge: What Really Went Wrong with Socialist Planned Economies?" *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 29107.

Harashima, Taiji (2012) "A Theory of Intelligence and Total Factor Productivity: Value Added Reflects the Fruits of Fluid Intelligence," *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 43151.

Hirsch, Werner Z. (1952) "Manufacturing Progress Functions," *The Review of Economics and Statistics*, Vol. 34, No. 2, pp. 143–155.

Hirsch, Werner Z. (1956) "Firm Progress Ratios," *Econometrica*, Vol. 24, No. 2, pp. 136–143.

Hollander, Samuel (2003) *The Sources of Increased Efficiency: A Study of DuPont Rayon Plants*, MIT Press, Cambridge, MA.

Joskow, Paul L. and George A. Rozanski (1979) "The Effects of Learning by Doing on Nuclear Plant Operating Reliability," *Review of Economics & Statistics*, Vol. 61, No. 2, pp. 161–168.

Lieberman, Marvin B. (1984) "The Learning Curve and Pricing in the Chemical Processing Industries," *The RAND Journal of Economics*, Vol. 15, No. 2, pp. 213–228.

Lieberman, Marvin B. (1987) "The Learning Curve, Diffusion, and Competitive Strategy," *Strategic Management Journal*, Vol. 8, No. 5, pp. 441–452.

McDonald, Alan and Leo Schrattenholzer (2001) "Learning Rates for Energy Technologies,"

Energy Policy, Vol. 29, No. 4, pp. 255–261.
Miketa, Asami and Leo Schrattenholzer (2004) "Experiments with a Methodology to Model the Role of R&D Expenditures in Energy Technology Learning Processes; First Results," *Energy Policy*, Vol. 32, pp. 1679–1692.
OECD/IEA (2000) *Experience Curves for Energy Technology Policy*, The Organization for Economic Co-operation and Development (OECD), Paris, France.
Papineau, Maya (2006) "An Economic Perspective on Experience Curves and Dynamic Economies in Renewable Energy Technologies," *Energy Policy*, Vol. 34, No. 4, pp. 422–432.
Pisano, Gary P., Richard M.J. Bohmer and Amy C. Edmondson (2001) "Organizational Differences in Rates of Learning: Evidence from the Adoption of Minimal Invasive Cardiac Surgery," *Management Science*, Vol. 47, No. 6, pp. 752–768.
Preston, Lee E. and E. C. Keachie (1964) "Cost Functions and Progress Functions: An Integration," *American Economic Review*, Vol. 54, No. 2, pp. 100–107.
Rapping, Leonard (1965) "Learning and World War II Production Functions," *The Review of Economic Statistics*, Vol. 47, No. 1, pp. 81–86.
Reis, Dayr A. (1991) "Learning Curves in Food Services," *The Journal of the Operational Research Society*, Vol. 42, No. 8, pp. 623–629.
Rizavi, Sayyid Salman and Catherine Sofer (2009) "Women's Relative Position and the Division of Household Work: A Study of French Couples," *European Journal of Economic and Social Systems*, Vol. 22, No. 2, pp. 13–26.
Searle, Allan D. and C. S. Goody (1945) "Productivity Increases in Selected Wartime Shipbuilding Programs," *Monthly Labor Review*, Vol. 61, No. 6, pp. 1132–1147.
Shannon, Claude (1948) "A Mathematical Theory of Communication," *Bell System Technical Journal*, Vol. 27, pp. 379–423 and 623–656.
Smith, Adam (1776) *The Wealth of Nations*, W. Strahan and T. Cadell, London.
Sorenson, Olav (2003) "Interdependence and Adaptability: Organizational Learning and the Longterm Effect of Integration," *Management Science*, Vol. 49, No. 4, pp. 446–463.
Stobaugh, Robert B. and Phillip. L. Townsend (1975) "Price Forecasting and Strategic Planning: The Case of Petrochemicals," *Journal of Marketing Research*, Vol. 12, pp. 19–29.
Sultan, Ralph G. (1976) *Pricing in the Electrical Oligopoly, Vol. 2: Business Strategy*, Division of Research, Graduate School of Business Administration, Harvard University.
Theil, Henri (1967) *Economics and Information Theory*, North-Holland, Amsterdam.
Theil, Henri (1972) *Statistical Decomposition Analysis: With Applications in the Social and Administrative Sciences*, North-Holland, Amsterdam.
van der Zwaan, Bob and Ari Rablc (2003) "Prospects for PV: A Learning Curve Analysis," *Solar Energy*, Vol. 74, No. 1, pp. 19–31.
van der Zwaan, Bob and Ari Rablc (2004) "The Learning Potential of Photovoltaics: Implications for Energy Policy," *Energy Policy*, Vol. 32, No. 13, pp. 1545–1554.
Wiersma, Eelke (2007) "Conditions That Shape the Learning Curve: Factors That Increase the Ability and Opportunity to Learn," *Management Science*, Vol. 53, No. 12, pp. 1903–1915.
Williams, Robert H. and Terzian, G. (1993), "A Benefit/Cost Analysis of. Accelerated Development of Photovoltaic Technology," *PU/CEES Report No. 281*, Center for Energy and Environmental Studies, Princeton University, NJ.
Womer, Norman Keith (1984) "Estimating Learning Curves from Aggregate Monthly Data," *Management Science*, Vol. 30, No. 8, pp. 982–992.
Womer, Norman Keith and J. Wayne Patterson (1983) "Estimation and Testing of Learning Curves," *Journal of Business & Economic Statistics*, Vol. 1, No. 4, pp. 265–272.
Wright, Theodore Paul (1936) "Factors Affecting the Cost of Airplanes," *Journal of the Aeronautical Sciences*, Vol. 3, No. 4, pp. 122–128.
Yelle, Louis E. (1979) "The Learning Curve: Historical Review and Comprehensive Survey,"

Decision Sciences, Vol. 10, No. 2, pp. 302–328.
Zimmerman, Martin B. (1982) "Learning Effects and the Commercialization of New Energy Technologies: The Case of Nuclear Power," *Bell Journal of Economics*, Vol. 13, No. 2, pp. 297–310.