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Sensitivity of Value at Risk estimation to Non-Normality of returns and Market capitalization

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Abstract

This paper investigates sensitivity of the VaR models when return series of stocks and stock indices are not normally distributed. It also studies the effect of market capitalization of stocks and stock indices on their Value at risk and Conditional VaR estimation. Three different market capitalized indices S&P BSE Sensex, BSE Mid cap and BSE Small cap indices have been considered for the recession and post-recession periods. It is observed that VaR violations are increasing with decreasing market capitalization in both the periods considered. The same effect is also observed on other different market capitalized stock portfolios. Further, we study the relationship of liquidity represented by volume traded of stocks and the market risk calculated by VaR of the firms. It confirms that the decrease in liquidity increases the value at risk of the firms.

Keywords Non-normality, market capitalization, Value at risk (VaR), CVaR, GARCH

JEL: C20, C22, G10

1. Introduction

The Value-at-risk (VaR) model pioneered by J.P. Morgan group in 1994 is a popular tool for managing market risks. Jorion (2001) describes VaR as a measure of worst expected loss over a given horizon under normal market condition at a given level of confidence. VaR asks a simple question how bad things can get. VaR is a function of two parameters confidence level ($x\%$) and time horizon (N). VaR is the loss corresponding to the $(100-x)$ th percentile distribution of the change in the value of the portfolio over the next N days. Among the main advantages of VaR are simplicity, wide applicability and universality.

As per Jorion (1990, 1997) and Morgan (1996), the VaR of a portfolio can be calculated as follows: let $r_1, r_2, r_3, \dots, r_n$ be identically distributed independent random variables representing the

financial returns of stocks. $F(r)$ is used to denote the cumulative distribution function, $F(r) = \Pr(r < r | t - 1)$ on the information set Ω_{t-1} that is available at time $t - 1$. Assuming that r_t follows the stochastic process:

$$r_t = \mu + \epsilon_t \quad (1)$$

$$\epsilon_t = z_t \sigma_t z_t \sim \text{iid} (0, 1)$$

Where $\sigma_t^2 = E(z_t^2 | \Omega_{t-1})$ and z_t has conditional distribution function $G(z)$, $G(z) = \Pr(z_t < z | \Omega_{t-1})$. The VaR with a given probability $\alpha \in (0, 1)$ is denoted by $\text{VaR}(\alpha)$, is defined as α quantile of the probability distribution of financial returns: $F(\text{VaR}(\alpha)) = \Pr(r_t < \text{VaR}(\alpha)) = \alpha$ or $\text{VaR}(\alpha) = \inf\{v | P(r_t \leq v) = \alpha\}$. To estimate σ_t , Morgan (1996) uses Exponential weighted moving average model (EWMA). The expression of this model is as follows:

$$\sigma_t^2 (1 - \lambda) \sum_{j=0}^{n-1} \lambda^j (\epsilon_{t-j})^2 \quad (2)$$

Where, $\lambda = 0.94$

$$\text{VaR}(\alpha) = F^{-1}(\alpha) = \mu + \sigma_t G^{-1}(\alpha) \quad (3)$$

Hence, a VaR model involves the specifications of $F(r)$ or $G(z)$

Conditional VaR

CVaR is a conditional VaR. VaR measures how worst things can get but CVaR measures the losses beyond VaR. It is also a function of two Parameters time horizon (N) and the confidence level ($x\%$).

$$\text{CVaR}(r) = E[r | r > \text{VaR}(r)] \quad (4)$$

Where, r represents return of indices. For calculating VaR, two parameters are important, one is to accurately map the distribution of the returns and second is to model the volatility of the return.

Last decade has witnessed plethora of literature on capturing these two above mentioned parameters to significantly improve the basic model of VaR. This study focuses on the importance of market capitalization of stocks and indices of stocks on VaR estimation model.

Halbelib and Pohlmeier (2012) considered the importance of market capitalization in VaR estimation. They compared various VaR models, their distribution pattern across different time windows and with this they also empirically proved the importance of market capitalization on VaR estimation. Dias (2013) investigated the importance of market capitalization on NYSE, AMEX and NASDAQ stocks, the result proved the importance of market capitalization on VaR estimation. Majority of the studies about VaR model are concentrated on (i) correctly modeling distribution of returns (ii) modeling volatility of the returns (iii) on comparison of different VaR models. Beder (1995), Hendricks (1996) and Pritsker (1997) compared various VaR models; they reported that no method performed significantly different from the other Ashley (2009) examined the extreme value theory and showed that the filtered historical simulation method performed better than other VaR estimation methods. According to Butler (1998) historical Simulation approach does not best utilize the information available. It also has the practical drawback that it only gives VaR estimates at discrete confidence intervals determined by the size of our data set.

The distribution of financial return has been documented to exhibit significantly excessive kurtosis (fat tails and peakness). Bollerslev (1987) indicated that normality assumption of returns is violated. Therefore, McAleer (2010a) proposed a risk management strategy consisting of choosing from among different combinations of alternative risk models to estimate VaR. This model gives a better estimate of VaR. Engle (1982) proposed the autoregressive conditional heterocedasticity (ARCH), considering variance that does not remain fixed but rather varies throughout a period. Bollerslev (1986) further extended the ARCH model to generalized model (GARCH). As in the GARCH family, alternative and more complex models have been developed for the pattern of the large memory. Harvey (1996), Giot and Laurent (2004) compared several volatility models, EWMA an asymmetric GARCH and realized volatility (RV). The models are estimated with the assumption that returns follow either normal or skewed t-Student distributions. They found that under a normal distribution, the RV model performed best. However, under a skewed t-distribution, the asymmetric GARCH and RV models provided very similar results.

Varma (1999) compared various model of VaR in Indian stock market. He did comparative analysis on NSE 50 index. He showed GARCH-GED model performed well in all common risk levels. Bhattacharyya & Madhav R (2012) did comparative analysis on VaR models for leptokurtic stock returns using 6 major Indices Sensex, Nifty, DJI, FTSE, HIS and Nikkei. Kuester (2006) used returns of NASDAQ index for VaR calculation. McNeil (2000) did back-testing on S&P500, DAX indices, BMW stock price, US Dollar-British pound exchange rate and gold prices Majority of the models performed analysis on large capitalized firms, major indices or highly traded currency which creates a research gap for estimation and validation of current VaR models for the mid cap and small cap firms or indices as mutual fund houses estimate VaR for different funds which are composed of different size of stocks so, is it correct to pool all assets together for calculating VaR.

Chuang (2012) investigated the relation between trading volume, stock return and stock volatility they had done analysis on 10 Asian stock markets. They found negative relation between trading volume and volatility in Japan and Taiwan. Copeland (1976) and Smirlock (1985) found significant relationship between trading volume and volatility Lamoureux (1990) proved that information contained in trading volume improves the prediction of volatility of stock return. Darrat (2003) finds evidence of a volume and volatility relation.

The study considers sensitivity of VaR models for various market capitalized index and stocks, when the returns are not-normally distributed. It empirically analyze the riskiness of different market capitalized stocks with the help of VaR and CVaR model and establishing relationship between market riskiness and share turnover. It examines the effect of market capitalization on VaR violations. This article is organized in five sections. Section 2 briefly discusses VaR methodologies used in the study and discusses back-testing model used. Section 3 discusses the data and methodology used, in section 4 results are reported and section 5 concludes the study.

2. VaR models

According to literature there are three types of VaR models (i) Parametric, (ii) Non-Parametric model and (iii) Semi-Parametric model. Parametric model has assumption of normal distribution of returns Morgan (1996). Non-parametric has historical simulation approach, and Semi-

parametric model has Monte Carlo approach. In this study we have used Parametric model assuming normal distribution, parametric model using conditional volatility with the help of GARCH (1,1) model and VaR estimation by fitting empirical distribution of the returns.

2.1 Parametric VaR estimation

Parametric VaR estimation model assumes the underlying distribution to be normal. In this model VaR is estimated as $1-\alpha$ quantile of standard normal distribution.

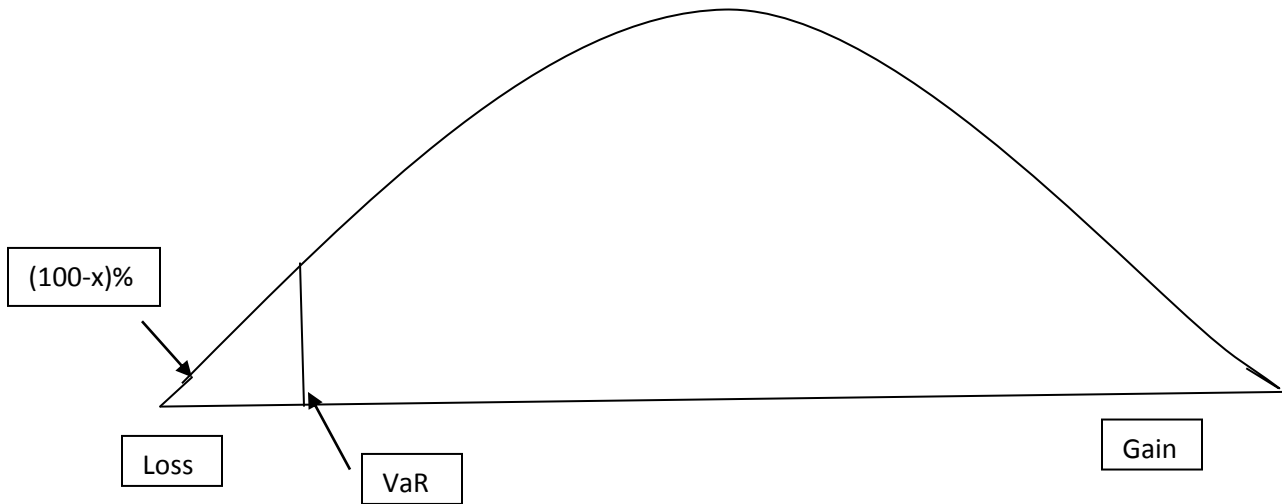


Fig.1

2.2 Parametric VaR using Garch (1, 1) volatility modeling with student t innovation

Underlying distribution of financial return has been documented to exhibit significantly excessive kurtosis (fat tails and peakness) Bollerslev (1987), therefore estimation of VaR by assuming normal distribution will not give accurate results therefore to model the volatility, Generalized autoregressive model(GARCH) has been used in VaR estimation. It estimates two equations: the first is mean equation, whereas second equation patterns the evolving volatility of returns. The most generalized formulation for the GARCH models is the GARCH (p, q) model represented by the following expression:

$$r_t = \mu_t + \epsilon_t \tag{5}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-1}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (6)$$

2.3 VaR estimation by fitting the empirical distribution of the returns.

It is known that distributions of stock returns generally possess kurtosis i.e. fatter tails than normal distribution, and they are skewed. The presence of excess-kurtosis or skewness or both indicates the non-normality of the underlying distribution. The approaches to handle non-normality fall under three broad categories; (i) using historical simulation method as there is no assumption of underlying distribution in this method (ii) fitting suitable non-normal or mixture distribution; (iii) or by modeling only the tails of return distribution like extreme value theory (EVT) method. If the specific form of the non-normality were known, one can easily estimate the VaR from the percentiles of the specific distributional form. The class of distributional forms considered would be quite large like t-distribution, mixture of two normal distribution, hyperbolic distribution, laplace distribution and so forth, Van den Goorbergh (1999). In this study underlying distribution of the stock return or index return is estimated with the help of @risk software¹ thereafter the VaR is estimated by the left most $1-\alpha$ percentile of the distribution. In the study, distribution fitted by the return series are not normal but they are distributed as Logistic, Weibull or Laplace distribution.

2.4 Properties of different distribution fitted by the return series.

2.4.1 Logistic Distribution

Logistic distribution is a continuous probability distribution. It has heavier tails as compared to normal distribution. This distribution is used in Logistic regression. If Z has standard Standard logistic distribution then for any $a \in \mathbb{R}$ and any $b > 0$,

$$x = a + bZ \quad (7)$$

has the logistic distribution with location parameters a and scale parameter b . the probability density function of the distribution is as follows:

¹ @Risk is window based software (from Palisade Corporation) for Monte Carlo simulation. It also supports a number of statistical distributions.

$$f(x) = \frac{\exp\left(\frac{x-a}{b}\right)}{b\left(1+\exp\left(\frac{x-a}{b}\right)\right)^2}, x \in \mathbb{R} \quad (8)$$

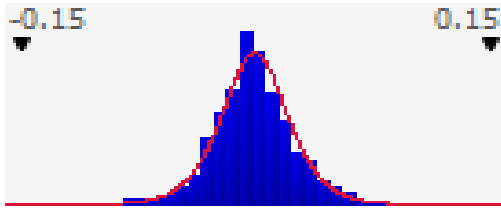


Fig 2 Logistic distribution

2.4.2 Laplace Distribution

It is a continuous probability distribution. It resembles normal distribution but it has higher spikes and slightly thicker tails than normal distribution. Suppose x has laplace distribution with location parameter a and scale parameter b . x has probability density function given as follows:

$$f(x) = \frac{1}{b} \exp\left(-\frac{|x-a|}{b}\right), x \in \mathbb{R} \quad (9)$$

- f is symmetric about a .
- f increases on $[0, a]$ and decreases on $[a, \infty]$. The mode is at $x = a$.

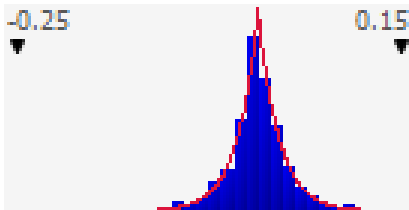


Fig: 3 Laplace distributions

2.4.3 Weibull distribution

A random variable x is said to have a Weibull distribution with parameters α and β ($\alpha > 0$, $\beta > 0$), the pdf of x is

$$f(x; \alpha, \beta) = \begin{cases} \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (10)$$

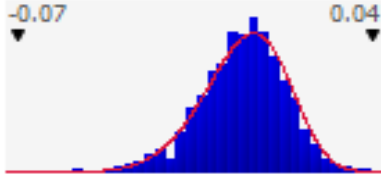


Fig:5 Weibull distribution

2.5 Back testing

Accuracy of VaR model is tested with the back testing procedure. It checks how many times losses in a day exceeded the 1-day 99% VaR. When actual losses exceeded VaR then it is referred to as exceptions. If exceptions happen to be around 1% in 99% VaR then, VaR model is accurate or fit for market risk estimation, if exceptions are 5% then the accuracy of the VaR model is doubted. Hence we can say VaR is underestimated. In this study Kupiec (1995) model is used to back-test the VaR accuracy. Suppose that the time horizon is one day and the confidence limit is $x\%$. If the VaR model used is accurate, the probability of the VaR being exceeded on any given day is $p = 1 - X$. Suppose that we look at a total of n days and we observe that the VaR limit is exceeded on m of the days where $m/n > p$. Here we test two hypotheses:

H_0 : The probability of an exception on any given day is p

H_1 : The probability of an exception on any given day is greater than p

It is assumed that exceptions are IID distributed and they follow Binomial distribution. From the properties of the binomial distribution, the probability of the VaR limit being exceeded on m or more days is:

$$= \sum_{m=0}^n \binom{n}{m} x^m a^{n-m} \quad (11)$$

The most often used confidence level in statistical tests is 5%. If the probability of the VaR limit being exceeded on more days is less than 5%, we reject the first hypothesis that the probability of an exception is p . If this probability of the VaR limit being exceeded on k or more days is greater than 5%, then the hypothesis is not rejected.

3. Data and Methodology

The period of analysis is considered from November 1st, 2005 till December 31st 2013. Three stock indices have been taken which represent different market capitalization. BSE Sensex 30 which represent highest market capitalized firms, BSE mid cap index representing firms with medium size and BSE small cap index representing small capitalized firms. BSE mid cap and small cap index is operational in India from April 2005 therefore period after April 2005 is considered. Daily closing prices of indices and stocks are taken from Bloomberg database .Daily log returns are calculated. Sample is divided into two periods recession period² and post recession period. VaR is calculated using 1000 trading days daily data. Value at risk is calculated with the help of three methods parametric VaR method assuming distribution to be normal, Garch (1,1) method for modeling conditional variance and Parametric VaR method using the empirical distribution of the return calculated with the help of @ risk software. To further investigate the effect of market capitalization on accuracy of VaR model, we have taken sample of 328 BSE 500 index firms. Firms are divided into 30 portfolios, where portfolio 1 means top 10% firms according to market capitalization, second portfolio means next 10% firms according to market capitalization so on and so forth.

3.1 Summary statistics

From Table 1 it is evident that returns are decreasing with decreasing market capitalization during normal market conditions. Jaraque –Bera, Anderson Darling test and Kolmogorov-Smirnov test proves that returns are not normally distributed in case of all the three indices. Variation in return is also highest for highest market capitalized index.

²Recession period is considered from 2007-2009

Table 1 Summary statistics for return series during post –recession & recession period

Post-Recession				Recession			
Index	Sensex	BSE Mid Cap Index	BSE Small Cap Index	Index	Sensex	BSE Mid Cap Index	BSE Small Cap Index
Mean	0.00023	3.54E-05	-0.000184	Mean	0.000631	0.00046	0.000366
Median	0.00041	0.000932	0.001202	Median	0.001379	0.00255	0.0028
Maximum	0.03704	0.034587	0.038664	Maximum	0.1599	0.11111	0.086601
Minimum	-0.04213	-0.04587	-0.06098	Minimum	-0.116044	-0.12076	-0.108357
Std. Dev.	0.01097	0.010385	0.010807	Std. Dev.	0.021069	0.01954	0.019459
Skewness	0.00778	-0.47691	-0.764167	Skewness	0.10076	-0.79575	-0.837959
Kurtosis	3.74065	3.913398	5.32771	Kurtosis	8.014637	7.93602	6.50578
Jarque-Bera	23.0272	73.1782	325.3465	Jarque-Bera	1056.813	1128.56	633.5371
P-value	0.00001	0	0	P-value	0	0	0
AD Statistics*	2.2	3.95	3.09	AD Statistics	11.08	17.84	15.73
KS Statistics**	0.04	0.05	0.06	KS Statistics	0.08	0.1	0.09

*Anderson Darling test **Kolmogorov-Smirnov test

From the Table 1 it is evident that volatility/standard deviation has increased almost twice during the recession period and volatility is highest for the Sensex which represent top market capitalized firms. This means highest market capitalized firms were more sensitive to global recession as compared to small capitalized firms. Skewness indicator used in distribution analysis is a sign of asymmetry and deviation from a normal distribution. Skewness more than zero means, right skewed distribution. It is observed from Table 1 that distribution for Sensex is positively skewed while distribution for mid Cap and small cap index are negatively skewed both in case of recession and post -recession period. If we look at the kurtosis of the series it is almost 3 post recession for large cap and mid cap index but more than three for small cap index which gives us the reason to think whether model for VaR calculation should be same for different market capitalized firms as high kurtosis leads to high probability for extreme values. The peaks

are more than three in case of recession for all three indices. If we compare the Kurtosis values during recession and post-recession one interesting observation is that kurtosis value is increasing with decreasing market capitalization after recession but during recession kurtosis is increasing with increasing market capitalization.

Table 2 Summary Statistics for thirty portfolios post-recession

Post Recession										
Portfolio	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	KS - Statistics	AD - Statistics	Jarque-Bera
1	7.88E-05	-0.0003	0.036462	-0.04154	0.010696	-0.01515	3.612125	0.03660	1.56360	15.65063
2	-7.8E-05	0.000289	0.04285	-0.053	0.012478	-0.01169	3.716783	0.03210	1.80370	21.43015
3	0.000265	0.000902	0.040224	-0.03545	0.011157	-0.04762	3.44903	0.03260	1.33010	8.779134
4	1.52E-05	0.00064	0.048034	-0.05426	0.01248	-0.19268	3.839021	0.03220	2.11780	35.51928
5	-0.00018	0.000716	0.039177	-0.06041	0.01363	-0.39113	3.937592	0.04210	2.65880	62.12596
6	0.000163	0.000684	0.036459	-0.03574	0.010416	-0.25863	3.402755	0.04030	2.08490	17.90694
7	0.000232	0.001112	0.048355	-0.04113	0.011723	-0.24539	3.580707	0.04370	2.21580	24.08691
8	0.000437	0.000996	0.03492	-0.03854	0.009281	-0.29852	3.947219	0.03900	2.60180	52.2364
9	0.00019	0.000641	0.03774	-0.04308	0.011611	-0.17106	3.6399	0.03570	1.96980	21.93849
10	9.11E-05	0.000296	0.044504	-0.04354	0.012016	-0.19215	3.483954	0.03670	1.50350	15.91234
11	0.000165	0.00102	0.039982	-0.0548	0.013478	-0.31784	3.559854	0.04080	1.93820	29.89673
12	-0.0003	7.97E-05	0.047444	-0.10533	0.014033	-0.6215	6.375543	0.03090	1.85420	539.1396
13	0.000177	0.00107	0.053367	-0.05434	0.012872	-0.33177	4.187597	0.03690	2.30710	77.11085
14	-0.00013	0.000565	0.042752	-0.05564	0.013186	-0.2672	3.647726	0.0231*	1.31570	29.38086
15	0.000124	0.001104	0.042873	-0.0572	0.012669	-0.32032	3.944568	0.04100	2.66180	54.276
16	-0.00048	0.000338	0.040029	-0.04468	0.012705	-0.25225	3.522147	0.03180	1.80980	21.96459
17	-0.00027	0.000218	0.039636	-0.06168	0.012498	-0.3695	4.269499	0.03970	2.37050	89.90563
18	-5.7E-05	0.000469	0.053626	-0.08092	0.011451	-0.55738	6.308657	0.04820	3.74230	507.9133
19	-0.0005	0.000359	0.03303	-0.05763	0.010572	-0.54719	4.832763	0.04880	4.22080	189.8614
20	0.000301	0.000604	0.039193	-0.04729	0.010277	-0.08127	3.762069	0.02580	0.75980	25.29861
21	-1.4E-05	0.000548	0.045569	-0.05722	0.012842	-0.3079	3.893222	0.03190	1.81150	49.04433
22	-6.2E-05	0.000644	0.037908	-0.05039	0.012137	-0.32846	3.45715	0.03120	1.62560	26.68881
23	-0.00016	0.000358	0.039834	-0.0533	0.012339	-0.30499	3.981346	0.03010	1.37430	55.62978
24	0.000365	0.000781	0.044727	-0.06277	0.012007	-0.53871	4.806199	0.03990	2.59170	184.2988
25	-8.8E-05	0.000673	0.05246	-0.06733	0.013144	-0.47157	4.700085	0.04960	3.03760	157.4914
26	0.000299	0.000678	0.042521	-0.049	0.011539	-0.21436	3.904548	0.03690	1.89710	41.75001
27	0.000565	0.001338	0.044068	-0.05051	0.01183	-0.42106	4.344141	0.05290	5.01430	104.8282
28	0.0008	0.001569	0.042541	-0.04422	0.01322	-0.26986	3.784603	0.03900	2.33340	37.78745
29	0.000929	0.001821	0.039971	-0.04863	0.012865	-0.33924	3.676746	0.04260	2.68380	38.263
30	0.002231	0.001886	0.212352	-0.17453	0.012742	1.830353	112.455	0.10340	---	499741.4
Combined	6.17E-05	0.00043	0.03305	-0.04224	0.010284	-0.17274	3.565242	0.08940	13.63360	18.28544

*Significance at 5%

From Table 2, it is evident that lower portfolio returns are more negatively skewed as compared to upper portfolio. Kurtosis is also higher for lower portfolio. Therefore in lower portfolio of stocks there is greater probability for extreme values of return towards negative side.

Table 3 Summary Statistics for ten portfolios during recession

Recession										
Portfolio	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	KS Statistics	AD Statistics	Jarque-Bera
1	0.000969	0.001562	0.114779	-0.09142	0.015918	0.015337	8.22392	0.0648	8.4592	1137.095
2	0.000991	0.002211	0.159304	-0.11027	0.021497	-0.05826	7.702621	0.0763	9.9536	922.0092
3	0.000961	0.001855	0.130423	-0.11783	0.019335	-0.20333	7.210121	0.0625	7.3354	745.437
4	0.000963	0.002753	0.122926	-0.10151	0.019281	-0.34995	6.564651	0.0693	9.5764	549.8585
5	0.001	0.00269	0.133752	-0.16822	0.023832	-0.74329	8.140332	0.0893	12.8340	1193.04
6	0.000795	0.002205	0.091477	-0.11212	0.017918	-0.73691	8.567528	0.0790	13.3755	1382.064
7	0.000734	0.002215	0.13425	-0.12278	0.021332	-0.38509	7.619923	0.0719	12.1209	914.0366
8	0.000923	0.001478	0.09246	-0.10638	0.017608	-0.51612	7.16306	0.0755	10.9405	766.5253
9	0.000522	0.002177	0.125777	-0.09558	0.019461	-0.23417	6.930422	0.0636	10.3987	652.8147
10	0.000557	0.001449	0.103113	-0.08685	0.016761	-0.49203	7.460121	0.0754	9.4127	869.2108
11	0.000983	0.002712	0.10946	-0.10764	0.020405	-0.76965	7.246917	0.0909	13.5923	850.2388
12	0.000621	0.001771	0.120311	-0.13193	0.020891	-0.58993	8.163323	0.0880	13.6793	1168.832
13	0.000588	0.001893	0.134019	-0.11764	0.019875	-0.54095	7.717253	0.0697	9.5338	975.9575
14	0.000798	0.002103	0.257716	-0.09623	0.019693	1.548202	33.3154	0.0808	—	38692.12
15	0.000911	0.001512	0.202555	-0.08196	0.017332	1.139837	23.31707	—	—	17415.84
16	0.000512	0.001732	0.104533	-0.13578	0.02142	-0.74585	8.753746	0.0912	15.3566	1472.115
17	0.00043	0.0009	0.104053	-0.09372	0.01734	-0.25664	6.790341	0.0731	9.3951	609.5888
18	0.000981	0.002558	0.096596	-0.14762	0.019287	-0.89619	9.409434	0.0860	15.7098	1845.561
19	0.000841	0.002155	0.143748	-0.11185	0.019032	-0.23356	9.640332	0.0664	10.9524	1846.342
20	0.00089	0.00113	0.144237	-0.09029	0.015582	0.235684	12.57474	0.0809	—	3829.077
21	0.000329	0.001575	0.092967	-0.10847	0.018947	-0.46488	6.96493	0.0723	11.3517	691.046
22	0.000232	0.001619	0.101033	-0.11666	0.020606	-0.52396	7.040881	0.0794	11.2071	726.1189
23	0.000293	0.001943	0.101799	-0.11895	0.022431	-0.85609	7.020232	0.1031	16.2640	795.5747
24	0.00025	0.000786	0.175337	-0.09518	0.019847	0.234683	11.2412	0.0856	—	2839.071
25	0.000451	0.002382	0.078762	-0.13385	0.021376	-0.76222	6.472844	0.0650	8.8562	599.3562
26	0.000612	0.001098	0.193012	-0.11822	0.018326	0.561534	18.86069	0.0731	—	10534.28
27	0.000516	0.000845	0.077654	-0.0966	0.017265	-0.38614	6.900841	0.0697	9.8864	658.8735
28	0.000528	0.001333	0.065103	-0.1382	0.01674	-1.25656	10.99658	0.0719	9.3240	2927.544
29	0.000894	0.000818	0.071387	-0.07345	0.016118	-0.35647	5.679147	0.0638	6.6280	320.2542
30	0.000216	0.000678	0.043499	-0.062	0.012544	-0.22105	4.453012	0.0435	3.1818	96.11257
Combined	0.00091	0.002381	0.121886	-0.09577	0.016618	-0.33575	8.316677	0.08	13.6336	1196.581

*Significance at 5%

From the Table 2 it is observed that none of the portfolio return series is found to be normally distributed. It is evident that after recession standard deviation was almost same for different portfolios but during recession period standard deviation was higher for high capitalized firms and lesser for smaller capitalized firms, but if we take all the firms together in a portfolio the standard deviation is on the lower side. During recession skewness is also increasing with decreasing market capitalization an indicator of extreme losses in smaller capitalized portfolios, but kurtosis value is on higher side in lower capitalized firm's portfolio. The combined portfolio has the kurtosis on the lower side. Therefore combining all the different market capitalized firms will not give correct VaR estimation.

4. Results

4.1 Parametric VaR

It is assumed that Rs100, 000 is invested in each index and portfolio of stocks. Since the returns on each index are not normally distributed we cannot use parametric method of VaR calculation. Table 4 and Table 5 suggest that if we use parametric method of VaR calculation for this scenario the VaR model fails to pass Kupic test.

Table 4 Parametric VaR model results during recession

Index	BSE SENSEX	BSE Mid Cap	BSE Small Cap
VaR	4845.877256	4507.602707	4497.436491
% Violation	1.89%	2.38%	3.08%
CVaR	6214.187072	6530.242219	6030.493007
Return	0.06%	0.05%	0.04%
Kupic Test	0.007401866	0.000120963	7.48E-08
Result	Model Rejected	Model Rejected	Model Rejected

Table 5 VaR model results Post Recession

Index	BSE SENSEX	BSE Mid Cap	BSE Small Cap
VaR	2533.409533	2416.072357	2536.416068
% Violation	1.19%	2.28%	2.38%
CVaR	3117.029942	2954.998392	3411.184404
Return	0.02%	3.54E-05	-0.02%
Kupic test	1.54	0.000299727	0.000120963
Result	Model Accepted	Model Rejected	Model Rejected

That means contemporary parametric VaR methodology is not suitable when returns are not normally distributed. The VaR violations are higher in both the cases.

4.2 Parametric VaR using Garch (1, 1) model with student t innovation

In this section we estimate VaR using parametric Garch (1,1) model to find out conditional volatility, using student t innovation. From Table 6 it is quite evident that percentage VaR violations are least for highest market capitalized index, and VaR violations increase as the market capitalization decreases, the model is accepted only in case of high capitalization index.

If we look at the extreme risk indicator, CVaR is lesser for small capitalized index during recession as compared to highest market capitalized index whereas, it is highest in post-recession.

Table 6 Parametric Garch (1, 1) VaR results

Index	Post-Recession			Recession		
	BSE SENSEX	BSEMID CAP	BSE SMALL CAP	BSE SENSEX	BSE MID CAP	BSE SMALL CAP
VaR	2550.131	2376.14309	2421.40702	5072.769	2573.5691	4497.436
% Violation	1.19%	2.28%	2.88%	1.39%	2.98%	3.08%
Kupic Test	0.3106751	0.00029972	7.35E-07	0.1395951	5.00E-15	7.48E-08
Result	Model Accepted	Model Rejected	Model Rejected	Model Accepted	Model Rejected	Model Rejected
CVaR	3117.0299	2954.99839	3251.89291	6680.6581	4254.0369	6030.493

4.3 VaR calculation using real distribution of returns

Since, the returns of the indices are not normally distributed therefore; VaR is calculated using empirical distribution of returns. The empirical distributions of stocks and indices returns are fitted using @ risk software. From table 7, it is evident that VaR violations are increasing with decreasing market capitalization both in recession and post-recession period, but this method is giving better estimate of VaR as compared to other two methods, as the model is acceptable for both high capitalization index and medium capitalization index. It is evident from the Table7 that VaR values are highest for the high cap index and lowest for mid cap indices in both recession and post- recession period. But CVaR is highest for small cap index post- recession.

Table 7 VaR results using actual distribution of Return

Index	Post- Recession			Recession		
	BSE SENSEX	BSEMID CAP	BSE SMALL CAP	BSE SENSEX	BSE MID CAP	BSE SMALL CAP
VaR	2762.6	2598.5	2638.3	5646	4999	5072
% Violation	0.79%	1.49%	2.28%	0.010924	0.017875	0.017875
CVaR	3325.7	3185.7	3625.901	7103.214	7144.435	6932.443
Kupic	0.7873	0.086	0.0003	0.42576	0.014731	0.014731
Result	Model Accepted	Model Accepted	Model Rejected	Model Accepted	Model Rejected	Model Rejected
Distribution fitted	Logistic	Logistic	Logistic	Laplace	Laplace	Laplace
AIC Value	-6249.21	-6357.67	-6318.37	-5073.31	-5266.8	-5229.42

Return distribution fitted for different market capitalized index Post-recession(return on X-axis and VaR violations on Y-axis)

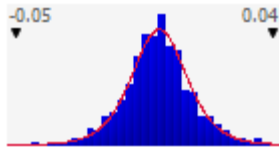


Fig.6 Sensex

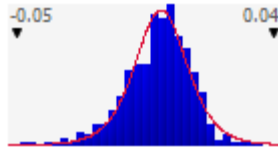


Fig.7 Mid-cap

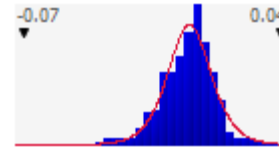


Fig.8 Small Cap

Return distribution fitted for different market capitalized index during recession

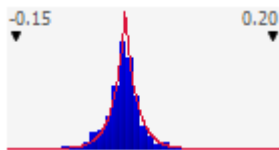


Fig.9 Sensex

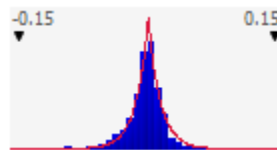


Fig.10 Mid-cap

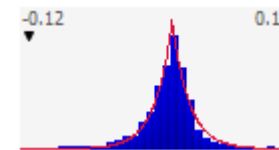


Fig.8 Small Cap

Therefore, from all the three methods it is evident that VaR calculations vary with the market capitalization. It is also evident that parametric method and parametric method using GARCH (1, 1) is underestimating VaR. Since VaR calculation on empirical distribution is performing best in capturing market risk therefore VaR calculation using empirical distribution fitting method is used for thirty different portfolios of the firms. Weights were assigned to each stock within the portfolio according to its market capitalization. Here the data is divided into two period's recession and post-recession and one combining both recession and post-recession period together, to check the VaR estimation considering both periods together. Thirty one portfolios has been created where thirty portfolios' represents decreasing market capitalization and last thirty first portfolio is the one consisting whole gamut of the stocks together for each period. It is again assumed that Rs100,000 lakhs is invested in each portfolio. None of the series is found to be normally distributed. Distribution fitted by most of the return series in Table 8 is either Logistic or Laplace. VaR model is accepted in most of the upper market capitalized portfolios and is rejected in lower portfolios. But if we are considering all the stocks together we find that model is acceptable according to Kupic test results. That means it is not correct to pool stocks of different market capitalization while calculating VaR. If we see in upper portfolio the percentage

of VaR violations are lesser as compared to lower portfolio. Kupic test results are tested at 5% level of significance.

Table 8 VaR calculation for recession and post- recession combined

Portfolio	VaR	% Violation	CVaR	Kupic Result	Result	Distribution Fitted	AIC value for distribution fitted
1	32148	1.41%	4370.78311	0.048386129	Model Rejected	Logistic	-6247.5521
2	48154	1.11%	6344.71838	0.344861992	Model Accepted	Laplace	-5942.7602
3	43109	1.16%	5510.65026	0.268641529	Model Accepted	Laplace	-6157.0792
4	44659	0.91%	6169.59471	0.694119404	Model Accepted	Laplace	-5947.5932
5	51620	1.41%	7002.44204	0.048386129	Model Rejected	Laplace	-5771.828
6	39061	1.41%	5436.94065	0.048386129	Model Rejected	Laplace	-6293.4139
7	45847	1.41%	6267.19426	0.048386129	Model Rejected	Laplace	-6060.5606
8	37016	1.31%	5220.085	0.105370897	Model Accepted	Laplace	-6544.9739
9	43521	1.06%	6191.19734	0.429301548	Model Accepted	Laplace	-6087.7742
10	34794	1.61%	4813.31009	0.00713992	Model Rejected	Logistic	-6008.1961
11	47000	1.31%	6571.29106	0.105370897	Model Accepted	Laplace	-5779.5812
12	48451	1.21%	6968.60105	0.202864977	Model Accepted	Laplace	-5732.0213
13	39175	2.16%	5451.34521	9.0416E-06	Model Rejected	Logistic	-5889.7631
14	35183	2.52%	4832.07761	8.11403E-09	Model Rejected	Logistic	-5826.9801
15	35183	1.41%	4983.72075	0.048386129	Model Rejected	Logistic	-5919.9633
16	47294	1.26%	6949.16221	0.148494123	Model Accepted	Laplace	-5901.7876
17	36168	1.86%	4892.27923	0.00035	Model Rejected	Logistic	-5953.8202
18	41547	1.51%	6086.69239	0.019700979	Model Accepted	Laplace	-6156.657
19	40811	1.31%	5846.56318	0.105370897	Model Accepted	Laplace	-6305.898
20	30781	1.76%	4100.86118	0.001264019	Model Rejected	Logistic	-6323.4463
21	44294	1.11%	6365.18807	0.344861992	Model Accepted	Laplace	-5888.2833
22	46024	1.36%	6197.4669	0.072500745	Model Accepted	Laplace	-5987.0434
23	48323	1.81%	6785.04079	0.000672749	Model Rejected	Laplace	-5967.1264
24	44019	1.26%	6166.40368	0.148494123	Model Accepted	Laplace	-6041.5995
25	48445	1.31%	7014.94385	0.105370897	Model Accepted	Laplace	-5861.6063
26	34948	1.76%	4891.67226	0.001264019	Model Rejected	Logistic	-6105.3143
27	40057	1.06%	5632.29931	0.42930	Model Accepted	Laplace	-6078.0279
28	35357	1.76%	5018.63973	0.001264019	Model Rejected	Logistic	-5834.2672
29	34774	1.41%	4763.66757	0.048386129	Model Rejected	Logistic	-5882.1872
30	36482	1.76%	4659.72877	0.00126	Model Rejected	Logistic	-5955.728

From Table 9 we observe that VaR violations are increasing with decreasing market capitalization in case of recession also, VaR model is acceptable for portfolio 1, 2, 3, 4 and 5, and it is rejected for most of the lower portfolios. In this case also model is accepted if market capitalization is not considered. Distribution fitted by portfolio series during recession in most of the cases is laplace. We get calculated AIC highly negative, this suggests that the density curves of the return is very narrow. Therefore normal parametric VaR methodologies are not suitable.

$$AIC = -2\text{Log}L + 2K \quad (12)$$

Table 9 VaR calculation for recession period

Portfolio	VaR	%Violation	CVaR	Kupic Result	Result	Distribution Fitted	AIC value for distribution fitted
1	4284.7	0.90%	5913.5489	0.66831	Model Accepted	Laplace	-5566.4692
2	5778	1.30%	7092.59267	0.20749	Model Accepted	Laplace	-4964.9149
3	5285	0.90%	6738.73317	0.66831	Model Accepted	Laplace	-5149.3028
4	5146	1.10%	6919.94196	0.41696	Model Accepted	Laplace	-5167.4892
5	6265	1.20%	8859.77912	0.99952	Model Accepted	Laplace	-4793.9902
6	4663.8	1.60%	6427.65331	0.04787	Model Rejected	Laplace	-5376.1383
7	5660	1.50%	7399.26044	0.08241	Model Accepted	Laplace	-5004.6501
8	4713	1.20%	6587.59473	0.30265	Model Accepted	Laplace	-5385.7628
9	5209	1.60%	6655.46067	0.04787	Model Rejected	Laplace	-5165.6504
10	4526.7	1.10%	6594.56384	0.41696	Model Accepted	Laplace	-5465.166
11	5490	1.80%	7856.99784	0.13444	Model Accepted	Laplace	-5106.6074
12	5490	1.40%	7856.99784	0.13444	Model Accepted	Laplace	-5078.8473
13	5369	1.60%	7036.795	0.04787	Model Rejected	Laplace	-5117.4462
14	4948	1.40%	6682.74861	0.13444	Model Accepted	Laplace	-5266.8827
15	3835	1.60%	5694.95451	0.04787	Model Rejected	Logistic	-5472.3535
16	5591	1.50%	8165.27963	0.08241	Model Accepted	Laplace	-5044.8317
17	4735.1	1.20%	6299.11992	0.30265	Model Accepted	Laplace	-5400.5025
18	4935	1.80%	6951.60094	0.01383	Model Rejected	Laplace	-5254.3074
19	5015	1.40%	6863.15204	0.13444	Model Accepted	Laplace	-5238.9174
20	4161.9	1.00%	5551.07429	0.54270	Model Accepted	Laplace	-5642.6647
21	5036	1.50%	7016.19128	0.08241	Model Accepted	Laplace	-5253.141
22	5529	1.00%	8154.36869	0.54270	Model Accepted	Laplace	-5070.4632
23	5858	1.90%	8095.86414	0.00690	Model Rejected	Laplace	-4947.117
24	5281	1.50%	6848.39774	0.08241	Model Accepted	Laplace	-5190.3362
25	5801	2.00%	7387.84614	0.00329	Model Rejected	Laplace	-4951.6033
26	4795.8	1.10%	6708.88536	0.41696	Model Accepted	Laplace	-5367.4219
27	4675.6	1.10%	6758.76352	0.41696	Model Accepted	Laplace	-5427.5867
28	4487.6	1.20%	7022.86928	0.30265	Model Accepted	Laplace	-5486.9004
29	3788	1.60%	5392.73918	0.02639	Model Rejected	Logistic	-5501.4133
30	3103.1	1.60%	3965.76655	0.02639	Model Rejected	Logistic	-5959.813
Combined	4296	1.50%	5581.54884	0.08241232	Model Accepted	Laplace	-5524.9227

From Table 10 it is observed that VaR model is accepted for higher market capitalized portfolio and is rejected in most of the lower portfolios. Distribution fitted post-recession in most of the cases is Logistic. CVaR is high for lower market capitalized portfolios and is lesser for upper market capitalized portfolios.

Table10: VaR calculation for Post- recession period

Portfolio	VaR	% Violation	CVaR	Kupic Result	Result	Distribution fitted	AIC value for distribution fitted
1	3214.8	2.80%	4370.78311	0.048386129	Model Rejected	Logistic	-6247.5521
2	4815.4	2.20%	6344.71838	0.344861992	Model Accepted	Logistic	-5942.7602
3	4310.9	2.30%	5510.65026	0.268641529	Model Accepted	Logistic	-6157.0792
4	4465.9	1.80%	6169.59471	0.694119404	Model Accepted	Logistic	-5947.5932
5	5162	2.80%	7002.44204	0.048386129	Model Rejected	Logistic	-5771.828
6	3906.1	2.80%	5436.94065	0.048386129	Model Rejected	Logistic	-6293.4139
7	4584.7	2.80%	6267.19426	0.048386129	Model Rejected	Logistic	-6060.5606
8	3701.6	2.60%	5220.085	0.105370897	Model Accepted	Logistic	-6544.9739
9	4352.1	2.10%	6191.19734	0.429301548	Model Accepted	Logistic	-6087.7742
10	3479.4	3.20%	4813.31009	0.00713992	Model Rejected	Logistic	-6008.1961
11	4700	2.60%	6571.29106	0.105370897	Model Accepted	Logistic	-5779.5812
12	4845.1	2.40%	6968.60105	0.202864977	Model Accepted	Logistic	-5732.0213
13	3917.5	4.30%	5451.34521	9.0416E-06	Model Rejected	Logistic	-5889.7631
14	3518.3	5.00%	4832.07761	8.11403E-09	Model Rejected	Logistic	-5826.9801
15	3518.3	2.80%	4983.72075	0.048386129	Model Rejected	Logistic	-5919.9633
16	4729.4	2.50%	6949.16221	0.148494123	Model Accepted	Logistic	-5901.7876
17	3616.8	3.70%	4892.27923	0.00035	Model Rejected	Logistic	-5953.8202
18	4154.7	3.00%	6086.69239	0.019700979	Model Rejected	Logistic	-6156.657
19	4081.1	2.60%	5846.56318	0.105370897	Model Accepted	Logistic	-6305.898
20	3078.1	3.50%	4100.86118	0.001264019	Model Rejected	Logistic	-6323.4463
21	4429.4	2.20%	6365.18807	0.344861992	Model Accepted	Logistic	-5888.2833
22	4602.4	2.70%	6197.4669	0.072500745	Model Accepted	Weibull	-5987.0434
23	4832.3	3.60%	6785.04079	0.000672749	Model Rejected	Logistic	-5967.1264
24	4401.9	2.50%	6166.40368	0.148494123	Model Accepted	Logistic	-6041.5995
25	4844.5	2.60%	7014.94385	0.105370897	Model Accepted	Logistic	-5861.6063
26	3494.8	3.50%	4891.67226	0.001264019	Model Rejected	Logistic	-6105.3143
27	4005.7	2.10%	5632.29931	0.42930	Model Accepted	Logistic	-6078.0279
28	3535.7	3.50%	5018.63973	0.001264019	Model Rejected	Logistic	-5834.2672
29	3477.4	2.80%	4763.66757	0.048386129	Model Rejected	Logistic	-5882.1872
30	3648.2	3.50%	4659.72877	0.00126	Model Rejected	Logistic	-6405.7325
Combined	2613	0.009	3172.23849	0.668312737	Model Accepted	Laplace	-5524.9227

4.4 Significance of market capitalization

The above results showed that VaR models performance depends on market capitalization. To statistically validate the results, a cross-sectional regression model is estimated by taking VaR violations of 30 different portfolios as dependent variable and market capitalization of the

portfolio of stocks as independent variable. From Table11 it is evident that the market capitalization is significant at 1% level. This indicates that VaR violations increase with the decrease in market capitalization of the portfolios.

Table11. Regression results for market capitalization and VaR violations

Post- Recession	
Constant	12.99*** (7.22E-04)
Market capitalization	-0.040*** (0.000713)
Recession	
Constant	14.33** (0.57)
Market capitalization	-0.033** (0.015)

** 5% level of significance

*** 1% level of significance

4.5 VaR and Volume Traded (liquidity)

A cross-sectional regression has been estimated for the dependent variable as VaR and independent variables- volume traded, beta and revenue on 328 BSE 500 companies. It has been observed that there exists inverse relationship between volumes traded of stocks, and market risk factor represented by VaR and positive relation is established between VaR and Beta. We conclude that highly traded stocks have lesser market risk. Results confirm the findings of Chuang(2012) that traded volume and volatile is negatively related in case of Japan and Taiwan stock exchange.

Table 12 Regression results for VaR and volume traded (liquidity)

Constant	-0.0522*** (0.0010)
Volume traded	-0.00344 *** (0.000596)
Beta	0.000437** (0.00022)
Revenue	4.54e-09 (2.56e-09)

** 5% level of significance

***1% level of significance

There is significant relation between volume traded of stocks and the volatility which is represented by VaR in Indian stocks of BSE 500 index.

5. Conclusion

VaR has become a very popular method of estimating market risk, in this paper we have considered (i) effect of market capitalization on VaR estimation (ii) modeling of non normality of return series of stock and stock indices and (iii) relation between stock market riskiness and stock turnover. None of the return series in estimation window is found to be normally distributed. The fitted distribution of return series is found to be, Logistic, Weibull and Laplace. Three indices BSE Sensex, BSE Mid Cap and BSE Small Cap have been taken in current study. Whole sample is divided into two periods' recession and post-recession. Since the VaR calculation using Variance-Covariance approach is not suitable due to non-normality of returns, VaR calculation have been done by modeling volatility with the help of GARCH (1,1) approach and modeling the best fitted empirical return distribution and finding out $1-\alpha$ quantile of return series for VaR estimation. It has been observed that VaR violations are increasing with decrease in market capitalization both in case of recession and post -recession period. It has been observed that fitting empirical distribution method gives better fit for VaR estimation. Further, in the case of thirty portfolios of BSE 500 stocks on the basis of market capitalization, the same results were obtained as market capitalization decreases, the VaR violations increases. Therefore we can conclude that market capitalization has impact on VaR estimation. To confirm the results further regression is run on VaR violations as dependent variable and market capitalization as independent variable .Market capitalization is coming out to be significant at 5% level in explaining VaR violations. It has been found that there is negative relation between volatility and VaR.

References

- Ashley, R. R. (2009). Frequency dependence in regression model coefficients: an alternative approach for modeling nonlinear dynamics relationships in timeseries. *Econometric Reviews* , 28, 4–20.
- Beder, T. (1996). Report card on value at risk: high potential but slow starter. *Bank Accounting & Finance* , 10, 14–25.
- Beder, T. (1995). VaR: seductive but dangerous. *Financial Analysts Journal* , 12-24.
- Bhattacharyya, M., & Madhav R, S. (2012). A Comparison of VaR Estimation Procedures for . *Journal of Mathematical Finance* , 2, 13-30.
- Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. *Review of Economics and Statistics* , 69, 542–547.
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics* , 21, 307–327.
- Breidt, F. C. (1998). The detection and estimation of long memory in stochastic volatility. *Journal of Econometrics* , 83, 325–348.
- Butler, J. S. (1998). Estimating value at risk with a precision measure by combining kernel estimation with historical simulation. *Review of Derivatives Research* , 1, 371–390.
- Chuang, W. I. (2012). The bivariate GARCH approach to investigating the relation between stock returns, trading volume, and return volatility. *Global Finance Journal* , 23(1), 1-15.
- Copeland, T. E. (1976). A model of asset trading under the assumption of sequential information arrival. *Journal of Finance* , 31.
- Darrat, A. F. (2003). Intraday trading volume and return volatility of the DJIA stocks: A note. *Journal of Banking and Finance* , 27, 2035–2043.
- Dias, A. (2013). Market capitalization and Value-at-Risk. *Journal of Banking & Finance* , 37, 5248–5260.
- Engle, R. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of UK inflation. *Econometrica* , 50, 987–1008.
- Giot, P. L. (2004). Modelling daily value-at-risk using realized volatility and ARCH type models. *Journal of Empirical Finance* , 11, 379–398.
- Halbleib, R. P. (2012). Improving the value at risk forecasts: Theory and evidence from the financial crisis. *Journal of Economic Dynamics & Control* , 36, 1212-1228.
- Harvey, A. S. (1996). Estimation of an asymmetric stochastic volatility model for asset returns. *Journal of Business and Economic Statistics* , 14, 429–434.

- Hendricks, D. (1996). Evaluation of value-at-risk models using historical data. *Federal Reserve Bank of New York Economic Policy Review* , 2, 39–70.
- Jorion, P. (1990). The exchange rate exposure of U.S. multinationals. *Journal of Business* , 63, 331–345.
- Jorion, P. (2001). *The New Benchmark for Managing Financial Risk*. McGraw-Hill.
- Jorion, P. (2001). Value at Risk: The New Benchmark for Controlling Market Risk. *Irwin, Chicago, IL*.
- Kuester, K. M. (2006). Value-at-risk prediction: a comparison of alternative strategies. *Journal of Financial Econometrics* , 4 (1), 53–89.
- Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives* , 2, 73–84.
- Lamoureux, C. G. (1990). Heteroskedasticity in stock return data: Volume versus GARCH effects. *Journal of Finance* , 45, 221–229.
- McAleer, M. J.-M.-A. (2010a). A decision rule to mini-mize daily capital charges in forecasting value-at-risk. *Journal of forecasting* , 29, 617-634.
- McNeil, A. F. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance* , 7, 271–300.
- Morgan, J. (1996). *Riskmetrics Technical Document* . 4th ed. J.P. Morgan, New York.
- Pritsker, M. (1997). Evaluating value at risk methodologies: accuracy versus computational time. *Journal of Financial Services Research* , 12, 201–242.
- Smirlock, M. &. (1985). A further examination of stock price changes and transaction volume. *Journal of Financial Research* , 8, 217–225.
- Van den Goorbergh, R. a. (1999). *Value-at-Risk Analysis of Stock Returns Historical Simulation, Variance Techniques or Tail Index Estimation*. No. 40, De Nederlandsche Bank.
- Varma, J. R. (1999). Value at Risk models in Indian stock market. *Working paper no.990705/1534* .

