Metzler paradox and home market effects in presence of internationally mobile capital and non-traded goods

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Abstract

In models of monopolistic competition with a single factor of production, imposition of tariff can lead (paradoxically) to a drop in the aggregate price index of the import competing sector. The present model first introduces an internationally mobile capital in such a set up. It is found that tariff attracts a capital inflow into the protected sector, which results in a reduction the price index. Interestingly, the tariff protected importing sector expands, although the domestic price index falls. However if there is a homogeneous non-traded good, along with the mobile capital, effect on the price index of the import competing sector becomes ambiguous. Further, the number of varieties produced by the import competing sector can actually fall and the import competing sector may actually contract.

JEL classification: F12, F13

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1. Introduction

Traditional trade theory has an extensive literature that deals with the effects of imposition of tariffs. In general, imposition of tariff has two effects, firstly, it enhances welfare by improving the terms of trade for the tariff imposing country, and secondly, it reduces welfare by causing the import competing sector to expand (and thus crowd out cheaper importables). Competitive trade theory identifies a situation where imposition of tariff can lower domestic price of the imports (that is when the first effect dominates the second effect). Better known as the Metzler Paradox (see Metzler (1949)), this can happen when the improvement in terms of trade, for the tariff imposing nation is so high that it actually lowers the domestic price of the output of the import competing sector and thus fail to protect it. Neary (1995) develops a model where capital is used as a specific factor in one of the industries, but is sluggishly mobile across countries. Imposition of tariff, in the short run (that is when capital stocks of the two countries do not respond to price changes), may under certain conditions lead to a Metzler Paradox type effect. A somewhat similar result is shown by Marjit (1993). It is shown that in a production structure exhibiting both Heckscher-Ohlin and specific factor features (this production structure is similar to that of Gruen-Corden (1970)), uniform tariffs may fail to protect some of the import competing sectors. This price lowering effect of the tariffs is due to the resource allocation between the Heckscher-Ohlin and specific factor production structures. Choi and Yu (1987) incorporates variable returns to scale\footnote{The scale economies in this model are external to the firm and internal to the industry.} in a two country, two commodity and two factor general equilibrium model and establishes sufficient conditions, for which tariffs may fail to protect the domestic import competing sector.

All these models are based on the assumptions of perfect competition and/or constant returns to scale. Departing from this tradition, trade in differentiated products has been modelled in the literature of international trade theory by a fairly extensive number of contributors.\footnote{Krugman (1979, 1980, 1981), Ethier (1982), Helpman (1981) and Venables (1982, 1987), builds models of intra industry trade where markets are monopolistically competitive, and trade in differentiated products occur due to scale economies and love for variety exhibited by the agents (See Dixit and Stiglitz (1977)).} Helpman and Krugman (1985) argues that increasing returns to scale and transportation costs would mean, the industry would tend to concentrate in a single coun-
try where most of its output is consumed so as to take advantage of the scale economies and also to reduce transportation costs. This has become famous in the literature of international trade as the home market effect.

Helpman and Krugman (1989) build up a variant of the model that is developed by Venables (1987) and associates home market effects with price lowering effect of tariffs. They consider a two country world in which there is both a differentiated goods sector and a homogeneous good, produced by a constant returns to scale technology and increasing returns to scale technology respectively by a single factor of production namely labour. While the homogeneous good can be traded in a costless manner, there is presence of transportation costs for trading the differentiated goods. In this setting, the home country is assumed to impose an ad valorem tariff unilaterally on the differentiated goods sector. Since the relatively cheaper varieties produced in home rises, this tends to reduce the aggregate price index. On the other hand, imposition of tariff on the foreign varieties makes them dearer and tends to raise the aggregate price index. The first effect dominates and thus imposition of tariff actually lowers the price index faced by home consumers. The strength of the result lies in the fact that such a price reducing effect of the tariffs does not remain a mere theoretical curiosum and would not require very restrictive conditions to hold.

The present model builds closely on Helpman and Krugman (1989), by introducing a mobile factor (capital) and a non-traded homogeneous good. Two distinct but related issues are addressed in the process. Firstly, the assumption of a traded homogeneous good is retained but only the single factor assumption of Venables (1987) and Helpman and Krugman (1989) is changed. Thus, it discusses the implications of capital mobility in presence of tariffs. Capital is used to start production of the varieties. All goods are traded and capital is fully mobile across nations. This in turn ensures complete factor price equalization between the home country and the foreign country. Even in such a situation home market effects take place and the aggregate price index falls. Imposition of tariff by the home country increases the number of varieties produced in Home and this gets manifested as a lower price index in the home economy. Thus, capital inflow is caused due to imposition of tariff, (reflected by a rise in the number of home varieties) which in turn improves welfare. This is accompanied by an

\[4^{\text{Thus capital is "footloose" as in Martin and Rogers (1995).}}\]
increase in the rent earned by capital, which also improves the welfare by enhancing the national income. The only potential source of welfare loss can be the reduction in tariff revenue (due to reduction in the foreign imported varieties). Welfare consequences of factor mobility has been discussed extensively in the literature of international economics. Kemp (1962, 1966) and Jones (1967) discusses the implications of factor mobility in presence of taxes and tariffs. In Johnson (1967) it is shown that factor accumulation may lead to welfare immiserisation in presence of distortionary tariffs. Brecher and Diaz Alejandro (1977) shows that capital inflow into a tariff protected import competing sector, reduces the welfare of the economy unambiguously by crowding out cheaper imports. Other contributions have been by Tan (1969), Bertrand and Flatters (1971), Khan (1981) and Grinols (1991). Sen et al (1997) discusses the issue of factor mobility in a set-up characterized by monopolistic competition and increasing returns to scale. Capital inflow into the differentiated sector increases the number of varieties and becomes a potential source of welfare gain. Chakraborty (2000) and Biswas (2013) shows that capital inflow into a tariff protected import competing sector, may lead to an increase in the import volume of the economy. In the present model, capital inflow raises the number of home produced varieties which leads to an improvement in welfare. This is because consumers now pays transportation costs for a lesser number of varieties (as in Helpman and Krugman (1989). However, imported brands fall, as a larger number of firms relocate their production in the Home country.

In the next section, it is assumed that the homogeneous good is non traded, while retaining the assumption of a mobile capital. The non traded good is produced using only labour and a constant returns to scale technology, while the differentiated good requires fixed units of capital to start the production (this accounts for the fixed costs) production of each additional unit of output requires only labour. Since the non traded good is both produced and consumed within each country, hence there is no channel through which the wage rates are equalized. Capital is allowed to be fully mobile and this equalizes the rental (the return to capital) across countries (See Kind et al (2000)). Trade in differentiated good is subject to transportation costs. In such a set up imposition of unilateral tariff by the home country may not lead to a drop in the price index via the home market effect. In general the effect of the tariff on the wage rate, the number of varieties produced and the price index for the differentiated good becomes ambiguous. This is because in Helpman and Krugman (1989), when the home country imposes tariff, only thing that adjusts to maintain the equilibrium is the number
of varieties produced in each country. However in the present model, the channels through which the adjustment takes place is not only the number of varieties but also the wage rate (which in turn implies that the rental and the per firm output adjusts). Imposition of tariff by the home country, cetiris paribus, increases the tariff revenue. Increase in this tariff revenue positively affects the consumer income (as we assume that the entire tariff revenue is rebated back to the consumers) whose demand for the non traded good rises pushing up its price in the home country. This in turn causes the wage rate to rise in the economy relative to the foreign wage rate. The difference lies in the assumption of the non traded good. Since labour is the only factor of production that is used in the production of non traded good, wage rate is free to adjust. Interestingly, no unambiguous effect can be predicted for the number of varieties produced by the import competing sector. This is important from the view point of trade policy. Imposition of tariff may actually fail to protect the import competing sector not only in terms of price (since the aggregate price index may fall) but also in terms of the varieties produced. Moreover, a particular parametrization is obtained for which wage rate of the economy actually falls.

This model is also closely related to Davis (1998). Davis (1998) introduces uniform transportation costs in an identical Helpman and Krugman (1985) model to arrive at the conclusion that manufacturing is spread across countries in proportion to their labour size when the homogeneous good is non traded in the equilibrium. The present model follows Davis (1998) in assuming the existence of trading costs in the homogeneous good, in fact it is assumed that these costs are prohibitive in nature. However, unlike Davis (1998), the present model extends the analysis to two factors of production (one of them being mobile internationally), and focusses on the price depressing effects of tariffs. Thus, this paper can be considered to link the price depressing effects of tariffs with models where the homogeneous good is subject to transportation costs.5

Section-II discusses the first model with mobile capital. Section-III then

5Head, Mayer and Ries(2002),and Crozet and Trionfetti (2007), discusses home market effects in the context of global agglomeration of the differentiated goods sector vis-a-vis the labour allocation across countries. The present model, is related to these papers as tariffs are used to relocate production, and thus depress the price index of the differentiated goods. Presence of a non traded good does however generates income effects that may prevent the price index to fall.
proceeds by introducing the non traded good in this set up. Finally Section IV concludes and discusses the implications for policy.

2. Basic model with mobile capital

We consider an (Home) economy where agents have a utility function given by

$$U = \log D + C$$

(1)

where the good $C$ is homogeneous good and the good $D$ is a composite good which compromises of varieties produced both by Home and Foreign. These varieties are denoted by $n_h$ and $n_f$ respectively.

$$D = \left( \sum_{h} D_h^\rho + \sum_{f} D_f^\rho \right)^{\frac{1}{\rho}}$$

(2)

The $D$ good is modelled as in Dixit-Stiglitz (1977). $D$ can be alternatively interpreted as a final good which is produced by intermediaries $n_h$ and $n_f$ that are produced in the home and foreign economies respectively (see Ethier (1982)). Since all agents are identical one can consider equation (1) and equation (2) indicating aggregate variables. Maximizing equation (1) subject to the budget constraint $Y = P_C C + P_D D$ yields $^6$

$$D = \frac{P_C}{P_D}$$

(3)

$$C = \frac{Y}{P_C} - 1$$

(4)

where $P_C$, $P_D$ and $Y$ represents the price of goods $C$, price of good $D$ and the national income at home respectively. All goods are traded.

The good $C$ is produced using only labour while the sector $D$ requires both capital and labour for production. One unit of capital is required to set up production and production of each additional unit of output requires one unit of labour.

This homogeneous good sector is assumed to be competitive and requires one unit of labour for production of each unit of output. Thus zero-profit condition of this sector, can be written as

$$P_C = w$$

(5)

$^6$Assuming an interior solution both $D, C > 0$
where \( w \) is the wage earned by the labourers. We choose the varieties produced in the home as the numeraire and thus price of each domestic brand, \( p_h \) is normalized to unity. As evident from (2) the D sector is monopolistically competitive and profit maximization by each producer implies,

\[
p_h(1 - \frac{1}{\sigma}) = w
\]

where \( \sigma = \frac{1}{1-\rho} \) is the elasticity of substitution among the varieties.

Equation (5) and equation (6) implies that

\[
P_C = w = \rho
\]

To understand the home market effects (i.e. the effect of tariff on the price index of the differentiated goods industry) it is assumed that Home imposes a tariff on the foreign varieties. On the other hand the good C can be traded costlessly across the world. Trade in the differentiated products is subject to transportation cost. Specifically, if one unit of good is shipped from a country, then only \( \frac{1}{\tau} \) units of the good reaches its destination, where \( \tau > 1 \) (See Kind et al (2000)). The foreign country is identical to the home, except the fact that it does not impose any tariffs on the varieties produced in the home good. (All variables of the foreign country are represented by asterisk.). Price of the composite good in the home and foreign are respectively given as

\[
P_{D}^{1-\sigma} = n_h p_h^{1-\sigma} + n_f (\tau p_f (1+t))^{1-\sigma} = n_h + n_f (\tau p_f (1+t))^{1-\sigma}
\]

\[
P_{D}^{*1-\sigma} = n_h (\tau p_h)^{1-\sigma} + n_f p_f^{1-\sigma} = n_h (\tau)^{1-\sigma} + n_f p_f^{1-\sigma}
\]

\( p_f \) represents the prices of the foreign brands and \( t \) is the tariff rate imposed by the home country on the varieties of the foreign country.

Uninhibited trade equalizes the price of good C in both countries. Thus,

\[
P_c = w = \rho = P_C^* = w^*
\]

However, the composite price indexes for the differentiated goods does not get equalized (see equation (8) and equation (9)) even in vicinity of free trade because of presence of transportation cost. Equation (10) also implies

\[
p_f = p_h = 1
\]
$x_h$ and $x_f$ are the outputs produced by the home and foreign firm while $r$ is the rental rate of capital. Free-entry in the differentiated goods sector implies that firms would break even and earn no supernormal profits. Thus

$$x_h = x_f = \sigma r$$

(12)

Full mobility of capital across countries guarantees that the rental is equal both in home and foreign.

Market clearing for a typical domestic firm would imply that (See Krugman (1979) for the derivation of the demand functions)

$$x_h = D_h + \tau D^*_h = \frac{p_h \rho}{P^1_D} - \tau (\tau p_h) - \sigma \frac{p \rho}{P^1_D}$$

Similarly for the foreign firm

$$x_f = \frac{\tau}{P^1_D} \rho [1 + t]^{-\sigma} - \sigma \frac{p_f \rho}{P^1_D}$$

Using equation (11) and equating the per firm output of the home and foreign country, as stated in equation (12), the ratio of price indices for the differentiated goods sector can be expressed in terms of tariff and transportation cost.

$$\left(\frac{p}{p^*}\right)^{\sigma - 1} = 1 - \frac{1 - \tau^{1 - \sigma}}{1 - \tau^{1 - \sigma}[1 + t]^{-\sigma}}$$

(13)

The above relation can be used to solve for the number of varieties produced in each country. (It is to be noted that the number of varieties produced in each country are not independent variables.) It is assumed that $K^H$ and $K^F$ are the capital stocks owned by the Home and Foreign economies respectively. Thus,

$$n_h + n_f = K^H + K^F$$

(14)

Solving equations (8), (9), (13), (14) we obtain the number of varieties produced in each country (See Appendix A for derivation),

$$n_h = \frac{(K^H + K^F)[(\tau (1 + t))^{1 - \sigma} - B]}{\tau^{1 - \sigma} B - (B + 1) + [\tau (1 + t)]^{1 - \sigma}}, \text{ where } B = \frac{1 - \tau^{1 - \sigma} [1 + t]^{-\sigma}}{1 - \tau^{1 - \sigma}}$$

(15)

\footnote{Substituting equation(7) into equation(3)we find that total expenditure on home goods is simply $\rho$}
\[
n_f = \frac{(K^H + K^F)[\tau(1 + t)^{1-\sigma}B - 1]}{\tau^{1-\sigma}B - (B + 1) + [\tau(1 + t)]^{1-\sigma}}
\]

The two countries are completely symmetric when there is free trade. This becomes clear, from equation (15) as it implies that in the vicinity of free trade \( n_h = n_f = \frac{1}{2} (K^H + K^F) \). As in Venables (1987), it is interesting to find out the effect of tariff on the aggregate price index of the differentiated good. The difference between that model and the present model is that unlike Venables (1987) the per firm output is not constant. To understand the effect on the price index, the effect of tariff on the number of varieties produced in Home is analysed. Differentiating equation (15) with respect to \( t \), at the vicinity of free trade, \((t = 0)\) (See Appendix-A)

\[
\frac{dn_h}{dt} = \frac{(K^H + K^F)}{4(1 - \tau^{1-\sigma})} [\tau^{1-\sigma}(\sigma - 1) + \frac{\sigma \tau^{1-\sigma}(1 + \tau^{1-\sigma})}{1 - \tau^{1-\sigma}}]
\]

(16)

which in turn implies that

\[
\hat{n}_h = \frac{dt}{2(1 - \tau^{1-\sigma})} [\tau^{1-\sigma}(\sigma - 1) + \frac{\sigma \tau^{1-\sigma}(1 + \tau^{1-\sigma})}{1 - \tau^{1-\sigma}}]
\]

(17)

(since at the vicinity of free trade, \( n_h = \frac{1}{2} (K^H + K^F) \))

Equation (17) clearly indicates that number of varieties produced by the home country rises unambiguously. Since capital is required to begin production of each variety, a higher number of home produced varieties would mean that a larger share of the global capital stock is now employed in the Home country. This is fairly intuitive, as the domestic sector receives protection, imported brands are crowded out of the economy, and thus domestic sector expands. Interestingly, unlike what happens in Brecher and Diaz Alejandro(1977) model, an inflow of capital into the protected sector increases the number of brands for which the domestic residents do not have to pay transportation costs (an increase in home produced varieties). Capital inflow, does conditionally increase welfare in Sen et all(1997)through increasing the available number of varieties in the home economy. In the present model, imposition of tariff actually makes it profitable for firms to relocate their production in Home rather than in the foreign country. This in turn reduces the aggregate price index faced by the consumers. The only factor that can raise the price index is the relatively higher price of the tariff ridden imports. To determine the magnitude of these changes, we differentiate (8) in the vicinity of free trade. (See Appendix-A)
\[
\frac{\dot{P}_D (\sigma - 1)}{dt} = \frac{-\tau^{1-\sigma} - \sigma - (\sigma - 1)\tau^{2(1-\sigma)}}{2(1 + \tau^{1-\sigma})(1 - \tau^{1-\sigma})} < 0
\] (18)

The RHS of equation (18) is negative. Thus home market effects lowers the aggregate price index faced by domestic consumers in this model. This, in turn brings unambiguous gain in terms of welfare. Hence tariffs, make the Home country attractive to mobile capital and though it reduces the number of import competing foreign brands, consumers are finally better off in terms of the differentiated good. As in Sen(1998), Chakraborty (2000) and Biswas (2013) an inflow of foreign capital has an ambiguous effect on the overall welfare of the economy. It is shown in the Appendix-A, that imposition of tariff increases the per firm output, the overall rental rate, and the total volume of the import competing sector \( (n_h x_h) \). Its effect on the tariff revenue is however ambiguous, since an imposition of tariff increases the revenue receipts directly, while reducing the demand for the imports (and thereby reducing the tariff revenue). The overall impact on the welfare thus remain indeterminant. Interestingly, though there is a Metzler paradox type effect, the total volume of the import competing sector \( (n_h x_h) \) rises.

3. Home market effects and non traded goods

Consider a variant of the above model. The only point of departure being that now the good \( C \) is assumed to be a non-traded good. Davis (1998) argues that when the homogeneous good is subject to trade costs, market size does not play a decisive role in the global distribution of the manufacturing sector. In the present analysis also home market effects get weakened when the homogeneous good sector is non traded (to keep matters simple, these trade costs are assumed to be prohibitive). More importantly, this has implications for Metzler Paradox type effects found in Helpman and Krugman(1989) and in the mobile capital model discussed above. We assume (as in the previous section) that agents have a quasilinear utility given by

\[
U = \log D + C
\] (19)

where the good \( C \) is homogeneous good produced using only labour and the good \( D \) is a composite good which compromises of varieties produced both by Home and Foreign. Capital and labour both are employed in its production. \( C \) is assumed to be non traded.
\[ D = \left( \sum_{1}^{n_h} D_h^\rho + \sum_{1}^{n_f} D_f^\rho \right)^{1/\rho} \] (20)

Since \( C \) is a non-traded good, equalization of wages in the two countries is no longer guaranteed. Thus in the Home and Foreign we have respectively

\[ P_C = w = \rho \] (21)

\[ P_C^* = w^* \] (22)

The second equality in equation (21) follows from assuming an identical production structure of the traded sector \( D \) as in the previous model. That is to start production of each variety one unit of capital is required, while after that each additional unit of output is produced by employing one unit of labour. Unlike equation (10), however, foreign wages are not equal to the wage rate in the home market. This in turn would imply that prices of domestic and foreign brands would not converge. Prices of home brands are normalized to unity. Prices of foreign brands are a constant markup over the foreign wages.

\[ p_f = \frac{w^*}{\rho} \] (23)

Moreover the per firm output of the home and foreign firms will be different

\[ x_h = \frac{r}{1 - \rho} \] (24)

\[ x_f = \frac{\rho r}{w^*(1 - \rho)} \] (25)

Now market clearing of the non traded good in Home implies that the total supply of output produced must be equal to the demand of good \( C \) as in equation (4)

\[ \bar{L} - n_h x_h = \frac{Y}{P_C} - 1 \]

Using equation (21) and (24) we can write this

\[ \bar{L} - n_h \frac{r}{1 - \rho} = \frac{\rho \bar{L} + r K^H + T}{\rho} - 1 \]
$K_H$ is the capital of Home country, while $L$ is the labour force. It is assumed that the home country imposes a tariff on the imports of the foreign varieties, $T$ is the tariff revenue generated by the imports and $T = n_f \tau p_f D_f$. Solving for $r$ we get

$$r = \frac{\rho - T}{n_h \frac{\rho}{1 - \rho} + K^H} \quad (26)$$

Similarly for the foreign country we have

$$r = \frac{w^*}{n_f \frac{\rho}{1 - \rho} + K^F} \quad (27)$$

where $K^F$ is the foreign capital stock. Free mobility of capital ensures that rental rate of capital are equalised along nations. Thus equations (26) and (27) relates that foreign wage rate with varieties produced in Home.

$$w^* = \frac{(\rho - T)(n_f \frac{\rho}{1 - \rho} + K^F)}{(n_h \frac{\rho}{1 - \rho} + K^H)} \quad (28)$$

Capital market clearing across the world implies that

$$n_h + n_f = K^H + K^F \quad (29)$$

Differentiating equation (29) yields

$$\delta_h \dot{n}_h + (1 - \delta_h) \dot{n}_f = 0 \quad (30)$$

where $\delta_h = n_h/2K$, is the relative number of home varieties with respect to total number of varieties produced in the world.

To understand the effect of tariffs on the varieties on the varieties produced in the home market, total differentiation of equation -(28)and using equation(30) gives.

$$\dot{w}^* = \frac{-T}{\rho - T} \dot{T} + \{ -n_h \rho \sigma + n_f (\frac{\delta_h}{1 - \delta_h}) \rho \sigma \} \dot{n}_h$$

$$= \frac{-T}{\rho - T} \dot{T} - [ \frac{(K^H + K^F)(\rho \sigma + 1)n_h \rho \sigma}{(n_f \rho \sigma + K^H)(n_h \rho \sigma + K^H)} ]$$

$$= -B_1 \dot{t} + \{ \frac{B_1 s_h}{1 - \delta_h} - \frac{(K^H + K^F)(\rho \sigma + 1)n_h \rho \sigma}{(n_f \rho \sigma + K^F)(n_h \rho \sigma + K^H)} \} \dot{n}_h + B_1 s_h (\sigma - 1) \dot{w}^*$$

$$+ \frac{B_1 d t}{1 + t} (\sigma + s_f (1 - \sigma))$$

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This on rearranging terms and after some manipulations yields
\[
\hat{w}^*\{1 - B_1(\sigma - 1)s_h\} - \frac{B_1s_h}{1 - \delta_h} - \frac{(K^H + K^F)(\rho\sigma + 1)n_h\rho\sigma}{(n_f\rho\sigma + K^F)(n_h\rho\sigma + K^H)}\hat{n}_h
\]
\[
= -\frac{B_1\hat{t}}{1 + t}[1 + t(1 - \sigma) + s_f t(\sigma - 1)]
\]
(31)

where \(B_1 = \frac{-T\alpha}{\alpha T}\)

Imposition of the tariff by the home country increases the tariff revenue that accrues to the nation. Given the number of varieties (\(\hat{n}_h = 0\)), an increase in the tariff revenue gets translated into a higher national income, since the entire tariff income is rebated to the consumers. The increased income raises the price of the non-traded good and in turn the wages of the home labourers relative to the foreign workers. This is reflected in equation (31) where keeping the number of varieties constant, an imposition of tariff lowers the foreign wage. Our choice of numeraire implies this can be interpreted as a decline in the relative wage of foreign workers vis-a-vis the wage rate of home workers.

Equation (31) involving two variables the change in wage rate of the foreign country and the change in the varieties produced in the home economy. To solve them explicitly we would require another equation involving these two terms. This is obtained from the zero profit condition involving the home firm and foreign firm. Free entry implies that in equilibrium, these firms would just break even. Consider the case of the home firm
\[
\frac{p_h x_h}{\sigma} - r = \frac{D_h + \tau D^*_h}{\sigma} - r = 0
\]
the second equality follows from the market clearing condition for the output produced by the home firm. Substituting the demand functions (See Krugman(1979)) we get the following equations for the home and foreign firm respectively.
\[
\rho P_d^{\sigma - 1} + \tau^{1-\sigma}w^* P_d^{\sigma - 1} = \sigma r
\]
(32)
\[
\tau^{1-\sigma}w^{1-\sigma} P_d^{\sigma - 1} - \rho^{\sigma}(1 + t)^{-\sigma} + w^{2-\sigma}\rho^{\sigma - 1}P_d^{\sigma - 1} = \sigma r
\]
(33)
These two equations yields
\[
\left(\frac{P_d}{P^*_d}\right)^{\sigma - 1} = \frac{w^{2-\sigma}\rho^{\sigma - 1} - \tau^{1-\sigma}w^*}{\rho - \tau^{1-\sigma}w^{1-\sigma}\rho^{\sigma}(1 + t)^{-\sigma}}
\]
(34)
Differentiating equation (34) and some algebraic manipulations yields

\[ w^*\{(\sigma - 1)(s_h^* - s_h) + (\sigma - 2) + (\sigma - 1)(\mu_1 + \mu_2)\} + \hat{n}_h\left(\frac{s_h^* - s_h}{1 - \delta_h}\right) \]

\[ = -[\mu_2\sigma + s_f(\sigma - 1)\frac{dt}{1 + t}] \]

(35)

\[ s_h = \frac{n_h p_h D_h}{\nu} \quad \text{and} \quad s_f = \frac{n_f p_f D_f}{\nu} \]

are the total expenditure shares of home and foreign varieties made by residents in the home economy. \( s_h^* \) and \( s_f^* \) are the analogous counterparts for the foreign economy.

\[ \mu_1 = \frac{\tau_1 - \sigma - \nu s_h^*}{\rho s_h^* (1 + \tau_1(1 + t))} \]

and \( \mu_2 = \frac{\tau_2 - \sigma - \nu s_f^*}{\rho s_f^* (1 + \tau_2(1 + t))} \) These equations lead us to the following proposition.

**Proposition 1.** The effect of imposition of tariff by the home country can either increase or decrease the foreign wage. Moreover the number of varieties produced in home may either increase or fall.

**Proof.** Solving equations (34) and (35) around zero tariffs i.e. by assuming that \( t = 0 \) we obtain

\[ w^* = -A(\mu_2\sigma + s_f(\sigma - 1)) + \left(\frac{s_h^* - s_h}{1 - \delta_h}\right)\frac{n_f \tau p_f D_f}{\alpha \rho} \]

\[ \hat{n}_h = -\left((\sigma - 1)(s_h^* - s_h) + (\sigma - 2)(\mu_1 + \mu_2)\right)\frac{n_f \tau p_f D_f}{\alpha \rho} + (\mu_2\sigma + s_f(\sigma - 1)) \]

(36)

(37)

The above two equations shows the effects of imposing tariff on the foreign wage rate and number of varieties produced in the home market. As shown in the appendix-B, stability analysis implies that the denominator of the above expressions is positive. However in general the sign of the numerator in either case cannot be determined. In contrast to the model of section-1, no unambiguous result is seen. More specifically unlike Venables (1987) varieties produced by the home may actually fall.

We concentrate on a specific situation, where around free trade, the two countries are identical (i.e. they are endowed with equal amount of capital and labour). The following proposition discusses the equilibrium.

**Proposition 2.** If both home and foreign have an equal endowment of capital and labour then imposition of tariff by the home leads to an unambiguous fall in the wages of the foreign country, while the effect on home produced number of varieties remains ambiguous.  

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Proof. Consider equation (34), substituting the value of the price indices we can solve for \( n_h \) in terms of the foreign wage rate at the vicinity of free trade \((t = 0)\) and assuming that \( K^H = K^F = K \)

\[
n_h = \frac{2K[G\tau^{1-\sigma}w^{1-\sigma}\rho^{-1} - w^{1-\sigma}\rho^{-1}]}{\tau^{1-\sigma} - w^{1-\sigma}\rho^{-1} + G\tau^{1-\sigma}w^{1-\sigma}\rho^{-1} - G}
\]

where \( G \) is the RHS of equation (34). Substituting \( w^* = \rho \), in equation (38) and \( n_h = K \), in equation (28) we can focus on a particular equilibrium situation which is completely symmetric around free trade, where \( n_h = n_f = K \), and \( w^* = \rho \). It is to be noted that in Appendix -B we have assumed that as firms enter into the market profits fall while as they exit, per firm profits rise. Thus there is only one possible equilibrium consistent with zero profits. Hence this is an unique equilibrium.

It is relatively straightforward, to check that \( s^*_h = \tau^{1-\sigma}/(1 + \tau^{1-\sigma}) < s_h = 1/(1 + \tau^{1-\sigma}) \) In this situation, wage rate in the foreign economy falls (see equation (36)), while the number of varieties produced in home can either increase or decrease.

Proposition 3. Imposition of tariff by the home country has an ambiguous effect on the price index. Interestingly total volume of output produced by the import competing sector may actually contract.

Proof. The price index of the differentiated goods sector is given by equation (ref 8). Thus one can express the change in the aggregate price index in terms of change in number of varieties produced in home and the change in the foreign wage rate.

\[
(1 - \sigma)\dot{P}_D = s_h\dot{n}_h + (1 - s_h)(\dot{n}_f + (1 - \sigma)\dot{w}^*)
\]

which after simplification and using equations (30) and (37) yields

\[
\frac{d\dot{P}_D}{dt} = -\frac{(\delta_h - s_h)}{(1 - \delta_h)(\sigma - 1)}\left\{[(\sigma - 1)(s_h^* - s_h) + (\sigma - 2)(\mu_1 + \mu_2)n_f\frac{\tau_{pf}D_f}{\rho} + (\mu_2\sigma + s_f(\sigma - 1))}\right\} + (1 - s_h)\left\{-A(\mu_2\sigma + s_f(\sigma - 1)) + \left(s_h^* - s_h\right) \frac{n_f\tau_{pf}D_f}{\rho}\right\}
\]
Thus the aggregate price index can either increase or fall depending on specific parameter values. Moreover the total output produced by the import competing sector \( (n_h x_h) \) may also go either way. (See Appendix-C for derivation)

4. Conclusion

Metzler (1949) had shown that imposition of a tariff can paradoxically fail to protect the import competing sector. This can happen when the offer curve of the country may be so inelastic that the tariff lowers the international price of the importables by a very large extent, thereby offsetting the increase in price caused due to the imposition of the tariff itself. Thus tariffs fail to protect the import competing sector in this situation. However, neo-classical trade theory considers this Metzler Paradox as a case of mere theoretical interest (See Marjit (1993) and Caves and Jones(1985)). Helpman and Krugman (1979) shows, price reducing effect of tariffs becomes more relevant, in a setting characterized by monopolistic competition and increasing returns to scale. In such a situation imposition of tariff is clearly desirable from the point of view of the policy maker. This is because the aggregate price index of the differentiated goods sector falls and the number of varieties produced in the home market also rises (which are relatively cheaper than their foreign counterpart, due to the presence of transportation costs). Imposition of tariff by the home economy causes a global shift in the distribution of production of the differentiated varieties across the world. This benefits the consumers of the home economy as they have to bear an additional transportation cost for a lesser number of varieties. The present model makes two important departure from the Helpman and Krugman (1979) model. In the first departure, a mobile capital is introduced into the model. Imposition of tariff,in presence of transportation costs, gives incentives to firms to locate thier production in the home economy rather than in the foreign. This benefits consumers by lowering the price index, just as in Helpman and Krugman(1979). Moreover, this has interesting implications for capital inflow into the tariff protected import competing sector. Johnson (1967) and Brecher and Diaz-Alejandro(1977) argues that capital inflow in presence of distortionary tariffs can be welfare immiserising. This may not be the case when Home market effects are present in the model.

The model is further extended to show that the situation may become more nuanced when we relax the assumptions of single factor of production and introduce a homogeneous non traded good. Imposition of the tariff has an
ambiguous effect on the aggregate price index. Also important from the policy perspective, is the fact that number of varieties produced by the home may actually fall. Policy makers, if they are interested in protecting the import competing sector may not thus be able to achieve it, when home market effects are present. This is in contrast to Venables (1987) and Helpman and Krugman (1989). Though our model shows that tariff may increase the aggregate price index of the differentiated goods industry, it also opens upon the channel that the industry may actually contract in terms of varieties and total output produced by the tariff protected import competing sector. (one can consider this as an example of Metzler Paradox in terms of quantities) This is quite paradoxical, since tariff protection is often sought to expand the import competing sector. Clearly, the effect on welfare is ambiguous and will depend on particular parameterization.

Appendix A.

From equation (11) equations (8) and (9) can be expressed as

\[ P_D^{1-\sigma} = n_h + n_f(\tau(1 + t))^{1-\sigma} \]

\[ P_D^{1-\sigma} = n_h(\tau)^{1-\sigma} + n_f \]

Substituting these into equation (13) and also using the fact that \( n_h = K^H + K^F - n_f \), equation(15) in the main text is obtained.

Now consider equation (16),

\[
\frac{dn_h}{dt} = \frac{K^H + K^F}{[\tau^{1-\sigma}B - (B + 1) + \{\tau(1 + t)\}^{1-\sigma}]^2} \left\{ \tau^{1-\sigma}B - (B + 1) \right. \\
+ \{\tau(1 + t)\}^{1-\sigma} \left\{ \tau^{1-\sigma}(1 + t)^{-\sigma}(1 - \sigma) - \frac{\sigma\tau^{1-\sigma}(1 + t)^{-\sigma}}{1 - \tau^{1-\sigma}} \right\} \\
- \{\tau(1 + t)\}^{1-\sigma} - B \left\{ \frac{\tau^{2-2\sigma}(1 + t)^{-\sigma}}{1 - \tau^{1-\sigma}} - \frac{\tau^{1-\sigma}(1 + t)^{-\sigma}}{1 - \tau^{1-\sigma}} \right\} \\
\left. + \tau^{1-\sigma}(1 - \sigma)(1 + t)^{-\sigma} \right\} \\
\]  

(A.1)
Now at the vicinity of free trade, we put \( t = 0 \), thus

\[
\frac{dn_h}{dt} = \frac{K^H + K^F}{2(1-\tau^{1-\sigma})^2} \left[ \left\{ \tau^{1-\sigma}(1-\sigma) - \frac{\sigma\tau^{1-\sigma}}{1-\tau^{1-\sigma}} \right\}(2\tau^{1-\sigma} - 2) - (\tau^{1-\sigma} - 1) \left\{ \frac{\tau^2(1-\sigma)}{1-\tau^{1-\sigma}} - \frac{\sigma\tau^{1-\sigma}}{1-\tau^{1-\sigma}} + \tau^{1-\sigma}(1-\sigma) \right\} \right]
\]

\[
= \frac{K^H + K^F}{4(1-\tau^{1-\sigma})^2} \left[ 2(\tau^{1-\sigma} - 1) \left\{ \tau^{1-\sigma}(1-\sigma) - \frac{\sigma\tau^{1-\sigma}}{1-\tau^{1-\sigma}} \right\} - (\tau^{1-\sigma} - 1) \left\{ \frac{\tau^2(1-\sigma)}{1-\tau^{1-\sigma}} - \frac{\sigma\tau^{1-\sigma}}{1-\tau^{1-\sigma}} + \tau^{1-\sigma}(1-\sigma) \right\} \right]
\]

\[
= \frac{K^H + K^F}{4(1-\tau^{1-\sigma})} \left[ \tau^{1-\sigma}(\sigma - 1) + \frac{\sigma\tau^{1-\sigma}(1 + \tau^{1-\sigma})}{1-\tau^{1-\sigma}} \right]
\]

(A.2)

This is equation (16) in the main text.

Near the free trade equilibrium, \( n_h = (K^H + K^F)/2 \) and also \( n_f = (K^H + K^F)/2 \). Substituting this in equation (16) will give equation-(17) Total differentiation of equation-(8) near free trade \( (t = 0) \) yields

\[
\hat{P}_d(1-\sigma) = s_h\hat{n}_h + (1-s_h)(\hat{n}_f + (1-\sigma))dt
\]

(A.3)

where \( s_h = \frac{n_h^{1-\sigma} - 1}{n_h^{1-\sigma} + n_f^{1-\sigma}(\tau p_f(1+t))^{1-\sigma}} = \frac{1}{1+\tau^{1-\sigma}} \). After some manipulations this can be expressed as

\[
\frac{(\sigma - 1)\hat{P}_d}{dt} = (1 - 2s_h)\frac{\hat{n}_h}{dt} + (\sigma - 1)(1 - s_h)
\]

\[
= \left( \frac{\tau^{1-\sigma} - 1}{\tau^{1-\sigma} + 1} \right) \frac{\hat{n}_h}{dt} + (\sigma - 1) \frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}}
\]

\[
= \frac{-1}{2(1+\tau^{1-\sigma})} \left[ \tau^{1-\sigma}(\sigma - 1) + \frac{\sigma(1 + \tau^{1-\sigma})}{1-\tau^{1-\sigma}} \right] + (\sigma - 1) \frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}}
\]

(A.4)

Rearranging the terms we get

\[
\frac{(\sigma - 1)\hat{P}_d}{dt} = \frac{1}{2(1+\tau^{1-\sigma})} \left[ \frac{\sigma(1 + \tau^{1-\sigma}) - (\sigma - 1)\tau^2(1-\sigma) - \sigma - \sigma\tau^{1-\sigma}}{1-\tau^{1-\sigma}} \right]
\]

\[
= \frac{-\tau^{1-\sigma} - \sigma - (\sigma - 1)\tau^2(1-\sigma)}{2(1+\tau^{1-\sigma})(1-\tau^{1-\sigma})}
\]

(A.5)

which is equation (18) in the text. To understand the effect on welfare, it is instructive to understand the effect of the tariffs on per-firm output. As
shown in the main text the per firm domestic output can be expressed as

\[ x_h = \rho P_d^{\sigma-1} + \tau^{1-\sigma} \rho P_d^{\sigma-1} \tag{A.6} \]

Differentiating both sides we get

\[ dx_h = \rho P_d^{\sigma-1}[(\sigma - 1)\hat{P}_d + \tau^{1-\sigma} \rho P_d^{\sigma-1}[(\sigma - 1)\hat{P}_d] \tag{A.7} \]

Differentiating equation (13) from the main text,

\[ (1 - \sigma)\hat{P}_d = (\sigma - 1)\hat{P}_d^* - \frac{\sigma\tau(1 + t)^{-\sigma+1}dt}{1 - \tau^{1-\sigma}(1 + t)^{-\sigma}} \tag{A.8} \]

From equation (9)

\[ (\sigma - 1)\hat{P}_d^* = \frac{1 - \tau^{1-\sigma}}{1 + \tau^{1-\sigma}\hat{n}_h} \tag{A.9} \]

when all the terms are evaluated around free trade. Substituting A.8 and A.9 into A.7 we get

\[
\begin{align*}
    dx_h &= \rho P_d^{\sigma-1}[(\sigma - 1)\hat{P}_d + \tau^{1-\sigma}(\sigma - 1)\hat{P}_d^*] \\
    &= \rho P_d^{\sigma-1}\left[\frac{1 - \tau^{1-\sigma}}{1 - \tau^{1-\sigma}\hat{n}_h} - \frac{\sigma\tau^{1-\sigma}}{1 - \tau^{1-\sigma}} dt + \frac{\tau^{1-\sigma}(1 - \tau^{1-\sigma})}{1 + \tau^{1-\sigma}} \hat{n}_h\right] \\
    &= \rho P_d^{\sigma-1}\left[\frac{1 - \tau^{2(1-\sigma)}}{1 - \tau^{1-\sigma}\hat{n}_h} - \frac{\sigma\tau^{1-\sigma}}{1 - \tau^{1-\sigma}} dt\right] \\
    &= dt\left[\tau^{1-\sigma}(\sigma - 1) + \frac{\sigma\tau^{1-\sigma}(1 + \tau^{1-\sigma})}{1 - \tau^{1-\sigma}} - \frac{\sigma\tau^{1-\sigma}}{1 - \tau^{1-\sigma}}\right] \\
    &= dt\left[\tau^{1-\sigma}(\sigma - 1) + \frac{\sigma\tau^{2(1-\sigma)}}{1 - \tau^{1-\sigma}}\right] > 0 \tag{A.10}
\end{align*}
\]

this implies that per firm output rises in the home economy, (as also the interest rate from equation (12)) along with an increase in the total number of home varieties produced. Thus the volume of the import competing sector expands. All these factors tend to increase the utility of the domestic consumers. However the net effect on the welfare of the home residents remains ambiguous. This is because the total tariff revenue \( T = tn_fp_fD_f \) may either rise or fall.

**Appendix B.**

For purpose of stability, it is assumed that firms enter into the market when existing firms earn supernormal profits and in turn diminishes profits
earned by each firm. On the other losses cause exit of firms from the industry and this reduces the losses made by existing firms in the industry. Thus profits earned by each of the firms are a decreasing function of the total number of firms operating in the market.

Consider the profit earned by the home firm

\[
\pi^h = \frac{D_h + \tau D^*_h - r}{\sigma} = \frac{\alpha \rho P^a - \tau \omega^* P_d^{a-\sigma} - \sigma r}{\sigma} \quad (B.1)
\]

Total differentiation of the above expression yields

\[
\frac{1}{\sigma} d\pi^h = \rho P_d^{a-1} (\sigma - 1) \frac{\tilde{P}_d}{\delta} + \tau^{1-\sigma} w^* P_d^{a-\sigma} (w^* + (\sigma - 1) P^*) - \sigma \tilde{r} \quad (B.2)
\]

After a little manipulation this yields

\[
= \rho P_d^{a-1} \left[ (\sigma - 1) \frac{\tilde{P}_d}{\delta} + \frac{(s^*_h - s_h)}{1 - \delta} \tilde{n}_h + \tilde{w}^*(\sigma - 1)(s^*_h - s_h) \right] + \tau^{1-\sigma} w^* P_d^{a-\sigma} \frac{\tilde{P}_d}{\delta}
\]

\[
+ \tau^{1-\sigma} w^* P_d^{a-\sigma} \frac{\tilde{w}^*}{\alpha} - \frac{\sigma \tilde{r}}{\tilde{r}}
\]

\[
= \sigma r \left[ (\sigma - 1) \frac{\tilde{P}_d}{\delta} - \tilde{r} \right] + \rho P_d^{a-1} \left[ \frac{(s^*_h - s_h)}{1 - \delta} \tilde{n}_h + \tilde{w}^*(\sigma - 1)(s^*_h - s_h) \right]
\]

\[
= \sigma r [\mu_2 + (\sigma - 2)] (-A) \tilde{n}_h + \tau^{1-\sigma} w^* P_d^{a-\sigma} (-A) \tilde{n}_h + \rho P_d^{a-1} \left[ \frac{(s^*_h - s_h)}{1 - \delta} \tilde{n}_h \right]
\]

\[
+ (\sigma - 1)(s^*_h - s_h)(-A) \tilde{n}_h \quad (B.3)
\]

Dividing both sides by \( \tilde{n}_h \) we get

\[
\frac{1}{\tilde{n}_h} d\pi^h = \sigma r [\mu_2 + (\sigma - 2)] (-A) + \tau^{1-\sigma} w^* P_d^{a-\sigma} (-A) + \rho P_d^{a-1} \left[ \frac{(s^*_h - s_h)}{1 - \delta} \right]
\]

\[
+ (\sigma - 1)(s^*_h - s_h)(-A) \quad (B.4)
\]

Now our assumption implies that LHS must be negative, which in turn
means

\[
[pP_d^{\sigma-1} + \tau^{1-\sigma} w* P_d^{\sigma-1}] \left[ (\alpha - 1)\mu_2 + (\sigma - 2) \right] (-A) + \tau^{1-\sigma} w* P_d^{\sigma-1} + pP_d^{\sigma-1} \left[ \frac{\dot{s}_h - \dot{s}_h}{1 - \delta_h} \right]
\]

\[+ (\sigma - 1)(s^*_h - s_h) (-A) \right] < 0
\]

or,

\[
pP_d^{\sigma-1} \left[ A \left\{ (\sigma - 1)\mu_2 + (\sigma - 1)(s_h^* - s_h) + (\sigma - 2) \right\} - \frac{s_h^* - s_h}{1 - \delta_h} \right]
\]

\[+ \tau^{1-\sigma} w* P_d^{\sigma-1} A((\sigma - 1)(\mu_2 + 1)) > 0
\]

or,

\[
A \left\{ (\sigma - 1)\mu_2 + (\sigma - 1)(s_h^* - s_h) + (\sigma - 2) \right\} - \frac{s_h^* - s_h}{1 - \delta_h}
\]

\[+ \frac{\tau^{1-\sigma} w* P_d^{\sigma-1} A((\sigma - 1)(\mu_2 + 1))}{pP_d^{\sigma-1}} > 0
\]

Now consider the last term in the left hand side of the inequality,

\[
\frac{\tau^{1-\sigma} w* P_d^{\sigma-1} A(\sigma - 1)(\mu_2 + 1)}{pP_d^{\sigma-1}} = \frac{\tau^{1-\sigma} w* P_d^{\sigma-1} A(\sigma - 1) \left[ \frac{\tau^{1-\sigma} w* P_d^{\sigma-1} A(\sigma - 1)}{\rho - \tau^{1-\sigma} w* P_d^{\sigma-1} A(\sigma - 1)} + 1 \right]}{pP_d^{\sigma-1}}
\]

\[= A(\sigma - 1)\mu_1
\]

\[
(\sigma - 1)(\mu_1 + \mu_2) + (\sigma - 1)(s_h^* - s_h) + (\sigma - 2) \middle\left. \frac{s_h^* - s_h}{1 - \delta_h} \middle) > 0
\]

\[i.e. \quad \vartheta > 0
\]

(B.6)

Appendix C.

\[
\hat{x}_h = \hat{r} \quad \text{From equation (24)}
\]

Now using equation(26)

\[
\hat{r} = \frac{-T}{\alpha \rho - T} \hat{T} + \frac{\hat{n}_h}{n_h \rho + K^H}
\]

\[= -B_1 \left\{ \hat{t} - \frac{s_h}{1 - \delta_h} \hat{n}_h - (\sigma - 1)s_h \hat{w}^* + \frac{dt}{1 + t} \left[ s_f(\sigma - 1) - \sigma \right] \right\}
\]

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So,
\[
\dot{x}_h = \dot{\hat{r}} = -B_1 \dot{\hat{t}} + \left\{ B_1 \frac{s_h}{1 - \delta_h} - \frac{1}{n_h \rho \sigma + KH} \right\} \dot{n}_h + B_1 (\sigma - 1) s_h \hat{w}^* + \frac{B_1 dt}{1 + t} \left[ s_f (1 - \sigma) + \sigma \right]
\]

Total output produced by the import competing sector in Home is given by \( n_h x_h \)
\[
\dot{\hat{n}}_h x_h = \left[ 1 + \frac{B_1 s_h}{1 - \delta_h} - \frac{1}{n_h \rho \sigma + KH} \right] \dot{n}_h + \frac{B_1 dt}{1 + t} \left[ s_f (1 - \sigma) + \sigma \right] - B_1 \dot{\hat{t}} + B_1 (\sigma - 1) \hat{w}^*
\]
\[
= \left[ 1 + \frac{B_1 s_h}{1 - \delta_h} - \frac{1}{n_h \rho \sigma + KH} \right] dt \left\{ - \left[ (\sigma - 1) (s_h^* - s_h) + (\sigma - 2) (\mu_1 + \mu_2) \right] n_f \frac{\tau p_f D_f}{\alpha \rho} + (\mu_2 \sigma + s_f (\sigma - 1)) \right\} + \frac{B_1 dt}{1 + t} \left[ s_f (1 - \sigma) + \sigma \right] - B_1 \dot{\hat{t}} + \frac{B_1 (\sigma - 1) dt}{\vartheta} \right\}
\]
which is indeterminate in its sign.

References


