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Abstract:

This paper develops a model of endogenous economic growth with special focus on the role of unionized labour market and on the interaction between the tax financed productive public expenditure and unemployment benefit policy of the government. We incorporate a 'Managerial' labour union in an otherwise identical Barro (1990) model; and use both 'Efficient Bargaining' model and 'Right to Manage' model to solve the negotiation problem between a labour union and an employers' association. Properties of growth rate maximizing income tax policy are derived in the steady state equilibrium; and the effects of unionization are analysed on the level of employment, growth rate, welfare and on tax rate respectively. This growth rate maximizing income tax rate appears to be higher than (equal to) the competitive output share of public input in the presence (absence) of unemployment benefit. Unionisation may be good or bad for the economy in the case of Efficient bargaining model; and the nature of the effect depends on the orientation of the labour union. However, this is always bad for both employment and growth in the case of a 'Right to Manage' model.

JEL classification: H41; H21; J51; J65; O41

Keywords: Labour union; Income tax; Public expenditure; Unemployment benefit; Efficient bargaining; Right to manage; Steady-state equilibrium; Endogenous growth

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1. Introduction

There exists a vast literature on the microeconomics of labour union behaviour and another on the theory of endogenous economic growth. However, a very small set of works combines these two streams to investigate the effect of unionisation on the long run growth rate of the economy.[1] However, those models do not introduce productive public expenditure; and hence can not analyse the role of interaction between tax financed public expenditure and unemployment benefit policy on the growth effect of unionisation.

The literature on endogenous economic growth with productive public expenditure starts with Barro (1990) and includes many other works[2]. These models analyse the optimality of various fiscal policies designed to finance productive public expenditure. However, all these models assume competitive labour market and full employment of labour. Hence these models can not analyse the distortionary role of providing unemployment benefit on the financing of productive public expenditure. There exists strong unionised labour market as well as huge amount of spending to run the unemployment benefit scheme in many countries, especially in the OECD countries. So it is important to derive optimal tax rate in a productive public expenditure model when labour market is unionised and the government finances unemployment allowance scheme.

Raurich and Sorolla (2003) attempts to analyse the growth effects of fiscal policies in the presence of unionised labour market when income taxes finance productive public capital accumulation and unemployment benefit. However, this model can not derive any analytical properties of optimum fiscal policies; and finally relies on the numerical solution. Only Kitaura (2010) derives analytical properties of optimum fiscal policies in the presence of labour union, productive public capital accumulation and unemployment subsidy. However, his model does not establish any link between unemployment rate and optimal tax policy. Additionally, there are few major limitations of each of these two works. In both these two works, the monopoly labour union maximises only the average income of workers but does not care for the size of membership. None of them introduces bargaining between the labour union and the employers' association; and so they can not analyse the growth effect of

[1] See, for example, Bräuninger (2000), Lingens (2003a, 2003b), Irmen and Wigger (2002/2003), Sorensen (1997), Palokangas (1996), Chang et al. (2007), Adjemian et al. (2010), Lai and Wang (2010), Ramos-Parreño and Sánchez-Losada (2002) etc.

[2] Futagami et al. (1993), Tsoukis and Miller (2003), Eicher and Turnovsky (2000), Kalaitzidakis and Kalyvitis (2004), Cassou and Lansing (1998), Devarajan et al. (1998), Benhabib and Velasco (1996), Economides and Philippopoulos (2008), Turnovsky (2000), Greiner (2005), Park and Philippopoulos (2003, 2004), Petraglia (2003), Clemens (2001), Chen (2003), Hung (2005), Gupta and Barman (2009, 2010, 2013), Barman and Gupta (2010), Agénor (2008), Dasgupta (1999), Baier and Glomm (2001), Cazzavillan (1996), Glomm and Ravikumar (1994), Bhattacharyya (2014) are few examples.

unionisation which is defined as an exogenous increase in the relative bargaining power of the labour union. Each of them develops Overlapping Generation model and hence can not analyse Ramsey optimal solution.

Also a set of models derive the properties of optimal unemployment benefit policies in dynamic general equilibrium models[3]. However, these models do not focus on the trade off between financing unemployment benefit and financing productive public expenditure; and hence can not analyse the role of productive public expenditure on the growth effect of unemployment benefit policy.

The present paper attempts to combine two different strands of literature. On the one hand, it investigates the growth effect and welfare effect of unionisation in the labour market. However, on the other hand, it attempts to analyse the optimality of an income tax policy designed to finance productive public expenditure in the presence of an unemployment benefit policy. The model developed here is an otherwise identical Barro (1990) model where the competitive full employment labour market assumption is replaced by wage bargaining between a labour union and an employers' association. This leads to an unemployment equilibrium causing a leakage of tax revenue from financing productive public expenditure to finance unemployment allowances. In this modified Barro (1990) framework, we use two alternative versions of bargaining models – the 'Efficient Bargaining Model' of McDonald and Solow (1981) and the 'Right to Manage Model' of Nickell and Andrews (1983).

We derive interesting results from this model. First, the optimum income tax rate in this model appears to be higher than (equal to) that obtained in Barro (1990) model in the presence (absence) of unemployment allowances. This optimum tax rate varies positively with the rate of unemployment benefit and with the level of unemployment. Secondly, the endogenous growth rate varies inversely with the rate of unemployment benefit. However, welfare level may not vary inversely with this rate; and there may exist a welfare maximising rate of benefit. These two results are valid in each of these two bargaining models. Thirdly, the nature of effects of unionisation in the labour market on employment level, economic growth and welfare level depends on the nature of the bargaining model considered. In the case of a 'Right to Manage Model', unionisation must have a negative effect on employment level and on economic growth irrespective of the orientation of labour union. However, this may not be true for the effect on welfare. In the case of an 'Efficient Bargaining Model', unionisation affects employment level and growth rate ambiguously; and the nature of this

[3] For example, Corneo and Marquardt (2000), Bräuninger (2005), Ono (2010) etc.

effect on employment (growth rate and welfare) depends solely (partially) on the nature of orientation of the labour union. Fourthly, the effects of unionisation on the optimum income tax rate are also different in these two models. In the 'Right to Manage' model, the optimum tax rate varies positively with the degree of unionisation. However, in the 'Efficient Bargaining Model', this may not be true when the labour union is employment oriented.

The paper is organized as follows. In section 2, we describe the basic model with 'Efficient Bargaining' and then derive various theoretical results. These results are compared to the corresponding results obtained from the 'Right to Manage' model in section 3. Concluding remarks are made in section 4.

2. The Model

2.1. Firms

The representative competitive firm produces the final good, Y, using private capital, K, labour, L, and public services, G. The production function of the final good is given by

$$Y = F(K, L, G) = AK^{\alpha}L^{\beta}G^{1-\alpha} \text{ where } A > 0;\ \alpha, \beta \in (0,1) \text{ and } \alpha + \beta < 1 \quad . \tag{1}$$

Here A is time independent. The Cobb-Douglas production function satisfies increasing returns to scale in term of all inputs but decreasing returns in terms of private inputs. So a positive bargaining power of employers' association leads to positive profit (rent) generated from the bargaining between the labour union and the employers' association. Following Chang et al. (2007), we assume that a fixed factor exists and is needed to set up a plant. So the number of firms is fixed in the short-run equilibrium; and is normalized to unity.

The representative firm's objective is to maximise its profit, π, defined as

$$\pi = (1-\tau)Y - wL - rK \quad . \tag{2}$$

Here w, r and τ stand for the wage rate of labour, rental rate on private capital and income tax rate respectively[4].

2.2. Capital Market

Private capital market is perfectly competitive. So rental rate on capital is determined by demand supply equality in this market. Profit maximizing behaviour of the competitive firm leads to the following demand function for capital.

$$r = (1-\tau)A\alpha K^{\alpha-1}L^{\beta}G^{1-\alpha} = \frac{(1-\tau)\alpha Y}{K} \quad . \tag{3}$$

[4] Here we are assuming that all firms and all inputs of production are properties of households. So profit income is also taxable. As there is a single final good, so its price is normalized to unity.

2.3. Labour Union's Objective Function

Following Pemberton (1988) and Chang et al. (2007), we consider a 'managerial' labour union with the utility function given by[5]

$$u_T = (w - w_c)^{\eta} L^{\mu} \quad . \tag{4}$$

Here u_T and w_c denote the utility of the labour union and the competitive wage rate respectively. η and μ are two non negative preference parameters. If $\eta > (<) (=) \mu$, then the labour union is said to be "wage oriented" ("employment oriented") ("neutral").[6]

In a competitive labour market, wage is equated to the marginal product of labour; and the labour force, normalized to unity, is fully employed[7]. So the competitive wage rate is given by the following equation.

$$w_c = (1 - \tau)\beta A K^{\alpha} G^{1-\alpha} \quad . \tag{5}$$

2.4. Employment and Wage Determination

In the basic model, we introduce the 'Efficient Bargaining' case. Both the wage rate and the level of employment are determined by bargaining between the nationwide labour union and the nationwide employers' association.[8] The result of the bargaining process can be obtained maximizing the 'generalised Nash product' function which is given by

$$\psi = (u_T)^{\phi} (\pi)^{(1-\phi)} \quad . \tag{6}$$

Bargaining disagreement results to zero employment, which, in turn, implies zero profit and zero utility. The parameter ϕ, satisfying $0 < \phi < 1$, is the relative bargaining power of the labour union. Using equations (2) and (3), we have[9]

$$\pi = (1 - \tau)(1 - \alpha)Y - wL \ . \tag{7}$$

Finally, incorporating equations (1), (4), (5) and (7) into equation (6), we obtain

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[5] Some models like Chang et al. (2007), Adjemian et al. (2010) etc. take the difference between bargained wage rate and the rate of unemployment benefit as the argument in the labour union's utility function. Contrary to this, the difference between bargained wage rate and competitive wage rate is used as an argument in the works of Irmen and Wigger (2003), Lingens (2003a) and Lai and Wang (2010).

[6] See Chang et al. (2007) to know more about these parameters.

[7] We assume that the population does not grow overtime.

[8] Details can be seen from Booth (1995). The 'Right to manage model' case is discussed in the next section.

[9] In competitive framework, an individual firm takes the rental rate of capital as given while taking decisions about labour employment. Here these nationwide employers' association and nationwide trade union are able to internalise the effect of their decision about labour employment on rental rate of capital. So they incorporate the capital demand function into the payoff function of the employers' association.

$$\psi = \{(w - [1-\tau]\beta AK^{\alpha}G^{1-\alpha})^{\eta}L^{\mu}\}^{\phi}\{(1-\tau)(1-\alpha)AK^{\alpha}L^{\beta}G^{1-\alpha} - wL\}^{(1-\phi)} \quad . \tag{8}$$

Here ψ is to be maximised with respect to w and L. Assuming an interior solution, we obtain[10]

$$\hat{L} = \left\{\frac{(1-\alpha)\{\phi\mu + \beta(1-\phi) - \phi\eta(1-\beta)\}}{\beta\{\phi\mu + (1-\phi)\}}\right\}^{\frac{1}{1-\beta}} \quad ; \tag{9}$$

and

$$w = \frac{\{\phi\mu + (1-\phi)\beta\}(1-\tau)\beta AK^{\alpha}G^{1-\alpha}}{\{\phi\mu + (1-\phi)\beta - \phi\eta(1-\beta)\}} = Xw_c \quad . \tag{10}$$

Here we assume $\phi\mu + (1-\phi)\beta > \phi\eta(1-\beta)$; and hence

$$X = \frac{\{\phi\mu + (1-\phi)\beta\}}{\{\phi\mu + (1-\phi)\beta - \phi\eta(1-\beta)\}} > 1 \quad . \tag{11}$$

X represents the ratio of bargained wage to competitive wage; and $X = 1$ when the union has no bargaining power, i.e., the employer is a monopsonist. We assume the following parametric restriction.

$$Condition\ A: \qquad -\left(\frac{1-\phi}{\phi} + \eta\right) < \frac{\mu - \eta}{\beta} < \left(\frac{\alpha}{1-\alpha-\beta}\right)\left[\frac{1-\phi}{\phi} + \eta\right] \quad .$$

This ensures that $0 < \hat{L} < 1$ and $w > 0$. Second order conditions of maximization of ψ are also satisfied[11].

Equation (9) shows that equilibrium level of employment is time independent; and equation (10) shows that bargained wage rate is proportional to and is greater than the competitive wage rate.

2.5. Government

The government spends the entire tax revenue to finance unemployment benefits as well as productive public expenditure; and so the balanced budget equation is given by

$$\tau Y = G + s(1-L) \quad . \tag{12}$$

Here s is the amount of unemployment benefit given to an unemployed worker.

2.6. Households

The representative household derives instantaneous utility from consumption of the final good only and not from leisure. She chooses the time path of consumption to maximise

[10] Derivation of equations (9) and (10) from the first order conditions are shown in the appendix A.
[11] For details, see appendix A.

her discounted present value of instantaneous utility subject to her intertemporal budget constraint. Mathematically the household's problem is given by the following.

$$Max \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t}\, dt \tag{13}$$

subject to, $\dot{K} = wL + rK + \pi + s(1-L) - c$; (14)

$K(0) = K_0$;

and $c \in [0, wL + rK + \pi + s(1-L)]$.

Here c is the control variable and K is the state variable. Here σ is the elasticity of marginal utility with respect to consumption; and ρ is the constant rate of discount. Capital is irreversible and does not depreciate. It is assumed that unemployment rate is same for all households; and the representative household saves and invests the rest of his income left after consumption.

Solving this dynamic optimisation problem we obtain the growth rate of consumption[12], denoted by γ, as given below:

$$\gamma = \frac{\dot{c}}{c} = \frac{r-\rho}{\sigma} \quad . \tag{15}$$

2.7. Optimum Tax Rate

We assume for simplicity that unemployment benefit per worker is proportional to the wage rate. So

$$s = bw \tag{16}$$

where b is a positive fraction. Using equations (1), (10), (12) and (16), we obtain

$$\frac{G}{K} = \left\{A\tau L^{\beta} - Ab(1-L)\beta X(1-\tau)\right\}^{\frac{1}{\alpha}} . \tag{17}$$

Using equations (3), (15) and (17) and then putting $L = \hat{L}$, we obtain

$$\gamma = \frac{A^{\frac{1}{\alpha}}(1-\tau)\alpha\hat{L}^{\beta}\left\{\tau\hat{L}^{\beta} - b\left(1-\hat{L}\right)\beta X(1-\tau)\right\}^{\frac{1-\alpha}{\alpha}} - \rho}{\sigma} \quad . \tag{18}$$

We now turn to derive the growth rate maximising income tax rate; and so we maximise the right hand side of equation (18) with respect to τ and then obtain

$$\tau^* = \frac{(1-\alpha)\hat{L}^{\beta} + b(1-\hat{L})\beta X}{\hat{L}^{\beta} + b(1-\hat{L})\beta X} = 1 - \frac{\alpha\hat{L}^{\beta}}{\hat{L}^{\beta} + b\left(1-\hat{L}\right)\beta X} \quad . \tag{19}$$

[12] Derivation of equation (15) is given in the appendix B.

Equation (19) shows that the growth rate maximizing tax rate, τ^*, varies positively with the rate of unemployment benefit, b. Here $\tau^* = 1 - \alpha$ when $b = 0$.

This is an important result because it differs from the corresponding result of Barro (1990) in the presence of a positive unemployment benefit. The Barro (1990) result states that growth rate maximising tax rate is identical to the elasticity of output with respect to productive public services. Barro (1990) does not consider unionised labour market and unemployment equilibrium. Our analysis shows that Barro (1990) result is valid even if there is an unionised labour market with unemployment equilibrium when the government does not finance any unemployment benefit. However, if the government finances unemployment benefit with a part of its tax revenue, then growth rate maximising tax rate will be higher than the elasticity of output with respect to productive public services. This is obvious because this tax revenue not only finances productive public expenditure but also finances unemployment benefits.

From equation (19), we have

$$\frac{\partial \tau^*}{\partial \hat{L}} = -\frac{\alpha b \beta X \hat{L}^{\beta-1}\left(\hat{L} + \beta[1 - \hat{L}]\right)}{\left\{\hat{L}^{\beta} + b(1 - \hat{L})\beta X\right\}^2} < 0 \quad . \tag{20}$$

This implies that τ^* varies inversely with the level of employment. This is so due to two reasons : (i) Higher level of employment leads to lower expenditure to provide unemployment benefit. (ii) Employment and output and hence employment and tax revenue (given the tax rate) are positively related to each others. In Kitaura (2010) too, growth rate maximising tax rate is higher than the elasticity of output with respect to productive public services. However, Kitaura (2010) does not show how τ^* varies with the unemployment level and with the bargaining power of the labour union.

We now turn to analyse its effect on the level of welfare, $\hat{\omega}$. Here

$$\hat{\omega} = \int_0^{\infty} \frac{c^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t}\, dt \quad ; \tag{21}$$

and it can be shown[13] that

$$\hat{\omega} = \frac{{K_0}^{1-\sigma}\left\{\frac{\rho\left[\hat{L}^{\beta} + b\beta X\left(1 - \hat{L}\right)\right] + \gamma\left[\sigma\hat{L}^{\beta} + \sigma b\beta X\left(1 - \hat{L}\right) - \alpha\hat{L}^{\beta}\right]}{\alpha\hat{L}^{\beta}}\right\}^{1-\sigma}}{[\rho - \gamma(1 - \sigma)](1 - \sigma)} + constant \quad . \tag{22}$$

[13] Derivation is found in appendix C.

Equation (22) shows that $\widehat{\omega}$ varies positively with γ if $\sigma\hat{L}^{\beta} + \sigma b\beta X(1-\hat{L}) > \alpha\hat{L}^{\beta}$ and $\rho > \gamma(1-\sigma)$. Since $\hat{L}$ is independent of tax rate, so the growth rate maximising tax rate is identical to the social welfare maximising tax rate. We now can establish the following proposition.

PROPOSITION 1: *The growth rate maximizing income tax rate and welfare maximizing income tax rates are identical; and this optimum tax rate exceeds (equals to) the elasticity of output with respect to productive public service when the rate of unemployment benefit is positive (zero). This optimum tax rate varies positively with the rate of unemployment benefit and with the level of unemployment.*

2.8. Growth Effect and Welfare Effect of Unemployment Benefit

Now we turn to analyse the effect of unemployment benefit on the growth rate of the economy. From equation (18), we obtain

$$\frac{\partial\gamma}{\partial b} = -\frac{A^{\frac{1}{\alpha}}(1-\tau)(1-\alpha)\hat{L}^{\beta}(1-\hat{L})\beta X(1-\tau)}{\sigma\{\tau\hat{L}^{\beta} - b(1-\hat{L})\beta X(1-\tau)\}^{\frac{2\alpha-1}{\alpha}}} < 0 \quad . \tag{23}$$

Equation (23) shows that, given the tax rate, the growth effect of providing unemployment benefit is always negative because denominator of equation (23) is positive as shown by equation (17). A rise in *b* raises expenditure on unemployment benefit; and given the tax rate, it causes productive public expenditure to fall[14]. So growth rate declines with a rise in *b*; and hence the growth rate maximising unemployment benefit rate is either zero or equal to a lower limit, $\bar{b}$, imposed by the political considerations.

From equation (22), we obtain

$$\frac{\partial\widehat{\omega}}{\partial b} = \widehat{\omega}\left\{\begin{array}{c}\frac{\partial\gamma}{\partial b}\left[\frac{(1-\sigma)}{[\rho-\gamma(1-\sigma)]} + \frac{(1-\sigma)[\sigma\hat{L}^{\beta} + \sigma b\beta X(1-\hat{L}) - \alpha\hat{L}^{\beta}]}{\rho[\hat{L}^{\beta} + b\beta X(1-\hat{L})] + \gamma[\sigma\hat{L}^{\beta} + \sigma b\beta X(1-\hat{L}) - \alpha\hat{L}^{\beta}]}\right] \\ + \frac{(1-\sigma)\{\rho\beta X(1-\hat{L}) + \gamma\sigma\beta X(1-\hat{L})\}}{\rho[\hat{L}^{\beta} + b\beta X(1-\hat{L})] + \gamma[\sigma\hat{L}^{\beta} + \sigma b\beta X(1-\hat{L}) - \alpha\hat{L}^{\beta}]}\end{array}\right\} . \tag{24}$$

Equation (24) shows that welfare effect of providing unemployment benefit consists of two different effects - a negative growth effect and a positive effect obtained from the increase in

[14] See equation (17).

initial disposable income. So optimum b is not necessarily equal to $\bar{b}$; and there may be an interior solution of b satisfying $1 > b > \bar{b}$ while maximizing welfare. We now establish the following proposition.

PROPOSITION 2: *Providing unemployment benefit must have a negative effect on economic growth though its welfare effect is not necessarily negative.*

2.9. Effects of Unionisation[15]

The economy is always in the steady state equilibrium without any transitional dynamics. At equilibrium, $\hat{L}$, τ^*, $\bar{b}$, all are time-independent; and γ and G/K are also so. So G, K and Y also grow at the same rate. w and π also grow at the same rate but r remains time-independent. $\tau^* Y$ and $bw(1-\hat{L})$ also grow at the same rate.

Now we turn to analyse how unionisation defined as an exogenous increase in the relative bargaining power of the labour union affects economy's employment, growth rate and welfare in the steady-state equilibrium. Chang et al. (2007), Palokangas (1996) etc. also make similar analysis in their models without considering the role of productive public expenditure.

From equation (9), we have

$$\frac{\partial \hat{L}}{\partial \phi} = \frac{(\mu-\eta)\hat{L}}{(\phi\mu+1-\phi)\{\phi\mu+(1-\phi)\beta-\phi\eta(1-\beta)\}} \gtreqless 0 \quad for \quad \mu \gtreqless \eta \quad . \tag{25}$$

Equation (25) shows that an increase in the relative bargaining power of the labour union will raise (lower) (not affect) the employment level of the economy if the labour union is employment oriented (wage oriented) (neutral). Chang et al. (2007) also obtains same result.

From equations (18) and (19) and putting $b = \bar{b}$, we obtain

$$\gamma|_{b=\bar{b}} = \frac{A^{\frac{1}{\alpha}}\alpha^2\hat{L}^{\frac{\alpha\beta+\beta}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\left[\hat{L}^{\beta}+\bar{b}(1-\hat{L})\beta X\right]\sigma} - \frac{\rho}{\sigma} \quad . \tag{26}$$

From equation (11), we obtain

$$\frac{\partial X}{\partial \phi} = \frac{\beta\eta(1-\beta)}{\{\phi\mu+(1-\phi)\beta-\phi\eta(1-\beta)\}^2} > 0 \quad . \tag{27}$$

From equation (26), we have

$$\left.\frac{\partial \gamma}{\partial \phi}\right|_{b=\bar{b}} = \left(\frac{A^{\frac{1}{\alpha}}\alpha^2(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\sigma}\right)\left(\frac{E_1\frac{\partial \hat{L}}{\partial \phi} - E_2\frac{\partial X}{\partial \phi}}{\left[\hat{L}^{\beta}+\bar{b}(1-\hat{L})\beta X\right]^2}\right) \quad ; \tag{28}$$

[15] In this section, we assume that optimum $b = \bar{b}$.

where

$$E_1 = \hat{L}^{\frac{\alpha\beta+\beta}{\alpha}}\bar{b}\beta X + \frac{\beta}{\alpha}\hat{L}^{\frac{2\alpha\beta+\beta-\alpha}{\alpha}} + \bar{b}(1-\hat{L})\beta X\left(\frac{\alpha\beta+\beta}{\alpha}\right)\hat{L}^{\frac{\alpha\beta+\beta-\alpha}{\alpha}} > 0 \tag{29}$$

and

$$E_2 = \hat{L}^{\frac{\alpha\beta+\beta}{\alpha}}\bar{b}(1-\hat{L})\beta > 0 \quad . \tag{30}$$

Equation (28) shows that the growth effect of unionisation is ambiguous in sign. It partly depends on the nature of orientation of the labour union. The first term of the last bracket of the R.H.S. of equation (28) depends solely on the sign of $\frac{\partial\hat{L}}{\partial\phi}$ whereas the second term inside that bracket is always negative. So an employment oriented labour union is necessary but not sufficient to ensure a positive growth effect of unionisation; and the growth effect is always negative if the union is not employment oriented. However, in Chang et al. (2007), an employment oriented labour union is necessary as well as sufficient to ensure a positive growth effect of unionisation; and the growth effect is negative if and only if the union is wage oriented.

The intuition behind this result can be explained as follows. Growth effect of unionisation in this model consists of two parts. First one comes from employment effect whose sign depends on the nature of orientation of the labour union; and this is same as that found in Chang et al. (2007). The second one is a negative tax effect; and it is special to the present model. Unionisation in the labour market raises negotiated wage rate. So unemployment benefit per worker, $\bar{b}w$, goes up. So government's expenditure to provide unemployment benefit is increased; and, to finance that expenditure, income tax rate has to rise. This reduces the after tax marginal productivity of private capital leading to this negative growth effect. This second effect does not exist in Chang et al. (2007) because they do not consider productive public expenditure.

We now turn to analyse its effect on the level of welfare, $\hat{\omega}$. From equation (22), we obtain

$$\left.\frac{\partial\hat{\omega}}{\partial\phi}\right|_{b=\bar{b}} = \hat{\omega}\left(\frac{E_3\frac{\partial\hat{L}}{\partial\phi} + E_4\frac{\partial X}{\partial\phi}}{\left[\hat{L}^{\beta} + \bar{b}(1-\hat{L})\beta X\right]^2}\right) \quad ; \tag{31}$$

where

$$E_3 = \frac{(1-\sigma)\{\rho[\beta\hat{L}^{\beta-1} - b\beta X] + \gamma[\sigma\beta\hat{L}^{\beta-1} - \sigma b\beta X - \alpha\beta\hat{L}^{\beta-1}]\}}{\rho[\hat{L}^{\beta} + b\beta X(1-\hat{L})] + \gamma[\sigma\hat{L}^{\beta} + \sigma b\beta X(1-\hat{L}) - \alpha\hat{L}^{\beta}]} - \frac{(1-\sigma)\beta}{\hat{L}}$$

$$+\left(\frac{(1-\sigma)}{[\rho-\gamma(1-\sigma)]}+\frac{(1-\sigma)\left[\sigma\hat{L}^{\beta}+\sigma b\beta X\left(1-\hat{L}\right)-\alpha\hat{L}^{\beta}\right]}{\rho\left[\hat{L}^{\beta}+b\beta X\left(1-\hat{L}\right)\right]+\gamma\left[\sigma\hat{L}^{\beta}+\sigma b\beta X\left(1-\hat{L}\right)-\alpha\hat{L}^{\beta}\right]}\right)$$

$$\cdot\quad\left(\frac{A^{\frac{1}{\alpha}}\alpha^{2}(1-\alpha)^{\frac{1-\alpha}{\alpha}}E_{1}}{\sigma\left[\hat{L}^{\beta}+\bar{b}\left(1-\hat{L}\right)\beta X\right]^{2}}\right)\quad. \tag{32}$$

and

$$E_{4}=\frac{(1-\sigma)\left\{\rho b\beta\left(1-\hat{L}\right)+\gamma\sigma b\beta\left(1-\hat{L}\right)\right\}}{\rho\left[\hat{L}^{\beta}+b\beta X\left(1-\hat{L}\right)\right]+\gamma\left[\sigma\hat{L}^{\beta}+\sigma b\beta X\left(1-\hat{L}\right)-\alpha\hat{L}^{\beta}\right]}$$

$$-\left(\frac{(1-\sigma)}{[\rho-\gamma(1-\sigma)]}+\frac{(1-\sigma)\left[\sigma\hat{L}^{\beta}+\sigma b\beta X\left(1-\hat{L}\right)-\alpha\hat{L}^{\beta}\right]}{\rho\left[\hat{L}^{\beta}+b\beta X\left(1-\hat{L}\right)\right]+\gamma\left[\sigma\hat{L}^{\beta}+\sigma b\beta X\left(1-\hat{L}\right)-\alpha\hat{L}^{\beta}\right]}\right)$$

$$\cdot\quad\left(\frac{A^{\frac{1}{\alpha}}\alpha^{2}(1-\alpha)^{\frac{1-\alpha}{\alpha}}E_{2}}{\sigma\left[\hat{L}^{\beta}+\bar{b}\left(1-\hat{L}\right)\beta X\right]^{2}}\right)\quad. \tag{33}$$

We can not sign E_3 and E_4 when $\sigma \neq 1$. In Chang et al. (2007), welfare effect of unionisation depends solely on the employment effect. However, our model shows that this is not necessarily true in the presence of productive public expenditure. The following proposition summarizes the major result.

PROPOSITION 3: *Unionisation raises (lowers) (does not affect) the level of employment in the efficient bargaining model when the labour union is employment oriented (wage oriented) (neutral). However, the growth effect of unionisation depends not only on the nature of orientation of the labour union but also on the negative taxation effect. An employment (wage) oriented labour union is necessary but not sufficient (sufficient but not necessary) to have a positive (negative) growth effect.*

Now we analyse the effect of unionisation on the optimal tax rate. From equation (19), we have

$$\left.\frac{\partial\tau^{*}}{\partial\phi}\right|_{b=\bar{b}}=\frac{\alpha\hat{L}\bar{b}(1-\alpha)\left\{\begin{matrix}\eta(1-\beta)\beta\left(1-\hat{L}\right)[1-\phi+\phi\mu]\\-(\mu-\eta)[\phi\mu+\beta(1-\phi)]\left[\hat{L}+\beta(1-\hat{L})\right]\end{matrix}\right\}}{\{\phi\mu+\beta(1-\phi)-\phi\eta(1-\beta)\}\left\{\hat{L}[1-\phi+\phi\mu]+\bar{b}(1-\alpha)(1-\hat{L})[\phi\mu+\beta(1-\phi)]\right\}^{2}}\quad. \tag{34}$$

The two terms of the denominator and the first term of the numerator in the R.H.S. of equation (34) are positive in sign but the sign of the second term of the numerator depends on

the nature of the orientation of the labour union. This equation (34) shows that an increase in ϕ leads to an increase (ambiguous change) in the optimal tax rate when the labour union is wage oriented or neutral (employment oriented).

Optimum tax rate and the level of employment are inversely related. As union becomes more powerful, then negotiated wage rate and hence unemployment benefit per worker are increased. This requires an increase in the optimum tax rate to finance the unemployment benefits. However, it may increase or decrease the employment level depending on labour unions' orientation; and thus may affect the optimum tax rate ambiguously. Employment level is decreased when the union is wage oriented. Models available in the existing literature do not incorporate the role of productive public input and of unionised labour market simultaneously; and hence the question of the effect of unionisation on optimum taxation does not arise there. This result is stated in the following proposition.

PROPOSITION 4: *Unionisation in the labour market raises optimal tax rate if the labour union is wage oriented or neutral. Otherwise, unionisation affects optimal tax rate ambiguously.*

3. Extension: 'The Right to Manage Model'

In 'The Right to manage model' of bargaining, firm's association and labour union bargain over wage only; and employment is determined from the labour demand function derived from the profit maximisation exercise of the firm. The inverted labour demand function is given by

$$w = (1-\tau)\beta AK^{\alpha}G^{1-\alpha}L^{\beta-1} = w_c L^{\beta-1} \quad . \tag{35}$$

Using equations (35) and (7), we have

$$\pi = (1-\tau)(1-\alpha-\beta)AK^{\alpha}L^{\beta}G^{1-\alpha} \quad . \tag{36}$$

So the 'generalised Nash product' function is modified as

$$\psi = \{(w-w_c)^{\eta}L^{\mu}\}^{\phi}\{(1-\tau)(1-\alpha-\beta)AK^{\alpha}L^{\beta}G^{1-\alpha}\}^{(1-\phi)} \quad . \tag{37}$$

Here ψ is to be maximised with respect to w only, subject to equation (35). Since equation (35) implies an inverse relationship between w and L, one can maximise ψ with respect to L instead of w subject to equation (35). From the first order condition of maximisation, we derive the level of employment and negotiated wage rate as given by[16]

[16] Second order condition of maximisation of ψ is also satisfied.

$$L^* = \left\{\frac{\{\phi\mu + \beta(1-\phi) - \phi\eta(1-\beta)\}}{\{\phi\mu + \beta(1-\phi)\}}\right\}^{\frac{1}{1-\beta}} \quad ; \tag{38}$$

and

$$w = w_c(L^*)^{\beta-1} \quad . \tag{39}$$

Condition A ensures that the negotiated wage rate is positive and the level of employment is a positive fraction.

Negotiated wage rates are same in both these two bargaining models; and this can be checked easily using equations (5), (10), (38) and (39).

Government's budget balance equation is same as equation (12). Representative household's optimisation problem is also represented by equations (13) and (14). Solving this dynamic optimisation problem, we obtain the similar expression of growth rate as given by equation (15). Using equations (1), (5), (12), (16) and (39), we obtain

$$\frac{G}{K} = \left\{A\tau L^{\beta} - Ab(1-L)\beta L^{\beta-1}(1-\tau)\right\}^{\frac{1}{\alpha}} \quad . \tag{40}$$

Using equations (40), (3) and (15) we obtain

$$\gamma = \frac{\dot{c}}{c} \quad = \frac{A^{\frac{1}{\alpha}}(1-\tau)\alpha L^{*\beta}\left\{\tau L^{*\beta} - b(1-L^*)\beta L^{*\beta-1}(1-\tau)\right\}^{\frac{1-\alpha}{\alpha}} - \rho}{\sigma} \quad . \tag{41}$$

Government chooses the tax rate to maximise the growth rate of consumption. Assuming an interior solution, we derive the following optimal tax rate

$$\bar{\tau} = \frac{(1-\alpha)L^* + b(1-L^*)\beta}{L^* + b(1-L^*)\beta} = 1 - \frac{\alpha L^*}{L^* + b(1-L^*)\beta} \quad . \tag{42}$$

From equations (1), (2), (3), (5), (14), (15), (16), (21) and (39), we obtain

$$\widehat{\omega} = \frac{{K_0}^{1-\sigma}\left\{\frac{\rho+\gamma\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] - \gamma\right\}^{1-\sigma}}{[\rho - \gamma(1-\sigma)](1-\sigma)} + constant \quad . \tag{43}$$

Like equation (22), equation (43) also shows that there exists a positive monotonic relationship between the welfare level and the growth rate since if $\frac{\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] > 1$; and this is always true for $\sigma > \alpha$. Since L^* is independent of tax rate, so the growth rate maximising tax rate is identical to the social welfare maximising tax rate.

Equation (41) shows that γ varies inversely with b. However, equation (43) shows that there may exist a welfare maximising interior solution of b. So propositions 1 and 2 are valid here too.

From equation (38), we have

$$\frac{\partial L^*}{\partial \phi} = -\frac{\beta\eta L^*}{[\phi\mu + \beta(1-\phi)]\{\phi\mu + (1-\phi)\beta - \phi\eta(1-\beta)\}} < 0 \quad . \tag{44}$$

This equation (44) implies that an increase in ϕ unambiguously lowers L^* for any set of values of parameters η and μ. This is contrary to the corresponding result obtained in the earlier model where the nature of the effect depends on the mathematical sign of $(\mu - \eta)$. This is so because, in 'The Right to Manage Model', two parties bargain only over wage and not over employment. The employer determines the level of employment according to its labour demand function.

Now using equations (41) and (42), we obtain

$$\gamma|_{b=\bar{b}} = \frac{\dot{c}}{c} = \frac{A^{\frac{1}{\alpha}}\alpha^2 L^{*\frac{(\beta+\alpha)}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\left[L^* + \bar{b}(1-L^*)\beta\right]\sigma} - \frac{\rho}{\sigma} \quad . \tag{45}$$

From equation (45), we have

$$\left.\frac{\partial \gamma}{\partial \phi}\right|_{b=\bar{b}} = \left(\frac{A^{\frac{1}{\alpha}}\alpha^2(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\sigma}\right)\left(\frac{\frac{\partial L^*}{\partial \phi}\left[\frac{\beta}{\alpha}L^{*\frac{\beta}{\alpha}}\{L^* + \bar{b}(1-L^*)\beta\} + \bar{b}\beta L^{*\frac{\beta}{\alpha}}\right]}{\left[L^* + \bar{b}(1-L^*)\beta\right]^2}\right) \quad . \tag{46}$$

Since all the terms of the right hand side of equation (46) are positive and $\frac{\partial L^*}{\partial \phi} < 0$, so equation (46) implies that unionisation unambiguously lowers the growth rate of the economy for any set of values of η and μ whereas the sign of the effect in the 'Efficient Bargaining' model depends partly on the sign of $(\mu - \eta)$.

To analyse its welfare effect, once again we assume that $\frac{\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] > 1$. So from equation (43), we obtain

$$\left.\frac{\partial \hat{\omega}}{\partial \phi}\right|_{b=\bar{b}} = \hat{\omega}\left(\begin{array}{c}\frac{\partial \gamma}{\partial \phi}\left[\frac{(1-\sigma)}{[\rho - \gamma(1-\sigma)]} + \frac{(1-\sigma)\left\{\frac{\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] - 1\right\}}{\left\{\frac{\rho+\gamma\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] - \gamma\right\}}\right] \\ -\frac{(1-\sigma)\left(\frac{\rho+\gamma\sigma}{\alpha}\right)\frac{b\beta}{L^{*2}}}{\left\{\frac{\rho+\gamma\sigma}{\alpha}\left[1 + b\beta\frac{(1-L^*)}{L^*}\right] - \gamma\right\}}\frac{\partial L^*}{\partial \phi}\end{array}\right) \quad . \tag{47}$$

Equation (47) shows that welfare effect of unionization is independent of labour union's orientation towards wage or employment which is not true in the efficient bargaining model.

Equations (42) and (44) show the inverse relationship between $\bar{\tau}$ and L^* and the inverse relationship between L^* and ϕ respectively. So there is a positive relationship between $\bar{\tau}$ and ϕ. This result is also different from the corresponding one obtained in the efficient bargaining model where the result depends on labour union's orientation.

We now state the major result in the form of the following proposition.

PROPOSITION 5: *In the 'Right to Manage Model', unionisation always lowers the level of employment as well as the rate of endogenous growth but raises the optimal tax rate. However, the welfare effect of unionisation, though independent of union's orientation, is ambiguous in sign.*

4. Conclusion

This paper, on the one hand, investigates the growth effect and welfare effect of unionisation in the labour market in the presence of productive public expenditure; and, on the other hand, analyses the properties of optimum income tax policy to finance productive public expenditure and unemployment benefit. The Barro (1990) model is extended by incorporating collective bargaining between the labour union and the employers' union resulting into an unemployment equilibrium. We use two alternative versions of bargaining models – the 'Efficient Bargaining Model' of McDonald and Solow (1981) and the 'Right to Manage Model' of Nickell and Andrews (1983).

Our major findings are as follows. First, the optimum rate of proportional income tax, that finances productive public expenditure as well as unemployment benefit, is found to be higher than that in the models of Barro (1990), Futagami et al. (1993); and its magnitude depends on the unemployment level, labour union's bargaining power and orientation of the labour union. Secondly, the endogenous growth rate of the economy varies inversely with the rate of unemployment benefit though social welfare may not. Both these two results are valid in each of these two bargaining models. Thirdly, how unionisation affects employment, economic growth and welfare depends on the nature of the bargaining model considered. In the 'Right to Manage Model', unionisation must have a negative effect on employment and growth regardless of the orientation of the labour union. However, welfare may increase due to unionisation. On the contrary, the nature of these effects at least partially depends on the nature of orientation of the labour union in the 'Efficient Bargaining Model'. Growth effects and welfare effects are not necessarily positive even if the union is employment oriented; and the growth effect is always negative if the union is neutral or wage oriented. Our results are different from those found in Chang et al. (2007). Fourthly, unionisation raises the optimal tax rate in the 'Right to Manage' model but affects it ambiguously in the 'Efficient Bargaining Model'. This point is not interesting in Chang et al. (2007) where there is no productive public expenditure to be financed by taxation.

However, our model is abstract and fails to consider many aspects of reality. We assume public expenditure as a flow variable and hence do not consider the role of public capital accumulation. We also rule out the possibility of human capital accumulation, population growth, technological progress, environmental degradation etc. Hence the allocation of government's budget and of household's income to education, R&D, pollution abatement etc. is not analysed here. One sector aggregative framework considered here fails to highlight the structural inter-relationship among different sectors. We ignore membership dynamics of labour union and the union's concern about worker's safety, health and workplace environment. We plan to do further research in future attempting to remove these limitations.

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Appendix

Appendix A

Derivation of equations (9) and (10):

From equations (8) and (5), we have

$$log\psi = \phi\eta\, log(w - w_c) + \phi\mu\, logL + (1-\phi)\, log\{(1-\tau)(1-\alpha)AK^{\alpha}L^{\beta}G^{1-\alpha} - wL\} \quad . \qquad (A.1)$$

The first order optimality conditions of maximization of $log\psi$ with respect to w and L are given by

$$\frac{\phi\eta}{w - w_c} + \frac{(1-\phi)(-L)}{\pi} = 0 \quad ; \qquad (A.2)$$

and

$$\frac{\phi\mu}{L} + \frac{(1-\phi)\{\frac{(1-\tau)(1-\alpha)\beta Y}{L} - w\}}{\pi} = 0 \quad . \qquad (A.3)$$

Using equations (A.2) and (A.3), we obtain

$$(\eta - \mu)w = \eta(1-\tau)(1-\alpha)\beta\frac{Y}{L} - \mu w_c \quad . \qquad (A.4)$$

Using equations (1), (5), (7), (A.2) and (A.4) we obtain equation (9) in the body of the paper.

Using equations (1), (7), (9) and (A.3) we obtain equation (10) in the body of the paper.

Second order conditions:

From equations (A.2) and (A.3), we obtain respectively

$$\frac{\partial^2 log\psi}{\partial w^2} = -\frac{\phi\eta}{(w - w_c)^2} - \frac{(1-\phi)L^2}{\pi^2} < 0 \quad ; \qquad (A.5)$$

and

$$\frac{\partial^2 log\psi}{\partial L^2} = -\frac{\phi\mu}{L^2} - \frac{(1-\phi)}{\pi}\left[(1-\tau)(1-\alpha)\beta(1-\beta)\frac{Y}{L^2}\right] - \frac{(1-\phi)}{\pi^2}\left[(1-\tau)(1-\alpha)\beta\frac{Y}{L} - w\right]^2 < 0 \quad . \qquad (A.6)$$

Again from equation (A.2), we have

$$\frac{\partial^2 log\psi}{\partial L \partial w} = -\frac{(1-\phi)}{\pi^2}\{(1-\tau)(1-\alpha)AK^{\alpha}L^{\beta}G^{1-\alpha}(1-\beta)\} \quad . \qquad (A.7)$$

From equations (7), (1), (5), (9) and (10), we have

$$\pi^2 = {w_c}^2 L^2 \left\{\frac{(1-\beta)(1-\phi)}{\{\phi\mu + \beta(1-\phi) - \phi\eta(1-\beta)\}}\right\}^2 \quad . \qquad (A.8)$$

Using equations (5), (9), (A.7) and (A.8), we have

$$\frac{\partial^2 log\psi}{\partial L \partial w} = -\frac{[1-\phi+\phi\mu]\{\phi\mu + \beta(1-\phi) - \phi\eta(1-\beta)\}}{(1-\beta)Lw_c(1-\phi)} \quad . \qquad (A.9)$$

From equations (5) and (10), we have

$$w - w_c = w_c\left\{\frac{\phi\eta(1-\beta)}{\phi\mu + \beta(1-\phi) - \phi\eta(1-\beta)}\right\} \quad . \qquad (A.10)$$

Using equations (A.5), (A.8) and (A.10), we have

$$\frac{\partial^2 log\psi}{\partial w^2} = -\frac{\{\phi\mu + \beta(1-\phi) - \phi\eta(1-\beta)\}^2}{{w_c}^2(1-\beta)^2\phi\eta(1-\phi)}[1-\phi+\phi\eta] \quad . \qquad (A.11)$$

From equations (A.6), (1), (5), (A.8), (10) we have

$$\frac{\partial^2 log\psi}{\partial L^2} = -\frac{1}{L^2}\left[\frac{(1-\phi+\phi\mu)[\phi\mu + \beta(1-\phi)]}{(1-\phi)}\right] \quad . \qquad (A.12)$$

Now using equations (A.9), (A.11) and (A.12), we have

$$\Rightarrow \left\{\frac{\partial^2 log\psi}{\partial w^2}\right\}.\left\{\frac{\partial^2 log\psi}{\partial L^2}\right\} - \left\{\frac{\partial^2 log\psi}{\partial L \partial w}\right\}^2 = \frac{\{\phi\mu + \beta(1-\phi) - \phi\eta(1-\beta)\}^2(1-\phi+\phi\mu)}{L^2{w_c}^2(1-\beta)^2(1-\phi)^2}$$

$$. \left\{\frac{[1-\phi+\phi\eta][\phi\mu + \beta(1-\phi)]}{\phi\eta} - [1-\phi+\phi\mu]\right\} \quad . \qquad (A.13)$$

Here, by assumption, $\phi\mu + \beta(1-\phi) - \phi\eta(1-\beta) > 0$.

$$\Rightarrow \frac{(1-\phi+\phi\eta)[\phi\mu + \beta(1-\phi)]}{\phi\eta} > [\phi\mu + (1-\phi)] \quad . \qquad (A.14)$$

Equation (A.13) and inequality (A.14) imply that

$$\left\{\frac{\partial^2 log\psi}{\partial w^2}\right\}.\left\{\frac{\partial^2 log\psi}{\partial L^2}\right\} - \left\{\frac{\partial^2 log\psi}{\partial w \partial L}\right\}^2 > 0 \quad .$$

Appendix B

Derivation of equation (15):

Using equations (13) and (14), we construct the Current Value Hamiltonian as given by

$$H_c = \frac{c^{1-\sigma} - 1}{1-\sigma} + \lambda[wL + rK + \pi + s(1-L) - c] \quad . \qquad (B.1)$$

Here λ is the co-state variable. Maximising equation (B.1) with respect to c, we obtain the following first order condition.

$$c^{-\sigma} - \lambda = 0 \quad ; \qquad (B.2)$$

Again from equation (B.1), we have

$$\frac{\dot{\lambda}}{\lambda} = \rho - r \quad ; \tag{B.3}$$

and from equation (B.2), we have

$$\frac{\dot{\lambda}}{\lambda} = -\sigma \frac{\dot{c}}{c} \quad . \tag{B.4}$$

Using equations (B.3) and (B.4), we have equation (15) in the body of the paper.

Appendix C

Derivation of equation (22):

From equation (21), we obtain

$$\widehat{\omega} = \frac{{c_0}^{1-\sigma}}{[\rho - \gamma(1-\sigma)](1-\sigma)} + constant \quad . \tag{C.1}$$

Here, $c(0) = c_0$.

From equations (2), (16), (14), (1), (10) we obtain

$$c_0 = K_0 \left\{ (1-\tau) A \hat{L}^{\beta} \left(\frac{G_0}{K_0}\right)^{1-\alpha} + (1-\tau) b\beta X (1-\hat{L}) A \left(\frac{G_0}{K_0}\right)^{1-\alpha} - \gamma \right\} \quad . \tag{C.2}$$

Using equations (3) and (15), we obtain

$$\frac{\rho + \sigma\gamma}{\alpha \hat{L}^{\beta}} = \left(\frac{G_0}{K_0}\right)^{1-\alpha} (1-\tau) A \quad . \tag{C.3}$$

Using equations (C.2) and (C.3), we obtain

$$c_0 = K_0 \left\{ \frac{\rho\left[\hat{L}^{\beta} + b\beta X(1-\hat{L})\right] + \gamma\left[\sigma \hat{L}^{\beta} + \sigma b \beta X\left(1-\hat{L}\right) - \alpha \hat{L}^{\beta}\right]}{\alpha \hat{L}^{\beta}} \right\} \quad . \tag{C.4}$$

Using equations (C.1) and (C.4), we obtain

$$\widehat{\omega} = \frac{{K_0}^{1-\sigma} \left\{ \frac{\rho\left[\hat{L}^{\beta} + b\beta X\left(1-\hat{L}\right)\right] + \gamma\left[\sigma \hat{L}^{\beta} + \sigma b \beta X\left(1-\hat{L}\right) - \alpha \hat{L}^{\beta}\right]}{\alpha \hat{L}^{\beta}} \right\}^{1-\sigma}}{[\rho - \gamma(1-\sigma)](1-\sigma)} + constant \quad . \tag{C.5}$$

Equation (C.5) is identical to the equation (22) in the body of the paper.