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Abstract

We investigate endogenous timing in a mixed duopoly in a differentiated product market. We find that private leadership is better than public leadership from a social welfare perspective if the private firm is domestic, regardless of the degree of product differentiation. Nevertheless, the public leadership equilibrium is risk-dominant, and it is thus robust if the degree of product differentiation is high. We also find that regardless of the degree of product differentiation, the public leadership equilibrium is risk-dominant if the private firm is foreign. These results may explain the recent revival of public financial institutions in Japan.

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1 Introduction

Until the 1980s, public enterprises played a leading role in the Japanese economy. For example, it was believed that lending by public financial institutions (e.g., the Development Bank of Japan) had a pump-priming effect on lending by private banks. Furthermore, public financing occupied an important position in the Japanese financial markets for over 40 years.[1] From the 1980s, public enterprises underwent major reforms.[2] The Koizumi Cabinet (April 2001–September 2006) changed this by declaring that public firms should play a complementary role to private firms, with the latter leading the markets rather than the former. As a result, major public institutions were substantially downscaled.

Such a situation can be described by a simple model in a homogeneous product market formulated by Pal (1998). He adopted the observable delay game formulated by Hamilton and Slutsky (1990) and investigated the endogenous role in mixed duopolies where public and private firms compete.[3] He showed that private leadership yields greater welfare than public leadership does. Although both public and private leadership equilibria exist in a mixed duopoly, he suggested that private leadership is more natural and robust. Subsequently, many researchers have proven that his conclusion is adequate. Matsumura and Ogawa (2010) showed

[1] See Horiuchi and Sui (1993). It is globally observed that public sectors play an important role in lending markets. See Bose et al. (2013).

[2] For example, three major state-owned public enterprises, the Japan Railway group, Japan Tobacco Incorporated, and Nippon Telegraph and Telephone Corporation, were privatized.

[3] The literature on mixed oligopolies has rich and diverse discussions on the observable delay game. Tomaru and Kiyono (2010) generalized the demand and cost functions. Matsumura (2003b) introduced foreign competition. Tomaru and Saito (2010) considered a subsidized mixed duopoly, and Bárcena-Ruiz (2007) investigated price competition and showed that Bertrand emerges in a mixed duopoly. Bárcena-Ruiz and Sedano (2011) discussed a different type of objective in a public firm. For the importance of sequential-move games in mixed oligopolies, see also Heywood and Ye (2009a), Ino and Matsumura (2010), Wang and Mukherjee (2012), and Wang and Lee (2013). For the recent development of mixed oligopolies, see also Ishida and Matsushima (2009), Bose et al. (2013), and Matsumura and Tomaru (2013).

that private leadership is risk-dominant in his model. They also adopted a partial privatization approach formulated by Matsumura (1998) and showed the advantage of private leadership, that is, the range for the degree of privatization that yields private leadership equilibrium is wider than that for public leadership equilibrium. Capuano and De Feo (2010) introduced the shadow cost of public funds into Pal's model and showed that unless the shadow cost is too high, one of the following results holds: (i) the unique equilibrium is a private leadership equilibrium; (ii) both public and private leadership equilibria exist and the private leadership equilibrium is risk-dominant.[4] Matsumura (2003a) used different endogenous timing games (two-production period models) formulated by Saloner (1987) and Pal (1991) and showed that only the private leadership equilibrium is robust. These results suggest that private leadership that is better for social welfare is more likely to occur in endogenous timing games.

However, public institutions have recently begun to once more lead the Japanese markets. Newly established public financial institutions such as Industrial Revitalization Corporation of Japan, Enterprise Turnaround Initiative Corporation of Japan, and Regional Economy Vitalization Corporation of Japan play leading roles in financial markets. The Nikkei, a Japanese newspaper, calls this situation "Kiko Capitalism (State Institution Capitalism)" (Nikkei, November 22, 2011). Furthermore, this type of capitalism is still expanding under the current Abe Cabinet (Nikkei, October 8, 2013). Following the literature results previously mentioned, welfare-maximizing public institutions are naturally followers rather than leaders. Thus, we question whether such state institution capitalism implies that the public sector does not seek to maximize welfare.

A notable feature of these recently established financial institutions is that they supply highly differentiated rather than homogeneous services. In addition to supplying money, these

[4] If the shadow cost is high, the unique equilibrium is a Cournot.

institutions provide consulting and auditing services and play an important role in business revitalization and firm reconstruction. These services are usually highly differentiated.

In this study, we introduce product differentiation using Dixit's (1979) model. First, we investigate a case where the private competitor is a domestic firm. We find that regardless of the degree of product differentiation, (i) two equilibria (i.e., private and public leadership) exist, and (ii) private leadership is better than public leadership for social welfare. Nevertheless, the public leadership equilibrium is risk-dominant, and thus, it is robust if the degree of product differentiation is high.

When the product is highly differentiated, the public firm's optimal output is less sensitive to the private leader's output. Thus, it becomes difficult for the private firm, as the leader, to reduce the public firm's output. Therefore, the private firm is reluctant to lead. This causes risk dominance of the public leadership equilibrium.

Our results contain the following important implications. From a social welfare perspective, public leadership is not the best solution. Nevertheless, even welfare-maximizing public institutions naturally choose a leading role when the degree of product differentiation is high. In such a situation, the public firm's commitment to not be the leader can improve welfare.[5]

Next, we investigate a case in which the private firm is foreign. In the literature on mixed oligopolies, it is known that foreign ownership in private firms often matters as long as the public firm maximizes domestic welfare.[6] We find that regardless of the degree of product differentiation, (i) two equilibria (private and public leadership) exist, (ii) public leadership is better than private leadership for social welfare, and (iii) the public leadership equilibrium

[5] For a discussion on this commitment, see Ino and Matsumura (2010).

[6] For pioneering works discussing foreign competition in mixed oligopolies, see Corneo and Jeanne (1994), Fjell and Pal (1996), and Pal and White (1998). Foreign ownership is important in the context of public policies in mixed oligopolies. See also Bárcena-Ruiz and Garzón (2005a, b), Heywood and Ye (2009b), and Lin and Matsumura (2012).

$1 \setminus 2$	Cooperation	Non-Cooperation
Cooperation	(A,a)	(B, b)
Non-Cooperation	(C,c)	(D, d)

Table 1: Payoff matrix of the example.

is risk-dominant, and thus, it is robust. Therefore, public leadership is more likely to emerge when the competitor is foreign. This may also explain the revival of public financial institutions in Japan, because there are foreign firms in corporate revitalization markets that are main competitors of Industrial Revitalization Corporation of Japan, Enterprise Turnaround Initiative Corporation of Japan, and Regional Economy Vitalization Corporation of Japan.

We now briefly explain the concept of risk dominance in Harsanyi and Selten (1988) and why we use this criterion as an equilibrium choice to investigate which of the two equilibria is robust. We explain the concept of risk dominance using a two-player, two-strategy example (Table 1). Suppose that there are two pure strategy Nash equilibria, (Cooperation, Cooperation) and (Non-Cooperation, Non-Cooperation), that is, $A > C$, $a > b$, $D > B$, and $d > c$ are satisfied. Suppose that $A > D$ and $a > d$, that is, both players prefer (Cooperation, Cooperation) to (Non-Cooperation, Non-Cooperation). This example is a (non-pure) coordination game under these assumptions. Players 1 and 2 agree to cooperate if they can communicate and trust each other. However, if it is unknown whether the rival will cooperate, each player faces risk; thus, players might choose their strategy to reduce this risk. (Non-Cooperation, Non-Cooperation) risk dominates (Cooperation, Cooperation) if the product of the deviation losses is higher for (Non-Cooperation, Non-Cooperation), that is, $(D-B)(d-c) > (A-C)(a-b)$. This inequality is more likely satisfied when C and b are small. When C and b are small, each player's payoff becomes small if the rival chooses Non-Cooperation. Thus, each player faces serious risk even if (s)he suspects that the other player expects that the equilibrium (Non-Cooperation, Non-

Cooperation) is chosen. To avoid this risk, each player chooses Non-Cooperation even if (s)he is risk neutral. Moreover, the risk-dominant equilibrium is evolutionary stable (Harsanyi and Selten, 1988).

Payoff dominance is another popular concept of equilibrium choice. In the previous example, the (Cooperation, Cooperation) payoff-dominates (Non-Cooperation, Non-Cooperation), because both players prefer (Cooperation, Cooperation) to (Non-Cooperation, Non-Cooperation). If B and c are large and C and b are small, (Non-Cooperation, Non-Cooperation) risk dominates (Cooperate, Cooperate). Therefore, risk dominance and payoff dominance may suggest a different robust equilibrium in the general context. Thus, whether we adopt risk dominance or payoff dominance is crucial.

In this study, we adopt risk dominance to select a robust equilibrium. There are two reasons why we adopt risk dominance rather than payoff dominance as the criterion of equilibrium choice. First, it is possible that neither of the two equilibria payoffs dominates the other. In the previous example, if we replace the assumption $a > d$ with $a < d$ (i.e., the so-called "battle of sexes"), we cannot determine which of the two equilibria is robust. Risk dominance has no such problem in the generic case. Second, Matsumura and Ogawa (2009) showed that in the observable delay game, if one equilibrium payoff-dominates the other, this equilibrium also risk-dominates the other. In other words, there is no conflict between two concepts in the observable delay game, in contrast to other games such as (non-pure) coordination games. Our result that public leadership equilibrium risk dominates private leadership equilibrium implies that private leadership equilibrium never payoff-dominates the public leadership one. Thus, we believe that adopting risk dominance is plausible in the general context, and that it is further reasonable in the observable delay game.[7]

[7] In the literature on the endogenous timing game, risk dominance is a fairly powerful and popular tool for equilibrium choice. See van Damme and Hurkens (2004), Amir and Stepanova (2006), and Hirata and

2 The Model

Firms 0 and 1 produce differentiated commodities, and the inverse demand function is given by $p_i = \alpha - \beta q_i - \beta\delta q_j$ $(i = 0, 1,\ i \neq j)$, where p_i and q_i are firm i's price and quantity respectively, α, β are positive constants and $\delta \in (0, 1)$. The marginal production costs are constant. Let m_i denote firm i's marginal cost. We assume that $\alpha > m_0 \geq m_1$.[8]

Firm 0 is a state-owned public firm, and its payoff is the total social surplus given by

$$SW = (p_0 - m_0)q_0 + (p_1 - m_1)q_1 + \left[\alpha(q_0 + q_1) - \frac{\beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)}{2} - p_0 q_0 - p_1 q_1\right] \ (\equiv V_0).$$

The quasi-linear utility function of a representative consumer, $U(q_0, q_1) = \alpha(q_0 + q_1) - \beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)/2 - (p_0 q_0 + p_1 q_1)$, provides the demand and consumer surplus functions adopted in this study. Firm 1 is a private firm and its payoff is its own profit, $\pi_1 = (p_1 - m_1)q_1$ $(\equiv V_1)$.

The game runs as follows. In the first stage, firm i $(i = 0, 1)$ simultaneously chooses whether to move early $(t_i = 1)$ or late $(t_i = 2)$. The basic game is then played using simultaneous play if both firms choose the same period, and sequential play otherwise. See Table 2 for the payoff matrix of the observable delay game in our environment, where V_i^F (res. V_i^L) denotes firm i's equilibrium payoff in the sequential-move game when it is the follower (res. leader), and V_i^N denotes each firm's equilibrium payoff in the simultaneous-move game (Nash).

Table 2: Payoff matrix of the observable delay game.

$0\backslash 1$	$t_1 = 1$	$t_1 = 2$
$t_0 = 1$	$\left(V_0^N, V_1^N\right)$	$\left(V_0^L, V_1^F\right)$
$t_0 = 2$	$\left(V_0^F, V_1^L\right)$	$\left(V_0^N, V_1^N\right)$

Matsumura (2011). For a convincing rationalization of this concept, see van Damme and Hurkens (2004).

[8]The assumption that $m_0 \geq m_1$ is popular in the literature and we believe that this is a reasonable assumption. For the theoretical and empirical discussion on the cost difference between public and private firms, see Matsumura and Matsushima (2004) and Megginson and Netter (2001), respectively. However, we can show that our results hold without this assumption unless the difference of these two costs is too large.

3 Fixed Timing Games

In this section, we discuss the second stage game given t_0 and t_1. Let $a_i \equiv \alpha - m_i$. We assume that the following three games have interior solutions.

3.1 Cournot ($t_0 = t_1 = 1$ or $t_0 = t_1 = 2$)

First, we consider the simultaneous-move game (Cournot competition). Each firm maximizes its payoff V_i with respect to q_i. The first-order conditions are

$$\begin{aligned} \frac{\partial V_0}{\partial q_0} &= a_0 - q_0\beta - q_1\beta\delta = 0, \\ \frac{\partial V_1}{\partial q_1} &= a_1 - 2q_1\beta - q_0\beta\delta = 0. \end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions for firms 0 and 1, respectively:

$$\begin{aligned} R_0 &= \frac{a_0 - q_1\beta\delta}{\beta}, \\ R_1 &= \frac{a_1 - q_0\beta\delta}{2\beta}. \end{aligned}$$

These functions lead to the following equilibrium quantities:

$$\begin{aligned} q_0^N &= \frac{2a_0 - \delta a_1}{\beta(2-\delta^2)}, \\ q_1^N &= \frac{a_1 - \delta a_0}{\beta(2-\delta^2)}. \end{aligned}$$

The resulting welfare and firm 1's profit are, respectively,

$$\begin{aligned} V_0^N &\equiv SW^N = \frac{4a_0^2 - 6a_0a_1\delta + 3a_1^2 + 2a_0a_1\delta^3 - a_0^2\delta^2 - a_1^2\delta^2}{2(2-\delta^2)^2\beta}, \\ V_1^N &\equiv \pi_1^N = \frac{(a_1 - a_0\delta)^2}{(2-\delta^2)^2\beta}. \end{aligned}$$

3.2 Stackelberg with Public Leadership ($t_0 = 1, t_1 = 2$)

Second, we consider the sequential-move game where firm 1 chooses $q_1 = R_1(q_0)$ and firm 0 maximizes its payoff, $V_0(q_0, R_1(q_0))$. We have

$$\begin{aligned} q_0^L &= \frac{4a_0 - 3a_1\delta}{\beta(4 - 3\delta^2)}, \\ q_1^F &= \frac{2(a_1 - a_0\delta)}{\beta(4 - 3\delta^2)}. \end{aligned}$$

The resulting welfare and firm 1's profit are, respectively,

$$\begin{aligned} V_0^L &\equiv SW^L = \frac{4a_0^2 - 6a_0a_1\delta + 3a_1^2}{2(4 - 3\delta^2)\beta}, \\ V_1^F &\equiv \pi_1^F = \frac{4(a_1 - a_0\delta)^2}{(4 - 3\delta^2)^2\beta}. \end{aligned}$$

3.3 Stackelberg with Private Leadership ($t_0 = 2, t_1 = 1$)

Third, we consider the sequential-move game where firm 0 chooses $q_0 = R_0(q_1)$ and firm 1 maximizes its payoff, $V_1(R_0(q_1), q_1)$. We have

$$\begin{aligned} q_0^F &= \frac{2a_0 - a_1\delta - a_0\delta^2}{2(1 - \delta)(1 + \delta)\beta}, \\ q_1^L &= \frac{a_1 - a_0\delta}{2(1 - \delta)(1 + \delta)\beta}. \end{aligned}$$

The resulting welfare and firm 1's profit are, respectively,

$$\begin{aligned} V_0^L &\equiv SW^L = \frac{4a_0^2 - 6a_0a_1\delta + 3a_1^2 - a_0^2\delta^2}{8(1 - \delta)(1 + \delta)\beta}, \\ V_1^F &\equiv \pi_1^F = \frac{(a_1 - a_0\delta)^2}{4(1 - \delta)(1 + \delta)\beta}. \end{aligned}$$

3.4 Result

Discussing the first stage choice, we present our main result.

Proposition 1 *(i) For any* $\delta \in (0,1)$*, both* $(t_0,t_1)=(1,2)$ *(public leadership) and* $(t_0,t_1)=(2,1)$ *(private leadership) constitute equilibria. (ii) There exists* $\delta^* \in (0,1)$ *such that the public (res. private) leadership equilibrium is risk-dominant if* $\delta <$ *(res.* $>$*)* δ^**.*

Proof

(i) The following inequalities directly indicate Proposition 1(i).

$$\begin{aligned}
V_0^L - V_0^N &= \frac{\delta^2(a_1-a_0\delta)^2}{2\beta(4-3\delta^2)(2-\delta^2)^2} > 0, \\
V_0^N - V_0^F &= -\frac{\delta^2(a_1-a_0\delta)^2(2-\delta)(2+\delta)}{8\beta(1-\delta)(1+\delta)(2-\delta^2)^2} < 0, \\
V_1^L - V_1^N &= \frac{\delta^4(a_1-a_0\delta)^2}{4\beta(1-\delta)(1+\delta)(2-\delta^2)^2} > 0, \\
V_1^N - V_1^F &= -\frac{\delta^2(a_1-a_0\delta)^2(8-5\delta^2)}{(4-3\delta^2)^2(2-\delta^2)^2} < 0.
\end{aligned}$$

(ii) In the mixed strategy equilibrium, the following two equalities regarding probabilities $Pr_i(\delta)$ that the firm i moves early are satisfied:

$$\begin{aligned}
&Pr_1(\delta)V_0^N + (1-Pr_1(\delta))V_0^L = Pr_1(\delta)V_0^F + (1-Pr_1(\delta))V_0^N \\
&\Longrightarrow \ Pr_1(\delta) = \frac{V_0^L - V_0^N}{V_0^L + V_0^F - 2V_0^N} = \frac{4(1-\delta)(1+\delta)}{3\delta^4 - 20\delta^2 + 20}.
\end{aligned}$$

$$\begin{aligned}
&Pr_0(\delta)V_1^N + (1-Pr_0(\delta))V_1^L = Pr_0(\delta)V_1^F + (1-Pr_0(\delta))V_1^N \\
&\Longrightarrow \ Pr_0(\delta) = \frac{V_1^L - V_1^N}{V_1^L + V_1^F - 2V_1^N} = \frac{(4-3\delta^2)^2\delta^2}{9\delta^6 - 4\delta^2 - 36\delta^2 + 32}.
\end{aligned}$$

$(t_0,t_1)=(1,2)$ risk dominates $(t_0,t_1)=(2,1)$ if $Pr_0(\delta)(1-Pr_1(\delta)) < (1-Pr_0(\delta))Pr_1(\delta)$, that is, $Pr_0(\delta) < Pr_1(\delta)$ (see Harsanyi and Selten 1988).

We have

$$\begin{aligned}
Pr_0(0) &= 0, \\
\frac{\partial Pr_0}{\partial \delta} &= \frac{8\delta(2-\delta^2)(4-3\delta^2)(15\delta^4-28\delta^2+16)}{(9\delta^6-4\delta^2-36\delta^2+32)^2} > 0, \\
Pr_0(1) &= 1, \\
Pr_1(0) &= \frac{1}{5}, \\
\frac{\partial Pr_0}{\partial \delta} &= -\frac{24\delta^3(2-\delta^2)}{(3\delta^4-20\delta^2+20)^2} < 0, \\
Pr_1(1) &= 0.
\end{aligned}$$

Therefore, there exists $\delta^* \in (0,1)$ such that $Pr_0(\delta^*) = Pr_1(\delta^*)$. For $\delta <$ (res. $>$) δ^*, $Pr_0 <$ (res. $>$) Pr_1. Q.E.D.

Solving $Pr_0(\delta) = Pr_1(\delta)$ for δ, we obtain $\delta^* \approx 0.6546$. As we discuss later, the private firm prefers public leadership to private leadership when δ is not so close to 1. When δ is smaller (i.e., the product is more differentiated), the public firm's optimal output is less sensitive to the private leader's output.[9] Thus, it becomes difficult for the private firm, as the leader, to reduce the public firm's output. Therefore, the private firm is reluctant to lead. In other words, the private firm has a stronger incentive to choose period 2 when δ is smaller. Given this incentive of the private firm, the public firm is also reluctant to choose period 2. If the private firm chooses period 2 and the public firm also chooses period 2, firms face Cournot competition. The public firm prefers the public leadership to Cournot, and thus, firm 2 has weaker incentive to choose period 2 when δ is smaller. Thus, public leadership payoff-dominates and risk-dominates private leadership when δ is small.

[9]Similarly, the private firm's optimal output is less sensitive to the public leader's output when δ is smaller. However, the private firm's optimal output is less sensitive to the rival's output and δ than the public firm's, and thus, a change in δ more significantly affects the leader's incentive when the follower is the public firm than when it is the private firm.

We now present a result on welfare.

Proposition 2 *(i) For any $\delta \in (0,1)$, private leadership is better than public leadership for social welfare. (ii) The private firm prefers public leadership to private leadership if and only if $\delta < \delta^{**} = 2\sqrt{2}/3 \approx 0.9428$.*

Proof The following first equation shown below directly indicates (i), and the second implies (ii):

$$\begin{aligned} SW^F - SW^L &= \frac{3(a_1 - a_0\delta)^2\delta^2}{8(4-3\delta^2)(1-\delta)(1+\delta)\beta} > 0, \\ \pi_1^L - \pi_1^F &= \frac{(a_1 - a_0\delta)^2(9\delta^2-8)\delta^2}{4(4-3\delta^2)^2(1-\delta)(1+\delta)\beta} \implies sign(\pi_1^L - \pi_1^F) = sign(9\delta^2 - 8). \quad \text{Q.E.D.} \end{aligned}$$

We have already explained why the private firm prefers public leadership to private leadership when δ is large (Proposition 2-ii). Proposition 2-i states that private leadership is more efficient than public leadership for social welfare. The private firm produces more aggressively in the private leadership case than in the Cournot case (simultaneous-move case) because the leader, the private firm, has an incentive to reduce the rival's output. This more aggressive behavior of the leader results in less aggressive behavior of the public firm. Because the public (resp. private) firm's output is larger (resp. smaller) than the first best output, changes in both firms' outputs improve welfare. In contrast, the leader, the public firm, produces less aggressively in the public leadership case than in the Cournot case because the leader has an incentive to increase the rival's output to improve welfare. This less aggressive behavior of the leader results in more aggressive behavior of the public firm. Thus, both public and private leadership improves welfare than the Cournot case. The improvement is significant because the absolute value of the slope of the public firm's reaction curve, $|\partial R_0/\partial q_1|$, is larger than that of the private firm's reaction curve, $|\partial R_1/\partial q_0|$. Therefore, the leader's strategic incentive for manipulating the follower is larger (and thus, the welfare-improving effect is stronger) in

the private leadership case than in the public leadership case. This leads to Proposition 2-ii.

Proposition 1-ii and 2-i imply that the less efficient equilibrium is risk-dominant when the degree of product differentiation is high. Private leadership yields greater welfare than public leadership does regardless of the degree of product differentiation. Private leadership yields larger profit for the private firm than the private leadership does unless the degree of product differentiation is low, and thus, it has a strong incentive to follow. This makes the less efficient equilibrium, namely, public leadership, more robust when δ is small. By contrast, if δ is large, both firms prefers the private leadership to the public one. In other words, there is no conflict of interest between the two firms for each firm's respective role. In this case the private leadership risk-dominates the public leadership.[10]

4 Foreign Competitor

In this study, we assume that the private firm is domestic. However, in the literature on mixed oligopolies, it is known that foreign ownership in private firms often changes the results drastically. If the private firm is domestic, a domestic welfare-maximizer is concerned with both consumer surplus and the profits of both firms. However, if the private firm is foreign (i.e., it is owned by foreign investors), the welfare-maximizer is only concerned with consumer surplus and the domestic public firm's profit. This changes the public firm's behavior directly, and it also changes the private firm's thorough strategic interaction between the firms. In this section, we briefly discuss the case in which the private firm is foreign.

When the private firm is foreign, the domestic social surplus is the sum of the consumer

[10]Even in the private duopoly, it is possible that both firms prefer to the more-efficient firm's leadership to the less-efficient firm's one. For the pioneering studies in this field, see Ono (1978,1982).

surplus and profit of the public firm. Thus, we modify the welfare function as follows.

$$SW = (p_0 - m_0)q_0 + \left[\alpha(q_0 + q_1) - \frac{\beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)}{2} - p_0 q_0 - p_1 q_1\right].$$

In this case, we find the following:

Proposition 3 *For any $\delta \in (0, 1)$, (i) both public and private leadership equilibria exist, (ii) the private firm prefers the public leadership to private leadership, (iii) public leadership yields larger domestic welfare than private leadership does, (iv)the public leadership equilibrium is always risk-dominant.*

The procedure to derive these results are similar to the results in the previous section, so we omit it. The formal proof is available upon request for the authors.

The result that both private and public leadership equilibria exist holds true whether the private firm is domestic or foreign. The result that public leadership equilibrium can be risk-dominant is strengthened when the private firm is foreign. As we later discuss, the public firm's optimal output is less sensitive to the private leader's output when the private firm is foreign than when it is domestic. Thus, it becomes more difficult for the private firm as the leader to reduce the public firm's output when it is foreign. Therefore, the private firm is further reluctant to lead. This is why public leadership risk-dominates private leadership regardless of δ.

However, the result that private leadership yields larger domestic welfare does not hold true under foreign ownership, in which public leadership yields larger domestic welfare than private leadership does. Similar to the domestic private firm case, the private firm here produces more aggressively in the private leadership case than in the Cournot case (simultaneous-move case) because the leader, the private firm, has an incentive to reduce the rival's output. This more-aggressive behavior of the leader results in less-aggressive behavior of the public firm. Because

the public (resp. private) firm's output is larger (resp. smaller) than the optimal output for domestic welfare, changes in both firms' outputs improve welfare. By contrast, the public firm produces less aggressively in the public leadership case than in the Cournot case because the leader, the public firm, has an incentive to increase the rival's output. This less-aggressive behavior of the leader results in more-aggressive behavior of the public firm. Thus, both public and private leadership improve welfare and the private firm's profits than the Cournot case does. In contrast to the domestic private firm case, we can show that the absolute value of the slope of the public firm's reaction curve is smaller than that of the private firm's in the foreign private firm case. Therefore, the leader's strategic incentive for manipulating the follower is larger (and thus, the welfare-improving and profit-enhancing effects are stronger) in the public leadership case than in the public leadership case.

The key in this intuitive explanation behind Proposition 3 (ii, iii, iv) is that foreign ownership in the private firm makes the public firm's optimal output less sensitive to the rival's output. We briefly explain this property. An increase in the private firm's output reduces the marginal value of the product for consumers. Thus, an increase in the private firm's output reduces the public firm's optimal output. This effect is common in both the domestic and foreign private firm cases. In addition, when the private firm is foreign, an increase in the private firm's output increases the domestic value for lowering the rival's price because a further reduction of the rival's price reduces the outflow to the foreign investors, $p_1 q_1$, more significantly. These two effects are canceled out in the foreign private firm case. Thus, foreign ownership in the private firm makes the public firm's optimal output less sensitive to the rival's output.

If we introduce partial foreign ownership in the private firm, we can show that Proposition 3 (i) holds regardless of the foreign ownership share, and Proposition 3(ii)–(iv) hold when the foreign ownership share is large. When the share of foreign ownership is small, results similar

to those presented in Section 3 hold. More detailed discussions for these results are available upon request for the authors.

5 Concluding Remarks

In this study, we investigated endogenous timing in a mixed duopoly. We introduced product differentiation using Dixit's (1979) model and found that regardless of the degree of product differentiation, two equilibria (private and public leadership) exist, and private leadership is better than public leadership for social welfare. These properties are seen in the previous studies that investigated homogeneous product markets. Thus, these results seem to suggest that product differentiation does not matter in this context. However, we showed that public leadership is risk-dominant and thus more robust than private leadership when the degree of product differentiation is high. This suggests that in highly differentiated markets, public firms are more likely to take a leadership position.

We believe that the recent revival of public financial institutions in Japan is well-explained by this result. Recently developed financial institutions, such as the Industrial Revitalization Corporation of Japan, provide highly differentiated services and lead the market. Our result also suggests that although welfare-maximizing public firms are more likely to lead in highly differentiated markets, from a welfare viewpoint, the private firm should be the market leader. In such a situation, the public firm's commitment to not be the leader can improve welfare. However, such a commitment reduces domestic welfare if the private firm is foreign. Public leadership equilibrium risk-dominates the private leadership one, and the former is better for social welfare. Thus, if the main competitors of the public firm are foreign, pubic leadership should not be restricted for domestic welfare.

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