Co-movement of commodity prices – results from dynamic time warping classification

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Abstract. Several factors are responsible for difficulties in describing the behaviour of commodity prices. Firstly, there are numerous different categories of commodities. Secondly, some categories overlap with other categories, while others indirectly compete in the market. Thirdly, although essentially commodity prices react to changes in economic conditions or exchange rates, to a large extent these prices depend on supply disturbances. However, in recent years commodity prices co-move, and researchers, beginning with Pindyck and Rotemberg (1990), have been trying to explain this phenomenon.

The objective of the article is to conduct the classification of the series of commodity prices in the pre-crisis and after-crisis periods. The results of such classification will reveal whether co-movement of commodity prices is the same in both periods. The analysis is based on monthly data from the period January 1990 to February 2014. All prices and price indices are published by the World Bank. The results obtained in dynamic time warping clustering reveal that co-movement of commodity prices is more evident in the pre-crisis period. There are only several paths which determine commodity prices.

Key words: Commodity prices, time series clustering, co-movement, dynamic time warping

JEL Classification: C38, Q02
AMS Classification: 91C20

1. Introduction

Several factors are responsible for difficulties in describing the behaviour of commodity prices. Firstly, there is a large number of different categories of commodities. Secondly, some categories overlap with other categories (for example, biofuel production and energy), while others indirectly compete in the market (for example, the development of one type of crops reduces the supply of other crops cultivated in a given area). Thirdly, although essentially commodity prices react to changes in economic conditions or exchange rates, to a large extent

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they depend on supply disturbances (such as droughts, floods, armed conflicts, etc.). In spite of such complex nature of the behaviour of commodities, the last decade noted their tendency to move together. Frankel [5] argues that the reason for such co-movement is the real interest rate, Akram [2] additionally investigates the role of the dollar exchange rates, while Svensson [22], Kilian [11]. Kilian [12] discusses the role of shifts in the global supply and demand. Krugman [13] explains the increase in food prices by biofuel production, as biofuel prices are correlated with oil prices. Numerous authors (e.g. Gilbert [6], Phillips and Yu [21], Irwin and Sanders [9], Pindyck and Rotemberg [20]) reason that co-movement is caused by speculations and the existence of price bubbles. From the methodological point of view, the assessment of price co-movement can be conducted with the use of several methods. One of the most common include cointegration (Papież and Śmiech [19], more recently replaced by the panel cointegration approach (Nazlioglu and Soytas [18]), threshold cointegration, (Natanelov et al. [17]) or the general equilibrium model (see e.g. Gohin and Chantert [7]). Other methods incorporate different statistical factor models, e.g FAVAR and PANIC (Byrne et al. [4]).

The objective of the paper is to conduct the classification of the series of commodity prices. The analysis is based on monthly data from three periods: before the global financial crisis, that is the period from 2001-01 to 2008-06, after the crisis, that is the period from 2009-01 to 2014-02, and the period covering the whole sample, that is from 2001-01 to 2014-02. The prices of 54 commodities taken into consideration in the analysis are listed by the World Bank in six categories i.e. energy, metals, beverages, food, raw materials and precious metals. Clustering was conducted with the use of dynamic time warping methods, which allows for the assessment of similarity between series shapes, that is a distance measure which identifies time-shifted patterns among series and seems to be appropriate for the analysis of co-movement of commodities. Eventually, three methods are used to classify time series: Ward's method, complete (hierarchical) and pam (division). The results of the classification are assessed by internal classification measuring the average silhouette width. The clustering conducted provides the answers to the following questions:

- Is moving together of commodity prices similar in intensity in the periods before and after the global financial crisis?
- How many clusters of commodity prices are there and how homogeneous are these clusters?
- Do commodities from the same category (e.g. energy commodities) belong to the same clusters, that is, do their prices behave in a similar manner?
To what extent do the clusters obtained in the study differ from the indices listed by the World Bank?

In comparison to the existing literature, our work differs in one important aspect, that is the methodology used. Related studies conducted so far assume linear correlations. The methodology used in this study allows us to stretch or compress two time series in order to draw comparisons, which offers a universal analysis of the nonlinear relationship and co-movement of commodity prices.

The rest of the paper is organised as follows. Section 2 describes methodology, empirical results are discussed in Section 3, and the conclusion is presented in the last section.

2. Methodology

Following the division suggested by Liao [16], three major time series clustering approaches include: raw data approaches, feature-based approaches and model-based approaches. The first ones deal with raw data in the time and frequency domain. They imply working with high dimensional space and are not effective if the raw data are highly noisy. In feature-based approaches certain features are extracted first to be clustered next. Kakizawa et al. 1998 characterize similarities of multivariate stationary time series in terms of their covariance or equivalently the spectral metrics. Model-based approaches assume that each time series is generated by a particular time series model. To obtain dissimilarity between series, the models are fitted and then discrepancies between them are looked for. Some authors suggest using some statistics of the errors associated with the estimates (Kumar and Patel [15]). The main disadvantage of the feature-based and model-based approaches is the obvious loss of information. What is more, the results of clustering in these methods depend on the feature selection and problems with parametric modelling. Alonso et al. [1] suggest another classification approach i.e. n clustering based on the models that generated the observations, but in respect of the forecasts at a specific future time.

One of the most widely used methods of assessing similarity in the raw data approach is Dynamic Time Warping (DTW) (Berndt and Clifford [3]). Given two time series, \( Q=q_1q_2\ldots q_n \) and \( R=r_1r_2\ldots r_m \), DTW aligns them in such a way as to minimize their difference. The metric establishes an \( n \times m \) cost matrix \( C \), which contains the distances (Euclidean) between two points \( q_i \) and \( r_j \). A warping path \( W=w_1w_2\ldots w_k \), where
maximize, is formed by a set of matrix components, respecting three rules: boundary condition, monotonicity condition and step size condition. Eventually, the path that minimizes the warping cost is considered as DTW distance:

$$\sum \min \left( d(r_t, q_i) \right)$$

The optimal warping path can be found using dynamic programming to evaluate the following expression:

$$\gamma(i, j) = \left\{ \begin{array}{ll} d(r_t, q_i) & \text{if cell } (i, j) \text{ is the current cell} \\ \gamma(i-1, j) + d(r_t, q_i) & \text{if cell } (i-1, j) \text{ is the current cell} \\ \gamma(i, j-1) + d(r_t, q_i) & \text{if cell } (i, j-1) \text{ is the current cell} \\ \gamma(i-1, j-1) + d(r_t, q_i) & \text{if cell } (i-1, j-1) \text{ is the current cell} \end{array} \right.$$  \hspace{1cm} (2)

where $d(r_t, q_i)$ is the distance found in the current cell, and $\gamma(i, j)$ is the cumulative distance of $d(r_t, q_i)$ and the minimum cumulative distances from the three nearby cells.

After determining the distance matrices, hierarchical or partitioning (crisp or fuzzy) clustering methods are used to find clusters. In order to evaluate an optimal number of clusters in the data and to assess which objects lie well within their cluster internal validity indices silhouette plot width (Kaufman and Rousseeuw [10]) is used. The silhouette is defined for each sample and takes values from -1 to 1. If the value is close to 1, the object is near the centre of the cluster it belongs to. Conversely, if the value is negative, the object is in an improper cluster. Finally, if the silhouette value is close to null, the sample is located near the frontier between its cluster and the nearest one. The higher the value of average silhouette width, the better the division of the series.

Adjusted Rand Index (ARI) can be next applied to compare the alternative classification results. The Adjusted Rand Index was proposed by Hubert and Arbie [8], who used the generalized hypergeometric distribution as the model of randomness. The index has expected value zero (for independent clustering) and maximum value of 1 (for identical clustering). The higher the adjusted Rand index, the greater agreement between the clustering results.

3. Data and empirical results

The data used in this study consist of monthly price indices from January 2001 to February 2014. All indices came from World Bank Commodity Price Data and are expressed in US dollars. Before the classification procedure, all price series are expressed as indices with their average values in 2007 equalling 1. The analysis is based on 54 series of variables, which are
assigned to World Bank classes.
The whole sample period is divided into two sub-periods: 2001:1-2008:6 and 2009:1-2014:2, thus the classification is based on the pre-crisis and post-crisis periods. The results are complemented by clustering series in the whole sample. The division is motivated by the disparate behaviour of commodity prices in these sub-periods.

DTW methods are used to classify time series. After obtaining dissimilarity metrics, Ward’s, complete (hierarchical group of methods) and pam (division) methods are used to find cluster.

![hclust (*, "ward")](image)

**Figure 1** The classification results in the first sub-period

The results of clustering for the period 2001-2008 are presented in Figure 1, and they yield three main clusters of time series (the average silhouette width is the biggest for three clusters in Ward’s method – see Table 1). The first cluster consists of 28 commodity prices including most energy commodities (their names in Fig. 1 begin with E), except for Gas US, metals (MM), (except for aluminium), and precious metals (PM). What is more, commodities belonging to the same category are close to each other, which means that their series paths are quite similar. The prices from the categories listed above are closest to one another, which means that their paths are the most similar. The second cluster includes the prices of Gas US

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2 The results of the complete method and pam method are available from the author upon request.
and Sugar EU, and it is hard to spot any connections between them. The last cluster consists of 24 commodity prices including most food, raw materials and beverages commodities. The silhouette plot indicates that most commodity prices have been assigned to proper clusters (Fig. 4). Only in three cases, the silhouette width is negative, which means that objects have been classified to improper clusters. Average silhouette width for this period equals 0.43.

The classification results in the second sub-period are presented in Figure 2. Although in this case the average silhouette width suggests the division into 2 clusters, we have opted for three cluster and, as a result, energy, metal, and precious metals commodities are in different groups. There are 14 commodity prices in the first cluster, including oil prices (except for WTI Oil, which is in the second cluster), some food and raw material commodity prices. There are 19 elements in the second cluster, including most food and raw material prices, gold, and tin. There are 23 prices in the third cluster, including most precious metals, metals and minerals, coal prices and the remaining food commodities. The silhouette plot (Fig.4) reveals that in the post-crisis period each cluster is less homogeneous then before. Average silhouette for clusters varies between 0.17 to 0.35. There are 5 commodity prices that seem to be classified to wrong clusters. Average silhouette width calculated for the post-crisis period is only 0.26, which suggests that the structure
obtained is artificial.

Finally, Figure 3 presents the results obtained for the whole sample. Here the average silhouette width suggests (see Table 1) the division into 4 groups (although the quality of division is rather poor). In the first cluster (17 elements) there are agricultural commodities (beverages, raw materials and other) and one industrial metal – aluminium. In the second cluster (18 elements) there are most industrial and precious metals, Australian Coal and some other agricultural commodities. The last two clusters are quite close to each other. The third consists of US Gas and Sugar UE, while the fourth contains most energy commodities and some food, especially oils (palms, soya, groundnut).

![hclust (*) , "ward")](image)

**Figure 3** The classification results in the whole sample period

<table>
<thead>
<tr>
<th>methods</th>
<th>Ward’s</th>
<th>complete</th>
<th>pam</th>
</tr>
</thead>
<tbody>
<tr>
<td>period\nr cluster</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2001-2014</td>
<td>0.235</td>
<td>0.287</td>
<td>0.322</td>
</tr>
<tr>
<td>2001-2008</td>
<td>0.374</td>
<td>0.427</td>
<td>0.301</td>
</tr>
<tr>
<td>2008-2014</td>
<td>0.371</td>
<td>0.257</td>
<td>0.221</td>
</tr>
</tbody>
</table>

**Table 1** The average silhouette width for 2, 3 and 4 clusters in different time period
In order to compare the results of classifications, the adjusted rand index are computed (see Table 2). The level of agreement of different classifications and the comparison of clusters and categories of different commodities (listed in the World Bank indices – symbol WB in table 2) are measured. As there are six different categories of commodities, the assumed division of the set of objects also consists of six clusters.

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WB</td>
<td>ward</td>
<td>compl.</td>
<td>WB</td>
<td>ward</td>
<td>compl.</td>
</tr>
<tr>
<td>WB</td>
<td>0.100</td>
<td>1.000</td>
<td>0.139</td>
<td>1.000</td>
<td>0.077</td>
<td>1.000</td>
</tr>
<tr>
<td>ward</td>
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<td>0.626</td>
<td>0.163</td>
<td>0.139</td>
<td>1.000</td>
<td>0.077</td>
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<tr>
<td>compl.</td>
<td>0.415</td>
<td>0.496</td>
<td>0.125</td>
<td>0.415</td>
<td>0.496</td>
<td>0.125</td>
</tr>
<tr>
<td>pam</td>
<td>0.415</td>
<td>0.960</td>
<td>0.125</td>
<td>0.496</td>
<td>0.688</td>
<td>0.114</td>
</tr>
</tbody>
</table>

Table 2 Adjusted Rand Index for different classification methods

The results obtained reveal that commodity classifications do not determine similar behaviour of commodity prices, which is clearly seen in low values of ARI for the first and the second (here the values are the highest) sub-periods as well as for the whole sample period. As far as various methods of obtaining clusters are concerned, they are relatively high (from 0.467 obtained for pair complete-ward in second sub-period to pair pam-complete in the first sub-period). Again, higher values of the similarity measure are obtained in the first
sub-period, which indicates that in this sub-period co-movement of indexes is more evident, and it is easily detected by different time series classification tools.

<table>
<thead>
<tr>
<th>Number of clusters</th>
<th>3 clusters</th>
<th>6 clusters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.419</td>
<td>0.730</td>
</tr>
</tbody>
</table>

Table 3 Adjusted Rand Index for the ward results and different periods

In order to compare the composition of clusters in different periods, ARI index is computed for 3 and 6 clusters. The results obtained reveal (see Table 3) that the composition of clusters in pre-crisis and post-crisis periods differ greatly (in the division into 3 cluster ARI equals 0.064 and into 6 clusters - 0.156). Relatively strong similarity of cluster composition in the pre-crisis sub-period and the whole period (ARI from 0.419 to 0.73 for 6 clusters) results from the fact that co - movement of all commodity prices in the pre-crisis period is stronger and more evident.

4. Conclusion and discussion

Dynamic time warping is used in the study to classify commodity price data in the pre-crisis and post-crisis periods. The results obtained reveal that co-movement of commodity prices is more evident in the pre-crisis period when the clusters are more homogeneous and consist of commodities from the same category (e.g. precious metals or energy commodities are located in the same cluster). Clusters obtained for the post-crisis period are less homogeneous. The internal classification measure demonstrates that the best division is obtained if only two or three clusters are considered in every period. Clusters obtained for the whole period sample indicate that there are only two patterns of behaviour of prices in the periods analysed (stronger in the first one). Comparing commodity categories with the results of clustering indicates that commodities which belong to one category do not always behave in the same way. It is especially evident in the second period, when certain energy commodities, metals or precious metals belong to different clusters. The results obtained might be of great importance to investors, as they demonstrate that at present co-movement of commodity prices is not as evident as it used to be. What is more, a well-diversified portfolio can consist of commodities from the same classes.
Concluding our study, it can be said that co-movement of commodity prices is recently not as evident as it used to be in the pre-crisis period. What might be the reason for such change in the investors’ behaviour? Of course, the lack of co-movement may result from the disappearance of its causes, which include, according to popular explanations, low interest rates and inflation expectations, shifts in global supply and demand, the risk resulting from geopolitical uncertainties and speculative bubbles. The first two seem still valid. In the post-crisis period real interest rates decreased. The crisis at first caused a dramatic demand slump, which gradually came back to the initial level. Due to difficulties with direct measuring, it is harder to refer to the remaining two causes of co-movement. It seems probable that the global financial crisis has lead to increasing geopolitical risks, so it is justified to assume that co-movement has been caused by speculations. Thus, it is most likely that the crisis has changed investors’ behaviour in the long run.

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**References**


