Timing of Earnings and Capital Structure

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Timing of Earnings and Capital Structure

Abstract. This paper shows that asymmetric information about the timing of earnings can affect corporate capital structure. It sheds some new light on two following questions: why may profitable firms be interested in issuing equity, and why does debt not necessarily signal a firm quality. These issues seem to be puzzling from the classical pecking-order theory or signalling theory point of view. The paper also contributes to the analysis of the link between debt-equity choice and subsequent performance after issue (short-term versus long-term) which has been widely discussed in empirical literature but did not get enough attention in theoretical research.

Keywords: Asymmetric information, Pecking-order theory, Signalling, Timing of earnings
1 Introduction

This paper builds on pecking order and signaling theories of capital structure. These theories directly relate to asymmetric information. The "Pecking-order theory" (POT) was put forth by Myers and Majluf (1984). According to this theory firms will use internal funds, if available, to finance profitable projects. In the case that internal funds are not available, they will issue debt. This creates the "pecking-order" where equity represents an inferior security. The evidence supports predictions of the pecking order theory such as the negative correlation between debt and profitability (Titman and Wessels, 1988; Rajan and Zingales, 1995; Fama and French, 2002; Frank and Goyal, 2007) and negative share price reaction on equity issue announcements (Masulis and Korwar, 1986; Antweiler and Frank, 2006). The evidence is mixed about whether firms always follow a pecking order hierarchy and whether the extent of asymmetric information reduces the incentive to issue equity (Shayam-Sunders and Myers (1999), Frank and Goyal (2003), Fama and French (2002), Lemmon and Zender (2008), Leary and Roberts (2010), Galpin (2004) and Chen and Zhao (2004)).

In the pecking order model, good quality firms have to use internal funds to avoid adverse selection problems and losing value. These firms cannot signal their quality by changing their capital structure. The signalling theory of capital structure offers models in which capital structure serves as a signal of private information (Ross, 1977; Leland and Pyle, 1978). Usually good quality firms increase leverage to signal quality. The empirical evidence supports such predictions of signaling theory as a negative market reaction on leverage-decreasing transactions and a positive reaction on leverage-increasing transactions excluding debt issues (Masulis, 1980; Antweiler and Frank, 2006;
Baker, Powell, and Veit, 2003). Second, the evidence does not support a positive market reaction to debt issues (Eckbo, 1986; Antweiler and Frank, 2006). The negative correlation between debt and profitability also contradicts signaling theory. Third, the evidence is mixed regarding the predictions of signaling theory about firms’ operating performance after issuing equity. Long-term underperformance of firms issuing equity compared to non-issuing firms (Jain and Kini, 1994; Loughran and Ritter, 1997) seems to be consistent with the spirit of signalling theory while better operating performance of firms issuing equity shortly after the issue compared to non-issuing firms does not support the theory. According to Jain and Kini (1994, Figure 1) the operating return on assets is higher for IPO firms in the first years after the issue and the operating cash flow on assets is higher in year "0" (immediately after issue). In Loughran and Ritter (1997) profit margins are higher in years 0 and +1, although there is different evidence about operating returns. In Mikkelson, Partch and Shah (1997, Table 3) IPO firms have higher performance in year 0.

The literature analyzing financing-investment games where firm insiders have private information usually deals with situations where firms differ in their qualities or overall intrinsic values. Typically, there are two types of firms: good (high value) and bad (low value). In the present paper, we analyze a signaling game where asymmetric information exists about the timing of earnings rather than the firm’s overall value. We argue that such a model can generate predictions which are not explained by existing theories.

Asymmetric information regarding the timing of earnings may take place because managers often have private information about: 1) the choice of inventory and depreciation methods; 2) estimation of pension liabilities; 3) capitalization of leases and marketing expenses; 4) recognition of sales not yet
shipped; and 5) delay in maintenance expenditures and delays in production. Financial literature considering the timing of earnings usually emphasizes the impact on firm value. The typical conclusion is, all other factors being equal, short-term cash flows lead to a higher firm value than long-term cash flows ("faster is better"). In the present paper, we focus on the link between the timing of earnings and the incentive to issue particular kinds of securities.

We build a two-stage investment-financing model where managers, representing initial shareholders, have a choice between debt (short- and long-term) and equity. We find that if information about the timing of earnings is asymmetric, a separating equilibrium may exist where a firm with late earnings issues debt and a firm with early earnings issues equity. This equilibrium implies that firms issuing equity have better operating performance at the moment of issue or soon after the issue. These firms also have lower operating performance in the long run. Leverage is negatively correlated with profitability because firms with higher profits in the first period issue equity in the first period. Firms with low rate of earnings growth issue equity and firms with high rate of earnings growth issue debt (Mohamed and Eldomiaty, 2008; and Chichti and Bougatef, 2010).

This paper shows that asymmetric information about the timing of earnings can affect corporate capital structure. It sheds some new light on two following questions: why may profitable firms be interested in issuing equity, and why does debt not necessarily signal a firm quality. These issues seem to be puzzling from the classical pecking-order theory or signalling theory point of view. The paper also contributes to the analysis of the link between debt-equity choice and subsequent performance after issue (short-term versus long-term) which has been widely discussed in empirical literature but did not get enough attention in theoretical research.
The rest of this paper is organized as follows. The next section provides a
description of the model, and analyzes optimal financing under asymmetric in-
formation about timing of earnings without managerial moral hazard. Section
3 analyzes the model with both asymmetric information and moral hazard.
Section 4 presents the model implications and empirical evidence. Section
5 discusses model extensions and robustness and the conclusion is drawn in
Section 6.

2 Model description and some preliminaries

Consider a firm with a two-stage investment project. The firm’s objective is to
maximize the wealth of initial shareholders (founders), whom we will call the
entrepreneur. In each stage $t = 1, 2$ an amount $b$ has to be invested. In each
stage, the project can either be successful or unsuccessful. If the former is the
case, the cash flow, $r_t$, equals 1 and if the latter is the case, the cash flow equals
0. In each period, the firm success depends on entrepreneur’s effort in that
period, and the firm’s intrinsic quality in that period. Regardless the level of
entrepreneur’s effort, some firms have better short-term earnings potential and
some firms have better long-term earnings potential. The entrepreneur’s effort
is $e_{jt}$, $e_{jt} \in \{0, 1\}$, where $j$ denotes the firm’s type, $j \in \{l, s\}$. If $e_{jt} = 0$, the
probability of success for either firm in period $t$ equals 0 and the entrepreneur
gets a private benefit equal to $c$.\footnote{This way of modelling the cost of effort is chosen for simplicity. Alternatively one can
assume that there is some cost for entrepreneur when providing a high effort. Qualitatively,
the results will be similar.} If $e_{j1} = 1$, the probability of success in
period 1 equals $\rho_{j1}$ and if $e_{j2} = 1$ the probability of success in period 2 equals
$\rho_{j2}$. Without loss of generality we assume $\rho_{j1} + \rho_{j2} = 1$. This implies that
if the entrepreneur delivers a high effort in both periods, the expected total cash flow over two periods is the same (equal to unity) for both firm types and they differ only in their timing of expected performances. Further we denote $\theta_j$ the probability of success in period 1 for type $j$ (the probability of success in period 2 is then $1 - \theta_j$). We assume $\theta_l < \theta_s$. It implies that $s$ (stands for "short-term") has better expected performance in period 1 and $l$ (stands for "long-term") has better expected performance in period 2.

We assume $b < 1/2$ with the $\theta$'s restricted to the interval $[b, 1 - b]$, which implies that, conditional on the entrepreneur's high effort, the investment has non-negative net-present value (NPV) in each period, i.e. the expected cash flow is at least equal to the amount of investment in period one ($b \leq \theta$) and in period two ($b \leq 1 - \theta$). Also we assume

$$2b > \max\{\theta, 1 - \theta\}$$

implying that the earnings from only one stage are not sufficient to cover the cost of investment in both stages. If the entrepreneur fails to obtain financing, his payoff equals 0. If financing is obtained and the entrepreneur delivers low effort in period $t$, the NPV of all benefits and costs in stage $t$ is $c - b$. Similarly if $e_{j1} = 1$, the NPV of stage 1 is $\theta_j - b$ and if $e_{j2} = 1$, the NPV of stage 2 is $1 - \theta_j - b$. We thus assume

$$c < \min\{\theta, 1 - \theta\}$$

This guarantees that high effort is socially optimal in both periods.

The entrepreneur's choice of effort and private benefits are non-observable and non-verifiable. Investors make decisions about providing financing for the firm taking into account their beliefs about the firm's type and their expectations about entrepreneur's level of effort. The firm's profit and it's capital
structure choice are observable and verifiable. There exists universal risk-neutrality in this economy. In addition, the competition among investors is perfect. This implies zero market profit and risk-neutral valuation of any security issued.

2.1 Financing strategies

The firm may use stage or up-front financing, and in both cases it can use equity or debt.

*Equity financing.* In the case of up-front equity financing (denote this strategy by $u$), the firm issues equity in the amount of $2b$ in the first period. The firm invests $b$ immediately and keeps $b$ for the second period. Alternatively, the firm may issue an amount of equity equal to $b$ (denote this strategy by $e$). Hence, in the second period, the firm has a choice between internal financing (the amount of internal financing is denoted by $f$) and external financing that is assumed to be debt financing (the amount of second-period debt equals $b - f$).\(^2\)

*Long-term debt* ($z$). The investment has two stages in our model so we assume that financing with long-term debt is up-front and the firm cannot distribute first-period cash to the shareholders (dividend covenants). This allows the firm to avoid the debt overhang problem in the second period when internal funds are not sufficient to cover the second-period investment and the firm has difficulty raising second-period financing in the presence of long-term claims. Alternative kinds of long-term debt financing are discussed in Section

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\(^2\)The introduction of possibility for other types of external financing in the second period will change nothing in the model’s main results. It will be shown that in the case of signalling equilibrium the value of any securities issued in the second period relies heavily on the firm’s expected performance in the second period that is the key for main results.
Short-term debt ($d$). In this case, the firm gets an amount $b$ from the market by issuing short-term debt. If $d$ was chosen and the firm’s profit is not sufficient to repay debt (profit equals 0) then there are two possibilities. First, the firm may be declared bankrupt. In this case, the shareholders get nothing and the creditors receive the liquidation value $V_L = \lambda(\mathit{Er}_2 - b)$, where $0 \leq \lambda \leq 1$. This equation shows that the liquidation value is proportional to the expected profit from the second stage of the project. For instance, if $\lambda$ is low, the cost of bankruptcy is high, and the liquidation value is low. Alternatively, the firm can continue to operate. This decision (to continue or to liquidate) is the result of a renegotiation between the entrepreneur and the creditors (Giammarino, 1989). The renegotiation is conducted in the following manner: the entrepreneur makes a "take-it-or-leave-it" offer to the creditors; the creditors may accept or reject the offer; if the creditors accept the continuation they get a fraction of the firm’s equity; if the offer is rejected the firm is liquidated. Note that if short-term debt is issued, it cannot be up-front: it makes no sense for the shareholders to keep cash in the presence of senior claims in the following period. Although there are some other ways of modelling different kinds of financing we believe that those suggested in the paper are very general and more importantly the results about pricing of securities (Lemma 1 below) are very intuitive. We discuss some other extensions in Section 6.

The sequence of events is illustrated in figure 1. We assume that the firm’s type is revealed to the entrepreneur in period 0 while financing, investment, and production take place in periods 1 and 2. The firm’s initial capital structure is 100% equity, with $n$ shares outstanding. Let $\alpha_t$ denote the proportion of equity owned by the entrepreneur in period $t$ (immediately after the issue of securities in period $t$, if it takes place). Clearly, $\alpha_0 = 1$. First-period outside
shareholders (strategies $e$ and $u$) discover the firm’s type immediately after acquiring shares. Since the stages are technologically dependant, if the entrepreneur is unable to obtain first-period financing there is no investment in either period and the entrepreneur’s utility equals $0$.³

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm’s type is revealed to the entrepreneur</td>
<td>Entrepreneur chooses $d, e, z$ or $u$</td>
<td>If external financing is needed, the entrepreneur issues second-period debt</td>
</tr>
<tr>
<td></td>
<td>Entrepreneur chooses $z_1$</td>
<td>The entrepreneur chooses $z_2$</td>
</tr>
<tr>
<td></td>
<td>Investment yields $r_1$</td>
<td>Investment yields $r_2$</td>
</tr>
<tr>
<td></td>
<td>If $d$ was played and $r_1 = 1$ the creditors are paid</td>
<td>It is distributed to the claimholders</td>
</tr>
<tr>
<td></td>
<td>Shareholders determines first-period dividends</td>
<td></td>
</tr>
<tr>
<td></td>
<td>If $d$ was played and $r_1 = 0$ the entrepreneur determines $\lambda$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>If the creditors reject the offer, the firm is liquidated and the creditors get $V_L$</td>
<td></td>
</tr>
</tbody>
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**Figure 1. The sequence of events.**

### 2.2 Symmetric information without moral hazard

This subsection provides: 1) some useful intuitions about benchmark pricing when the market knows the firm’s type and the entrepreneur’s effort is

³Throughout this article we use the concept of Perfect-Bayesian equilibria. In some cases a complete description of off-equilibrium investors beliefs about the firm type can be ommitted for brevity. They are avilable upon request.
verifiable; 2) formulas which will be utilized throughout this paper. Since information is symmetric, the choice of financing is arbitrary: any financial structure will lead to a first-best strategy for the entrepreneur. Since the optimal scenario occurs when the firm invests in both stages of the project and the entrepreneur’s effort is high in both periods, the firm’s total expected cash flow equals 1. The total amount of investment in the two periods equals $2b$, and given competitive capital markets this sum equals the expected payoff to investors. Thus, the entrepreneur’s expected payoff is $1 - 2b$ regardless of the financing strategy, as is usually the case under perfect market conditions. A bankruptcy never occurs in this case because the firm’s continuation is Pareto-efficient. In a renegotiation the creditors will receive the fraction $\lambda$ of firm’s equity: this makes them indifferent between liquidation and continuation.

Denote the face value of first-period debt by $D_1$, the second-period debt face value by $D_2$, the price of issued shares by $p$, the face value of long-term debt by $L$, and the value of the firm for the entrepreneur by $V$.

Lemma 1 shows that under symmetric information about the timing of earnings and the absence of moral hazard problem, the value of the firm for entrepreneur equals $1 - 2b$ (total expected earnings if the entrepreneur delivers a high level of effort minus the cost of investment). It also explains the pricing of securities under symmetric information.

**Lemma 1.** Under symmetric information without moral hazard:

\[
p = \frac{1 - 2b}{n} \tag{3}
\]

\[
D_1 = \frac{b - \lambda(1 - \theta)(1 - \theta - b)}{\theta} \tag{4}
\]

\[
D_2 = \frac{b - f}{1 - \theta} \tag{5}
\]

\[
L = \frac{2b}{1 - \theta + \theta^2} \text{ if } b \leq \frac{1 - \theta + \theta^2}{2} \tag{6}
\]
\[ L = \frac{2b - \theta^2 + (1 - \theta)^2}{\theta(1 - \theta)} \text{ if } b > \frac{1 - \theta + \theta^2}{2} \quad (7) \]

\[ V = 1 - 2b \quad (8) \]

**Proof.** See Appendix 1.

Note that \( p \) depends only on the firm’s total profit and not on its profit profile over time. This result is not surprising when using equity financing (strategy \( u \) or \( e \)) since total cash flow is the same for both types when the entrepreneurs effort is high in both periods. It follows from (4)-(7) that: 1) the face value of short-term debt is negatively linked to the expected performance in the first period, and positively related to both the amount of borrowing and the cost of bankruptcy, which decreases in \( \lambda \) as mentioned earlier; 2) if \( \lambda \) is sufficiently low, the face value of short-term debt depends more heavily on first-period performance than on second-period performance; 3) conversely, second-period short-term debt only depends on second-period performance; and 4) the value of long-term debt depends on both first-period and second-period performance. For example, if the cost of investment is relatively small (eq. (6)) and the firm is solvent when total earnings over both periods equals 1 or 2, the face value of long-term debt is positively linked to the amount of financing over both periods, and inversely related to the probability of solvency over both periods.

### 2.3 Asymmetric information without moral hazard

Consider the situation with asymmetric information about the firm’s type but without moral hazard, i.e. let us assume that the entrepreneur always delivers a high effort.
**Proposition 1.** If there is no moral hazard, strategy $u$ is a first-best pooling equilibrium.

*Proof.* Consider the situation where both firm types play $u$ and use internal financing for the second stage. Also, if a firm uses external financing for stage 2, the market believes that the firm is $s$. From (3) $\alpha_1 = \frac{n}{n+2b/p} = 1 - 2b$. Since both types use internal financing for stage 2, total dividends equal $r_1 + r_2$. Thus, the entrepreneur’s expected payoff is $\alpha_1 E(r_1 + r_2) = 1 - 2b$ since total expected cash flow for each firm is equal to 1. According to equation (8), this is equal to the first-best firm value for the entrepreneur. If $l$ deviates and borrows in the second period it suffers from the fact that it will be perceived by the market as type $s$. Type $s$ is indifferent between internal financing and borrowing in the second period because the interest rate would correspond to type $s$ according to market beliefs described above. This situation constitutes an equilibrium. *End Proof.*

The idea behind this proposition was discussed in the introduction. According to (3), in an environment without moral hazard, the share price in the first period is the same for all firm types: this completely eliminates the problem of asymmetric information under up-front financing. $l$ can finance the second stage of the project internally, thereby avoiding the lemon problem. If $s$ attempts to borrow in the second period the market will correctly realize the firm’s type, effectively eliminating it’s ability to earn informational rents in this period.\(^4\)

\(^4\)For more information regarding this case see Miglo and Zenkevich (2006). Up-front financing is not allowed in that paper.
3 Asymmetric information with moral hazard

In this case, the valuation of securities issued by the firm is based on the market’s belief about the entrepreneur’s effort. If investors believe the entrepreneur’s effort will be low, they will either reduce the share price, increase the interest rate charged, or refuse to finance the project. The investors’ beliefs are based on their calculation of the entrepreneur’s incentives. Therefore, the choice of financing should send a credible signal to the market about the entrepreneur’s effort level. However, some agency costs will arise under all types of financing. Under equity financing, agency costs arise because the entrepreneur’s fraction of equity is reduced, decreasing the incentive to provide high effort. Under short-term debt financing, agency costs arise when default occurs in the first period and creditors obtain a high fraction of equity, reducing the incentive for entrepreneurial effort in the second period. Agency costs may also arise if the face value of debt is excessively high, leading the entrepreneur to provide low first-period effort. Similarly, under long-term debt financing the problem may appear when first-period earnings are low, and the entrepreneurs payoff for high effort in the second period is diluted by the creditors claims.

In order to find an equilibrium in the model with both asymmetric information and moral hazard we will first establish some preliminary results. We know from previous section that \( u \) is optimal financing under asymmetric information without moral hazard. With moral hazard the entrepreneur’s effort depends on private benefits from low effort. If these benefits are small, the low effort will be chosen and vice versa. The following lemma shows the conditions under which \( u \) is the first-best financing (i. e. the entrepreneur’s effort is high in both periods) when information about the firm’s type is symmetric.

**Lemma 2.** When information about firm’s type is symmetric, the entre-
preneur’s effort is high in both periods under strategy u if and only if

\[ \theta \leq 1/2 \text{ and } (1 - 2b)\theta \geq c \quad (9) \]

or

\[ \theta > 1/2 \text{ and } (1 - 2b)(1 - \theta) \geq c \quad (10) \]

Proof. See Appendix 2.

Proposition 1 and Lemma 2 imply that when both asymmetric information and moral hazard are presented, a first-best equilibrium can exist if the cost of low effort is sufficiently high, or the private benefits from low effort are small. Comparing conditions (9) and (10) for each type leads us to the following proposition:

**Proposition 2.** Strategy u is a first-best pooling equilibrium if and only if: 1) \( \theta_l > 1/2 \) and \( c \leq (1 - 2b)(1 - \theta_s) \); 2) \( \theta_l \leq 1/2 < \theta_s \) and \( c \leq \min\{(1 - 2b)(1 - \theta_s), (1 - 2b)\theta_l\} \); and 3) \( \theta_s \leq 1/2 \) and \( c \leq (1 - 2b)\theta_l \).

Proof. See Appendix 3.

Next we consider situations where the equilibrium described in Proposition 2 does not exist. These are:

\[ \theta_l > 1/2 \text{ and } c > (1 - 2b)(1 - \theta_s) \quad (11) \]

\[ \theta_l < 1/2 < \theta_s \text{ and } c > \min\{(1 - 2b)(1 - \theta_s), (1 - 2b)\theta_l\} \quad (12) \]

\[ \theta_s < 1/2 \text{ and } c > (1 - 2b)\theta_l \quad (13) \]

### 3.1 Efficient separating equilibria

An equilibrium is efficient if financing is obtained for both stages, the entrepreneur’s effort is high in both periods (respectively, the incentive constraints
should hold for both firm types in each period) and his expected payoff equals $1 - 2b$.

The general intuition concerning the role of asymmetric information in our model is as follows. The prices of securities can be affected by the “lemon” effect in both periods.\(^5\) Intuitively, \(l\) would seem to have an informational advantage in the first period: lower profits in this period mean that this type of firm can capitalize on the adverse selection problem. On the other hand, in the second period, the informational advantage passes to \(s\). We show that \(l\) and \(s\) face very different incentives regarding financial decisions. The point is that the price of first-period equity is type-independent because both types face the same total profit over the two periods. As a result, if \(l\) were to issue equity in the first period, they would always be mimicked by \(s\). \(s\) stands to gain in the second period by being perceived as growing and, therefore, as expecting high profits in the second period. The implication is that \(l\) is at a disadvantage for issuing equity in the first period. This is the main engine driving the results of this article.

To signal its type, \(l\) can issue short-term debt. In particular, if the cost of bankruptcy is high enough (or when non-recourse debt issued), first-period interest rates will be relatively high compared to second period rates, since \(l\) is considered “bad” in the first period and “good” in the second. Given such an interest rate profile, we show that if \(s\) issues short-term debt, it will be beneficial to creditors, but not to the firm. This is because creditors benefit in the first period due to high interest rates and the fact that \(s\) does well in that period.

The analysis below develops the above ideas but first we argue that \(u\) is

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\(^5\)We use the term "lemon" to describe a situation where private information leads to the underpricing of a "good" type. See Akerlof (1970) for a classical example.
never played in an efficient signaling equilibrium. Recall that in an environment without moral hazard, upfront equity is a good strategy because it can mitigate problems related to asymmetric information about timing of earnings (Proposition 1). The result holds when there is moral hazard but its extent is relatively small (Proposition 2). If pooling with \( u \) is not an equilibrium described in Proposition 2, the private benefits from low effort are relatively high for at least one firm type (conditions (11)-(13)). Even if for one firm type private benefits are low and he can use \( u \), these benefits will be high for the other firm type (which does not play \( u \) in equilibrium). Therefore, that type will mimic the type playing \( u \). Since the first-period share price is always \( \frac{1-2b}{n} \) (type-independent), it will not suffer from the adverse selection problem, but will gain by providing low effort. We thus have the following result.

**Proposition 3.** \( u \) is never played in an efficient signalling equilibrium.

**Proof.** See Appendix 4.

Proposition 3 is consistent with Neher (1999). This paper argues that upfront financing is less important (and respectively stage financing is more important) when the entrepreneurial moral hazard problem becomes more important. The author also discusses empirical evidence consistent with this prediction.

Let us analyze other strategies. Let \( V_x(\theta, \tilde{\theta}) \) be the entrepreneur’s final payoff if strategy \( x \) is played, the firm is of type \( \theta \) but is perceived as type \( \tilde{\theta} \), given a high effort in both periods.

**Lemma 3.** \( V_c(\theta_s, \theta_l) > 1 - 2b \) and \( V_c(\theta_l, \theta_s) < 1 - 2b \).

**Proof.** See Appendix 5.

The idea behind Lemma 3 is that when effort is high in both periods, the first-period share price is type-independent as follows from (3). Because the first-best share price in period one is the same for all types, \( s \) benefits from its
informational advantage in the second period (when it is really a “lemon”). This implies that an efficient separating equilibrium where \( l \) plays \( e \) does not exist. The analysis below develops this idea.

**Proposition 4.** An efficient separating equilibrium where \( l \) plays \( e \) does not exist.

*Proof.* Suppose the opposite is true: that such an equilibrium exists. By Lemma 3, the payoff to \( s \) if it mimics \( l \) is greater than \( 1 - 2b \) because \( \theta_s > \theta_l \).

*End proof.*

Now consider strategy \( z \). The difficulty involved in \( l \) separating itself by playing \( z \) is similar to the case of strategy \( e \). Since the value of long-term debt depends on firm’s performance in both periods and the values of both types (under high effort in both periods) are equal, then intuitively, \( l \) does not have an advantage when issuing long-term debt. This leads to the following proposition.

**Proposition 5.** An efficient separating equilibrium where \( l \) plays \( z \) does not exist.

*Proof.* See Appendix 6.

Let us turn to strategy \( d \).

**Proposition 6.** The set of parameters for which \( V_d(\theta_s, \theta_l) < 1 - 2b \) is not empty.

*Proof.* The following is an example proving the proposition: \( \lambda = 0 \) and \( \theta_l < 1/2 \). First note that when \( s \) mimics \( l \) it will never use internal financing in the second period because the second-period interest rates for type \( l \) are advantageous given the high performance of this type in the second period. Thus,
\[ V_d(\theta_s, \theta_l) = \theta_s(1 - \frac{b}{\theta_l}) + (1 - \theta_s)(1 - \frac{b}{1 - \theta_l}) \]  \hspace{1cm} (14) \\

This means that the probability that \( r_1 = 1 \) equals \( \theta_s \). Since the firm is perceived as type \( l \), the debt face value is \( \frac{b}{\theta_l} \) (see (4)). Hence, the entrepreneur’s first-period expected earnings are \( \theta_s(1 - \frac{b}{\theta_l}) \). The reasoning is similar in period 2.

From (14) we have:

\[ V_d(\theta_s, \theta_l) < 1 - 2b \Leftrightarrow \frac{\theta_s}{\theta_l} + \frac{1 - \theta_s}{1 - \theta_l} > 2 \Leftrightarrow \]  \hspace{1cm} (15) \\

\[ \Phi(\theta_l) \equiv \theta_s(1 - \theta_l) + (1 - \theta_s)\theta_l - 2\theta_l(1 - \theta_l) > 0, \]  \hspace{1cm} (16)

where \( \Phi \) is convex with roots \( \theta_l = 1/2 \) and \( \theta_l = \theta_s \). Therefore \( V_d(\theta_s, \theta_l) < 1/2 \) if \( \theta_l < 1/2 \). [Note that since (16) is strictly positive, Proposition 6 may hold when \( \lambda > 0 \) (\( \lambda \) is sufficiently small) by continuity\(^6\)] End proof.

Intuitively, by analogy with the perfect information case (see the remarks in Section 3 about the value of short-term debt for the case when \( \lambda \) is sufficiently low), a downward sloping interest rate profile (\( \theta_l < 1/2 \)) is suitable for performance-improving firms and not for firms with a lower rate of profit growth (\( \theta_s > \theta_l \)), which are better off with upward sloping interest rate profile.

\[ \text{Corollary 1. The only efficient separating equilibrium which may exist, where both debt and equity are issued, is one where } l \text{ plays } d \text{ and } s \text{ plays } e. \]

\[ \text{Proof. It follows from Propositions 3-6 that the only candidate for an efficient separating equilibrium is one where } l \text{ plays } d \text{ and } s \text{ plays } e. \]

\(^6\)Also note that inequality (14) is one of the most interesting and technically sound results of our research.
example is the situation where \( \lambda \) is sufficiently small or equal \( 0 \), \( \theta_l < 1/2 < \theta_s \) and \( c > \min\{(1-2b)(1-\theta_s), (1-2b)\theta_l\} \). First-best pooling equilibrium with \( u \) does not exist as follows from (12), \( l \) does not mimick \( s \) by Lemma 3 and \( s \) does not mimick \( l \) as follows from the proof of Proposition 6. \textit{End proof.}

### 3.2 Other separating equilibria with debt and equity

From the previous subsection we see that firms issuing equity (\( s \)) have a lower rate of profit growth than firms issuing debt (\( l \)). This is of interest when we explain the long-term afterissuing underperformance of firms issuing equity. The problem, with our analysis thus far, is that we have not considered an inefficient separating equilibria. The general intuition regarding this equilibria is as follows. If both types invest only in the first stage of the project and provide high effort in that period (the issued claims will obviously depend only on the first-period expected performance), \( l \) will mimic \( s \) (recall that \( s \) is the low profit type in this period). A situation where a firm only invests in the second stage is impossible because the stages are technologically dependant. Thus, at least one type will invest in both stages, provide high effort in the first period, and provide high effort in the second period when \( r_1 = 1 \) (and possibly when \( r_1 = 0 \)). Otherwise, the investors will be unable to provide financing for both periods because cash from only one period is insufficient to cover the total investment, by (1). In equilibrium, \( l \) is unable to use strategy \( e \) and invest in both stages since it will be mimicked by \( s \). This leads to the following proposition:

\textbf{Proposition 7.} \textit{The only inefficient separating equilibria which may exist, where both debt and equity are issued, are the following: 1) \( l \) plays \( d \) and invests in both stages (high effort in both stages) and \( s \) plays \( e \), invests in}
the first stage, and provides high effort in that stage; 2) $l$ plays $z$ and invests in both stages (high effort in the first stage and also in the second stage when $r_1 = 1$) and $s$ plays $e$ and invests only in the first stage; and 3) $s$ plays $z$ and invests in both stages (high effort in the first stage, and also in the second stage when $r_1 = 1$) and $l$ plays $e$ and only invests in the first stage.

Proof. See Appendix 7.

One can see from Proposition 7 that in any equilibrium a firm issuing equity has lower performance in the second period relative to a firm issuing debt.

4 Implications and empirical evidence

This paper sheds some new light on discussions related to the following issues.

(i) Firms issuing equity underperform in the long-run as compared to non-issuing firms. First, let us summarize the analysis of the model. When the cost of entrepreneurial effort is relatively low, up-front financing is the optimal strategy. Otherwise, a separating equilibrium may exist where firms issuing equity have higher performance in the first period and lower performance in the second period than firms issuing debt. This equilibrium means that firms issuing equity underperform in the long-run as compared to non-issuing firms (measured as a decline of profit, profit to assets ratio or profit per share). This is implied by Corollary 1 and Proposition 6: only the types with higher first-period expected profit and lower second period profit issue equity in the first period. This conclusion is confirmed by empirical findings (see for example Jain and Kini (1994), Pagano, Panetta and Zingales (1995), Cai and Wei (1997), Mikkelson, Partch and Shah (1997)) or Purnanandam and Swaminathan (2004) for IPO firms and Loughran and Ritter (1997) for SEO (seasoned equity issues) firms. Also some recent papers found that firms with
low rate of earnings growth issue equity and firms with high rate of earnings
growth issue debt (Mohamed and Eldomiaty, 2008; and Chichti and Bougatef,
2010).

(ii) The performance of firms issuing equity exceeds the performance of the
non-issuing firms at the time of issue. The paper provides a new theoretical
result that have hitherto not been tested. The model predicts that the per-
formance of firms issuing equity exceeds the performance of the non-issuing
firms at the time of issue (or in the near future after issue). In the equilibrium
described in Corollary 1 and in 2 of the 3 equilibria from Proposition 7 the
following holds: the absolute performance of firms issuing equity exceeds the
performance of non-issuing firms at the time of issue, or in the near future
after issue. While this point was not the main focus of the empirical research
cited above, some authors did stress the point that issuing firms outperform
non-issuing firms just before issue, and others documented that issuing firms
outperform non-issuing firms in the year of issue and in the first year after
issue (Mikkelson, Partch and Shah (1997) and Jain and Kini (1994)).

(iii) A new motive for issuing equity. This paper suggests a new motive for
issuing equity (Corollary 1 and Proposition 7) that has not been explored in
existing literature. When the firm knows that it will be high-profitable in the
near future and low-profitable in the long-term, the entrepreneur may want to
issue equity. An alternative interpretation is that equity should be issued when
asymmetric information exists regarding the timing of earnings. Possible tests
of these predictions will be based on identifying firms and industries with high
degree of asymmetric information regarding the timing of earnings. One can
use the spread in analysts’ valuations of firms’ shares as a proxy for the extent
of asymmetric information regarding the firms’ total values and the spread in
the forecasts of future earnings (long-term spread versus short-term spread)
as a proxy for asymmetric information about future rates of earnings growth. Also firms manipulating earnings prior to issue (as in Theo et al, 1998) can be seen as ones with high degree of asymmetric information about timing of earnings since earnings management can often be seen as a redistribution of earnings between periods rather than accounting fraud (Degeorge, Patel and Zeckhauser (1999), Miglo (2010)).

5 The model extensions and robustness

*Mixed financing.* Allowing mixed financing provides little usefulness for the analysis of operating performance of firms issuing equity versus that of non-issuing firms. The reason is that most empirical literature on this topic does not differentiate issuers according to fractions of equity in capital structure. Even a marginally small issue of shares puts a firm into the category of issuing firms. Thus it will be hard to interpret the equilibrium in terms of existing empirical evidence. However, allowing for mixed financing is important with regard to the conclusions about the negative correlation between debt and profitability and more interestingly about the conditions of existence of this phenomena which constitute an addition to the literature on this phenomena.

The main results of the model are robust when the possibility of mixed financing is allowed. The main insight that firms with an increasing profit profile are at a disadvantage when issuing equity while stagnating firms can "hide" their low second-period performance by issuing equity (the price of which is type irrelevant), holds under mixed financing. We can show that if an equilibrium exists where firms with higher second-period performance issue more equity, then there also exists a separating equilibrium where these firms issue less equity, but not vice versa. Thus, the latter equilibrium prevails (see...
analogous reasoning in Goswami, Noe and Rebello, 1995).

**Long-term debt.** Alternative ways of modelling long-term debt do not affect the paper main result. Suppose, for example, that in the first period a firm may issue long-term debt \( D_z \) which should be repaid in the second period. Consider symmetric information case and suppose that second-period debt is senior (or both debts have the same priority). Since first-period profit is distributed in total as dividend, the only source of payoff for both first-period and second-period creditors is the firm’s second-period profit (only when it equals to 1). It implies,

\[
D_z + D^2_z \leq 1
\]  

(17)

where \( D^2_z \) denote the second-period debt face value. Also we have (analogously to (5))

\[
D^2_z = b / (1 - \theta)
\]  

(18)

Expected payoff of first-period creditors equals \( D_z (1 - \theta) \). Since it should be equal to \( b \) we have

\[
D_z = b / (1 - \theta)
\]  

(19)

However, when \( \theta \geq 1 - 2b \) (18) and (19) contradict (17). In other words the firm may not be able to obtain second-period financing at all. Other scenarios (different priority of debts, for example) also imply some conditions which cannot be feasible. This means that long-term debt may not dominate equity as it automatically happens, for example, in Goswami, Noe and Rebello (1995).

**Different profit distribution functions.** Now we briefly comment on the model’s robustness with respect to possible generalizations of projects’ profit distribution functions.\(^7\) For example, one can consider situation where firm profits are ordered by first-order dominance. One can show that the basic

\(^7\)Recall that we use the Bernoulli function in the model.
results such as propositions 1, 3, 4 and 5 and Lemma 3 hold. This provides an idea why growing types avoid playing equity with subsequent conclusions about operating performance of firms issuing equity. In addition intuitively, if interest rate profile is downward sloping (growing economy) stagnating firms will prefer equity and otherwise they will prefer debt. However, since, first of all, $V_d$ becomes non-linear, the determination of exact conditions for the existence of different types of equilibrium, especially for the case of multiple type economy become very difficult technically. Nevertheless, numerical calculations for some classes of distribution functions confirm the results found in this paper.

Firms differ in their overall values. In the model, different types of firm have the same overall values and differ only in their timing of earnings. An interesting extensions is related to the situation where firms differ not only in their timing of earnings but also in their total values. To illustrate the main idea and to show that the main result of the article may still hold, suppose that the firms are of two types, type $s$ and type $l$, with respective probabilities of success $\theta_{st}$ and $\theta_{lt}$ in stage $t$. Suppose a type $s$ issues equity for each stage of investments and distributes period 1 earnings as dividends. In stage 2, investors require a fraction of equity $\alpha_2$ such that: $\alpha_2 \theta_{s2} = b$. In stage 1, investors require a fraction $\alpha_1$ of equity such that: $\alpha_1 \theta_{s1} + (1 - \alpha_2)\theta_{s2} = b$. Now consider the payoff of shareholders of $l$ in case decides to mimic $s$. This equals $(1 - \alpha_1)\theta_{l1} + (1 - \alpha_1)(1 - \alpha_2)\theta_{l2}$. If a signaling equilibrium exists, the shareholders’ payoff for type $l$ is $\theta_{l1} + \theta_{l2} - 2b$ (the true value of $l$). Thus, a separating equilibrium exists if $(1 - \alpha_1)\theta_{l1} + (1 - \alpha_1)(1 - \alpha_2)\theta_{l2} \leq \theta_{l1} + \theta_{l2} - 2b$. This can be simplified to:

$$\frac{\theta_{s1} + \theta_{s2} - 2b}{\theta_{l1} + \theta_{l2} - 2b} < \frac{\theta_{s1} + \theta_{s2} - b}{\theta_{l1} + \theta_{l2}(1 - b/\theta_{s2})}$$

(20)

If the extent of asymmetric information regarding firms’ total values is
sufficiently small and if \( \theta_{s1} > \theta_{l1} \) and \( \theta_{s2} < \theta_{l2} \), then (20) holds. In an extreme case, for example, when \( \theta_{s1} + \theta_{s2} = \theta_{l1} + \theta_{l2} \), Equation (20) becomes \( \theta_{s2} < \theta_{l2} \). Here, the value of shares in period depends on the firm’s total value and not on the firm’s performance in a particular period, while the value of shares in period 2 depends on period 2’s performance. The firm with low overall value can benefit from overvaluation in period 1 but can have a loss from period 2 undervaluation. When asymmetric information about a firm’s overall value is relatively small and information about the timing of earnings is high, the latter effect can dominate.

The separating equilibrium described above implies that firms issuing equity have better operating performance at the moment of issue or soon after the issue as in the basic model. These firms also have lower operating performance in the long run. Leverage is negatively correlated with profitability because firms with higher profits in the first period issue equity.

Multiple types. For simplicity the basic model had two types of firms. Our analysis shows that the results may hold even in a multiple types environment.\(^8\) Let the distribution of types be exponential truncated: \( f(\theta) = K \exp(-\gamma \theta) \),\(^9\) where \( \theta \) is the expected profit in the first period. Let \( y \) is the average first-period profit in the economy. High \( y \) corresponds to a stagnating economy (low second period profit) and a low \( y \) indicates a growing economy. Theoretically possible equilibria are: semi-separating, pooling with debt or pooling with equity. If the equilibrium is semi-separating, firms with \( \theta < \theta^* \) issue debt and firms with \( \theta > \theta^* \) issue equity. This equilibrium is consistent with our results since it implies that the average first-period performance of

\(^{8}\)It is wellknown that calculations become siginificantly more complicated in that case.

\(^{9}\)Where \( K = \frac{\gamma}{\gamma - 1} \). \( K \) is a constant that allows us to keep the cumulative probability equal to 1.
firms issuing equity is higher that that of non-issuing firms. The results of numerical analysis are presented in the Table below.

Table 1. Equilibrium with multiple types.

The density of types is \( f(\theta) = K \exp(-\gamma \theta) \) where \( \theta \) is the expected profit in the first period. \( y \) is the average first-period profit in the economy. High \( y \) corresponds to a stagnating economy (low second period profit) and a low \( y \) indicates a growing economy. Theoretically possible equilibria are: semi-separating, pooling with debt or pooling with equity. If several equilibriums exist, the one with minimal mispricing is chosen. If the equilibrium is semi-separating, firms with \( \theta < \theta^* \) issue debt and firms with \( \theta > \theta^* \) issue equity.

\[ a) \quad b = 0.4 \]

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<th>( \gamma )</th>
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<th>0</th>
<th>2</th>
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<th>6</th>
<th>8</th>
<th>10</th>
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<td>0.4868</td>
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<td>( (1 - y)/y, \text{ economy rate of growth} )</td>
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<td>1.0542</td>
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<td>1.2408</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta^* )</td>
<td>-</td>
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<td>0.5594</td>
<td>0.5396</td>
<td>0.5297</td>
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<tr>
<td>( 1 - F(\theta^*), \text{ proportion of firms, issuing equity} )</td>
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<td>0.0005</td>
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</table>

\[ b) \quad b = 0.25 \]
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<tr>
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### 6 Conclusions

This paper examines optimal financing in a dynamic setting (two-stage investment process) under asymmetric information. The analysis is based on the idea that firms have private information about their profit profiles over time. It is shown that separating equilibria may exist where firms issuing equity have higher performance in the first period and lower performance in the second period than firms issuing debt. The paper contributes to POT by explaining why firms can issue equity as a signal. It contributes to signalling theory by explaining why debt does not necessarily signal a firm quality. The paper suggests an explanation for why firms issuing equity underperform (operating underperformance) non-issuing firms in the long run. It also provides new insights on important capital structure phenomena, such as the negative correlation between debt and profitability. To our knowledge, this is the first attempt to simultaneously explain all of
these phenomena. Finally, this paper provides some new theoretical results which have not yet been tested. These are: 1) the decision about the issuance of standard securities, such as debt and equity, can be affected by the private information about timing of earnings; 2) firms issuing equity have higher performance shortly after the issue; and 3) up-front financing is less likely (stage financing is more likely) when moral hazard problem is important.

Appendix 1

Equity financing. Denote the total amount of funds raised in the first period by $b_1, b_1 \in \{b, 2b\}$, the number of shares issued by $\Delta n$, the dividend per share in period $t$ by $w_t$, total dividend in period $t$ by $W_t$, and cash retained in period $t$ (analogous to being invested in zero coupon bonds) by $m_t$. The equilibrium relationships are:

1) first-period total investment equals first-period total financing:

$$b_1 = p \Delta n$$

$$b_1 = b + m_1$$

2) market valuation of shares (share price equals expected dividends per share):

$$p = E(w_1 + w_2)$$

3) total dividend in period $t$:

$$W_1 = w_1(n + \Delta n)$$

$$W_2 = w_2(n + \Delta n)$$

4) earnings in period $t$:

$$r_1 + m_1 = W_1 + f + m_2$$
\[
\max\{m_2 + r_2 - D_2, 0\} = W_2 \tag{27}
\]

First-period earnings (the sum of cash or investment in zero-coupon bonds in period 1 and cash flow from the project) can be used to pay out dividends, finance the second stage of the project, or invest in zero-coupon bonds in period 2. On the other hand, second-period earnings are distributed, in total, to the shareholders.

5) Market valuation of second-period debt (recall that the firm raises \(b - f\) externally in the second period):\(^{10}\)

\[
b - f = E \min\{m_2 + r_2, D_2\} \tag{28}
\]

Using equations (22), (24), (25), (26), (27), (28) and the identity

\[
\min(X, Y) + \max(0, X - Y) = X \tag{29}
\]

with \(X = m_2 + r_2\) and \(Y = D_2\), we can transform (23) into:

\[
p = \frac{Er_1 + Er_2 - 2b + b_1}{n + \Delta n}
\]

This equation together with (21) produces

\[
p = 1 - \frac{2b}{n} \tag{30}
\]

\(^{10}\) Further, we assume for brevity that \(D_2 > 0\) which implies \(m_2 = 0\). Note that no results are affected by this assumption. To see this, suppose that \(D_2 > 0\) and \(0 < m_2 < b - f\). Then \(D_2 = \frac{b - f - \theta m_2}{1 - \theta}\). The entrepreneur’s second-period expected payoff is \(V_2 = (1 - \theta)(1 + m_2 - \frac{b - f - \theta m_2}{1 - \theta}) = 1 - \theta - b + f + m_2\). Now suppose that the entrepreneur invests \(m_2\) in the second stage of the project. Then \(D_2' = \frac{b - f - m_2}{1 - \theta}\) and the entrepreneur’s expected payoff equals \(V_2' = (1 - \theta)(1 - \frac{b - f - m_2}{1 - \theta}) = 1 - \theta - b + f + m_2 = V_2\). The idea is analogous for the case \(m_2 > b - f\).
For second-period debt, we get from (28) that:

\[ D_2 = \frac{b - f}{1 - \theta} \quad (31) \]

*Long-term debt.* \( L \) is determined by the following equation:

\[ 2b = E \min\{ m_2 + r_2, L \} \quad (32) \]

Recall that long-term debt is issued with dividend covenants. Therefore, the firm uses its initial resources to finance the second stage, and must invest first-period earnings in zero-coupon bonds. We can thus rewrite (32) as:

\[ 2b = E \min\{ r, L \} \]

where \( r \) denotes the firm’s total cash flow over the two periods. Note that \( r \) equals 2 with probability \( \theta(1 - \theta) \), equals 1 with probability \( \theta^2 + (1 - \theta)^2 \) and 0 otherwise. Two cases are possible. If \( L \leq 1 \) the probability that the creditors get the face value equals the probability that \( r_1 + r_2 \geq 1 \). Otherwise they get nothing. Thus:

\[ 2b = (1 - \theta + \theta^2) L \quad (33) \]

If \( L > 1 \), we have

\[ 2b = \theta(1 - \theta) L + \theta^2 + (1 - \theta)^2 \quad (34) \]

*Short-term debt.* Denote the face value of first-period debt by \( D_1 \). We have the following relationship:

\[ b = E \min\{ r_1, D_1 \} + \Pr(r_1 < D_1) \lambda EW_2 \quad (35) \]

Equation (35) takes into account the fact that creditors receive the fraction \( \lambda \) of equity when first-period cash flow is insufficient to pay short-term debt. This equation can be rewritten as
\[ b = \theta D_1 - \lambda (1 - \theta) E(W_2 \mid r_1 < D_1) \]  

(36)

If \( r_1 < D_1 \) (default), \( f = m_2 = 0 \). Using (27), (28) and (29) with \( X = r_2 \) and \( Y = D_2 \) we get:

\[ E(W_2 \mid r_1 < D_1) = 1 - \theta - b \]  

(37)

(36) and (37) imply

\[ D_1 = \frac{b - \lambda (1 - \theta)(1 - \theta - b)}{\theta} \]  

(38)

Finally, note that regardless of how the investment is financed, the value of the firm for the entrepreneur is:

\[ V = 1 - 2b \]  

(39)

For example, if equity is issued the entrepreneur’s expected payoff equals:

\[ \alpha_1 E(m_1 + r_1 - f - m_2 + \max(0, m_2 + r_2 - D_2)) \]  

(40)

where

\[ \alpha_1 = \frac{n}{n + \Delta n} \]  

(41)

From (21), (30) and (41) we have

\[ \alpha_1 = \frac{1 - 2b}{1 - 2b + b_1} \]  

(42)

Taking into account (22), (42) and (29) with \( X = m_2 + r_2 \) and \( Y = D_2 \) we get that (40) equals \( 1 - 2b \).

**Appendix 2**

The second-period incentive constraint (IC) is

\[ \alpha_2 E \max\{m_2 + r_2 - D, 0\} \geq c + \alpha_2 E\{m_2 - D, 0\} \]  

(43)
where $D$ denotes the total face value of debt in the second period. The left side of (43) shows the entrepreneur’s expected payoff if $e_2 = 1$ and the right side shows his payoff if $e_2 = 0$. If $D > 0$ then $m_2 = 0$ and (43) can be rewritten as

$$\alpha_2 E \max\{r_2 - D, 0\} \geq c$$  \hspace{1cm} (44)

If $D = 0$ then (43) becomes: $\alpha_2 E (m_2 + r_2) \geq c + \alpha_2 Em_2$ which also corresponds to (44). Note that the left side of (44) depends on the first-period dividend policy. If first-period dividends are high, the firm will borrow more in the second period and the IC will be stronger. The entrepreneur’s optimal decision is to invest as much as possible with internally generated funds given that both investment in the second period and high effort are socially optimal by (2).

If the second-period IC holds and the entrepreneur provides a high effort in the first period, the entrepreneur’s expected payoff equals the first-best firm value which is equal to $1 - 2b$ by (39). Therefore, the first-period IC is

$$1 - 2b \geq c + E[\alpha_1 W_1 + \alpha_2 W_2 \mid e_1 = 0]$$  \hspace{1cm} (45)

Under strategy $u$ the firm is always able to finance the second stage of the project internally. Thus, $D = 0$ in (44) and the second-period IC is:

$$\alpha_2 E r_2 \geq c$$  \hspace{1cm} (46)

Given that $r_1 = W_1 = 0$ when first-period effort is low, we can rewrite (45) as

$$1 - 2b \geq c + \alpha_2 E r_2$$  \hspace{1cm} (47)

From (42) $\alpha_1 = \alpha_2 = 1 - 2b$ and we can rewrite (46) and (47) as:

$$(1 - 2b) \theta \geq c$$  \hspace{1cm} (48)

$$(1 - 2b)(1 - \theta) \geq c$$  \hspace{1cm} (49)
If a firm has a growing profit profile, the consequences of entrepreneurial moral hazard are less pronounced in the first period because the expected profit from high effort is relatively low, and visa versa for the other type. Formally, if $\theta \leq 1/2$ the first condition is stronger. On the other hand if $\theta > 1/2$ the second condition is stronger. Hence we have: $u$ is optimal if and only if

$$\theta \leq 1/2 \text{ and } (48) \text{ or } \theta > 1/2 \text{ and } (49)$$

(50)

Appendix 3

Proof. If $\theta_s \leq 1/2$ then from (50) $u$ is the first-best strategy for each type when $c < (1 - 2b)\theta_j, j = l, s$. Proposition 2 follows from $\theta_s > \theta_l$. If $\theta_l > 1/2$ then, from (50) $u$ is the first-best strategy for both types if $c < (1 - 2b)(1 - \theta_j), j = l, s$. Again, Proposition 2 follows from $\theta_s > \theta_l$. Now consider $\theta_s > 1/2 \geq \theta_l$. From (50) $u$ is feasible for both types if $c < (1 - 2b)(1 - \theta_s)$ and $c < (1 - 2b)\theta_l$. Note that in all cases, the off-equilibrium beliefs supporting these equilibria can be the same as those described in the previous proposition.

End proof.

Appendix 4

Proof. Suppose the opposite is true, such that an equilibrium exists where $l$ plays $u$. First-period IC

$$c < (1 - 2b)\theta_l$$

From (11)-(13) this is only possible when $\theta_l > 1/2$ or $\theta_l < 1/2 < \theta_s$ and

$$c > (1 - 2b)(1 - \theta_s)$$

(51)

The latter implies that if $s$ mimics $l$ and cheats (provides low effort) in the second period, its total payoff is $(1 - 2b)\theta_s + c$ and this is greater than $1 - 2b$ by

\[11\] Obviously, if $\theta = 1/2$ both conditions are equivalent.
Thus $s$ will mimic $l$ and such an equilibrium does not exist. The proof is analogous for the case when $s$ plays $u$. End proof.

**Appendix 5**

**Proof.** Consider $V_e(\theta_l, \theta_s)$. From Lemma 1, $p = \frac{1 - 2b}{n}$ and $\alpha_1 = \frac{1 - 2b}{1 - b}$.

Recall that $l$ finances internally if $r_1 = 1$. Thus

$$V_e(\theta_l, \theta_s) = \frac{1 - 2b}{1 - b}(\theta_l(1 - b + 1 - \theta_l) + (1 - \theta_l)^2(1 - \frac{b}{1 - \theta_s})) \quad (52)$$

Lemma 3 follows from $\theta_l < \theta_s$ and (52). The proof is analogous for $V_e(\theta_s, \theta_l)$. End proof.

**Appendix 6**

Let $L(\theta)$ denote the perfect information face value of long-term debt if the firm is of type $\theta$, assuming that $z$ is a first-best strategy for $\theta$ under symmetric information (it would invest in both periods and provide high effort in both periods).

**Lemma 5.** $L(\theta_s) \gtrsim L(\theta_l)$ if $\theta_s + \theta_l \gtrsim 1$.

**Proof.** $L(\theta_s)$ and $L(\theta_l)$ are both less than 1. Otherwise, a high effort will not be provided in the second stage when $r_1 = 0$. Thus, Lemma 3 follows directly from (33). End proof.

**Corollary 1.** 1) $V_l(\theta_s, \theta_l) \gtrsim 1 - 2b$ If $\theta_s + \theta_l \gtrsim 1$; 2) $V_l(\theta_l, \theta_s) \gtrsim 1 - 2b$ if $\theta_s + \theta_l \gtrsim 1$.

**Proof.** Suppose $\theta_s + \theta_l < 1$ and consider $V_l(\theta, \theta)$. This is equal to:

$$V_l(\theta_s, \theta_l) = \theta_s(1 - \theta_s)(2 - L(\theta_l)) + (\theta_s^2 + (1 - \theta_s)^2)(1 - L(\theta_l))$$
By Lemma 3
\[ V_l(\theta_s, \theta_l) > \theta_s(1 - \theta_s)(2 - L(\theta_s)) + (\theta_s^2 + (1 - \theta_s)^2)(1 - L(\theta_s)) = V_l(\theta_s, \theta_s) = 1 - 2b \]

This proves the first part of the corollary. The proof is analogous for the second part. End proof.

Proof of Proposition 4. Consider a separating equilibrium where \( l \) plays \( l \) and \( s \) plays \( c \). Then from (33) \( L = \frac{2b}{1 - \theta_l + \theta_l^2} \). Suppose that \( \theta_l + \theta_s < 1 \). In this case \( s \) will mimic \( l \) by Corollary 2. Thus:
\[ \theta_l + \theta_s > 1 \]
which implies \( \theta_s > 1/2 \). From (11), (12) and (53) we get \( c > (1 - 2b)(1 - \theta_s) \).
Now consider the IC of type \( s \) in the second period. From (3) \( p = \frac{1 - 2b}{n} \) and \( \alpha_1 = \frac{n}{n + b/p} = \frac{1 - 2b}{1 - b} \). Type \( s \) earns \( 1 - \theta_s - b \) in the second period. The entrepreneur will provide a high effort only if \( c < \frac{(1 - 2b)(1 - \theta_s - b)}{1 - b} \). However, this contradicts the condition \( c > (1 - 2b)(1 - \theta_s) \). End proof.

Appendix 7.

\( l \) plays \( e \) and \( s \) plays \( d \). If \( l \) provides a high effort in both periods it will be mimicked by \( s \) because of the "lemon" argument (Lemma 2). Consider the case when \( l \) only obtains first-period financing (and provides a high effort in this period). We have:
\[ b = p \Delta n \]  
\[ p = \frac{\theta_l}{n + \Delta n} \]  
(54) and (55) imply \( p = \frac{\theta_l - b}{n} \) and \( \alpha_1 = \frac{\theta_l - b}{\theta_l} \). The equilibrium payoff of \( l \) is obviously \( \theta_l - b \). Suppose that \( s \) provides a high effort in the first period. Then
\[ D_1 = \frac{b - \lambda(1 - \theta_s)(1 - \theta_s - b)}{\theta_s} \]. If \( l \) mimics \( s \) it has at least \( \theta_l(1 - b/\theta_s) > \theta_l - b \). Thus,
such a situation is impossible. Now if $s$ provides low effort in the first period and is subsequently liquidated, the entrepreneur gets $c$. The IC for $l$ is

$$c < \theta_l - b$$

If $s$ mimics $l$ it gets $\frac{\theta_l - b}{\theta_l} \theta_s > \theta_l - b > c$ (its equilibrium payoff). The latter inequality follows from (56). Thus such an equilibrium is impossible.

The cases where $l$ or $s$ provide low effort in the first period and high effort in the second period are impossible. The firm’s total earnings are $1 - \theta$, which is less than the total investment by (1). The last observation also holds for the situations considered below.

**$l$ plays $e$ and $s$ plays $z$.** Consider the case when $l$ only obtains first-period financing (and provides high effort in this period). The situation where the effort of $s$ is high in the first period and low in the second (under both cash-flow realizations), or its effort is low in the first period and high in the second are impossible by (1): the earnings from only one stage are not sufficient to cover the total cost of investment ($2b$). Now suppose that $s$ provides high effort in both periods. The incentive constraint for $l$ is given by (56). If $\theta_l + \theta_s > 1$, then $l$ mimics $s$ and gets a higher payoff than its equilibrium payoff by Corollary 1. Consider $\theta_l + \theta_s < 1$. It implies $\theta_l < 1/2$. From (12) and (13) $c > (1 - 2b)\theta_l$. The latter contradicts (56). The only possible case where $s$ provides high effort in the first and second periods is when $r_1 = 1$.

**$l$ plays $z$ and $s$ plays $e$.** The case when the effort of $l$ is high in the first period and low in the second, under both states, is impossible given the previous argument. Now consider the case when $l$ provides high effort in both periods provided $r_1 = 1$. The payoff to $l$ equals $2\theta_l - \theta_l^2 - 2b + c(1 - \theta_l)$. Suppose that $s$ exerts high effort in both periods. In this case, $l$ will mimic $s$. $l$ will provide high effort in the second period only if $r_1 = 1$, and will get
\[
\frac{1-2b}{1-b} (\theta_l (1 - b + 1 - \theta_l)) + c(1 - \theta_l).
\]
This is more than his payoff in equilibrium. Thus, \(l\) will deviate. Finally, the only possible cases are those where \(s\) obtains financing for the first period and provides high effort in that period, and where \(l\) provides a high effort in both periods provided that \(r_1 = 1\).

\(l\) plays \(d\) and \(s\) plays \(e\). First consider the following case: \(s\) provides high effort in both periods and \(l\) provides high effort only in the first period. We have \(p = \frac{1-2b}{n}\). If \(l\) mimics \(s\) and provides low effort in the second period it gets \(\frac{1-2b}{1-b} \theta_l + c\) which is more than its equilibrium payoff of \(\theta_l - b\). Now consider the case when both types provide high effort in the first period and low effort in the second period. We have \(p = \frac{\theta_s - b}{n}\). Hence, \(l\) mimics \(s\), and gets \(\frac{\theta_s - b}{\theta_s} \theta_l > \theta_l - b\).

Finally, note that strategy \(u\) does not play an important role. If \(u\) is played in equilibrium then by (1) the effort should be high in both periods. However, such a situation is impossible given that (11)-(13) should hold (analogously to Proposition 3). \textit{End Proof.}

### References


