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# Exploring the Two Commodity World

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This paper attempts to explore the nature of the goods in a Two Commodity world. The analysis suggests that the only possibility that the two goods have same income elasticity is the case when both goods have unit income elasticities. Moreover, if both the goods have equal income elasticities, then these goods will belong to the same category and will be in equal relation to each other. The analysis further suggests that if one of the good has zero income elasticity, it will always be a substitute to the other good which will always be an elastic good. These results are supported by the CES and Quasi-linear utility functions.

JEL classification: D11

Key Words: Consumer Behaviour, Demand Functions, Two Commodity, Utility Functions

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## **I. Introduction**

The theory of consumer behavior has been a major area of discussion in Economics as it is a basic building block for many economic studies. It plays a practically important role in Welfare Economics, International Trade, General Equilibrium, Public Finance, etc. For many years the economists have been trying to explain and predict the complex and random human behavior with regards to his consumption pattern. Because of the complexities and randomness in human behavior certain axioms are made to present the consumer preferences in a consistent way. It is assumed that the consumer has preference relation on  $X=\mathbb{R}^2_+$  which is rational, continuous, strictly convex and locally non-satiated. Given his limited income ( $y>0$ ) and the (strictly positive) market prices of the commodities he plans the expenditure in a way that gives him the maximum possible satisfaction.

The traditional theory usually presents the consumer behavior by assuming the two-commodity case. Though the assumption seems to be unrealistic, it greatly facilitates the analysis in drawing conclusions about the behavior of the consumer. These conclusions seem to be reasonable and realistic. Secondly, we can cover all the commodities used by a consumer in a two-commodity case by taking the first commodity as the commodity under analysis and the second commodity as a composite good that takes into account for all other commodities. In this case we can also take the second commodity as the money income that is not spent on the first commodity.

The theory also provides certain properties and conditions, which have to be satisfied by the optimal consumption bundles. This paper is an attempt to discuss the implications of these conditions on the consumption bundles in a two-commodity world. In particular, the objective of the paper is to explore the nature of the two goods. That is, given

the axioms, properties and conditions provided by the theory, if we know the nature of one good can we make any inference about the nature of the other good?

With this objective we start by reviewing the consumption theory briefly in the next section. Section III presents the results regarding the nature of the goods implied by the conditions. These conditions will be analyzed simultaneously in Section IV and the suggested results will be presented. The final section contains the conclusions.

## **II. The Theory of Consumer Behavior**

Let us assume that there are only two goods  $Q_1$  and  $Q_2$  for consumption. The satisfaction derived from consuming these goods can be represented by a continuous utility function, that is,

$$U = f(q_1, q_2)$$

where  $U$  is the utility or satisfaction and  $q_1$  and  $q_2$  are the amount of the goods consumed. The objective of the consumer is to get the maximum possible satisfaction. That is, he wants to achieve

$$U^* = f(q_1^*, q_2^*)$$

where  $q_1^*$  and  $q_2^*$  are the optimal consumption bundles and  $U^*$  represent the maximum utility that the consumer can achieve.

In achieving maximum satisfaction, the consumer is constrained by his limited income and the market prices of the goods. The constraint, known as the budget constraint, is written as

$$P_1q_1 + P_2q_2 = y$$

where  $y$  is his income and  $P$ 's are the prices of the two goods. The constraint simply tells us that the sum of the expenditures on the two goods must be equal to the consumer's income.

Given the budget constraint, the optimal consumption bundles can be derived by setting the Lagrangian,

$$L = f(q_1, q_2) + \lambda\{y - P_1q_1 - P_2q_2\}$$

The first order condition for optimality requires that

$$L_1 = f_1 - \lambda P_1 = 0$$

$$L_2 = f_2 - \lambda P_2 = 0$$

$$L_\lambda = y - P_1q_1 - P_2q_2 = 0$$

These equations are simultaneously solved to get the demand functions for the goods, i.e.,

$$q_i = q_i(P_i, P_j, y)$$

where  $i=1,2$ ;  $j=1,2$ ; and  $i \neq j$ . The function tells us that the demand for a good depends on its own price, the price of the other good, and the income of the consumer. Thus any change in these variables will change the quantity demanded. The degree of change, however, depends on the elasticities of quantity demanded with respect to prices and income. These elasticities are defined and classified as follows:

**(a) Own Price Elasticity**

$$e_{ii} = \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} > -1 \text{ (the good is inelastic)}$$

$$e_{ii} = \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} = -1 \text{ (the good is unit elastic)}$$

$$e_{ii} = \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} < -1 \text{ (the good is elastic)}$$

**(b) Cross Price Elasticity**

$$e_{ij} = \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} > 0 \text{ (good i is substitute to good j)}$$

$$e_{ij} = \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} = 0 \text{ (good i is independent to good j)}$$

$$e_{ij} = \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} < 0 \text{ (good i is complement to good j)}$$

### (c) Income Elasticity

$$\eta_i = \frac{\partial q_i}{\partial y} \cdot \frac{y}{q_i} > 1 \text{ (good } i \text{ is a luxury)}$$
$$\eta_i = \frac{\partial q_i}{\partial y} \cdot \frac{y}{q_i} = 1 \text{ (good } i \text{ is unit income elasticity)}$$
$$\eta_i = \frac{\partial q_i}{\partial y} \cdot \frac{y}{q_i} < 1 \text{ (good } i \text{ is a necessity)}$$

These elasticities have to satisfy some conditions which are discussed below.

#### A. Cournot Condition

The condition requires that

$$S_1 e_{11} + S_2 e_{21} = -S_1$$

$$S_1 e_{12} + S_2 e_{22} = -S_2$$

Where  $S_1 = P_1 q_1 / y$  and  $S_2 = P_2 q_2 / y$  are the respective shares of each good in the total expenditure. The condition, which is derived from budget constraint, tells us that any change in the price of one good will affect the consumption of both the goods and that the consumption will be adjusted to satisfy the budget constraint.

#### B. Engel Condition

The condition requires that

$$S_1 \eta_1 + S_2 \eta_2 = 1$$

where  $0 < S_1, S_2 < 1$  and  $S_1 + S_2 = 1$ . The condition, also derived from the budget constraint, tells us that the weighted sum of the income elasticities of the two goods must be equal to unity. The weights are the shares of the corresponding good in total expenditure.

#### C. Homogeneity Condition

The condition requires that

$$e_{11} + e_{12} + \eta_1 = 0$$

$$e_{21} + e_{22} + \eta_2 = 0$$

The condition, derived from the demand functions of the two goods, says that the demand functions are homogeneous of degree zero in prices and income. That is, if the prices of the two goods and income of the consumer are changed by the same proportion, the consumption pattern will not be affected.

### III. Results Implied by the Conditions

The two conditions - Cournot and Engle, discussed above provide some useful information regarding the nature of two goods. This section presents these results.

The cournot condition can be written alternatively as

$$S_1(1+e_{11}) + S_2e_{21} = 0$$

$$S_1e_{12} + S_2(1+e_{22}) = 0$$

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We can infer some useful conclusions from these equations regarding the nature of two goods. These are,

(a)  $e_{11} > -1 \Rightarrow (1+e_{11}) > 0 \Rightarrow e_{21} < 0$

if Good 1 is inelastic, Good 2 will be complement to Good 1

(b)  $e_{11} = -1 \Rightarrow (1+e_{11}) = 0 \Rightarrow e_{21} = 0$

if Good 1 is unit elastic, Good 2 will be independent to Good 1

(c)  $e_{11} < -1 \Rightarrow (1+e_{11}) < 0 \Rightarrow e_{21} > 0$

if Good 1 is elastic, Good 2 will be substitute to Good 1

The same conclusion can be derived by looking at the budget constraint.

$$P_1q_1 + P_2q_2 = y$$

$$(a) \quad e_{11} > -1 \Rightarrow \text{if } P_1 \uparrow \Rightarrow (P_1 q_1) \uparrow \Rightarrow (P_2 q_2) \downarrow \Rightarrow q_2 \downarrow \Rightarrow e_{21} < 0$$

If Good 1 is inelastic the increase in its price will result an increase in its expenditure. Since income is constant the expenditure on Good 2 has to decrease to satisfy the budget constraint. But the price of Good 2 is also constant which implies that the consumption of Good 2 has to go down to maintain the identity. Hence the increase in the price of Good 1 will result in a decrease in the consumption of Good 2 suggesting that Good 2 is complement to Good 1.

Similar kind of explanation can be given for the other two cases. Similarly, by observing the second equation we can get the picture of the other good. The results implied by the Cournot condition can be summarized as follows:

$$e_{11} > -1 \Leftrightarrow e_{21} < 0, \quad e_{22} > -1 \Leftrightarrow e_{12} < 0$$

$$e_{11} = -1 \Leftrightarrow e_{21} = 0, \quad e_{22} = -1 \Leftrightarrow e_{12} = 0$$

$$e_{11} < -1 \Leftrightarrow e_{21} > 0, \quad e_{22} < -1 \Leftrightarrow e_{12} > 0$$

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Next, the Engle condition can be solved further as

$$S_2 \eta_2 = 1 - S_1 \eta_1$$

or

$$\eta_2 = \frac{1 - S_1 \eta_1}{1 - S_1}$$

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Thus we can determine the elasticity of Good 2 given the elasticity of Good 1. That is,

$$\eta_1 < 1 \Rightarrow \eta_2 > 1$$

$$\eta_1 = 1 \Rightarrow \eta_2 = 1$$

$$\eta_1 > 1 \Rightarrow \eta_2 < 1$$

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Similar conclusions can be derived alternatively. Since



$$S_1 = P_1 q_1 / y$$

Therefore

$$\frac{dS_1}{dy} = \frac{P_1 q_1}{y^2} (\eta_1 - 1) = \begin{cases} < 0 & \text{if } \eta_1 < 1 \\ = 0 & \text{if } \eta_1 = 1 \\ > 0 & \text{if } \eta_1 > 1 \end{cases}$$

This suggests that the share of a good in total expenditure is affected by the income elasticity of that good. We also know that

$$S_1 + S_2 = 1$$

Hence if the share of a good decreases by an increase in income the share of the other good must increase to satisfy the identity. This means that the goods must have opposite elasticities. The only possible case which will leave the shares unchanged is the case when the two goods have unit income elasticities.

### III. Some Further Results

So far the results have been derived from the conditions separately. However, the combination of these conditions provides some interesting results regarding the nature of two goods. we proceed with the homogeneity condition by assigning different values to income elasticities and finding the effect on the corresponding price elasticities. The condition is reproduced below

$$e_{11} + e_{12} + \eta_1 = 0$$

$$e_{21} + e_{22} + \eta_2 = 0$$

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#### Case I:

Let us assume that  $\eta_1 = 1$ . This implies that  $\eta_2 = 1$  from 4. Equation 5 can now be written as

$$(1+e_{11}) + e_{12} = 0$$

$$e_{21} + (1+e_{22}) = 0$$

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We can get some useful results from equations 6 and 2. These are.

(a) From 6  $e_{11} > -1 \Rightarrow (1+e_{11}) > 0 \Rightarrow e_{12} < 0$

From 2  $e_{12} < 0 \Rightarrow (1+e_{22}) > 0 \Rightarrow e_{22} > -1$

From 2  $e_{11} > -1 \Rightarrow e_{21} < 0$

That is, if Good 1 is inelastic it is complement to Good 2. Moreover, Good 2 will also be inelastic and will be complement to Good 1.

(b) From 6  $e_{11} = -1 \Rightarrow (1+e_{11}) = 0 \Rightarrow e_{12} = 0$

From 2  $e_{12} = 0 \Rightarrow (1+e_{22}) = 0 \Rightarrow e_{22} = -1$

From 2  $e_{11} = -1 \Rightarrow e_{21} = 0$

Thus both the goods will be unit elastic and will be independent to each other.

(c) From 6  $e_{11} < -1 \Rightarrow (1+e_{11}) < 0 \Rightarrow e_{12} > 0$

From 2  $e_{12} > 0 \Rightarrow (1+e_{22}) < 0 \Rightarrow e_{22} < -1$

From 2  $e_{11} < -1 \Rightarrow e_{21} > 0$

Thus both the goods will be elastic and will be substitute to each other.

Hence the above analysis suggests that if the income elasticities of the two goods are equal, both the goods will belong to the same category and will be in equal relation to each other. An example of this kind of preferences may be a CES utility function. That is,

$$U = A[\alpha q_1^{-\rho} + (1-\alpha)q_2^{-\rho}]^{-1/\rho}$$

The demand functions for the two goods are

$$q_1 = \frac{y}{P_2 + P_1 \left( \frac{P_1}{P_2} * \frac{1-\alpha}{\alpha} \right)^{\frac{1}{1+\rho}}}$$

$$q_2 = \frac{y}{P_1 + P_2 \left( \frac{P_1}{P_2} * \frac{1-\alpha}{\alpha} \right)^{\frac{1}{1+\rho}}}$$

It can be verified that income elasticities of both the goods are equal to unity.

$$\eta_1 = \eta_2 = 1$$

The own price elasticities are

$$e_{11} = - \frac{P_1 + P_2 \left( \frac{1}{1+\rho} \right) \left( \frac{P_1 * 1 - \alpha}{P_2 \alpha} \right)^{\frac{1}{1+\rho}}}{P_1 + P_2 \left( \frac{P_1 * 1 - \alpha}{P_2 \alpha} \right)^{\frac{1}{1+\rho}}}$$

$$e_{22} = - \frac{P_1 \left( \frac{1}{1+\rho} \right) + P_2 \left( \frac{P_1 * 1 - \alpha}{P_2 \alpha} \right)^{\frac{1}{1+\rho}}}{P_1 + P_2 \left( \frac{P_1 * 1 - \alpha}{P_2 \alpha} \right)^{\frac{1}{1+\rho}}}$$

The cross price elasticities are

$$e_{12} = - \frac{P_2 \left( \frac{\rho}{1+\rho} \right) \left( \frac{P_1 * 1 - \alpha}{P_2 \alpha} \right)^{\frac{1}{1+\rho}}}{P_1 + P_2 \left( \frac{P_1 * 1 - \alpha}{P_2 \alpha} \right)^{\frac{1}{1+\rho}}}$$

$$e_{21} = - \frac{P_1 \left( \frac{\rho}{1+\rho} \right)}{P_1 + P_2 \left( \frac{P_1 * 1 - \alpha}{P_2 \alpha} \right)^{\frac{1}{1+\rho}}}$$

It can be seen that the values of these price elasticities depend on the value of  $\rho$  (the substitution parameter). Hence

(a) If  $-1 < \rho < 0$ , then  $e_{11}, e_{22} < -1$  and  $e_{12}, e_{21} > 0$ , that is, both the goods will be elastic and will be substitute to each other.

(b) If  $\rho = 0$ , then  $e_{11}, e_{22} = -1$  and  $e_{12}, e_{21} = 0$ , that is, both the goods will be unit elastic and will be independent to each other.

(c) If  $\rho > 0$ , then  $e_{11}, e_{22} > -1$  and  $e_{12}, e_{21} < 0$ , that is, both the goods will be inelastic and will be complement to each other.

Hence, whatever is the value of  $\rho$ , the two goods will always belong to the same category and will be in equal relation to each other.

**Case II:**

Now assume that  $\eta_1 = 0$ . This means that  $\eta_2 > 1$  from 4. From the homogeneity condition it follows that

$$e_{11} + e_{12} = 0$$

$$e_{21} + (1+e_{22}) < 0$$

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Once again, we can get some useful results by combining equations 7 and 2. These are

From 7  $e_{11} < 0 \Rightarrow e_{12} > 0$

From 2  $e_{12} > 0 \Rightarrow e_{22} < -1$

From 2  $e_{11} > -1 \Rightarrow e_{21} < 0, e_{11} = -1 \Rightarrow e_{21} = 0, e_{11} < -1 \Rightarrow e_{21} > 0$

Hence whatever Good 1 is, it will always be a substitute to Good 2 which will always be an elastic good. The relationship of Good 2 to Good 1,  $e_{21}$ , will depend on the nature of Good 1,  $e_{11}$ . An example of this kind of preferences may be a Quasi-linear utility function. That is,

$$U = \ln q_1 + q_2$$

The demand functions are

$$q_1 = P_2 / P_1$$

$$q_2 = (y - P_2) / P_2$$

It can be verified that the income elasticities are

$$\eta_1 = 0$$

$$\eta_2 = y / (y - P_2) > 1$$

The own price elasticities are

$$e_{11} = -1$$

$$e_{22} = -y / (y - P_2) < -1$$

The cross price elasticities are

$$e_{12} = 1 > 0$$

$$e_{21} = 0$$

It can be seen that Good 1 is a substitute to Good 2 which is an elastic good. In this particular case, Good 1 is unit elastic and Good 2 is independent of Good 1.

### Case III:

Finally, assume that  $\eta_1 < 1$ . This implies that  $\eta_2 > 1$  from 4. The homogeneity condition becomes

$$(1+e_{11}) + e_{12} > 0$$

$$e_{21} + (1+e_{22}) < 0$$

8

Once again we can proceed by repeating the earlier exercise, that is, combine equations 8 and 2.

(a) From 8  $e_{11} < -1 \Rightarrow (1+e_{11}) < 0 \Rightarrow e_{12} > 0$

From 2  $e_{12} > 0 \Rightarrow (1+e_{22}) < 0 \Rightarrow e_{22} < -1$

From 2  $e_{11} < -1 \Rightarrow e_{21} > 0$

That is, both the goods will be elastic and will be substitute to each other.

(b) From 8  $e_{11} = -1 \Rightarrow (1+e_{11}) = 0 \Rightarrow e_{12} > 0$

From 2  $e_{12} > 0 \Rightarrow (1+e_{22}) < 0 \Rightarrow e_{22} < -1$

From 2  $e_{11} = -1 \Rightarrow e_{21} = 0$

That is, Good 1 is now unit elastic but still substitute to Good 2 whereas Good 2 is elastic and independent to Good 1.

(c) From 8  $e_{11} > -1 \Rightarrow (1+e_{11}) > 0 \Rightarrow e_{12} = ?$  (uncertain)

From 2  $e_{11} > -1 \Rightarrow e_{21} < 0$

Hence in this case, we cannot make inferences with that much certainty as we made earlier. All we can say is that as long as Good 1 is either elastic or unit elastic, it will always be a substitute to Good 2 which will always be an elastic good and can be either independent

or substitute to Good 1. But if Good 1 becomes inelastic we will be unable to make any conclusions.

## **V. Conclusions**

The purpose of the paper was to explore the nature of the goods in a Two Commodity world. Various properties and conditions provided by the theory guided us in this exercise. Some useful and interesting results were found in the process.

The analysis suggests that the only possibility that the two goods have same income elasticity is the case when both goods have unit income elasticities. Moreover, if both the goods have equal income elasticities, then these goods will belong to the same category and will be in equal relation to each other. The analysis further suggests that if one of the good has zero income elasticity, it will always be a substitute to the other good which will always be an elastic good. These results are supported by the CES and Quasi-linear utility functions.

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