Voting power in the Electoral College: The noncompetitive states count, too

Brams, Steven J and Kilgour, D. Marc

New York University

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Steven J. Brams
Department of Politics
New York University
New York, NY 10003
USA
steven.brams@nyu.edu

D. Marc Kilgour
Department of Mathematics
Wilfrid Laurier University
Waterloo, Ontario N2L 3C5
CANADA
mkilgour@wlu.ca

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Abstract

In U.S. presidential elections, voters in noncompetitive states seem not to count—and so have zero voting power, according to the Banzhaf and other voting-power indices—because they cannot influence the outcome in their states. But because the electoral votes of these states are essential to a candidate's victory, it seems that they do count, but in a different way.

We measure the power of voters in noncompetitive states by modeling how these states structure the contest in the competitive states, as illustrated in the 2012 election. Barack Obama's lead of 46 electoral votes over Mitt Romney in the 41 noncompetitive states and the District of Columbia gave him 5.5 times as many ways of winning in the 9 competitive states as Romney had. Also, Romney's winning coalitions were weaker by two additional measures: They were 2.4 times more vulnerable, and 5.5 times more fragile than Obama's. Compared with being tied with Romney in the noncompetitive states, Obama's lead in these states contributed very substantially to his victory.

Voting Power in the Electoral College: The Noncompetitive States Count, Too

1. Introduction

The role of the Electoral College in U.S. presidential elections has been controversial since the Constitution was adopted in 1789. It became more so starting in the 1830s, when states began to require that all their electoral votes be cast for the plurality-vote winner in their state, which is true today in all states except Maine and Nebraska. The controversy was exacerbated by the 2000 election, when, for the third time—the two previous instances were in 1876 and 1888—the popular-vote winner (Al Gore) was not the electoral-vote winner (George Bush).[1]

The winner-take-all rule in states has not only produced divided verdicts, like that in 2000, but it also has had a profound effect on how candidates campaign in presidential elections. Especially in recent elections, candidates have spent almost all their campaign resources in the competitive states, which have the tightest races, based on state polls, and in which a candidate's expenditures can make a difference between victory and defeat in these states.

In 2012, there was a consensus among pollsters and analysts that nine states were "up for grabs," based on their state polls, though in the actual election Mitt Romney won only one of these states (North Carolina).[2] Before the election, it was believed that none

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[1] In 1824, there was no majority electoral-vote winner, so the election was decided in the House of Representatives. However, the popular-vote winner was uncertain, because electors in six states were legislatively appointed rather than popularly elected, so one cannot determine who the popular-vote winner was (Andrew Jackson received a plurality of electoral votes but lost the election to John Quincy Adams in the House).

[2] A tenth state, Pennsylvania, received some attention from both candidates, but only at the end of the campaign. In fact, the margin of victory that Pennsylvania gave Obama (5.39%) was slightly less than the margin he received in two other states—New Hampshire (5.58%) and Iowa (5.81%)—so in retrospect it probably should have been included among the competitive states.

of the other 41 states or the District of Columbia could be wrested from the candidate who was clearly ahead in the polls.

Effectively, then, the voters in the noncompetitive states did not count—under no circumstances could they, as individuals, influence the outcomes in their states and, therefore, their states in the Electoral College. Consequently, they had zero voting power, as measured, for example, by the relative Banzhaf index, to which we will return later.

But noncompetitive states *do* influence the outcome, because their electoral votes are essential to a candidate's victory. We resolve the paradox of how a noncompetitive state can influence the outcome, even though its voters have no voting power, by showing how the distribution of electoral votes in the noncompetitive states sets up the contest in the competitive states.

More specifically, the nine states that were believed competitive in 2012 collectively had 110 of the 538 electoral votes. But because Barack Obama had a 237–191 electoral-vote lead over Mitt Romney in the 41 noncompetitive states and the District of Columbia, he needed only 33 of the 110 electoral votes in the competitive states to win with a majority of 270 electoral votes, whereas Romney needed 79. As we will show, there were 5.5 more ways that Obama could win a majority of electoral votes than could Romney.

Additionally, in 251 of Obama's 431 winning coalitions (58.2%), no competitive state—even the largest, Florida—could change the outcome by changing its vote, whereas in 75 of Romney's 76 winning coalitions (98.7%), at least one state's defection could rob him of victory, making Romney's winning coalitions much more vulnerable.

We measure the *vulnerability* of a candidate's winning and tied coalitions by the proportion of them in which a single competitive state, by switching to the other candidate, can change the winner to the other candidate or create a tie. Their fragility is measured by the expected number of states it takes to change the outcome in winning and tied coalitions.

By these measures, Romney's winning coalitions were 2.4 times more vulnerable, and 5.5 times more fragile, than Obama's, which is attributable to the lead that Obama enjoyed in the noncompetitive states. In sum, the noncompetitive states count, but they exercise power in a different way from the competitive states.

The paper proceeds as follows. In section 2, we give a hypothetical 5-state example, in which there are $2^5 = 32$ possible splits of the 5 states (each state may vote for one of the other of the two major-party candidates). It illustrates the advantages that the leading candidate receives—in opportunities to win and in diminished vulnerability and fragility—when he or she needs fewer electoral votes from the competitive states to win. We compare these results with those in which each candidate needs exactly the same number of electoral votes—a simple majority—to win. We then make these calculations for the nine competitive states in 2012, for which it is not feasible to list the $2^9 = 512$ possible splits.

In section 3, we calculate the relative Banzhaf index of voting power for the competitive states, and individual voters in them, as players. We show that voters in the larger competitive states had substantially greater influence, especially in making Romney the winner, than voters in the smaller states. For example, all but two of

Romney's 76 winning coalitions (97.4%), but only 79 of Obama's 431 winning coalitions (18.3%), depended upon each candidate's winning Florida.

In section 4 we draw several conclusions. In particular, we argue that the noncompetitive states, far from being ciphers, exercise power indirectly, and in a different way from, the competitive states. They set up the race in the competitive states, which in 2012 gave Barack Obama an overwhelmng advantage.

2. Winningness, Vulnerability, and Fragility

To illustrate our analysis, we start with a simple hypothetical example. Assume there are two candidates, X and Y, and 5 competitive states, (A, B, C, D, E), which have, respectively, (2, 3, 4, 5, 6) electoral votes (total: 20). The winner in each state wins all the electoral votes of that state.

We assume that X has a substantial lead over Y in the noncompetitive states and needs only 6 of the 20 electoral votes to win, whereas Y needs 15. In Table 1, we list the 32 possible *splits* (i.e., coalitions) of the states, from X's winning all 5 states to Y's winning all 5. Because the states are competitive, we assume each is equally likely to vote for X or Y.

Table 1 about here

This does *not* mean that every voter in a competitive state is equally likely to vote for X or Y. In fact, most voters in competitive states, whom we call *partisan*, decide well before an election how they will vote. At the time of the election, a relatively few undecided voters in these states, whom we call *nonpartisan*, may be thought of as deciding, with equal likelihood, whether to vote for the Democratic or the Republican candidate. If there is an even split of partisan voters in the competitive states, the equiprobable choices of the nonpartisan voters make these states, in the parlance of electoral politics, "too close to call."

For each of the 32 splits of the 5 competitive states, we have indicated with an asterisk in Table 1 which state(s) are *critical* to the winning coalitions: If that state changed its vote, the winning coalition would lose. Notice that in every split, either X wins with at least 6 electoral votes, or X receives 5 or fewer electoral votes, in which case Y wins with at least 15. A tie is not possible in this example.

As we will show later, if each candidate could have won with a simple majority of 11 of the 20 electoral votes, then a 10-10 split, in which both candidates tie, is possible. However, we will not treat ties as losing but, instead, as halfway between winning and losing, in a manner to be made precise later.[3]

If N is an integer satisfying $0 \leq N \leq 32$, define a candidate's *winningness index*, $W(N)$, as the percentage of the 32 splits in which he or she wins with at least N votes. Table 1 shows that when X needs at least 6 votes, and Y at least 15, to win, then

$$W(6) = 26/32 = 81.25\%; \quad W(15) = 6/32 = 18.75\%.$$

Observe that $W(6) + W(15) = 100\%$, because ties are not possible: Either one or the other of the two candidates wins.

The difference in winningness of X and Y when ties are not possible is

$$W(N) - [1 - W(N)] = 2W(N) - 1,$$

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[3] In the case of a tie in the Electoral College, or if no candidate wins a majority of electoral votes because of a split among more than two candidates, the election is decided by a simple majority in the House of Representatives, wherein each state has one vote (12th Amendment, 1804).

which equals 62.5% when $N = 6$. This is the percentage gap in the ability of X vs. Y to win in our 5-state example.

Viewed another way, because X wins in 26 splits and Y in 6, X has 26/6 = 4 1/3 times more opportunities to win in our model, in which all states vote independently for each candidate with probability ½, so all possible splits are equiprobable. As we pointed out earlier, however, this does not imply that every voter votes, with equal probability, for one or the other candidate. In the aggregate, however, every state, because it is competitive, is a priori equally likely to cast its electoral votes for either candidate.

There are significant differences between X and Y's winning coalitions. Define a candidate's *vulnerability index*, $V(N)$, to be the conditional probability, given that the candidate needs at least N electoral votes to win, that his or her winning coalition includes at least one critical state. Table 1 shows that 13 of X's 26 winning coalitions, and 6 of Y's 6 winning coalitions, include a critical state, so

$$V(6) = 13/26 = ½ = 50\%; \quad V(15) = 6/6 = 100\%.$$

Because these conditional probabilities reflect different scales (note that they do not have a common denominator), they do not sum to a fixed percentage, though they both have an upper bound of 100% (i.e., at most all a candidate's winning coalitions can be vulnerable). The percentage gap in our example is $V(15) - V(6) = 50\%$; in ratio terms, Y's winning coalitions are twice as vulnerable as X's.

$V(N)$ does not take into account the number of members of a winning coalition that render it vulnerable. Another useful measure is a candidate's *fragility index*, $F(N)$, or the expected the number of critical states in a winning coalition, given that the candidate needs at least N electoral votes to win, that make a winning coalition vulnerable.

Multiplying the number of each candidate's winning coalitions by the number of states that make it vulnerable, and dividing this result by the number of winning coalitions of each candidate, we obtain from Table 1 the following fragility values:

$F(6) = [13(0) + 8(1) + 5(2)]/26 = 18/26 \approx 0.69;$

$F(15) = [1(1) + 3(3) + 2(4)]/6 = 18/6 = 3.$

In ratio terms, *Y*'s vulnerable coalitions are (18/5)/(18/26) = 26/6 = 4 1/3 times as fragile as *X*'s.

It is no accident that the ratio of *X*'s winningness to *Y*'s, and *Y*'s fragility to *X*'s, are equal,

$$\frac{F(6)}{F(15)} = \frac{W(15)}{W(6)} = 4\ 1/3,$$

even though winningness measures the proportion of winning coalitions and fragility measures the expected number of coalition members that render a winning coalition vulnerable. Because any defection that changes *Y* from winning to losing can be reversed to create a defection that changes *X* from winning to losing, and vice versa, both *X* and *Y* face the same number of possible defections, making the numerators of $F(6)$ and $F(15)$ the same (18 in our example). The denominators of $F(6)$ and $F(15)$ count the number of winning coalitions of *X* and *Y*, but they are inverted as $W(15)$ and $W(6)$ in the quotient shown above.

Although these ratios convey the same information about *X*'s advantage over *Y*, the actual values of $W(N)$ and $F(N)$ describe different aspects of *X*'s advantage. In particular, *X*'s winningness (81.25%) does not directly shed light on *X*'s robustness—that an average of only 0.69 of the 5 states (13.8%) render *X*'s coalitions vulnerable, whereas an average of 3 states (60%) render *Y*'s coalitions vulnerable. And neither winningness nor fragillty directly illuminate X's lack of vulnerability—that only half of *X*'s 26 winning coalitions are vulnerable, whereas all of *Y*'s 6 winning coalitions are vulnerability.

In sum, our three indices measure the strength of a candidate in different ways, where "strength" is increasing in $W(N)$ and decreasing in $V(N)$ and $F(N)$. As useful as they are, however, these indices do *not* measure the power of the noncompetitive states, compared with the competitive states, in deciding an election.

For this purpose, we show in Table 2 the 16 splits in which each candidate wins with at least 11 electoral votes, or ties with 10, of the 20 electoral votes (we assume in Table 2 the winning candidate to be *X*.) We call this situation, which assumes that the noncompetitive states split their electoral votes evenly so it takes a simple majority of electoral votes in the competitive states to win, the *baseline*. We will compare the index values we just computed, in which *X* needs 6 and *Y* needs 15 electoral votes to win, with their baseline values.

Table 2 about here

In two of the 17 splits in Table 2 (#7 and #16), there is no winner because there is a 10-10 tie between *X* and *Y*. In this split, all 5 states are critical, because if one state defects from either *X* or *Y*, it causes its tied coalition to lose.

We count a critical defection of a state from a tied coalition of *X* (#7 and #16 in Table 2) as ½ of a defection from a winning coalition, based on the assumption that it is equally likely that this defection will be from the other tied coalition and, therefore, not cause one's tied coalition to lose. By the same token, if a state's defection causes a winning coalition to tie with the other coalition (as is possible in splits #4, #6, #8, #10, and #13), it also counts as ½ a defection, because it does not cause the winning coalition to become losing but instead prevents it from winning.

In sum, either making or breaking a tie through a defection has 50% of the value of defection from a winning coalition that causes it to lose. Using the formulas given previously for winningness, vulnerability, and fragility, the baselines values when X and Y need at least 11 electoral votes to win are as follows:

$$W(11) = 50\%; \quad V(11) \approx 78.1\%; \quad F(11) \approx 1.69.$$

In Table 3, we pull together the index values of winningness, vulnerability, and fragility—including the differences in these values and their ratios when $N = 6$ and $N = 15$—and show their percent changes from the baseline figures given above. In the case of winningness, the change for both X and Y from the baseline figure of 50% is 31.25/50 = 62.5%, which is up for X and down for Y.

Table 3 about here

The baseline figure for vulnerability is 78.1%, where the shift down for X to 50% is 28.1/78.1 $\approx$ 36.0%, and the shift up to 100% for Y is 21.9/78.1 $\approx$ 28.0%. The baseline figure for fragility is 1.69, and the shift down for X to 0.69 is 1/1.69 $\approx$ 59.2%, and the shift up for Y to 3 is 1.31/1.69 $\approx$ 77.5%.

To summarize, five of the six of the changes from the baselines, which are shown in parentheses in Table 3, are greater or equal to 50%, with only the decrease in the vulnerability of X less (36.0%). These shifts from the baselines, ranging from 36.0% to

77.5%, indicate that *X*'s lead in the noncompetitive states has a significant and sometimes dramatic effect on the candidates' political strength.

Only for winningness is the effect symmetric, helping *X* and hurting *Y* by the same amount. This index is probably *primus inter pares* as an indicator of the boost the leading candidate receives from being ahead in the noncompetitive states, although this is not to deny the importance of vulnerability and fragility as indicators of the robustness of the coalition supporting *X*.

We attribute *X*'s strength over what it would be if the playing field were level to the *setup power* of the noncompetitive states—their power to tilt the election in favor of *X*. This power is total in the extreme case in which the noncompetitive states have already decided the election, because then none of the competitive states would be critical in any of the winning coalitions.

How do these figures compare with those in the 2012 presidential election that gave Barack Obama a head start of 237–191 = 46 electoral votes in the 41 noncompetitive states and the District of Columbia? We give in Table 4 the values of the different indices, based on the 507 winning and 5 tied coalitions—in which each candidate receives 269 electoral votes—of the 9 competitive states.

Table 4 about here

The 9 competitive states in 2012 had collectively 110 electoral votes, with Obama needing 33, and Romney 79, electoral votes to win a simple majority of 270 electoral votes. Had the playing field been level, each candidate would have needed 56 electoral votes from the competitive states to win. Because it is infeasible to display the 512 winning and tied coalitions, as we did for the 5-state example in Tables 1 and 2, we give in Table 4 only the summary statistics we did for the 5-state example in Table 3.

Winningness favors Obama over Romney by 69.4% and a ratio of 5.5. As for vulnerability and fragility, Romney's winning and tied coalitions are 2.4 times more vulnerable, and 5.5 times more fragile (same as the winningness ratio), than Obama's.

The candidates' baseline values on our three indices, whereby each candidate needs 56 of the 110 electoral votes in the 9 competitive states to win, are as follows:

$$W(56) = 0.50\%; \quad V(56) \approx 79.3\%; \quad F(56) \approx 2.02.$$

The percent changes in the 2012 values from those given above indicate Obama to be 69.4% ahead in winningness; his winning and tied coalitions are 48.7% less vulnerable and 63.4% less fragile than Romney's. Romney is behind Obama on winningness by the same percentage that Obama is ahead (69.4%); Romney's winning and tied coalitions are 24.5% more vulnerable and 49.8% more fragile than Obama's.[4]

Compared with the baselines, the increase in Obama's winningness, and the decreases in his vulnerability and fragility, are greater than those for *X* in our 5-state hypothetical example. But *Y*'s percent increases in vulnerability and fragility are greater than Romney's in 2012. Overall, both the 5-state and 9-state examples demonstrate that a candidate who needs less than half as many votes to win, compared to his or her opponent, enjoys a very substantial advantage according to all our indices.

The head start that Obama received from the noncompetitive states, however, was not critical, because he won the electoral votes of all the competitive states except North

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[4] Romney's winning coalitions are also considerably larger than Obama's, averaging 4.00 states compared with 2.85 states for Obama.

Carolina (15 electoral votes). Even if the electoral votes of the competitive states had split 55-55, Obama still would have won with 293 electoral votes (he actually won with 332 electoral votes, and by almost 5 million votes of 129 million cast).

Before the 2012 election, Obama's advantages were discussed by Nate Silver in several articles in the *New York Times*. In fact, Silver correctly predicted the outcomes in all 50 states, based on state polls (http://en.wikipedia.org/wiki/Nate_Silver). In retrospect, the election was not as close as many analysts thought it would be, though the candidates campaigned as if it were, fighting fiercely to the end.

Although the voting power of the competitive states is not the main focus of our study, we next calculate the Banzhaf power of these states, considering first the states as players and then individual voters in them as players. Unlike the usual calculation, our calculation takes into account that a state's defection that makes or breaks a tie counts for half as much as a defection that causes a winning coalition to receive fewer than half the votes.[5]

We show that the Banzhaf power of these states changes somewhat when the electoral votes needed to win are not a simple majority but, instead, favor one candidate over the other. More significant, the power of voters in large competitive states far outstrips that of voters in small competitive states, in violation of the principle of one person, one vote.[6]

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[5] A variant of this calculation treats a tied coalition as losing, so a defection from a tied coalition is not counted as half a critical defection. But this loss in critical defections is exactly made up for by counting defections that create ties as full defections, so the Banzhaf calculation (described in the next section) is not affected. However, our winningness, vulnerability, and fragility indices would change if we did not take into account tied splits.

[6] Many analysts have commented on the normative implications of this violation, but we do not pursue this matter here because the primary purpose of our study is to show how, and in what ways, the noncompetitive states matter.

3. The Banzhaf Index of Voting Power for the Competitive States

John F. Banzhaf III proposed an index of voting power in Banzhaf (1965), which he later applied to the Electoral College (Banzhaf, 1968). This index measures the ability of a voter, by changing his or her vote, to change the outcome in his or her state and, in turn, for that state to change the outcome in the Electoral College.[7]

We first define the *relative Banzhaf index*, $B(J)$, for states as players. This is the number of vulnerable winning coalitions in which state J is critical, $C(J)$—or the number of its critical defections—divided by the total number of critical defections over all the competitive states (over which the index K in the denominator runs):[8]

$$B(J) = \frac{C(J)}{\sum_K C(K)}.$$

As before, we assume all competitive states vote independently and are equally likely to support Obama or Romney in the 2012 election, rendering equiprobable all 512 splits of these states in which one candidate or the other wins or ties. Equivalently, $B(J)$ can be interpreted as the probability that a randomly chosen critical defection is cast by state J when all critical defections are assumed to be equiprobable.

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[7] The Banzhaf index, and a related index due to Johnston (1978), have been applied to members of other federal institutions, including the President, Representatives, and Senators (Brams, Affuso, and Kilgour, 1989). Different voting-power indices, such as the Shapley-Shubik (1954) index, are analyzed and compared in Felsenthal and Machover (1998) and Holler and Nurmi (2013). These indices have been applied to many voting bodies, as well as other institutions connected by a set of rules, including, for example, the European Union Council of Ministers and the European Parliament (Cichocki and Zyczkwoski, 2010).

[8] The *absolute Banzhaf index* of state J is the number of its critical defections—the numerator of the right side of $B(J)$—divided by the total number of coalitions that include J, which is 2^{n-1} because half of all coalitions include J. This index gives the unconditional probability that J will be critical, whereas the relative Banzhaf index normalizes the absolute Banzhaf index, so the voting power of all states sums to 1.

We next consider how voting power is affected when the voters, rather than the states, are considered to be the players. The Banzhaf power of a voter in competitive state J measures the relative degree to which he or she can influence the outcome in a presidential election via the outcome in state J, and the state, in turn, in the Electoral College.

Assume partisan voters in each competitive state split 50-50, and independent voters are equally likely to support Obama or Romney, and so, by extension, is the state. Banzhaf (1968) showed that if the number of voters in a state is sufficiently large, the relative power of *each voter* in competitive state J scales according to $1/\sqrt{n_J}$, where n_J is the number of voters in J.[9]

The relative Banzhaf index of an individual voter in J, compared with an individual voter in other competitive states, is

$$\beta(J) = \frac{C(J)/\sqrt{n_J}}{\sum_K C(K)/\sqrt{n_K}}.$$

Like the $B(J)$'s, the $\beta(J)$'s sum to 1. They reflect the relative ability of an individual voter in state J, compared to what his or her power would be living in another competitive state, to change the national outcome by casting a *doubly decisive vote*—

[9] This effect is also known as the "Penrose square root law,"

http://en.wikipedia.org/wiki/Penrose_square_root_law

because it was first shown in Penrose (1946), though not in the context of the Electoral College. It would be more accurate to call this the *inverse* square root law, as Miller (2013) does, because $\sqrt{n_J}$, is a divisor, not a multiplier. See also Edelman (2003) and Miller (2008).

breaking a tie in his or her state, and the state's being critical in the Electoral College—so the voter, through his or her state, is critical in the Electoral College.

A voter's criticality in a presidential election is, of course, minuscule. What $\beta(J)$ measures is the *scaled power* of a voter in state J—it is scaled relative to the voting power of voters in other states, showing how an individual's voting power depends on the size of his or her state as well as the state's criticality in the Electoral College.

Even the smallest state, Wyoming, which has a population of less than one million, is more than large enough to make the scaling factor, $1/\sqrt{n_J}$, an accurate measure of the ability of an individual voter in Wyoming, compared with voters in different-size states, to be critical. True, the population of each state is not a precise reflection of the number of its citizens who actually voted in 2012, but for purposes of measuring the relative power of citizens in different states, it is a good approximation.

We start by calculating $B(J)$ in our 5-state example, wherein the states are assumed to be the players and have the same numbers of voters (2, 3, 4, 5, and 6) as electoral votes (see Table 5), which is too few to invoke the inverse square root law.[10] These calculations can be made directly from Tables 1 and 2 when X and Y need

(i) different numbers (6 for X, and 15 for Y)

(ii) the same number (11 for both candidates)

of the 20 electoral votes to win.

Table 5 about here

[10] For the 9 competitive states in the 2012 election, which have from several hundred thousand to many millions of voters, we later calculate and compare the scaled power of the voters as players.

In case (ii), more of the 32 winning coalitions are vulnerable than in case (i) (24 vs. 19), and there are also more critical defections in case (ii) (54 vs. 36). Thus, by putting the candidates on a par in the competitive states, one increases the vulnerability of winning coalitions and the criticality of states in them.

The power values of the 5 states are quite close, if not identical, in the two cases. Only the power of states *C* and *D* differ in cases (i) and (ii); for the other three states, the power values are the same. In fact, the values in the smallest (*A*) and largest (*E*) states are not only identical but also proportional to the electoral votes of the states: *E* has thrice the weight (i.e., numbers of voters), and thrice the voting power, of *A*.

But in neither case (i) nor (ii) are the power values equal to the weight proportions. For example, *A* has 2/20 = 1/10 of the weight but 1/9 of the voting power in both cases. Also observe that states with different numbers of electoral votes can have the same voting power, so the greater weight of a state does not necessarily give it with more voting power.

B(*J*)'s of the 9 competitive states in the 2012 election are shown in the top half of Table 6. As in the 5-state hypothetical example, we give these values when the two candidates need

(i) different numbers (33 for Obama, 79 for Romney)

(ii) same number (56 for both candidates)

of the 110 electoral votes to win. As in the 5-state example, there are more winning and tied coalitions (512) that are vulnerable (404) in case (ii) than in case (i) (255), and there are also more critical defections in case (ii) (1,032) than in case (i) (404). Once again, leveling the playing field increases the vulnerability of winning coalitions and the criticality of states in them.

Table 6 about here

The $B(J)$'s of the 9 states are never the same in cases (i) and (ii), unlike the 5-state example. In case (i), the proportion of voting power of the largest competitive state, Florida (0.242), is somewhat less than its proportion of electoral votes ($29/110 \approx 0.264$), but in case (ii), Florida's proportion of voting power (0.306) exceeds its electoral-vote proportion. Indeed, Florida is the only state to exercise more voting power in case (ii) than in case (i), suggesting how a large competitive state benefits from an even split of the noncompetitive states.

How do the $B(J)$'s for states compare with the $\beta(J)$'s, for voters, based on the populations of the 9 competitive states? Because the $\beta(J)$'s of individual voters are very small numbers, we give *normalized values*, $N\beta(J)$'s (N for "normalized"), fixing the power of a voter in the state with the least voting power, Iowa, at 1. Iowa voters are more disadvantaged than Nevada voters, which also has 6 electoral votes, because Iowa, based on the 2010 census, has more voters per representative. Iowa's voters also do not benefit as much, proportionally, as New Hampshire voters do from every state's having 2 electoral votes for its 2 senators, which gives New Hampshire even fewer voters per representative than Iowa or Nevada.

As shown in the bottom half of Table 6, voters in the largest states enjoy the greatest benefit, with Florida voters having 1.76 times as much voting power, per capita, as Iowa voters in the 2012 election. This advantage of living in a large state would be magnified if each candidate had needed the same number of electoral votes from the competitive states to win (i.e., 56), because then Florida's per-capita advantage over Iowa voters would then be 2.64.[11] Although individual voters in Florida are less likely to be critical in their state, their loss in criticality is more than compensated for by the many more electoral votes (29) they, rather than Iowa voters (6), can potentially swing.

The Banzhaf power values aggregate the critical defections of states, and the critical defections of voters in them, for both candidates. In doing so, they hide the fact that some states and their voters were much more critical for Romney than for Obama.

Florida, for example, was critical in 74 of Romney's 76 winning coalitions (97.4%)—only the grand coalition of all 9 states, and the 8 states other than Florida, if they had they voted for Romney, were invulnerable to Florida's defection—whereas Florida was critical in only 79 of Obama's 431 winning coalitions (18.3%).

In sum, Romney desperately needed to win Florida, whereas Obama could afford to lose it, in that 81.7% of Obama's winning coalitions did not include it. Nevertheless, both candidates devoted substantially more resources to Florida than any other competitive state. In the election itself, Florida turned out to be the state with the closest winning margin (Obama won by 0.88%).

The nine competitive states plus Pennsylvania had the ten closest margins in the country, indicating in retrospect that the candidates knew well where to spend their campaign resources to greatest advantage. Unfortunately for Mitt Romney, the cards

[11] If the largest state, California, were competitive, the voting power of its voters since the 1960s has exceeded that of voters in the smallest states by more than 3:1 (Banzhaf, 1968; Owen, 1975; Miller, 2013). A 2-person noncooperative game-theoretic model of optimal resource allocation under the Electoral College also shows that if California were a competitive state, its attractiveness per voter would exceed that of the smallest states also by more than 3:1, based on the "3/2's rule" (Brams and Davis, 1973, 1974).

were stacked against him in the noncompetitive states and, it turned out, the competitive states as well.

4. Conclusions

Contrary to conventional wisdom, the noncompetitive states in a U.S. presidential election *do* count, but in a different way from the competitive states. True, the major-party candidates target, almost exclusively, voters in competitive states, because it is these states that determine the outcome in most presidential elections.

In particular, the largest competitive states—Ohio and Florida in 2012—received, even per capita, inordinate attention. Their voters were, according to the Banzhaf index, far more critical than voters in small competitive states.

But it is the noncompetitive states who have what we called setup power: In 2012, there were 41 such states plus the District of Columbia that structured the outcome to substantially favor Barack Obama over Mitt Romney. More specifically, they afforded Obama 5.5 times as many ways to win the election in the 9 competitive states, and they rendered Romney's winning coalitions 2.4 times more vulnerable, and 5.5 more fragile, than Obama's, assuming that each competitive state was equally likely to vote for Obama or Romney.

If the noncompetitive states had split evenly, each candidate would have stood a 50% chance of winning the election, assuming that all the competitive states had been truly competitive (they were not, as we saw). Obama's benefited even further from his 46 electoral-vote lead in the noncompetitive states, which increased his winningness index, and decreased his vulnerability and fragility indices, substantially from the neutral case of an even split of the noncompetitive states. This shift in favor of Obama can be attributed to the setup power of the noncompetitive states, even though none of their voters could change, individually, the outcome in their state or the nation.

The head start that the noncompetitive states typically give one candidate does matter, even if the voters in these one-sided states cannot change the outcome. To be sure, individual voters in the competitive states actually have only a minuscule chance of influencing the outcome in their states and, through them, the nation, but because their states are seen as up for grabs, they become the so-called battleground states.

While their influence is substantial, however, the setup power of the noncompetitive states should not be underestimated. In 2012, they gave Barack Obama an overwhelming advantage, putting Mitt Romney in a catch-up position from which he could not win even if had won the same number of electoral votes in the competitive states as Obama did.

Table 1

32 Splits, and Critical States in *X* and *Y*'s Winning Coalitions, in 5-State Example Wherein *X* Needs 6, and *Y* Needs 15, Electoral Votes to Win

Split	*X*	*Y*	Winner	# States in Winning Coalition	# States Critical
1	*ABCDE*		*X*	5	0
2	*BCDE*	*A*	*X*	4	0
3	*ACDE*	*B*	*X*	4	0
4	*ABDE*	*C*	*X*	4	0
5	*ABCE*	*D*	*X*	4	0
6	*ABCD*	*E*	*X*	4	0
7	*CDE*	*AB*	*X*	3	0
8	*BDE*	*AC*	*X*	3	0
9	*BCE*	*AD*	*X*	3	0
10	*BCD*	*AE*	*X*	3	0
11	*ADE*	*BC*	*X*	3	0
12	*ACE*	*BD*	*X*	3	0
13	*ACD*	*BE*	*X*	3	0
14	*ABE**	*CD*	*X*	3	1
15	*ABD**	*CE*	*X*	3	1
16	*ABC**	*DE*	*X*	3	1
17	*DE**	*ABC*	*X*	2	1
18	*CE**	*ABD*	*X*	2	1
19	*C*D**	*ABE*	*X*	2	2
20	*BE**	*ACD*	*X*	2	1
21	*B*D**	*ACE*	*X*	2	1
22	*B*C**	*ADE*	*X*	2	2
23	*AE**	*BCD*	*X*	2	1
24	*A*D**	*BCE*	*X*	2	1
25	*A*C**	*BDE*	*X*	2	2
26	*AB*	*C*D*E**	*Y*	3	3
27	*E**	*ABCD*	*X*	1	1
28	*D*	*A*B*C*E**	*Y*	4	4
29	*C*	*A*B*D*E**	*Y*	4	4
30	*B*	*AC*D*E**	*Y*	4	3
31	*A*	*BC*D*E**	*Y*	4	3
32		*ABCDE**	*Y*	5	1

Note: A defection by a state with an asterisk in one of *X*'s or *Y*'s winning coalitions causes it to lose. 13 of *X*'s 26 winning coalitions (50%), and 6 of *Y*'s 6 winning coalitions (100%), are vulnerable, which are the *V*(6) and *V*(15) values. *X*'s winning coalitions have an average of 16/26 ≈ 0.69 critical states, and *Y*'s have an average of 18/6 = 3 critical states, which are the *F*(6) and *F*(15) values.

Table 2

16 Splits (#7 and #16 Are the Same), and Critical States in *X*'s Winning and Tied Coalitions, in 5-State Example Wherein *X* (and *Y*) Need 11 Electoral Votes to Win

Split	*X*	*Y*	Winner	# States in *X*'s Winning or Tied Coalition	# States Critical
1	*ABCDE*		*X*	5	0
2	*BCDE*	*A*	*X*	4	0
3	*ACDE*	*B*	*X*	4	0
4	*ABD(E)*	*C*	*X*	4	½
5	*ABCE**	*D*	*X*	4	1
6	*AB(C)D**	*E*	*X*	4	1½
7	*A*B*D**	*C*E**	Tie	3	1½
8	*C(D)E**	*AB*	*X*	3	1½
9	*BD*E**	*AC*	*X*	3	2
10	*(B)C*E**	*AD*	*X*	3	2½
11	*B*C*D**	*AE*	*X*	3	3
12	*AD*E**	*BC*	*X*	3	2
13	*(A)C*E**	*BD*	*X*	3	2½
14	*A*C*D**	*BE*	*X*	3	3
15	*A*B*E**	*CD*	*X*	3	3
16	*C*E**	*A*B*D**	Tie	2	1
17	*D*E**	*ABC*	*X*	2	2

Note: A defection by a state with an asterisk in one of *X*'s winning or tied coalitions causes it to lose and is counted as 1, except in tied splits #7 and #16, where it counts as ½. A defection by a parenthesized player in splits #4, #6, #8, #10, and #13 causes *X* to tie with the *Y* coalition and is also counted as ½. 13½ of *X*'s 16 winning and tied coalitions (78.1%) are vulnerable to either kind of defection (#4 is half vulnerable), which is its *V*(11) value; the 16 coalitions have an average of 27/16 ≈ 1.69 critical states, which is its *F*(11) value.

Table 3

Index Values, and Percent Changes from Baselines Values, in 5-State Example

Index	*X* $N = 6$	*Y* $N = 15$	Absolute Difference	Ratio
Winningness: $W(N)$ **—Change from $W(11)$**	81.25% (+62.5%)	18.75% (–62.5%)	62.5%	4.3 (X/Y)
Vulnerability: $V(N)$ **—Change from $V(11)$**	50% (–36.0%)	100% (+50%)	50%	2 (Y/X)
Fragility: $F(N)$ **—Change from $F(11)$**	0.69 (–59.2%)	3 (+77.5%)	1.77	4.3 (Y/X)

Table 4

Index Values, and Percent Changes from Baselines Values, in 2012 Presidential Election for 9 Competitive States

Index	Obama $N = 33$	Romney $N = 79$	Absolute Difference	Ratio
Winningness: $W(N)$ —Change from $W(56)$	84.7% (+69.4%)	15.3% (–69.4%)	69.4%	5.5 (O/R)
Vulnerability: $V(N)$ —Change from $V(56)$	40.7% (–48.7%)	98.7% (+24.5%)	51.0%	2.4 (R/O)
Fragility: $F(N)$ —Change from $F(56)$	0.73 (–63.4%)	4.02 (+49.8%)	3.29	5.5 (R/O)

Table 5

Relative Banzhaf Power Values of 5 States in Hypothetical Example

States as Players	*A* (2 votes)	*B* (3 votes)	*C* (4 votes)	*D* (5 votes)	*E* (6 votes)
***X* needs 6, *Y* needs 15**	4/36 = 1/9 0.111	4/36 = 1/9 0.111	8/36 = 2/9 0.222	8/36 = 2/9 0.222	12/36 = 1/3 0.333
***X* and *Y* need 11**	6/54 = 1/9 0.111	6/54 = 1/9 0.111	10/54 = 5/27 0.185	14/54 = 7/27 0.259	18/54 = 1/3 0.333

Note: When *X* needs 6 and *Y* needs 15 votes to win, a total of 19 winning coalitions or tied of *X* and *Y* are vulnerable, in which the 5 states are critical a total of 36 times. When *X* and *Y* both need 11 votes to win, a total of 24 winning or tied coalitions of *X* and *Y* are vulnerable, in which the 5 states are critical a total of 54 times.

Table 6

Relative Banzhaf Power Values of 9 Competitive States in 2012 Election

Case	NH (4)	NV (6)	IA (6)	CO (9)	WI (10)	VA (13)	NC (15)	OH (18)	FL (29)
States as Players: *B(J)*									
***O/R* need 33/79**	0.040	0.055	0.055	0.081	0.090	0.122	0.138	0.176	0.242
***O/R* need 56/56**	0.035	0.047	0.047	0.078	0.085	0.116	0.128	0.159	0.306
Voters as Players: $N\beta(J)$									
***O/R* need 33/79**	1.08	1.06	1.00	1.14	1.20	1.36	1.41	1.63	1.76
***O/R* need 56/56**	1.14	1.06	1.00	1.29	1.33	1.153	1.154	2.36	2.64

Note: When *O*/*R* need 33/79 votes to win, a total of 255 winning or tied coalitions are vulnerable, in which the 9 states are critical a total of 631 times. When *O*/*R* need 56/56 votes to win, a total of 404 winning or tied coalitions of *X* and *Y* are vulnerable, in which the 9 states are critical a total of 1,032 times.

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