Efficient Microlending without Joint Liability

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Abstract

Peer-group mechanisms have been widely used by micro-credit institutions to minimize default risk. However, there are costs associated with establishing and maintaining liability groups. In the case when output is fully observable, we propose a dynamic individual lending mechanism. Assuming that risky borrowers discount the future costs and benefits relatively higher, our mechanism performs equally well in repayment rates, distinguishes safe and risky borrowers through differentiated interest rates and payment schedules. In case of unobservable types, it is able to eliminate adverse selection problem, and it reaches the first best outcome of the case that types of borrowers are publicly known. It improves wealth of individuals, and hence achieves a net welfare-superior outcome when compared with joint liability. Individual lending further saves from internal costs of group formation, and broadens the fractions of society into which microfinance institutions penetrate. We also identify unique welfare maximizing contract in our mechanism. Finally, we introduce a history dependent success probabilities, and show existence of efficient individual contract in that environment.

Keywords: Microfinance, Graamen bank, joint liability, adverse selection, microlending, group lending, individual lending

JEL Classification Numbers:

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1 Introduction

Microfinance institutions (MFI) mostly rely on joint liability (JL) mechanisms and peer-group effects to minimize the risk, reduce monitoring costs and achieve high turnout rates (Yunus, 1999). Participants are required to form a liability group (of varying sizes) to be eligible for a credit. It is argued that this innovation was instrumental in microfinance's success (Armendariz de Aghion and Morduch, 2010; Daley-Harris, 2009; Morduch, 1999) and provide social collateral. Empirical evidence on that is mixed though. Al-Azzam *et al.* (2012) documented evidence for positive effects of peer monitoring on repayment rates. Attanasio *et al.* (2011), Gine and Karlan (2014) and Godquin (2007) found that the repayment rate does not significantly differ between individual and group lending. Wydick (1999) also found that group pressure is not significant in dealing with moral hazard. Gine *et al.* (2006) observed that "the group-based mechanisms that are frequently employed can induce moral hazard (or more risk-taking behavior) instead of reducing it.". Benarjee (2013) also argues about that the theory is missing that monitoring the others with no control on them increases free riding of others safe choices, and increase risk taking. Fischer (2010) with an experiment also concludes that JL stimulates risk taking further. Thus, we conclude that a model of group lending with *ex ante*types may fail to capture this effect of altering behaviors of individuals after group formation. With this in hand, we also know that Graamen, ACCION and BancoSol have been the pioneers of the group lending with different mechanisms, whereas, now they are converting from group to individual lending. The trend is same for Europe, El Salvador, Peru and many other countries.

There are potential sources of inefficiencies pertaining the group lending mechanisms. Group requirement could cause an inefficiency by leaving out those who would have succeeded in availability of credit but do not have access to it because they couldn't form a group[1]. Indeed, Gine and Karlan (2014) found that "The conversion to individual liability did lead to larger lending groups, hence further outreach"(82). Furthermore, microcredit institutions offer a single loan scheme with a flat interest rate to all borrowers. However, both within groups and across groups, debtors differ in ability and potential to succeed. If credits were offered to individuals, adverse selection could be the result. Peer-groups are meant to induce cooperative behavior and keep turnout high. But there are costs associated with forming and maintaining groups, which raises an efficiency concern. Bhatta and Tang (1998) also mention about that in some cases MFI also involves in group formation mechanism which adds more administrative costs to the institution.

Several studies investigated properties of joint-liability mechanisms. Besley and Coate (1995) showed that JL schemes may have positive and negative effects on repayment rates. Successful members will have an incentive to cover for those who couldn't pay back. Liability works in other way if the whole group defaults ("strategic default"). Armendariz de Aghion (1999) showed that individual lending could outperform group lending if peer monitoring is very costly and social sanctioning abilities are limited. Another problem Besley and Coate (1995) points to is collusion among borrowers in group lending, which demands more complex punishment mechanisms with potentially further monitoring costs. Our mechanism replaces individual liability while maintaining same expected return for the bank. So, it does not suffer from potential drawbacks of group liability.

Another strand of studies argue about the efficiency of JL and societal characteristics. Aghion and Morduch (2000) infer that group lending may be a poor mechanism for the industrialized societies. Ladman and Acfha (1990) and Arene (1993) conclude that lack of cooperative behavior among members of the society give rise to failures of gruop lending in Bolivia and Nigeria, respectively. Moreover, Gine *et al* (2011) effect on religion on repayment rates of group lending. Ahlin and Townsend (2007) shows that social structures that decrease penalties may reduce the repayment. Cassar *et al.* (2007) and Azzam *et al.* (2011) infer that the specific nature of society affects repayment performance differently. The latter found

[1] In general there are large number of peers requested for group lending. For instance, Graamen requires 5 people, Indian system for 6-10 people, and 5 for Turkey case.

that percentage of relatives in group may worsen the repayment rate, since it is hard to enforce relatives in some socities. These require a stirct analysis of the traditions and life style of the society in designing group mechanism which does not seem very practical. Bhatt and Tang (1998) emphasize the effect of geographic proximity on the success of group lending due to potentially high costs of communication and monitoring of peers. Noballa (1992) and Azzam *et al.* (2011) represent the evidence of this kind of negative impacts. Addilitonally, JL in general depends on that *ex-ante* borrowers know each others types which is a hidden information in terms of banks. This assumption is even harder to the members who are not geographically close. All these reveal the difficulties for implementing "true" group lending mechanism for different societies, and thus pave the way for individual lending.

For borrowers' side, group formation costly, internal cost of this. The amount of this cost may prevent the borrower to go to bank and it depends on the social environment she has. If they do not know each other for a long time, they may not have an idea about the risk of the others projects, which is assumed in theory. and they cannot trust. Also as discussed, Close relatives cannot apply sanctions to each others in some societies.

Rai and Sjostrom (2004) argued that a JL mechanism with a cross-reporting scheme leads to a better risk-sharing and less defaults. Bhole and Ogden (2010) similarly discussed the importance of cross-reporting mechanisms and found that in the absence of sanctions, group lending does worse than individual lending in repayment rates. Bond and Rai (2008) relied on social sanctions to enforce payments. Given the costly nature of such sanctions, it would be an improvement if we can substitute individual liability instead of group liability while maintaining same returns. Our dynamic mechanism addresses this problem. Bond and Rai (2008) proposes "cosigned loans" as an alternative to group loans when sanctioning abilities differ among individuals. Our individual lending mechanism produces better results in terms of welfare than JL schemes, independent of sanctioning abilities.

Individual lending as an alternative to group-lending were not studied as extensively as group lending mechanisms. One notable exception is Armendariz de Aghion and Morduch (2000). They propose mechanisms with more stringent direct monitoring and restricted payment schedules that brings an effect equivalent to peer groups. Assuming that riskier types discount the future more, our mechanism introduces differentiating interest rates and repayment schedules which achieves same outcome without further direct monitoring.

Bhatt and Tang (1998) extensively discusses transaction costs in group lending. They point to the "...costs associated with screening potential group members, group formation, agreeing on formal or informal group rules..." (624). Shatragom and Bayer (2013) argues in similar vein. The mechanism we develop addresses transaction cost concerns in two ways. We require no group formation so there is no fixed cost of eligibility. Also in equilibrium, the loan contract that bank offers leads to a self-selection of agents (safe and risky types) with respect to their success probabilities. This differentiation reduces bank's monitoring costs and improves welfare.

In this paper, we address these issues by proposing a dynamic credit mechanism. In our model, risk-neutral individuals borrow in period 0 and pay back in (not necessarily equal) periodic installments, raising revenue in each period from their initial investment. We assume that the creditor can perfectly observe debtors' revenues whereas types of individuals are private information for others and bank. We show that in a dynamic setting where agents discount future benefits and costs differently, it is possible to offer individual credit contracts with differing interest rates and pay-back schemes which improve welfare and perfectly distinguishes among types of individuals in equilibrium. Following Rai and Sjostrom (2004), bank's expected revenues from both safe and risky types of individuals are equal so bank is equally likely to extend credit to both types. This screening through contracts remedies potential adverse selection problem and achieves an improvement in welfare when compared with the equivalent JL case. Our mechanism allows for individual lending and hence, saves on monitoring costs associated to group lending. It also extends the reach of credit opportunities by relaxing the group requirement. Benarjee (2013) addresses the new directions for future reseach on theoretical side. He discusses the significance of identifying optimal dynamic credit contract under the presence of behavioral aspects. In line with Banerjee (2013)'s agenda, we also investigate the impact of JL in dynamic contracts and show that individual liability mechanism we offer strictly dominates an equivalent JL mechanism in terms of welfare. The dynamic aspects of our mechanism are essential. In the practice, microfinance institutions sign a dynamic repayment scheme with their borrowers in weekly or monthly terms. In many practical cases, payments starts in general as soon as borrowing is done in the following week or month. Fischer (2011) hints that just a 2 month delay for repayment makes a large effect on income of borrowers. Indeed, the wedge between discount factors allowed a welfare-superior individual liability credit contract. Ghatak and Guinnane (1999) and Chowdhury (2005, 2007) work in sequential environments. In our dynamic model, we assume that output realizations occur every period and borrowers discount future. In case of that types of borrowers are not private information by the lender, there is no problem and bank can charge perfectly desired interest rates to each type. This yields the first best outcome with distinct rates for distinct types. On the other side, if it cannot observe the types of borrowers, a unique contract may pose adverse selection problem. We assume types of agents are not seen by lender, and differentiate the payment schemes. We introduce two schemes and first alter period-wise payments of only one type of contract. Using our assumption that safe type discounts less than risky type, we solve adverse selection problem. Hence, we achieve exactly to the best outcome in which types are observed by the bank. We also improve welfare of both individuals and society when compared with the JL case in which bank gets same expected repayment. Second, we also alter the other contract and introduce a new payment scheme which further increase individual welfares. It is also the welfare-maximizing contracts for the society. We finally extend our environment to a more general one. We relax the history independent success probabilities, and add a memory to the likelihood of success of projects. In this new environment, we also show the existence of same kind of contract with individual lending. To the best of our knowledge, this study is the first in the microlending literature, by incorporating dynamic payment scheme and discount factors of individuals, which at the end reaches the first best solution with individual lending mechanism.

Our mechanism screens safe and risky types under an individual lending scheme without a loss to the bank and society. Ghatak (1999,2000) showed that with JL and self-selection, safe borrowers gather together to form credit cooperatives and risky borrowers are screened out. We achieve the same equilibrium outcome without JL. Ghatak and Guinnane (1999) discusses moral hazard problems in group-lending. We've already discussed the cost issues of such peer-monitoring mechanisms. Thus, there is a clear efficiency gain from replacing JL with individual liability in case of observable outputs.Table 1 represents the summary of parameters.

Parameter	Definition
ρ	Competetive interest rate for 2 periods
β_i	Discount factor of type i agents
p_i^t	Success probability of type i agent for period t
R_i	Interest rate charged to type i agents
R_t^i	Interest rate charged to type i agents at period t
X	Output in case of success in each period
θ	Fraction of safe tyoe agents in society
γ	History effect on the success at $t = 2$

Table 1: Summary of parameters

The rest of the paper is organized as follows: Section 2 introduces the model. Section 3 introduces the benchmark individual contract shceme with welfare discussions. Section 4 identifies the welfare maximizing one among these individual contracts. Finally, in Section 5, we change the environment with history dependency and identify the contract in this environment. Section 6 concludes.

2 Model

There are two types of potential borrowers, safe and risky. θ fraction of all borrowers are of safe type whereas $(1-\theta)$ fraction of them are risky. Types are private information of each borrowers herself. Neithr other borrowers nor the lender does not know the type of any borrower. Each type borrows from the same lender (MFI), and invest their funds in their businesses. We assume output for projects are same regardless of types. Investment of each borrower either yields a revenue of X with a positive probability or nothing for each period. Hence, if a borrower succeeds for both periods, she receives $2X$ as output. We assume that the lender observes this output for each period. Following Rai and Sjstrm (2004), we assume poor people have no assets, then they are not supposed to repay anything in case of failure. Borrowers are required to pay (gross) interest on their loan only when the outcome is success. Let p_s and p_r be the success probabilities of safe and risky types, respectively and for each period. We assume that $p_s > p_r$.

Let ρ be the two-period market interest rate, which is defined as the total amount a borrower need to pay back for \$1 credit. Following Rai and Sjostrom (2004), we assume that microcredit institution in a competetive market sets interest rates R such that the expected revenue is market rate. This may be also called as zero profit constraint (ZPC) for MFI. Thus, it does not extract profit for itself, and supposed to maximize welfare of society. In the case of a uniform credit contract, this corresponds to $pR = \rho$ where $p = \theta p_s + (1-\theta)p_r$ (i.e. average success probability for the society). Let R^s and R^r be the interest bank is willing to charge to safe and risky types, respectively if it can observe the types. That is, there rates are able to perfectly differentiate the types and there is no adverse selection problem. Then, ZPC implies $p_s R^s = p_r R^r = pR = \rho$. We note that since $p_s > p_r$, $R_s < R_r$.

We construct a dynamic framework in which agents borrow in period 0, invest in their businesses and repay the interest in two (not necessarily equal) periodic installments. R_t^i denotes the payment type i is supposed to make at time t according to her contract. Following diagram illustrates the timeline:

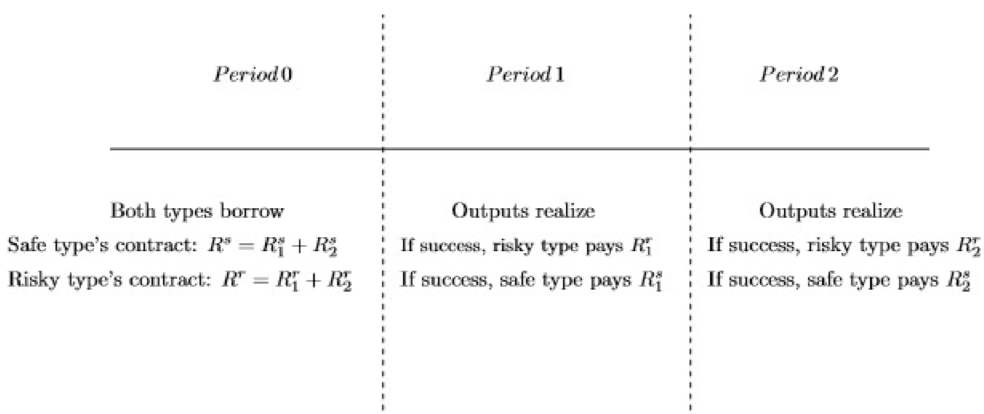

Figure 1: Timeline

The success or failure of type i at $t = 1$ does not affect her success and fail probability for the following period. The intiution behind this is that, a borrower may produce some hand-made products like in a village economy, and try to sell those. Having made the investment, the probability that she can sell her products is approximtely same for each time period. Agents are risk-neutral. In a given period, net utility of a borrower type i is $p_i(X - R_t^i)$. Safe and risky borrowers discount future benefits with β_s and β_r respectively. We assume that $\beta_s \geq \beta_r$, which is, as we will show, a necessary condition for existence of differentiated contracts that are incentive compatible. One can interpret this as follows: safe individuals are more likely succeed in future periods so they tend be concerned less about immediate gratification. With these in hand, expected utility of agent i with individual contract (H_1^i, H_2^i) as follows:

$$EU_i(H_1^i, H_2^i) = p_i\beta_i(X - H_1^i) + p_i\beta_i^2(X - H_2^i) \ \ \forall\, i \in \{s, r\}$$

Given the fraction of safe types in the society θ and payment schemes for both type (H_1^s, H_2^s) and (H_1^r, H_2^r), welfare of the society is given as follows:

$$\begin{aligned} EW_{soc}(H_1^s, H_2^s, H_1^r, H_2^r) = \theta(p_s\beta_s(X - H_1^s) + p_s\beta_s^2(X - H_2^s)) + \\ (1-\theta)(p_r\beta_r(X - H_1^r) + p_r\beta_r^2(X - H_2^r)) \end{aligned}$$

Given that types are not observable and $p_sR^s = p_rR^r = pR = \rho$, if agents were not to discount future benefits and costs, safe type's contract has strictly better terms. Both types would prefer safe type's contract. It creates the adverse selection problem in static environment, and gives incetive to the bank to charge a global rate R. Eventually, since $R_s < R < R_r$, safe types are worse off and risky types are better off. In what follows, in dynamic environment, we will demonstrate that, if $\beta_s \geq \beta_r$, there exists an incentive compatible contract scheme such that agents of each type prefer their own contracts (agents are screened according to their types). We also show that such a scheme requires $R_1^s > R_2^s$. Intuitively, we exploit the difference between discount factors. By sufficiently increasing the early installment of safe type's contract, it is possible to make it unattractive to risky type even though $R^r > R^s$, i.e. risky type's contract requires a higher undiscounted total payment. Later, we show the welfare gain of our individual lending mechanism over the corresponding JL schemes.

3 Dynamic Contract with Observable Output

We now propose a dynamic contracting mechanism which addresses hidden types i.e. the adverse selection issue. Initially, we will fix risky types repayment scheme $\{R_1^r, R_2^r\} = \{\frac{R_r}{2}, \frac{R_r}{2}\}$, i.e. a standard repayment schedule in practice, and establish the existence of $\{R_1^s, R_2^s\}$ that satisfies the requirements provided in the following proposition.

Without loss of generality, we fix outside option to 0 for all types. Hence, individual rationality constraints for safe and risky types are as follows:

$$IR(s) : EU_s(R_1^s, R_2^s) \geq 0 \tag{1}$$

$$IR(r) : EU_r(R_1^r, R_2^r) \geq 0 \tag{2}$$

Solving IR's of both types we get the minimum level of output:

$$X \geq \frac{R_1^s(1-\beta_s) + \beta_s R_s}{1-\beta_s}$$

$$X \geq \frac{R_r}{2}$$

We endogenously get $\frac{R_r}{2} \leq X$ which is the condition that period-wise payment for risky type cannot be greater than period-wise payment. It is also received in aggregate terms for risky type. In addition, since $R_s < R_r$, the result in aggregate terms is valid for safe type, too. We additionally assume $X \geq R_t^s$ for $t = 1, 2$, since poor people are assumed to have no endownments. We also need incentive compatibility constraints to induce each agent to prefer the credit contract suited for her own type:

$$IC(s) : EU_s(R_1^s, R_2^s) \geq EU_s(R_1^r, R_2^r) \tag{3}$$
$$IC(r) : EU_r(R_1^r, R_2^r) \geq EU_r(R_1^s, R_2^s) \tag{4}$$

where we fix $R_t^r = \frac{R_r}{2}$ in the contract as in most of real-world contracts. We only allow to choose the reallocation of safe type's payment scheme. By solving these IC's,

$$\frac{\frac{p_s}{2p_r}(1+\beta_s) - \beta_s}{1-\beta_s} R_s \geq R_1^s \geq \frac{\frac{p_s}{2p_r}(1+\beta_r) - \beta_r}{1-\beta_r} R_s \tag{5}$$

As long as $\beta_s \geq \beta_r$, there exists R_1^s which can implement the contract. The consequences are represented in following proposition.

Proposition 1 *For all $\beta_i, p_i \in (0,1)$, $\beta_s \geq \beta_r$ and $p_s > p_r$ with sufficiently large output level X, there exists a two-period repayment schedule $\{R_1^s, R_2^s, R_1^r, R_2^r\}$ s.t.*

1. *$\{R_1^r, R_2^r\} = \{\frac{R^r}{2}, \frac{R^r}{2}\}$*
2. *Safe type agents prefer $\{R_1^s, R_2^s\}$*
3. *Risky type agents prefer $\{R_1^r, R_2^r\}$*
4. *Bank's expected revenue from both type is the same*

where R_1^s satisfies (5), and $R_2^s = R_s - R_1^s$ with ZPC.

Proof. See appendix. ∎

Proposition 1 shows the existence of a contracting scheme that screens agents according to their types. We note that MFI can implement this scheme with only chosing one variable R_1^s. In the next section, we investigate the welfare properties of this mechanism. We also note that the case $\beta_s = \beta_r$, the contract is unique. We also note that there may be cases in which $R_1^s > R_s$. This means safe type may pay more than total amount in the first period, then she receives some amount back at the final period. Our assumption $X > R_1^s$ makes this kind of situation possible. On the other hand, there is an upper bound for β_r which gives rise to that $R_1^s \leq R_s$ always holds[2].

We now turn to welfare issues. In what follows, we first show that our becnhmark payment interval maximizes the expected welfare of th society when we set R_1^s ot its lower bound. By doing so, we receive the maximum welfare case of our becnhmark contract. We then compare the differentiated repayment schemes we propose is superior to JL contracts in terms of both social and individual welfares.

[2]See Appendix.

Claim: $R_1^{s*} = \frac{\frac{p_s}{2p_r}(1+\beta_r)-\beta_r}{1-\beta_r} R_s$ must be the choice in the benchmark contract in order to get the maximal level of safe type utility, and hence maximize the societal welfare.

Since societal welfare in this case is as follows,

$$EW_{soc}(R_1^s, R_2^s, \frac{R_r}{2}, \frac{R_r}{2}) = \theta p_s \beta_s((\beta_s+1)X - \beta_s R_s + (\beta_s - 1)R_1^s)$$
$$+(1-\theta)p_r\beta_r((\beta_r+1)X - R_r\frac{\beta_r+1}{2})$$

Since $\beta_s < 1$, the minimal value of R_1^s the one that maximizes the expected utility of the safe type. Due to uniform scheme of risky type, it is also welfare maximizing choice for the society.

3.1 Individual vs. Joint Liability

Since our contract has the ability to differentiate types as in the case that types are common knowloedge, it improves the wealth when compared with the uniform contract with (R_1, R_2) and where $R_1 + R_2 = R$. An alternative mechanism could make use of JL within a dynamic contract.

In JL, positive assortive matching is one possibility. Ghatak (1999) and van Tassel (1999) argue that it is more likely to occur in static environment. Moreover, the socially optimal matching may change in dynamic environment. Thus, we are not sure which is the welfare superior. As a result we take both cases of JL into account. In a 2-person group scheme, expected welfare of type i when she borrows with a group of j type is given as follows:

$$EU_i^{JL}(R_i^1, R_i^2, j) = p_i p_j \beta_i (X - R_1^i) + p_i(1-p_j)\beta_i(X - R_1^i - R_1^j) \quad (6)$$
$$+ p_i p_j \beta_i^2 (X - R_1^i) + p_i(1-p_j)\beta_i^2(X - R_2^i - R_2^j) \quad (7)$$

where j is type of her peer in the group. Positive assortive matching means $j = i$, whereas, negative asortive matching is that j and i are different types. As a result, social welfare in case of a general matching of i and j types is defined as follows:

$$EW_{soc}^{JL}(R_i^1, R_i^2, R_j^1, R_j^2, i, j) = \theta_i(p_i p_j \beta_i (X - R_1^i) + p_i(1-p_j)\beta_i(X - R_1^i - R_1^j) \quad (8)$$
$$+ p_i p_j \beta_i^2 (X - R_1^i) + p_i(1-p_j)\beta_i^2(X - R_2^i - R_2^j))+ \quad (9)$$
$$(1-\theta_i)(p_j p_i \beta_j (X - R_1^j) + p_j(1-p_i)\beta_j(X - R_1^j - R_1^i) \quad (10)$$
$$+ p_j p_i \beta_j^2 (X - R_1^j) + p_j(1-p_i)\beta_j^2(X - R_2^j - R_2^i)) \quad (11)$$

where θ_i is the fraction of i tyoe in the society. On the other hand, following proposition shows that individual lending is welfare-superior to both cases of group lending.

Proposition 2 *For a given periodic repayment dynamic contract with differing time preferences of borrowers, individual liability credit scheme provided in Proposition 1 welfare dominates a JL scheme both individually and socially.*

Proof. See Appendix. ■

The proposition above implies that we need not concern ourselves with JL schemes once we implement a dynamic repayment scheme with varying time preferences of agents. This result is quite intuitive[3]. In

[3] The result is valid for all types of JL, not necessarily for only full liability case.

an environment where output is observable, JL mechanisms could be used to address adverse selection issues. However, the difference in time preferences allow us to construct a pair of contracts such that each agent self-selects herself to the contract suitable for her type. Implementing JL in this context brings no improvement in selection while burdening each agent with a partner's risks. Hence, individual liability mechanisms are superior in welfare[4]. We also illustrate the result with an example of initial parameters.

Example 1: In this example we show that our benchmark contract with payment scheme $(R_1^{s*}, R_2^{s*}, \frac{R_r}{2}, \frac{R_r}{2})$ improves welfare both individually and thus societally when compared with positive (i,i) and negative (i,j) assortive matching cases of JL. Exogenous parameters are $(\theta, p_s, p_r, \rho, \beta_r, \beta_s, X)$,

Exogenous parameters						
β_s	β_r	ρ	p_r	p_s	X	θ
0.9	0.7	1.05	0.7	0.8	1.2	0.6

Using $p_s R_s = p_r R_r = pR = \rho$, we calculate endogenous parameters,

Endogenous paramters			
R	R_s	R_r	p
1.3816	1.3125	1.5000	0.7600

Then selecting benchmark contract for safe types in case of individual liability to maximize societal welfare, we calculate $R_1^{s*} = 1.1875$, thus $R_2^{s*} = 0.1250$. We also note that $R_1^r = R_2^r$. With these in hand, we calcaluate and compare welfares with the JL in case of both types of matchings.

Welfare	Individual contract	JL (i,i) matching	JL (i,j) matching
Society	0.5735	0.3860	0.3386
Safe agent	0.7059	0.5184	0.3978
Risky agent	0.3749	0.1874	0.2499

As illustrated, individual contract scheme we proposed improves the welfare in significant levels. In our scheme society has expected welfare as 0.5735, whereas the levels are 0.3860 and 0.3386 in positive and negative assortive matching cases respectively of gruop lending with full liaibility. Additionally, individual based utilities are *ex antr* increased for both types. Safe type agent receives 0.7059 in our scheme, whereas she gets 0.5184 in maximal case of JL. The result is valid by comparing individual vs group lending with numbers 0.3749 and 0.2499, respectively. The impact of individual improvements on the societal increase depends on the composition of society i.e. θ. Moreover, the effect of the individual contract on each type is related to their p_i and β_i values.

4 The Contract with Welfare Evaluations

So far we restricted ourselves the optimal choice of safe type's contract $\{R_1^s, R_2^s\}$ keeping $\{R_1^r, R_2^r\} = \{\frac{R^r}{2}, \frac{R^r}{2}\}$. We demonstrated the welfare gain associated with differentiating safe type's contract. Now we

[4]Indeed, as argued in introduction, there are further transaction costs associated with JL mechanisms which we do not account for in our welfare calculations. Thus, the welfare improvement we characterized is a lower bound at best

let $R_1^r \neq R_2^r$, so the problem becomes choosing $\{R_1^r, R_1^s\}$. Since $R_2^i = R_i - R_1^i$, we do not count them as choice variables. Following proposition characterizes the new pair of efficient contracts and then get the ones that are welfare maxizing. We again have same IC and IR constraints, but not with $\frac{R_r}{2}$ but with (R_1^r, R_2^r). Solving those we get following restrictions:

$$X \geq \frac{R_1^i(1-\beta_i) + \beta_i R_i}{1-\beta_i} \quad \forall\ i \in \{s, r\}$$

$$R_s \frac{\beta_r(\frac{p_s}{p_r} - 1)}{1-\beta_r} \leq R_1^s - R_1^r \leq R_s \frac{\beta_s(\frac{p_s}{p_r} - 1)}{1-\beta_s}$$

As long as $\beta_s \geq \beta_r$, the contract is pmlementable. Following proposition states that it can be different from our benchmark contracting scheme.

Proposition 3 *For all $p_i, \beta_i \in (0,1)$ and $\beta_s \geq \beta_r$ with $p_s > p_r$ with sufficiently large output level X, there exists a two-period repayment schedule $\{R_1^s, R_2^s, R_1^r, R_2^r\}$ s.t. condition in Proposition 1 are satisfied and $\{R_1^s, R_2^s\}$ is not necessarily $\{R^r/2, R^r/2\}$*

Proof. See Appendix. ■

We also note that the case $\beta_s = \beta_r$, the contract is unique. The merit of flexibility in both types' contracts becomes apparent when we consider the welfare improvement. The societal welfare can be rewritten as:

$$EW_{soc}(R_1^s, R_2^s, R_1^r, R_2^r) = K + \theta p_s \beta_s(\beta_s - 1) R_s \frac{(1+\beta_r)\frac{p_s}{2p_r} - \beta_r}{1-\beta_r} + (1-\theta) p_r \beta_r(\beta_r - 1)\frac{R^r}{2}$$

where $K = \theta p_s \beta_s(1+\beta_s)X - \theta p_s \beta_s^2 R_s + (1-\theta) p_r \beta_r(1+\beta_r)X - (1-\theta) p_r \beta_r^2 R_r$.

As can be seen, we have negative coefficients with R_1^s and R_1^r, and restriction containing difference of the two. By taking $R_1^r = 0$, and also setting minimum R_1^s, we maximize the societal welfare, as well as that of individuals. Following proposition represents the result.

Proposition 4 *Fully differentiated contract pair with $\{R_1^{s*}, R_2^{s*}, R_1^{r*}, R_2^{r*}\}$ yields the maximum welfare both individually and socially, where $R_1^{r*} = 0$ and $R_1^{s*} = R_s \frac{\beta_r(\frac{p_s}{p_r} - 1)}{1-\beta_r}$. From ZPC, payments at $t = 2$ is found by $R_2^i = R_i - R_1^i$ for both types.*

Proof. See Appendix. ■

Thus, allowing one type contract with payment only at $t = 2$, and reallocating the other compared with benchmark contract, we identify the welfare maximizing one for both individuals and the society. Proposition 4 implies that it is possible to improve welfare further if we allow risky type's contract to have varying period installments. In practice, it is common among microcredit institutions to demand uniform installments. Our result, however, suggests otherwise in the presence of heterogeneous time preferences. We illutrate welfare increase with an example of benchmark paramaters:

Example 2: We illustrate that our new contract (with R_1^r choice) further improve the welfare in both individual and societal levels when compared with the benchmark contract we represented. We use same parameter values as in Example 1.

Welfare	Contract with (R_1^{s*}, R_1^{r*})	Benchmark contract
Society	0.6498	0.5735
Safe	0.7596	0.7059
Risky	0.4851	0.3749

We observe that for this set of parameters, *ex ante* societal welfare jumps from 0.5735 to 0.6498 with this further choice in the contracting scheme. Moreover, both risky and safe types receive increases in their utilities as 0.0537 and 0.1102, respectively. We note that relative levels of individual welfare improvements depend on p_i and β_i values. In addition, θ determines the effect of individual enhancements on the overall improvement of welfare.

5 History dependent probabilities

We now turn our attention into a scenario in which the outcome of the first period -being either success or fail- has an affect on the probability of success in the second period. This is an attempt to introduce a more realistic model. We define γ as the memory dependence of the projects of both types. At $t = 2$, the probability of success and fail for each type is now conditional on the outcome at $t = 1$. We introduce symmetric effect of past on $t = 2$ with γ. Now $p_{1,i} \neq p_{2,i}$ there $p_{t,i}$ is the success probability of type i at time t. If agent of type i succeeds at $t = 1$, $p_{2,i} = p_{1,i} + \gamma$, otherwise, it is $p_{2,i} = p_{1,i} - \gamma$. Thus the nature of dependency is same for each type and bad or good news from period 1 affects in same amount.

The second periods probabilty of success is now kind of an expected probability of success for either type, and it is equal to $p_2^i = 2p_1^i\gamma + p_1^i - \gamma$. We can also summarize these like below:

$$p_{2,i}(Success\ at\ t = 2|Success\ at\ t = 1) = p_{1,i} + \gamma$$
$$p_{2,i}(Success\ at\ t = 2|Failure\ at\ t = 1) = p_{1,i} - \gamma$$

Note that $p_s > p_r$ implies $p_{s,2} > p_{r,2}$. In this environment, expected utility of agent type i is the following:

$$EU_i = \beta_i p_i(X - R_1^s) + \beta_i^2(p_i(p_i + \gamma) + (1 - p_i)(p_i - \gamma))(X - R_2^i) \tag{12}$$

This extension alters ZPC of bank, and IR's as well as IC constraints of the individuals. Here is ZPC of the bank with this manipulation:

$$p_i R_1^i + (p_i(p_i + \gamma) + (1 - p_i)(p_i - \gamma))R_2^s = \rho \quad \forall\ i\ \in \{s, r\} \tag{13}$$

Solving ZPC above for both types gives the second periods' repayment i.e.

$$R_2^i = \frac{(R_i - R_1^i)p_i}{2\gamma p_i + p_i - \gamma}$$

IR(s):

$$p_s\beta_s(X - R_1^s) + \beta_s^2 \underbrace{(2p_s\gamma + p_s - \gamma)}_{p_{s,2}}(X - R_2^s) \geq 0$$

IR(r):

$$p_r\beta_r(X - R_1^r) + \beta_r^2 \underbrace{(2p_r\gamma + p_r - \gamma)}_{p_{r,2}}(X - R_2^r) \geq 0$$

Solving the individual rationality constraints above gives a lower bound for the level of output X :

$$X \geq \frac{p_i R_1^i + R_2^i(2p_i\gamma\beta_i + p_i\beta_i - \gamma)}{p_i + p_i\beta_i + 2p_i\gamma\beta_i - \gamma} \tag{14}$$

where $R_2^i = \frac{(R_i - R_1^i)p_i}{2\gamma p_i + p_i - \gamma}$.

The new incentive compatibility constraints are now;

IC(s):

$$p_s\beta_s(X - R_1^s) + \beta_s^2(2p_s\gamma + p_s - \gamma)R_2^s \; \geq \; p_s\beta_s(X - R_1^r) + \beta_s^2(2p_s\gamma + p_s - \gamma)(X - R_2^r)$$

IC(r):

$$p_r\beta_r(X - R_1^r) + \beta_r^2(2p_r\gamma + p_r - \gamma)R_2^r \; \geq \; p_r\beta_r(X - R_1^s) + \beta_r^2(2p_r\gamma + p_r - \gamma)(X - R_2^s)$$

Note that the only difference from the previous incentive compatibility constraints is in the second periods' probability of success. Following proposition states the existence of the efficient individual lending contracting schemes in this environment.

Proposition 5 *For all $p_i, \beta_i \in (0,1)$, $p_s > p_r$, and sufficiently large output level X, if history effect is appropriate*[5]*, there exists a fully differentiated contract pair for both types; who has history dependent probabilities of success $(p_{s,2}, p_{r,2})$, which gives the maximum welfare both individually and societally.*

Proof. Solving $IC(s)$, and $IC(r)$, above; gives

$$R_1^s(1 - \beta_s) - R_1^r(1 + \frac{\beta_s p_{s,2} p_r}{p_{r,2} p_s}) \leq \frac{\beta_s \rho(p_{s,2} - p_{r,2})}{p_s p_{r,2}}$$

$$R_1^s(1 + \frac{\beta_r p_{r,2} p_s}{p_{s,2} p_r}) - R_1^r(1 - \beta_r) \geq \frac{\beta_r \rho(p_{s,2} - p_{r,2})}{p_r p_{s,2}}$$

Note that, to be able to maximize social welfare which is

$$EW_{soc}^{'(R_1^s, R_2^s, R_1^r, R_2^r) = \theta\left(p_s\beta_s(X - R_1^s) + \beta_s^2(2p_s\gamma + p_s - \gamma)R_2^s\right) +}$$
$$(1 - \theta)\left(p_r\beta_r(X - R_1^r) + \beta_r^2(2p_r\gamma + p_r - \gamma)R_2^r\right)$$

we need to take R_1^s and R_1^r as small as possible [6]. For $IC(r)$ to be satisfied, R_1^s should be nonzero, however, $R_1^r = 0$ poses no threat for the IC constraints. Now that we are sure, we can set $R_1^r = 0$ this gives new boundaries for R_1^s; which is;

$$\frac{\frac{\beta_r \rho(p_{s,2} - p_{r,2})}{p_r p_{s,2}}}{(1 + \frac{\beta_r p_{r,2} p_s}{p_{s,2} p_r})} \leq R_1^s \leq \frac{\frac{\beta_s \rho(p_{s,2} - p_{r,2})}{p_s p_{r,2}}}{(1 - \beta_s)}$$

As long as the upper bound is greater than the lower bound for R_1^s, we can ensure that there exists a repayment schedule which deals with the adverse selection problem as well as maximizes social and

[5] We have some limitations on γ: $0 \leq \gamma$, $p_i + \gamma \leq 1$, $p_i - \gamma \geq 0$ and $0 < p_2^i < 1$ for all types.

[6] As in the previous propositions, more specifically as in Proposition 2 and 4

individual welfare. Need to show that

$$\frac{\frac{\beta_r \rho(p_{s,2}-p_{r,2})}{p_r p_{s,2}}}{(1+\frac{\beta_r p_{r,2} p_s}{p_{s,2} p_r})} \leq \frac{\frac{\beta_s \rho(p_{s,2}-p_{r,2})}{p_s p_{r,2}}}{(1-\beta_s)}$$

By some algebra, we get

$$2\beta_s \beta_r p_s p_{r,2} + \beta_s p_{s,2} p_r - \beta_r p_{r,2} p_s \geq 0$$

Note that, $p_r p_{s,2} \geq p_s p_{r,2}$ implies the inequality above [7]. This ensures that the upper bound of R_1^s is strictly greater than the lower bound. Hence, we can choose R_1^s as the lower bound which satisfies the IC by construction and maximizes both social and individual welfare [8]. Summing up all, the repayment schedule $\{R_1^{s*}, R_2^{s*}, R_1^{r*}, R_2^{r*}\}$ where

$$R_1^{s*} = \frac{\beta_r \rho(p_{s,2} - p_{r,2})}{p_{s,2} p_r + \beta_r p_{r,2} p_s}$$

$$R_2^{s*} = \frac{(R_s - R_1^s) p_s}{2\gamma p_s + p_s - \gamma}$$

$$R_1^{r*} = 0$$

$$R_2^{r*} = \frac{R_r p_r}{2\gamma p_r + p_r - \gamma}$$

identify the optimal contract that solves adverse selection problem and maximizes the social welfare as well as that of individuals. ∎

We note that, since period-wise success probabilities are not uniform in this environment. R_t^i also depends upon (p_1^i, p_2^i), since the outcome of first period affects the second.

6 Conclusion

In this paper, we introduced a dynamic credit contract with individual lending to tackle adverse selection issues in micro-credits. We first assumed that output is fully observable to the lender and showed that in a dynamic setting, it is possible to formulate an individual lending scheme that screens safe and risky borrowers, generates the same expected revenue for the lender and improves welfare of the society. This has profound implications for the evolution of microcredit programs. There has been a shift from group lending to individual lending in many leading microcredit institutions (Armendariz de Aghion and Morduch 2005, 119-120). Given that forming and maintaining a peer group has transaction costs and the empirical evidence about its effectiveness is mixed, a switch to robust individual lending mechanisms should be desirable. We offer one such mechanism, based on the premise that borrowers have different time preferences.

Our mechanism perfectly screens borrowers with differentiated contracts. Bank is still equally likely to extend contracts to all sorts of borrowers, so the reach of credit is not harmed. Indeed, by saving from the costs associated to JL mechanisms, our mechanism is likely to extend the reach of credit opportunities. This is a very desirable result and in accordance with objectives of micro-credit programs.

[7] Easily it can be seen by plugging $p_{i,2}$ into $p_r p_{s,2} > p_s p_{r,2}$ where $p_{i,2} = (2p_i\gamma + p_i - \gamma)$ for all $i \in \{s, r\}$

[8] Note that, $\gamma = 0$ is the benchmark case which is consistent with the result here as long as safe type discounts less than the risky type does.

In our mechanism, possibility of renegotiation in the interim is irrelevant due to ZPC of MFI. On the other hand, future research could generalize our mechanism to T periods, or even to infinite horizon. A natural extension could be renewal of credits. Borrowers might repeat the game in our mechanism more than once and reputation concerns might play a significant part. Throughout our analysis, we also kept the population of borrowers constant. The impact of our mechanism on the safe-risky composition of society over time could be another line of research.

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8 Appendix

8.1 Proof of Proposition 1

Note that the expected utility of an agent is the following:

$$EU_i(H_1, H_2) = p_i\beta_i(X - H_1) + p_i\beta_i^2(X - H_2) \ \forall i \in \{s, r\}$$

where H_t is the payment scheme $\forall\ t \in \{1, 2\}$. Recall that the bank sets it. For such a contract to exist, need to see that there is an interval for R_1^s to be chosen, so that we can choose it and satisfy the conditions (with sufficiently large X of course, to be able to satisfy agents to participate.) Remember, the boundaries coming from individual compatibility constraints are[9]:

$$\frac{\frac{p_s}{2p_r}(1+\beta_r) - \beta_r}{1-\beta_r} R_s \leq R_1^s \leq \frac{\frac{p_s}{2p_r}(1+\beta_s) - \beta_s}{1-\beta_s} R_s$$

and the lower boundary of the output level X is either[10]

$$\frac{R_1^s(1-\beta_s) + \beta_s R_s}{1+\beta_s} \leq X \quad \text{or} \quad \frac{R_r}{2} \leq X$$

With the assumption of the output level X being sufficiently large, each agent's contract yields nonnegative benefits in expected terms and which ensures participation. Therefore, *Individual rationality* constraints are not an issue here. As long as we can show that the upper bound is greater than the lower bound of R_1^s, we are done. But, with $\beta_s \geq \beta_r$ and $p_s \geq p_r$, one can easily see that;

$$\frac{\frac{p_s}{2p_r}(1+\beta_r) - \beta_r}{1-\beta_r} R_s \leq \frac{\frac{p_s}{2p_r}(1+\beta_s) - \beta_s}{1-\beta_s} R_s$$

Hence, there exists R_1^s that satisfy individual compatibility constraints. This basically completes the proof. Now that, we claim $\frac{\frac{p_s}{2p_r}(1+\beta_r)-\beta_r}{1-\beta_r} \geq \frac{1}{2}$.

$$\frac{p_s}{p_r}(1+\beta_r) - 2\beta_r \geq 1 - \beta_r$$

$$\frac{p_s}{p_r}(1+\beta_r) \geq 1 + \beta_r \Rightarrow R_1^s > \frac{R_s}{2} \tag{15}$$

Also note that from IC_r (4), if we assume the first instalment R_1^s cannot be greater than the total repayment R^s, then we will have an upper bound for risky types discounting which is $\beta_r \leq \frac{2p_r}{p_s} - 1$. However, it is not that strict. We can also assume that the safe type borrower can pay more than the repayment at the first stage and get some back at the second stage.

[9] where $IC(s) : EU_s(R_1^s, R_2^s) \geq EU_s(R_1^r, R_2^r)$ and $IC(r) : EU_r(R_1^r, R_2^r) \geq EU_r(R_1^s, R_2^s)$

[10] where $IR(s) : EU_s(R_1^s, R_2^s) \geq 0$ and $IR(r) : EU_r(R_1^r, R_2^r) \geq 0$

8.2 Proof of Proposition 2

Under individual liability, we have

$$EU_i(R_1^i, R_2^i) = p_i\beta_i(X - R_1^i) + p_i\beta_i^2(X - R_2^i) \tag{16}$$

The joint liability contract yields the following utility for type i who has the peer j,

$$\begin{aligned} EU_i^{JL}(R_1^i, R_2^i, R_1^j, R_2^j, j) &= p_i p_j \beta_i(X - R_1^i) + p_i(1 - p_j)\beta_i(X - R_1^i - R_1^j) \\ &+ p_i p_j \beta_i^2(X - R_1^i) + p_i(1 - p_j)\beta_i^2(X - R_2^i - R_2^j) \end{aligned}$$

Under joint liability, each borrower i has to cover for her associate's payment R^j in case of failure [11].

We need to show $EU_i() > EU_i^{JL}()\ \forall R^i, R^j \in \mathbb{R}_{++}$. The expression becomes:

$$\begin{aligned} p_i(X - R_1^i) + p_i\beta_i(X - R_2^i) > [p_i p_j + p_i(1 - p_j)](X - R_1^i) + \beta_i[p_i p_j + p_i(1 - p_j)](X - R_2^i) \\ - p_i(1 - p_j)R_1^j - \beta_i p_i(1 - p_j)R_2^j \end{aligned}$$

Since $p_i p_j + p_i(1 - p_j) = p_i$, this reduces to

$$p_i(1 - p_j)R_1^j + \beta_i p_i(1 - p_j)R_2^j > 0$$

which is satisfied for sure.

8.3 Proof of Proposition 3

Here, rather than fixing R_1^r to $\frac{R_r}{2}$, we choose both $R_1^s$and R_1^r where they are the repayment in the first period for the safe type and repayment in the first period for the risky type, respectively. From the ZPC, the secnd period's repayment levels are determined. Under the light of this new set-up, individual rationality and incentive compatibility constraints, in the same order presented in Proposition 1, are the followings:

$$\begin{aligned} &IR(s) : \beta_s p_s(X - R_1^s) + \beta_s^2 p_s(X - r_2^s) \geq 0 \\ &IR(r) : \beta_r p_r(X - R_1^r) + \beta_r^2 p_r(X - R_2^r) \geq 0 \\ &IC(s) : \beta_s p_s(X - R_1^s) + \beta_s^2 p_s(X - R_2^s) \geq \beta_s p_s(X - R_1^r) + \beta_s^2 p_s(X - R_2^r) \\ &IC(r) : \beta_r p_r(X - R_1^r) + \beta_r^2 p_r(X - R_2^r) \geq \beta_r p_r(X - R_1^s) + \beta_r^2 p_r(X - R_2^s) \end{aligned}$$

We still have $p_s R_s = p_r R_r = pR = \rho$ as the moneylender's behaviour. Solving the participation conditions i.e. IRs gives a new lower boundary for the output level X; which is

$$max\left\{\frac{R_1^s(1 - \beta_s) + \beta_s R_s}{(1 + \beta_s)}, \frac{R_1^r(1 - \beta_r) + \beta_r R_r}{(1 + \beta_r)}\right\}$$

[11] which occurs with probability $p_i(1 - p_j)$

After achieving the level of Output to satisfy the players participate in the mechanism, we need to lead either type to behave as themselves. Incentive Compatibility constraints capture this requirement. Hence, solving the last two inequality above yields;

$$R_s \frac{\beta_r(\frac{p_s}{p_r} - 1)}{1 - \beta_r} \leq R_1^s - R_1^r \leq R_s \frac{\beta_s(\frac{p_s}{p_r} - 1)}{1 - \beta_s}$$

Here again, since the both boundaries are positive, as long as the upper bound is greater than the lower bound, we can find a pair $\{R_1^s, R_1^r\}$, that satisfies the IC constraints and solves the adverse selection problem. But, as an assumption we have $\beta_s \geq \beta_r$, and this ensures that there is a positive interval for $R_1^s - R_1^r$. This concludes the proof so far.

8.4 Proof of Proposition 4

Recall from proposition 1 and 2 we could have solved the adverse selection problem and increased the social welfare compared to group lending with full liabilty by an individual based contract.[12]. Now we search further for a better contract by manipulating the first period payment of the risky type just as we did for the safe type instead of fixing it. As long as we have sufficiently large enough output level X; which is specified in proposition 3, remember from the same proposition, we had a positive interval for $R_1^s - R_1^r$ [13]. We now try to maximize agents' payoffs by not violating the constraints. As in the previous propositions, the lower R_1^i where $i \in \{s, r\}$, the higher expected social payoff. Hence, $R_1^r = 0$ [14] will maximize the risky types' payoff and together with the inequality above $(R_1^s - R_1^r)$ brings up a new condition for R_1^s; which is:

$$R_1^s \geq R_s \frac{\beta_r(\frac{p_s}{p_r} - 1)}{1 - \beta_r}.$$

To maximize EW; R_1^s needs to bind as well. Hence the contract $(H^s(R_1^s, R_2^s), H^r(R_1^r, R_2^r))$ maximizes social as well as individual wealth where

$$R_1^{s*} = R_s \frac{\beta_r(\frac{p_s}{p_r} - 1)}{1 - \beta_r}$$

$$R_2^{s*} = R_s(1 - \frac{\beta_s(\frac{p_s}{p_r} - 1)}{1 - \beta_s})$$

$$R_1^{r*} = 0, \quad R_2^{r*} = R_r$$

This completes the proof.

[12] $R_1^r = \frac{R_r}{2}$ and $R_1^s = \frac{\frac{p_s}{2p_r}(1+\beta_r) - \beta_r}{1-\beta_r} R_s$

[13] that also ensures the existence of such contract

[14] note that, we are not allowed to choose $R_1^s = 0$ because of the existence of such a contract.