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Abstract

The aim of the paper is to analyse the conditional dependence structure between precious metal returns using a copula-DCC-GARCH approach. Conditional correlation matrices are used to identify the states of the precious metals market by assuming that a given state of the market corresponds to a typical pattern of the conditional dependence structure. Cluster analysis allows for pointing at transition points between the market states, that is the points of drastic change in the conditional dependence structure. The application of the methodology described above to the period between 1997 and 2013 indicates three market states of four major precious metals (gold, silver, platinum and palladium). The results obtained reveal a sudden increase in dependencies between precious metals at the turn of April and May 2004.

Keywords: precious metals, dependence structure, copula-GARCH, market states.

JEL Classification: C58, Q02

1. Introduction

Gold, platinum, palladium and silver are the most common precious metals. For centuries they have been primarily used to make jewellery, whereas nowadays they also play an important part in various industries, for example, gold is used in electronics, telecommunication and aviation, silver is used in the electronic and electrical industries (mobile phones, computer hardware), and, to a smaller extent, in photography, while platinum is mostly used in the chemical and petrochemical industries as a catalyst and in the motor industry for building catalytic converters, as well as in the electronic and electrical industries. In recent years these metals have become an important means of thesaurisation, and are now frequently used as an investment through, for instance, the purchase of gold bars or the purchase of Exchange traded funds. Such funds allow for the investment in metals without the need of their actual possessing (for example, in January 2005 the iShares Gold Trust and in April 2006 the iShares Silver Trust were created).

The existing research on precious metals focuses mainly on gold and silver. The analysis of the prices of precious metals can be divided into two areas. The first one covers the analysis of the relationship between prices of precious metals. Ciner (2001) finds evidence of the disappearance of the long term relationship between gold and silver in the 1990s. Their conclusion is contested by Lucey and Tully (2006), who say that this relationship strengthens and weakens over time but is prevalent in the long run. Similarly, Sari et al. (2007) notice a strong relationship between gold and silver. Śmiech and Papież (2012) show that causality between the prices of gold, silver, platinum and copper change in the period 2000-2011. Also Papież and Śmiech (2012) examine causality in mean and variance between commodity prices (including metal prices) and financial market prices.

The second area of analysis is connected with examining the volatility of returns of the precious metals. Hammoudeh and Yuan (2008) show that gold and silver have similar volatility persistence globally, but there is no leverage effect in gold and silver prices. Sari et al. (2010) examine the co-movements and information transmission among the spot prices of four precious metals (gold, silver, platinum, and palladium), the oil price, and the US dollar/euro exchange rate. Hammoudeh et al. (2010) examine the conditional volatility and correlation dependence for four major precious metals, and they find that almost all of them are weakly responsive to news spilled over from other metals in the short run. Morales and Andreosso-O'Callaghan (2011) find that in terms of volatility spillovers, an asymmetric effect is observed; gold tends to dominate the markets and the evidence favouring the case of other precious metals influencing the gold market is weak. Cochran et al. (2012) show that events taking place during the post-September 2008 period increased the volatility in gold, platinum, and silver returns. Sensoy (2013) claims that the turbulent year of 2008 had no significant effect on the volatility levels of gold and silver, although caused an upward shift in the volatility levels of palladium and platinum. Using the consistent dynamic conditional correlations, he shows that precious metals became strongly correlated with each other during the last decade, which reduces the diversification benefits across them and indicates a convergence to a single asset class.

The objective of this study to identify the states of the precious metals market (gold, silver, platinum and palladium) and to present their temporal evolution in the period from September 22, 1997 to February 13, 2014. Since our sample period covers the recent global financial crisis, we want to examine whether the market states are affected by the financial crisis. The process of identifying the states of the precious metals market and analysing their temporal evolution is based on the conditional dependence structure using a copula-DCC-GARCH methodology.

This allows us to address several questions, which might be of interest to both investors and researchers:

- Is the dependence between prices in the precious metals markets stable or does it undergo changes?
- Are the changes in relations between precious metals prices evolutionary or drastic?
- What are the causes of drastic changes in relations between these prices?

The paper contributes to the existing literature in the following aspects. Firstly, most analyses of the precious metals market conducted so far are based on standard multivariate GARCH (MGARCH) models (see e.g. Hammoudeh et al., 2010; Morales and Andreosso-O'Callaghan, 2011; Sensoy, 2013; Silvennoinen and Thorp, 2013), which assume that standardized innovations follow a multivariate elliptical distribution. In case of the multivariate normal distribution all marginal distributions must be normal, and multivariate Student's t distribution imposes, also often unrealistically, the same degrees of freedom for all marginal distributions.

However, a copula-based multivariate GARCH model used in this study allows for modelling the conditional dependence structure when standardized innovations are non-elliptically distributed. Thus, it makes it possible to model the volatility of particular metals using univariate GARCH models with different standarized residual distribution. Generally, copulas allow the researcher to specify the models for the marginal distributions separately from the dependence structure that links these distributions to form a joint distribution. They offer a greater flexibility in modelling and estimating margins than while using parametric multivariate distributions (see e.g. Nelsen, 1999; Joe, 1997).

Secondly, at present a copula-GARCH methodology is widely used in the analysis of financial time series (see e.g. Aloui et al., 2013; Philippas and Siriopoulos, 2013; Lee and Long, 2009; Li and Yang, 2013; Patton, 2006; Serban et al., 2007; Wu et al., 2012; Zolotko and Okhrin, 2014 and for a review Patton, 2012). However, in most studies on the precious metals market, copula methodologies are used to analyse the dependencies between single metal markets and other markets (see e.g. Reboredo, 2013a, 2013b). This study is based on conditional correlations using a copula-GARCH methodology to investigate the dynamics of conditional dependence structure between precious metals. It also attempts to identify the states of the market on the basis of these conditional correlations and to follow their temporal evolution. To the best of our knowledge, such approach has not been applied to investigate dependencies in the precious metals markets so far.

The paper is organised as follows. Section 2 describes the data and econometric methodologies employed. Empirical results are discussed in Section 3, and the conclusions are presented in the last section.

2. Methodology

The dynamic relationship between precious metals is analysed with the use of a copula-DCC-GARCH model for daily log-returns. In this approach, multivariate joint distributions of the return vector $r_t = (r_{1,t},...,r_{k,t})'$, $t = 1,...,T$, conditional on the information set available at time $t-1$ (Ω_{t-1}) is modelled using conditional copulas introduced by Patton (2006). This model takes the following form:

$$r_{1,t} \mid \Omega_{t-1} \sim F_{1,t}(\cdot \mid \Omega_{t-1}),..., r_{k,t} \mid \Omega_{t-1} \sim F_{k,t}(\cdot \mid \Omega_{t-1}) \quad (1)$$

$$r_t \mid \Omega_{t-1} \sim F_t(\cdot \mid \Omega_{t-1}) \quad (2)$$

$$F_t(r_t \mid \Omega_{t-1}) = C_t\left(F_{1,t}(r_{1,t} \mid \Omega_{t-1}),..., F_{k,t}(r_{k,t} \mid \Omega_{t-1}) \mid \Omega_{t-1}\right) \quad (3)$$

where C_t denotes the copula, while F_t and $F_{i,t}$ respectively the joint cumulative distribution function and the cumulative distribution function of the marginal distributions at time *t*. Using a copula allows for separate modelling of marginal distributions and the dependence structure of vector r_t. In the empirical study elliptical copulas are used to describe the dynamics of the dependence structure, while conditional marginal distributions are modelled with the use of ARMA-GARCH models.

Conditional correlation matrices are taken as the parameters of conditional copulas. Their dynamics is modelled with the use of the dynamic conditional correlation model (DCC):

$$r_t = E(r_t \mid \Omega_{t-1}) + y_t , \tag{4}$$

$$y_t = D_t \varepsilon_t \text{ and } H_t = D_t R_t D_t , \tag{5}$$

where $D_t = diag(h_{11,t}^{1/2}, ..., h_{kk,t}^{1/2})$ is diagonal matrix of conditional standard deviations. Conditional variances $h_{ii,t}$ are modelled with the use of one-dimensional GARCH(p,q) processes:

$$h_{ii,t} = \omega_i + \sum_{k=1}^{p} \alpha_{ik} y_{i,t-k}^2 + \sum_{k=1}^{q} \beta_{ik} h_{ii,t-k} . \tag{6}$$

If ε_t is a i.i.d. vector of random variables with zero mean and unit variance, R_t is a conditional correlation matrix of standardized residuals:

$$\varepsilon_{i,t} = \frac{y_{it}}{\sqrt{h_{ii,t}}} . \tag{7}$$

In the DCC(m, n) model the dynamics of matrices is described as follows (see (Engle, 2002)):

$$R_t = \left\{ diag(Q_t)^{-1/2} \right\} Q_t \left\{ diag(Q_t)^{-1/2} \right\}, \tag{8}$$

where Q_t is a symmetric positive definite $k \times k -$ dimensional covariance matrix of standardized residuals given by the equation:

$$Q_t = \left(1 - \sum_{k=1}^{m} c_k - \sum_{k=1}^{n} d_k \right) \overline{Q} + \sum_{k=1}^{m} c_k (\varepsilon_{t-k} \varepsilon'_{t-k}) + \sum_{k=1}^{n} d_k Q_{t-k} . \tag{9}$$

In specification $\overline{Q}$ denotes unconditional covariance matrix of standardized residuals, $c_k, k = 1,...,m$, $d_k, k = 1,...,n$ are scalars which capture respectively the effect of previous shocks and previous dynamic correlation on the current conditional correlation.

In the empirical study conditional correlation matrices are modelled with the use of DCC(1, 1) model.

The states of the precious metals market are identified on the basis of conditional correlation matrices. It is assumed that a given market state corresponds to a typical pattern of the conditional dependence structure described by a conditional correlation matrix R_t. Transition points between market states, corresponding to drastic changes in the conditional dependence structure, are identified using Ward's method of cluster analysis and a similarity measure suggested by Münnix et al. (2012), which allows us to quantify the difference of the correlation structure for two points in time.

3. Data and empirical results

The data used in this study consist of the daily (five working days per week) spot prices of gold (Gold), silver (Silv), platinum (Plat) and palladium (Pall) from September 22, 1997 to February 13, 2014 (see Fig. 1). The source of data is Bloomberg, and all prices of precious metals are measured in US dollars per troy ounce. As usual, price return series are computed on a continuous compounding basis as $r_{i,t} = 100 \times (\log(P_{i,t} / P_{i,t-1}))$, where $P_{i,t}$ and $P_{i,t-1}$ are current and one-period lagged spot prices of precious metals. After eliminating the mismatching transaction days, we finally obtain 4185 log-returns for each series (see Fig. 2).

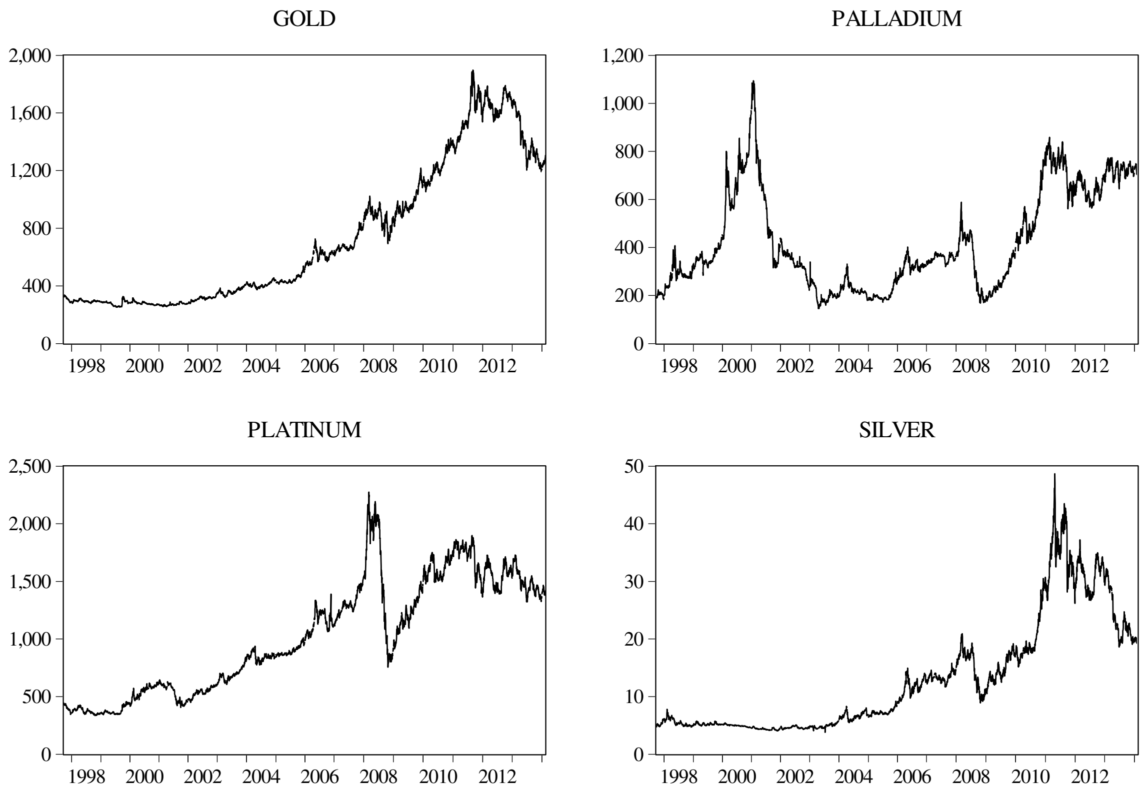

Fig. 1. Price series of major precious metals between September 1997 and February 2014.

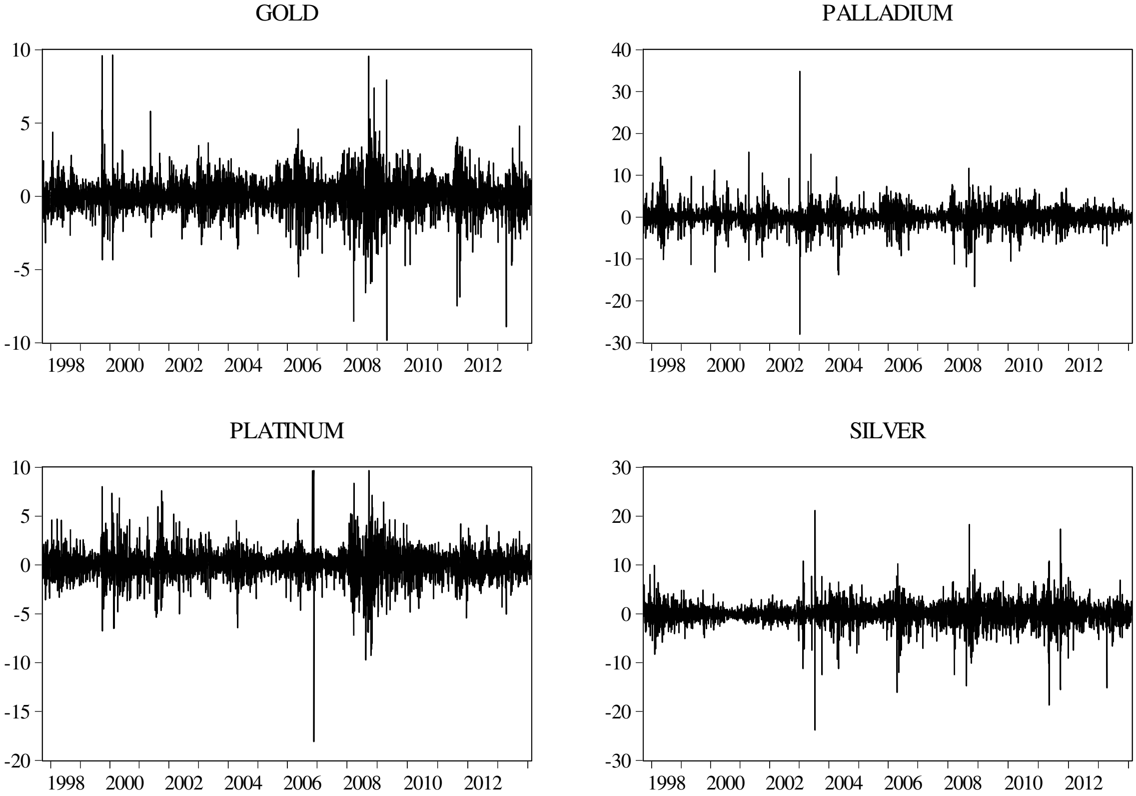

Fig. 2. Returns of major precious metals between September 1997 and February 2014.

The descriptive statistics for the metals' price levels in U.S. dollar and the returns are reported (in level form) in Table 1a and Table 1b, respectively.

Table 1a. Descriptive statistics for levels.

	GOLD	PALLADIUM	PLATINUM	SILVER
Mean	730.39	425.16	1021.36	13.02
Median	520.75	354.50	929.00	8.83
Max	1896.50	1094.00	2276.00	48.70
Min	252.90	144.00	335.00	3.78
Std. Dev.	485.76	205.61	489.60	9.80
C.V.	1.50	2.07	2.09	1.33
Skewness	0.79	0.71	0.24	1.16
Kurtosis	2.22	2.38	1.81	3.34

C.V. is the coefficient of variation.

Table 1b. Descriptive statistics for returns

	GOLD	PALLADIUM	PLATINUM	SILVER
Mean	0.03323	0.03129	0.02779	0.03486
Median	0.02535	0.00000	0.00000	0.04999
Max	9.64163	34.83067	9.67362	21.14351
Min	-9.84051	-27.98695	-18.08837	-23.78497
Std. Dev.	1.18972	2.37340	1.50888	2.20188
C.V.	0.02793	0.01318	0.01842	0.01583
Skewness	-0.10254	0.25457	-0.40458	-0.60756
Kurtosis	11.66950	22.93386	12.87393	16.53494

C.V. is the coefficient of variation.

The statistics for the metals' returns generally follow those for their prices. Gold return has the highest historical volatility followed by platinum, as measured by the coefficient of variation, while palladium return has the lowest among the four metals. In term of historical return means, silver has the highest average return followed by gold and palladium, while platinum has the lowest averages.

Table 2. Copula-DCC-GARCH parameters for log-returns.

Parameter	Estimate	Std. Error	t value	Pr(>|t|)
(Gold).mu	0.029086	0.012316	2.361737	0.018190
(Gold).ar1	-0.049451	0.015856	-3.118691	0.001817
(Gold).omega	0.020068	0.005796	3.462679	0.000535
(Gold).alpha1	0.073493	0.012870	5.710575	0.000000
(Gold).beta1	0.914029	0.014309	63.878167	0.000000
(Gold).shape	4.830476	0.386835	12.487167	0.000000
(Silver).mu	-0.002054	0.026160	-0.078522	0.937413
(Silver).ar1	-0.084966	0.023533	-3.610577	0.000306
(Silver).omega	0.014193	0.007579	1.872632	0.061119
(Silver).alpha1	0.042194	0.003504	12.041960	0.000000
(Silver).beta1	0.956806	0.003013	317.543829	0.000000
(Platinum).mu	0.046781	0.017618	2.655337	0.007923
(Platinum).ar1	-0.524437	0.127566	-4.111113	0.000039
(Platinum).ma1	0.541134	0.124888	4.332960	0.000015
(Platinum).omega	0.055136	0.020305	2.715404	0.006620
(Platinum).alpha1	0.112217	0.023435	4.788354	0.000002
(Platinum).beta1	0.866600	0.029305	29.571831	0.000000
(Platinum).skew	0.993258	0.020038	49.569488	0.000000
(Platinum).shape	5.425853	0.443948	12.221817	0.000000
(Palladium).mu	0.038276	0.023363	1.638314	0.101356
(Palladium).ar1	0.050001	0.016266	3.073882	0.002113
(Palladium).omega	0.207753	0.046007	4.515633	0.000006
(Palladium).alpha1	0.212891	0.027570	7.721894	0.000000
(Palladium).beta1	0.784233	0.024263	32.322346	0.000000
(Palladium).shape	3.900838	0.259408	15.037464	0.000000
(Joint)dcca1	0.021455	0.002566	8.360567	0.000000
(Joint)dccb1	0.973780	0.003508	277.586697	0.000000
(Joint)mshape	29.502754	3.478219	8.482143	0.000000

In the empirical study different variants of the AR-GARCH specification are considered for individual returns. Eventually, on the basis of information criteria, Student's t AR(1)-GARCH(1,1) model has been assumed for gold, normal AR(1)-GARCH(1,1) model has been assumed for silver, skewed Student's t ARMA(1,1)-GARCH(1,1) model has been assumed for platinum, and Student's t AR(1)-GARCH(1,1) has been assumed for palladium. On the other hand, Gauss and Student's t copulas have been considered in the analysis of the dynamics of dependencies between the rates of return, and, again on the basis of information criteria[1], Student's t has been chosen.

Conditional correlation matrices obtained with the use of the estimated model are applied to analyse the precious metals market. Dynamic correlations in this marked are presented in Fig. 3.

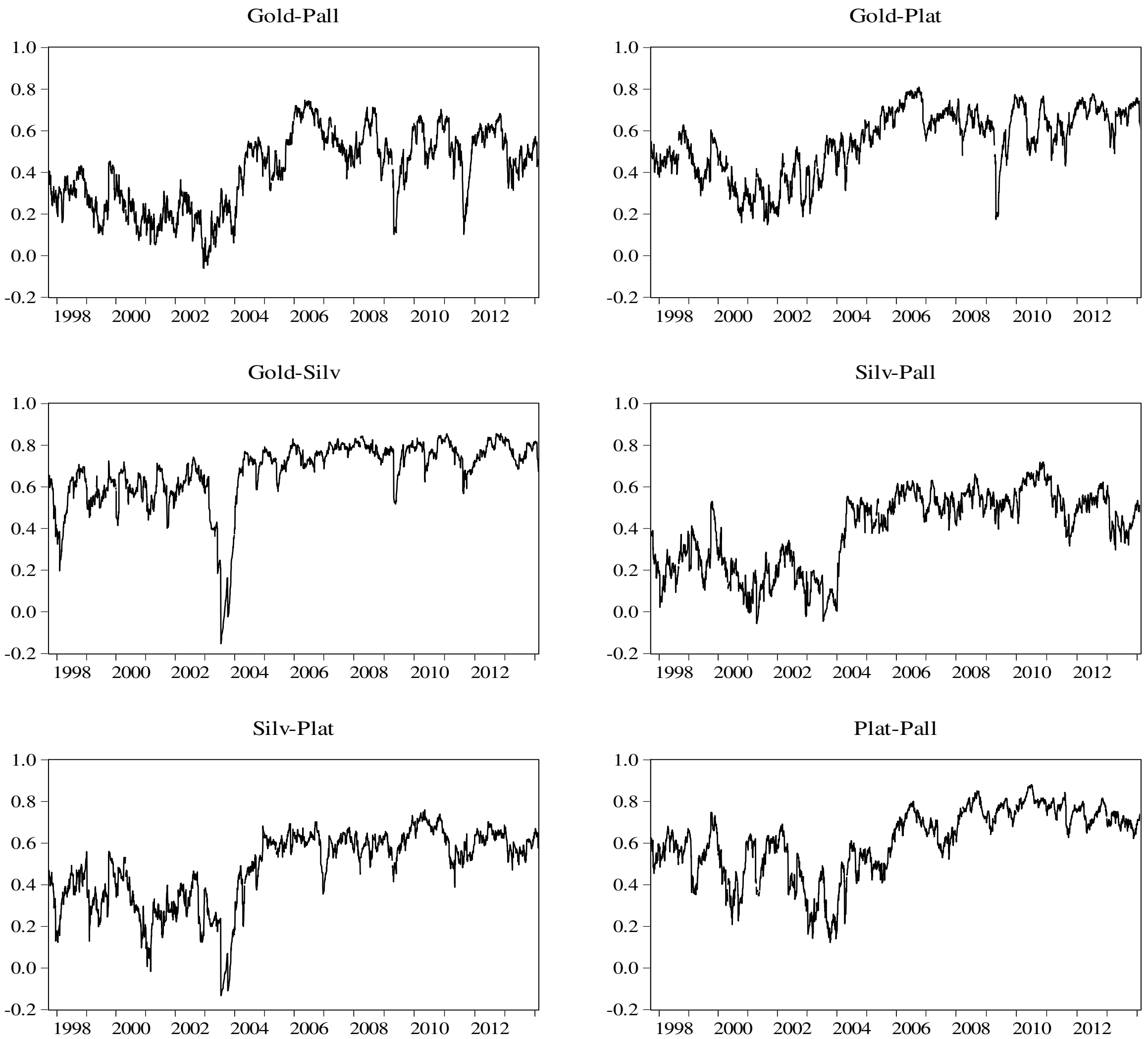

Fig. 3. Dynamic correlations of precious metal returns.

Figure 4 shows the temporal evolution of the market states in the period from September 22, 1977 to February 13, 2014, obtained as a result of clustering conditional correlation matrices with Ward's method of cluster analysis. The left panel illustrates the division into two clusters (Rousseeuw's Silhouette internal cluster quality index equals 0, 5512), while the right panel illustrates the division into three clusters (Rousseeuw's Silhouette

[1] The parameters of the model are assessed using R package "rmgarch" (version 1.2-6), developed by Alexios Ghalanos. The results can be obtained from the author on request.

internal cluster quality index equals 0,3169). Structural changes in the precious metals markets in the analysed period are identified with the use of a similarity measure of correlation matrices ((Münnix et al., 2012)) and presented in Fig. 5 (the left panel illustrates the similarity of matrices distant from each other by a multiple of one quarter, the right panel – by a multiple of one year). Light shading denotes similar conditional correlation matrices and dark shading denotes dissimilar ones. If we assume that a point on the diagonal designates "now", then the similarity to previous times from this point can be found on the vertical line below this point, or the horizontal line to the left of this point.

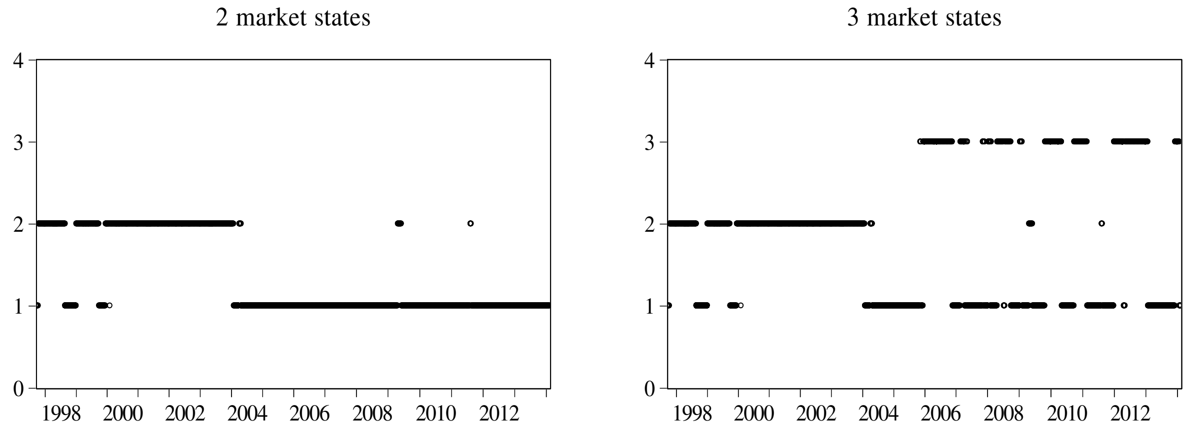

Fig. 4. Temporal evolution of the market state

After analysing the dynamics of conditional correlation matrices (Fig. 3), it can be said that bilateral correlation between precious metals markets increased considerably in 2004 and has remained at this level since then. A similar conclusion can be drawn from the analysis of similarity maps showing conditional correlation matrices (Fig. 5). Particular areas in the matrix show how similar dependence measures in two periods are: the first one is on the horizontal axis and the second one on the vertical axis. Figure 3 presents the results for two options: the first option covers correlations calculated for sub-periods with 70 observations, while the second – for sub-periods with 250 observations. A low value of a similarity measure of correlation matrices (considerably darker shading) for 2004 together with its rise and maintaining this high level (lighter shading) indicate structural changes in precious metals markets in 2004. It confirms Sensoy's (2013) hypothesis that precious metals will be a single asset class in the near future. This change in the precious metals markets in 2004 is also evident in clustering results (Fig. 2). Divisions into two and three clusters indicate a stable change of the market state in April 2004 (4/29/2004). For the last decade this market has not returned (with few exceptions) to the state from before 4/29/2004. The more detailed analysis of this decade (the division into 3 clusters) reveals two basis states with numerous transition points between them. On the basis of the results obtained, it can be concluded that the global financial crisis from 2008 has not considerably affected the precious metals market.

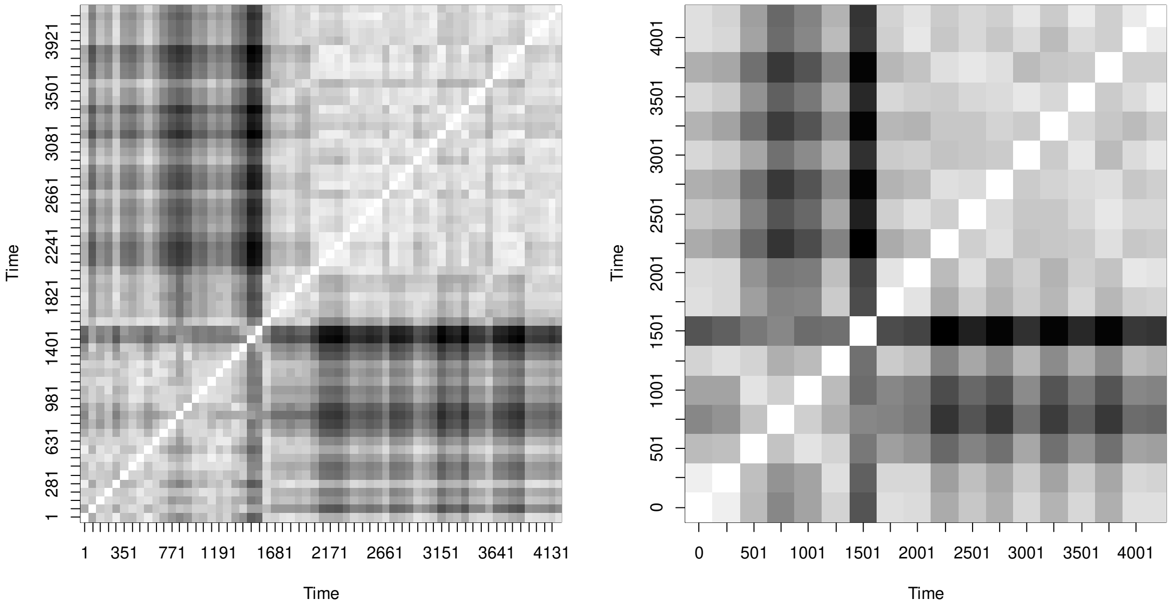

Fig. 5. Correlation/similarity matrix for the precious metals market.

Conclusion

The objective of this study is to analyse the conditional dependence structure between precious metals using the copula-DCC-GARCH methodology and to follow their temporal evolution. The results obtained in the study reveal that the dependence structure is not stable over time. Internal clustering criteria applied to Ward's method prove that two (which seems to be better choice) or three typical patterns of the conditional dependence are plausible. If two market states are assumed, the transition point takes place in April 2004. The state of the precious metals market before and after this moment is stable, with rare and transitory changes. Conditional correlations in the first period are lower than in the second period. If three market states are assumed, the one till April 2004 is stable, but later we observe two patterns which change frequently. The similarity between this two last state are however quite high. Summing up, the results obtained indicate that the dependence structure of precious metals undergoes only one drastic structural change in April 2004. It confirms Sensoy's (2013) hypothesis that precious metals will be a single asset class in the near future. A unique opportunity to test this thesis was the global financial crisis, which, however, did not affect the correlation structure of precious metals returns. The results obtained might be of great importance to investors, as they demonstrate that drastic changes of the correlation structure of the precious metals market is currently highly unlikely.

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