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Beginning with the writing of A.C. Pigou [2] and Joan Robinson [3], third-degree price discrimination has been treated extensively in the literature. A monopolist who operates in two or more markets may charge different prices to different groups of consumers for the same product. Also, a manufacturer may sell a product to different distributors at a uniform price, but may induce the distributors to resell the product in different markets at different mark-ups [1 and 5]. Although it is commonly assumed (see, for example, [4]) that the markets of a firm practicing price discrimination are independent, third-degree price discrimination is sometimes practice in interdependent market (that is, the quantity sold in one market is a function of the quantity sold in each of the remaining markets). A familiar example is the "singles bar," where individuals congregate in order to meet members of the opposite sex. Frequently the price of admission for males differs from the price of admission for females. In addition, the markets which are served by the singles bar interdependent, since the presence of a relatively large number of females (males) serves to attract additional males (females). Many other examples of such interdependence can be given.

The purpose of this note is to develop a model of third-degree price discrimination in which the quantities sold in each market are interdependent. We demonstrate that in this situation the price differential may be explained not only in terms of price elasticity differentials, as is typically the case, but also in terms of the market interdependency.

Let us assume that a monopolist practices third-degree price discrimination in two interdependent markets.[1] Total profits, *TPro*, are equal to[2]

(1) $TPro = P_1(q_1,q_2) \cdot q_1(P_1,q_2) + P_2(q_2,q_1)\ ) \cdot q_2(P_2,q_1) - TC(Q)$,

where P_1 = price in Market 1, q_1 = output in Market 1, P_2 = price in Market 2, q_2 = output in Market 2, TC = total cost, and Q ($Q = q_1 + q_2$) = total output.

Partially differentiating Equation (1) with respect to q_1 and q_2 yields

(2) $\partial\, TPro/\partial q_1 = P_1(1 + 1/e_1) + \partial \mathrm{Rev}_2/\partial q_1 - MC(1 + \partial q_2/\partial q_1)$ and
(3) $\partial\, TPro/\partial q_2 = P_2(1 + 1/e_2) + \partial \mathrm{Rev}_1/\partial q_2 - MC(1 + \partial q_1/\partial q_2)$,

where e_1 and e_2 = price elasticity of demand in Markets 1 and 2, respectively; REV refers to the total revenue; and MC refers to marginal cost. After setting Equations (2) and (3) equal to zero to maximize profits, it is simple to show that

(4) $[P_1(1 + 1/e_1) + \partial \mathrm{Rev}_2/\partial q_1]/(1 + \partial q_2/\partial q_1) = [P_2(1 + 1/e_2) + \partial \mathrm{Rev}_1/\partial q_2]/(1 + \partial q_1/\partial q_2)$.

Given maximum profits,

$$(5)\quad P_1 = [P_2(1 + 1/e_2)(1 + \partial q_2/\partial q_1) - \partial \mathrm{Rev}_2/\partial q_1(1 + \partial q_1/\partial q_2)$$
$$+ \partial \mathrm{Rev}_1/\partial q_2(1 + \partial q_2/\partial q_1)]/[(1 + 1/e_1)(1 + \partial q_1/\partial q_2)]$$

and

$$(6)\quad P_2 = [P_1(1 + 1/e_1)(1 + \partial q_1/\partial q_2) - \partial \mathrm{Rev}_1/\partial q_2(1 + \partial q_2/\partial q_1)$$
$$+ \partial \mathrm{Rev}_2/\partial q_1(1 + \partial q_1/\partial q_2)]/[(1 + 1/e_2)(1 + \partial q_2/\partial q_1)].$$

Here P_1 may be less than, equal to or greater than P_2 regardless of the size of e_1 and e_2. Even if $e_1 = e_2$, P_1 may not (and probably would not) be equal to P_2. If $e_1 = e_2$, letting $(1 + 1/e_1) = X$. we have

$$(7)\quad P_1 = [P_2(X)(1 + \partial q_2/\partial q_1) + \partial \mathrm{Rev}_1/\partial q_2(1 + \partial q_2/\partial q_1)]/[X(1 + \partial q_1/\partial q_2)] - (\partial \mathrm{Rev}_2/\partial q_1)(X^{-1})$$

and

$$(8)\quad P_2 = [P_1(X)(1 + \partial q_1/\partial q_2) + \partial \mathrm{Rev}_2/\partial q_1(1 + \partial q_1/\partial q_2)]/[X(1 + \partial q_2/\partial q_1)] - (\partial \mathrm{Rev}_1/\partial q_2)(X^{-1}).$$

Not only are price differentials attributable to price elasticity differentials, but also to interdependent utility functions per se. As can be seen in Equations (7) and (8), even if price elasticities are equal, price differentials are to be expected except under the most extraordinary conditions. Clearly, these results can be generalized to the "*n*" interdependent market case; moreover, there a myriad of possible applications of this analysis.

NOTES

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1. The model presented her must be distinguished from the seemingly similar, but actually quite different, analysis in [4]. There are at least two important fundamental differences between the present model and that in [4]. To begin with, Heinrich von Stackleberg [4, p.65] distinguishes "….two forms of price discrimination according to whether the market division is brought into being by the monopolist himself or is a datum for him." The model in [4] deals exclusively with the former case. In particular, the model in [4], which can result in first-degree discrimination (see [6]), is concerned with the case "…in which it is assumed that not only price fixing, but also the division of the market depends upon the entrepreneur" [4, p. 65]. By contrast, to [4], the model presented here deals with the general case where segmentation is a "datum" for the monopolist; in contrast to [4], the market in this analysis is not arbitrarily segmented at will by the monopolist.

The second major difference, perhaps the more important one, is that the von Stackleberg paper [4] does *not* deal with market interdependence; in point of fact, as is common, [4] expressly deals with a case where the firm's markets are *independent*. By contrast, this paper deals expressly with the case where the firm' markets are *interdependent*. In fact, von Stackleberg's basic profit equation [4, p.68] is a special case of the more general Equation (1) of this paper.

2. It is assumed here that there are no marginal cost differentials from operation in Markets 1 and 2.

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