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Abstract

In this paper, we model an overlapping generation economy affected by an unexpected immigration shock and determine how households would insure themselves against "immigration risks" efficiently. We use the model to study the impact of immigration on (i) the welfare of various generations, (ii) the distributions of income among factors of production, and (iii) the optimal design of the intergenerational welfare state. In particular, we construct a system of public education and public pensions that mimics efficient complete market allocation. We also show the impact of immigration shocks in a small open economy. In this case, our model suggests that the external capital flows can act as substitutes for the missing private insurance markets. Our analysis delivers a set of predictions that we find useful for understanding certain aspects of the Spanish experience during 1996 and 2007.

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1. Introduction

In most countries of the developed world, the combination of declining fertility rates and increasing life expectancies makes immigration flows to developed countries increasingly important. In fact, immigration is the main source of population growth in most of these countries. In addition, but perhaps no less important, in a densely populated world subject to strong climatic pressures, the risk of large population displacements is greater than ever before. All these aspects make it increasingly important that the institutions of a country are adapted to migration shocks.

Given this concern, we are interested in the following questions. What are the intergenerational economic effects of a large immigration flow? How do households insure themselves against an immigration shock? How does an immigration shock affect the welfare of the various generations in the receiving country, both current and future generations? In particular, how does immigration impact intergenerational arrangements such as public education and pensions, which make up the core of the contemporary welfare state? To begin answering these questions, we develop a simple theoretical framework with overlapping generations that live for three periods in which they accumulate human capital in the first, work in the second and retire and live off the return from their investments in the third. The latter includes both physical capital and the resources they lent the young people to invest in human capital because we allow for this type of lending-borrowing relationship to be established through financial markets in our baseline model.

We take immigration shock to be an increase in the size of the middle-aged generation that engenders, among other things, a reduction in the average human capital of the labor force. In other words, the immigrants are new middle-aged workers who are somewhat less skilled than the average natives. The shock lasts one period after which the economy moves along its new growth path with a larger number of middle-aged workers. We assume that the children of the immigrants perfectly integrate. Hence, after one period, the children of immigrants accumulate as much per-capita human capital as the offspring of the native workers with the same level of skill. In the context of our model, one period lasts approximately 25-30 years.

Because we are interested in determining how households would insure themselves against the "immigration risk," we assume financial markets are sequentially complete in the baseline model. Because there are always two possible states of the world in the next period—one with and one without immigration—there are two financial assets agents buy from and sell to each other in every period. One asset pays one unit of consumption only when there is an immigration shock, and the other pays a unit of consumption only when there is no immigration shock. Through these two assets, which are accessible to all individuals living in the country, young and middle-aged people insure themselves from the impact of an immigration shock. In particular, young people, who will be middle-aged and working in the next period, would like to insure themselves against the negative impact that the arrival of immigrants may have on their wages; they do so by purchasing insurance from the currently middle age people. The latter, who are saving for retirement, can use the extra payoff they would receive from their capital investment if the immigration shock were realized in the next period to provide such insurance. The elderly do not accumulate further assets because we assume that they must die without debt and there is no bequest motive.

The buying and selling of insurance takes place at the same time and through the same instruments that the middle aged and young use to lend/borrow to/from each other. More precisely, middle-aged individuals invest in physical capital (by purchasing assets issued by the competitive firms that carry out production in the next period) and in human capital (by purchasing assets issued by the young agents to finance their own education). Because the capital invested in the firms pays off more when there is immigration, it compensates for the lower payoff from the human capital investment accruing to the middle age. This assures that both young and middle age people implement as much consumption smoothing as it is feasible in the benchmark complete markets economy; this consumption takes place when they are middle age and elderly, respectively.

This does not imply perfect consumption smoothing or that some ex-ante notion of efficiency is satisfied at the equilibrium of our benchmark model. This is because agents cannot insure against the risk of being born in a period of high immigration beforehand. This is a feature of the world that is well captured by OLG models. Young agents born in a period with a positive immigration shock are worse off than they would be otherwise because they must compete with the offspring of the immigrants both to borrow funds for investing in human capital this period and in supplying labor to the market in the next period. We assume that this type of risk cannot be insured away either. It would be insurable if parents were altruistic and internalized the future welfare of their children via bequests. Instead, we assume that parents are selfish and do not leave anything to their children. Hence, the latter must bear the cost of being born in the "wrong" period. The extension to the case in which a bequest motive leads parents to purchase insurance for the future generations is an interesting venue for future research.

The key channels through which immigration affects welfare in this economy is that it increases the labor supply of unskilled workers in the face of a predetermined stock of physical capital and skilled workers. This lowers the wages of unskilled workers and increases both the return on physical capital and the wages of skilled workers, which shifts income from one part of the population to another. In this sense, factor prices move around because we have assumed there is zero mobility of both physical capital and skilled labor in the benchmark model. If there were perfect mobility of capital and skilled labor, both factors of production would flow into the country from the outside on the footsteps of unskilled immigrant labor and the capital intensity ratios would remain unchanged. In this case, factor prices would be unaffected by immigration, which would amount to nothing more than an increase in the size of the economy. Under constant returns to scale in production, which we assume, this does not affect the welfare of the native agents. The capital intensity ratios and the wage per unit of human capital remain constant. Hence, the salaries of the native do not change at all. This case is trivial, and we do not consider it.

Nevertheless, if there are frictions in the international financial markets and capital adjustment is not instantaneous, i.e., it takes time for the capital stock of the country to be built up to restore the initial capital intensity ratio, then immigration causes a redistribution between generations as outlined above. The latter observation suggests that the larger is the trade deficit following an immigration shock, the quicker will be the adjustment toward the old capital intensity ratio and the smaller the redistribution away from native workers and toward native owners of capital. This is an interesting result because it suggests that the trade deficit that follows an immigration shock and borrowing from aboard can be a substitute for the missing internal insurance markets.

We are not the first to claim a link between immigration flows and international capital flows. Other authors have explored this relationship in dynamic general equilibrium models designed to quantify the impact of immigration on capital inflows and, therefore, on trade deficits in small open economies that have experienced a large immigration wave. Izquierdo, Jimeno and Rojas (2010) and Gavilán, de Cos, Jimeno and Rojas (2011) constructed and calibrated a large-scale overlapping generation model for Spain, which is a country that has received a massive wave of immigration in recent years [1]. In both papers, the results indicate

[1] Spain, which has traditionally been an out-migration country, suffered an immigration boom

that interest rates and immigration are the main factors responsible for the investment boom and the build-up of a sizable external imbalance witnessed in the Spanish economy over the period from the mid-1990s to 2008. Another example of a large immigration shock was Israel following the collapse of the Soviet Union in the 1990's[2]. Cohen, Eckstein and Weiss (2012) and Cohen and Hsieh (2001) show that the average effective wages of native Israelis fell and the return to capital increased during the height of the influx in 1990 and 1991. By 1997, however, both average wages and the return to capital had returned to pre-immigration levels due to an investment boom induced by the initial increase in the return to capital. The investment boom was largely financed by external borrowing. The model elaborated in Wilson (2003), is calibrated to the Canadian economic environment in the years that led to the Great War in which Canada experienced a dramatic shift in migration patterns[3]. Again, the results suggest that up to three-quarters of the increase in the capital formation rate and the foreign capital inflow rate in the Canadian economy over the period of 1899-1911 can be attributed to the dramatic inflow of immigrants over this period.

All these papers are quantitative macroeconomic studies[4] of the impact of immigration on the host country based on the calibration of a general equilibrium model in which immigration is a deterministic process. Another example of this type of model for a closed economy framework is Canovas and Ravn (2000) in which the authors use an infinitely lived agents model to demonstrate that the reunification of Germany (an event similar to a mass migration of low-skilled agents who hold no capital into a foreign country) generated a significant redistribution to high-skilled workers and entrepreneurs. Although the papers listed above forget human capital accumulation, Eberhard (2012) analyzes the effect in a closed economy of an unexpected influx of immigrants on the price of skill and hence on the earnings, human capital accumulation, and educational attainment of native workers. Nevertheless, Eberhard (2012) abstracts from the effects of migration on

from the late nineties to 2007. The foreign population in Spain increased from 0.35 million (1% of the total population) in 1995 to 5.22 million (11% of the total population) in 2008.

[2]From late 1989 through 1996, 670 thousand Russian Jews immigrated to Israel, which increased the total population of Israel by 11 percent and the labor force by 14 percent.

[3]Canada went from being a net supplier of migrants to a net receiver of immigrants with a net immigration rate of 15.1% in the period of 1901-1911 (as a percentage of the 1901 population).

[4]There is an extensive literature of microeconometric studies of the impact of immigrants on the labor market performance of native workers. For instance, see Card (2001), Borjas (1999), Ottaviano and Peri (2006), Hatton and Tani (2005), Gandal, Hanson and Slaughter (2204) and Cortes (2008).

physical capital accumulation.

In contrast to the previous literature, our paper has mostly a normative value. We focus on the design of optimal policies that allow agents to insure against an immigration risk. In fact, we next ask whether government policies can be used to substitute for the credit and insurance markets of the baseline model when these are either absent or largely incomplete as is often the case. To do this, we build on previous results presented in Boldrin and Montes (2005, 2009), which answered the question in the affirmative for the case of no immigration shock, and adapt their framework to the particular circumstances at hand. Under uncertainty, we need to use the welfare state to also allocate risk efficiently between generations and heterogeneous agents and not only allow for intergenerational trade as in deterministic case. In the present case, we show that pension payments and social security contributions must be negatively indexed to the size of the immigration flow, but educational expenditures and the issuance of public debt financing should be positively correlated. Intuitively, this is because social security contributions play the role that the repayment of debt plus interest—by the currently middle-aged generation to those that lend them money to invest in human capital—plays in the model with sequentially complete financial markets. The pension payments are nothing but these contributions as received by the elderly: they correspond to the payoff from the securities that were traded to finance the human capital investment of the young generation in the previous period. Likewise, the educational investment (financed via the issuance of bonds) corresponds to the issuance of the same securities in this period, and hence it should increase because the size of the young generation is larger than expected.

Other authors (Shiller (1999), Bohn (1998, 1999)) have stressed the positive role of an unfunded social security system as an instrument to efficiently reallocate the economic impact of aggregate shocks across various generations. They argued that, if the returns to capital and wages are imperfectly correlated and driven by an aggregate shock, an unfunded social security system that endows retired households with a claim to labor income may serve as such a risk sharing tool between generations. Krueger and Kubler (2005) note that the potentially positive intergenerational risk sharing role of social security needs to be traded off against the standard crowding-out effect that unfunded social security has on private savings and thus capital formation. In a realistically calibrated economy with stochastic production, they find that the intergenerational risk sharing role of an unfunded social security system is dominated in its importance by the adverse effect on physical capital accumulation that arises from the introduction of such a system. Sanchez-Marcos and Sanchez (2004) confirm the findings of Krueger and Kubler (2005) for the case of demographic uncertainty.

An important difference between all of these papers and the economy in our paper is that the authors of these papers abstract from the accumulation of human capital and, therefore, from the negative effect that missing credit markets has on education. As we show in section 4 (and in more detail in Boldrin and Montes (2005)), when credit markets for education are absent and even in the presence of government-financed education, there is too much investment in physical capital with respect to the complete market allocation. This is because public education allows the working generation to invest in the human capital of future generations, but it does not allow the former investors to collect the market return from their beneficiaries. This will generally lead to an inefficiency: investment in physical capital is too high and there is less intergenerational consumption smoothing than under the complete market allocation. In this sense, the introduction of a PAYGO system in which social security contributions correspond to the capitalized value of education services received is a tool for "efficiently" crowding-out physical capital.

The rest of the paper proceeds as follows. In Section 2, we describe the benchmark model. In Section 3, we show the effects of the absence of credit and insurance markets. In Section 4, we look at the efficient welfare state in the presence of immigration and in a closed economy with incomplete markets. In Section 5, we look at an open economy with incomplete financial markets but with public education and pensions. Section 6 concludes the paper with some practical considerations about the Spanish experience.

2. The basic model

We use an OLG model with two types of agents in each generation who live for three periods: youth, middle age, and old age. Agents differ in the level of productive skills they inherit: high (H) and low (L).

There is aggregate uncertainty due to an immigration flow that may increase the size of the low-skill middle-aged group, which affects the total supply of labor, the wage rates, the return on capital, the aggregate output and the size of future generations.

We use the superscripts, y, m and o to denote young, middle-aged and elderly people, respectively, and the superscripts $i = H, L$ to denote high and low human capital. The population structure in period t is (N_t^y, N_t^m, N_t^o), with $N_t^y = N_t^{yH} + N_t^{yL}$, $N_t^m = N_t^{mH} + N_t^{mL}$ and $N_t^o = N_t^{oH} + N_t^{oL}$. Additionally, $N_t^{mH} = N_{t-1}^{yH}$,

$N_t^{mL} = (1+z_t)N_{t-1}^{yL}$ and $N_t^{yi} = (1+n)\,N_t^{mi}$ for $i = H, L$, where $-1 < n$, and z_t is the realization of the immigration shock in period t. For simplicity, we assume that the shock z follows a two-state Markow process with state space $Z = \{\bar{z}, 0\}$, $\bar{z} > 0$. The notation $\pi(z_{t+1}|z_t)$ denotes the probability of $z_{t+1} \in Z$ given z_t.

In each period $t = 0, 1, ...$, a new generation $N_t^y = (1+n)\,N_t^m$ is born. Each type $i = H, L$ is born with a per-capita endowment of basic knowledge, h_t^{yi}, which is an input to the production of future human capital, according to $h_{t+1}^{mi} = h(d_t^i, h_t^{yi})$. We denote the physical resources invested in the education of a young individual of type i in period t with d_t^i; we assume $h_t^{yH} > h_t^{yL}$. The function $h(d, h^y)$ is a constant returns to scale neoclassical production function. During the second period of life, individuals work and decide how much of their income to consume, how much to save, and how to allocate the latter among various financial instruments. When they are elderly, individuals have no decisions to make: they consume all their income and then die. We assume agents draw utility from consumption when middle age and elderly. We also assume immigrants enter the country with the same human capital as the low-skill middle-aged natives and with zero capital or financial assets. Consumption when young, leisure, and the welfare of descendants do not affect lifetime utility.

The initial conditions are: K_0, for the capital stock; $(N_0^{yi}, N_0^{mi}, N_0^{oi})$, for the population; h_0^{mi}, for the human capital of the middle age individuals; $A_{-1}^{yi}(0)$ and $A_{-1}^{mi}(0)$, for the portfolios of middle-aged and elderly people, respectively; and $A_{-1}^{f}(0)$, for that of the representative firm, which owns K_0. Finally, we assume there are no immigrants in the first period.

The preferences of an individual of type i, born in period $t-1$, are

$$E_{t-1}\left\{u(c^{mi}(z_t)) + \delta E_t\left[u(c^{oi}(z_{t+1}))\right]\right\},$$

where δ is the period discount factor, and E is the expectation operator. The function $u : \Re_+ \longrightarrow \Re$ is assumed to be strictly increasing, strictly concave and C^2.

2.1. Market structure

Normalize to one the price of output in the initial period, in which the state is $z = 0$; write $p_t(z)$ for the price of output in period t and state $z \in Z$ in all subsequent periods. We assume sequentially complete financial markets, *i.e.,* given the current state z_t and the set Z of possible future states, for all $z \in Z$, there is a competitive market in which contingent claims $A_t(z)$ are traded with a payoff in the units of the next period's consumption, $b[A_t(z), z_{t+1}] = 1$ if $z_{t+1} = z$, and zero otherwise. We assume agents cannot die in debt, i.e., we impose $A_t^{mi}(z) \geq 0$ for all t and z with $i = H, L$. Let $q(z, z_t)$ be the price, in units of consumption at t, of asset $A(z)$ in period t and state z_t. To save notation, the symbol $A_t(z)$ also indicates the number of units of that asset traded in a given period.

2.2. Firms

There is a representative firm, which uses physical capital and the two types of human capital to produce output according to $Y_t = F(K_t, H_t, L_t)$, where $H_t = h_t^{mH} N_t^{mH}$, $L_t = h_t^{mL} N_t^{mL}$, and $F(K, H, L)$ is a constant returns to scale neoclassical production function. We assume a full depreciation of capital and that high-skill workers are more productive than low-skill workers, everything else equal, i.e., $F_H(K, X, X) > F_L(K, X, X)$. Firms last one period and own the physical capital, which they finance by issuing state-contingent securities. More specifically, in each period t the representative firm issues securities $A_t^f(z)$ at a price of $q(z, z_t)$, for $z \in \{\bar{z}, 0\}$, with the proceedings of which they purchase K_{t+1}, which is used for production in the next period. In period $t+1$, after the realization of the shock, the firm hires workers, carries out production, pays off wages, honors its financial liabilities and then dissolves.

Let $w^i(z_t)$ be the nominal wage in period t and state $z_t \in Z$ for an agent of type $i = H, L$. Write $w^i(z_t)/p(z_t) = \omega^i(z_t)$ and $\varphi(z_t) = p(z_t)F_K(K_t, H_t, L(z_t))$. The problem of the firm is

$$\max_{A_t^f(z_{t+1}), H_{t+1}, L_{t+1}} E_t \left\{ p(z_{t+1}) \left[Y(z_{t+1}) - \omega^H(z_{t+1})H_{t+1} - \omega^L(z_{t+1})L(z_{t+1}) - A_t^f(z_{t+1}) \right] \right\}$$

subject to,

$$\begin{aligned} Y(z_{t+1}) &= F(K_{t+1}, H_{t+1}, L(z_{t+1})) \\ K_{t+1} &= \sum_{z \in Z} q(z, z_t) A_t^f(z). \end{aligned}$$

The first order conditions for H, L and for $A^f(z)$ are

$$\omega^H(z_{t+1}) = F_H(K_{t+1}, H_{t+1}, L(z_{t+1})) \quad \forall\ z_{t+1} \in Z \tag{1.a}$$

$$\omega^L(z_{t+1}) = F_L(K_{t+1}, H_{t+1}, L(z_{t+1})) \quad \forall\ z_{t+1} \in Z \tag{1.b}$$

$$q(z, z_t) = \frac{\pi(z|z_t) p_{t+1}(z)}{\sum_{z \in Z} \pi(z|z_t) \varphi_{t+1}(z)} \quad \forall\ z \in Z. \tag{1.c}$$

2.3. Consumers

For a native agent of type $i = H, L$ born in period $t-1$ when the state is z_{t-1}, the lifetime optimization problem is

$$\max_{d^i(z_{t-1}), A^{yi}_{t-1}(z), A^{mi}_t(z)} E_{t-1}\left\{u(c^{mi}(z_t)) + \delta E_t\left[u(c^{oi}(z_{t+1}))\right]\right\}$$

subject to,

$$\begin{aligned}
d^i(z_{t-1}) + \sum_{z\in Z} q(z, z_{t-1})A^{yi}_{t-1}(z) &\leq 0 & (2.a)\\
c^{mi}(z_t) + \sum_{z\in Z} q(z, z_t)A^{mi}_t(z) &= \omega^i(z_t)h^{mi}_t + A^{yi}_{t-1}(z_t) \quad \forall z_t \in Z & (2.b)\\
c^{oi}(z_{t+1}) &= A^{mi}_t(z_{t+1}) \qquad \forall z_{t+1} \in Z & (2.c)\\
h^{mi}_t &= h\left(d^i(z_{t-1}), h^{yi}_{t-1}\right) & (2.d)
\end{aligned}$$

The first order conditions for the choice of $\mathcal{A}^{yi}(z_{t-1}) = \{A^{yi}_{t-1}(z)$, for all $z \in Z\}$ and $d^i(z_{t-1})$ reduce to

$$\begin{aligned}
q(z, z_{t-1}) &= \frac{\pi(z|z_{t-1})u'(c^{mi}_t(z))}{\sum_{z\in Z}\pi(z|z_{t-1})u'(c^{mi}_t(z))\omega^i_t(z)h_d\left(d^i(z_{t-1}), h^{yi}_{t-1}\right)} \quad \forall z \in Z & (3.a)\\
1 &= \sum_{z\in Z} q(z, z_{t-1})\omega^i_t(z)h_d\left(d^i(z_{t-1}), h^{yi}_{t-1}\right). & (3.b)
\end{aligned}$$

For each of the $A^{mi}_t(z)$, the first order condition reads

$$q(z, z_t) = \frac{\pi(z|z_t)\delta u'(c^{oi}_{t+1}(z))}{u'(c^{mi}(z_t))} \quad \forall z \in Z. \qquad (3.c)$$

For a middle-aged immigrant who arrives in the state of the world z_t with human capital $\bar{h}^m_t = h^{mL}_t$ and $A^y_{t-1}(z_t) = 0$, the maximization problem is

$$\max_{\bar{A}^m_t(z)} u(\bar{c}^m(z_t)) + E_t\left[\delta u(\bar{c}^o(z_{t+1}))\right]$$

subject to,

$$\begin{aligned}
\bar{c}^m(z_t) + \sum_{z\in Z} q(z, z_t)\bar{A}^m_t(z) &= \omega^L(z_t)\bar{h}^m_t & (4.\mathrm{a})\\
\bar{c}^o(z_{t+1}) &= \bar{A}^m_t(z_{t+1}) \quad \forall z_{t+1} \in Z. & (4.\mathrm{b})
\end{aligned}$$

The first order conditions that determine $\bar{A}^m(z_t)$ are analogous to those in (3.c):

$$q(z,z_t) = \frac{\pi(z|z_t)\delta u'(\bar{c}^o_{t+1}(z))}{u'(\bar{c}^m(z_t))} \quad \forall z \in Z. \tag{4.c}$$

2.4. Financial markets

It should be clear from the budget constraint that the net financial position of the young is non-positive (i.e., $\sum_{z\in Z} q(z,z_{t-1})A^{yi}_{t-1}(z) \leq 0$ for $i = H, L$) and the net financial position of the middle–aged is non-negative (i.e., $\sum_{z\in Z} q(z,z_t)A^{mi}_t(z) \geq 0$ for $i = H, L$ and $\sum_{z\in Z} q(z,z_t)\bar{A}^m_t(z) \geq 0$). When the latter is positive, it corresponds to aggregate national savings, which is invested in the physical capital of firms and in the education of the young agents. The first order conditions for the profit maximization of the representative firm imply

$$q(z,z_t) = \frac{\pi(z|z_t)p_{t+1}(z)}{\sum_{z\in Z}\pi(z|z_t)\varphi_{t+1}(z)} \text{ for each } z \in Z. \tag{1.c}$$

Multiplying $(1.c)$ by $F_K(K_{t+1}, H_{t+1}, L_{t+1}(z))$ and aggregating over $z \in Z$ we obtain

$$\sum_{z\in Z} q\left(z, z_t\right) F_K(K_{t+1}, H_{t+1}, L_{t+1}(z)) = 1. \tag{5}$$

2.5. Competitive equilibrium

A *competitive equilibrium* is a mapping from the current state of the world into a distribution of quantities and prices at all t. Given an initial condition (K_0, H_0, L_0), z_0, $\left(N^{oi}_0, N^{mi}_0, N^{yi}_0\right)$, $\left(A^{yi}_{-1}\left(z_0\right), A^{mi}_{-1}\left(z_0\right), A^{f}_{-1}\left(z_0\right)\right)$, with $i = H, L$, and a sequence of exogenous basic knowledges $\left\{h^{yH}_t(z), h^{yL}_t(z)\right\}_{t=0}^{\infty}$, a competitive equilibrium is a collection of the following:

1. choices of the native, $\left\{d^i_t(z), c^{mi}_t(z), c^{oi}_t(z), A^{yi}_t\left(z\right), A^{mi}_t\left(z\right)\right\}_{t=0}^{\infty}$, $i = H, L$, and immigrant, $\left\{\bar{c}^m_t(z), \bar{c}^o_t(z), \bar{A}^m_t\left(z\right)\right\}_{t=0}^{\infty}$, households;

2. choices of the representative firm, $\left\{K_t(z), H_t(z), L_t(z), A^f_t\left(z\right)\right\}_{t=0}^{\infty}$; and

3. prices, $\{p_t(z), q(z,z_t)\}_{t=0}^{\infty}$ and $\left\{\omega^H_t(z), \omega^L_t(z), \varphi_t\left(z\right)\right\}_{t=0}^{\infty}$

such that, for all t and $z \in Z$, the consumers and the firm maximize their payoffs and the markets clear.

In each period t and state z there are three sets of markets to clear:
i) Output market:

$$C_t^m(z) + C_t^o(z) + D_t(z) + K_{t+1}(z) = F(K_t, H_t, L_t(z)). \tag{6.a}$$

where $C_t^m(z)$ and $C_t^o(z)$ are the aggregate consumption of the middle-aged and old, respectively, in period t and state z, and $D_t(z)$ is aggregate physical resources invested in education in period t and state z.
ii) Labor market:

$$\begin{aligned} H_t &= h_t^{mH} N_{t-1}^{yH}, \\ L_t(z) &= h_t^{mL}(1+z) N_{t-1}^{yL}. \end{aligned} \tag{6.b}$$

iii) Capital market:

$$\begin{aligned} \sum_{z\in Z} q(z, z_t) A_t^f(z) &= K_{t+1}, \\ A_t^f(z) &= \sum_{i=H,L} A_t^{mi}(z) N_{t-1}^{yi} + \bar{A}_t^m(z) z_t N_{t-1}^{yL} + \sum_{i=H,L} A_t^{yi}(z) N_t^{yi}, \\ \sum_{i=H,L} d^i(z_t) N_t^{yi} &= -\sum_{i=H,L}\sum_{z\in Z} q(z, z_t) A_t^{yi}(z) N_t^{yi}. \end{aligned} \tag{6.c}$$

For each state $z \in Z$, the payoff from security $A_t^f(z)$ is

$$\begin{aligned} b[A_t^f(z), z_{t+1} &= z] A_t^f(z) = F(K_{t+1}, H_{t+1}, L_{t+1}(z)) - \omega_{t+1}^H(z) H_{t+1} - \omega_{t+1}^L(z) L_{t+1}(z) \\ &= F_K(K_{t+1}, H_{t+1}, L_{t+1}(z)) K_{t+1}. \end{aligned} \tag{6.d}$$

2.6. Numerical Evaluation

In Appendix A, we study an analytical illustration of our model in which the following specification of our key functions is used[5]: $u(c) = \log c$, $F(K, H, L) = AK^{\alpha}H^{\theta}L^{1-\alpha-\theta}$ and $h(d^i, h^{yi}) = B(d^i)^{\beta}(h^{yi})^{1-\beta}$.

Our main purpose in this section is to consider the practical implications of an immigration shock in a world with complete markets using a numerical evaluation of the economy in Appendix A. To do this, we assign reasonable values to each parameter and provide a numerical computation of the impact of an immigration shock. We compare two economies: an economy with no immigration $(z_0, z_1, z_2, ...) = (0, 0, 0, ...)$ and another with only one immigration shock $(z_0, z_1, z_2, ...) = (0, \bar{z}, 0, ...)$. We normalize $N_0^m = 1$ with $N_0^{mH} = 0.6$ and assume an annual growth rate of the population equal to 0. Recall that a period in this model is approximately 30 years. We assume $\bar{z} = 0.4$ and $\pi(\bar{z}|z_t) = \pi(0|z_t) = 0.5$. With respect to the production technology, α is fixed to 0.3, $\theta = 0.45$, and the scale parameter A is fixed at 1. In the human capital technology, we set $\beta = 0.13$, which corresponds to an elasticity of output of 0.058 with respect to education for a high human capital worker and 0.0325 for a low human capital worker. The discount factor δ is set to match an average ratio of investment over output $I/Y = 21.9\%$. This yields a value of $\delta = 0.904$, which corresponds to an annual discount factor of 0.996641. We set the scale parameter B equal to 4.35 to obtain an annual rate of aggregate output growth equal to 3% along the balance growth path. With these parameter values we have an annual interest rate of 4.1%. The fraction of total output spent to finance education (D/Y) is equal to 6.3%, which is on the low side for the US but not for most European countries, including Spain.

Assume the same initial conditions for both economies and assume they are on their balance growth path from the start. In Table 1, we show the welfare changes expressed as equivalent variation in consumption caused by an immigration shock in period $t = 1$ (where G_{t-1}^i, $i = H, L$, denote the generation born in period $t-1$ with skill i). To disaggregate the effects of the shock along the life cycle of an

[5] We have used a particular case of a more general production function, $Y = AK^{\alpha}E^{1-\alpha}$, with E a CES aggregate of various types of labor of the form $E = \left[aH^{\frac{\sigma-1}{\sigma}} + (1-a)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$, where $\sigma > 0$ measures the elasticity of substitution between workers with various skills and a $(1-a)$ is high (low) skill-specific productivity level. We have assumed $\sigma = 1$, and therefore we get $E = H^a L^{1-a}$. The sign of F_{HL} depends on the elasticity of substitution. If $\sigma < 1/\alpha$ then $F_{HL} > 0$ and, therefore, immigration has a positive effect on high human capital wages. In any case, with immigration, the capital labor ratio (K/E) falls, which increases the return of physical capital and decreases the marginal productivity of aggregate labor (E).

individual, we also shown the change in consumption of middle-aged and elderly agents induced by the immigration shock in Table 1. First, notice that the middle-aged and elderly generations that are alive when the shock hits consume more in this period than in the economy with no shock because the output is much higher during that period and the insurance mechanism redistributes this extra income to both middle-aged and elderly people. Therefore, elderly generations that are alive when immigrations arrive are better off.

Table 1: Welfare gains of an immigration shock in t=1.

	Equivalent variation in consumption[6] (%)		Change in consumption (%)			
t	G^H_{t-1}	G^L_{t-1}	c^{mH}_t	c^{mL}_t	c^{oH}_t	c^{oL}_t
0	0.48	0.48	0	0	0	0
1	0.69	0.69	1.01	1.01	1.01	1.01
2	13.16	-19.17	13.25	-19.11	0.34	0.34
3	13.03	-19.29	13.05	-19.25	13.05	-19.25
4	13.00	-19.29	13.00	-19.28	13.00	-19.28
5	12.99	-19.30	12.99	-19.29	12.99	-19.29
6	12.99	-19.30	12.99	-19.30	12.99	-19.30
7	12.99	-19.30	12.99	-19.30	12.99	-19.30

Nevertheless, future generations with low human capital are worse off in the economy with a shock because insurance against the immigration shock cannot be purchased before being born, and they pay the price of having low human capital income competing with them. The immigration shock negatively affects the low human capital workers of future generations because their wages are smaller along the transition to the new balanced growth path and positively effects the high human capital workers of future generations because their wages are higher along the same path. Depending on the choice of parameter values, a second effect may or may not push in the same direction: immediately after the shock hits, there is "too much" L and "too little" of both H and K relative to the balanced growth

[6] We compute the fraction of additional consumption along the life cycle that we must give to or take away from an individual in an economy with no shock so that the welfare in the economy with no immigration shock is the same as in the economy with an immigration shock.

ratios. As H and K accumulates toward the desired level and to the extent that this takes place at various speeds depending on the parameter values, there may be "too little" H and "too much" K for some periods during the transition. This further increases the wages of the high human capital workers, and decreases the marginal productivity of capital relative to their levels in the economy without a shock.

It is important to note that such effects depend on the presence or absence of certain financial markets that various generations may use to purchase insurance:

i) The low human capital workers who are alive during the transition periods are worse off because the immigration shock took place before they were born, and so they could not insure against it. Because there are "too many" of them during the transition, their lifetime income is lower than in the absence of the shock.

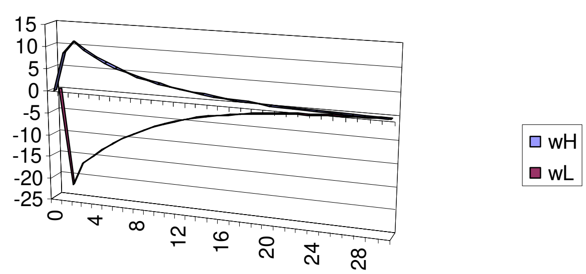
ii) The rapid increase in the stock of K immediately after the immigration shock hits is due to the presence of the financial assets that allow the living middle-aged workers to profit from the shock. Because they share in the bounty with the owners of physical capital, their disposable income increases, and this allows them to immediately invest more in K, which immediately surges. Simple calculations show that, absent the financial instruments that allow middle-aged workers to share in it, the extra output that accrues to firms in the form of an increase in the productivity of K would go to only the owners of K, i.e., the elderly retirees. They would consume it all instead of investing part in tomorrow's stock of capital, which is instead what the middle-aged people do with their share. In an economy without this type of financial assets, this means that the rate of return on capital increases immediately after the shock hits and then slowly decreases toward its long run position. In our economy, instead, the rate of return on capital increases at the time of the shock and decreases a period later (when investment surges) to converge from below to its long run position because high human capital is accumulated much more slowly than is physical capital. To put it differently, the hypothesis of sequentially complete financial markets has testable aggregate implications.

In Table 2, we show the effect of the shock on the annual rate of return on capital and growth rates. In Figure 1, we show the effect of the shock on wages. As argued, the immigration shock has a positive effect on the investment rate in the period in which the immigrants arrive and a temporarily negative (positive) effect on low (high) human capital labor productivity. Capital productivity increases in the period in which the immigrants arrive and then decreases and stays temporarily below its value in the economy without shocks. After the adjustment is completed, the growth rate and the marginal productivities resume their original long-run levels.

Table 2: Annual return on capital r_t and growth rate of GDP g_Y.

t	r_t (%)	g_Y (%)
0	4.10	3.02
1	4.39	3.31
2	4.08	3.16
3	4.09	3.02
4	4.10	3.02

Figure 1: Percentage deviations of wages per efficient unit of human capital.



Immigration shock in period t=1.

It is important to note that ours is a model of endogenous growth driven by constant returns to scale in the three reproducible factors. The long run growth rate does not depend on the size of the economy, which increases after the immigration shock, but only on the technology and preference parameters. Once the low-skill labor supply increases, equilibrium dictates that an extra amount of output must be allocated to endow these workers and their offspring with physical capital and high human capital in all periods following the shock period. Hence, savings rates must temporarily increase until the factor proportions return to the balanced growth levels after which growth resumes at the same rate as in the economy without shock. Because the additional investment that is made necessary by the shock reduces (increases) the consumption of low (high) human capital agents for a few periods after which consumption grows at the same rate as in the original equilibrium path, the new path will always stay below (above) the original one for the low (high) human capital individuals. Hence, consumption levels will differ forever despite growing at the same rate. This explains the lower (higher) steady state utility for the representative low (high) human capital agent in the economy with the immigration shock[7].

3. Equilibrium when credit and insurance markets are missing

The results obtained here are consistent with those reported in Boldrin and Montes (2005). Nevertheless, adding heterogeneity to each generation and adding uncertainty regarding the size and composition of the next cohort of middle-aged workers enriches the model and makes it possible to ask a number of new and interesting question. More precisely, one would want to distinguish the study of what happens (1) when there are not markets to insure against unexpected immigration shocks from (2) what happens when there are no markets for lending/borrowing across generations and over time. This leads us to consider the following two cases separately.

1. Young people cannot trade the security $A^y(z)$, but they can borrow to invest d in human capital. Because there are no state contingent assets, they must repay their debt at a fixed interest rate regardless of whether there

[7] To simplify the analysis, we have considered only a one-off immigration shock. It is easy to see that, if we considered a model with a negative shock (part of the low skilled workers emigrates to a foreign country), the labor supply of unskilled workers decreases in the face of a predetermined stock of physical capital and skilled workers. Therefore, the effects of a negative shock on factor prices, aggregate variables and the welfare of future generations are opposite. In this sense, the transition to the new balance growth path could be faster and the effects of a positive shock could be partially reversed by a subsequent negative shock (of a similar size). Previous simulations confirm these results.

is an immigration shock in the next period. In other words, they can borrow, but they cannot insure. In this case, even if the middle-aged people attempted to trade in state contingent $A^m(z)$ assets, this would not work because of a lack of a counterpart. The only entity they could trade those assets with is the firm, which cannot insure them against anything because it has no compensating sources of income in bad states. The income of elderly people is now equal to the fixed return on d plus the random return on capital investment. Because these are linearly independent returns and there are only two states of the world, middle-aged people can still use a portfolio composed of "educational bonds" and "shares of the firm" to fully insure their elderly consumption (for certain parameter configurations this may require taking a negative position in one of the two assets, which is an impossibility in this environment). However, this result is special because it follows from the simplifying assumption of only two immigration states, which makes spanning possible with just two assets. In either case, when insurance markets are absent, the young people bear all the risk because they must reimburse a fixed amount $d(1+r)$, when they reach middle age regardless of the state of the world. This implies that, when there is immigration, low human capital middle-aged natives have less income to consume and save than in the complete markets case. As in the world with complete markets, their wage bill is lower, but their debt payment is higher, which leaves less for $c^{mL}(z_t) + \sum_{z \in Z} q(z, z_t) A_t^{mL}(z)$. Therefore, their lifetime utility is lower. On the contrary, high human capital middle-aged natives have more income to consume and save than in the complete markets case (their wage bill is higher, as in the world with complete markets, but their debt payment is lower). Additionally, aggregate net labor income is lower, and therefore, total investment decreases.

2. Young people cannot borrow at all. Hence, middle-aged people can invest in only the physical capital. Obviously, this implies that there is a much lower level of human capital in the economy and that there is no growth. In this case, "workers" bear all the downside risk (i.e., they either do "normal" or do "worse"), but the owners of capital bear all the upside (i.e., they either do "normal" or do "better").

Because both these results are quite straightforward, we skip the mathematical details and move on to consider whether and how such inefficiencies could be alleviated with some type of "welfare state" intervention.

4. The Welfare State of a Closed Economy

4.1. Missing credit and insurance markets

Let us begin with case 2 of the previous section in which all credit and insurance markets other than the market for purchasing physical capital have been shut down. In this case, $F_K(K, H, L)$, $F_H(K, H, L)$ and $F_L(K, H, L)$ are all affected by the immigration shock, and no factor owner can insure against it. We want to derive policies capable of implementing the sequentially complete market allocation (SCMA) of Section 2. They turn out to be not very different from those derived in Boldrin and Montes (2005), apart from the fact that contributions and benefits are now state contingent.

In particular, in Boldrin and Montes (2005), the young "borrow" from the middle-aged via the public education system, and they pay back the debt at the market interest rate via a social security tax the proceeds of which finance pension payments[8]. Under uncertainty, we need to use the welfare state to also allocate risk efficiently between generations and heterogeneous agents and not just to allow for intergenerational trade as in the deterministic case. Think of what happens when there is an unexpected flow of immigrants ($z = \bar{z}$): the marginal productivity of low human capital labor decreases, and the marginal productivity of high human capital and physical capital increase. If we simply levy a social security contribution in the amount of $t_t^{pi} = d_{t-1}^{i*}(1 + r_t^*)$ where r_t is the market interest rate (starred symbols refer to the SCMA quantities in the remainder of the paper) and nothing else, the disposable per capita income of the low (high) human capital middle-aged individuals decreases (increases) compared to the SCMA. Furthermore, the amount by which the savings of this group decrease is not compensated by the increased saving of the high human capital workers, which implies an under-investment in physical capital compared to the SCMA.

Therefore, there are potential gains from risk sharing among various agents. We should stress here a relatively delicate point: an immigration shock causes aggregate uncertainty (it increases aggregate output), but part of that uncertainty is insurable because it affects the three factors of production differently. In particular, the native low human capital workers face the risk of a reduced per capita income, and the native high human capital workers and capital owners face a

[8] As in Boldrin and Montes (2005), we assume exogenous debt constraint. See Andolfatto and Gervais (2006) and Lochner and Monge (2011) for problems related to endogenous debt constraint.

larger per capita income. If there is no immigration, the opposite is true. Said it differently, it is often the case that aggregate risk obtains from the composition of various individual risks and this has clear redistributional consequences. Some people gain and some loose even from "aggregate" shocks. Then, insurance must work the following way: when there is immigration, the elderly (the owners of capital) pay something to the native middle-aged people, and vice versa in the other periods. Additionally, depending on parameter values, the native high human capital workers may or may not have to transfer something to the native low human capital workers: this will depend on how large their income gains are relative to the aggregate increase in output. Therefore, to implement the SCMA, the social planner needs to emulate the way in which intergenerational insurance markets would work.

Assume a period-by-period balanced budget and introduce two tax and transfer schemes; we call the first a "pension scheme" and the second an "education scheme." For each $z_t \in Z$, write

$$\sum_{i=H,L} t^{pi}(z_t) N_{t-1}^{yi} + \bar{t}^{p}(z_t) z_t N_{t-1}^{yL} = \sum_{i=H,L} b^{i}(z_t) N_{t-2}^{yi} + \bar{b}(z_t) z_{t-1} N_{t-2}^{yL}$$

for the pension scheme and

$$\sum_{i=H,L} t^{ei}(z_t) N_{t-1}^{yi} + \bar{t}^{e}(z_t) z_t N_{t-1}^{yL} = \sum_{i=H,L} e^{i}(z_t) N_{t}^{yi}$$

for the education scheme. Let us start from the last equation. Here, $e^{i}(z_t)$ denotes the educational transfer received from each member i of the current young generation when the aggregate shock is z_t. On the other side of the budget constraint, we find the contributions provided by the middle-aged natives $(t^{ei}(z_t))$ and by the middle-aged immigrants $(\bar{t}^{e}(z_t))$. In the optimal policy, we treat working native differently because they receive a different net income during middle age. The optimal policy also dictates treating young people differently in light of their different endowment of basic knoldwedge without differentiating between native and immigrants.

The budget constraint for the pension scheme can be interpreted similarly, but we need treating natives and immigrants differently on both sides. They make different contributions ($t^{pi}(z_t)$ and $\bar{t}^{p}(z_t)$, respectively) and receive different benefits when retired, $b^{i}(z_t)$ and $\bar{b}(z_t)$. Again, this mimics what would have happened in an economy similar to that of section 2 when markets were dynamically complete. The important point is that the contribution and benefit rates are state contingent in both schemes, i.e., they change depending on the immigration flow. The latter is an aggregate variable, and hence the state contingent policy does not depend on any private information but on a state variable that should, at least in principle, be observable by the policy maker.

Under these policies, the budget constraints for a representative member of the generation born in period $t-1$ become

$$\begin{aligned}
&d^i(z_{t-1}) \leq e^i(z_{t-1}) \\
&c^{mi}(z_t) + s^i(z_t) = \omega^i(z_t) h(d^i(z_{t-1}), h^{yi}_{t-1}) - t^{ei}(z_t) - t^{pi}(z_t) \quad \forall z_t \in Z \\
&c^{oi}(z_{t+1}) = s^i(z_t) R(z_{t+1}) + b^i(z_{t+1}) \quad \forall z_{t+1} \in Z
\end{aligned}$$

The symbol $s^i(z_t)$ is the investment in physical capital an individual of type $i = H, L$ makes in period t and state z_t, and $R(z_{t+1}) = (1 + r(z_{t+1}))$ is the return factor on saving in state z_{t+1}.

For an immigrant who arrives in period t, the budget constraints read

$$\begin{aligned}
\bar{c}^m(z_t) + \bar{s}(z_t) &= \omega^L(z_t)\bar{h}^m_t - \bar{t}^e(z_t) - \bar{t}^p(z_t), \\
\bar{c}^o(z_{t+1}) &= \bar{s}(z_t) R(z_{t+1}) + \bar{b}(z_{t+1}) \quad \forall z_{t+1} \in Z.
\end{aligned}$$

If we set $e^i(z_{t-1}) = d^{i*}(z_{t-1})$ (starred symbols refer to the SCMA), i.e., we transfer educational resources to the young generation up to the point at which the expected return on education is equal to the expected return on physical capital,

$$\sum_{z \in Z} \pi(z|z_{t-1}) p_t(z) R_t(z) = \sum_{z \in Z} \pi(z|z_{t-1}) p_t(z) \omega^i_t(z) h_d\left(d^i(z_{t-1}), h^{yi}_{t-1}\right),$$

we reach the efficient level of human capital in period t. In Boldrin and Montes (2005), we show (in a world with no immigration shocks and with homogenous agents) that, in a deterministic world, this policy, $t^{pi}(z_t) = d^{i*}_{t-1} R^*(z_t)$ and $b^i(z_t) = t^{ei}_{t-1} R^*(z_t)$ implements the efficient CMA overall. Pension benefits received (social security contributions paid) should correspond to the capitalized value of the lifetime contributions to aggregate human capital accumulation paid (educational services received). However, this policy is not enough to implement the appropriate amount of intergenerational risk sharing when random shocks affect the size of the working population. We need to add a second mechanism that allocates risk between generations.

Comparison of the last budget restrictions with the budget restrictions of the SCMA, $(2.a)$-$(2.d)$, shows that, if the lump-sum tax-transfer amounts satisfy

$$t^{pi}(z_t) = -A^{yi*}_{t-1}(z_t), \qquad \bar{t}^p(z_t) = 0,$$

$$\begin{aligned} b^i(z_{t+1}) &= A^{mi*}_t(z_{t+1}) - \lambda^{i*}(z_t)K^*_{t+1}R^*(z_{t+1}), \\ \bar{b}(z_{t+1}) &= \bar{A}^{m*}_t(z_{t+1}) - \bar{\lambda}^*(z_t)K^*_{t+1}R^*(z_{t+1}), \end{aligned}$$

and

$$t^{ei}(z_t) = \tilde{A}^{mi*}(z_t) - \lambda^{i*}(z_t)K^*_{t+1}, \quad \bar{t}^e(z_t) = \tilde{\bar{A}}^{m*}(z_t) - \bar{\lambda}^*(z_t)K^*_{t+1},$$

where $\lambda^{i*}(z_t)$ and $\bar{\lambda}^*(z_t)$ with $\Sigma_i\lambda^{i*}(z_t)N^{yi}_{t-1} + \bar{\lambda}^*(z_t)z_tN^{yL}_{t-1} = 1$ are the shares of each type of middle-aged individual in the aggregate investment, then the SCMA is achieved. Note how public policy operates here: the pension system implements the efficient investment in physical and human capital by "crowding-out" private savings through social security contributions.

We can interpret the efficient pension system as one with two components, which are described below.

$$\begin{aligned} b^i(z_t) &= \underbrace{t^{ei}(z_{t-1})R^*(z_t)}_{\hat{b}^i(z_t)} + \underbrace{\left(A^{mi*}_{t-1}(z_t) - \tilde{A}^{mi*}(z_{t-1})R^*(z_t)\right)}_{\tilde{b}^i(z_t)}, \\ t^{pi}(z_t) &= \underbrace{d^{i*}(z_{t-1})R^*(z_t)}_{\widehat{t}^{pi}(z_t)} - \underbrace{\left(A^{yi*}_{t-1}(z_t) + d^{i*}(z_{t-1})R^*(z_t)\right)}_{\tilde{t}^{pi}(z_t)}. \end{aligned}$$

The first component $(\hat{b}^i(z_t), \widehat{t}^{pi}(z_t))$ is used to repay the capitalized value of the educational debt to the lender. The second component $\left(\tilde{b}^i(z_t), \tilde{t}^{pi}(z_t)\right)$ is an insurance contract through which the native middle-aged and elderly generations share the immigration risk[9]. The signs of $\tilde{b}^i(z_t)$ and $\tilde{t}^{pi}(z_t)$ depend on the realization of the shock: when immigration is positive, $\tilde{b}^i(\bar{z}) < 0$ for $i = H, L$ and $\Sigma_i\tilde{t}^{pi}(\bar{z})N^{yi}_{t-1} > 0$, which reflects a transfer from retirees to workers; the opposite occurs in the other case. Additionally, depending on the technological parameters

[9]For an immigrant we have $\widehat{\bar{t}}^p(z_t) = \widetilde{\bar{t}}^p(z_t) = 0$, $\widehat{\bar{b}}(z_t) = \bar{t}^e(z_{t-1})R^*(z_t)$ and $\widetilde{\bar{b}}(z_t) = \bar{A}^{m*}_{t-1}(z_t) - \widetilde{\bar{A}}^{m*}(z_{t-1})R^*(z_t)$.

values, the native high human capital workers may or may not have to transfer something to the native low human capital workers: this will depend on how large their income gains are relative to the aggregate increase in output.

Consider now the case in which, instead of financing education via taxation, the government issues one-period, ear-marked debt in the amount of $\Sigma_{i=H,L} d^{i*}(z_t) N_t^{yi}$ in each period. In the following period, the government pays back $\Sigma_{i=H,L} d^{i*}(z_t) R(z_{t+1}) N_t^{yi} + \Sigma_{i=H,L} \tilde{b}^i(z_{t+1}) N_{t-1}^{yi} + \widetilde{\bar{b}}(z_{t+1}) z_t N_{t-1}^{yL}$ to the debt holders (where $\tilde{b}^i(z_{t+1}) = A_t^{mi*}(z_{t+1}) - \tilde{A}^{mi*}(z_t) R^*(z_{t+1})$ and $\widetilde{\bar{b}}(z_{t+1}) = \bar{A}_t^{m*}(z_{t+1}) - \widetilde{\bar{A}}^{m*}(z_t) R^*(z_{t+1})$). Also in this case, the repayment is financed by a tax on the middle-aged individuals who is also computed by adding two components. The first component is proportional to the previous use of public education financing. The second component again, is for intergenerational insurance. Notice that the net present value of this tax is effectively a lump sum for the middle-aged worker because it depends on only actions taken when the workers are young and on the realization of an exogenous state of the world. In particular, it is not affected by individual's labor supply decisions. In this scheme, the government effectively acts as a (somewhat special) financial institution that issues the missing securities and uses its taxing power to enforce repayment that are efficiently state contingent[10].

4.2. Missing insurance markets

The previous analysis shows that in case 1 of section 3, i.e., when agents have access to credit markets to finance education but insurance is not offered, a PAYGO pension system that always transfers resources from workers to retirees is not efficient. In the absence of private insurance markets, we need a system of intergenerational tax transfers contingent on the realization of the immigration shock. Call this $\tilde{t}^{pi}(z_t), \tilde{b}^i(z_t), \widetilde{\bar{b}}(z_t)$ for $i = H, L$. The balanced budget of this system reads

$$\sum_{i=H,L} \tilde{t}^{pi}(z_t) N_{t-1}^{yi} + \sum_{i=H,L} \tilde{b}^i(z_t) N_{t-2}^{yi} + \widetilde{\bar{b}}(z_t) z_{t-1} N_{t-2}^{yL} = 0.$$

[10]It is important to note that, in general and particularly in Europe, the generosity of the welfare state plays an important role in migration decisions. Nevertheless, it is noteworthy that the present value of net welfare state receipts is zero for the individuals under the optimal welfare state policy described in this paper.

The budget constraints for a member $i = L, H$ of the generation born in period $t-1$ become

$$
\begin{aligned}
& d^i(z_{t-1}) \leq e^i(z_{t-1}) \\
& c^{mi}(z_t) + s^i(z_t) = \omega^i(z_t) h(d^i(z_{t-1}), h^{yi}_{t-1}) - d^i(z_{t-1}) R(z_t) + \tilde{t}^{pi}(z_t) \quad \forall z_t \in Z \\
& c^{oi}(z_{t+1}) = s^i(z_t) R(z_{t+1}) + \tilde{b}^i(z_{t+1}) \quad \forall z_{t+1} \in Z,
\end{aligned}
$$

where $s^i(z_t)$ includes investment in both physical and human capital. For an immigrant who arrives in period t, the budget constraints read

$$
\begin{aligned}
& \bar{c}^m(z_t) + \bar{s}(z_t) = \omega^L(z_t) \bar{h}^m_t \\
& \bar{c}^o(z_{t+1}) = \bar{s}(z_t) R(z_{t+1}) + \tilde{\bar{b}}(z_{t+1}) \quad \forall z_{t+1} \in Z.
\end{aligned}
$$

Market clearing is

$$
\sum_{i=H,L} s^i(z_t) N^{yi}_{t-1} + \bar{s}(z_t) z_t N^{yL}_{t-1} = K_{t+1} + \sum_{i=H,L} d^i(z_t) N^{yi}_t.
$$

To implement the SCMA, we must set

$$
\begin{aligned}
\tilde{b}^i(z_t) &= A^{mi*}_{t-1}(z_t) - \tilde{A}^{mi*}(z_{t-1}) R^*(z_t) \\
\tilde{\bar{b}}(z_t) &= \bar{A}^{m*}_{t-1}(z_t) - \tilde{\bar{A}}^{m*}(z_{t-1}) R^*(z_t) \quad \text{and} \\
\tilde{t}^{pi}(z_t) &= A^{yi*}_{t-1}(z_t) + d^{i*}(z_{t-1}) R^*(z_t).
\end{aligned}
$$

Numerical Evaluation (Continued). To provide a better understanding of how intergenerational insurance operates, we return to the log and Cobb Douglas economy of Appendix A. On the one hand, in the SCMA, the return on the savings of the middle-aged generation in period $t+1$ is $E_t \left\{ \varphi_{t+1}(z) \right\} / p(z_{t+1})$ when the shock is z_{t+1}. In an economy without insurance markets, the return on the savings is $R(z_{t+1}) = \varphi(z_{t+1}) / p(z_{t+1})$. On the other hand, in the SCMA, the net income during middle age is equal to $E_{t-1} \left\{ p_t(z) \omega^i_t(z) h^{mi}_t - d^i_{t-1} \varphi_t(z) \right\} / p(z_t)$, but in an economy without insurance, the net income during middle age is equal to $\omega^i(z_t) h^{mi}_t - d^i_{t-1} R(z_t)$. Let $I^i(z_t) = \omega^i(z_t) h^{mi}_t - d^i(z_{t-1}) R(z_t)$ be the labor income net of educational debt. To implement the SCMA, we must pick

$$
\begin{aligned}
\tilde{b}^i\left(z_{t+1}\right) &= s^i(z_t)R(z_{t+1})\left[\frac{E_t\varphi_{t+1}\left(z\right)/p\left(z_{t+1}\right)}{R(z_{t+1})}-1\right] \quad \text{and} \\
\tilde{t}^{pi}(z_t) &= I^i(z_t)\left[\frac{E_{t-1}\left\{p_t\left(z\right)I^i(z)\right\}/p\left(z_t\right)}{I^i(z_t)}-1\right].
\end{aligned}
$$

The public policy that mimics the efficient insurance contract dictates that the welfare state compensate for the deviations from the expected net total income. In this sense, elderly individuals in period $t+1$ must pay/receive a percentage of her capital income equal to the percent of deviation between the expected return factor on savings in real terms, $E_t\varphi_{t+1}\left(z\right)/p\left(z_{t+1}\right)$, and the real return factor on savings in period $t+1$, $R(z_{t+1})$, when the shock is z_{t+1}. When there is immigration, this difference is negative, and therefore, the elderly generation must pay something to the working generation. Regarding the middle-aged generation in period t, the efficient insurance contract says that working people must pay/receive a percentage of her net labor income in real terms equal to the percentage deviation between the expected labor income net of educational debt in real terms and the real net labor income in period t when the shock is z_t. In the economy of the example of Appendix A, $\tilde{b}_t^i\left(z\right)/(s^i(z_{t-1})R_t(z)) = \tilde{t}_t^{pH}(z)/I_t^H(z)$. With the parameter values of section 2.6, these ratios are set to 0.0385 when there is no immigration and to -0.0357 when an immigration shock takes place. So, the efficient insurance contract in this economy dictates that, when there is immigration, the elderly and high-skilled workers must pay 3.57% of their total net income to the low-skilled workers. This percentage increases in the aggregate increase in output—it increases in the share of low human capital income in the aggregate income and in the magnitude of the immigration shock—and decreases in the probability that an immigration occurs[11]. When there is no immigration, these individuals must receive 3.85% of their net income. This percentage increases in the share of low human capital income in the total output, in the magnitude of the immigration shock and in the probability that an immigration occurs. For the low human capital workers, we have $\tilde{t}_t^{pL}(0)/I_t^L(0) = -0.1356$ and $\tilde{t}_t^{pL}(\bar{z})/I_t^L(\bar{z}) = 0.186$.

[11] Tedious algebra shows that $\tilde{b}_t^i\left(\bar{z}\right)/(s^i(z_{t-1})R_t(\bar{z})) = \tilde{t}_t^{pH}(\bar{z})/I_t^H(\bar{z}) = -\pi(0|z_{t-1})\frac{(1-\alpha-\theta)}{1+\bar{z}}\bar{z}$, $\tilde{b}_t^i\left(0\right)/(s^i(z_{t-1})R_t(0)) = \tilde{t}_t^{pH}(0)/I_t^H(0) = \pi(\bar{z}|z_{t-1})\frac{(1-\alpha-\theta)}{1+(\alpha+\theta)\bar{z}}\bar{z}$, but $\tilde{t}_t^{pL}(\bar{z})/I_t^L(\bar{z}) = \pi(0|z_{t-1})\frac{(\alpha+\theta)+\beta(1-\alpha-\theta)\Psi(z_{t-1})}{1-\beta\Psi(z_{t-1})(1+\bar{z})}\bar{z}$ and $\tilde{t}_t^{pL}(0)/I_t^L(0) = -\pi(\bar{z}|z_{t-1})\frac{(\alpha+\theta)+\beta(1-\alpha-\theta)\Psi(z_{t-1})}{(1-\beta\Psi(z_{t-1}))(1+(\alpha+\theta)\bar{z})}\bar{z}$.

5. The Welfare State of an Open Economy.

How should the previous analysis be altered in the case of an open economy? First, notice that, if capital mobility is instantaneous and physical capital (and high human capital) flows into the country at the same speed at which low-skill immigrants do such that equality is restored between the internal rate of return on capital and the one established on the international capital markets, the immigration shock has no relevance whatsoever because neither the wage of the native workers nor the return on capital of the native capital owners will be affected by the arrival of new workers. In this context, efficient and frictionless capital markets may act as insurance devices that render the state-contingent assets essentially redundant. This is an interesting result because it suggests that, in the light of the simulations presented earlier, the trade deficit that follows an immigration shock is beneficial to both the native and immigrant households in terms of consumption and overall utility.

This observation helps explain, at least in part, what we observed in Spain from 1996 until the arrival of the global financial crisis in 2007-2008[12]: as the flow of immigration into the country continued and even increased, Spain's external trade deficit ballooned but productivity did not move. Along the lines of our model, these facts have the following interpretation: capital flew into Spain at a very high rate if not at the same rate at which labor was entering, which prevented the capital labor ratio and the real wage of unskilled workers from falling. Analysis of the actual data is difficult but not impossible by means of a model as simplified as this, in which there is no distinction between durable and non-durable goods and one period lasts roughly thirty years of which we have observed at most half since the immigration shock first hit. To put it differently, if one takes our model literally, there is no sense in which it can be used to study the Spanish case because we have not yet completely observed even a single "model period" in the actual data.

Nevertheless, a simple back-of-the-envelope calculation based on our model should tell us how much the immigration shock contributed to the Spanish trade deficit of 1996-2007. Therefore, assume that capital entered Spain at roughly the rate needed year after year to keep the internal capital labor ratio constant. We do know that, on average, the educational level of immigrants is very similar to

[12] As we have noted in the introduction, similar results can be obtained in other examples of large immigration shocks like for Israel in early nineties or Canada with the turn of the twentieth century.

that of natives, but immigrants are more likely to be overeducated in their current jobs than are natives[13]. Let us assume for simplicity that the skill distribution of the immigrant workers is similar to the skill distribution of the native workers. This assumption can provide us with a useful benchmark.

The following is the bottom line of the calculation. In Spain, the annual K/Y ratio was approximately 2.9 without housing and 4.0 with housing in 1996. Employment in 1996 was still approximately 12.8 M, and it was 20.3 M in 2007, of which 2.7 M were immigrants. The K/Y ratio increased slowly during the expansion and reached approximately 3.1 in 2007. Hence, the immigrants added almost 21% to the original work force, but they were approximately 13% of the 2007 work force. Take a number in between (i.e., 17%) to account for the fact that this process took place over roughly a decade (a little less, in fact). Because a period in our model is about three times as long as the amount of time considered in the data, everything should be scaled accordingly.

This implies that, if (i) the immigrants came without any K, (ii) the saving rate of the natives remained constant (it roughly did), which means that the national savings supplied the K needed by the natives if Spain was in a steady state before 1996, and, (iii) the final capital-labor ratio for immigrants is similar to those for natives, then Spain would have had to borrow the resources needed to increase its original stock of K by approximately 17% from abroad. In fact, the number is larger because, on top of the 2.7 M immigrant workers, we have approximately another 4.0 M native workers who became employed and were not officially employed when the expansion period started. The quantitative problem with the native workers is more complicated because a part of them were most likely already working in the underground economy (hence, their K already existed), part had accumulated savings they invested in their own K, etc. In any case, because the employment growth attributable to natives would only add to our estimated demand for capital to be imported from abroad, the reference value of 17% based only on the immigrant inflow is a very reasonable lower bound.

In summary, when applied to the 1996-2007 period, our simplified model predicts that Spain should have imported an amount of capital equal to at least 17%

[13] According to the Spanish Labor Force Survey, the immigrant population presents, on average, a very similar educational level as that of natives. In 2007, 57% of Spanish employees had completed at least high school, which is the same number we found for the immigrant employees. Although it is true that native employees are more likely to have a college level education that are immigrant employees (35.8% to 20.7%), immigrants are more likely to be overeducated in their current jobs than are natives (in the sense that their level of education is above of their occupational category).

of its initial capital stock if the international capital markets were functioning properly. Notice that "imported" here means "net import" because there is no export in our model: import in the model is equal to the trade deficit in the national income accounts. This means that Spain had to import a little less than 1/5 of its original stock of capital over a period of approximately 11 years, which is approximately 1.5-1.6% of its capital stock per year every year between 1996 and 2007. Because the K/Y averaged approximately 3.0 (4.0 when housing is considered) during those years, this implies that something between a lower bound of 4.5% and an upper bound of 6.4% of GNP had to be imported each year. That yields a cumulated total import of between 50% and 71% of the Spanish GNP, everything else equal. Between 1996 and 2007, the actual trade deficit adds up to 50.7% of GNP with an annual average of 4.2%. Is this mere chance? Maybe.

6. Conclusions

We have carried out a straightforward exercise. We built the simplest dynamic model in which immigration shocks have both aggregate and distributional effects, which affect some parts of the population differently from others. In particular, in our model, a positive immigration shock increases the labor supply of unskilled middle-aged workers, which makes the high-skill workers and the (older) owners of capital better off apart from increasing GNP. Next, we asked how a system of (sequentially) complete financial markets would handle such shocks and characterized the properties of the sequentially complete market allocations (SCMA). Finally, we asked two types of questions. First, if financial markets were not complete in a practically meaningful sense, what type of welfare policies could bring back the SCMA? Second, if international capital markets were frictionless, how much capital should a country import to insure itself against the immigration shock? Our analysis provides us with four interesting lessons, which we can apply to the case of Spain (1996-2007).

Lesson number 1: immigration shocks have large impacts not only on aggregate output but also on its composition and income distribution. Absent complete financial markets, such impacts should be properly managed by well-planned government policies of the form we have described. It is at least dubious that such policies were implemented in Spain during the period under consideration. The absence of such policies has clear detrimental effects not only on welfare but also on human capital accumulation and overall economic growth.

Lesson number 2: the trade deficit and borrowing from abroad can be a substitute for the missing internal insurance markets. Spain received a very large number of immigrants, and this would have had a dramatic impact on productivity and income distribution if the trade deficit had not allowed the country to accumulate capital stock much faster than the national savings rate allowed. This generated a large trade deficit but increased output, wages, consumption and overall welfare. The same could be said of other countries that have increased their trade deficit after suffering heavy immigration shocks, such as in Israel in the early 1990s.

Lesson number 3: the debate over the impact of immigration on Spanish society and the economy seems to be missing some key factors. In the model presented here, we have outlined some of these factors and paid particular attention to education and pensions. In particular, we have shown that an optimal policy response to a large immigration flow requires a reduction of pension payments and an increase in education investments.

Lesson number 4: even simple stylized models can facilitate the investigation of difficult issues in economic policy and are capable of shedding new light on issues that are often forgotten or considered too complicated to be addressed formally.

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Appendix A: Analytical illustration

In this analytic example, we consider an economy with logarithmic utility and Cobb Douglas production functions: $u(c) = \log c$, $F(K, H, L) = AK^{\alpha} H^{\theta} L^{1-\alpha-\theta}$ and $h\left(d^{i}, h^{yi}\right) = B(d^{i})^{\beta} \left(h^{yi}\right)^{1-\beta}$.

Write $W^{i}(z_t) = \omega^{i}(z_t) h(d^{i}_{t-1}, h^{yi}_{t-1}) + A^{yi}_{t-1}(z_t)$, which is the net income of individual i in period t and state z_t. From (3.c), we have, for a native middle-aged of type i,

$$q(z, z_t) = \frac{\pi(z|z_t)\delta}{A^{mi}_{t}(z)} \left[W^{i}(z_t) - \widetilde{A}^{mi}(z_t)\right] \ \forall z \in Z,$$

where $\widetilde{A}^{mi}(z_t) = \sum_{z \in Z} q(z, z_t) A^{mi}_{t}(z)$. Multiplying this by $A^{mi}_{t}(z)$ and aggregating it in $z \in Z$, we arrive at the total demand for contingent securities of native middle-aged individuals of either type,

$$\widetilde{A}^{mi}(z_t) = \frac{\delta}{1+\delta} W^{i}(z_t).$$

The demand for consumption in middle age and the demand for each component $A^{mi}_{t}(z)$ of $\mathcal{A}^{mi}(z_t)$ are

$$c^{mi}(z_t) = \frac{1}{1+\delta} W^{i}(z_t) \qquad \text{and} \qquad A^{mi}_{t}(z) = \frac{\delta}{1+\delta} W^{i}(z_t) \frac{\pi(z|z_t)}{q(z, z_t)}.$$

Write $\bar{W}(z_t) = \omega^{L}(z_t) h^{mL}_{t}$. For an immigrant in t, we obtain

$$\widetilde{\bar{A}}^{m}(z_t) = \frac{\delta}{1+\delta} \bar{W}(z_t), \ \ \bar{c}^{m}(z_t) = \frac{1}{1+\delta} \bar{W}(z_t) \ \text{ and } \ \bar{A}^{m}_{t}(z) = \frac{\delta}{1+\delta} \bar{W}(z_t) \frac{\pi(z|z_t)}{q(z, z_t)}.$$

Using condition (3.b) and the first order conditions for the firm, we have

$$d^{H}\left(z_{t-1}\right) N^{yH}_{t-1} = \frac{\beta\theta}{\alpha} K_t \quad \text{and} \quad d^{L}\left(z_{t-1}\right) N^{yL}_{t-1} = \frac{\beta(1-\alpha-\theta)}{\alpha} \Psi\left(z_{t-1}\right) K_t,$$

where $\Psi\left(z_{t-1}\right) = E_{t-1}\{p_t(z)\left(1+z\right)^{-\alpha-\theta}\}/E_{t-1}\{p_t(z)\left(1+z\right)^{1-\alpha-\theta}\}$. From (3.$a$), we have $p_t\left(\bar{z}\right) c^{mi}_{t}\left(\bar{z}\right) = p_t\left(0\right) c^{mi}_{t}\left(0\right)$, for $i = H, L$. Then, using the condition for the firm (1.c) and the consumer budget restriction (2.c), we obtain the demand for each component $A^{yi}_{t-1}\left(z\right)$ of $\mathcal{A}^{yi}(z)$:

$$A^{yi}_{t-1}\left(z\right) = \frac{E_{t-1}\left\{p_t\left(z\right) \omega^{i}_{t}(z) h(d^{i}_{t-1}, h^{yi}_{t-1}) - d^{i}_{t-1} \varphi_t(z)\right\}}{p_t\left(z\right)} - \omega^{i}_{t}(z) h(d^{i}_{t-1}, h^{yi}_{t-1}).$$

It is important to note that, in the SCMA, the net income during middle age in equilibrium is equal to $E_{t-1}\left\{p_t(z)\,\omega_t^i(z)h(d_{t-1}^i, h_{t-1}^{yi}) - d_{t-1}^i\varphi_t(z)\right\}/p(z_t)$. Now, using (6.$c$)-(6.$d$) and taking into account that $A_t^{mi}(z) = \widetilde{A}^{mi}(z_t)\pi(z|z_t)/q(z,z_t)$ and $\bar{A}_t^m(z) = \widetilde{\bar{A}}^m(z_t)\pi(z|z_t)/q(z,z_t)$, we have the aggregate demand for each component $A_{t-1}^y(z)$ of $\mathcal{A}^y(z)$:

$$\sum_{i=H,L} A_{t-1}^{yi}(z)N_{t-1}^{yi} = -D(z_{t-1})\frac{\pi(z|z_{t-1})}{q(z,z_{t-1})} + K_t\left[\frac{\varphi_t(z)}{p_t(z)} - \frac{\pi(z|z_{t-1})}{q(z,z_{t-1})}\right].$$

Finally, from (1), we obtain the equilibrium prices for each period t and state $z \in Z$:

$$\begin{aligned}
\omega_t^H(z) &= \theta A K_t^{\alpha} H_t^{\theta-1} L_t(z)^{1-\alpha-\theta} \\
\omega_t^L(z) &= (1-\alpha-\theta)\, A K_t^{\alpha} H_t^{\theta} L_t(z)^{-\alpha-\theta} \\
\varphi_t(z) &= p_t(z)\,\alpha A K_t^{\alpha-1} H_t^{\theta} L_t(z)^{1-\alpha-\theta} \\
q(z,z_t) &= \frac{\pi(z|z_t)p_{t+1}(z)}{E_t\left\{p_{t+1}(z)\,\alpha A K_{t+1}^{\alpha-1} H_{t+1}{}^{\theta} L_{t+1}(z)^{1-\alpha-\theta}\right\}}.
\end{aligned}$$

Note also that, in equilibrium, $p_t(\bar{z})c_t^{mi}(\bar{z}) = p_t(0)c_t^{mi}(0)$. Substituting the values of $c_t^{mi}(\bar{z})$ and $c_t^{mi}(0)$, we arrive to $p_t(\bar{z}) = p_t(0)(1+\bar{z})^{\alpha+\theta}/(1+(\alpha+\theta)\bar{z})$, where $(1+\bar{z})^{\alpha+\theta} < 1+(\alpha+\theta)\bar{z}$ and we normalize $p_t(0) = 1$.

Given initial conditions for $\left(K_0, H_0, L_0, N_0^{oi}, N_0^{mi}, N_0^{yi}, A_{-1}^{yi}(z_0), A_{-1}^{mi}(z_0), A_{-1}^{f}(z_0)\right)$, the following system describes the dynamic of the economy for a given sequence of shocks $(z_0, z_1, ...)$ and a sequence of endowments of basic knowledges $\left\{h_t^{yH}, h_t^{yL}\right\}_{t=0}^{\infty} = \left\{H_t/N_t^{yH}, L_t/N_t^{yL}\right\}_{t=0}^{\infty}$,

$$K_{t+1} = \Omega(z_t, z_{t-1})\, A K_t^{\alpha} H_t^{\theta} L_t^{1-\alpha-\theta}, \tag{A.1}$$

$$H_{t+1} = B\left[\frac{\beta\theta\Omega(z_t,z_{t-1})}{\alpha}\right]^{\beta} A^{\beta} K_t^{\alpha\beta} H_t^{\theta\beta+1-\beta} L_t^{(1-\alpha-\theta)\beta}, \tag{A.2}$$

$$L(z_{t+1}) = (1+z_{t+1})\, B\left[\frac{\beta(1-\alpha-\theta)\Psi(z_t)\,\Omega(z_t,z_{t-1})}{\alpha}\right]^{\beta} A^{\beta} K_t^{\alpha\beta} H_t^{\theta\beta} L_t^{1-\alpha\beta-\theta\beta}, \tag{A.3}$$

where

$$\begin{aligned}\Omega\left(z_t, z_{t-1}\right) &= \frac{\delta}{(1+\delta)\,\Theta\left(z_t\right)}\left[1-\frac{\left(1+(\alpha+\theta)\,z_t\right)\alpha\Theta\left(z_{t-1}\right)}{\left(1+z_t\right)}\left[\pi(0)+\frac{\pi(\bar{z})\,(1+\bar{z})}{(1+(\alpha+\theta)\,\bar{z})}\right]\right], \\ \Theta\left(z_{t-1}\right) &= 1+\frac{\beta}{\alpha}\left(\theta+(1-\alpha-\theta)\Psi\left(z_{t-1}\right)\right),\end{aligned}$$

and

$$\Psi\left(z_{t-1}\right)=\frac{\pi(0|z_{t-1})+\pi(\bar{z}|z_{t-1})\frac{1}{1+(\alpha+\theta)\bar{z}}}{\pi(0|z_{t-1})+\pi(\bar{z}|z_{t-1})\frac{(1+\bar{z})}{1+(\alpha+\theta)\bar{z}}}.$$

Given a sequence of shocks $(z_0, z_1, ..)$, the evolution of the factor intensity ratios $\tilde{k}=K/L$ and $\tilde{h}=H/L$ are given by

$$\begin{aligned}\tilde{k}\left(z_{t+1}\right) &= \frac{\left(A\Omega\left(z_t, z_{t-1}\right)\right)^{1-\beta}}{\left(1+z_{t+1}\right)B\left(\frac{\beta(1-\alpha-\theta)\Psi(z_t)}{\alpha}\right)^{\beta}}\tilde{k}_t^{\alpha(1-\beta)}\tilde{h}_t^{\theta(1-\beta)} \\ \tilde{h}\left(z_{t+1}\right) &= \frac{1}{\left(1+z_{t+1}\right)}\left(\frac{\theta}{(1-\alpha-\theta)\Psi\left(z_t\right)}\right)^{\beta}\tilde{h}_t^{(1-\beta)}.\end{aligned}$$

Set $(z_t, z_{t+1}, z_{t+2}, ...)=(0,0,0,...)$. The rays

$$\begin{aligned}\tilde{h}^* &= \frac{\theta}{(1-\alpha-\theta)\Psi\left(0\right)} \qquad \text{and} \\ \tilde{k}^* &= \left[\frac{\left(\Omega\left(0,0\right)\right)^{1-\beta}}{B\left(\frac{\beta(1-\alpha-\theta)\Psi(0)}{\alpha}\right)^{\beta}}\left(\frac{\theta}{(1-\alpha-\theta)\Psi\left(0\right)}\right)^{\theta(1-\beta)}\right]^{\frac{1}{1-\alpha(1-\beta)}}\end{aligned}$$

define a balanced growth path. For all the initial conditions $(H_0, K_0, L_0)\in\Re_+^2$, the iteration of $(A.1)-(A.3)$ leads (H_t, K_t, L_t) to the rays $\tilde{h}^*$ and $\tilde{k}^*$.

Along the balanced growth path, the three stocks of capital expand (or contract) at the factor

$$1+g^*=\Omega\left(0,0\right)A\left[\frac{\left(\Omega\left(0,0\right)\right)^{1-\beta}}{B\left(\frac{\beta(1-\alpha-\theta)\Psi(0)}{\alpha}\right)^{\beta}}\right]^{\frac{\alpha-1}{1-\alpha(1-\beta)}}\left(\frac{\theta}{(1-\alpha-\theta)\Psi\left(0\right)}\right)^{\frac{\theta\beta}{1-\alpha(1-\beta)}}.$$