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Abstract

Empirical evidence from sequential auctions shows that prices of identical goods tend to decline between rounds. In this paper, I show how expectations-based reference-dependent preferences and loss aversion can rationalize this phenomenon. I analyze two-round sealed-bid auctions with symmetric bidders having independent private values and unit demand. Equilibrium bids in the second round are history-dependent and subject to a "discouragement effect": the higher the winning bid in the first auction is, the less aggressive the behavior of the remaining bidders in the second auction. When choosing his strategy in the first round, however, a bidder conditions his bid on being pivotal and hence expects not to be discouraged. Equilibrium behavior, therefore, leads the winner of the first round to overestimate the bid of his highest opponent and hence the next-round price so that equilibrium prices decline. Moreover, sequential and simultaneous auctions are not bidder-payoff equivalent nor revenue equivalent.

JEL classification: D03; D44; D81; D82.

Keywords: Reference-Dependent Preferences; Loss Aversion; Sequential Auctions; Afternoon Effect.

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1 Introduction

Sequential auctions are often used to sell multiple lots of the same or similar goods. How should one expect prices to vary from one round to the next? Weber (1983) and Milgrom and Weber (2000) showed that with symmetric, risk-neutral, unit-demand bidders having independent private values, the law of one price should hold and on average prices should be the same across different rounds. Intuitively, if they were not, then demand from the rounds with a higher expected price would shift towards those rounds with a lower expected price, due to arbitrage opportunities. To see why, consider a two-round second-price auction. In the last round, all bidders still participating in the auction will bid their valuations since this is a weakly-dominant strategy. In the first round, it is optimal for bidders to shade their bids to account for the option value of participating in the second round. Bidders with a higher valuation also have a higher option value and, therefore, they shade their bids by a greater amount than do bidders with a lower valuation. In the second round, the number of bidders is lower, but the number of objects is lower as well. The first fact has a negative effect on the competition for an object while the second one has a positive effect. Remarkably, in equilibrium these two effects exactly offset each other. As a result, all gains to waiting are arbitraged away and the expected prices in both rounds are the same. The intuition for this result is very general, holds also for more than two rounds and does not depend on the specific type of auction.[1]

However, this neat theoretical result does not seem to be supported by the data. Ashenfelter (1989), McAfee and Vincent (1993) and Ginsburgh (1998) document a puzzling *declining price anomaly* or *afternoon effect* (reflecting that later auctions often take place in the afternoon whereas earlier ones are in the morning) in sequential auctions for identical bottles of wine. Declining price patterns have been also found by Beggs and Graddy (1997) for artwork, Ashenfelter and Genesove (1992) for condominiums, Van den Berg *et al.* (2001) for flowers and Lambson and Thurston (2006) for fur. There is also experimental evidence of declining prices in sequential auctions; see Keser and Olson (1996), Février *et al.* (2007) and Neugebauer and Pezanis-Christou (2007). Moreover, while declining prices are more frequent, increasing prices have also been documented; see, for example, Chanel *et al.* (1996), Gandal (1997) and Deltas and Kosmopoulou (2004).[2] It is quite an interesting result that, in a variety of different types of auctions, price direction throughout an auction can be predicted. Declining prices have been documented in more instances than rising prices have. Declining prices do not occur in every auction, but they seem to be an empirically robust feature of sequential auctions.

[1] Technically, with independent private values, the price sequence of any standard auction is a martingale, so that the expected price in round $k+1$, conditional on p_k, the price in round k, is equal to p_k.

[2] Milgrom and Weber (2000) showed that if bidders' signals are affiliated and values are interdependent, then the equilibrium price sequence is a sub-martingale and the expected value of p_{k+1}, conditional on p_k, is higher than p_k. Mezzetti (2011) showed that affiliated signals are not necessary to explain increasing-price sequences: interdependent values with informational externalities — that is, when a bidder's value is increasing in all bidders' private signals — even with independent signals, push prices to increase between rounds.

In this paper, I study two-round sealed-bid auctions with symmetric bidders having independent private values and unit demand and I argue that reference-dependent preferences and loss aversion provide an explanation for the afternoon effect. More generally, I show that reference-dependent preferences with expectations as the reference point induce an endogenous form of interdependence in the bidders' payoffs even though values are private and independent. Indeed, the derivation of the equilibrium strategies resembles that of the standard reference-free model with interdependent (common) values. The reason is that even though a bidder's valuation does not depend directly on his competitors' types, these affect the likelihood of him winning the auction and hence his reference point.

Section 2 introduces the model of bidders' preferences and the solution concept. Following Köszegi and Rabin (2006), I assume that in addition to classical *consumption utility*, a bidder also derives *gain-loss utility* from the comparison of his consumption to a reference point equal to his lagged expectations regarding the same outcomes, with losses being more painful than equal-sized gains are pleasant. To account for the intrinsic dynamic nature of sequential auctions, I develop a dynamic version of the Choice Acclimating Personal Equilibrium (*CPE)* introduced in Köszegi and Rabin (2007) that I call Sequential Choice Acclimating Personal Equilibrium (*SCPE).* In a *SCPE*, a decision maker correctly predicts his (possibly stochastic) strategy at each point in the future, folds-back the game tree using backward induction, and then applies the same (static) *CPE* as in Köszegi and Rabin (2007) at every stage of the game.[3]

Sections 3 and 4 analyze sequential first-price and second-price auctions, respectively. First, I show that expectations-based reference-dependent preferences create an informational externality that renders equilibrium bids history-dependent, even if bidders have independent private values. Intuitively, learning the outcome of the previous auction modifies a bidder's expectations about how likely he is to win in the current one. Since expectations are the reference point, the optimal bid in the second auction depends also on what a bidder learns from the first one as this modifies his reference point. More precisely, I identify what I call the *discouragement effect*: the higher the winning bid in the first auction is, the less aggressive the bidding strategy of the remaining bidders in the second auction. The intuition is that, from the point of view of a bidder who lost the first auction, the higher the type of the winner is, the less likely he is to win in the second one; this in turn lowers the bidder's reference point who does not feel a strong attachment to the item and therefore reduces his equilibrium bid.

Notice that the history dependence arising in my model has the opposite effect of the one stemming from interdependent (common) values. With interdependent values, since in equilibrium he conditions his bid on himself having the highest signal, if a bidder loses the current

[3] The original notion of *CPE* in Köszegi and Rabin (2007) is related to the models of "disappointment aversion" of Bell (1985), Loomes and Sugden (1986), and Gul (1991), where outcomes are also evaluated relative to a reference lottery that is identical to the chosen lottery; likewise the notion of *SCPE* introduced in this paper is related to the notion of dynamic disappointment aversion proposed in Artstein-Avidan and Dillenberger (2011).

auction he learns that the winner had a higher signal than his; this in turn makes a losing bidder revise his estimate of the value of the good upward and therefore he will bid more aggressively in subsequent auctions. The discouragement effect instead goes in the opposite direction by pushing bidders to bid less aggressively in later rounds.

When choosing his bid in the first round, however, a bidder conditions his bid on himself having the highest type and hence expects not to be discouraged. Indeed, in the first round it is optimal for a bidder to bid up to a point where he is indifferent between barely winning in the current round (by being tied with his strongest opponent) and winning in the next round. However, ex-post the winner never barely wins. Equilibrium behavior, therefore, leads the winner of the first round to overestimate the bid of his highest opponent and hence the next-round price so that equilibrium prices tend to decline.

With risk-neutral bidders having independent private values, sequential and simultaneous auctions are revenue-equivalent for the seller and payoff-equivalent for the bidders. In Section 5 I show that these equivalences break down if bidders are expectations-based loss-averse. The key difference between sequential and simultaneous auctions is the timing of information. Sequential auctions provide bidders, in between rounds, with the opportunity to update their beliefs about the intensity of competition. Such feedback, however, is absent in simultaneous auctions. In the classical model this difference is irrelevant since bidding strategies in sequential auctions are history-independent. Loss-averse bidders, instead, update their reference point based on the outcome of the previous round. I show that bidders with high (resp. low) values prefer sequential (simultaneous) auctions since they are more likely to receive good (bad) news between rounds. Furthermore, sequential auctions generate a higher (resp. lower) revenue than simultaneous ones when the number of bidders is large (small).

For most of the paper, when dealing with sequential sealed-bid auctions, I assume that the winning bid in the first round is publicly announced by the seller prior to the second round. This assumption is inconsequential in the classical reference-free model when bidders have independent private values, but it is not when bidders have reference-dependent preferences. Therefore, in Section 6 I analyze sequential auctions with no bid announcement and I show that the equilibrium strategies are radically different. If the winning bid from the first round is not publicly revealed, a losing bidder must use his own past bid to update his expectations about how likely he is to win in the second one. As auctions without bid announcement provide them with a noisier feedback mechanism, thus exposing them to greater risk, loss-averse bidders react by bidding less aggressively so that the seller's expected revenue decreases. Nevertheless, the afternoon effect still arises in equilibrium. The reason, in this case, is that bidders are willing to pay a premium in the first round in order to avoid having to go to the second round and being discouraged.

Many different explanations for the afternoon effect have been proposed. Ashenfelter (1989) hypothesized risk aversion as a plausible explanation for the declining-price pattern. However, McAfee and Vincent (1993) argue that risk aversion is not a convincing explanation. They studied two-round first-price and second-price auctions with independent private values, and showed that equilibrium prices decline only if bidders display increasing absolute risk aversion. Under the more plausible assumption of decreasing absolute risk aversion, a monotone symmetric pure-strategy equilibrium fails to exist and prices need not decline. Black and De Meza (1992) and Février *et al.* (2005) argue that declining prices are no anomaly if the winning bidder in one auction is allowed to purchase all subsequent lots at the same price.[4] Bernhardt and Scoones (1994), Engelbrecht-Wiggans (1994) and Gale and Hausch (1994) consider sequential auctions of "stochastically equivalent" objects — that is, when each bidders' valuations are identically distributed across the objects, but are not perfectly correlated — and show that in this case equilibrium bidding implies declining prices. Eyster (2002) models the behavior of an agent who has a taste for rationalizing past actions by taking current actions for which those past actions were optimal. He shows that this taste for consistency gives rise to an "unsunk-cost fallacy" that can rationalize declining prices in sequential English auctions. Other studies have emphasized demand complementarity (Menezes and Monteiro, 2003), supply uncertainty (Jeitschko, 1999), order-of-sale effects (Chakraborty *et al.*, 2006) and budget constraints (Pitchick and Schotter, 1988) in accounting for the declining price anomaly.

More recently, Mezzetti (2011) introduced a special case of risk aversion, called *aversion to price risk,* according to which a bidder prefers to win an object at a certain price rather than at a random price with the same expected value.[5] Under this different notion of risk aversion, in sequential auctions with independent private values a monotone equilibrium in pure strategies always exists and in equilibrium prices decline. Although aversion to price risk and loss aversion are both able to explain the afternoon effect, the intuition behind the result is quite different. In Mezzetti (2011), the afternoon effect is due to the bidders' dislike of uncertainty over money; in my model, instead, the afternoon effect arises because bidders dislike uncertainty over consumption. The different intuition translates also into different testable predictions. When bidders are averse to price risk, the equilibrium strategies do not depend on the history of the game and therefore the seller's information revelation policy does not affect revenue; with loss aversion, instead, bidders' strategies are history-dependent and the seller is always better off by committing to publicly reveal the history of the winning bids.

Section 7 concludes the paper by recapping the results of the model and pointing out some of its limitations as well as possible avenues for future research. All proofs are relegated to Appendix A.

[4] However, Ashenfelter (1989) finds declining prices also for the case of bidders with unit demand.

[5] Similarly to the model of reference-dependent preferences of Köszegi and Rabin (2006), Mezzetti's notion of aversion to price risk assumes separability of a bidder's payoff between the utility from winning the object and the disutility from paying the price.

2 Model

2.1 Environment

Suppose 2 identical items are sold to $N > 2$ bidders via a series of sealed-bid auctions. More specifically, one item is sold using a sealed-bid auction and the winning bid is publicly announced; then, the remaining item is sold using a sealed-bid auction. Announcing the winning bid from former auctions prior to the current one is in accord with government procurement statutes and with actual practice in some auctions.

I assume bidders want at most one unit and have independent private values. Each bidder's valuation (type) θ_i is drawn independently from the same distribution F with continuous positive density f everywhere on the support $\left[0, \overline{\theta}\right]$. I will consider two types of games. In the first one, the goods are sold sequentially via a series of first-price auctions. In the second one, the goods are sold sequentially via a series of second-price auctions. Both auctions have a zero reserve price. Throughout the paper, I restrict attention to symmetric equilibria in pure and (strictly) monotone strategies.[6] It is convenient to think of the auctions as being held in different periods of the day, the first one in the morning and the second one in the afternoon; however I assume the auctions are held in a short enough time so that bidders do not discount payoffs from the second auction.

2.2 Bidders' Preferences

Bidders have expectations-based reference-dependent preferences as formulated by Köszegi and Rabin (2006). In this formulation, a bidder's utility function has two components. First, if he wins the auction at price p, a type-θ bidder experiences *consumption utility* $\theta - p$. Consumption utility can be thought of as the classical notion of outcome-based utility. Second, the bidder also derives utility from the comparison of his actual consumption to a reference point given by his recent expectations (probabilistic beliefs).[7] Slightly departing from the original model of Köszegi and Rabin (2006), I assume that bidders have reference-dependent preferences only with respect to their valuation for the item, but not with respect to the price they might pay; in other words, bidders are risk neutral over money. Although restrictive, this assumption is reasonable when bidders' income is already subject to large background risk as argued by Köszegi and Rabin (2009). Relatedly, Novemsky and Kahneman (2005) propose that money given up in purchases

[6] In a symmetric equilibrium, the final allocation is efficient: the first good will go to the bidder with the highest value and the second one to the bidder with the second-highest value.

[7] Recent experimental evidence lends support to Köszegi and Rabin's (2006) expectations-based model of reference-dependent preferences and loss aversion; see, for instance, Abeler *et al.* (2011), Ericson and Fuster (2011), Gill and Prowse (2012), Karle *et al.* (forthcoming) and Sprenger (2011). More pertinently, Banerji and Gupta (2014), Eisenhuth and Ewers (2012) and Ratan (2013) provide experimental evidence supporting the model of Köszegi and Rabin (2006) in the context of sealed-bid auctions.

is not generally subject to loss aversion. Hence, for a riskless consumption outcome (θ, p) and riskless reference point r^{θ}, a bidder's total utility is given by

$$U\left[(\theta, p) \mid \left(r^{\theta}\right)\right] = \theta - p + \mu\left(\theta - r^{\theta}\right) \tag{1}$$

where

$$\mu(x) = \begin{cases} \eta x & \text{if} \quad x \geq 0 \\ \eta \lambda x & \text{if} \quad x < 0 \end{cases}$$

is *gain-loss utility,* with $\eta > 0$ and $\lambda > 1$. The parameter η captures the relative weight a consumer attaches to gain-loss utility while λ is the coefficient of loss aversion.

Because in many situations expectations are stochastic, Köszegi and Rabin (2006) extend the utility function in (1) to allow for the reference point to be a probability distribution H. In this case a consumer's overall utility from the outcome (θ, p) can be written as

$$U\left[(\theta, p) \mid H\right] = \theta - p + \int_{r^{\theta}} \mu\left(\theta - r^{\theta}\right) dH \tag{2}$$

In words, a bidder compares the realized consumption outcome with each possible outcome in the reference lottery. For example, if he expected to win the auction with probability q, then winning the auction feels like a gain of $\theta(1-q)$ while losing the auction results in a loss of θq. Thus, the weight on the loss (gain) in the overall experience is equal to the probability with which he was expecting to win (lose) the auction.

2.3 Solution Concept

Each bidder learns his valuation before submitting his bids and, therefore, maximizes his interim expected utility. If the distribution of the reference point is H and the distribution of consumption outcomes is G, a type-θ bidder's interim expected utility is given by

$$EU\left[G|H\right] = \int_{\{\theta,p\}} \int_{\{r^{\theta}\}} U\left[(\theta, p) \,|r^{\theta}\right] dH dG.$$

For each auction in which he participates, after placing a bid, a bidder basically faces a lottery between winning or losing the auction and the probabilities and potential payoffs depend on his own as well as other players' bids. The final outcome is then evaluated with respect to any possible outcome from this lottery as a reference point. As laid out in Köszegi and Rabin (2007), Choice Acclimating Personal Equilibrium (*CPE)* is the most appropriate solution concept for such decisions under risk when uncertainty is resolved after the decision is made so that the decision maker's strategy determines the distribution of the reference point as well as the distribution of final consumption outcomes; that is, $H = G$.

A strategy for bidder i is a pair of bidding functions $\boldsymbol{\beta}_i = (\beta_1, \beta_2)$, one for each auction.

Fixing all other bidders' strategies, $\boldsymbol{\beta}_{-i}$, bidder i's strategy $\boldsymbol{\beta}_i$ induces a distribution over the set of final consumption outcomes. For $k = 1, 2$, let $\Gamma_k \left(\boldsymbol{\beta}_i, \boldsymbol{\beta}_{-i}\right)$ denote the distribution over final consumption outcomes from auction k point of view. Similarly, let EU_k denote a bidder expected utility from auction k point of view if he plans to bid according to $\boldsymbol{\beta}_i$ and expects his rivals to bid according to $\boldsymbol{\beta}_{-i}$. To account for the intrinsic dynamic nature of sequential auctions, I introduce a slightly modified version of *CPE.*

Definition 1. *A strategy profile $\boldsymbol{\beta}^*$ constitutes a Sequential Choice Acclimating Personal Equilibrium (SCPE) if for all i, and for $k = 1, 2$:*

$$EU_k \left[\boldsymbol{\beta}_i^*, \boldsymbol{\beta}_{-i}^* | \Gamma_k \left(\boldsymbol{\beta}_i^*, \boldsymbol{\beta}_{-i}^*\right)\right] \geq EU_k \left[\widetilde{\boldsymbol{\beta}}_i, \boldsymbol{\beta}_{-i}^* | \Gamma_k \left(\widetilde{\boldsymbol{\beta}}_i, \boldsymbol{\beta}_{-i}^*\right)\right]$$

for any $\widetilde{\boldsymbol{\beta}}_i \neq \boldsymbol{\beta}_i^*$.

In words, in a *SCPE* a bidder correctly predicts his (possibly stochastic) strategy at each point in the future, then folds-back the game tree using backward induction and applies the same (static) *CPE* as in Köszegi and Rabin (2007) at every stage of the game. Notice that, at stage k, a bidder's reference point is given by his stage-k expectations, $\Gamma_k \left(\boldsymbol{\beta}_i^*, \boldsymbol{\beta}_{-i}^*\right)$, about his final consumption at the end of all auctions. Furthermore, in the second round bidders update their reference point based on the outcome of the first round.[8] The following assumption, maintained for the remainder of the paper, guarantees that all bidders participate in the auction for any realization of their own type, and that the equilibrium bidding functions derived in the next sections are strictly increasing:

Assumption 1 (*No dominance of gain-loss utility*) $\Lambda \equiv \eta (\lambda - 1) \leq 1$.

This assumption places, for a given η (λ), an upper bound on λ (η) and ensures that an agent's equilibrium expected utility is increasing in his type.[9] What it requires is that the weight a bidder places on expected gain-loss utility does not (strictly) exceed the weight he puts on consumption utility. Finally, notice that risk neutrality is embedded in the model as a special case (for either $\eta = 0$ or $\lambda = 1$).

[8]The concept of *SCPE* introduced in this paper coincides with a special case of the dynamic version of Preferred Personal Equilibrium (*PPE*) introduced by Köszegi and Rabin (2009). In their dynamic model, people have a reference point for every period in which they expects to consume — so that consumption levels in different periods are treated like different dimensions — and are loss-averse over changes in beliefs about present as well future consumption. If consumption takes place only in the last period and the weight on prospective gain-loss utility (or "news" utility) is equal to 1, my solution concept is equivalent to theirs.

[9]Herweg *et al.* (2010) first introduced Assumption 1 and referred to it as "no dominance of gain-loss utility". Assumption 1 has been used also by Lange and Ratan (2010), Eisenhuth (2012) and Eisenhuth and Ewers (2012).

3 First-price Auctions

Consider a situation in which two identical items are sold sequentially via first-price auctions. In this case, a symmetric equilibrium consists of two bidding functions (β_1, β_2), one for each auction. I assume that both functions are strictly increasing and differentiable. The first-round bidding strategy is a function $\beta_1 : \left[0, \overline{\theta}\right] \to \mathbb{R}_+$ that depends only on the bidder's type. The bid in the second auction, instead, might depend also on the price paid in the first auction. Since we are focusing on a symmetric equilibrium, it is useful to take the point of view of one of the bidders, say bidder i with type θ_i and to consider the order statistics associated with the types of the other bidders. Let $Y_1^{(N-1)} \equiv Y_1$ be the highest of $N-1$ values, $Y_2^{(N-1)} \equiv Y_2$ be the second-highest and so on. Also, let F_1 and F_2 be the distributions of Y_1 and Y_2 respectively, with corresponding densities f_1 and f_2. Since the first-period bidding function β_1 is assumed to be invertible, after the first auction is over and its winning bid is revealed the valuation of the winning bidder is commonly known to be $y_1 = \beta_1^{-1}(p_1)$. Thus, the second-period strategy can be described as a function $\beta_2 : \left[0, \overline{\theta}\right] \times \left[0, \overline{\theta}\right] \to \mathbb{R}_+$ so that a bidder with value θ bids $\beta_2(\theta, y_1)$ if $Y_1 = y_1$. To find an equilibrium that is sequentially rational, I start by looking at the bidder's problem in the second auction.

3.1 Second-period strategy

Consider a bidder with type θ who plans to bid as if his type were $\widetilde{\theta} \neq \theta$ when all other $N-2$ remaining bidders follow the equilibrium strategy $\beta_2(\cdot, y_1)$. His expected payoff is

$$\begin{aligned} EU_2\left(\widetilde{\theta}, \theta; y_1\right) = {} & F_2\left(\widetilde{\theta}|y_1\right)\left[\theta - \beta_2\left(\widetilde{\theta}, y_1\right)\right] \\ & + F_2\left(\widetilde{\theta}|y_1\right)\eta\left\{(\theta - 0)\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]\right\} \\ & + \left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]\eta\lambda\left\{(0 - \theta)F_2\left(\widetilde{\theta}|y_1\right)\right\} \end{aligned} \tag{3}$$

where $F_2\left(\widetilde{\theta}|y_1\right)$ is the probability that Y_2, the second highest valuation among $N-1$, is less than $\widetilde{\theta}$ conditional on $Y_1 = y_1$ being the highest. The first term on the right-hand-side of (3), $F_2\left(\widetilde{\theta}|y_1\right)\left[\theta - \beta_2\left(\widetilde{\theta}, y_1\right)\right]$, is standard expected consumption utility. The other terms capture expected gain-loss utility and are derived as follows. A bidder of type θ who bids as if his type were $\widetilde{\theta}$ expects to win the auction with probability $F_2\left(\widetilde{\theta}|y_1\right)$ and if he wins he gets consumption utility θ; thus, winning the auction feels like a gain of $\eta(\theta - 0)$ compared to the outcome of losing the auction and getting 0, which the bidder expected to happen with probability $\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]$. Similarly, with probability $\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]$ the bidder loses the auction and gets 0; thus, losing the auction feels like as loss of $\eta\lambda(0 - \theta)$ compared to the outcome of winning the auction and getting θ, which the bidder expected to happen with probability $F_2\left(\widetilde{\theta}|y_1\right)$. Collapsing terms we can re-write (3) as

$$EU_2\left(\widetilde{\theta},\theta;y_1\right)= \quad F_2\left(\widetilde{\theta}|y_1\right)\left[\theta-\beta_2\left(\widetilde{\theta},y_1\right)\right]-\Lambda\theta F_2\left(\widetilde{\theta}|y_1\right)\left[1-F_2\left(\widetilde{\theta}|y_1\right)\right]$$

where $\Lambda \equiv \eta(\lambda-1)$ is the weight on expected gain-loss utility. Notice that expected gain-loss utility is always negative as, since $\lambda > 1$, losses are felt more heavily than equal-size gains.

Differentiating $EU_2\left(\widetilde{\theta},\theta;y_1\right)$ with respect to $\widetilde{\theta}$ yields the first-order condition:

$$f_2\left(\widetilde{\theta}|y_1\right)\left[1-2F_2\left(\widetilde{\theta}|y_1\right)\right]\theta\Lambda=f_2\left(\widetilde{\theta}|y_1\right)\left(\theta-\beta_2\left(\widetilde{\theta},y_1\right)\right)-\beta_2'\left(\widetilde{\theta},y_1\right)F_2\left(\widetilde{\theta}|y_1\right)$$

where β_2' is the derivative of β_2 with respect to its first argument.

Substituting $\theta=\widetilde{\theta}$ into the first-order condition and re-arranging results in the following differential equation

$$\frac{\partial}{\partial\theta}\left\{\beta_2(\theta,y_1)F_2(\theta|y_1)\right\}=f_2(\theta|y_1)\,\theta\left\{1-\Lambda\left[1-2F_2(\theta|y_1)\right]\right\} \tag{4}$$

together with the boundary condition that $\beta_2(0,y_1)=0$.

Because the different values are drawn independently, we have that

$$F_2(\theta|y_1)=\frac{F(\theta)^{N-2}}{F(y_1)^{N-2}}$$

and substituting into (4) yields

$$\beta_2^*(\theta,y_1)=\frac{\int_0^\theta x\left\{1-\Lambda\left[1-2\frac{F(x)^{N-2}}{F(y_1)^{N-2}}\right]\right\}dF(x)^{N-2}}{F(\theta)^{N-2}}. \tag{5}$$

The complete bidding strategy is to bid $\beta_2^*(\theta,y_1)$ if $\theta<y_1$ and to bid $\beta_2^*(y_1,y_1)$ if $\theta\geq y_1$. The latter might occur if a bidder of type $\theta\geq y_1$ underbid in the first period causing a lower type to win (of course this is an off-equilibrium event).

The expression in (5) can be re-written as:

$$(1-\Lambda)\frac{\int_0^\theta x dF(x)^{N-2}}{F(\theta)^{N-2}}+\Lambda\frac{\int_0^\theta 2x\frac{F(x)^{N-2}}{F(y_1)^{N-2}}dF(x)^{N-2}}{F(\theta)^{N-2}}.$$

Therefore, the bid under loss aversion is a convex combination of the risk-neutral bid and a term that depends on the bidder's expectations about how likely he is to win the auction (reference point).

The first thing worth noticing is that, even if bidders have independent private values, the equilibrium bidding strategy in the second period is history-dependent, as it is a function of y_1;

with risk-neutral preferences ($\Lambda = 0$), instead, this is not the case:

$$\beta_2^{RN}(\theta) = \frac{\int_0^\theta x dF(x)^{N-2}}{F(\theta)^{N-2}}.$$

Under risk neutrality, a bidder submits a bid equal to his estimation of the highest valuation of his opponents, conditional on his own valuation being the highest. Because of this conditioning, bids are independent of the prior history of the game. With reference-dependent preferences, instead, the second-round equilibrium bid is decreasing in the first-round price, as shown in the following lemma.

Lemma 1. *(Discouragement Effect) If $\Lambda > 0$, then $\frac{\partial \beta_2^*(\theta, y_1)}{\partial y_1} < 0\ \forall \theta$.*

According to the result in Lemma 1, the higher is the type of the winner in the first round, the less aggressively the remaining bidders will bid in the second round. The rationale for this negative effect, which I call the *discouragement* effect, is as follows. From the perspective of a bidder who lost the first auction, the higher is the type of the winner, the less likely this bidder is to win in the second auction; with expectations-based reference-dependent preferences a bidder who thinks that most likely he is not going to win does not feel a strong attachment to the item and this pushes him to bid more conservatively. Thus, revealing the first-period winner's bid (and hence his type) creates an informational externality. However, notice that the effect of this informational externality on the second-period bids is exactly the opposite of the one that arises with interdependent (common) values. Indeed, with interdependent values the higher is the type of the first-round winner, the higher is the value of the object to all remaining bidders who in turn bid more aggressively in the second auction. Therefore, by analyzing the distribution of bids in the second auction, one can use the discouragement effect to empirically test the implications of loss aversion against the implications of the classical risk-neutral model with either private values (where there is no history dependence) or with common values (where the higher is the winning price in the first auction, the more aggressively the remaining bidders behave in the second auction).

Figure 1 displays the bidding strategy $\beta_2^*(\theta, y_1)$ for two different values of y_1 assuming $\theta \overset{U}{\sim} [0,1]$, $\Lambda = \frac{1}{2}$ and $N = 4$: the dashed curve is for the case $y_1 = \frac{1}{2}$ while the solid one is for the case $y_1 = \frac{3}{4}$. As we would expect from Lemma 1, for $\theta \leq \frac{1}{2}$ $\beta_2^*\left(\theta, \frac{3}{4}\right)$ is always below $\beta_2^*\left(\theta, \frac{1}{2}\right)$.

Furthermore, we have that

$$\frac{\partial \beta_2^*(\theta, y_1)}{\partial \Lambda} = \frac{\int_0^\theta x\left[2\frac{F(x)^{N-2}}{F(y_1)^{N-2}} - 1\right] dF(x)^{N-2}}{F(\theta)^{N-2}}$$

implying that there exists $0 < \widehat{\theta}(y_1) < \overline{\theta}$ such that $\frac{\partial \beta_2^*(\theta, y_1)}{\partial \Lambda} > 0 \Leftrightarrow \theta > \widehat{\theta}(y_1)$. In other words,

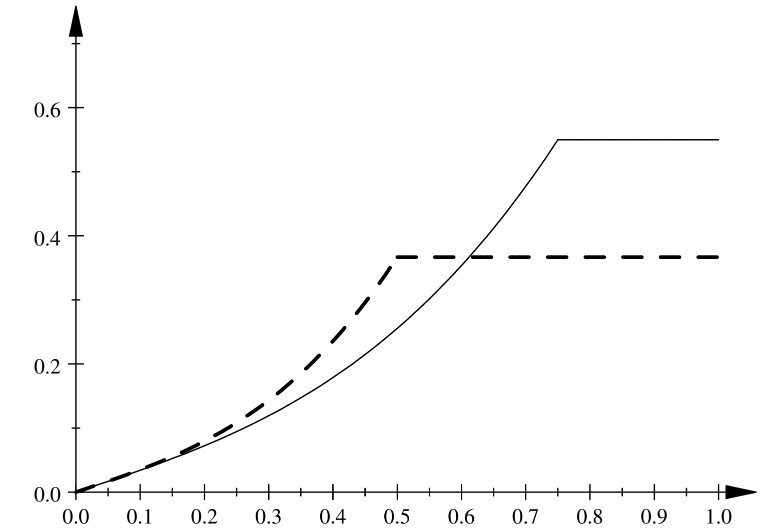

Figure 1: The effect of y_1 on $\beta_2^*(\theta, y_1)$ for $N = 4$ and $\Lambda = \frac{1}{2}$ with θ distributed uniformly on $[0, 1]$.

under loss aversion bidders with relatively high types overbid compared to the risk-neutral benchmark whereas bidders with relatively low types underbid.

I end this section with an example where I compute the second-round bidding strategy explicitly for the case of uniformly-distributed types.

Example 1. *Suppose that* $\theta \overset{U}{\sim} [0, 1]$ *and let* y_1 *be the type of the winner in the first round. Then, in the second round the remaining bidders will play the following strategy:*

$$\beta_2^*(\theta, y_1) = \begin{cases} (1-\Lambda)\left(\frac{N-2}{N-1}\right)\theta + \Lambda\left(\frac{2N-4}{2N-3}\right)\frac{\theta^{N-1}}{y_1^{N-2}} & \text{if} \quad \theta < y_1 \\ (1-\Lambda)\left(\frac{N-2}{N-1}\right)y_1 + \Lambda\left(\frac{2N-4}{2N-3}\right)y_1 & \text{if} \quad \theta \geq y_1 \end{cases}.$$

It is easy to see that, for $\theta < y_1$, $\beta_2^*(\theta, y_1)$ *is (i) increasing in* θ*, (ii) decreasing in* y_1 *and (iii) increasing in* Λ *if and only if* $\theta > \left(\frac{N-2}{N-1}\frac{2N-3}{2N-4}\right)^{\frac{1}{N-2}} y_1$.

3.2 First-period strategy

Consider a bidder with type θ who plans to bid as if his type were $\widetilde{\theta} > \theta$ when all other $N-1$ bidders follow the strategy $\beta_1(\cdot)$.[10] Furthermore, suppose that all bidders expect to follow the equilibrium strategy $\beta_2^*(\theta, y_1)$ in the second auction, regardless of what happens in the first one (sequential rationality). The bidder's expected total utility at the beginning of the first round is

[10] The analysis is virtually identical for the case $\widetilde{\theta} < \theta$.

$$
\begin{aligned}
EU_1\left(\widetilde{\theta},\theta\right) &= F_1\left(\widetilde{\theta}\right)\left[\theta-\beta_1\left(\widetilde{\theta}\right)\right]+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)\left[\theta-\beta_2^*\left(\theta,y_1\right)\right] f_1\left(y_1\right) dy_1 \\
&\quad -\Lambda\theta\left[F_1\left(\widetilde{\theta}\right)+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1\right]\left[1-F_1\left(\widetilde{\theta}\right)-\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1\right]
\end{aligned} \tag{6}
$$

where $F_1\left(\widetilde{\theta}\right)$ is the probability that Y_1, the highest valuation among $N-1$, is less than $\widetilde{\theta}$, and $F_2\left(\theta|y_1\right)$ and Λ are defined as before.

The first line on the right-hand-side of (6) is the sum of expected consumption utilities in periods 1 and 2. The second line captures expected gain-loss utility. Indeed, $F_1\left(\widetilde{\theta}\right)+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1$ is the sum of the probability with which a bidder of type θ expects to win the first auction given that he pretends to be of type $\widetilde{\theta}$ and of his expectation, in the first round, of the probability of winning in the second round given that he pretends to be of type $\widetilde{\theta}$ in the first auction but expects to behave as his real type in the second one. Hence, in accordance with the definition of *SCPE* in Section 2, a bidder's reference point in the first round is given by his overall probability of consumption.

Differentiating $EU_1\left(\widetilde{\theta},\theta;y_1\right)$ with respect to $\widetilde{\theta}$ yields the following first-order condition:

$$
\begin{aligned}
0 &= f_1\left(\widetilde{\theta}\right)\left[\theta-\beta_1\left(\widetilde{\theta}\right)\right]-\beta_1'\left(\widetilde{\theta}\right)F_1\left(\widetilde{\theta}\right)-F_2\left(\theta|\widetilde{\theta}\right)\left[\theta-\beta_2^*\left(\theta,\widetilde{\theta}\right)\right]f_1\left(\widetilde{\theta}\right) \\
&\quad -\Lambda\theta\left[f_1\left(\widetilde{\theta}\right)-F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\right]\left[1-F_1\left(\widetilde{\theta}\right)-\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1\right] \\
&\quad -\Lambda\theta\left[F_1\left(\widetilde{\theta}\right)+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1\right]\left[-f_1\left(\widetilde{\theta}\right)+F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\right].
\end{aligned}
$$

Substituting $\theta=\widetilde{\theta}$ and re-arranging results in the following differential equation

$$
\frac{d}{d\theta}\left\{\beta_1\left(\theta\right)F_1\left(\theta\right)\right\}=f_1\left(\theta\right)\beta_2^*\left(\theta,\theta\right)
$$

together with the boundary condition that $\beta_1\left(0\right)=0$. Solving the differential equation yields

$$
\beta_1^*\left(\theta\right)=\frac{\int_0^{\theta}\beta_2^*\left(s,s\right)f_1\left(s\right)ds}{F_1\left(\theta\right)}. \tag{7}
$$

The first thing worth noticing is that the bidding function in (7) resembles the bidding function in the classical reference-free model with interdependent values (see Milgrom and Weber, 2000 and Mezzetti, 2011). Furthermore, $\beta_1^*\left(\theta\right)$ depends on Λ only indirectly, through $\beta_2^*\left(s,s\right)$. Indeed, just like in the standard model with reference-free preferences, in the first round a bidder chooses his optimal bid by conditioning on himself having the highest type. This is because a small change in his bid only matters when the bidder wins or is close to winning.[11] When conditioning on having the highest type, however, a bidder expects that if he were to lose the current auction, he would win the next one for sure and this is why expected gain-loss utility does not directly appear into the first-period bidding function. Finally, it is easy to check that for $\Lambda = 0$ we get back to the risk-neutral benchmark:

$$\beta_1^{RN}(\theta) = \frac{\int_0^\theta \beta_2^{RN}(s) f_1(s)\, ds}{F_1(\theta)}$$

where $\beta_2^{RN}(s)$ does not depend of the type of the winner of the first auction.

Let $y_1 = \beta_1^{-1}(p_1)$. Then, the expected equilibrium price in the second auction conditional on the price of the first auction is

$$\mathbb{E}[p_2|p_1] = \mathbb{E}[p_2|\beta_1(y_1)] = \mathbb{E}\left[\beta_2^*\left(Y_1^{(N-1)}, y_1\right)|Y_1^{(N-1)} \leq y_1\right] = \frac{\int_0^{y_1} \beta_2^*(\theta, y_1) f_1(\theta)\, d\theta}{F_1(y_1)}.$$

The following proposition delivers the first main result of the paper.

Proposition 1. *(Afternoon Effect) If $\Lambda > 0$, then the price sequence in a two-round sequential first-price auction is a supermartingale and the afternoon effect arises in equilibrium. That is,*

$$p_1 = \beta_1^*(y_1) > \mathbb{E}[p_2|\beta_1^*(y_1)] = \mathbb{E}[p_2|p_1].$$

The intuition behind Proposition 1 is that, just like in the reference-independent case, in equilibrium bidders must be indifferent between winning in the first auction or in the second one. Hence, in the first auction a bidder bids the expectation of the second-round price conditional on himself having the highest type and being the price-setter. However, by conditioning his first-period bid on him having the highest type, a bidder expects not to feel discouraged in the second auction. And because the discouragement effect depresses bids in the second auction, the expectation of the second-round price conditional on being the price-setter is higher than the second-round expected price conditional on the first-round price. In essence, optimal equilibrium behavior leads the current price setter to overestimate the bid of his highest opponent and hence the next-round price. Example 2 illustrates the afternoon effect for the case of uniformly-distributed types.

[11] If a bidder deviates from the symmetric equilibrium strategy by slightly overbidding, there are only two possible consequences. First, if he was already going to win then he still wins, but pays a slightly higher price. Second, his deviation might make him win the current round when he was otherwise going to lose. In the latter case, however, it must be that the highest opposing type is so close that the bidder was almost certain to win in the second round.

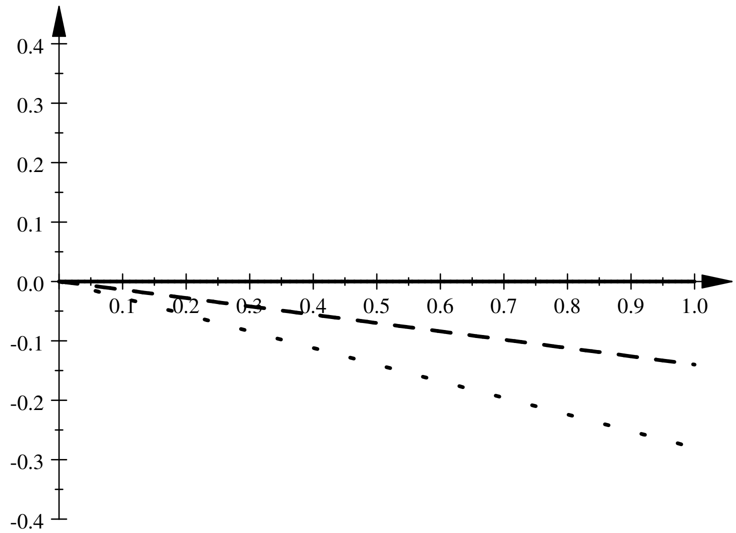

Figure 2: $\mathbb{E}[p_2|p_1] - p_1$ as a function of y_1 for three different values of Λ (0, $\frac{1}{2}$, 1) when $N = 4$ and θ is distributed uniformly on $[0, 1]$

Example 2. *Suppose that* $\theta \overset{U}{\sim} [0, 1]$. *The first-round equilibrium bid and price are*

$$\beta_1^*(\theta) = (1 - \Lambda)\left(\frac{N-2}{N}\right)\theta + \Lambda\left(\frac{2N-4}{2N-3}\right)\left(\frac{N-1}{N}\right)\theta$$

and

$$p_1 = \beta_1^*(y_1) = (1 - \Lambda)\left(\frac{N-2}{N}\right)y_1 + \Lambda\left(\frac{2N-4}{2N-3}\right)\left(\frac{N-1}{N}\right)y_1.$$

The conditional second-round expected price is

$$\mathbb{E}[p_2|p_1] = \mathbb{E}[p_2|\beta_1^*(y_1)] = (1 - \Lambda)\left(\frac{N-2}{N}\right)y_1 + \Lambda 2\left(\frac{N-2}{2N-3}\right)^2 y_1.$$

Hence,

$$\mathbb{E}[p_2|p_1] - p_1 = \Lambda\left[2\left(\frac{N-2}{2N-3}\right)^2 - \left(\frac{2N-4}{2N-3}\right)\left(\frac{N-1}{N}\right)\right]y_1$$

and

$$\mathbb{E}[p_2|p_1] < p_1 \Leftrightarrow 2\left(\frac{N-2}{2N-3}\right)^2 < \left(\frac{2N-4}{2N-3}\right)\left(\frac{N-1}{N}\right) \Leftrightarrow N^2 - 3N + 3 > 0.$$

In Figure 2 I plot the difference $\mathbb{E}[p_2|p_1] - p_1$ as a function of y_1 for three different values of Λ when $N = 4$ and θ is distributed uniformly on $[0, 1]$: i) $\Lambda = 0$ (solid), ii) $\Lambda = \frac{1}{2}$ (dashed) and iii) $\Lambda = 1$ (dotted). Two effects are evident from the plot: 1) for a given value of y_1, the higher is Λ the stronger is the afternoon effect and 2) for a fixed strictly positive Λ, the higher is y_1 the stronger is the afternoon effect.

It is useful to compare the logic behind the afternoon effect in my model with the learning effect that arises in common value-auctions with informational externalities. In the symmetric equilibrium of a common-value auction, a bidder conditions his estimate of the value of the item (and hence his bid) on his strongest rival having a (weakly) lower signal than his. In this case, since a bidder revise his estimate of the value of the good upward when losing the first auction, the equilibrium price sequence drifts upward. Conversely, with the informational externalities that arise in a private-value auction with expectations-based loss aversion, when losing the first auction a bidder becomes more pessimistic about how likely he is to win the second one (compared to his first-round expectations); this creates a discouragement effect that pushes bidders to behave less aggressively and, in turn, generates a declining price path in equilibrium.

4 Second-Price Auctions

In this section I assume that two identical items are sold using a sequence of second-price sealed-bid auctions. I continue to focus on symmetric strategies (b_1, b_2) that are strictly increasing and to assume that the winning bid of the first round is publicly disclosed by the seller prior to the second one.[12] I begin by analyzing the bidder's problem in the second round.

4.1 Second-period strategy

Fixing the bidding strategies of the other players, let $\Phi(b_2|y_1)$ denote the probability with which a bidder of type θ expects to win with a bid equal to b_2 conditional on y_1 being the type of the first-round winner. The payment he has to make if he wins the auction is given by the second largest bid and follows the distribution $\Phi(\cdot|y_1)$. Then, the bidder's expected utility is

$$EU_2(b_2, \theta; y_1) = \int_0^{b_2} (\theta - p)\, d\Phi(p|y_1) - \theta\Lambda\Phi(b_2|y_1)\left[1 - \Phi(b_2|y_1)\right] \tag{8}$$

Differentiating (8) with respect to b_2 yields the first-order condition:

$$\theta - b_2 - \theta\Lambda\left[1 - 2\Phi(b_2|y_1)\right] = 0.$$

In a symmetric equilibrium, $\Phi(b_2|y_1) = F_2(\theta|y_1)$ and hence we obtain:

$$b_2^*(\theta, y_1) = \theta\left\{1 - \Lambda\left[1 - 2\frac{F(\theta)^{N-2}}{F(y_1)^{N-2}}\right]\right\}.$$

While it is well known that without loss aversion ($\Lambda = 0$) in a symmetric equilibrium a bidder submits a bid equal to his own valuation, the above expression shows that this is not the case

[12] Notice that in a second-price auction the winning bid is not the price the winner actually ends up paying. This an important point because if the seller were to reveal the winning price of the first auction, then the bidders would infer the type of the highest remaining bidder and a symmetric equilibrium in monotone strategies would fail to exist.

with reference-dependent preferences. Indeed, we have

$$\frac{\partial b_2^*\left(\theta,y_1\right)}{\partial\Lambda}=\left[2\frac{F\left(\theta\right)^{N-2}}{F\left(y_1\right)^{N-2}}-1\right]\theta>0\Leftrightarrow\frac{F\left(\theta\right)}{F\left(y_1\right)}>\left(\frac{1}{2}\right)^{\frac{1}{N-2}}.$$

Therefore, higher (lower) types bid higher (lower) than their valuation. Furthermore, as for first-price auctions, we have the following result.

Lemma 2. *(Discouragement Effect II) If $\Lambda>0$, then $\frac{\partial b_2^*(\theta,y_1)}{\partial y_1}<0\ \forall\theta$.*

The intuition is like in Lemma 1: the higher the type of the winner in the first auction, the less likely a remaining bidder is to win in the second one and, therefore, he bids less aggressively.

4.2 First-period strategy

As shown by Lange and Ratan (2010) for the case of single-unit auctions, if bidders are not loss-averse over money, first-price and second-price auctions are revenue equivalent. The reason is that, when bidders are risk-neutral over money expected gain-loss utility depends only on the probability with which a bidder expects to win the auction and this is the same in both formats. It is easy to see that the same intuition applies also to multi-unit auctions. Hence, we can use the revenue equivalence theorem to derive the first-round equilibrium bidding function.

In the first auction a type-θ bidder wins with probability $F_1\left(\theta\right)$ and, if he wins, the price he pays is $b_1^*\left(y_1\right)$, the highest among his rivals' bids. Thus, his expected first-round payment is

$$F_1\left(\theta\right)\int_0^\theta b_1^*\left(y_1\right)f_1\left(y_1|\theta\right)dy_1.$$

In a first-price auction, instead, the winning bidder pays his own bid and therefore his expected payment in the first round is:

$$F_1\left(\theta\right)\beta_1^*\left(\theta\right)=F_1\left(\theta\right)\left[\frac{\int_0^\theta\beta_2^*\left(s,s\right)f_1\left(s\right)ds}{F_1\left(\theta\right)}\right]$$

where the equality follows from (7). From revenue equivalence it follows that

$$\int_0^\theta b_1^*\left(y_1\right)f_1\left(y_1|\theta\right)dy_1=\frac{\int_0^\theta\beta_2^*\left(s,s\right)f_1\left(s\right)ds}{F_1\left(\theta\right)}$$

and differentiating both sides of the equality with respect to θ yields

$$b_1^*\left(\theta\right)=\beta_2^*\left(\theta,\theta\right).$$

Therefore, the equilibrium bid in the first of two sequential second-price auctions is equal to the second round's bid of two sequential first-price auction where, in the latter, the bidder conditions his bid on himself having the highest type. Finally, notice that the afternoon effect arises in equilibrium since, by revenue equivalence, in each round the seller's expected revenue from a second-price auction is equal to the expected revenue from a first-price auction.

5 Sequential vs. Simultaneous Auctions

In this section I focus on simultaneous auctions; that is, auctions in which all the items are allocated after only one round of bidding. I derive the equilibrium bidding strategy in a two-unit discriminatory (pay-your-bid) auction. In a discriminatory auction, bidders submit sealed bids and the highest bidders each receive one object and each pays his own bid. This procedure generalizes the single-object first-price auction, and is the procedure most commonly used for the sale of U.S. Treasury bills.[13] As before, I continue to look for an equilibrium in symmetric monotone strategies.

Consider a bidder with type θ who plans to bid as if his type were $\widetilde{\theta} \neq \theta$ when all other $N-1$ bidders follow the strategy $\beta(\cdot)$. His expected utility is

$$EU\left(\theta, \widetilde{\theta}\right) = F_2\left(\widetilde{\theta}\right)\left[\theta - \beta\left(\widetilde{\theta}\right)\right] - \Lambda\theta F_2\left(\widetilde{\theta}\right)\left[1 - F_2\left(\widetilde{\theta}\right)\right] \tag{9}$$

where $F_2\left(\widetilde{\theta}\right) \equiv F_1\left(\widetilde{\theta}\right) + (N-1)\left[1 - F\left(\widetilde{\theta}\right)\right] F\left(\widetilde{\theta}\right)^{N-2}$ is the probability that Y_2, the second highest valuation among $N-1$, is less than $\widetilde{\theta}$ and Λ is defined as before. Notice that it is not necessary for a bidder to outbid all his competitors in order to be awarded an object; it is enough to outbid $N-2$ of them.

Differentiating (9) with respect to $\widetilde{\theta}$ yields the first-order condition:

$$0 = f_2\left(\widetilde{\theta}\right)\left[\theta - \beta\left(\widetilde{\theta}\right)\right] - \beta'\left(\widetilde{\theta}\right) F_2\left(\widetilde{\theta}\right) - \Lambda\theta f_2\left(\widetilde{\theta}\right)\left[1 - 2F_2\left(\widetilde{\theta}\right)\right].$$

Then, substituting $\theta = \widetilde{\theta}$ into the FOC and re-arranging results in the following differential equation

$$\frac{d}{d\theta}\left\{\beta(\theta) F_2(\theta)\right\} = f_2(\theta)\,\theta\left\{1 - \Lambda\left[1 - 2F_2(\theta)\right]\right\}$$

[13] An alternative procedure is the uniform-price auction. In a uniform-price auction, the bidders submit sealed bids and the winning bidders all pay the same price, equal to the highest rejected bid. This procedure generalizes the single-object second-price auction. The analysis for the uniform-price auction is virtually identical and hence omitted.

together with the boundary condition that $\beta(0) = 0$. Solving the differential equation yields

$$\beta^*(\theta) = \frac{\int_0^\theta s\{1 - \Lambda[1 - 2F_2(s)]\} f_2(s)\, ds}{F_2(\theta)}.$$

Again, the equilibrium bidding function can be re-written as a convex combination of the risk-neutral bid and a term that depends on the bidder's expectations (reference point):

$$\beta^*(\theta) = (1 - \Lambda)\frac{\int_0^\theta s f_2(s)\, ds}{F_2(\theta)} + \Lambda\frac{\int_0^\theta 2sF_2(s) f_2(s)\, ds}{F_2(\theta)}.$$

Now I compare the bidders' equilibrium utility and the seller's expected revenue under simultaneous and sequential auctions. I do the comparison for first-price (sequential) auctions and discriminatory (simultaneous) auctions, but the same results apply for second-price (sequential) and uniform-price (simultaneous) auctions by revenue equivalence.

Let $V^{sim}(\theta)$ and $V^{seq}(\theta)$ denote a bidder's expected utility in equilibrium in a simultaneous and sequential auction, respectively. With independent private values and under risk neutrality ($\Lambda = 0$), it is well known that a bidder's equilibrium expected utility in a simultaneous auction is the same as in a sequential auction. Under loss aversion, instead, we have:

Proposition 2. *(Bidder-payoff Equivalence) If $\Lambda > 0$, then $V^{seq}(\theta) \geq V^{sim}(\theta)$ if and only if*

$$\int_0^\theta F_2(s)\, s f_2(s)\, ds \geq \int_0^\theta \int_0^x F_2(s|x)\, s f_2(s|x)\, ds f_1(x)\, dx + \int_\theta^{\overline{\theta}} \int_0^\theta F_2(s|x)\, s f_2(s|x)\, ds f_1(x)\, dx.$$

It is easy to see that the inequality in Proposition 2 cannot bind for every type unless $F_2(s) = F_2(s|x)$ implying that, generically, sequential auctions and simultaneous ones are not bidder-payoff equivalent. Notice also that a bidder's ex-ante probability of obtaining an item is the same under both formats and this implies, trivially, that a bidder's expected gain-loss utility is also the same under both formats. Hence, the difference between $V^{\mathrm{seq}}(\theta)$ and $V^{sim}(\theta)$ is simply given by the difference in the expected payments. Indeed, recall that with risk-neutral bidders the condition in Proposition 2 is

$$F_2(\theta)\left[\frac{\int_0^\theta s f_2(s)\, ds}{F_2(\theta)}\right] \geq F_1(\theta)\left[\frac{\int_0^\theta \int_0^x s f_2(s|x)\, ds f_1(x)\, dx}{F_1(\theta)}\right] + \int_\theta^{\overline{\theta}} \int_0^\theta \frac{s f_2(s|x)\, ds}{F_2(\theta|x)} F_2(\theta|x)\, f_1(x)\, dx$$

which always binds since both sides are simply equal to $\Pr[Y_2 \leq \theta] \times \mathbb{E}[Y_2 | Y_2 \leq \theta]$. With reference-dependent preferences, however, the expected payments in the two formats depend also on the bidder's beliefs about how likely he is to win (represented by the additional cdf term under the integral) since these determine his reference point.

Figure 3 shows how $V^{\mathrm{seq}}(\theta) - V^{sim}(\theta)$ varies with θ for five different values of N (3, 4, 5, 10

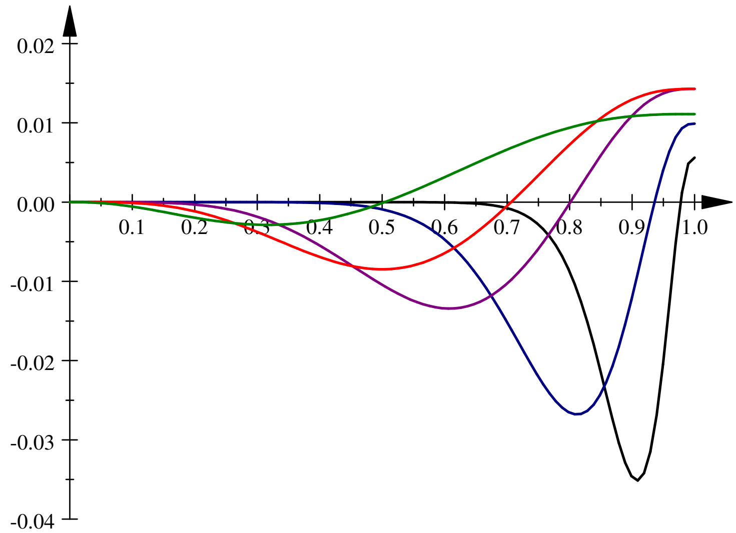

Figure 3: $V^{\text{seq}}(\theta) - V^{sim}(\theta)$ for $N = 3, 4, 5, 10$ and 20 with θ distributed uniformly on $[0, 1]$.

and 20) when the bidders' types are uniformly distributed on $[0, 1]$ (a darker color corresponds to a higher value for N). For a given N there exists a cutoff type θ^* who is indifferent between the two formats. Furthermore, the value of the cutoff θ^* is increasing in N.

According to Figure 3, low-type bidders prefer simultaneous auctions whereas high-type ones prefer sequential ones. The intuition for these opposing preferences is reminiscent of the result in Köszegi and Rabin (2009) about a loss-averse decision maker's dislike of interim partial information because it exposes her to possibly unnecessary bad news due to fluctuations in beliefs. More precisely, Köszegi and Rabin (2009) consider an example with two equiprobable possible consumption levels, $c \in \{0, 1\}$ and an expectations-based loss-averse agent who has access to a signal $\varsigma \in \{0, 1\}$, where the signal is accurate ($\varsigma = c$) with probability $w > \frac{1}{2}$. They show that unless $w = 1$, the agent prefers not to receive the signal. In an auction, however, a bidder's expected consumption depends on his type and higher types are more likely to win. Hence, during the course of a sequential auction higher types are more likely to receive good news (and be less discouraged) whereas lower types are more likely to receive bad news (and be more discouraged). Therefore, bidders with higher types prefer sequential auctions while bidders with lower types prefer simultaneous ones.

Next I compare the seller's expected revenue between the two formats. It is now convenient to take the point of view of the seller and consider the order statistics of the values of all N bidders. Let $Z_1^{(N)} \equiv Z_1$ be the highest of N values, $Z_2^{(N)} \equiv Z_2$ be the second-highest and so on. Also, let M_1 and M_2 be the distributions of Z_1 and Z_2 respectively, with corresponding densities m_1 and m_2. Under risk neutrality ($\Lambda = 0$), the two auction formats are revenue-equivalent, both yielding an expected revenue equal to $2E[Z_3]$ (Milgrom and Weber, 2000). Under loss aversion, instead, we have:

Proposition 3. *(Revenue Equivalence) If $\Lambda > 0$, then $\mathbb{E}[R^{sim}] \geq \mathbb{E}[R^{seq}]$ if and only if*

$$\int_0^{\overline{\theta}} \int_0^{\theta} \frac{F_2(s)\, s f_2(s)}{F_2(\theta)} ds \left[m_1(\theta) + m_2(\theta)\right] d\theta \geq \int_0^{\overline{\theta}} \int_0^{\theta} \int_0^{x} \frac{F_2(s|x)\, s f_2(s|x)}{F_1(\theta)} ds f_1(x)\, dx m_1(\theta)\, d\theta + \int_0^{\overline{\theta}} \int_0^{x} \int_0^{\theta} \frac{F_2(s|x)\, s f_2(s|x)}{F_2(\theta|x)} ds \frac{f_1(\theta)}{F_1(x)} d\theta m_1(x)\, dx.$$

As for the previous result about bidder-payoff equivalence, it is easy to see that the condition in Proposition 3 cannot bind for every N unless $F_2(s) = F_2(s|x)$, in which case both sides reduce to $2E[Z_3]$. Therefore, which format yields a higher revenue depends on the number of bidders. Sequential auctions yield a higher revenue when the number of bidders is relatively high. For example, straightforward calculations show that with two objects if $\theta \overset{U}{\sim} [0,1]$, simultaneous auctions yield a higher expected revenue than sequential ones for $N = 3$ whereas for $N = 4$ the two formats yield the same expected revenue. For $N \geq 5$ sequential auctions yield a higher expected revenue than simultaneous ones.

Gathering together the results from this section and the previous ones, we obtain the following corollary:

Corollary 1. *(Comparison of Different Auction Formats) If $\Lambda > 0$, revenue equivalence holds within formats but not between. That is, sequential first-price auctions are revenue-equivalent to sequential second-price auctions and discriminatory auctions are revenue-equivalent to uniform-price auctions. However, sequential first-price auctions are not revenue-equivalent to discriminatory auctions and sequential-second price auctions are not revenue-equivalent to uniform-price auctions.*

Recall that in equilibrium a bidder's expected probability of consumption and expected gain-loss utility are the same under all four types of auctions considered in this paper. Thus, the non-equivalence result in Corollary 1 is due to the effect that sequential (partial) information revelation has on the expected payments of a loss-averse bidder.

6 Sequential Auctions without Announcements of the Winning Bid

In the classical reference-free model with independent private values, the optimal bidding strategy does not depend on the (public) history of the winning bids.[14] However, this is no longer the case with expectations-based reference-dependent preferences. Hence, some questions naturally arise: Is equilibrium bidding different if the seller commits to not revealing the history

[14] This also implies, trivially, that the equilibrium is exactly the same with and without price announcement.

of winning bids? Does the rationale for the afternoon effect with expectations-based reference-dependent preferences rely on the history of winning bids being publicly available? And, finally, would the seller be better off by not disclosing the history of winning bids? I answer these questions in the context of sequential first-price auctions.

6.1 Second-period strategy

Consider a bidder of type θ who plans to bid as if his type were $\widetilde{\theta} \neq \theta$ when all other $N-2$ remaining bidders follow the strategy β_2. Let σ be the type that the bidder pretended to be in the first auction. If he lost the first auction, he knows that $y_1 > \sigma$. Then his expected second-round payoff is

$$EU_2\left(\widetilde{\theta}, \theta; \sigma\right) = \varphi\left(\sigma\right) F\left(\widetilde{\theta}\right)^{N-2}\left[\theta - \beta_2\left(\widetilde{\theta}, \sigma\right)\right] - \Lambda\theta\varphi\left(\sigma\right) F\left(\widetilde{\theta}\right)^{N-2}\left[1 - \varphi\left(\sigma\right) F\left(\widetilde{\theta}\right)^{N-2}\right] \quad (10)$$

where $\varphi\left(\sigma\right) = \frac{(N-1)[1-F(\sigma)]}{1-F(\sigma)^{N-1}}$. Thus, $\varphi\left(\sigma\right) F\left(\widetilde{\theta}\right)^{N-2}$ denotes the probability that the second highest of $N-1$ draws is below $\widetilde{\theta}$ given that the highest is above σ; or, in other words, the probability that a bidder who pretends to be of type $\widetilde{\theta}$ in the second auction wins this auction given that he pretended to be of type σ in the first auction and lost it.[15] Notice that, crucially, the second-round bid might, in principle, depend also on σ.

As first conjectured by Milgrom and Weber (2000) and later shown by Mezzetti *et al.* (2008), with no bid announcement and affiliated values it is optimal for a bidder of type θ to behave according to his type in the second auction if and only if he behaved as type $\sigma \leq \theta$ in the first auction. By contrast, if a bidder of type θ behaves as if his type were higher than θ in the first auction, he might want to over-bid in the second auction as well. This happens because, as Milgrom and Weber (2000) pointed out, "a bidder might choose a bid a bit higher in the first round in order to have a better estimate of the winning bid, should he lose". Recall that, with interdependent values, a better estimate of the winning bid is also a better estimate of the value of the object for sale. In our case, however, values are private and independent; therefore, it is optimal for a bidder in the second auction to bid according to his true type, no matter what he did in the first one.

[15] Technically, we have that

$$\frac{(N-1)\left[1-F\left(\sigma\right)\right] F\left(\widetilde{\theta}\right)^{N-2}}{1-F\left(\sigma\right)^{N-1}} = \frac{\int_{\sigma}^{\overline{\theta}} \int_0^{\widetilde{\theta}} h\left(y_1, y_2\right) dy_2 dy_1}{\int_{\sigma}^{\overline{\theta}} \int_0^{\widetilde{y}_1} h\left(\widetilde{y}_1, \widetilde{y}_2\right) d\widetilde{y}_2 d\widetilde{y}_1}$$

where $h\left(y_1, y_2\right) = (N-1)(N-2) f\left(y_1\right) f\left(y_2\right) F\left(y_2\right)^{N-3}$ is the joint density of Y_1 and Y_2.

Differentiating $EU_2\left(\widetilde{\theta},\theta;\sigma\right)$ with respect to $\widetilde{\theta}$ yields the first-order condition:

$$\begin{aligned} 0 &= \theta\left(N-2\right)F\left(\widetilde{\theta}\right)^{N-3}f\left(\widetilde{\theta}\right)-\theta\left(N-2\right)F\left(\widetilde{\theta}\right)^{N-3}f\left(\widetilde{\theta}\right)\Lambda\left[1-2\varphi\left(\sigma\right)F\left(\widetilde{\theta}\right)^{N-2}\right] \\ & -\frac{\partial\beta_2\left(\widetilde{\theta},\sigma\right)}{\partial\widetilde{\theta}}F\left(\widetilde{\theta}\right)^{N-2}-\beta_2\left(\widetilde{\theta},\sigma\right)\left(N-2\right)F\left(\widetilde{\theta}\right)^{N-3}f\left(\widetilde{\theta}\right). \end{aligned}$$

Hence, $\varphi\left(\sigma\right)$ enters the FOC only through the reference point, but it does not affect the "direct" part of a bidder's payoff and since $\Lambda\leq 1$ the "direct" part carries a higher weight than the reference-dependent part. Substituting $\theta=\widetilde{\theta}$ into the FOC and re-arranging results in the following differential equation

$$\frac{\partial}{\partial\theta}\left\{\beta_2\left(\theta,\sigma\right)F\left(\theta\right)^{N-2}\right\}=\theta\left\{1-\Lambda\left[1-2\varphi\left(\sigma\right)F\left(\theta\right)^{N-2}\right]\right\}\left(N-2\right)F\left(\theta\right)^{N-3}f\left(\theta\right)$$

together with the boundary condition that $\beta_2\left(0,\sigma\right)=0$. Thus, the equilibrium bidding function is

$$\widehat{\beta}_2\left(\theta,\sigma\right)=\frac{\int_0^\theta x\left\{1-\Lambda\left[1-\frac{2(N-1)(1-F(\sigma))F(x)^{N-2}}{1-F(\sigma)^{N-1}}\right]\right\}dF\left(x\right)^{N-2}}{F\left(\theta\right)^{N-2}}.$$

The equilibrium bidding strategy is a function of the type that the bidder mimicked in the previous auction since, if the seller does not publicly reveal the first-round winning bid, a bidder who lost the first auction must use his own bid from the previous round in order to infer where he stands in the ranking of the remaining bidders' values. Hence, the equilibrium strategy depends on the (private) history of the game and, as the following lemma shows, a slightly different form of discouragement effect arises in equilibrium.

Lemma 3. *(Discouragement Effect III) If $\Lambda>0$, then $\frac{\partial\widehat{\beta}_2(\theta,\sigma)}{\partial\sigma}<0\ \forall\theta$.*

The intuition for this result slightly differs from the one behind Lemmas 1 and 2. When the winning bid from the first auction is not publicly revealed, a bidder can only use his own first-round bid to formulate an expectation about how likely he is to win in the second one. The higher the type he pretended to be in the first auction, the less likely he feels to win in the current one since not winning the first auction, given that he pretended to have a high type, is bad news about how fierce competition is. This, in turn, implies that the higher is the type a bidder pretended to be in the first auction, the less aggressive his bidding will be in the second auction. Recall that when the seller announces the winning bid after the first auction, the equilibrium bid in the second auction is:

$$\beta_2^*\left(\theta,y_1\right)=\frac{\int_0^\theta x\left\{1-\Lambda\left[1-2\left(\frac{F(x)}{F(y_1)}\right)^{N-2}\right]\right\}dF\left(x\right)^{N-2}}{F\left(\theta\right)^{N-2}}.$$

Comparing the second-round equilibrium strategies with and without bid announcement yield the following result.

Lemma 4. *(Effect of information I) Equilibrium bidding in the second auction is more aggressive when the seller does not reveal the winning bid of the first auction if and only if*

$$\frac{(N-1)\left[1-F(\sigma)\right]}{1-F(\sigma)^{N-1}} > \frac{1}{F(y_1)^{N-2}}. \tag{11}$$

First, notice that condition (11) can hold only if $y_1 > \sigma$. The term on the left-hand-side of (11) represents what a bidder learns from losing in the first round about how fierce competition is in the second round: he knows that the first-round winner's type is above σ. Similarly, the term on the right-hand-side of (11) represents what a bidder learns from losing in the first round about how fierce competition is in the second round: he knows that all remaining bidders' types are below y_1. Hence, with no bid announcement, bidders are asymmetrically informed about the intensity of competition in the second round whereas with bid announcement they all have the same information. It is easy to see that the right-hand-side of condition (11) is decreasing in y_1 implying that, for a fixed σ, the higher is the type of the winner in the first auction, the more aggressive second-round bidding behavior is when the winning bid is not revealed. Similarly, the left-hand-side of condition (11) is decreasing in σ implying that, for a fixed y_1 a bidder who pretended to be a low type in the first auction behaves more aggressively in the second one when the winning bid is not revealed. Of course, as I am about to show next, in equilibrium a bidder will behave according to his type in both auctions so that $\sigma = \theta$.

6.2 First-period strategy

Consider a bidder of type θ who plans to bid as if his type were $\widetilde{\theta} > \theta$ when all other $N-1$ bidders follow the strategy β_1.[16] Suppose that all bidders expect to follow the equilibrium strategy $\widehat{\beta}_2$ in the second auction. Then, the bidder will solve the following problem:

$$\begin{aligned} EU_1\left(\widetilde{\theta},\theta\right) &= F_1\left(\widetilde{\theta}\right)\left[\theta-\beta_1\left(\widetilde{\theta}\right)\right]+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)\left[\theta-\widehat{\beta}_2\left(\theta,\widetilde{\theta}\right)\right] f_1\left(y_1\right)dy_1 \\ &\quad -\Lambda\theta\left[F_1\left(\widetilde{\theta}\right)+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right)dy_1\right]\left[1-F_1\left(\widetilde{\theta}\right)-\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right)dy_1\right] \end{aligned}$$

[16] The analysis is virtually identical for the case $\widetilde{\theta} < \theta$.

Differentiating $EU_1\left(\widetilde{\theta}, \theta\right)$ with respect to $\widetilde{\theta}$ yields the following first-order-condition:

$$\begin{aligned} 0 &= f_1\left(\widetilde{\theta}\right)\left[\theta - \beta_1\left(\widetilde{\theta}\right)\right] - \beta_1'\left(\widetilde{\theta}\right) F_1\left(\widetilde{\theta}\right) \\ &\quad - (N-1) f\left(\widetilde{\theta}\right) F(\theta)^{N-2}\left[\theta - \beta_2\left(\theta, \widetilde{\theta}\right)\right] - \frac{\partial \beta_2\left(\theta, \widetilde{\theta}\right)}{\partial \widetilde{\theta}} (N-1)\left[1 - F\left(\widetilde{\theta}\right)\right] F(\theta)^{N-2}. \end{aligned}$$

Substituting $\theta = \widetilde{\theta}$ and re-arranging results in the following differential equation

$$f_1(\theta)\beta_2(\theta,\theta) - (N-1)\left[1 - F(\theta)\right] F(\theta)^{N-2} \left.\frac{\partial \beta_2\left(\theta, \widetilde{\theta}\right)}{\partial \widetilde{\theta}}\right|_{\widetilde{\theta}=\theta} = \frac{d}{d\theta}\left\{\beta_1(\theta) F_1(\theta)\right\} \tag{12}$$

together with the boundary condition that $\beta_1(0) = 0$. Notice that, crucially, $\frac{\partial \beta_2\left(\theta, \widetilde{\theta}\right)}{\partial \widetilde{\theta}} \neq 0$. That is, by mimicking another type in the first auction, a bidder is not just affecting the probability of getting to the second auction — like in the classical reference-free model — but he is also affecting his own future bid in the second auction. This occurs because, with no bid announcement between auctions, a player's bid in the current round affects his reference point in the next one. Solving the differential equation in (12) yields

$$\widehat{\beta}_1(\theta) = \frac{\int_0^\theta \widehat{\beta}_2(s,s) f_1(s)\, ds}{F_1(\theta)} - \frac{\int_0^\theta \left\{ \left.\frac{\partial \widehat{\beta}_2\left(s, \widetilde{\theta}\right)}{\partial \widetilde{\theta}}\right|_{\widetilde{\theta}=s} \frac{1-F(s)}{f(s)} \right\} f_1(s)\, ds}{F_1(\theta)}. \tag{13}$$

Notice that

$$\frac{\partial \widehat{\beta}_2\left(\theta, \widetilde{\theta}\right)}{\partial \widetilde{\theta}} = -\frac{2(N-1)\Lambda f\left(\widetilde{\theta}\right)\left[1 - F_2\left(\widetilde{\theta}\right)\right]\int_0^\theta x F(x)^{N-2}\, dF(x)^{N-2}}{\left[1 - F\left(\widetilde{\theta}\right)^{N-1}\right]^2 F(\theta)^{N-2}} < 0$$

where $F_2\left(\widetilde{\theta}\right) = F\left(\widetilde{\theta}\right)^{N-1} + (N-1)\left[1 - F\left(\widetilde{\theta}\right)\right] F\left(\widetilde{\theta}\right)^{N-2}$.

The following lemma shows that, compared to the case analyzed in Section 3, bidders behave less aggressively in the first auction when the seller commits to not revealing the winning bid.

Lemma 5. *(Effect of information II) Equilibrium bidding in the first round is more aggressive when the seller commits to publicly reveal the winning bid prior to the second round; that is,* $\beta_1^*(\theta) - \widehat{\beta}_1(\theta) \geq 0\ \forall \theta$ *and the inequality is strict if* $\theta < \overline{\theta}$.

The intuition behind Lemma 5 is the following. When anticipating that the seller will not reveal the winning bid of the first auction prior to the second one, a bidder knows that his bid in the first auction — in case he does not win— will determine his reference point in the second one. A high bid in the first auction, hence, implies also a high reference point in the second auction. Having a high reference point in the second auction, however, exposes the bidder to a greater disappointment in case he were to lose the second auction as well. Therefore, if the seller commits to not revealing the first-round winning price, bidders bid less aggressively in the first auction. Furthermore, the seller's total expected revenue is higher when she commits to revealing the first round's winning bid.

Proposition 4. *(Revenue) The seller's expected revenue is higher when she commits to disclose the winning bid from the first auction prior to the second one.*

From an ex-ante perspective, bidders going into the second round without knowing the type of the winner in the first round are exposed to much more uncertainty about competition compared to bidders who know the type of the first-round winner. Indeed, in the latter case every bidder going into the second auction knows that all of his competitors' types are below a certain cutoff type while in the former a bidder only knows that the winner has a higher type than his. An expectations-based loss-averse bidder dislikes uncertainty in his consumption outcomes because he dislikes the possibility of a resulting loss more than he likes the possibility of a resulting gain (so he is "first-order" risk averse; see Köszegi and Rabin, 2007). As auctions without bid announcements expose bidders to greater risk, they react by bidding less aggressively. Therefore, compared to the analysis in Section 3, if the seller does not reveal the winning bid of the first auction prior to the second one, her expected revenue decreases.[17]

Mezzetti *et al.* (2008) show that, under the assumption of affiliated private values, the seller's expected revenue in a sequential auction with winning-bid announcement is the same as in a sequential auction with no bid announcement, and is lower than in a simultaneous auction. By contrast, if bidders have independent private values and expectations-based reference-dependent preferences, sequential auctions with winning-bid announcement always yield a higher revenue than sequential auctions with no bid announcement (Proposition 4) and might yield a higher revenue than simultaneous auctions (Proposition 3).

The following proposition shows that even with no bid announcement, however, the afternoon effect still arises in equilibrium.

Proposition 5. *(Afternoon Effect II) If $\Lambda > 0$, then the price sequence in a two-round first-price auction without bid announcement is a supermartingale and the afternoon effect arises in equilibrium. That is,*

$$p_1 = \widehat{\beta}_1 (y_1) > \mathbb{E} \left[p_2 | \widehat{\beta}_1 (y_1) \right] = \mathbb{E} \left[p_2 | p_1 \right].$$

Like in Section 3 in equilibrium prices decline because of the discouragement effect, but the intuition is slightly different. When the seller commits to not disclosing the winning bid, in the first round bidders are willing to pay a positive premium — equal to the second term on

[17] This result is akin to the famous "Linkage Principle" of Milgrom and Weber (1982): auctioneers have an incentive to pre-commit to revealing all available information.

the right-hand-side of (13) — in order to avoid having to go to the second round and being discouraged.

Summing up, with expectations-based reference-dependent preferences the equilibrium bidding strategy changes depending on whether the seller commits to publicly reveal the winning bids from the previous rounds. With winning-bid announcement first-round bids are always higher whereas bids in the second round can be higher or lower than without bid announcement. Furthermore, the seller's expected revenue is higher when she commits to disclose the previous round's winning bid. In either case, however, equilibrium prices follow a declining path.

7 Conclusions

In this paper I have proposed a novel, preference-based explanation for the afternoon effect observed in sequential auctions by positing that bidders are expectations-based loss-averse. My explanation based on loss aversion has the advantage of applying very generally, without requiring any additional modification of the auction environment. This is important since, as the evidence suggests, declining prices have been found in many different settings, even with no option to buy additional units and with identical objects.

Expectations-based reference-dependent preferences create an informational externality, the discouragement effect, that renders the equilibrium strategy history-dependent: the higher is the type of the winner in the first auction, the less aggressively the remaining bidders will bid in the second one. The effect of this informational externality on the second-round bids is the opposite of the one that arises in models with common values where the higher is the signal of the first-round winner, the higher is the estimated value of the object for all remaining bidders who in turn bid more aggressively. Therefore, by looking at the distribution of bids in the second auction, one can use the discouragement effect to empirically test the implications of loss aversion against the implications of the classical model with either private values (no history dependence) or common values (the higher the winning price in the first auction, the more aggressively bidders behave in the second auction).

In equilibrium a bidder must be indifferent between winning in the first auction or in the second one. Hence, in the first auction he chooses the optimal bid conditional on having the highest type and being the price-setter. By conditioning his bid in the first auction on having the highest type, however, a bidder expects not to feel discouraged in the second auction. Thus, in equilibrium bidders bid more aggressively in the first auction and prices decline.

In addition to rationalizing the afternoon effect, loss aversion delivers new testable implications that are of independent interest. For example, when bidders are expectations-based loss-averse simultaneous and sequential auctions are not revenue-equivalent nor bidder-payoff equivalent. Furthermore, in sequential auctions the seller always achieves a higher expected revenue by committing to reveal the previous round's winning bid.

Despite being able to explain the afternoon effect and to generate new testable predictions, there are several dimensions along which the model could be improved. First, I have departed from the original model of expectations-based reference-dependent preferences of Köszegi and Rabin (2006) by assuming that bidders are loss-averse only over consumption, but not over money. Admittedly restrictive, however, this assumption considerably simplifies the analysis. For example, with loss aversion over money first-price and second-price auctions are not revenue-equivalent anymore and the analysis of the second-price auction becomes much more intricate. Furthermore, in some of the auctions discussed in the Introduction the goods up for sale are not sought after by the bidders for their consumption value, but rather for commercial purposes (i.e., a production or a resale motive). In this case what bidders care about is the monetary value of the goods and a model of reference-dependent preferences where gains and losses are evaluated with respect to the overall gains from trade $(\theta - p)$ might be more appropriate.

There are several interesting directions for future research. One would be to study sequential dynamic (open) auctions, like English and Dutch auctions. If bidders are risk-neutral and have independent private values, it is well-known that the English (resp. Dutch) auction is strategically equivalent to the second-price (resp. first-price) sealed-bid auction. This equivalence, however, is unlikely to hold when bidders are expectations-based loss-averse.[18]

Another interesting extension would be to analyze a model where loss-averse bidders have interdependent values. Mezzetti (2011) showed that the informational externality arising from the interdependence between the bidders' values generates an increasing price sequence. The current paper argues that expectations-based loss aversion also creates an informational externality which, however, pushes prices to decline. Which effect will dominate would likely depend on the strength of loss aversion as well as on how sensitive a bidder's value is to his rivals' signals.

[18]See Ehrhart and Ott (2014) for a first analysis of single-object dynamic auctions with expectations-based loss-averse bidders.

A Proofs

Proof of Lemma 1: We have

$$\begin{aligned}
\frac{\partial \beta_2^*(\theta, y_1)}{\partial y_1} &= -\frac{2\Lambda (N-2)^2 F(y_1)^{N-3} f(y_1) \int_0^\theta x F(x)^{2N-5} f(x)\, dx}{[F(y_1) F(\theta)]^{2(N-2)}} \\
&< 0. \blacksquare
\end{aligned}$$

Proof of Proposition 1: We have

$$\begin{aligned}
\beta_1^*(y_1) &= \frac{\int_0^{y_1} \beta_2^*(\theta, \theta) f_1(\theta)\, d\theta}{F_1(y_1)} \\
&> \frac{\int_0^{y_1} \beta_2^*(\theta, y_1) f_1(\theta)\, d\theta}{F_1(y_1)} \\
&= \mathbb{E}[p_2 | p_1]
\end{aligned}$$

where the inequality follows from Lemma 1. ■

Proof of Lemma 2: We have

$$\begin{aligned}
\frac{\partial b_2^*(\theta, y_1)}{\partial y_1} &= -\frac{2\Lambda (N-2)\theta F(\theta)^{N-2}}{F(y_1)^{N-1}} \\
&< 0. \blacksquare
\end{aligned}$$

Proof of Proposition 2: We have that

$$V^{sim}(\theta) = F_2(\theta)[\theta - \beta^*(\theta)] - \Lambda\theta F_2(\theta)[1 - F_2(\theta)]$$

and

$$\begin{aligned}
V^{seq}(\theta) &= F_1(\theta)[\theta - \beta_1^*(\theta)] + \int_\theta^{\overline{\theta}} F_2(\theta | y_1)[\theta - \beta_2^*(\theta, y_1)] f_1(y_1)\, dy_1 \\
&\quad -\Lambda\theta \left[F_1(\theta) + \int_\theta^{\overline{\theta}} F_2(\theta|y_1) f_1(y_1)\, dy_1\right]\left[1 - F_1(\theta) - \int_\theta^{\overline{\theta}} F_2(\theta|y_1) f_1(y_1)\, dy_1\right].
\end{aligned}$$

Hence,

$$\begin{aligned}
V^{seq}(\theta) - V^{sim}(\theta) &= F_2(\theta)\beta^*(\theta) - F_1(\theta)\beta_1^*(\theta) - \int_\theta^{\overline{\theta}} F_2(\theta|y_1)\beta_2^*(\theta, y_1) f_1(y_1)\, dy_1 \\
&= (1-\Lambda)\int_0^\theta s f_2(s)\, ds + \Lambda \int_0^\theta 2F_2(s) s f_2(s)\, ds \\
&\quad - (1-\Lambda)\int_0^\theta \int_0^x s f_2(s|x)\, ds f_1(x)\, dx - \Lambda \int_0^\theta \int_0^x 2F_2(s|x) s f_2(s|x)\, ds f_1(x)\, dx \\
&\quad - (1-\Lambda)\int_\theta^{\overline{\theta}} \int_0^\theta s f_2(s|x)\, ds f_1(x)\, dx - \Lambda \int_\theta^{\overline{\theta}} \int_0^\theta 2F_2(s|x) s f_2(s|x)\, ds f_1(x)\, dx.
\end{aligned}$$

Notice that

$$\int_0^{\theta} sf_2(s)\,ds = \int_0^{\theta}\int_0^{x} sf_2(s|x)\,dsf_1(x)\,dx + \int_{\theta}^{\overline{\theta}}\int_0^{\theta} sf_2(s|x)\,dsf_1(x)\,dx$$

implying that

$$V^{seq}(\theta) - V^{sim}(\theta) \geq 0 \;\Leftrightarrow$$

$$\int_0^{\theta} F_2(s)\,sf_2(s)\,ds \geq \int_0^{\theta}\int_0^{x} F_2(s|x)\,sf_2(s|x)\,dsf_1(x)\,dx + \int_{\theta}^{\overline{\theta}}\int_0^{\theta} F_2(s|x)\,sf_2(s|x)\,dsf_1(x)\,dx$$

and this concludes the proof. ■

Proof of Proposition 3: We have that

$$\mathbb{E}[R^{\mathrm{seq}}] = \int_0^{\overline{\theta}} \beta_1^*(\theta)\,m_1(\theta)\,d\theta + \int_0^{\overline{\theta}}\int_0^{x} \frac{\beta_2^*(\theta,x)\,f_1(\theta)}{F_1(x)}d\theta m_1(x)\,dx$$

and

$$\mathbb{E}\left[R^{sim}\right] = \int_0^{\overline{\theta}} \beta^*(\theta)\,[m_1(\theta)+m_2(\theta)]\,d\theta.$$

Hence,

$$\mathbb{E}[R^{sim}] - \mathbb{E}[R^{\mathrm{seq}}] =$$

$$\int_0^{\overline{\theta}}\left[\frac{(1-\Lambda)\int_0^{\theta} sf_2(s)\,ds + \Lambda\int_0^{\theta} 2F_2(s)\,sf_2(s)\,ds}{F_2(\theta)}\right][m_1(\theta)+m_2(\theta)]\,d\theta$$

$$-\int_0^{\overline{\theta}}\left[\frac{(1-\Lambda)\int_0^{\theta}\int_0^{x} sf_2(s|x)\,dsf_1(x)\,dx + \Lambda\int_0^{\theta}\int_0^{x} 2F_2(s|x)\,sf_2(s|x)\,dxf_1(x)\,dx}{F_1(\theta)}\right]m_1(\theta)\,d\theta$$

$$-\int_0^{\overline{\theta}}\left[\frac{(1-\Lambda)\int_0^{x}\int_0^{\theta} \frac{sf_2(s|x)}{F_2(\theta|x)}dsf_1(\theta)\,d\theta + \Lambda\int_0^{x}\int_0^{\theta} 2F_2(s|x)\,\frac{sf_2(s|x)}{F_2(\theta|x)}dsf_1(\theta)\,d\theta}{F_1(x)}\right]m_1(x)\,dx.$$

Notice that

$$\begin{aligned}\int_0^{\overline{\theta}}\int_0^{\theta} \frac{sf_2(s)}{F_2(\theta)}ds\,[m_1(\theta)+m_2(\theta)]\,d\theta \quad &= \quad \int_0^{\overline{\theta}}\int_0^{\theta}\int_0^{x} \frac{sf_2(s|x)}{F_1(\theta)}dsf_1(x)\,dxm_1(\theta)\,d\theta \\ &\quad + \int_0^{\overline{\theta}}\int_0^{x}\int_0^{\theta} \frac{sf_2(s|x)}{F_2(\theta|x)}ds\frac{f_1(\theta)}{F_1(x)}d\theta m_1(x)\,dx\end{aligned}$$

implying that

$$\mathbb{E}\left[R^{sim}\right] - \mathbb{E}[R^{\mathrm{seq}}] \geq 0 \;\Leftrightarrow$$

$$\int_0^{\overline{\theta}}\int_0^{\theta}\frac{F_2(s)\,sf_2(s)}{F_2(\theta)}ds\,[m_1(\theta)+m_2(\theta)]\,d\theta \;\; \geq \;\; \int_0^{\overline{\theta}}\int_0^{\theta}\int_0^{x}\frac{F_2(s|x)\,sf_2(s|x)}{F_1(\theta)}ds f_1(x)\,dx m_1(\theta)\,d\theta + \int_0^{\overline{\theta}}\int_0^{x}\int_0^{\theta}\frac{F_2(s|x)\,sf_2(s|x)}{F_2(\theta|x)}ds\frac{f_1(\theta)}{F_1(x)}d\theta m_1(x)\,dx$$

and this concludes the proof. ■

Proof of Lemma 3: We have

$$\frac{\partial\widehat{\beta}_2(\theta,\sigma)}{\partial\sigma}=-\frac{2(N-1)\Lambda f(\sigma)[1-F_2(\sigma)]\int_0^{\theta}xF(x)^{N-2}\,dF(x)^{N-2}}{\left[1-F(\sigma)^{N-1}\right]^2F(\theta)^{N-2}}<0$$

where $F_2(\sigma)=F(\sigma)^{N-1}+(N-1)[1-F(\sigma)]F(\sigma)^{N-2}$. ■

Proof of Lemma 4: Immediate by inspection. ■

Proof of Lemma 5: We have

$$\beta_1^*(\theta)-\widehat{\beta}_1(\theta)=\frac{\int_0^{\theta}\left[\beta_2^*(s,s)-\widehat{\beta}_2(s,s)+\left.\frac{\partial\widehat{\beta}_2\left(s,\widetilde{\theta}\right)}{\partial\widetilde{\theta}}\right|_{\widetilde{\theta}=s}\frac{1-F(s)}{f(s)}\right]f_1(s)\,ds}{F_1(\theta)}.$$

A sufficient condition for the above expression to be non-negative is

$$\begin{aligned}
\beta_2^*(s,s)-\widehat{\beta}_2(s,s) &\geq -\left.\frac{\partial\widehat{\beta}_2\left(s,\widetilde{\theta}\right)}{\partial\widetilde{\theta}}\right|_{\widetilde{\theta}=s}\frac{1-F(s)}{f(s)}\\
&\Leftrightarrow \frac{2\Lambda\left\{\frac{1}{F(s)^{N-2}}-\frac{(N-1)[1-F(s)]}{1-F(s)^{N-1}}\right\}\int_0^{s}xF(x)^{N-2}\,dF(x)^{N-2}}{F(s)^{N-2}}\geq\\
&\frac{2\Lambda(N-1)[1-F(s)][1-F_2(s)]\int_0^{s}xF(x)^{N-2}\,dF(x)^{N-2}}{\left[1-F(s)^{N-1}\right]^2F(s)^{N-2}}\\
&\Leftrightarrow \frac{1}{F(s)^{N-2}}-\frac{(N-1)[1-F(s)]}{1-F(s)^{N-1}}\geq\frac{(N-1)[1-F(s)][1-F_2(s)]}{\left[1-F(s)^{N-1}\right]^2}.\\
&\Leftrightarrow \frac{1-F_2(s)}{(N-1)[1-F(s)]F(s)^{N-2}}\geq\frac{1-F_2(s)}{1-F(s)^{N-1}}\\
&\Leftrightarrow 1\geq F_2(s)
\end{aligned}$$

and this concludes the proof. ■

Proof of Proposition 4: We know from Lemma 5 that first-round bidding is more aggressive with price announcement and this implies, trivially, that the first-round expected revenue is higher with price announcement. Hence, it suffices to show that the second-round expected revenue is also higher with price announcement; that is:

$$\int_0^{\overline{\theta}}\int_0^{y_1}\frac{\beta_2^*(\theta,y_1)\,f_1(\theta)\,d\theta}{F_1(y_1)}m_1(y_1)\,dy_1 \geq \int_0^{\overline{\theta}}\widehat{\beta}_2(\theta,\theta)\,m_2(\theta)\,d\theta.$$

Substituting and re-arranging yields:

$$\begin{aligned}&(1-\Lambda)\int_0^{\overline{\theta}}\int_0^{y_1}\frac{X(\theta)\,f_1(\theta)\,d\theta}{F_1(y_1)}m_1(y_1)\,dy_1+\Lambda\int_0^{\overline{\theta}}\int_0^{y_1}\frac{2\widehat{X}(\theta)\,f_1(\theta)\,d\theta}{F(y_1)^{N-2}\,F_1(y_1)}m_1(y_1)\,dy_1 \geq \\ &(1-\Lambda)\int_0^{\overline{\theta}}X(\theta)\,m_2(\theta)\,d\theta+\Lambda\int_0^{\overline{\theta}}\frac{2(N-1)\left[1-F(\theta)\right]}{1-F(\theta)^{N-1}}\widehat{X}(\theta)\,m_2(\theta)\,d\theta\end{aligned}$$

where $X(\theta)=\frac{\int_0^{\theta}x dF(x)^{N-2}}{F(\theta)^{N-2}}$ and $\widehat{X}(\theta)=\frac{\int_0^{\theta}xF(x)^{N-2}dF(x)^{N-2}}{F(\theta)^{N-2}}$.

Notice that

$$\int_0^{\overline{\theta}}\int_0^{y_1}\frac{X(\theta)\,f_1(\theta)\,d\theta}{F_1(y_1)}m_1(y_1)\,dy_1=\int_0^{\overline{\theta}}X(\theta)\,m_2(\theta)\,d\theta$$

which further implies that

$$\int_0^{\overline{\theta}}\underbrace{\int_0^{y_1}\frac{\widehat{X}(\theta)\,f_1(\theta)\,d\theta}{F_1(y_1)}}_{\Omega(y_1)}m_1(y_1)\,dy_1=\int_0^{\overline{\theta}}\widehat{X}(\theta)\,m_2(\theta)\,d\theta.$$

The result then follows since

$$\int_0^{\overline{\theta}}\frac{2\Omega(y_1)}{F(y_1)^{N-2}}m_1(y_1)\,dy_1 \geq \int_0^{\overline{\theta}}\frac{2(N-1)\left[1-F(\theta)\right]}{1-F(\theta)^{N-1}}\widehat{X}(\theta)\,m_2(\theta)\,d\theta$$

as

$$\frac{1}{F(s)^{N-2}} \geq \frac{(N-1)\left[1-F(s)\right]}{1-F(s)^{N-1}} \Leftrightarrow 1 \geq F_2(s)$$

which holds $\forall s \in \left[0,\overline{\theta}\right]$. ■

Proof of Proposition 5: We have

$$\begin{aligned}\widehat{\beta}_1(y_1) &= \frac{\int_0^{y_1}\widehat{\beta}_2(\theta,\theta)\,f_1(\theta)\,d\theta}{F_1(y_1)}-\frac{\int_0^{y_1}\left\{\left.\frac{\partial\widehat{\beta}_2(\theta,\widetilde{\theta})}{\partial\widetilde{\theta}}\right|_{\widetilde{\theta}=\theta}\frac{1-F(\theta)}{f(\theta)}\right\}f_1(\theta)\,d\theta}{F_1(y_1)} \\ &> \frac{\int_0^{y_1}\widehat{\beta}_2(\theta,\theta)\,f_1(\theta)\,d\theta}{F_1(y_1)} \\ &= \mathbb{E}\left[p_2|p_1\right]\end{aligned}$$

where the inequality follows from Lemma 3. ■

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