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What happens if in the Principal Component Analysis the Pearsonian is replaced by the Brownian coefficient of correlation?

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Abstract

The Brownian correlation has been recently introduced by Székely et al. (2007; 2009), which has an attractive property that when it is zero, it guarantees independence. This paper investigates into the effects and advantages, if any, of replacement of the Pearsonian coefficient of correlation (r) by the Brownian coefficient of correlation (say, ρ), other things remaining the same. Such a replacement and analysis of its effects have been made by the Host-Parasite Co-evolutionary algorithm of global optimization applied on six datasets.

Keywords: Brownian correlation, Principal Component Analysis, Global Optimization, Host-Parasite Co-evolutionary algorithm, Iris Flower Dataset, 1985 Auto Imports Database, Levy distribution, outliers.

JEL Classification: C13, C43, C45, C61, C63, C87

1. Introduction

Principal Component Analysis (PCA) is a statistical analysis of fundamental importance when dimensionality reduction of the multivariate data or construction of best representative composite indices is needed. It finds its application in almost any branch of science ranging from anthropology to zoology. Given a dataset $X(m)$ in n observations on m variables, it obtains a set of composite variables $Z(m_1)$; $m_1 \leq m$ such that:

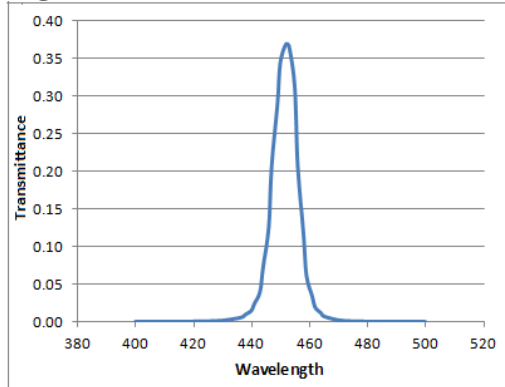
- (1) Z_k is a linear combination of original variables, X_j ; $j=1, m$, that is, $Z_k = \sum_j X_j a_{j,k}$; $k=1, m_1$, where $m_1 \leq m$.
- (2) The sum of squared (Pearsonian) coefficients of correlation between a given composite variable Z_k and all original variables X_j ; $j=1, m$, that is, $\sum_j r^2(X_j, Z_k)$; $k=1, m_1$, is maximized. Each Z_k is associated with a local optimum, although one of them (practically, almost always) attains a unique global optimum.
- (3) Any two composite variables, Z_k and $Z_{k'}$; $k, k'=1, m_1$; $k \neq k'$ are orthogonal to each other, that is, the dot product $Z_k Z_{k'} = 0$.

This paper investigates into the question as to replacement of the Pearsonian coefficient of correlation (r) by the Brownian coefficient of correlation (say, ρ), other things remaining the same.

2. The Brownian Coefficient of Correlation

The Pearsonian coefficient of correlation needs no introduction, but the Brownian coefficient of correlation (also known as the distance correlation) was introduced in 2005 by G.J. Székely in his lectures and later published by him and his coauthors (Székely et al., 2007; Székely and Rizzo, 2009). It was shown that the Brownian correlation is a generalization of the Pearsonian correlation in the sense that the former is applicable to the relationship between two sets of variables of the same or different dimensions, equal to or larger than one, while the latter is limited to two sets of variable each being a singleton. Secondly, as it is well known, the Pearsonian correlation is limited to the linear relationship between two variables and, therefore, Pearsonian uncorrelatedness ($r=0$) does not imply independence. The Brownian correlation addresses this problem such that $\rho=0$ implies independence and the relationship among the variables need not be linear. However, while the Pearsonian coefficient of correlation ranges between $[-1, 1]$, the Brownian coefficient of correlation ranges between $[0, 1]$. In this aspect, it behaves much like the squared value of Pearsonian coefficient of correlation.

Fig.1. Plot of Eckerle and NIST Data



In severely nonlinear data when the Pearsonian coefficient of correlation may be near zero, the Brownian coefficient of correlation may be quite large. Székely and Rizzo (2009) cite a case from Eckerle and NIST (1979) where the Pearsonian coefficient of correlation (r) between wavelength (x) and transmittance (y) is 0.0356, statistically showing linear independence of y with x . However, the Brownian coefficient of correlation (ρ) is 0.4275431, showing significant dependence of y on x . We have obtained $r=0.03844$ and $\rho=0.42744$ from the data (35 observations) provided by Eckerle and NIST (1979).

The computational steps of the Brownian coefficient of correlation are given below.

- i. Let X and Y be two sets of p and q number of variables (respectively), each in n observations, such that $x_{i,j} \in X; i=1, n; j=1, p$ and $y_{i,j} \in Y; i=1, n; j=1, q$. Moreover, $p, q \geq 1$.

- ii. Compute

$$a_{i,j} \in A : a_{i,j} = \|X_i - X_j\| = \left(\sum_{k=1}^p (x_{i,k} - x_{j,k})^2 \right)^{1/2} ; i, j = 1, n.$$

$$b_{i,j} \in B : b_{i,j} = \|Y_i - Y_j\| = \left(\sum_{k=1}^q (y_{i,k} - y_{j,k})^2 \right)^{1/2} ; i, j = 1, n.$$

$$\bar{a}_i = \sum_{j=1}^n a_{i,j} / n ; \bar{b}_i = \sum_{j=1}^n b_{i,j} / n ; \quad \bar{a}_j = \sum_{i=1}^n a_{i,j} / n ; \bar{b}_j = \sum_{i=1}^n b_{i,j} / n$$

$$\bar{a} = \sum_{i=1}^n \bar{a}_i / n = \sum_{j=1}^n \bar{a}_j / n ; \quad \bar{b} = \sum_{i=1}^n \bar{b}_i / n = \sum_{j=1}^n \bar{b}_j / n$$

- iii. Centralize A to A' and B to B' such that

$$a'_{i,j} = a_{i,j} - (\bar{a}_i + \bar{a}_j) + \bar{a} \quad \text{and} \quad b'_{i,j} = b_{i,j} - (\bar{b}_i + \bar{b}_j) + \bar{b}$$

- iv. Compute $\text{cov}(X, Y)$, $\text{var}(X)$ and $\text{var}(Y)$, the last two ($\text{var}(X)$ and $\text{var}(Y)$) being the special cases of the first ($\text{cov}(X, Y)$).

$$\text{Cov}(X, Y) = \left((1/n^2) \sum_{i=1}^n \sum_{j=1}^n a'_{i,j} b'_{i,j} \right)^{0.5} ; \quad \text{Var}(X) = \left((1/n^2) \sum_{i=1}^n \sum_{j=1}^n a'_{i,j} a'_{i,j} \right)^{0.5} ;$$

$$\text{Var}(Y) = \left((1/n^2) \sum_{i=1}^n \sum_{j=1}^n b'_{i,j} b'_{i,j} \right)^{0.5}$$

- v. Compute the Brownian coefficient of correlation $\rho(X, Y)$

$$\rho(X, Y) = \text{Cov}(X, Y) / (\text{Var}(X) \cdot \text{Var}(Y))^{0.5}$$

3. Principal Component Analysis based on Brownian correlation

For simplicity, let X be a set of m ($m \geq 1$) subsets of variables, or $X = [X_1, X_2, \dots, X_m]$. Let every $X_j; j=1, m$ be one-dimensional and having n observations (replicates). Thus, $X_j = X_j(n, 1) \forall j=1, m$. Our objective is to find $Z(n, 1) = Z_1 = \sum_{j=1}^m X_j w_{j,1}$. That is, Z_1 (the first principal component scores associated with the global optimum) is a linear combination (weighted sum) of $X_j; j=1, m$. We have to obtain $w_1 = [w_{1,1}, w_{2,1}, \dots, w_{m,1}]$ such that $\sum_{j=1}^m \rho^2(Z_1, X_j)$ is maximized. It may be noted that $\rho^2(Z_1, X_j)$ is the squared Brownian coefficient of correlation between Z_1 and X_j .

4. Optimization of sum of squared Brownian correlation among Z_1 and X_j

As formulated in the preceding section, our task is to find the (global) maximum of $\sum_{j=1}^m \rho^2(Z_1, X_j)$ with $w_1 = [w_{1,1}, w_{2,1} \dots, w_{m,1}]$ as the decision variables. This may be accomplished by a suitable method of global optimization (such as the Genetic algorithm, Particle Swarm, the Differential Evolution, Host-Parasite Co-evolutionary algorithm, etc.). If, subsequently, other principal component scores are to be obtained, additional decision variables $w_j = [w_{1,j}, w_{2,j}, \dots, w_{m,j}]$; $j \neq 1$ and an independence constraint $\rho(Z_1, Z_j) = 0$ may be incorporated in the optimization problem. Presently, we keep the issue of subsequent principal components scores (Z_2, Z_3 etc.) and orthogonality (or independence) among them in abeyance.

5. Materials and Methods

In this investigation we have tested the effects of replacement of the Pearsonian correlation by the Brownian correlation in constructing the leading PC or the Principal Component scores (Z_1) on six datasets. These datasets are: (i) Iris flower dataset, (ii) Motor Car dataset, (iii) Sinusoidal dataset, (iv) Crime Incidence in India – 2011 dataset, (v) the dataset with outliers in variables, and (vi) Lévy Distributed Probability Density Function dataset.

For obtaining the leading PC scores we have used the Host-Parasite Coevolutionary algorithm of global optimization (Mishra, 2013) to directly work out the weight vector with which Z_{1P} (the leading Pearsonian PC scores) is computed as $Z_{i,1P} = \sum_{j=1}^m X_{i,j} w_{j,1P}$; $i = 1, 2, \dots, n$ and $\sum_{j=1}^m r^2(Z_{1P}, X_{i,j})$ is maximized for the Pearsonian correlation, r). The results are identical to the ones that we get using the SPSS package software that applies the traditional method (of eigen-decomposition of the Pearsonian correlation matrix among the variables, X). Similarly, we have computed Z_{1B} (the leading Brownian PC scores) as $Z_{i,1B} = \sum_{j=1}^m X_{i,j} w_{j,1B}$; $i = 1, 2, \dots, n$ and $\sum_{j=1}^m \rho^2(Z_{1B}, X_{i,j})$ is maximized for the Brownian correlation, ρ). But, these results (based on the Brownian correlation) have no counterpart to be compared with, since there is no software or alternative algorithm to be used for this purpose. We have developed our own (Fortran) program for this purpose.

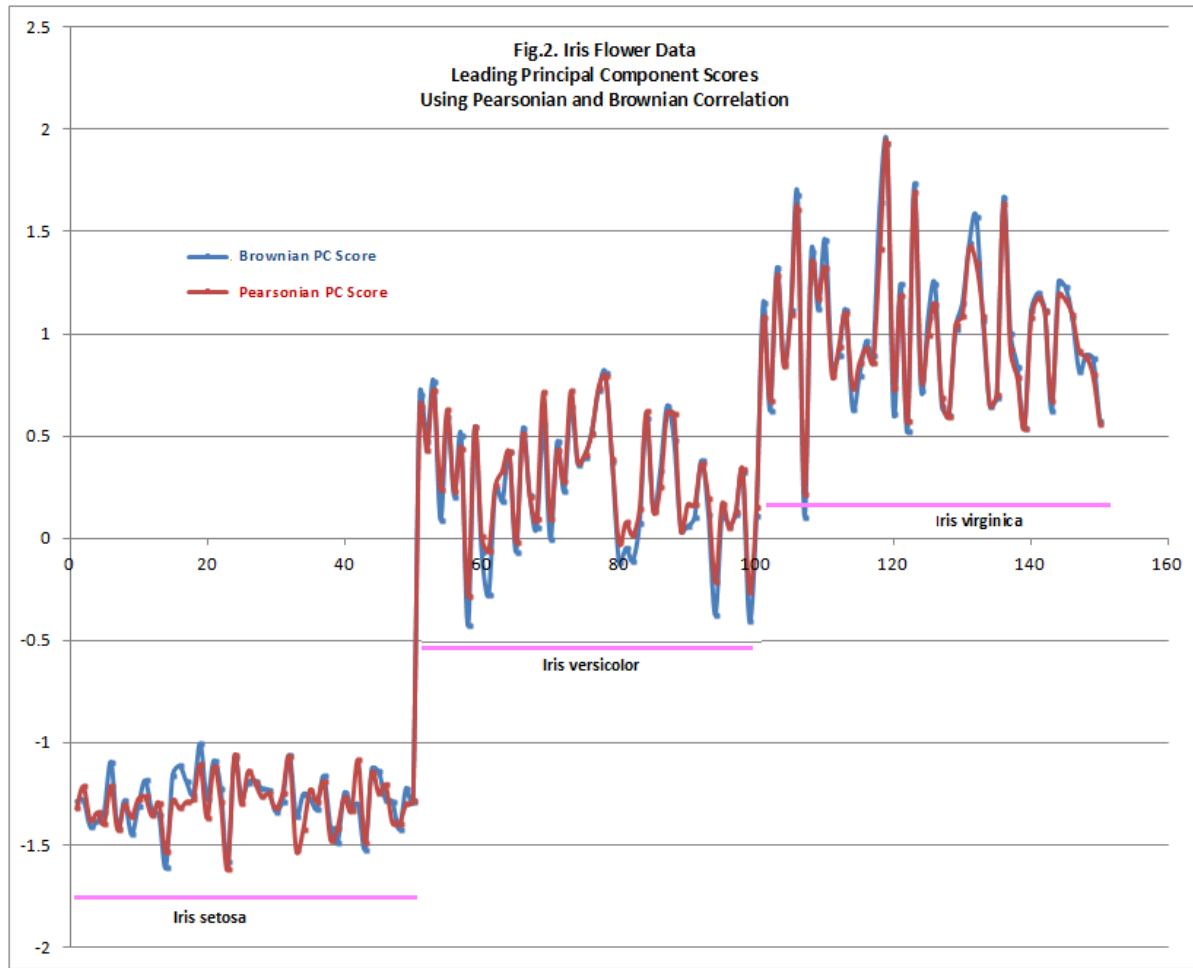
We have compared the performance of the leading PC scores based on the Brownian correlation with the leading PC scores based on the Pearsonian correlation. We have also presented the correlation matrices (both Pearsonian and Brownian) and their eigenvectors and eigenvalues with our observations on them. Wherever necessary or facilitating, we have given illustrative graphical presentation.

6. Findings

For the different datasets our findings are presented below.

6.1. Iris Flower Dataset: Anderson (1936) and Fisher (1936) collected and analyzed the data relating to the flowers of different species of Iris. Since Fisher applied his (linear) discriminant analysis on this dataset, it has been used by many researchers and now it is considered as a classic dataset.

Description of Data: X_1 = sepal length, X_2 = sepal width, X_3 = petal length, X_4 = petal width. Flowers of different species of Iris. No. of observations: *Iris setosa* (50), *Iris versicolor* (50), *Iris virginica* (50); total no. of observation: 150.



Findings: There is not much difference between the Brownian and Pearsonian leading PC scores. The coefficients of correlation (r and p) between them are (0.99699, 0.99727) respectively (see Table.1.1 and Table.1.2). However, the Brownian PC scores have relatively larger variance. They have been plotted in Fig.2. We observe that the three clusters of (the species of) Iris are clearly discernible. The details are given in Table.A1 in the appendix.

Table.1.1. Pearsonian Correlation matrix of Iris Flower Dataset						
Measures	Sepal length	Sepal width	Petal length	Petal width	Score(Brown)	Score(Pearson)
Sepal length	1.00000	-0.11757	0.87175	0.81794	0.91406	0.89017
Sepal width	-0.11757	1.00000	-0.42844	-0.36613	-0.39130	-0.46014
Petal length	0.87175	-0.42844	1.00000	0.96287	0.99214	0.99156
Petal width	0.81794	-0.36613	0.96287	1.00000	0.96608	0.96498
Score(Brown)	0.91406	-0.39130	0.99214	0.96608	1.00000	0.99699
S core(Pearson)	0.89017	-0.46014	0.99156	0.96498	0.99699	1.00000

The eigenvalues (λ) and the associated eigenvectors (V) of the Pearsonian and the Brownian correlation matrices (of X1 through X4) are given in Table.1.3 and Table.1.4. The Pearsonian leading PC scores explain 72.963% of the total variance in the data and it is commensurate with $72.963=100(\lambda_1/4)$. However, the Brownian leading PC scores explain 75.483% of the total variance in the data while $76.936 = 100(\lambda_1/4)$ is larger than that. This is a discrepancy which will be discussed at the end of this paper.

Table.1.2. Brownian Correlation matrix of Iris Flower Dataset						
Measures	Sepal length	Sepal width	Petal length	Petal width	Score(Brown)	Score(Pearson)
Sepal length	1.00000	0.31053	0.85852	0.82660	0.90101	0.87966
Sepal width	0.31053	1.00000	0.54157	0.51300	0.52353	0.55212
Petal length	0.85852	0.54157	1.00000	0.97363	0.99236	0.99208
Petal width	0.82660	0.51300	0.97363	1.00000	0.97399	0.97297
Score(Brown)	0.90101	0.52353	0.99236	0.97399	1.00000	0.99727
Score(Pearson)	0.87966	0.55212	0.99208	0.97297	0.99727	1.00000

Table.1.3. Eigenvalues and Eigenvectors of Pearsonian Correlation matrix of Iris Flower Dataset				
Measures	λ_1	λ_2	λ_3	λ_4
λ	2.918499	0.914030	0.146758	0.020713
$\lambda/4$	0.729625	0.228507	0.036690	0.005178
V	V_1	V_2	V_3	V_4
1	0.521065	0.377420	-0.719575	0.261262
2	-0.269349	0.923295	0.244389	-0.123500
3	0.580413	0.024493	0.142153	-0.801444
4	0.564857	0.066940	0.634254	0.523619

Table.1.4. Eigenvalues and Eigenvectors of Brownian Correlation matrix of Iris Flower Dataset				
Measures	λ_1	λ_2	λ_3	λ_4
λ	3.077445	0.738238	0.161808	0.022509
$\lambda/4$	0.769361	0.184560	0.040452	0.005627
V	V_1	V_2	V_3	V_4
1	0.504328	-0.413653	-0.748353	0.120470
2	0.357569	0.898947	-0.247231	0.053989
3	0.560082	-0.092801	0.305698	-0.764359
4	0.551458	-0.110331	0.534222	0.631132

6.2. Motor Car Dataset: The “1985 Auto Imports Database” was compiled by J.C. Schlimmer from various sources such as (1) 1985 Model Import Car and Truck Specifications, 1985, Ward's Automotive Yearbook; (2) Personal Auto Manuals, Insurance Services Office, 160 Water Street, New York, (3) Insurance Collision Report, Insurance Institute for Highway Safety, Watergate 600, Washington. The data pertains to 26 attributes of 205 models of automobile of different makes, with some information missing in case of some models. The dataset is archived on the website of Donald Bren School of Information and Computer Sciences (1996). A subset of this

dataset was used by Kibler et al. (1989) to predict price of car using all numeric and Boolean attributes. In the present study we use only a smaller subset of data for 195 models - for which the complete numerical information on body and engine characteristics (13 variables) is available. These 13 variables are used for constructing the leading principal component scores with Pearsonian and Brownian correlation. The dataset also has information on price of car, which we use for regression analysis.

Description of Data: X_1 = Wheelbase; X_2 = Length; X_3 = Width; X_4 = Height; X_5 = Curb height; X_6 = Engine size; X_7 = Bore; X_8 = Stroke; X_9 = Compression ratio; X_{10} = Horse power; X_{11} = Peak rpm; X_{12} = City mpg; X_{13} = Highway mpg. Also, P= price of car (of a particular model/make). Total no. observations: 195.

Table.2.1. Pearsonian Correlation matrix of Motor Car Dataset

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	SB	SP
X1	1.0000	0.8792	0.8190	0.5925	0.7827	0.5697	0.4982	0.1717	0.2477	0.3755	-0.3523	-0.4991	-0.5664	0.8115	0.8063
X2	0.8792	1.0000	0.8581	0.4962	0.8817	0.6875	0.6094	0.1187	0.1602	0.5838	-0.2810	-0.6897	-0.7193	0.9210	0.9136
X3	0.8190	0.8581	1.0000	0.3158	0.8673	0.7403	0.5443	0.1864	0.1910	0.6168	-0.2516	-0.6471	-0.6922	0.8942	0.8907
X4	0.5925	0.4962	0.3158	1.0000	0.3077	0.0313	0.1893	-0.0555	0.2612	-0.0844	-0.2641	-0.1024	-0.1512	0.3275	0.3260
X5	0.7827	0.8817	0.8673	0.3077	1.0000	0.8576	0.6458	0.1728	0.1554	0.7603	-0.2789	-0.7722	-0.8127	0.9680	0.9649
X6	0.5697	0.6875	0.7403	0.0313	0.8576	1.0000	0.5831	0.2120	0.0246	0.8427	-0.2190	-0.7106	-0.7321	0.8582	0.8645
X7	0.4982	0.6094	0.5443	0.1893	0.6458	0.5831	1.0000	-0.0668	0.0031	0.5685	-0.2777	-0.5920	-0.6000	0.7089	0.7201
X8	0.1717	0.1187	0.1864	-0.0555	0.1728	0.2120	-0.0668	1.0000	0.1999	0.1000	-0.0683	-0.0276	-0.0365	0.1469	0.1477
X9	0.2477	0.1602	0.1910	0.2612	0.1554	0.0246	0.0031	0.1999	1.0000	-0.2144	-0.4446	0.3314	0.2679	0.0706	0.0382
X10	0.3755	0.5838	0.6168	-0.0844	0.7603	0.8427	0.5685	0.1000	-0.2144	1.0000	0.1057	-0.8341	-0.8129	0.7862	0.7996
X11	-0.3523	-0.2810	-0.2516	-0.2641	-0.2789	-0.2190	-0.2777	-0.0683	-0.4446	0.1057	1.0000	-0.0695	-0.0170	-0.2451	-0.2378
X12	-0.4991	-0.6897	-0.6471	-0.1024	-0.7722	-0.7106	-0.5920	-0.0276	0.3314	-0.8341	-0.0695	1.0000	0.9723	-0.8467	-0.8497
X13	-0.5664	-0.7193	-0.6922	-0.1512	-0.8127	-0.7321	-0.6000	-0.0365	0.2679	-0.8129	-0.0170	0.9723	1.0000	-0.8764	-0.8793
SB	0.8115	0.9210	0.8942	0.3275	0.9680	0.8582	0.7089	0.1469	0.0706	0.7862	-0.2451	-0.8467	-0.8764	1.0000	0.9987
SP	0.8063	0.9136	0.8907	0.3260	0.9649	0.8645	0.7201	0.1477	0.0382	0.7996	-0.2378	-0.8497	-0.8793	0.9987	1.0000

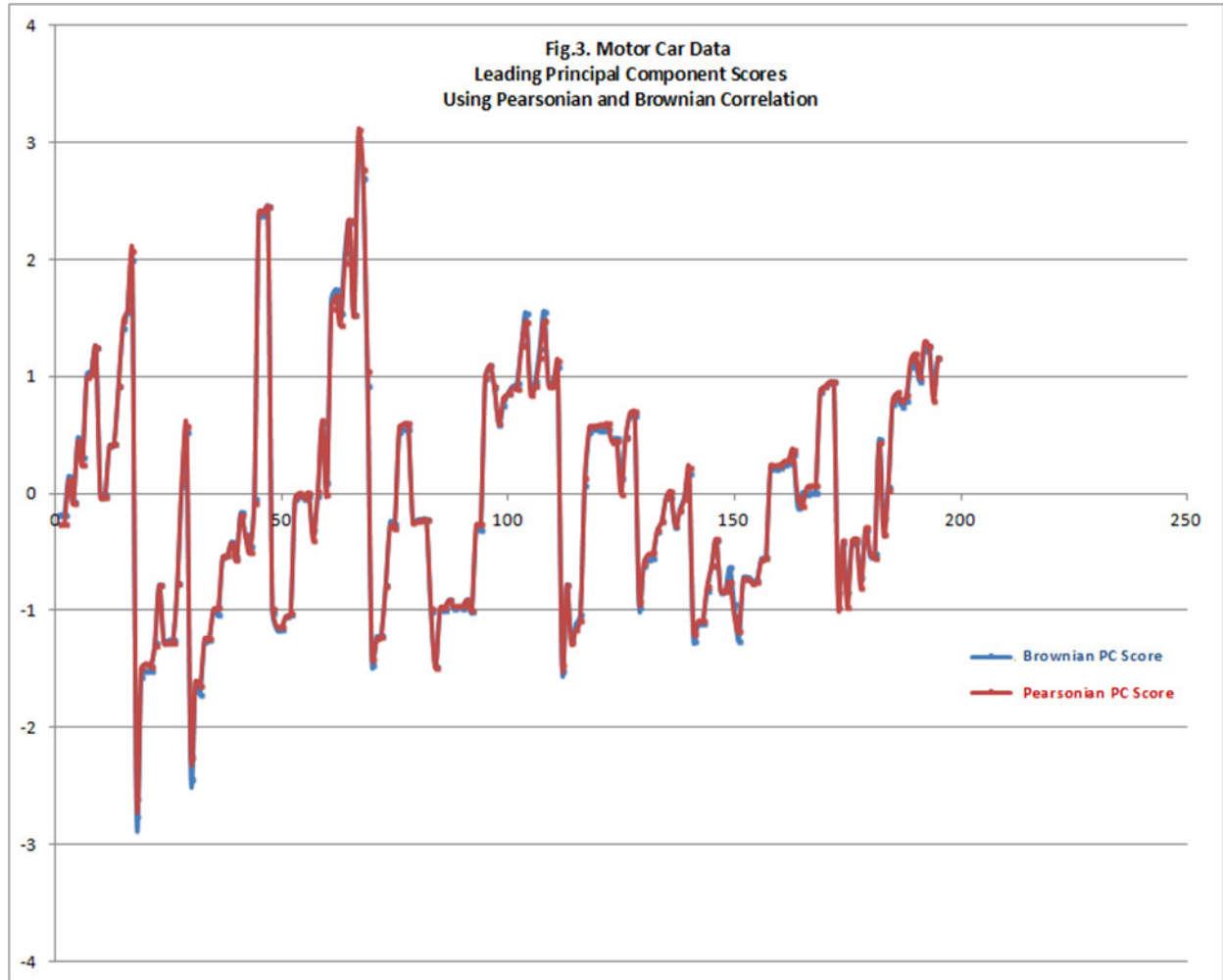
Table.2.2. Brownian Correlation matrix of Motor Car Dataset

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	SB	SP
X1	1.0000	0.8833	0.8363	0.6077	0.7777	0.6208	0.5305	0.2415	0.3077	0.4897	0.3260	0.5231	0.5777	0.8124	0.8057
X2	0.8833	1.0000	0.8711	0.5566	0.8614	0.7044	0.6187	0.2287	0.2260	0.6200	0.2758	0.6632	0.6934	0.8971	0.8886
X3	0.8363	0.8711	1.0000	0.4319	0.8599	0.7475	0.5913	0.2644	0.2746	0.6442	0.2525	0.6573	0.6939	0.8934	0.8877
X4	0.6077	0.5566	0.4319	1.0000	0.3979	0.2358	0.2663	0.1639	0.2865	0.2031	0.2849	0.2267	0.2553	0.4051	0.4034
X5	0.7777	0.8614	0.8599	0.3979	1.0000	0.8642	0.6725	0.2314	0.2538	0.7830	0.2612	0.7973	0.8305	0.9655	0.9600
X6	0.6208	0.7044	0.7475	0.2358	0.8642	1.0000	0.6670	0.2943	0.1880	0.8407	0.2472	0.7604	0.7680	0.8749	0.8774
X7	0.5305	0.6187	0.5913	0.2663	0.6725	0.6670	1.0000	0.3232	0.1623	0.5982	0.3262	0.6126	0.6269	0.7296	0.7387
X8	0.2415	0.2287	0.2644	0.1639	0.2314	0.2943	0.3232	1.0000	0.3230	0.2349	0.2553	0.2014	0.2105	0.2342	0.2336
X9	0.3077	0.2260	0.2746	0.2865	0.2538	0.1880	0.1623	0.3230	1.0000	0.2460	0.4866	0.3341	0.2886	0.2136	0.2018
X10	0.4897	0.6200	0.6442	0.2031	0.7830	0.8407	0.5982	0.2349	0.2460	1.0000	0.1838	0.8835	0.8572	0.8098	0.8180
X11	0.3260	0.2758	0.2525	0.2849	0.2612	0.2472	0.3262	0.2553	0.4866	0.1838	1.0000	0.1765	0.1507	0.2385	0.2310
X12	0.5231	0.6632	0.6573	0.2267	0.7973	0.7604	0.6126	0.2014	0.3341	0.8835	0.1765	1.0000	0.9622	0.8443	0.8473
X13	0.5777	0.6934	0.6939	0.2553	0.8305	0.7680	0.6269	0.2105	0.2886	0.8572	0.1507	0.9622	1.0000	0.8692	0.8720
SB	0.8124	0.8971	0.8934	0.4051	0.9655	0.8749	0.7296	0.2342	0.2136	0.8098	0.2385	0.8443	0.8692	1.0000	0.9983
SP	0.8057	0.8886	0.8877	0.4034	0.9600	0.8774	0.7387	0.2336	0.2018	0.8180	0.2310	0.8473	0.8720	0.9983	1.0000

Findings: There is not much difference between the Brownian and Pearsonian leading PC scores. The coefficients of correlation (r and ρ) between them are (0.9987, 0.9983) respectively (see Table.2.1 and Table.2.2). However, the Brownian PC scores have slightly larger variance. They have been plotted in Fig.3. The details are given in Table.A2 in the appendix.

The eigenvalues (λ) and the associated eigenvectors (V) of the Pearsonian and the Brownian correlation matrices (of X_1 through X_{13}) are given in Table.2.3 and Table.2.4. The Pearsonian leading PC scores explain 52.275% of the total variance in the data and it is commensurate with $52.275=100(\lambda_1/13)$.

However, the Brownian leading PC scores explain 53.372% of the total variance in the data while $56.24 = 100(\lambda_1/13)$ is larger than that. We will discuss this issue in due course.



**Table.2.3. Eigenvalues and Eigenvectors of
Pearsonian Correlation matrix of Motor Car Dataset**

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}	λ_{13}
λ	6.79575	2.26793	1.19702	0.90126	0.54167	0.41122	0.29691	0.23572	0.11800	0.09561	0.06642	0.05221	0.02026
$\lambda/13$	0.52275	0.17446	0.09208	0.06933	0.04167	0.03163	0.02284	0.01813	0.00908	0.00735	0.00511	0.00402	0.00156
V	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}
1	0.30931	-0.27976	-0.11059	0.23657	0.03915	0.10455	0.40046	0.10371	-0.44033	-0.38093	0.45934	-0.12544	0.09249
2	0.35044	-0.14973	-0.10612	0.13945	-0.03890	0.03488	0.18606	-0.04672	-0.33081	0.72326	-0.31254	-0.17013	-0.15925
3	0.34168	-0.11574	0.06598	0.08612	-0.19615	0.18832	0.45207	0.00794	0.75923	-0.00905	-0.04267	-0.03559	0.01070
4	0.12507	-0.39131	-0.47143	0.42114	0.14273	-0.16432	-0.50627	0.24956	0.25027	-0.03532	-0.03790	-0.01831	0.00417
5	0.37014	-0.05089	0.06846	-0.02685	-0.11576	0.07599	-0.10589	-0.01229	-0.12722	-0.01652	-0.07715	0.88890	0.10913
6	0.33161	0.08570	0.25140	-0.19342	-0.09412	0.16752	-0.19349	0.54041	-0.13587	-0.31785	-0.47013	-0.27144	0.03864
7	0.27624	0.01574	-0.15412	-0.41893	0.20758	-0.78729	0.22161	0.04549	0.05254	-0.07175	-0.04562	0.00915	-0.00382
8	0.05666	-0.11483	0.74402	0.42011	0.42772	-0.25259	-0.04029	-0.04268	0.04290	0.03531	-0.00466	0.00535	-0.01493
9	0.01466	-0.52734	0.25505	-0.15205	-0.61057	-0.22257	-0.23903	-0.34131	-0.05589	-0.07921	0.02831	-0.16395	0.02218
10	0.30674	0.30925	0.15195	-0.09647	-0.18850	-0.04820	-0.31050	0.27819	0.09012	0.32844	0.65737	-0.08987	-0.10008
11	-0.09120	0.44412	-0.05086	0.55528	-0.51295	-0.38361	0.14712	0.09715	-0.08360	-0.10860	-0.14690	0.04078	0.01169
12	-0.32596	-0.28694	0.08301	-0.06516	-0.10462	-0.04639	0.19544	0.45282	-0.01066	0.01041	0.04496	0.21963	-0.70246
13	-0.33730	-0.23692	0.08421	-0.06844	-0.08616	-0.08213	0.17766	0.46932	0.00389	0.31098	0.04953	0.07023	0.66949

**Table.2.4. Eigenvalues and Eigenvectors of
Brownian Correlation matrix of Motor Car Dataset**

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}	λ_{13}
λ	7.3118	1.5780	1.1786	0.8022	0.6626	0.4610	0.3460	0.2306	0.1255	0.1133	0.0943	0.0677	0.0285
$\lambda/13$	0.5624	0.1214	0.0907	0.0617	0.0510	0.0355	0.0266	0.0177	0.0097	0.0087	0.0073	0.0052	0.0022
V	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}
1	0.3035	-0.1934	-0.3604	0.0499	0.0275	-0.2785	0.0976	-0.1050	0.5287	-0.3602	-0.4486	-0.1274	-0.1120
2	0.3294	-0.0520	-0.3082	0.0600	-0.0102	-0.1461	0.0648	-0.2434	0.0703	-0.1577	0.7391	0.3238	0.1565
3	0.3271	-0.0180	-0.2060	0.0751	0.0265	-0.4147	0.0718	0.0260	-0.7677	0.0227	-0.2672	0.0680	-0.0146
4	0.1768	-0.4051	-0.4587	-0.0550	0.2472	0.6533	-0.2087	0.1591	-0.1277	0.1147	-0.0639	0.0277	-0.0116
5	0.3493	0.0957	-0.0872	-0.0099	-0.0412	-0.1432	-0.0583	0.0884	0.1161	0.5368	0.2455	-0.6699	-0.1303
6	0.3237	0.1768	0.0993	0.1007	-0.1371	-0.1134	-0.3184	0.5923	0.2476	0.2150	-0.0988	0.4921	0.0261
7	0.2771	0.0391	0.1070	0.2743	-0.4411	0.3907	0.6700	0.1734	-0.0390	-0.0743	-0.0254	-0.0352	-0.0007
8	0.1291	-0.3006	0.4047	0.7557	0.3258	-0.0033	-0.1486	-0.1644	0.0136	0.0249	0.0158	-0.0348	-0.0237
9	0.1404	-0.4720	0.3830	-0.4397	0.3729	-0.1783	0.3655	0.3078	0.0401	-0.0039	0.1126	0.0197	0.0608
10	0.3105	0.2521	0.2287	-0.1121	0.0550	0.1609	-0.3371	0.1964	-0.1466	-0.6547	0.1114	-0.3441	0.1147
11	0.1343	-0.5199	0.2471	-0.2013	-0.6526	-0.0192	-0.3358	-0.2565	-0.0550	0.0274	-0.0374	0.0026	0.0118
12	0.3184	0.2288	0.2170	-0.2301	0.1536	0.1841	0.0136	-0.3485	-0.0061	0.0459	-0.0375	0.2384	-0.7118
13	0.3246	0.2328	0.1500	-0.1711	0.1567	0.1562	0.0419	-0.4093	0.0851	0.2444	-0.2797	0.0667	0.6486

Relationship between motor car price and the car's engine and body characteristics: If we use car price as the dependent variable and the leading principal component score as an explanatory variable in a linear regression model, we obtain

$$\hat{P} = 13248.014 + 6768.322S_B; R^2=701 \quad ; \quad \hat{P} = 13248.014 + 6768.322S_P; R^2= 0.706$$

(316.162) (316.162) (313.730) (314.538)

for Brownian and Pearsonian principal component scores, respectively. The figures in the parentheses are standard error of estimates of the regression coefficients. It appears that the two are statistically indistinguishable. However, a lower value of R^2 in case of the Brownian regression equation may be due to its proneness to incorporate nonlinearity in the explanatory variable, S_B .

6.3. Sinusoidal Data: The sinusoidal data (9 variables: X_1 through X_9) have been generated by us to introduce severe nonlinearities in the dataset. They are different sine or cosine functions of basic variable (1, 2, 3, through 50) perturbed by (uniformly distributed) random numbers between 0 and unity. Their movement along the horizontal axis (1 through 50) has been presented in Fig.4(a).

Findings: There is not much difference between the Brownian and Pearsonian leading PC scores. The coefficients of correlation (r and ρ) between them are (0.9987, 0.9983) respectively (see Table.4.1 and Table.3.2). However, the Brownian PC scores have slightly smaller variance. They have been plotted in Fig.4(b). The details are given in Table.A3 in the appendix.

The eigenvalues (λ) and the associated eigenvectors (V) of the Pearsonian and the Brownian correlation matrices (of X_1 through X_9) are given in Table.3.3 and Table.3.4. The Pearsonian leading PC scores explain 22.949% of the total variance in the data and it is commensurate with $22.949=100(\lambda_1/9)$. However, the Brownian leading PC scores explain 22.404% of the total variance in the data while $32.986 = 100(\lambda_1/9)$ is substantially larger than that. The possible explanation of this observed discrepancy will be given in due course.

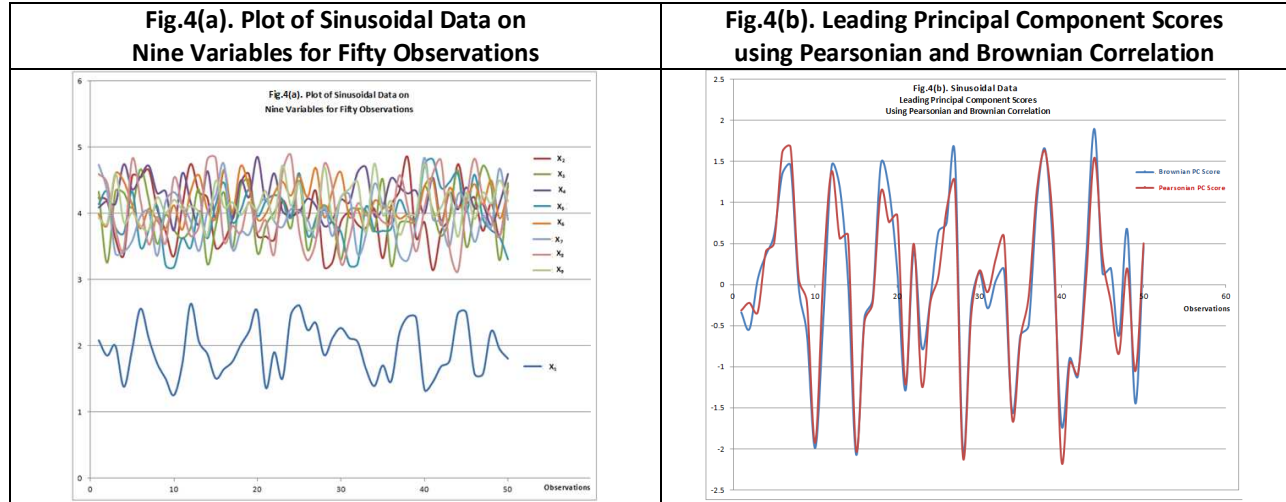


Table.3.1. Pearsonian Correlation matrix of Sinusoidal Dataset

	X1	X2	X3	X4	X5	X6	X7	X8	X9	SB	SP
X1	1.000000	0.477999	0.036105	0.054236	-0.075043	0.055156	-0.169786	-0.252138	-0.284375	0.691472	0.692272
X2	0.477999	1.000000	0.121609	0.117210	0.009134	0.029593	-0.195106	-0.048875	-0.283840	0.706035	0.663524
X3	0.036105	0.121609	1.000000	-0.073124	0.183494	0.336459	-0.064162	-0.046778	0.021822	0.264908	0.085322
X4	0.054236	0.117210	-0.073124	1.000000	-0.104353	-0.125137	-0.138354	-0.242769	-0.390480	0.339441	0.515656
X5	-0.075043	0.009134	0.183494	-0.104353	1.000000	0.252031	0.026387	0.162591	-0.009649	0.000100	-0.136356
X6	0.055156	0.029593	0.336459	-0.125137	0.252031	1.000000	-0.145654	-0.211004	0.139617	0.259056	0.042260
X7	-0.169786	-0.195106	-0.064162	-0.138354	0.026387	-0.145654	1.000000	-0.015072	0.239745	-0.518213	-0.462006
X8	-0.252138	-0.048875	-0.046778	-0.242769	0.162591	-0.211004	-0.015072	1.000000	0.108076	-0.402119	-0.407898
X9	-0.284375	-0.283840	0.021822	-0.390480	-0.009649	0.139617	0.239745	0.108076	1.000000	-0.600414	-0.687379
SB	0.691472	0.706035	0.264908	0.339441	0.000100	0.259056	-0.518213	-0.402119	-0.600414	1.000000	0.954728
SP	0.692272	0.663524	0.085322	0.515656	-0.136356	0.042260	-0.462006	-0.407898	-0.687379	0.954728	1.000000

Table.3.2. Brownian Correlation matrix of Sinusoidal Dataset

	X1	X2	X3	X4	X5	X6	X7	X8	X9	SB	SP
X1	1.000000	0.445287	0.187440	0.176104	0.226644	0.175839	0.247937	0.274121	0.289344	0.671368	0.662206
X2	0.445287	1.000000	0.275675	0.228791	0.157907	0.170978	0.240721	0.233906	0.344412	0.678935	0.634896
X3	0.187440	0.275675	1.000000	0.206049	0.271758	0.415916	0.251077	0.196363	0.185312	0.314755	0.197056
X4	0.176104	0.228791	0.206049	1.000000	0.215051	0.190766	0.244889	0.278645	0.390223	0.390847	0.520346
X5	0.226644	0.157907	0.271758	0.215051	1.000000	0.288959	0.228980	0.279306	0.166121	0.170292	0.206381
X6	0.175839	0.170978	0.415916	0.190766	0.288959	1.000000	0.287810	0.246804	0.195513	0.290597	0.164029
X7	0.247937	0.240721	0.251077	0.244889	0.228980	0.287810	1.000000	0.147723	0.276358	0.538614	0.460901
X8	0.274121	0.233906	0.196363	0.278645	0.279306	0.246804	0.147723	1.000000	0.206043	0.376501	0.389649
X9	0.289344	0.344412	0.185312	0.390223	0.166121	0.195513	0.276358	0.206043	1.000000	0.548356	0.632674
SB	0.671368	0.678935	0.314755	0.390847	0.170292	0.290597	0.538614	0.376501	0.548356	1.000000	0.936263
SP	0.662206	0.634896	0.197056	0.520346	0.206381	0.164029	0.460901	0.389649	0.632674	0.936263	1.000000

Table.3.3. Eigenvalues and Eigenvectors of Pearsonian Correlation matrix of Sinusoidal Dataset

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9
λ	2.065385	1.624454	1.167268	1.053080	0.898750	0.753980	0.567224	0.455192	0.414667
$\lambda/9$	0.229487	0.180495	0.129696	0.117009	0.099861	0.083776	0.063025	0.050577	0.046074
V	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9
1	0.481700	-0.097258	-0.010123	-0.501194	-0.112449	-0.198313	-0.216908	-0.305350	0.561202
2	0.461696	-0.135288	0.275504	-0.370222	-0.156784	0.136067	0.554838	0.195259	-0.408045
3	0.059369	-0.542656	-0.036833	0.111104	-0.060755	0.776374	-0.206298	-0.196039	0.029332
4	0.358806	0.275228	-0.157660	0.572991	-0.134996	0.142654	0.487445	-0.148183	0.379734
5	-0.094880	-0.423166	0.362276	0.332175	-0.486778	-0.436820	0.019075	-0.355257	-0.126525
6	0.029406	-0.615632	-0.278844	0.121941	0.175223	-0.273148	0.171769	0.559849	0.281532
7	-0.321475	0.126998	-0.261257	-0.221793	-0.788456	0.150370	0.038844	0.298574	0.167774
8	-0.283825	0.050795	0.754032	-0.031100	0.110092	0.176117	0.124520	0.219820	0.490273
9	-0.478295	-0.158839	-0.227958	-0.309368	0.198545	-0.000714	0.563970	-0.480770	0.098531

Table.3.4. Eigenvalues and Eigenvectors of Brownian Correlation matrix of Sinusoidal Dataset

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9
λ	2.968768	1.120953	0.924676	0.902607	0.763621	0.675246	0.604829	0.538365	0.500935
$\lambda/9$	0.329863	0.124550	0.102742	0.100290	0.084847	0.075027	0.067203	0.059818	0.055659
V	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9
1	0.343545	0.357240	0.486353	-0.132822	-0.183718	-0.028951	0.139495	0.525209	0.412651
2	0.355119	0.392337	0.393442	0.097039	0.223422	0.138632	-0.221430	-0.198575	-0.630991
3	0.335935	-0.412562	0.184455	0.240025	0.455300	0.232231	-0.350501	-0.151213	0.467288
4	0.327793	0.159618	-0.665199	-0.044539	0.141665	0.117187	-0.346207	0.508743	-0.100596
5	0.311297	-0.344173	-0.005012	-0.377014	-0.504023	0.595395	0.057406	-0.115834	-0.131254
6	0.332048	-0.511669	0.048284	0.147949	-0.196355	-0.274964	0.517824	0.314518	-0.349993
7	0.327597	-0.065431	-0.067013	0.480012	-0.607502	-0.419401	-0.286512	-0.161431	0.019820
8	0.316668	-0.020031	-0.071905	-0.688139	0.138912	-0.529933	-0.101519	-0.321002	0.084505
9	0.347572	0.371245	-0.346602	0.213006	0.089518	0.159539	0.570059	-0.401992	0.236295

6.4. Crime Incidents in India 2011 Data: The dataset is provided by the National Crime Records Bureau, Ministry of Home Affairs, Govt. of India on its website.

Description of variables: This dataset has seven variables (each in 35 observations for the states and union territories of India). X_1 = Murder; X_2 = Attempt to Commit Murder; X_3 = Culpable Homicide Not Amounting to Murder; X_4 = Rape; X_5 = Kidnapping & Abduction; X_6 = Dacoity; X_7 = Preparation & Assembly For Dacoity.

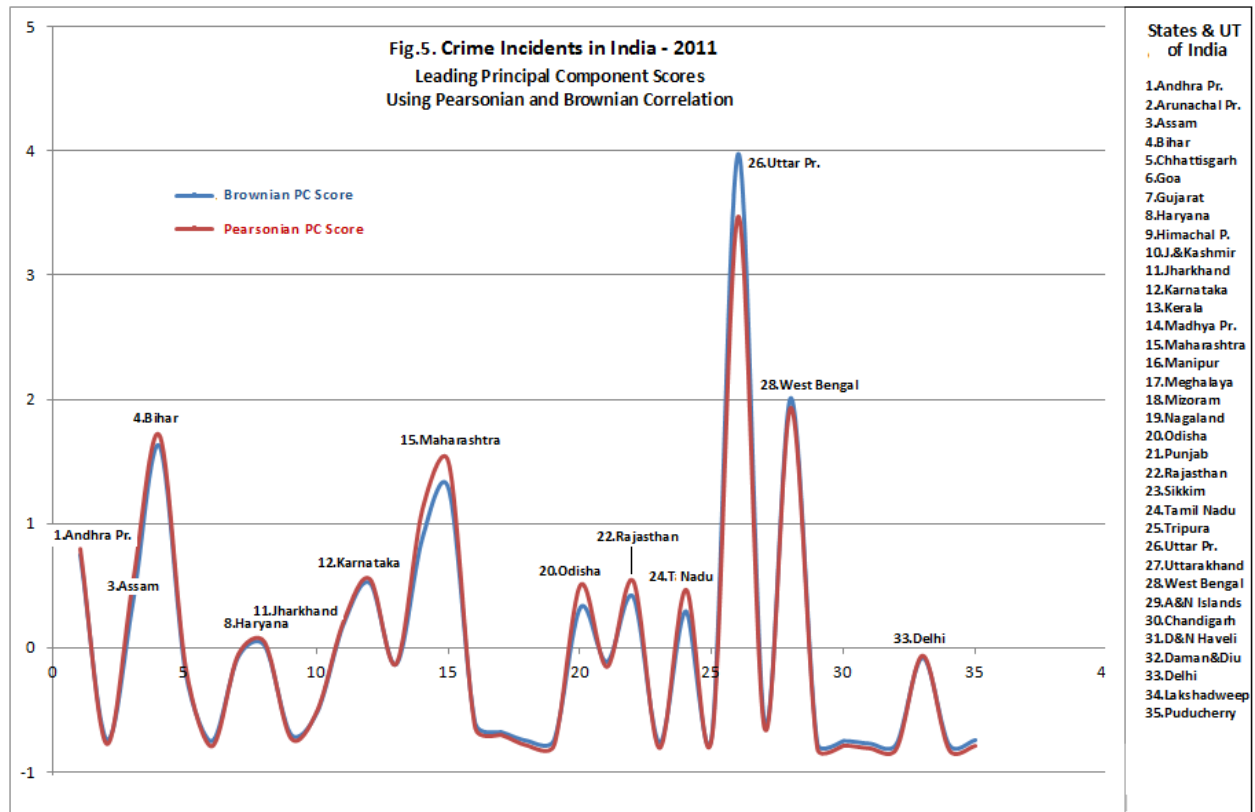


Table.4.1. Pearsonian Correlation matrix of Crime Incidence Dataset

	X1	X2	X3	X4	X5	X6	X7	SB	SP
X1	1.000000	0.952748	0.762417	0.768416	0.817072	0.750916	0.322254	0.951722	0.965378
X2	0.952748	1.000000	0.752003	0.715526	0.797357	0.650701	0.354426	0.925068	0.937870
X3	0.762417	0.752003	1.000000	0.505564	0.843174	0.428564	0.265564	0.881343	0.821074
X4	0.768416	0.715526	0.505564	1.000000	0.633816	0.526476	0.470646	0.766260	0.810667
X5	0.817072	0.797357	0.843174	0.633816	1.000000	0.540617	0.293448	0.902621	0.886100
X6	0.750916	0.650701	0.428564	0.526476	0.540617	1.000000	0.343141	0.702146	0.743570
X7	0.322254	0.354426	0.265564	0.470646	0.293448	0.343141	1.000000	0.462047	0.474430
SB	0.951722	0.925068	0.881343	0.766260	0.902621	0.702146	0.462047	1.000000	0.992830
SP	0.965378	0.937870	0.821074	0.810667	0.886100	0.743570	0.474430	0.992830	1.000000

Table.4.2. Brownian Correlation matrix of Crime Incidence Dataset

	X1	X2	X3	X4	X5	X6	X7	SB	SP
X1	1.000000	0.940148	0.761567	0.850679	0.796399	0.820574	0.499891	0.958803	0.963778
X2	0.940148	1.000000	0.761491	0.801443	0.770022	0.710575	0.548970	0.926337	0.932280
X3	0.761567	0.761491	1.000000	0.675050	0.792049	0.594823	0.523484	0.866787	0.823615
X4	0.850679	0.801443	0.675050	1.000000	0.752195	0.694113	0.540678	0.881868	0.896625
X5	0.796399	0.770022	0.792049	0.752195	1.000000	0.664563	0.423464	0.865649	0.859378
X6	0.820574	0.710575	0.594823	0.694113	0.664563	1.000000	0.500087	0.810772	0.819527
X7	0.499891	0.548970	0.523484	0.540678	0.423464	0.500087	1.000000	0.595239	0.582965
SB	0.958803	0.926337	0.866787	0.881868	0.865649	0.810772	0.595239	1.000000	0.996319
SP	0.963778	0.932280	0.823615	0.896625	0.859378	0.819527	0.582965	0.996319	1.000000

**Table.4.3. Eigenvalues and Eigenvectors of
Pearsonian Correlation matrix of Crime Incidence Dataset**

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ	4.706050	0.914373	0.611295	0.420157	0.195834	0.128903	0.023388
$\lambda/7$	0.672293	0.130625	0.087328	0.060022	0.027976	0.018415	0.003341
V	V ₁	V ₂	V ₃	V ₄	V ₅	V ₆	V ₇
1	0.445009	0.118239	0.181549	-0.090992	0.286485	0.013008	0.815171
2	0.432329	0.107970	0.045497	-0.081574	0.669808	-0.304322	-0.501453
3	0.378490	0.328745	-0.463797	0.311209	-0.021786	0.649678	-0.118984
4	0.373692	-0.250344	0.020210	-0.784371	-0.298612	0.260005	-0.158949
5	0.408464	0.250049	-0.270028	0.105833	-0.545348	-0.623624	0.014307
6	0.342763	-0.135306	0.743395	0.419699	-0.273098	0.153989	-0.192685
7	0.218697	-0.850167	-0.351999	0.292625	0.087483	-0.070109	0.085359

**Table.4.4. Eigenvalues and Eigenvectors of
Brownian Correlation matrix of Crime Incidence Dataset**

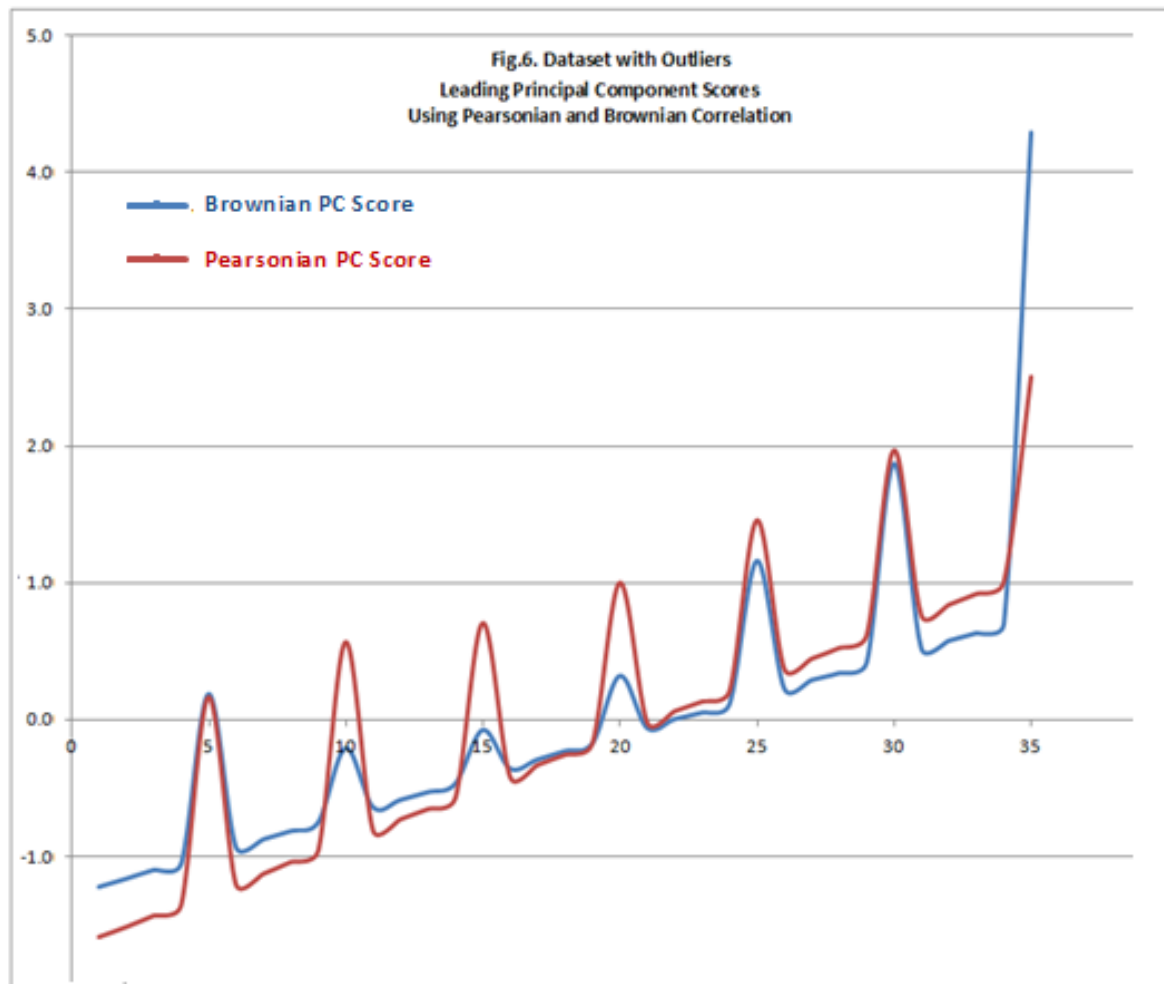
	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ	5.174474	0.656804	0.450269	0.295293	0.232572	0.157547	0.033040
$\lambda/7$	0.739211	0.093829	0.064324	0.042185	0.033225	0.022507	0.004720
V	V ₁	V ₂	V ₃	V ₄	V ₅	V ₆	V ₇
1	0.419658	-0.177682	-0.201783	0.132612	0.315844	0.048299	0.794936
2	0.408761	-0.080167	-0.011430	0.297761	0.568397	0.359839	-0.533982
3	0.375214	-0.057734	0.625572	-0.348122	0.216238	-0.542567	-0.047069
4	0.391635	-0.039225	-0.137466	0.621331	-0.462217	-0.453969	-0.142831
5	0.384349	-0.291710	0.330283	-0.175593	-0.550978	0.568172	0.029418
6	0.365937	-0.025155	-0.658475	-0.596185	-0.089692	-0.137390	-0.222510
7	0.284612	0.933490	0.078991	-0.015547	-0.071711	0.161312	0.099736

Findings: There is not much difference between the Brownian and Pearsonian leading PC scores. The coefficients of correlation (r and ρ) between them are (0.992830, 0.996319) respectively

(see Table.4.1 and Table.4.2). However, the Brownian PC scores have larger variance. They have been plotted in Fig.5). The details are given in Table.A4 in the appendix.

The eigenvalues (λ) and the associated eigenvectors (V) of the Pearsonian and the Brownian correlation matrices (of X1 through X7) are given in Table.4.3 and Table.4.4. The Pearsonian leading PC scores explain 67.229% of the total variance in the data and it is commensurate with $67.229=100(\lambda_1/7)$. However, the Brownian leading PC scores explain 72.392% of the total variance in the data while $73.9211=100(\lambda_1/7)$ is marginally larger than that. The possible explanation of this observed discrepancy will be given in due course.

6.5. Dataset with Outliers: This dataset contains 7 variables (each one in 35 observations) that are linearly correlated with very large correlation among them, except that each one has a single large (of a magnitude of about 10 times of its cohorts) outlier. We subject this dataset to construction of the leading principal component scores by using Pearsonian and Brownian correlation coefficients.



Findings: There is some difference between the Brownian and Pearsonian leading PC scores. The coefficients of correlation (r and ρ) between them are (0.910506, 0.949876) respectively (Table.5.1 and Table.5.2). They have been plotted in Fig.6. The details are given in Table.A5 in the appendix. The

Brownian PC scores have larger variance, although it cannot be concluded that they ameliorate the problems due to outliers or they represent the variables (with outliers) better than the Pearsonian leading PC score.

The eigenvalues (λ) and the associated eigenvectors (V) of the Pearsonian and the Brownian correlation matrices (of X1 through X7) are given in Table.5.3 and Table.5.4. The Pearsonian leading PC scores explain 30.750 of the total variance in the data and it is commensurate with $30.751=100(\lambda_1/7)$. However, the Brownian leading PC scores explain 58.360% of the total variance in the data while $67.497 = 100(\lambda_1/7)$ is considerably larger than that. The possible explanation of this observed discrepancy will be explored in due course.

Table.5.1. Pearsonian Correlation matrix of Dataset with Outliers

	X1	X2	X3	X4	X5	X6	X7	SB	SP
X1	1.000000	0.333274	0.257309	0.246588	0.275624	0.318127	0.360049	0.719453	0.790055
X2	0.333274	1.000000	0.149002	0.145658	0.168815	0.192087	0.223476	0.432582	0.600698
X3	0.257309	0.149002	1.000000	0.103183	0.113524	0.134531	0.151032	0.308415	0.468284
X4	0.246588	0.145658	0.103183	1.000000	0.097512	0.105096	0.118198	0.290887	0.427888
X5	0.275624	0.168815	0.113524	0.097512	1.000000	0.097911	0.106620	0.381527	0.453212
X6	0.318127	0.192087	0.134531	0.105096	0.097911	1.000000	0.104480	0.464015	0.500756
X7	0.360049	0.223476	0.151032	0.118198	0.106620	0.104480	1.000000	0.823015	0.555852
SB	0.719453	0.432582	0.308415	0.290887	0.381527	0.464015	0.823015	1.000000	0.910506
SP	0.790055	0.600698	0.468284	0.427888	0.453212	0.500756	0.555852	0.910506	1.000000

Table.5.2. Brownian Correlation matrix of Dataset with Outliers

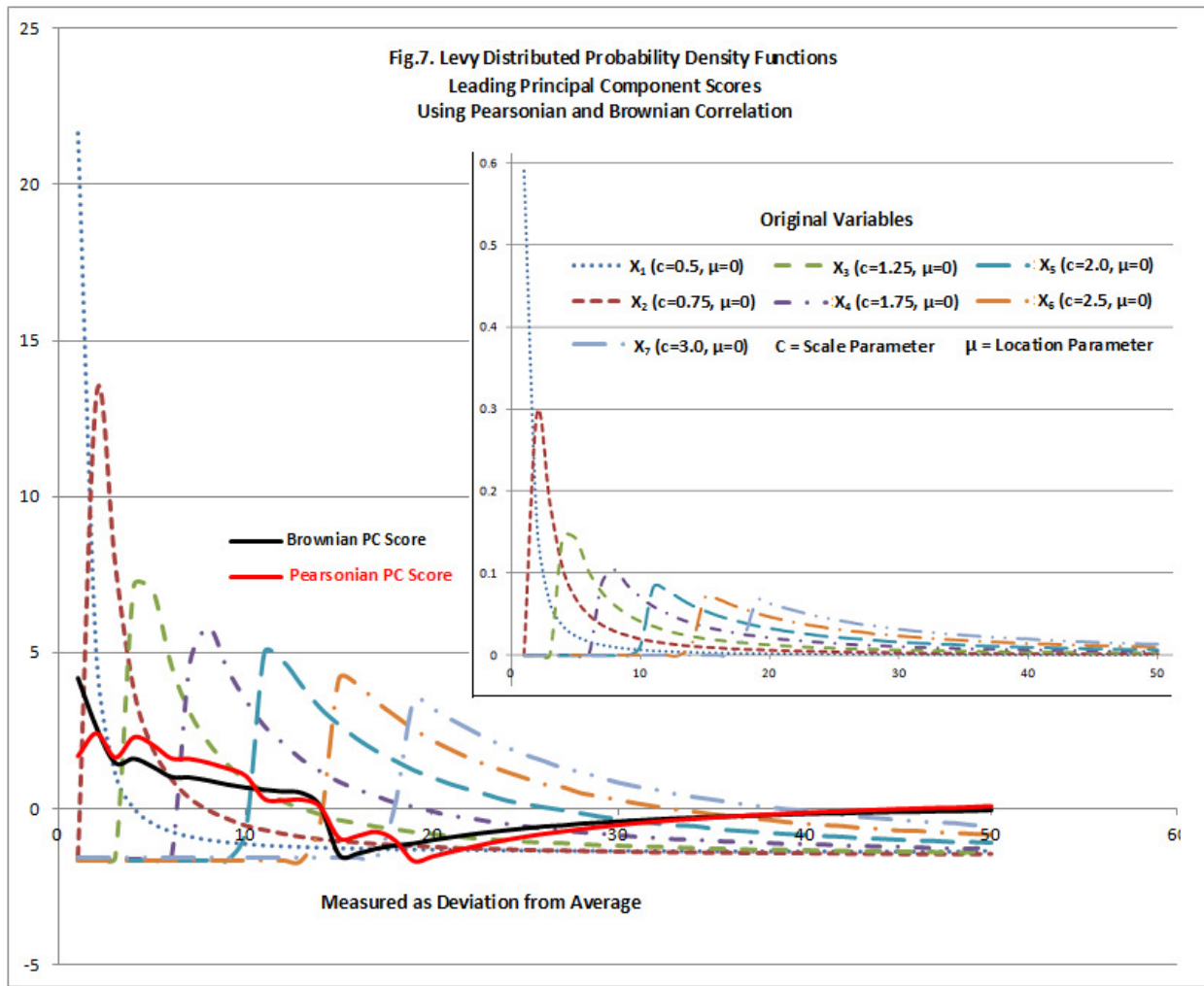
	X1	X2	X3	X4	X5	X6	X7	SB	SP
X1	1.000000	0.789665	0.750109	0.703358	0.667044	0.636991	0.612285	0.894048	0.857865
X2	0.789665	1.000000	0.733230	0.685645	0.650741	0.616587	0.592989	0.793885	0.834236
X3	0.750109	0.733230	1.000000	0.649538	0.613027	0.583419	0.557450	0.735454	0.763273
X4	0.703358	0.685645	0.649538	1.000000	0.574242	0.542829	0.519851	0.696234	0.705768
X5	0.667044	0.650741	0.613027	0.574242	1.000000	0.512619	0.491103	0.696388	0.673816
X6	0.636991	0.616587	0.583419	0.542829	0.512619	1.000000	0.467770	0.696247	0.653841
X7	0.612285	0.592989	0.557450	0.519851	0.491103	0.467770	1.000000	0.812590	0.644645
SB	0.894048	0.793885	0.735454	0.696234	0.696388	0.696247	0.812590	1.000000	0.949876
SP	0.857865	0.834236	0.763273	0.705768	0.673816	0.653841	0.644645	0.949876	1.000000

Table.5.3. Eigenvalues and Eigenvectors of Pearsonian Correlation matrix of Dataset with outliers

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ	2.152533	0.906245	0.905057	0.896032	0.870888	0.753702	0.515544
$\lambda/7$	0.307505	0.129464	0.129294	0.128005	0.124413	0.107672	0.073649
V	V ₁	V ₂	V ₃	V ₄	V ₅	V ₆	V ₇
1	0.538493	0.014697	0.007084	0.003407	-0.068165	0.183408	-0.819428
2	0.409431	0.027749	0.047809	0.038439	-0.347471	-0.833329	0.112517
3	0.319179	0.363108	0.034708	0.214613	0.826841	-0.148375	0.115464
4	0.291645	-0.624408	-0.647375	-0.184998	0.216205	0.007993	0.157895
5	0.308908	-0.470763	0.744400	-0.205451	0.105925	0.155698	0.226176
6	0.341314	0.502643	-0.125485	-0.652742	-0.173306	0.260185	0.302166
7	0.378867	0.054891	-0.086534	0.670780	-0.320860	0.397376	0.367634

Table.5.4. Eigenvalues and Eigenvectors of Brownian Correlation matrix of Dataset with outliers							
	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ	4.724762	0.539186	0.492721	0.427024	0.348319	0.262176	0.205812
$\lambda/7$	0.674966	0.077027	0.070389	0.061003	0.049760	0.037454	0.029402
V	V ₁	V ₂	V ₃	V ₄	V ₅	V ₆	V ₇
1	0.416111	0.030670	-0.045894	-0.079184	0.159636	0.320837	-0.830129
2	0.408857	0.039555	-0.069779	-0.101422	0.229522	0.693142	0.531968
3	0.393810	0.063031	-0.093190	-0.186466	0.662084	-0.586447	0.123332
4	0.375607	0.096218	-0.193867	-0.625877	-0.616283	-0.188480	0.070893
5	0.360742	0.164631	-0.535195	0.691006	-0.240479	-0.135274	0.052057
6	0.347109	0.423596	0.776229	0.236361	-0.169752	-0.100441	0.052719
7	0.336007	-0.881889	0.240396	0.153612	-0.130501	-0.093730	0.046580

6.6. Lévy Distributed Probability Density Function Dataset: This dataset has 7 variables (Lévy-distributed probability density functions with the same location parameter, $\mu=0$, but varying scale parameters described below.



Description: X1=Lévy PDF ($c=0.5$, $\mu=0$); X2=Lévy PDF ($c=0.75$, $\mu=0$); X3=Lévy PDF ($c=1.25$, $\mu=0$); X4=Lévy PDF ($c=1.75$, $\mu=0$); X5=Lévy PDF ($c=2.0$, $\mu=0$); X6=Lévy PDF ($c=2.5$, $\mu=0$); X7=Lévy PDF ($c=3.0$, $\mu=0$). They all have 50 observations each and plotted in the inset of Fig.7 as ‘original variables’ against the observation number (1 through 50) in the horizontal axis. Their Brownian and Pearsonian leading principal component scores as well as the transformed (original variables measured as signed deviations from their respective Brownian average) are plotted in Fig.7. It may be noted that the principal component scores are unique up to scale factor; they mean the one and the same up to multiplication by any nonzero k , negative or positive, since they are derived through using the squared values of correlation coefficients.

Findings: There is some difference between the Brownian and Pearsonian leading PC scores. The coefficients of correlation (r and ρ) between them are (0.885419, 0.955735) respectively (see Table.6.1 and Table.6.2). The details are given in Table.A6 in the appendix. The Brownian PC scores have larger variance, although it cannot be concluded that they represent the variables better than the Pearsonian leading PC score.

The eigenvalues (λ) and the associated eigenvectors (V) of the Pearsonian and the Brownian correlation matrices (of X_1 through X_7) are given in Table.6.3 and Table.6.4. The Pearsonian leading PC scores explain 33.111% of the total variance in the data and it is commensurate with $33.111=100(\lambda_1/7)$. However, the Brownian leading PC scores explain 51.582% of the total variance in the data while $59.32 = 100(\lambda_1/7)$ is considerably larger than that. The possible explanation of this observed discrepancy will be explored in due course.

Table.6.1. Pearsonian Correlation matrix of Lévy PDF Dataset

	X1	X2	X3	X4	X5	X6	X7	SB	SP
X1	1.000000	0.221632	-0.044305	-0.142426	-0.195994	-0.224126	-0.230510	0.727126	0.398615
X2	0.221632	1.000000	0.248500	-0.083527	-0.236534	-0.314331	-0.343052	0.551320	0.631029
X3	-0.044305	0.248500	1.000000	0.281389	-0.112608	-0.311533	-0.402016	0.364388	0.595124
X4	-0.142426	-0.083527	0.281389	1.000000	0.318013	-0.090133	-0.295947	0.045643	0.165073
X5	-0.195994	-0.236534	-0.112608	0.318013	1.000000	0.392853	0.016171	-0.364405	-0.427726
X6	-0.224126	-0.314331	-0.311533	-0.090133	0.392853	1.000000	0.503399	-0.768704	-0.787706
X7	-0.230510	-0.343052	-0.402016	-0.295947	0.016171	0.503399	1.000000	-0.564906	-0.758809
SB	0.727126	0.551320	0.364388	0.045643	-0.364405	-0.768704	-0.564906	1.000000	0.885419
SP	0.398615	0.631029	0.595124	0.165073	-0.427726	-0.787706	-0.758809	0.885419	1.000000

Table.6.2. Brownian Correlation matrix Lévy PDF Dataset

	X1	X2	X3	X4	X5	X6	X7	SB	SP
X1	1.000000	0.543886	0.342699	0.314039	0.341826	0.364742	0.363316	0.717816	0.544026
X2	0.543886	1.000000	0.612421	0.463055	0.467230	0.509518	0.521185	0.723720	0.730972
X3	0.342699	0.612421	1.000000	0.646473	0.525622	0.559687	0.589850	0.665574	0.694461
X4	0.314039	0.463055	0.646473	1.000000	0.648226	0.558311	0.574386	0.578283	0.537635
X5	0.341826	0.467230	0.525622	0.648226	1.000000	0.677774	0.554493	0.645913	0.661594
X6	0.364742	0.509518	0.559687	0.558311	0.677774	1.000000	0.722742	0.909340	0.880468
X7	0.363316	0.521185	0.589850	0.574386	0.554493	0.722742	1.000000	0.741749	0.777842
SB	0.717816	0.723720	0.665574	0.578283	0.645913	0.909340	0.741749	1.000000	0.955735
SP	0.544026	0.730972	0.694461	0.537635	0.661594	0.880468	0.777842	0.955735	1.000000

Table.6.3. Eigenvalues and Eigenvectors of Pearsonian Correlation matrix of Lévy PDF Dataset

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ	2.317735	1.585835	0.937160	0.754292	0.576703	0.484076	0.344198
$\lambda/7$	0.331105	0.226548	0.133880	0.107756	0.082386	0.069154	0.049171
V	V_1	V_2	V_3	V_4	V_5	V_6	V_7
1	0.261831	-0.334383	0.728222	-0.164964	0.479844	0.134440	0.117420
2	0.414493	-0.164932	0.025601	0.785576	-0.229017	0.351670	0.084184
3	0.390909	0.322387	-0.407504	0.148603	0.693660	-0.203329	0.180577
4	0.108429	0.656567	0.111594	-0.232877	-0.025743	0.695058	-0.081911
5	-0.280953	0.504355	0.457731	0.314748	-0.100631	-0.348247	0.476135
6	-0.517407	0.057659	0.109183	0.424339	0.420538	0.096670	-0.592269
7	-0.498426	-0.261290	-0.262842	0.014600	0.220216	0.450759	0.601931

Table.6.4. Eigenvalues and Eigenvectors of Brownian Correlation matrix of Lévy PDF Dataset

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ	4.152402	0.880731	0.572471	0.506443	0.383676	0.281065	0.223213
$\lambda/7$	0.593200	0.125819	0.081782	0.072349	0.054811	0.040152	0.031888
V	V_1	V_2	V_3	V_4	V_5	V_6	V_7
1	0.280995	-0.772653	0.290840	-0.284994	0.346749	-0.193698	0.022027
2	0.371631	-0.433004	-0.316830	0.226435	-0.554737	0.453312	-0.097566
3	0.394281	0.059439	-0.618752	0.136276	0.076403	-0.640461	0.153481
4	0.388326	0.280178	-0.291196	-0.487858	0.402649	0.407779	-0.345676
5	0.388941	0.262628	0.301223	-0.498398	-0.464776	-0.076616	0.467684
6	0.406687	0.202743	0.450506	0.272035	-0.147727	-0.280378	-0.645063
7	0.400009	0.152981	0.231713	0.537146	0.407711	0.309892	0.460556

7. On the Issue of Discrepancy Observed in Brownian Correlation based PC Scores

Throughout the analysis we have found a discrepancy between the explanatory (variance-explaining) performance of the leading Brownian PC and λ_1/m of the Brownian correlation matrix (m being the order of the correlation matrix concerned). The variance-explaining performance of the leading Brownian PC is lower than λ_1/m , although the magnitude of this gap is different for the different datasets.

This discrepancy may be due to (i) our inability to find a global optimum solution (by the Host-Parasite Co-evolutionary algorithm used for obtaining the solution), or (ii) inherent decomposability of the Brownian correlation matrix into (orthogonal) eigenvectors and the associated eigenvalues. The plausibility of the first cause is meagre since this procedure (which directly and iteratively optimizes the sum of squared coefficients of correlation between the PC scores and the variables that they are constituted of) always returns the results identical to those that we obtain by the traditional method (proceeding through singular value decomposition of the correlation matrix among the variables, as suggested by Hotelling, 1933). In other words, *the results of the traditional method of constructing PC scores are identical to the ones obtained through direct global optimization - without proceeding through the singular value decomposition of the Pearsonian correlation matrix among the variables and using the eigenvector as the weight vector on the variables under consideration*. Further, we also find that correlation between the Pearsonian and the Brownian Leading PCs is always very high (greater

than 0.9). These reasons suggest that the second cause is more plausible. It may be noted that in the eigenvalue decomposition of a matrix R , it is presumed that R (Pearsonian correlation matrix) can be decomposed as $R = R_1 + R_2 + \dots + R_m$ where $R_j = \lambda_j V_j V_j'$ or R_j is a product of the j^{th} eigenvalue and the cross product of the (associated) j^{th} eigenvector (V_j) and its transpose (V_j'). Since the eigenvectors are mutually orthogonal to each other ($V_j V_{k; j \neq k} = 0 \forall j, k$), so are their cross product, R_j . Now, since the Brownian correlation is not a product moment of X , but the product moment of the distances among different observations on X , the Brownian principal components scores (of variables) may not preserve the orthogonal decomposability properties as the Pearsonian principal component scores do. We have found that the alternative scores (say, ξ_1) obtained by using the eigenvalues and eigenvectors of the Brownian correlation matrix are suboptimal and do not maximize $\sum_{j=1}^m \rho^2(\xi_1, X_j)$, that is, $\sum_{j=1}^m \rho^2(\xi_1, X_j)$ is far less than $\sum_{j=1}^m \rho^2(Z_1, X_j)$.

8. Leading Principal Component of Transformed variables

The Brownian correlation is inherently nonlinear and so far we have constructed the leading Principal Component scores by linear aggregation. Now, suppose, we suitably transform the basic variables in such a manner that their product moment matrix is equal to the Brownian correlation matrix, then the possible objection to (illegitimate) mixing up of linear aggregation and nonlinear correlation may be largely addressed. In other words, let $U = f(X)$ be transformation of X such that $[f(X)][f(X)]' = [\rho] = \langle U, U \rangle$, where $[\rho]$ is the (symmetric positive semi-definite) Brownian correlation matrix, and ε is the eigenvectors (properly scaled by the corresponding eigenvalues) of $[\rho]$. The principal component scores are, then, $U\varepsilon$.

Such a transformation is possible. Let $[r]$ be the Pearsonian correlation matrix obtained from X . Let $[L]$ and $[E]$ be the eigenvalues of $[r]$. The matrix $[L]$ is diagonal and the eigenvectors in $[E]$ are of unit length. Similarly, $[\lambda]$ and $[\varepsilon]$ be the eigenvalues and eigenvectors of $[\rho]$. Then the transformation $U = X[E][L]^{-0.5}[[\varepsilon][\lambda]^{0.5}]'$ would provide $[\rho] = \langle U, U \rangle$.

The results of such transformations are provided in tables B1 through B6 in the appendix. The Pearsonian correlation matrices of the transformed data are the same as the Brownian correlation of the original data. The leading principal component scores ($U\varepsilon$) obtained by using the Pearsonian correlation matrix derived from U and the leading principal component scores (Xa) obtained through the Pearsonian correlation matrix derived from X are almost identical (that is, $U\varepsilon \cong kXa$; $k \cong 1$).

9. Concluding Remarks

In this study we attempted to construct the leading principal component scores in which the Brownian correlation is used and compared the results of such endeavor with the traditional principal component scores based on time tested method that uses the Pearsonian correlation. The testing was done on six datasets, some of which contain highly nonlinear variables. Although the leading PC scores based on the Brownian correlation explain somewhat larger proportion of total variation in the data, we found that such a modest improvement does not

provide sufficient attractive features so that one should replace the traditional one (based on the Pearsonian correlation) by the new one (based on the Brownian correlation). A suitable transformation of data shows that the Brownian correlation based PC scores with transformed variables and Pearsonian correlation based PC scores with the original data are almost identical. Therefore, the Brownian correlation is not likely to provide any attractive feature in the Principal Component analysis if we approach the problem through eigen-decomposition.

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Appendix

Table.A1. Iris Flower Dataset						
Sl no.	Sepal length	Sepal width	Petal length	Petal width	Score(Brown)	Score(Pearson)
1	5.1	3.5	1.4	0.2	-1.28638	-1.32123
2	4.9	3.0	1.4	0.2	-1.27942	-1.21404
3	4.7	3.2	1.3	0.2	-1.40977	-1.37930
4	4.6	3.1	1.5	0.2	-1.38228	-1.34147
5	5.0	3.6	1.4	0.2	-1.33949	-1.39424
6	5.4	3.9	1.7	0.4	-1.09355	-1.21093
7	4.6	3.4	1.4	0.3	-1.41777	-1.42585
8	5.0	3.4	1.5	0.2	-1.28303	-1.30265
9	4.4	2.9	1.4	0.2	-1.44796	-1.36203
10	4.9	3.1	1.5	0.1	-1.30862	-1.27434
11	5.4	3.7	1.5	0.2	-1.18377	-1.26383
12	4.8	3.4	1.6	0.2	-1.33278	-1.35707
13	4.8	3.0	1.4	0.1	-1.35352	-1.29425
14	4.3	3.0	1.1	0.1	-1.61060	-1.53616
15	5.8	4.0	1.2	0.2	-1.15690	-1.28275
16	5.7	4.4	1.5	0.4	-1.11183	-1.31978
17	5.4	3.9	1.3	0.4	-1.19006	-1.28791
18	5.1	3.5	1.4	0.3	-1.24923	-1.27785
19	5.7	3.8	1.7	0.3	-1.00371	-1.10763
20	5.1	3.8	1.5	0.3	-1.27361	-1.36713
21	5.4	3.4	1.7	0.2	-1.08701	-1.11682
22	5.1	3.7	1.5	0.4	-1.22029	-1.28758
23	4.6	3.6	1.0	0.2	-1.58377	-1.61856
24	5.1	3.3	1.7	0.5	-1.07021	-1.06102
25	4.8	3.4	1.9	0.2	-1.26039	-1.29933
26	5.0	3.0	1.6	0.2	-1.19423	-1.13871
27	5.0	3.4	1.6	0.4	-1.18460	-1.19665
28	5.2	3.5	1.5	0.2	-1.22532	-1.26515
29	5.2	3.4	1.4	0.2	-1.23328	-1.24823
30	4.7	3.2	1.6	0.2	-1.33738	-1.32156
31	4.8	3.1	1.6	0.2	-1.28427	-1.24855
32	5.4	3.4	1.5	0.4	-1.06096	-1.06856
33	5.2	4.1	1.5	0.1	-1.35948	-1.52557
34	5.5	4.2	1.4	0.2	-1.25180	-1.42710
35	4.9	3.1	1.5	0.2	-1.27146	-1.23096
36	5.0	3.2	1.2	0.2	-1.32308	-1.28804
37	5.5	3.5	1.3	0.2	-1.16275	-1.19314
38	4.9	3.6	1.4	0.1	-1.41359	-1.47445
39	4.4	3.0	1.3	0.2	-1.48825	-1.41745
40	5.1	3.4	1.5	0.2	-1.24609	-1.26581
41	5.0	3.5	1.3	0.3	-1.31030	-1.33393
42	4.5	2.3	1.3	0.3	-1.30098	-1.08403

43	4.4	3.2	1.3	0.2	-1.52059	-1.48980
44	5.0	3.5	1.6	0.6	-1.12646	-1.14606
45	5.1	3.8	1.9	0.4	-1.13994	-1.24676
46	4.8	3.0	1.4	0.3	-1.27921	-1.20749
47	5.1	3.8	1.6	0.2	-1.28663	-1.39126
48	4.6	3.2	1.4	0.2	-1.42258	-1.39688
49	5.3	3.7	1.5	0.2	-1.22071	-1.30066
50	5.0	3.3	1.4	0.2	-1.29099	-1.28572
51	7.0	3.2	4.7	1.4	0.70605	0.64278
52	6.4	3.2	4.5	1.5	0.47331	0.42666
53	6.9	3.1	4.9	1.5	0.77069	0.72399
54	5.5	2.3	4.0	1.3	0.09141	0.23773
55	6.5	2.8	4.6	1.5	0.59905	0.62743
56	5.7	2.8	4.5	1.3	0.20509	0.22676
57	6.3	3.3	4.7	1.6	0.50561	0.43553
58	4.9	2.4	3.3	1.0	-0.42675	-0.28431
59	6.6	2.9	4.6	1.3	0.54552	0.54134
60	5.2	2.7	3.9	1.4	-0.07106	0.00667
61	5.0	2.0	3.5	1.0	-0.27688	-0.06429
62	5.9	3.0	4.2	1.5	0.24855	0.25710
63	6.0	2.2	4.0	1.0	0.18082	0.32794
64	6.1	2.9	4.7	1.4	0.42210	0.41979
65	5.6	2.9	3.6	1.3	-0.06517	-0.01946
66	6.7	3.1	4.4	1.4	0.53901	0.51071
67	5.6	3.0	4.5	1.5	0.21012	0.20434
68	5.8	2.7	4.1	1.0	0.05023	0.09265
69	6.2	2.2	4.5	1.5	0.56111	0.71472
70	5.6	2.5	3.9	1.1	-0.00242	0.09621
71	5.9	3.2	4.8	1.8	0.47245	0.43037
72	6.1	2.8	4.0	1.3	0.23221	0.27787
73	6.3	2.5	4.9	1.5	0.64606	0.72002
74	6.1	2.8	4.7	1.2	0.36396	0.36921
75	6.4	2.9	4.3	1.3	0.39925	0.40993
76	6.6	3.0	4.4	1.4	0.51824	0.51005
77	6.8	2.8	4.8	1.4	0.72097	0.73305
78	6.7	3.0	5.0	1.7	0.81141	0.79250
79	6.0	2.9	4.5	1.5	0.37405	0.38785
80	5.7	2.6	3.5	1.0	-0.11531	-0.02349
81	5.5	2.4	3.8	1.1	-0.04732	0.07631
82	5.5	2.4	3.7	1.0	-0.10860	0.01368
83	5.8	2.7	3.9	1.2	0.07628	0.14091
84	6.0	2.7	5.1	1.6	0.58831	0.61905
85	5.4	3.0	4.5	1.5	0.13624	0.13067
86	6.0	3.4	4.5	1.6	0.33036	0.25036
87	6.7	3.1	4.7	1.5	0.64855	0.61183
88	6.3	2.3	4.4	1.3	0.48345	0.60938

89	5.6	3.0	4.1	1.3	0.03930	0.04060
90	5.5	2.5	4.0	1.3	0.05907	0.16538
91	5.5	2.6	4.4	1.2	0.10227	0.16281
92	6.1	3.0	4.6	1.4	0.38180	0.36438
93	5.8	2.6	4.0	1.2	0.11658	0.19633
94	5.0	2.3	3.3	1.0	-0.37365	-0.21130
95	5.6	2.7	4.2	1.3	0.11194	0.16836
96	5.7	3.0	4.2	1.2	0.06322	0.05330
97	5.7	2.9	4.2	1.3	0.11654	0.13285
98	6.2	2.9	4.3	1.3	0.32537	0.33627
99	5.1	2.5	3.0	1.1	-0.40428	-0.26117
100	5.7	2.8	4.1	1.3	0.10858	0.14978
101	6.3	3.3	6.0	2.5	1.15365	1.07613
102	5.8	2.7	5.1	1.9	0.62588	0.67551
103	7.1	3.0	5.9	2.1	1.32494	1.28656
104	6.3	2.9	5.6	1.8	0.86174	0.84019
105	6.5	3.0	5.8	2.2	1.11632	1.08969
106	7.6	3.0	6.6	2.1	1.67854	1.60545
107	4.9	2.5	4.5	1.7	0.10668	0.21412
108	7.3	2.9	6.3	1.8	1.40004	1.34325
109	6.7	2.5	5.8	1.8	1.12243	1.17071
110	7.2	3.6	6.1	2.5	1.46173	1.31836
111	6.5	3.2	5.1	2.0	0.84078	0.79586
112	6.4	2.7	5.3	1.9	0.89578	0.93501
113	6.8	3.0	5.5	2.1	1.11760	1.09907
114	5.7	2.5	5.0	2.0	0.63430	0.73515
115	5.8	2.8	5.1	2.4	0.79548	0.85623
116	6.4	3.2	5.3	2.3	0.96355	0.92765
117	6.5	3.0	5.5	1.8	0.89532	0.85844
118	7.7	3.8	6.7	2.2	1.64741	1.41553
119	7.7	2.6	6.9	2.3	1.92684	1.93147
120	6.0	2.2	5.0	1.5	0.60787	0.73728
121	6.9	3.2	5.7	2.3	1.24476	1.18881
122	5.6	2.8	4.9	2.0	0.52473	0.57056
123	7.7	2.8	6.7	2.0	1.73479	1.69050
124	6.3	2.7	4.9	1.8	0.72518	0.77781
125	6.7	3.3	5.7	2.1	1.08041	0.99221
126	7.2	3.2	6.0	1.8	1.24221	1.14016
127	6.2	2.8	4.8	1.8	0.64794	0.68556
128	6.1	3.0	4.9	1.8	0.60279	0.59562
129	6.4	2.8	5.6	2.1	1.02630	1.04333
130	7.2	3.0	5.8	1.6	1.15199	1.08726
131	7.4	2.8	6.1	1.9	1.44205	1.42114
132	7.9	3.8	6.4	2.0	1.57460	1.34470
133	6.4	2.8	5.6	2.2	1.06346	1.08671
134	6.3	2.8	5.1	1.5	0.64581	0.65000

135	6.1	2.6	5.6	1.4	0.68776	0.70153
136	7.7	3.0	6.1	2.3	1.66914	1.63281
137	6.3	3.4	5.6	2.4	1.00381	0.91959
138	6.4	3.1	5.5	1.8	0.84222	0.78543
139	6.0	3.0	4.8	1.8	0.54172	0.53955
140	6.9	3.1	5.4	2.1	1.11424	1.08049
141	6.7	3.1	5.6	2.4	1.20008	1.17544
142	6.9	3.1	5.1	2.3	1.11616	1.10950
143	5.8	2.7	5.1	1.9	0.62588	0.67551
144	6.8	3.2	5.9	2.3	1.25608	1.19047
145	6.7	3.3	5.7	2.5	1.22902	1.16572
146	6.7	3.0	5.2	2.3	1.08258	1.09126
147	6.3	2.5	5.0	1.9	0.81879	0.91278
148	6.5	3.0	5.2	2.0	0.89724	0.88745
149	6.2	3.4	5.4	2.3	0.88146	0.80089
150	5.9	3.0	5.1	1.8	0.57717	0.56045

Table.A2. Motor Car Dataset															
Sl	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉	X ₁₀	X ₁₁	X ₁₂	X ₁₃	Score(Br.)	Score(Pear)
1	88.6	168.8	64.1	48.8	2548	130	3.47	2.68	9.00	111	5000	21	27	-0.17394	-0.26243
2	88.6	168.8	64.1	48.8	2548	130	3.47	2.68	9.00	111	5000	21	27	-0.17394	-0.26243
3	94.5	171.2	65.5	52.4	2823	152	2.68	3.47	9.00	154	5000	19	26	0.15419	0.11602
4	99.8	176.6	66.2	54.3	2337	109	3.19	3.40	10.00	102	5500	24	30	-0.05990	-0.08275
5	99.4	176.6	66.4	54.3	2824	136	3.19	3.40	8.00	115	5500	18	22	0.48040	0.44286
6	99.8	177.3	66.3	53.1	2507	136	3.19	3.40	8.50	110	5500	19	25	0.31033	0.25098
7	105.8	192.7	71.4	55.7	2844	136	3.19	3.40	8.50	110	5500	19	25	1.02202	0.98971
8	105.8	192.7	71.4	55.7	2954	136	3.19	3.40	8.50	110	5500	19	25	1.05217	1.01947
9	105.8	192.7	71.4	55.9	3086	131	3.13	3.40	8.30	140	5500	17	20	1.24919	1.24719
11	101.2	176.8	64.8	54.3	2395	108	3.50	2.80	8.80	101	5800	23	29	-0.00804	-0.03633
12	101.2	176.8	64.8	54.3	2395	108	3.50	2.80	8.80	101	5800	23	29	-0.00804	-0.03633
13	101.2	176.8	64.8	54.3	2710	164	3.31	3.19	9.00	121	4250	21	28	0.41612	0.41001
14	101.2	176.8	64.8	54.3	2765	164	3.31	3.19	9.00	121	4250	21	28	0.43120	0.42489
15	103.5	189.0	66.9	55.7	3055	164	3.31	3.19	9.00	121	4250	20	25	0.92785	0.91278
16	103.5	189.0	66.9	55.7	3230	209	3.62	3.39	8.00	182	5400	16	22	1.41723	1.47016
17	103.5	193.8	67.9	53.7	3380	209	3.62	3.39	8.00	182	5400	16	22	1.55909	1.58389
18	110.0	197.0	70.9	56.3	3505	209	3.62	3.39	8.00	182	5400	15	20	2.00732	2.07183
19	88.4	141.1	60.3	53.2	1488	61	2.91	3.03	9.50	48	5100	47	53	-2.75293	-2.60594
20	94.5	155.9	63.6	52.0	1874	90	3.03	3.11	9.60	70	5400	38	43	-1.56155	-1.49250
21	94.5	158.8	63.6	52.0	1909	90	3.03	3.11	9.60	70	5400	38	43	-1.51252	-1.45179
22	93.7	157.3	63.8	50.8	1876	90	2.97	3.23	9.41	68	5500	37	41	-1.51745	-1.47569
23	93.7	157.3	63.8	50.8	1876	90	2.97	3.23	9.40	68	5500	31	38	-1.27717	-1.30167
24	93.7	157.3	63.8	50.8	2128	98	3.03	3.39	7.60	102	5500	24	30	-0.77313	-0.78301
25	93.7	157.3	63.8	50.6	1967	90	2.97	3.23	9.40	68	5500	31	38	-1.25410	-1.28105
26	93.7	157.3	63.8	50.6	1989	90	2.97	3.23	9.40	68	5500	31	38	-1.24807	-1.27510
27	93.7	157.3	63.8	50.6	1989	90	2.97	3.23	9.40	68	5500	31	38	-1.24807	-1.27510
28	93.7	157.3	63.8	50.6	2191	98	3.03	3.39	7.60	102	5500	24	30	-0.75775	-0.76997
29	103.3	174.6	64.6	59.8	2535	122	3.34	3.46	8.50	88	5000	24	30	0.06945	0.12306
30	95.9	173.2	66.3	50.2	2811	156	3.60	3.90	7.00	145	5000	19	24	0.52781	0.57434
31	86.6	144.6	63.9	50.8	1713	92	2.91	3.41	9.60	58	4800	49	54	-2.43666	-2.25186
32	86.6	144.6	63.9	50.8	1819	92	2.91	3.41	9.20	76	6000	31	38	-1.61318	-1.60274
33	93.7	150.0	64.0	52.6	1837	79	2.91	3.07	10.10	60	5500	38	42	-1.71020	-1.64719
34	93.7	150.0	64.0	52.6	1940	92	2.91	3.41	9.20	76	6000	30	34	-1.25625	-1.23694
35	93.7	150.0	64.0	52.6	1956	92	2.91	3.41	9.20	76	6000	30	34	-1.25187	-1.23261
36	96.5	163.4	64.0	54.5	2010	92	2.91	3.41	9.20	76	6000	30	34	-0.98625	-0.98140
37	96.5	157.1	63.9	58.3	2024	92	2.92	3.41	9.20	76	6000	30	34	-1.03470	-0.97168
38	96.5	167.5	65.2	53.3	2236	110	3.15	3.58	9.00	86	5800	27	33	-0.54091	-0.54248

39	96.5	167.5	65.2	53.3	2289	110	3.15	3.58	9.00	86	5800	27	33	-0.52638	-0.52814
40	96.5	175.4	65.2	54.1	2304	110	3.15	3.58	9.00	86	5800	27	33	-0.40729	-0.42295
41	96.5	175.4	62.5	54.1	2372	110	3.15	3.58	9.00	86	5800	27	33	-0.53518	-0.57050
42	96.5	175.4	65.2	54.1	2465	110	3.15	3.58	9.00	101	5800	24	28	-0.15942	-0.17945
43	96.5	169.1	66.0	51.0	2293	110	3.15	3.58	9.10	100	5500	25	31	-0.34342	-0.36368
44	94.3	170.7	61.8	53.5	2337	111	3.31	3.23	8.50	78	4800	24	29	-0.45208	-0.50345
47	96.0	172.6	65.2	51.4	2734	119	3.43	3.23	9.20	90	5000	24	29	-0.04120	-0.08108
48	113.0	199.6	69.6	52.8	4066	258	3.63	4.17	8.10	176	4750	15	19	2.37985	2.41705
49	113.0	199.6	69.6	52.8	4066	258	3.63	4.17	8.10	176	4750	15	19	2.37985	2.41705
50	102.0	191.7	70.6	47.8	3950	326	3.54	2.76	11.50	262	5000	13	17	2.46032	2.45538
51	93.1	159.1	64.2	54.1	1890	91	3.03	3.15	9.00	68	5000	30	31	-1.01497	-0.98957
52	93.1	159.1	64.2	54.1	1900	91	3.03	3.15	9.00	68	5000	31	38	-1.16250	-1.13902
53	93.1	159.1	64.2	54.1	1905	91	3.03	3.15	9.00	68	5000	31	38	-1.16113	-1.13767
54	93.1	166.8	64.2	54.1	1945	91	3.03	3.15	9.00	68	5000	31	38	-1.04544	-1.04388
55	93.1	166.8	64.2	54.1	1950	91	3.08	3.15	9.00	68	5000	31	38	-1.02865	-1.02304
60	98.8	177.8	66.5	53.7	2385	122	3.39	3.39	8.60	84	4800	26	32	-0.04151	-0.03510
61	98.8	177.8	66.5	55.5	2410	122	3.39	3.39	8.60	84	4800	26	32	-0.01773	0.00769
62	98.8	177.8	66.5	53.7	2385	122	3.39	3.39	8.60	84	4800	26	32	-0.04151	-0.03510
63	98.8	177.8	66.5	55.5	2410	122	3.39	3.39	8.60	84	4800	26	32	-0.01773	0.00769
64	98.8	177.8	66.5	55.5	2443	122	3.39	3.39	22.70	64	4650	36	42	-0.31207	-0.39959
65	98.8	177.8	66.5	55.5	2425	122	3.39	3.39	8.60	84	4800	26	32	-0.01361	0.01175
66	104.9	175.0	66.1	54.4	2670	140	3.76	3.16	8.00	120	5000	19	27	0.60126	0.63041
67	104.9	175.0	66.1	54.4	2700	134	3.43	3.64	22.00	72	4200	31	39	0.09529	-0.00705
68	110.0	190.9	70.3	56.5	3515	183	3.58	3.64	21.50	123	4350	22	25	1.66577	1.58017
69	110.0	190.9	70.3	58.7	3750	183	3.58	3.64	21.50	123	4350	22	25	1.75087	1.68780
70	106.7	187.5	70.3	54.9	3495	183	3.58	3.64	21.50	123	4350	22	25	1.53925	1.44225
71	115.6	202.6	71.7	56.3	3770	183	3.58	3.64	21.50	123	4350	22	25	2.07028	1.96564
72	115.6	202.6	71.7	56.5	3740	234	3.46	3.10	8.30	155	4750	16	18	2.31539	2.33506
73	96.6	180.3	70.5	50.8	3685	234	3.46	3.10	8.30	155	4750	16	18	1.53429	1.52440
74	120.9	208.1	71.7	56.7	3900	308	3.80	3.35	8.00	184	4500	14	16	3.03552	3.10651
75	112.0	199.2	72.0	55.4	3715	304	3.80	3.35	8.00	184	4500	14	16	2.69391	2.76848
76	102.7	178.4	68.0	54.8	2910	140	3.78	3.12	8.00	175	5000	19	24	0.92468	1.04696
77	93.7	157.3	64.4	50.8	1918	92	2.97	3.23	9.40	68	5500	37	41	-1.46713	-1.42132
78	93.7	157.3	64.4	50.8	1944	92	2.97	3.23	9.40	68	5500	31	38	-1.21958	-1.24025
79	93.7	157.3	64.4	50.8	2004	92	2.97	3.23	9.40	68	5500	31	38	-1.20313	-1.22402
80	93.0	157.3	63.8	50.8	2145	98	3.03	3.39	7.60	102	5500	24	30	-0.78115	-0.79196
81	96.3	173.0	65.4	49.4	2370	110	3.17	3.46	7.50	116	5500	23	30	-0.23589	-0.26859
82	96.3	173.0	65.4	49.4	2328	122	3.35	3.46	8.50	88	5000	25	32	-0.25259	-0.29817
83	95.9	173.2	66.3	50.2	2833	156	3.58	3.86	7.00	145	5000	19	24	0.52629	0.56973
84	95.9	173.2	66.3	50.2	2921	156	3.59	3.86	7.00	145	5000	19	24	0.55349	0.59744
85	95.9	173.2	66.3	50.2	2926	156	3.59	3.86	7.00	145	5000	19	24	0.55486	0.59879
86	96.3	172.4	65.4	51.6	2365	122	3.35	3.46	8.50	88	5000	25	32	-0.22992	-0.25059
87	96.3	172.4	65.4	51.6	2405	122	3.35	3.46	8.50	88	5000	25	32	-0.21896	-0.23976
88	96.3	172.4	65.4	51.6	2403	110	3.17	3.46	7.50	116	5500	23	30	-0.21431	-0.22209
89	96.3	172.4	65.4	51.6	2403	110	3.17	3.46	7.50	116	5500	23	30	-0.21431	-0.22209
90	94.5	165.3	63.8	54.5	1889	97	3.15	3.29	9.40	69	5200	31	37	-0.99982	-0.98214
91	94.5	165.3	63.8	54.5	2017	103	2.99	3.47	21.90	55	4800	45	50	-1.46867	-1.49507
92	94.5	165.3	63.8	54.5	1918	97	3.15	3.29	9.40	69	5200	31	37	-0.99187	-0.97430
93	94.5	165.3	63.8	54.5	1938	97	3.15	3.29	9.40	69	5200	31	37	-0.98639	-0.96888
94	94.5	170.2	63.8	53.5	2024	97	3.15	3.29	9.40	69	5200	31	37	-0.90557	-0.91283
95	94.5	165.3	63.8	54.5	1951	97	3.15	3.29	9.40	69	5200	31	37	-0.98282	-0.96537
96	94.5	165.6	63.8	53.3	2028	97	3.15	3.29	9.40	69	5200	31	37	-0.96892	-0.96532
97	94.5	165.3	63.8	54.5	1971	97	3.15	3.29	9.40	69	5200	31	37	-0.97734	-0.95995
98	94.5	170.2	63.8	53.5	2037	97	3.15	3.29	9.40	69	5200	31	37	-0.90201	-0.90932
99	95.1	162.4	63.8	53.3	2008	97	3.15	3.29	9.40	69	5200	31	37	-1.00706	-0.99360
100	97.2	173.4	65.2	54.7	2324	120	3.33	3.47	8.50	97	5200	27	34	-0.29921	-0.26091
101	97.2	173.4	65.2	54.7	2302	120	3.33	3.47	8.50	97	5200	27	34	-0.30524	-0.26687
102	100.4	181.7	66.5	55.1	3095	181	3.43	3.27	9.00	152	5200	17	22	0.99136	0.99365
103	100.4	184.6	66.5	56.1	3296	181	3.43	3.27	9.00	152	5200	17	22	1.09530	1.09930
104	100.4	184.6	66.5	55.1	3060	181	3.43	3.27	9.00	152	5200	19	25	0.90716	0.91952
105	91.3	170.7	67.9	49.7	3071	181	3.43	3.27	9.00	160	5200	19	25	0.59446	0.59946
106	91.3	170.7	67.9	49.7	3139	181	3.43	3.27	7.80	200	5200	17	23	0.75720	0.81744
107	99.2	178.5	67.9	49.7	3139	181	3.43	3.27	9.00	160	5200	19	25	0.86222	0.85477

108	107.9	186.7	68.4	56.7	3020	120	3.46	3.19	8.40	97	5000	19	24	0.92134	0.90631
109	107.9	186.7	68.4	56.7	3197	152	3.70	3.52	21.00	95	4150	28	33	0.94989	0.89724
110	114.2	198.9	68.4	58.7	3230	120	3.46	3.19	8.40	97	5000	19	24	1.27771	1.25652
111	114.2	198.9	68.4	58.7	3430	152	3.70	3.52	21.00	95	4150	25	25	1.54298	1.46385
112	107.9	186.7	68.4	56.7	3075	120	3.46	2.19	8.40	95	5000	19	24	0.89855	0.84578
113	107.9	186.7	68.4	56.7	3252	152	3.70	3.52	21.00	95	4150	28	33	0.96497	0.91213
114	114.2	198.9	68.4	56.7	3285	120	3.46	2.19	8.40	95	5000	19	24	1.23610	1.15596
115	114.2	198.9	68.4	58.7	3485	152	3.70	3.52	21.00	95	4150	25	25	1.55805	1.47873
116	107.9	186.7	68.4	56.7	3075	120	3.46	3.19	8.40	97	5000	19	24	0.93642	0.92119
117	107.9	186.7	68.4	56.7	3252	152	3.70	3.52	21.00	95	4150	28	33	0.96497	0.91213
118	108.0	186.7	68.3	56.0	3130	134	3.61	3.21	7.00	142	5600	18	24	1.08234	1.13327
119	93.7	157.3	63.8	50.8	1918	90	2.97	3.23	9.40	68	5500	37	41	-1.50608	-1.46434
120	93.7	157.3	63.8	50.8	2128	98	3.03	3.39	7.60	102	5500	24	30	-0.77313	-0.78301
121	93.7	157.3	63.8	50.6	1967	90	2.97	3.23	9.40	68	5500	31	38	-1.25410	-1.28105
122	93.7	167.3	63.8	50.8	1989	90	2.97	3.23	9.40	68	5500	31	38	-1.11018	-1.16335
123	93.7	167.3	63.8	50.8	2191	98	2.97	3.23	9.40	68	5500	31	38	-1.02927	-1.08413
124	103.3	174.6	64.6	59.8	2535	122	3.35	3.46	8.50	88	5000	24	30	0.07254	0.12696
125	95.9	173.2	66.3	50.2	2818	156	3.59	3.86	7.00	145	5000	19	24	0.52526	0.56957
126	94.5	168.9	68.3	50.2	2778	151	3.94	3.11	9.50	143	5500	19	27	0.55876	0.58047
127	89.5	168.9	65.0	51.6	2756	194	3.74	2.90	9.50	207	5900	17	25	0.54204	0.58841
128	89.5	168.9	65.0	51.6	2756	194	3.74	2.90	9.50	207	5900	17	25	0.54204	0.58841
129	89.5	168.9	65.0	51.6	2800	194	3.74	2.90	9.50	207	5900	17	25	0.55410	0.60032
133	99.1	186.6	66.5	56.1	2658	121	3.54	3.07	9.31	110	5250	21	28	0.46531	0.44228
134	99.1	186.6	66.5	56.1	2695	121	3.54	3.07	9.30	110	5250	21	28	0.47531	0.45228
135	99.1	186.6	66.5	56.1	2707	121	2.54	2.07	9.30	110	5250	21	28	0.13552	-0.00345
136	99.1	186.6	66.5	56.1	2758	121	3.54	3.07	9.30	110	5250	21	28	0.49258	0.46932
137	99.1	186.6	66.5	56.1	2808	121	3.54	3.07	9.00	160	5500	19	26	0.66533	0.69607
138	99.1	186.6	66.5	56.1	2847	121	3.54	3.07	9.00	160	5500	19	26	0.67602	0.70662
139	93.7	156.9	63.4	53.7	2050	97	3.62	2.36	9.00	69	4900	31	36	-0.97261	-0.92551
140	93.7	157.9	63.6	53.7	2120	108	3.62	2.64	8.70	73	4400	26	31	-0.61069	-0.58859
141	93.3	157.3	63.8	55.7	2240	108	3.62	2.64	8.70	73	4400	26	31	-0.56354	-0.51800
142	97.2	172.0	65.4	52.5	2145	108	3.62	2.64	9.50	82	4800	32	37	-0.55031	-0.50726
143	97.2	172.0	65.4	52.5	2190	108	3.62	2.64	9.50	82	4400	28	33	-0.32087	-0.31128
144	97.2	172.0	65.4	52.5	2340	108	3.62	2.64	9.00	94	5200	26	32	-0.23337	-0.23586
145	97.0	172.0	65.4	54.3	2385	108	3.62	2.64	9.00	82	4800	24	25	-0.02231	-0.02723
146	97.0	172.0	65.4	54.3	2510	108	3.62	2.64	7.70	111	4800	24	29	-0.02806	0.01912
147	97.0	173.5	65.4	53.0	2290	108	3.62	2.64	9.00	82	4800	28	32	-0.28523	-0.27355
148	97.0	173.5	65.4	53.0	2455	108	3.62	2.64	9.00	94	5200	25	31	-0.13182	-0.14396
149	96.9	173.6	65.4	54.9	2420	108	3.62	2.64	9.00	82	4800	23	29	-0.02334	-0.04670
150	96.9	173.6	65.4	54.9	2650	108	3.62	2.64	7.70	111	4800	23	23	0.16923	0.21753
151	95.7	158.7	63.6	54.5	1985	92	3.05	3.03	9.00	62	4800	35	39	-1.26272	-1.19711
152	95.7	158.7	63.6	54.5	2040	92	3.05	3.03	9.00	62	4800	31	38	-1.10431	-1.08515
153	95.7	158.7	63.6	54.5	2015	92	3.05	3.03	9.00	62	4800	31	38	-1.11117	-1.09191
154	95.7	169.7	63.6	59.1	2280	92	3.05	3.03	9.00	62	4800	31	37	-0.82870	-0.79066
155	95.7	169.7	63.6	59.1	2290	92	3.05	3.03	9.00	62	4800	27	32	-0.61481	-0.61509
156	95.7	169.7	63.6	59.1	3110	92	3.05	3.03	9.00	62	4800	27	32	-0.39004	-0.39321
157	95.7	166.3	64.4	53.0	2081	98	3.19	3.03	9.00	70	4800	30	37	-0.83661	-0.83670
158	95.7	166.3	64.4	52.8	2109	98	3.19	3.03	9.00	70	4800	30	37	-0.83081	-0.83313
159	95.7	166.3	64.4	53.0	2275	110	3.27	3.35	22.50	56	4500	34	36	-0.62717	-0.75563
160	95.7	166.3	64.4	52.8	2275	110	3.27	3.35	22.50	56	4500	38	47	-0.94192	-1.04617
161	95.7	166.3	64.4	53.0	2094	98	3.19	3.03	9.00	70	4800	38	47	-1.25534	-1.17890
162	95.7	166.3	64.4	52.8	2122	98	3.19	3.03	9.00	70	4800	28	34	-0.71320	-0.73370
163	95.7	166.3	64.4	52.8	2140	98	3.19	3.03	9.00	70	4800	28	34	-0.70827	-0.72883
164	94.5	168.7	64.0	52.6	2169	98	3.19	3.03	9.00	70	4800	29	34	-0.74458	-0.76647
165	94.5	168.7	64.0	52.6	2204	98	3.19	3.03	9.00	70	4800	29	34	-0.73499	-0.75700
166	94.5	168.7	64.0	52.6	2265	98	3.24	3.08	9.40	112	6600	26	29	-0.55107	-0.56765
167	94.5	168.7	64.0	52.6	2300	98	3.24	3.08	9.40	112	6600	26	29	-0.54147	-0.55818
168	98.4	176.2	65.6	52.0	2540	146	3.62	3.50	9.30	116	4800	24	30	0.21746	0.24081
169	98.4	176.2	65.6	52.0	2536	146	3.62	3.50	9.30	116	4800	24	30	0.21636	0.23973
170	98.4	176.2	65.6	52.0	2551	146	3.62	3.50	9.30	116	4800	24	30	0.22047	0.24378
171	98.4	176.2	65.6	52.0	2679	146	3.62	3.50	9.30	116	4800	24	30	0.25556	0.27842
172	98.4	176.2	65.6	52.0	2714	146	3.62	3.50	9.30	116	4800	24	30	0.26515	0.28789
173	98.4	176.2	65.6	53.0	2975	146	3.62	3.50	9.30	116	4800	24	30	0.34610	0.37853

174	102.4	175.6	66.5	54.9	2326	122	3.31	3.54	8.70	92	4200	29	34	-0.11058	-0.02856
175	102.4	175.6	66.5	54.9	2480	110	3.27	3.35	22.50	73	4500	30	33	0.01331	-0.11541
176	102.4	175.6	66.5	53.9	2414	122	3.31	3.54	8.70	92	4200	27	32	0.00123	0.05219
177	102.4	175.6	66.5	54.9	2414	122	3.31	3.54	8.70	92	4200	27	32	0.01063	0.07221
178	102.4	175.6	66.5	53.9	2458	122	3.31	3.54	8.70	92	4200	27	32	0.01329	0.06410
179	102.9	183.5	67.7	52.0	2976	171	3.27	3.35	9.30	161	5200	20	24	0.87615	0.88528
180	102.9	183.5	67.7	52.0	3016	171	3.27	3.35	9.30	161	5200	19	24	0.91870	0.91564
181	104.5	187.8	66.5	54.1	3131	171	3.27	3.35	9.20	156	5200	20	24	0.95119	0.95712
182	104.5	187.8	66.5	54.1	3151	161	3.27	3.35	9.20	156	5200	19	24	0.95633	0.95136
183	97.3	171.7	65.5	55.7	2261	97	3.01	3.40	23.00	52	4800	37	46	-0.84414	-0.96862
184	97.3	171.7	65.5	55.7	2209	109	3.19	3.40	9.00	85	5250	27	34	-0.42201	-0.40347
185	97.3	171.7	65.5	55.7	2264	97	3.01	3.40	23.00	52	4800	37	46	-0.84331	-0.96781
186	97.3	171.7	65.5	55.7	2212	109	3.19	3.40	9.00	85	5250	27	34	-0.42119	-0.40266
187	97.3	171.7	65.5	55.7	2275	109	3.19	3.40	9.00	85	5250	27	34	-0.40392	-0.38561
188	97.3	171.7	65.5	55.7	2319	97	3.01	3.40	23.00	68	4500	37	42	-0.71743	-0.80501
189	97.3	171.7	65.5	55.7	2300	109	3.19	3.40	10.00	100	5500	26	32	-0.30709	-0.29211
190	94.5	159.3	64.2	55.6	2254	109	3.19	3.40	8.50	90	5500	24	29	-0.53451	-0.51149
191	94.5	165.7	64.0	51.4	2221	109	3.19	3.40	8.50	90	5500	24	29	-0.50686	-0.54783
192	100.4	180.2	66.9	55.1	2661	136	3.19	3.40	8.50	110	5500	19	24	0.47118	0.43137
193	100.4	180.2	66.9	55.1	2579	97	3.01	3.40	23.00	68	4500	33	38	-0.20990	-0.35514
194	100.4	183.1	66.9	55.1	2563	109	3.19	3.40	9.00	88	5500	25	31	0.06111	0.03572
195	104.3	188.8	67.2	56.2	2912	141	3.78	3.15	9.50	114	5400	23	28	0.77604	0.80326
196	104.3	188.8	67.2	57.5	3034	141	3.78	3.15	9.50	114	5400	23	28	0.82171	0.86229
197	104.3	188.8	67.2	56.2	2935	141	3.78	3.15	9.50	114	5400	24	28	0.75075	0.78995
198	104.3	188.8	67.2	57.5	3042	141	3.78	3.15	9.50	114	5400	24	28	0.79230	0.84493
199	104.3	188.8	67.2	56.2	3045	130	3.62	3.15	7.50	162	5100	17	22	1.08472	1.14278
200	104.3	188.8	67.2	57.5	3157	130	3.62	3.15	7.50	162	5100	17	22	1.12765	1.19911
201	109.1	188.8	68.9	55.5	2952	141	3.78	3.15	9.50	114	5400	23	28	0.95958	0.99744
202	109.1	188.8	68.8	55.5	3049	141	3.78	3.15	8.70	160	5300	19	25	1.22622	1.30180
203	109.1	188.8	68.9	55.5	3012	173	3.58	2.87	8.80	134	5500	18	23	1.26588	1.26068
204	109.1	188.8	68.9	55.5	3217	145	3.01	3.40	23.00	106	4800	26	27	0.95680	0.79760
205	109.1	188.8	68.9	55.5	3062	141	3.78	3.15	9.50	114	5400	19	25	1.16697	1.16217

Table.A3. Sinusoidal Dataset											
Sl	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉	Score(Brown)	Score(Pearson)
1	2.0837	4.2353	4.3292	4.0904	4.1422	3.9829	4.7411	4.5986	3.9351	-0.337441	-0.311286
2	1.8541	4.1915	3.2582	4.1899	4.3320	3.8189	4.3377	4.4643	3.8473	-0.538507	-0.218996
3	1.9994	3.6198	4.3375	4.1483	3.7891	4.6002	3.3976	3.7702	4.6008	0.061252	-0.334053
4	1.3837	3.3781	4.3133	4.7420	3.7048	4.4836	3.4238	3.4706	3.8691	0.364762	0.407034
5	1.9511	4.5662	4.0955	4.3636	4.3368	4.1238	3.5894	4.8252	4.0103	0.609176	0.510047
6	2.5612	4.5431	4.6683	4.5552	3.4883	3.7691	3.9627	4.1986	3.8160	1.352704	1.618107
7	2.1086	4.6512	4.1957	4.7175	3.7525	4.0692	4.0314	3.5325	3.7539	1.453773	1.677951
8	1.7348	4.0675	3.5432	4.3068	4.1192	3.8883	3.3629	3.9518	4.2416	-0.033888	0.080931
9	1.5065	3.7316	3.9810	4.3329	3.2210	3.7644	4.1856	3.5475	4.0060	-0.610669	-0.214704
10	1.2573	3.3584	3.7540	3.7481	3.1882	4.1226	4.3259	4.5405	4.2041	-1.990575	-1.923801
11	1.7246	4.1046	3.6241	4.6056	3.6256	3.7500	4.1309	4.1175	4.0622	-0.407947	0.130207
12	2.6322	4.7450	3.7855	4.1694	3.4801	4.3316	3.8871	3.6584	4.1042	1.441995	1.381882
13	2.0621	4.3156	4.3592	3.8445	4.0382	4.5804	3.4367	3.8129	4.0146	1.184961	0.563772
14	1.8791	4.2367	3.2403	4.6382	3.6257	4.0495	4.0247	4.8212	3.6610	0.050395	0.609045
15	1.5112	3.4910	3.8038	3.9307	4.1306	3.9202	4.2733	4.8377	4.4899	-2.072868	-2.019629
16	1.6518	3.5881	4.3199	4.6517	4.4673	4.6639	4.7464	3.5748	4.0936	-0.406312	-0.457361
17	1.7610	3.9197	3.4747	3.9237	3.8715	4.0219	3.4690	3.8142	4.1663	-0.173838	-0.220847
18	2.0136	4.4562	4.4243	4.4978	4.0509	4.7200	3.7309	3.7082	3.9769	1.469388	1.128241
19	2.2197	4.5856	4.4887	4.2467	4.3618	4.3798	3.6868	4.1732	4.1462	1.154814	0.762270
20	2.5207	3.6700	3.4058	4.8536	3.9622	3.9076	4.0618	3.7211	4.0655	0.168860	0.844616
21	1.3715	3.6554	3.8140	4.1879	4.1889	3.9913	4.2248	3.8971	4.3366	-1.289759	-1.213762
22	1.9048	3.6063	4.0161	4.6047	4.2792	4.2978	3.8869	3.3820	3.9040	0.405438	0.502145
23	1.5089	4.1864	4.2004	4.0401	4.2139	4.4783	3.7998	4.6189	4.7220	-0.774427	-1.235220
24	2.4648	3.9409	3.7769	3.9755	3.9379	4.2695	4.0705	4.8825	4.1152	-0.199345	-0.232100

25	2.6104	4.0564	4.4911	4.0244	4.6120	4.5664	4.0326	3.7293	4.4612	0.640811	0.102681
26	2.2400	3.9058	3.4525	4.2232	3.6707	4.2454	3.9085	3.2905	3.7562	0.750211	0.915919
27	2.3430	4.3348	3.9327	4.1118	3.9066	4.6888	3.6346	3.5922	3.8081	1.596170	1.251684
28	1.8607	3.1990	4.1295	3.8114	4.0561	3.9393	4.1108	4.7483	4.6985	-2.028398	-2.099501
29	2.1214	3.2734	3.9152	3.9105	3.8588	4.3526	3.6619	4.3791	3.8977	-0.248547	-0.370087
30	2.2698	3.8770	3.3021	4.2093	3.7168	4.6214	4.2444	3.2473	4.2505	0.151837	0.173487
31	2.1173	4.0114	4.0465	4.1677	3.2163	3.9461	4.2888	3.6079	4.3656	-0.286422	-0.088996
32	2.0616	3.8351	4.0729	4.6203	3.2402	4.0584	3.3906	4.1492	4.4709	0.049310	0.317923
33	1.6516	3.7818	4.0430	4.6904	4.1050	3.7441	3.6889	3.9700	3.7097	0.183634	0.578401
34	1.3956	4.0700	3.7404	3.9725	3.7471	4.1872	4.4494	4.0215	4.7488	-1.553858	-1.639446
35	1.7059	3.3272	4.5193	4.0814	3.7378	4.0890	4.1434	3.8772	3.9771	-0.615917	-0.648026
36	1.4636	4.1047	3.2159	4.5279	3.7705	4.0785	3.9561	3.8587	4.1870	-0.495547	-0.141531
37	2.1891	4.3371	3.8465	4.3971	4.2060	3.9199	3.3812	4.5791	3.6793	1.016687	1.151814
38	2.4384	4.8442	3.8776	4.3019	3.9639	3.9791	3.3013	4.0922	3.9668	1.651053	1.630686
39	2.4249	3.6340	3.8810	4.3401	3.9354	3.8813	3.8557	3.4267	4.0039	0.377956	0.643057
40	1.3428	3.8686	4.4143	4.0009	4.7477	4.4126	4.8444	4.0901	4.7466	-1.712407	-2.145952
41	1.4547	3.1468	4.0471	4.5406	4.8227	4.5071	3.9225	4.6013	3.9690	-0.895527	-0.948567
42	1.6920	3.7372	3.6634	3.8872	4.3905	3.8158	3.8826	4.7977	4.0939	-1.109418	-1.085991
43	1.7749	3.9428	3.9479	4.3180	4.4757	4.3852	3.5086	3.4772	4.3180	0.330109	0.070397
44	2.4863	4.7421	4.6521	4.0632	4.6023	4.0975	4.2879	3.1249	3.6476	1.898624	1.547493
45	2.5014	4.1742	3.5020	4.3939	4.0202	4.1966	4.1206	4.1394	4.3147	0.139512	0.339895
46	1.5804	4.2400	4.0922	4.0834	4.5901	4.4423	3.5738	4.8312	3.9163	0.201991	-0.209040
47	1.5751	3.7389	4.7221	4.1356	3.9224	4.1362	3.9769	4.2264	4.3136	-0.620030	-0.836694
48	2.2233	4.1546	4.4104	3.8274	3.7488	4.5009	3.8050	3.9350	4.1471	0.673622	0.200528
49	1.9518	3.6631	3.2961	4.1534	3.6438	3.9252	4.6823	3.6540	4.4967	-1.447854	-1.050840
50	1.8075	4.4237	4.4625	4.5947	3.3065	4.3341	3.9091	4.3481	4.1868	0.470457	0.506216

Table.A4. Crime Incidents in India – 2011 Dataset

State/UT	Murder (Sec. 302 IPC)	Attempt to Commit Murder (Sec. 307 IPC)	C.H. Not Amount- ing to Murder (Sec. 304, 308 IPC)	Rape (Sec. 376 IPC)	Kidnapping & Abduction (Sec. 363- 310, 371- 373 IPC)	Dacoity (Sec.395- 398 IPC)	Preparation & Assembly For Dacoity (Sec.399- 402 IPC)	Score (Brown)	Score (Pearson)
1.Andhra Pr.	2808	2229	171	1442	2154	126	7	0.749764	0.794320
2.Arunachal Pr.	65	29	2	42	93	13	0	-0.735533	-0.777157
3.Assam	1303	504	48	1700	3764	305	4	0.367099	0.529381
4.Bihar	3198	3327	348	934	4268	556	105	1.626906	1.712828
5.Chhattisgarh	1110	747	28	1053	472	68	7	-0.198115	-0.154705
6.Goa	48	22	6	29	28	2	0	-0.748896	-0.797532
7.Gujarat	1126	478	43	439	1614	221	24	-0.070253	-0.055458
8.Haryana	1062	851	60	733	959	167	176	0.020274	0.042718
9.Himachal P.	130	50	6	168	212	1	0	-0.694342	-0.731526
10.J.&Kashmir	169	494	29	277	1077	14	0	-0.514322	-0.509559
11.Jharkhand	1747	718	83	784	941	309	40	0.192686	0.207980
12.Karnataka	1820	1837	85	636	1395	214	399	0.523504	0.552124
13.Kerala	365	521	105	1132	299	71	245	-0.130657	-0.131914
14.Madhya Pr.	2511	2340	139	3406	1288	118	117	0.887699	1.116751
15.Maharashtra	2818	2105	144	1701	1669	773	291	1.276130	1.485005
16.Manipur	78	245	4	53	169	1	154	-0.605028	-0.648949
17.Meghalaya	170	51	3	130	87	49	0	-0.676070	-0.705606
18.Mizoram	26	24	8	77	6	1	0	-0.747646	-0.792724
19.Nagaland	46	43	11	23	34	7	0	-0.737980	-0.787135
20.Odisha	1477	1621	51	1112	1139	417	84	0.327749	0.502767
21.Punjab	842	997	112	479	681	28	143	-0.110480	-0.155850
22.Rajasthan	1461	1566	100	1800	3204	28	72	0.411694	0.535415
23.Sikkim	14	7	8	16	10	0	0	-0.759842	-0.810890

24.Tamil Nadu	1877	2962	28	677	1984	101	11	0.295429	0.466029
25.Tripura	163	75	0	205	154	11	0	-0.686939	-0.715228
26.Uttar Pr.	4951	4653	1454	2042	8500	379	39	3.973624	3.473496
27.Uttarakhand	178	189	54	129	314	13	1	-0.594947	-0.653215
28.West Bengal	2109	2242	486	2363	4285	236	939	2.007857	1.930806
29.A&N Islands	14	6	2	13	15	1	0	-0.767162	-0.814402
30.Chandigarh	24	40	6	27	58	6	2	-0.746446	-0.791264
31.D&N Haveli	14	2	0	4	9	7	0	-0.768208	-0.813733
32.Daman&Diu	6	1	0	1	3	4	0	-0.772847	-0.819133
33.Delhi	543	386	71	572	3767	33	25	-0.077412	-0.065369
34.Lakshadweep	0	1	0	0	0	0	0	-0.777023	-0.824161
35.Puducherry	32	22	12	7	12	5	10	-0.740266	-0.794110
Source: National Crime Records Bureau, Ministry of Home Affairs, Govt. of India (http://ncrb.nic.in/CD-CII2011/cii-2011/Table%203.1.pdf)									

Table.A5. Dataset with Outliers

Sl	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	Score (Brown)	Score (Pearson)
1	1.077682	1.140908	1.208296	1.151883	0.822192	0.719490	0.983262	-1.228140	-1.588607
2	1.923077	1.960607	2.226456	2.064075	2.156102	1.499589	2.572422	-1.168314	-1.515159
3	2.962812	2.829700	3.264841	3.159331	3.180563	3.304192	3.687762	-1.103764	-1.432080
4	4.080219	4.049193	4.255646	4.326654	3.500006	3.490373	3.570235	-1.060687	-1.357769
5	49.411562	5.081494	4.893908	5.136103	4.832139	5.258435	4.505863	0.183137	0.163046
6	6.008522	5.842828	5.809820	5.803793	6.141950	6.424315	6.008884	-0.936988	-1.203804
7	7.041500	6.817300	6.944475	7.044302	6.599157	6.589016	7.422431	-0.879339	-1.127030
8	8.011244	8.141910	8.036765	7.671292	8.035854	8.502820	7.912820	-0.819698	-1.041212
9	8.944017	9.117921	9.079594	9.334909	8.894629	9.480762	9.496862	-0.755880	-0.959126
10	9.922213	98.398512	9.922357	9.824893	10.208457	9.663454	10.018876	-0.212391	0.565465
11	11.021235	10.850046	10.744802	11.283774	10.635725	11.326977	11.170139	-0.645498	-0.810919
12	12.031965	12.182981	11.750493	11.901474	12.011181	12.594397	11.498643	-0.590484	-0.728639
13	13.092702	12.939893	12.746827	12.795517	13.291158	12.972235	12.468223	-0.534694	-0.654560
14	13.917489	14.077809	13.899319	13.931400	14.223924	13.951558	14.087580	-0.473131	-0.574977
15	15.097872	14.995657	14.989473	15.149472	14.965230	15.175330	14.348062	-0.080067	0.703821
16	16.060008	15.803115	15.813045	16.030555	16.200695	15.875041	15.391044	-0.362741	-0.421715
17	17.055419	17.070173	17.271701	17.126990	17.077891	16.969294	16.917957	-0.295905	-0.332128
18	18.036488	17.925449	18.217657	17.917049	17.600922	17.946274	18.584179	-0.234437	-0.257982
19	18.911146	19.022213	19.276058	19.211489	19.273300	18.893385	19.342020	-0.178161	-0.177531
20	19.906232	20.063513	19.952000	20.296821	20.438697	19.711310	19.381258	0.315949	0.998294
21	21.004311	21.043895	21.034252	21.030676	21.315756	20.841754	20.816167	-0.066159	-0.022824
22	22.051297	22.097930	21.888662	21.817636	22.451371	22.490388	22.098432	-0.000058	0.060622
23	23.050831	22.938382	22.939871	22.726158	23.135141	23.220819	22.438843	0.046916	0.130427
24	24.065438	23.807062	23.742039	23.977633	24.035376	23.860401	23.904998	0.107762	0.206821
25	25.006791	25.170981	24.815775	25.000573	24.932705	24.556066	24.377850	1.157007	1.454063
26	26.008554	25.985657	26.049708	25.918773	25.802752	26.403516	25.432177	0.220856	0.367000
27	26.946083	26.926664	26.922245	26.812549	27.137024	26.973472	27.314238	0.285576	0.443887
28	27.948304	27.967381	27.735514	28.322362	28.173216	28.196977	27.404016	0.336006	0.521695
29	29.099590	28.839757	29.216105	29.378184	29.298987	28.957820	29.540645	0.410901	0.612222
30	30.022379	29.830491	30.127099	29.891538	29.751847	29.333455	29.816559	1.867513	1.964431
31	31.047042	30.964575	30.995462	30.852886	30.564621	31.161013	30.753473	0.513778	0.758011
32	32.074665	31.858398	32.068824	32.223393	31.552488	31.889945	31.847920	0.572783	0.837578
33	33.037495	32.948731	33.029843	33.269463	32.813184	33.046659	32.497468	0.628593	0.916808
34	34.002371	34.185251	34.171760	34.078895	34.363046	34.016002	33.538122	0.689824	1.001054
35	35.010063	34.838184	34.994360	35.161942	35.158680	35.264930	35.069607	4.289935	2.501275

Table.A6. Lévy Distributed Probability Density Function Dataset

Sl	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	Score (Brown)	Score (Pearson)
1	0.5906	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	4.178488	1.695382
2	0.1591	0.2938	0.0000	0.0000	0.0000	0.0000	0.0000	2.589793	2.421982
3	0.0645	0.1855	0.0000	0.0000	0.0000	0.0000	0.0000	1.467566	1.646117
4	0.0360	0.1054	0.1448	0.0000	0.0000	0.0000	0.0000	1.602804	2.298040

5	0.0237	0.0688	0.1425	0.0000	0.0000	0.0000	0.0000	1.345085	2.057081
6	0.0168	0.0492	0.1024	0.0000	0.0000	0.0000	0.0000	1.020808	1.620680
7	0.0128	0.0369	0.0771	0.0879	0.0000	0.0000	0.0000	0.998423	1.603635
8	0.0100	0.0292	0.0610	0.1044	0.0000	0.0000	0.0000	0.902487	1.477564
9	0.0082	0.0239	0.0496	0.0856	0.0000	0.0000	0.0000	0.774206	1.300282
10	0.0068	0.0195	0.0411	0.0713	0.0115	0.0000	0.0000	0.676237	1.061663
11	0.0056	0.0167	0.0352	0.0597	0.0827	0.0000	0.0000	0.608008	0.325490
12	0.0049	0.0143	0.0298	0.0520	0.0793	0.0000	0.0000	0.551036	0.276416
13	0.0044	0.0123	0.0260	0.0447	0.0696	0.0000	0.0000	0.505242	0.300167
14	0.0038	0.0108	0.0229	0.0394	0.0606	0.0146	0.0000	0.076050	0.070464
15	0.0033	0.0096	0.0199	0.0347	0.0540	0.0711	0.0000	-1.475440	-0.925603
16	0.0031	0.0088	0.0182	0.0312	0.0482	0.0688	0.0000	-1.434086	-0.860886
17	0.0027	0.0078	0.0164	0.0283	0.0435	0.0621	0.0000	-1.275902	-0.727867
18	0.0024	0.0071	0.0150	0.0253	0.0397	0.0567	0.0218	-1.179105	-0.998228
19	0.0023	0.0067	0.0135	0.0235	0.0359	0.0514	0.0665	-1.112057	-1.661399
20	0.0021	0.0060	0.0124	0.0214	0.0330	0.0470	0.0633	-1.003513	-1.520444
21	0.0019	0.0054	0.0114	0.0194	0.0303	0.0434	0.0587	-0.913393	-1.369768
22	0.0018	0.0049	0.0104	0.0184	0.0281	0.0400	0.0545	-0.826010	-1.230395
23	0.0016	0.0047	0.0096	0.0169	0.0260	0.0372	0.0503	-0.754151	-1.101107
24	0.0015	0.0044	0.0090	0.0158	0.0238	0.0346	0.0468	-0.686653	-0.984189
25	0.0014	0.0041	0.0084	0.0147	0.0224	0.0322	0.0436	-0.624901	-0.883202
26	0.0012	0.0038	0.0078	0.0135	0.0211	0.0298	0.0410	-0.564852	-0.793973
27	0.0012	0.0036	0.0076	0.0127	0.0200	0.0287	0.0380	-0.534670	-0.717601
28	0.0011	0.0034	0.0071	0.0120	0.0187	0.0263	0.0363	-0.473252	-0.640932
29	0.0011	0.0032	0.0065	0.0114	0.0174	0.0251	0.0340	-0.442851	-0.575648
30	0.0010	0.0029	0.0062	0.0107	0.0164	0.0236	0.0318	-0.404412	-0.508083
31	0.0010	0.0028	0.0059	0.0100	0.0155	0.0223	0.0301	-0.370467	-0.452349
32	0.0009	0.0025	0.0057	0.0095	0.0147	0.0211	0.0288	-0.340445	-0.406121
33	0.0009	0.0026	0.0053	0.0089	0.0139	0.0200	0.0271	-0.311243	-0.354273
34	0.0008	0.0025	0.0050	0.0087	0.0137	0.0191	0.0258	-0.288214	-0.317614
35	0.0008	0.0023	0.0047	0.0081	0.0127	0.0181	0.0247	-0.263413	-0.276967
36	0.0007	0.0022	0.0046	0.0078	0.0120	0.0174	0.0235	-0.245184	-0.239846
37	0.0007	0.0021	0.0042	0.0076	0.0115	0.0165	0.0223	-0.222112	-0.202816
38	0.0007	0.0020	0.0041	0.0072	0.0110	0.0159	0.0215	-0.206645	-0.176282
39	0.0007	0.0019	0.0040	0.0068	0.0105	0.0150	0.0206	-0.182976	-0.142655
40	0.0006	0.0019	0.0039	0.0067	0.0101	0.0143	0.0200	-0.164704	-0.117466
41	0.0006	0.0018	0.0036	0.0065	0.0097	0.0138	0.0188	-0.151904	-0.087695
42	0.0006	0.0017	0.0035	0.0060	0.0094	0.0135	0.0182	-0.144969	-0.072067
43	0.0005	0.0016	0.0033	0.0059	0.0088	0.0127	0.0173	-0.124528	-0.039443
44	0.0006	0.0015	0.0033	0.0056	0.0086	0.0124	0.0167	-0.116032	-0.023136
45	0.0005	0.0015	0.0031	0.0055	0.0082	0.0117	0.0161	-0.098235	0.001264
46	0.0005	0.0014	0.0030	0.0052	0.0079	0.0116	0.0154	-0.096129	0.015630
47	0.0005	0.0014	0.0028	0.0049	0.0077	0.0110	0.0149	-0.080882	0.034336
48	0.0005	0.0014	0.0027	0.0047	0.0073	0.0108	0.0147	-0.076134	0.043579
49	0.0005	0.0013	0.0028	0.0047	0.0073	0.0104	0.0138	-0.064110	0.066534
50	0.0013	0.0015	0.0039	0.0055	0.0070	0.0101	0.0136	-0.042659	0.091750

Table.B1. Transformed Iris Flower Dataset						
Sl no.	Sepal length	Sepal width	Petal length	Petal width	Score(Brown)	Score(Pearson)
1	-1.247908	-0.409229	-1.402788	-1.385778	-1.325658	-1.32123
2	-0.629788	-1.242008	-1.297983	-1.175388	-1.218104	-1.21404
3	-1.128450	-1.154195	-1.339647	-1.261441	-1.383916	-1.37930
4	-1.048737	-1.354322	-1.190768	-1.235037	-1.345959	-1.34147
5	-1.494989	-0.361034	-1.394747	-1.432252	-1.398909	-1.39424
6	-1.648892	0.434334	-1.314571	-1.303563	-1.214984	-1.21093
7	-1.551650	-0.948060	-1.272623	-1.224736	-1.430628	-1.42585
8	-1.172695	-0.617934	-1.311653	-1.352479	-1.307011	-1.30265
9	-0.911809	-1.797463	-1.181732	-1.147754	-1.366597	-1.36203
10	-0.762465	-1.117316	-1.266827	-1.359024	-1.278610	-1.27434
11	-1.296653	0.118455	-1.432539	-1.469921	-1.268063	-1.26383
12	-1.344563	-0.778443	-1.212478	-1.365654	-1.361615	-1.35707
13	-0.696251	-1.343175	-1.291182	-1.311936	-1.298584	-1.29425
14	-1.150667	-1.791623	-1.360839	-1.306952	-1.541302	-1.53616
15	-1.434109	0.829112	-1.726658	-1.566679	-1.287047	-1.28275
16	-2.187827	1.334502	-1.555581	-1.497024	-1.324202	-1.31978
17	-1.666889	0.400025	-1.545550	-1.275984	-1.292225	-1.28791
18	-1.269630	-0.392605	-1.388874	-1.252370	-1.282135	-1.27785
19	-1.203722	0.538602	-1.377956	-1.384216	-1.111341	-1.10763
20	-1.741823	0.014187	-1.369154	-1.389267	-1.371706	-1.36713
21	-0.810962	-0.262606	-1.279025	-1.353708	-1.120561	-1.11682
22	-1.604647	-0.101927	-1.342565	-1.212525	-1.291889	-1.28758
23	-1.865721	-0.733516	-1.542865	-1.417234	-1.623980	-1.61856
24	-0.981780	-0.599102	-1.162463	-0.919570	-1.064570	-1.06102
25	-1.331065	-0.752711	-1.039244	-1.386338	-1.303684	-1.29933
26	-0.532606	-1.140310	-1.203209	-1.186037	-1.142526	-1.13871
27	-1.211639	-0.576108	-1.226081	-1.092558	-1.200654	-1.19665
28	-1.155225	-0.316109	-1.365758	-1.389533	-1.269390	-1.26515
29	-1.000827	-0.457424	-1.410828	-1.339304	-1.252407	-1.24823
30	-1.114952	-1.128463	-1.166413	-1.282126	-1.325985	-1.32156
31	-0.867871	-1.176658	-1.174453	-1.235651	-1.252734	-1.24855
32	-0.863404	-0.246512	-1.366687	-1.073103	-1.072136	-1.06856
33	-2.086888	0.463698	-1.455721	-1.782946	-1.530676	-1.52557
34	-2.007456	0.858113	-1.574373	-1.676557	-1.431885	-1.42710
35	-0.784187	-1.100692	-1.252913	-1.225617	-1.235087	-1.23096
36	-0.868398	-0.909142	-1.459537	-1.245126	-1.292355	-1.28804
37	-0.899673	-0.079633	-1.543393	-1.366323	-1.197139	-1.19314
38	-1.561452	-0.462201	-1.387946	-1.568800	-1.479390	-1.47445
39	-1.075206	-1.673301	-1.252152	-1.184193	-1.422201	-1.41745
40	-1.084512	-0.533390	-1.332368	-1.349339	-1.270054	-1.26581
41	-1.362313	-0.485725	-1.425904	-1.248616	-1.338403	-1.33393
42	0.103538	-2.501303	-1.170229	-0.744306	-1.087664	-1.08403
43	-1.393001	-1.407824	-1.277501	-1.270862	-1.494788	-1.48980
44	-1.413979	-0.410122	-1.210928	-0.869076	-1.149902	-1.14606

45	-1.745547	0.065120	-1.124261	-1.283439	-1.250941	-1.24676
46	-0.739694	-1.309927	-1.263354	-1.045120	-1.211538	-1.20749
47	-1.715602	0.006141	-1.325323	-1.529570	-1.395919	-1.39126
48	-1.212134	-1.230161	-1.261187	-1.271476	-1.401563	-1.39688
49	-1.384837	0.033912	-1.411823	-1.473061	-1.305021	-1.30066
50	-1.018297	-0.759249	-1.356723	-1.302250	-1.290028	-1.28572
51	0.792093	1.281413	0.314186	0.177253	0.644935	0.64278
52	0.232271	0.773623	0.336902	0.305610	0.428093	0.42666
53	0.850084	1.097910	0.476979	0.337066	0.726415	0.72399
54	0.889641	-1.258046	0.320862	0.435015	0.238523	0.23773
55	0.960543	0.335790	0.424631	0.475192	0.629536	0.62743
56	0.294018	-0.382381	0.504780	0.190150	0.227521	0.22676
57	-0.027533	0.855597	0.474345	0.378755	0.436986	0.43553
58	0.235311	-1.742480	-0.013476	0.020880	-0.285257	-0.28431
59	0.933273	0.519824	0.363413	0.168182	0.543153	0.54134
60	-0.036720	-0.972675	0.288477	0.392561	0.006688	0.00667
61	0.968083	-2.171736	0.131998	0.183567	-0.064504	-0.06429
62	0.095650	0.059699	0.292594	0.397263	0.257963	0.25710
63	1.554621	-1.017940	0.188219	0.093826	0.329034	0.32794
64	0.475132	0.122309	0.538648	0.278995	0.421200	0.41979
65	0.006443	-0.411381	-0.006881	0.205729	-0.019524	-0.01946
66	0.672941	0.869314	0.215772	0.231852	0.512425	0.51071
67	-0.155403	-0.168200	0.527973	0.367158	0.205022	0.20434
68	0.588266	-0.514757	0.224020	-0.136020	0.092960	0.09265
69	1.644878	-0.722847	0.505081	0.732672	0.717116	0.71472
70	0.698974	-0.949851	0.189225	0.091566	0.096536	0.09621
71	-0.260313	0.426511	0.655453	0.669449	0.431807	0.43037
72	0.624256	-0.087094	0.133196	0.237185	0.278798	0.27787
73	1.274366	-0.205780	0.677320	0.578230	0.722434	0.72002
74	0.677473	-0.043678	0.523495	0.055513	0.370448	0.36921
75	0.743407	0.325006	0.231609	0.182586	0.411307	0.40993
76	0.743655	0.652032	0.249163	0.272046	0.511762	0.51005
77	1.255814	0.589950	0.464060	0.337415	0.735506	0.73305
78	0.793670	0.837910	0.616656	0.634041	0.795151	0.79250
79	0.356229	0.037235	0.457787	0.423052	0.389145	0.38785
80	0.631984	-0.783502	-0.089058	-0.054457	-0.023566	-0.02349
81	0.765188	-1.175710	0.164870	0.138654	0.076562	0.07631
82	0.782410	-1.200911	0.093212	0.012141	0.013728	0.01368
83	0.535825	-0.498664	0.136358	0.144585	0.141386	0.14091
84	0.679298	-0.160155	0.843519	0.601760	0.621118	0.61905
85	-0.331770	-0.337286	0.569404	0.360878	0.131107	0.13067
86	-0.459980	0.717551	0.408327	0.339790	0.251199	0.25036
87	0.664718	0.911669	0.402920	0.344575	0.613880	0.61183
88	1.613108	-0.547391	0.386118	0.432556	0.611423	0.60938
89	-0.129958	-0.235757	0.269167	0.127921	0.040734	0.04060
90	0.571847	-0.992569	0.295512	0.348347	0.165935	0.16538

91	0.452668	-0.842146	0.499902	0.144025	0.163360	0.16281
92	0.311736	0.246470	0.468228	0.242556	0.365596	0.36438
93	0.699221	-0.622825	0.206778	0.181025	0.196990	0.19633
94	0.482393	-1.790675	-0.021516	0.067354	-0.212006	-0.21130
95	0.351234	-0.625395	0.364936	0.251029	0.168926	0.16836
96	-0.015553	-0.159260	0.292283	-0.009241	0.053479	0.05330
97	0.121623	-0.275375	0.318872	0.167501	0.133296	0.13285
98	0.567040	0.155919	0.273040	0.176306	0.337393	0.33627
99	0.217562	-1.449762	-0.226901	0.137918	-0.262045	-0.26117
100	0.276021	-0.416690	0.273802	0.217730	0.150279	0.14978
101	-0.164535	1.116716	1.350249	1.489794	1.079730	1.07613
102	0.437766	-0.279369	0.926690	0.995704	0.677773	0.67551
103	1.100013	1.319774	1.109152	1.118180	1.290867	1.28656
104	0.605108	0.435086	1.072574	0.756854	0.843001	0.84019
105	0.544690	0.820561	1.189613	1.239642	1.093336	1.08969
106	1.572426	1.802531	1.409789	1.085617	1.610826	1.60545
107	-0.021644	-1.390447	0.764182	0.828664	0.214836	0.21412
108	1.518439	1.340559	1.269634	0.739991	1.347747	1.34325
109	1.602430	0.259460	1.155902	0.928961	1.174627	1.17071
110	0.156925	2.284399	1.183532	1.381157	1.322775	1.31836
111	0.238843	0.992750	0.732223	0.934421	0.798529	0.79586
112	0.975867	0.245045	0.917888	1.000755	0.938138	0.93501
113	0.817465	1.031836	0.940319	1.136339	1.102753	1.09907
114	0.641157	-0.621343	0.928924	1.219534	0.737616	0.73515
115	0.170262	-0.063511	0.983584	1.619409	0.859095	0.85623
116	0.094494	0.975233	0.910169	1.317716	0.930762	0.92765
117	0.618078	0.728334	0.960724	0.726695	0.861312	0.85844
118	0.372208	2.974183	1.359333	0.868597	1.420267	1.41553
119	2.266254	1.415100	1.640835	1.508225	1.937937	1.93147
120	1.491007	-0.849048	0.835235	0.691918	0.739754	0.73728
121	0.553409	1.432258	1.037571	1.305837	1.192790	1.18881
122	0.071782	-0.316248	0.853870	1.093287	0.572467	0.57056
123	2.004625	1.613550	1.458254	1.035123	1.696159	1.69050
124	0.891407	0.109569	0.693711	0.891786	0.780416	0.77781
125	0.261587	1.362662	1.038500	0.989407	0.995535	0.99221
126	0.940065	1.628499	1.079091	0.627533	1.143977	1.14016
127	0.639827	0.149187	0.644007	0.852206	0.687854	0.68556
128	0.238348	0.338697	0.697117	0.755503	0.597620	0.59562
129	0.787024	0.436763	1.106275	1.203552	1.046822	1.04333
130	1.292304	1.312620	0.961124	0.461175	1.090898	1.08726
131	1.734800	1.291834	1.160018	0.933663	1.425901	1.42114
132	0.578520	3.084290	1.116841	0.628746	1.349204	1.34470
133	0.765303	0.453387	1.120189	1.336960	1.090345	1.08671
134	0.806673	0.209589	0.754784	0.434438	0.652173	0.65000
135	0.992318	-0.198712	1.096374	0.346945	0.703875	0.70153
136	1.594670	1.877436	1.128178	1.390047	1.638278	1.63281

137	-0.319708	1.198522	1.092682	1.340631	0.922671	0.91959
138	0.370997	0.776529	0.968765	0.680221	0.788061	0.78543
139	0.145665	0.245577	0.660088	0.759258	0.541352	0.53955
140	0.742252	1.240540	0.849185	1.103039	1.084106	1.08049
141	0.509719	1.138480	1.047846	1.483194	1.179382	1.17544
142	0.685311	1.248057	0.703778	1.390540	1.113221	1.10950
143	0.437766	-0.279369	0.926690	0.995704	0.677773	0.67551
144	0.474224	1.364869	1.173776	1.288907	1.194454	1.19047
145	0.174702	1.429158	1.094154	1.523038	1.169628	1.16572
146	0.672340	0.954809	0.815628	1.420699	1.094911	1.09126
147	1.191980	-0.130707	0.790719	1.104967	0.915837	0.91278
148	0.561137	0.735850	0.815317	1.014195	0.890427	0.88745
149	-0.395169	1.080200	0.983994	1.217872	0.803570	0.80089
150	0.070979	0.186765	0.854037	0.735433	0.562326	0.56045

Table.B2. Transformed Motor Car Dataset

SI	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉	X ₁₀	X ₁₁	X ₁₂	X ₁₃	BScore(Br.)	Score(Pear)
1	-0.557695	-0.560356	-0.146444	-0.513944	-0.271215	-0.560962	-0.716676	-2.515760	-0.766157	0.038160	0.037223	1.014680	1.071615	-0.2631	-0.26243
2	-0.557695	-0.560356	-0.146444	-0.513944	-0.271215	-0.560962	-0.716676	-2.515760	-0.766157	0.038160	0.037223	1.014680	1.071615	-0.2631	-0.26243
3	-1.702349	-0.353443	-1.326105	0.556531	0.118531	0.682443	0.142128	0.967187	-1.290366	1.225774	0.436121	0.920523	0.940506	0.116321	0.11602
4	-0.034923	0.004314	-0.076952	-0.436483	0.276623	-0.245663	0.363586	0.760022	-0.663253	-0.322803	-0.188415	-0.290734	-0.140092	-0.08297	-0.08275
5	0.360787	0.367434	0.295826	-0.077228	0.128460	0.405459	0.386871	0.934705	-1.203876	0.727066	-0.280634	0.726382	0.713397	0.440997	0.44286
6	0.065997	-0.037735	0.183077	-0.363431	0.328979	-0.137232	0.601966	0.714285	-0.908458	0.616997	-0.332011	0.371801	0.646468	0.251627	0.25098
7	1.553952	0.771444	0.056945	0.520315	1.143640	0.695724	2.478081	1.449389	-0.494838	-0.095138	-0.124468	0.422554	1.011014	0.992757	0.99871
8	1.532572	0.840256	0.110853	0.558834	1.013546	0.838738	2.496417	1.450577	-0.491582	-0.125395	-0.075438	0.482009	1.070722	1.022098	1.01947
9	1.327226	1.453378	-0.030926	0.659246	1.447420	1.104633	2.386295	1.640640	-0.971537	0.239847	0.061555	1.140484	1.070662	1.250403	1.24719
10	0.430393	0.631928	1.113071	-0.394716	0.282851	-0.344997	0.256405	-1.041600	-1.266510	-0.280734	-0.446037	-0.613559	-0.415629	-0.03642	-0.03633
11	0.430393	0.631928	1.113071	-0.394716	0.282851	-0.344997	0.256405	-1.041600	-1.266510	-0.280734	-0.446037	-0.613559	-0.415629	-0.03642	-0.03633
12	-0.368389	0.176148	0.159339	1.610256	0.417397	-0.033479	-0.672669	-0.035542	0.592004	1.234471	-0.016149	0.580446	0.725268	0.411061	0.41001
13	-0.379080	0.214554	0.186293	1.625515	0.352350	0.033028	-0.663501	-0.034948	0.593671	1.219393	0.008365	0.610174	0.755122	0.425982	0.42489
14	0.409043	0.572734	0.497705	2.231281	0.836145	0.508228	0.133331	0.451383	0.523809	0.816597	0.211362	1.081888	1.086408	0.915134	0.91278
15	0.740499	0.943253	1.729543	0.220742	1.822076	2.223173	0.612326	0.563207	-0.119260	1.965203	0.102607	0.946156	1.023879	1.47394	1.47016
16	0.594588	0.857620	1.728060	0.014870	1.818585	2.195011	1.337321	0.357907	0.133324	1.655549	0.216570	1.418002	1.501052	1.587969	1.58389
17	1.758461	1.832091	1.567790	0.659549	2.230335	2.480441	2.339062	1.015154	0.269002	1.791689	0.214643	1.274939	1.407654	2.077163	2.07183
18	-2.622087	-2.104943	-2.322306	-0.607458	-2.600052	-1.233905	-2.150355	-1.470440	0.045160	-1.990992	-0.858584	-2.679429	-3.279284	-2.679429	-2.60594
19	-1.545582	-1.263436	-1.342662	-0.826671	-1.580749	-0.936355	-0.296587	-0.930151	-0.169597	-1.162784	-0.555760	-1.474160	-1.902203	-1.49634	-1.49250
20	-1.565157	-1.316016	-1.204309	-0.752922	-1.414402	-0.807118	-0.232027	-0.906020	-0.174551	-1.286956	-0.520424	-1.410159	-1.834469	-1.45552	-1.45179
21	-1.682534	-1.512533	-1.382533	-1.164816	-1.548141	-1.083434	-0.037601	-0.652415	-0.305078	-1.142572	-0.586321	-1.074929	-1.629268	-1.47949	-1.47569
22	-1.453624	-1.263193	-1.286024	-1.134582	-1.380365	-1.335070	-0.191764	-0.478660	-0.708933	-0.712048	-0.575229	-1.022082	-0.908668	-1.30502	-1.30167
23	-1.363881	-0.504815	-0.676220	-1.235340	-0.845434	-0.880535	-0.524068	0.074907	-1.077092	0.215260	-0.821793	-0.055132	-0.145879	-0.78503	-0.78301
24	-1.507881	-1.211416	-1.239306	-1.142081	-1.499273	-1.253987	-0.144658	-0.504523	-0.680587	-0.746803	-0.534546	-0.944630	-0.836588	-1.28435	-1.28105
25	-1.512157	-1.196054	-1.233124	-1.135978	-1.525292	-1.227385	-0.140790	-0.504285	-0.687920	-0.752834	-0.527440	-0.932739	-0.824646	-1.27838	-1.27510
26	-1.512157	-1.196054	-1.233124	-1.135978	-1.525292	-1.227385	-0.140790	-0.504285	-0.687920	-0.752834	-0.527440	-0.932739	-0.824646	-1.27838	-1.27510
27	-1.412695	-0.472590	-1.039224	-0.393227	-1.250608	-0.931227	-0.833130	-0.481429	0.048742	-1.057596	0.197182	-0.793590	0.009186	-0.088996	-0.76997
28	0.787291	0.537792	0.468418	1.127574	0.037310	0.315102	-1.149935	1.512119	-0.184960	0.314456	-0.649716	-0.844463	-0.603022	0.123381	0.12306
29	-0.330109	0.021407	0.148231	-1.440367	0.516732	0.618132	-0.320772	0.890763	1.072761	1.135111	-0.735686	1.759630	1.498438	0.575817	0.57434
30	-2.431211	-2.717191	-3.147280	-1.004002	-2.428449	-0.554136	-1.022464	-0.956788	1.474394	-1.977741	-0.643962	-1.702238	-2.449139	-2.25746	-2.25186
31	-2.333460	-1.736437	-2.067928	-2.379819	-1.849800	-0.653715	-0.653242	-0.268089	-1.406510	-0.580129	-0.671112	-1.236405	-1.071346	-1.66087	-1.60274
32	-1.314798	-1.176916	-1.849856	-0.839204	-1.879927	-1.079276	-0.295117	-0.819169	-0.670701	-1.187351	-0.457577	-1.624616	-2.107107	-1.65143	-1.64719
33	-0.964436	-0.949425	-1.354124	-1.176258	-1.581441	-0.779694	-0.267891	0.324891	-1.522369	-0.200572	-0.650999	-1.099886	-1.389030	-1.24012	-1.23694
34	-0.967546	-0.938252	-1.346283	-1.171819	-1.600364	-0.760347	-0.265024	0.325644	-1.521884	-0.204959	-0.643688	-1.091238	-1.380345	-1.23578	-1.23261
35	-0.756168	-0.887741	-0.590435	-1.012775	-0.964894	-0.545404	-0.061013	0.790006	-1.600640	0.583137	-0.582622	-1.203534	-1.449833	-0.98393	-0.98140
36	-0.042460	-0.484431	-0.758765	-1.038176	-1.040666	0.001974	-0.836016	1.224681	-1.911762	-0.317829	-0.628626	-1.833382	-1.986406	-0.97418	-0.97168
37	-0.393230	-0.738298	-0.301055	-1.332068	-0.610205	-0.194391	0.089928	0.860108	-0.675827	-0.326380	-0.588048	-0.627800	-0.490163	-0.54388	-0.54248
38	-0.402622	-0.701289	-0.275082	-1.317364	-0.672887	-0.130302	0.098763	0.860680	-0.674219	-0.340910	-0.565224	-0.599153	-0.461395	-0.5295	-0.52814
39	-0.296778	-0.853558	0.072354	-1.007762	-0.297506	0.022498	0.132695	1.047640	-0.760495	-0.653921	-0.505264	-0.581298	-0.411231	-0.42404	-0.42295
40	-0.989469	-0.953983	0.782869	-0.918664	-0.581298	-0.080429	-0.615806	1.012503	-0.980189	-0.481343	-0.510627	-0.634970	-0.612540	-0.57197	-0.57050
41	-0.217554	-0.401422	0.159594	-0.971997	-0.108898	0.149244	0.020034	1.181614	-1.140585	-0.259475	-0.373360	0.014463	-0.156107	-0.17991	-0.17945
42	-0.748321	-0.542415	-0.657435	-1.332374	-0.321038	-0.494353	0.411467	0.616620	-0.270445	-0.156491	-0.458174	0.134702	0.147519	-0.36441	-0.36348
43	-0.709793	-0.684243	0.399037	0.338670	-0.617936	-0.930020	-1.846562	-0.145497	-0.410488	-0.026810	-0.578819	0.176360	0.052960	-0.50475	-0.50345
44	-0.211301	-0.137680	1.174064	-0.374192	-0.598231	-0.324652	-0.275491	-0.481634	0.143505	-0.212338	-0.226199	0.687668	0.643002	-0.08128	-0.08108
45	0.553716	1.091670	2.257205	0.496643	1.359107	2.598631	2.271005	2.502860	2.332255	2.905605	0.282285	2.694193	2.480347	2.423267	2.41705
46	0.553716	1.091670	2.257205	0.496643	1.359107	2.598631	2.271005	2.502860	2.332255	2.905605	0.282285	2.694193	2.480347	2.423267	2.41705
47	0.025952	0.638085	0.947218	0.422923	2.560033	3.278379	3.353122	-2.348444	0.374811	0.053547	2.245557	3.119142	2.659708	2.461696	2.45538
48	-0.364764	-0.898675	-1.402760	0.121864	-0.908825	-1.229774	-1.115543	-0.153285	-0.829074	-0.443407	-0.604981	-0.334431	-1.086048	-0.99212	-0.99557
49	-0.744123	-1.010445	-1.499923	0.085738	-1.007594	-0.967930	-1.018094	-0.323784	-0.424933	-0.800126	-0.615763	-1.145960	-0.970859	-1.14196	-1.13902
50	-0.745095	-1.006954	-1.497472	0.087125	-1.013508	-0.961884	-1.017261	-0.323730	-0.424781	-0.781497	-0.615335	-1.143257	-0.948145	-1.1406	-1.13767
51	-0.789438	-1.183524	-1.156600	0.268258	-0.721641	-0.895156	-0.854645	-0.245894	-0.439540	-1.096685	-0.543302	-1.001935	-0.817029	-1.04557	-1.04388
52	-0.706057	-1.139500	-1.072996	0.194825	-0.697464	-0.877944	-0.908078	-0.300274	-0.350729	-1.108887	-0.569672	-0.994608	-0.807557	-1.02547	-1.02304
53	0.245707	-0.365480	-0.141018	0.232500	0.080554	-0.390177	-0.087802	0.315070	0.725740	-0.379385	-0.572396	0.115885	0.287455	-0.03519	-0.03510
54	0.549970	-0.242114	-0.106454	0.534161	0.152547	-0.099338	-0.372085	0.564952	0.568224	-0.378982	-0.562355	-0.214994	0.097055	-0.00712	-0.00749
55	0.245707	-0.365480	-0.141018	0.232500	0.080554	-0.390177	-0.087802	0.315070	0.725740	-0.379385	-0.572396	0.115885	0.287455	-0.03519	-0.03510
56	0.549970	-0.242114	-0.106454	0.534161	0.152547	-0.099338	-0.372085	0.564952	0.568224	-0.378982	-0.562355	-0.214994	0.097055	-0.00712	-0.00749
57	0.245707	-0.365480	-0.141018	0.232500	0.080554	-0.390177	-0.087802	0.315070	0.725740	-0.379385	-0.572396	0.115885	0.287455	-0.03519	-0.03510
58	0.549970	-0.242114	-0.106454	0.534161	0.152547	-0.099338	-0.372085	0.564952	0.568224	-0.378982	-0.562355	-0.214994	0.097055	-0.00712	-0.00749
59	0.245707	-0.365480	-0.141018	0.232500	0.080554	-0.390177	-0.087802	0.315070	0.725740	-0.379385	-0.572396	0.115885	0.287455		

71	1.791644	1.871107	1.457227	1.242461	1.312933	1.011122	2.345940	1.855148	2.262174	0.841404	3.089396	1.297930	1.201421	1.970702	1.96564
72	2.024041	1.803324	1.588012	2.565399	1.816640	1.646186	3.126012	0.674324	0.659284	2.150823	0.548002	1.750090	1.574567	2.34107	2.35066
73	1.216347	0.218255	-0.057826	0.632723	0.474657	2.005513	1.689946	-0.837843	0.329768	2.034166	0.718019	2.690554	2.446737	1.528324	1.52440
74	2.431491	1.797022	2.854877	2.644935	2.697464	2.709107	3.226318	0.973629	2.072035	3.089771	0.523222	2.159463	1.919425	3.10651	
75	2.525008	1.082355	1.819106	1.916489	2.462495	2.747747	2.545158	0.485408	1.846479	3.641458	0.540975	2.378305	2.183378	2.775403	2.74848
76	0.882464	2.037770	0.777741	0.029087	1.711188	1.138060	0.145772	-0.450321	0.442090	0.882456	-0.294424	0.911212	0.634955	1.049459	1.04696
77	-1.530140	-1.479836	-1.503000	-1.140829	-1.552470	-0.976386	0.149796	-0.469577	-0.247774	-1.162944	-0.556002	-1.034328	-1.550931	-1.42498	-1.42132
78	-1.306168	-1.212275	-1.393693	-1.124018	-1.415280	-1.197046	-0.801942	-0.475526	-0.650946	-0.740426	-0.531001	-0.961695	-0.816419	-1.24045	-1.24025
79	-1.317830	-1.170377	-1.364289	-1.107371	-1.486240	-1.124493	0.008059	-0.474877	-0.649116	-0.748075	-0.504258	-0.935265	-0.783851	-1.22717	-1.22402
80	-1.351711	-0.557169	-1.095489	-1.249888	-0.864011	-0.820610	-0.578472	0.056436	-1.105194	0.171002	-0.801001	-0.023626	-0.109531	-0.794	-0.79196
81	-1.290001	-0.407776	-0.368002	-1.299043	-0.046827	-0.512716	0.548761	0.120804	-0.415732	-0.126287	-0.683526	0.515891	0.577879	-0.26929	-0.26859
82	-0.809935	-0.825871	-0.233273	-0.909236	-0.355171	-0.904034	0.118783	-0.150640	0.668802	-0.193443	-0.614746	0.524236	0.713488	-0.29894	-0.29817
83	-0.354228	0.039357	0.128414	-1.345437	0.480807	0.628533	-0.275418	0.810025	0.992385	1.124307	-0.699237	1.754475	1.494035	0.571194	0.56973
84	-0.354462	0.108913	0.187463	-1.355986	0.382749	0.737177	-0.271768	0.800809	1.012785	1.098015	-0.665733	1.802964	1.545154	0.598976	0.59744
85	-0.355433	0.112404	0.190113	-1.354599	0.376834	0.743224	-0.270935	0.800143	1.012937	1.096644	-0.663505	1.805647	1.547868	0.600332	0.59879
86	-0.412017	-0.645465	-0.129466	-0.552000	-0.301230	-0.542201	-0.240703	0.139032	0.477723	-0.171011	-0.403486	0.223982	0.473947	-0.25123	-0.25059
87	-0.419792	-0.626723	-0.193343	-0.540903	-0.348537	-0.493833	-0.234035	0.139464	0.478936	-0.181977	-0.585857	0.245602	0.495660	-0.24038	-0.23976
88	-0.891305	-0.239353	-0.349716	-0.942917	0.011844	-0.155721	0.188608	0.410433	-0.406493	-0.102758	-0.674248	0.213475	0.336167	-0.22246	-0.22209
89	-0.891305	-0.239353	-0.349716	-0.942917	0.011844	-0.155721	0.188608	0.410433	-0.406493	-0.102758	-0.674248	0.213475	0.336167	-0.22246	-0.22209
90	-0.552839	-1.104512	-0.711706	-0.261761	-0.658018	-0.853887	-0.992311	0.053527	-0.330938	-0.789331	-0.622665	-1.106863	-0.107789	-0.98467	-0.98214
91	-1.483473	-0.200761	-1.296828	-0.531234	-1.344500	-0.773372	-0.824206	0.181634	1.186990	-1.732939	2.301458	-2.010443	-2.452907	-1.48892	-1.49507
92	-0.584746	-1.084262	-0.702894	-0.253715	-0.692315	-0.818619	-0.987377	0.056431	-0.330059	-0.772882	-0.609739	-1.091188	-1.062098	-0.9768	-0.97430
93	-0.562363	-1.070296	-0.693092	-0.248166	-0.715969	-0.794435	-0.984043	0.065857	-0.329452	-0.802765	-0.600825	-1.080378	-1.051242	-0.97138	-0.96888
94	-0.785195	-1.199219	-0.585554	-0.279838	-0.658264	-0.823542	-0.709796	-0.018186	-0.249078	-0.528352	-0.816391	-0.816391	-0.808785	-0.91518	-0.91283
95	-0.564890	-1.061218	-0.686722	-0.245640	-0.731344	-0.778715	-0.981876	0.065997	-0.329058	-0.806329	-0.595030	-1.073352	-1.044186	-0.96785	-0.96537
96	-0.800696	-1.086023	-0.651323	-0.413055	-0.876492	-0.858629	-0.770145	-0.091230	-0.221827	-0.844130	-0.557933	-0.857474	-0.861234	-0.9678	-0.96532
97	-0.568778	-1.042752	-0.647690	-0.239011	-0.754997	-0.754531	-0.978542	0.066213	-0.328451	-0.811812	-0.586116	-1.062542	-1.033229	-0.9642	-0.95995
98	-0.787722	-1.190141	-0.521383	-0.276232	-0.673438	-0.807822	-0.707429	-0.018046	-0.248684	-0.527354	-0.809364	-0.809364	-0.801729	-0.91166	-0.90932
99	-0.794875	-0.959952	-0.636643	-0.472735	-0.993800	-0.934187	-0.789319	-0.107624	-0.191270	-0.783275	-0.599952	-0.935465	-0.948914	-0.99616	-0.99360
100	-0.150393	-0.459890	-0.707025	-0.240474	0.003022	0.085269	-0.505040	0.549336	0.150220	-0.313229	-0.619908	-0.477094	-0.270340	-0.26158	-0.26091
101	-0.146117	-0.475252	-0.808807	-0.290178	0.029041	0.058666	-0.504208	0.549498	0.157552	-0.307198	-0.488985	-0.282282	-0.26755	-0.26687	-0.26687
102	0.591411	0.660723	0.899661	0.406610	0.976847	1.371121	0.213755	0.335623	-0.527789	1.507790	0.260950	0.938879	0.952691	0.996206	0.99365
103	0.721416	0.782898	1.131162	0.721515	0.923292	1.765871	0.145303	0.499716	-0.615639	1.342140	0.369444	0.995124	0.997103	1.102131	1.09930
104	0.403711	0.452378	0.944470	0.474621	1.071862	1.483664	0.339625	0.268090	-0.266673	1.181120	0.256595	0.698850	0.813013	0.921887	0.91952
105	-0.065756	-0.154221	-0.568337	-0.102700	0.381460	1.380435	0.564422	-0.844069	-0.050681	1.173331	0.385248	1.588121	1.673736	0.601002	0.59946
106	-0.464498	0.440333	-0.726173	-0.104310	1.048803	1.855609	0.458443	-0.848473	-0.228376	1.513885	0.267441	1.924982	1.719866	0.819546	0.81744
107	-0.290651	0.410924	0.201781	-0.614177	0.650128	1.036466	1.379483	0.554388	0.258165	1.257406	0.319514	1.516836	1.499527	0.856965	0.85477
108	1.470429	1.680807	0.990662	1.203313	0.525433	0.201093	0.911093	0.779585	-0.040297	0.104413	-0.335085	0.458680	0.744863	0.908641	0.90631
109	1.288452	1.024785	0.865803	0.825141	0.448623	0.338976	0.189253	0.907207	2.519188	0.093901	2.491566	1.664496	2.010264	0.899554	0.89724
110	1.598099	2.126329	1.874252	2.031835	0.931543	0.419373	1.384930	1.340848	0.020458	-0.707029	-0.278044	0.296328	0.592940	1.259556	1.25652
111	1.845416	1.741056	1.974516	1.708010	0.970455	0.251232	0.521725	1.697250	2.044152	0.364800	2.570010	0.851036	0.201197	1.467417	1.46385
112	1.832072	2.088302	0.988677	2.396412	0.478535	-0.048455	1.414742	-1.784411	-1.179680	-0.149731	0.063336	0.102199	0.492844	0.847956	0.84578
113	1.277762	1.063191	0.892757	0.840400	0.383576	0.405434	0.198422	0.907801	2.520856	0.078822	2.516081	0.196224	0.240118	0.914475	0.91213
114	1.594051	2.488948	1.970975	2.897662	0.771801	-0.119741	2.211945	-1.491605	-0.323936	0.121042	0.222507	0.567777	1.158933	1.51596	
115	1.854726	1.799462	2.003470	1.724070	0.905407	0.317740	0.530893	1.697844	2.045820	0.371901	2.602524	0.880764	0.231051	1.482538	1.47873
116	1.459739	1.646413	0.936016	0.840400	0.383576	0.267400	0.920262	0.780179	-0.040429	0.089334	-0.10571	0.488407	0.774717	0.923562	0.92119
117	1.277762	1.063191	0.892757	0.840400	0.383576	0.405434	0.198422	0.907801	2.520856	0.078822	2.516081	0.196224	0.240118	0.914475	0.91213
118	1.187958	2.035421	1.045053	1.040444	1.145891	1.176180	1.290565	0.485580	-0.335208	0.478612	-0.543530	0.397691	0.614370	1.136184	1.13327
119	-1.490833	-1.483720	-1.362005	-1.152527	-1.597977	-1.032183	-0.030509	-0.651977	-0.307709	-1.154209	-0.569921	-1.051960	-1.606269	-1.46811	-1.46434
120	-1.363881	-0.504815	-0.067420	-1.235340	-0.845434	-0.880535	-0.524068	0.074907	-1.077092	-0.125640	-0.821793	-0.053132	-0.145879	-0.78503	-0.78301
121	-1.507881	-1.211416	-1.243906	-1.142081	-1.499723	-1.253987	-0.144658	-0.504523	-0.688587	-0.468803	-0.534546	-0.944630	-0.836588	-1.28435	-1.28105
122	-1.523078	-1.449873	-0.012711	-0.082406	-1.073522	-1.714585	0.029577	0.376915	-0.726410	-1.147121	-0.454006	-0.805547	-0.679776	-1.16634	-1.16335
123	-1.523080	-1.426556	-0.675748	-0.297143	-1.311167	-0.871717	0.101783	-0.397019	-0.694099	-1.067466	-0.342805	-0.706216	-0.560027	-1.08692	-1.08413
124	0.804162	0.545699	0.645411	1.112610	0.043328	0.317335	-1.180854	1.501322	-0.167228	0.312889	-0.655436	-0.843538	-0.601670	0.127788	0.12696
125	-0.334442	0.036989	0.137186	-1.384563	0.504565	0.612628	-0.288938	0.798976	1.009662	1.126253	-0.711463	1.747292	1.489245	0.571033	0.56957
126	1.080348	0.713441	0.233328	-0.240477	0.668580	0.749523	0.419087	-1.584889	0.458676	0.565891	-0.083800	0.787631	1.297814	0.581965	0.58047
127	-0.006362	0.235473	0.625590	-1.832535	1.550144	1.995213	-0.478449	-0.251632	-1.222342	1.727395	0.471422	0.575974	0.731684	0.589929	0.58841
128	-0.006362	0.235473	0.625590	-1.832535	1.550144	1.995213	-0.478449	-0.251632	-1.222342	1.727395	0.471422	0.575974	0.731684	0.589929	0.58841
129	-0.014914	0.266198	0.647153	-1.820328	1.498106	2.048419	-0.471114	-0.251157	-1.221007	1.715332	0.491033	0.599576	0.755567	0.601865	0.60032
130	1.008422	0.527370	0.892476	0.400736	0.944880	0.426307	-0.088492	-0.101851	-0.500990	-0.406400	-0.119812	-0.018365	0.445140	0.44415	

173	-0.001878	0.213458	0.623013	-0.245903	-0.020711	0.652456	-0.508319	0.168503	1.134336	0.313316	-0.157755	0.747782	0.732556	0.379507	0.37853
174	-0.166103	-0.054364	-0.670702	-0.113619	0.207337	-0.506443	-0.415663	0.881683	1.680952	-0.096329	-0.634577	0.102695	-0.145763	-0.02863	-0.02856
175	0.282103	0.098988	-0.332100	0.196077	-0.155956	-1.201881	-0.164782	0.549818	0.890553	-0.377509	2.627504	-0.298848	-0.630044	-0.11571	-0.11541
176	-0.237513	0.043258	-0.594557	-0.989389	0.111889	-0.646007	-0.302508	0.828120	1.585129	0.056632	-0.580270	0.433449	0.220206	0.052327	0.05219
177	-0.054668	0.102096	-0.582161	1.151125	0.168311	-0.516024	-0.463192	0.962349	1.497199	0.040664	-0.580882	0.292119	0.106778	0.072396	0.07221
178	-0.246064	0.073983	-0.572993	1.001596	0.059851	-0.607401	-0.395174	0.878596	1.586463	0.044569	-0.568058	0.457331	0.244089	0.064264	0.06410
179	-0.308469	0.598169	0.292073	0.024744	1.086296	0.868952	1.424096	0.308454	-0.063099	1.221321	0.311611	1.268072	0.878075	0.887558	0.88528
180	-0.304165	0.661730	0.321957	0.038162	1.062444	0.891284	1.410885	0.326952	-0.103925	1.260344	0.330675	1.236289	1.038333	0.917993	0.91564
181	-0.251618	0.738361	0.986494	0.002160	1.032161	1.146388	1.002213	0.672149	-0.284164	1.163091	0.323939	1.022195	0.654122	0.959582	0.95712
182	-0.291600	0.935129	0.958767	0.573505	1.030393	1.071277	0.937062	0.718290	-0.358276	1.038805	0.304132	0.991915	0.790721	0.953811	0.95136
183	-0.589237	-1.144597	-0.169661	-0.080402	-0.949248	-0.811418	-0.366572	0.561649	0.716678	-1.572946	2.683440	-1.959186	-1.380183	-0.97112	-0.96862
184	0.039753	-0.470592	-0.427095	-0.004426	-0.148681	-0.167583	-0.433166	0.702959	-0.323544	-0.408568	-0.530247	-0.805696	-0.481353	-0.40451	-0.40347
185	-0.589820	-1.142502	-0.105491	-0.080410	-0.952796	-0.807790	-0.366072	0.561681	0.716769	-1.573769	2.684977	-1.957564	-1.378555	-0.9703	-0.96781
186	0.039169	-0.468497	-0.425625	-0.003593	-0.152229	-0.163956	-0.432666	0.702992	-0.323453	-0.409391	-0.528910	-0.804074	-0.479724	-0.40369	-0.40266
187	0.026924	-0.424505	-0.394751	0.013885	-0.226737	-0.087775	-0.422164	0.703673	-0.321542	-0.486662	-0.500830	-0.770023	-0.445527	-0.3866	-0.38541
188	-0.644695	-0.839050	-1.215321	0.283780	-0.457527	-0.820387	-0.655386	0.403332	0.837317	-1.283009	2.776413	-1.208408	-1.433677	-0.80709	-0.80501
189	0.026924	-0.194867	-0.270868	-0.385759	0.028972	0.086355	-0.302498	0.746794	-0.750348	-0.307819	-0.213831	-0.683815	-0.561837	-0.29286	-0.29271
190	0.095869	-0.350108	-0.254575	-0.580133	-0.494226	-0.136463	-1.183262	0.636248	-1.141140	0.274199	-0.653482	-0.574705	-0.544740	-0.51281	-0.51149
191	-0.746309	-0.701204	-0.275168	-1.138451	-0.625326	-0.782910	-0.440574	0.133809	-0.822357	0.037616	-0.624704	0.093837	0.003654	-0.54924	-0.54783
192	0.577949	0.210251	0.298480	0.076937	0.442099	0.328090	0.572129	1.059312	-1.063026	0.500281	-0.246257	0.344476	0.546490	0.432478	0.43137
193	-0.242330	-0.347410	-0.839201	0.524456	-0.386829	-0.882916	0.179670	0.871696	0.749207	-1.254090	2.923457	-0.610309	-0.686877	-0.56005	-0.55712
194	0.295915	-0.085894	0.223277	-0.005579	0.099285	0.118969	0.713488	0.948108	-0.553768	0.746787	-0.324596	-0.262826	0.021406	0.035812	0.03574
195	1.467143	0.990596	1.736490	0.065030	0.951390	0.887178	0.543940	-0.064866	0.263357	-0.114631	-0.139070	0.067960	0.186161	0.805326	0.80326
196	1.681130	1.152277	1.812393	0.311735	0.880453	1.222920	0.355389	0.109449	0.152748	-0.142837	-0.085488	-0.049827	0.104927	0.864513	0.86229
197	1.450594	0.971026	1.738281	0.069091	0.900733	0.941027	0.548093	-0.082484	0.305494	-0.109726	-0.130053	0.133795	0.060900	0.791983	0.78995
198	1.667496	1.122233	1.866832	0.311635	0.847536	1.258631	0.377041	0.092970	0.194350	-0.095019	-0.083157	0.007900	-0.028476	0.8471	0.84493
199	0.805699	1.975312	1.153060	0.715851	1.744261	1.087860	0.213805	0.246938	-0.270964	0.543386	-0.307851	0.940740	0.787852	1.145717	1.14278
200	1.021629	1.230010	1.240662	0.959781	1.685151	1.411510	0.022786	0.404646	0.381877	0.517922	-0.258726	0.817548	0.701190	1.202191	1.19911
201	1.653212	1.510865	1.570440	0.049392	0.995863	0.680826	1.533992	-0.007887	0.641975	0.022710	-0.180961	0.106126	0.251294	1.000005	0.99744
202	1.206150	2.288039	1.389427	0.151501	1.803415	1.105412	1.263708	0.628488	0.447051	0.595488	-0.176185	0.538251	0.588613	1.305152	1.30180
203	1.443151	1.503297	1.490251	0.711321	1.312324	0.874309	1.950319	-0.436887	0.674468	1.067241	0.067021	0.355518	0.604062	1.263926	1.26068
204	0.385303	0.861476	0.197533	0.710516	0.311850	-0.004168	1.807066	1.443488	0.202123	0.354933	3.410973	0.509022	-0.054757	0.799468	0.79760
205	1.836719	1.765822	1.702150	0.104869	0.986798	0.613815	1.438713	0.130941	0.344239	0.314221	-0.128989	0.324968	0.755911	1.165163	1.16217

Table.B3. Transformed Sinusoidal Dataset

SI	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉	Score(Brown)	Score(Pearson)
1	0.379768	0.591644	0.864886	-0.583739	0.437819	-0.706206	2.086528	1.164571	-0.655893	-0.314447	-0.311286
2	-0.222814	0.487962	-1.770506	-0.231000	0.910600	-1.291688	1.026590	0.887420	-0.958380	-0.221219	-0.218996
3	0.158524	-0.865345	0.885310	-0.378477	-0.441734	1.497563	-1.443534	-0.544974	1.637564	-0.337445	-0.334053
4	-1.457372	-1.437488	0.825761	1.726260	-0.651721	1.081300	-1.374693	-1.163250	-0.883275	0.411166	0.407034
5	0.031762	1.374938	0.289824	0.384787	0.922557	-0.203191	-0.939577	1.632199	-0.396816	0.515225	0.510047
6	1.632960	1.320257	1.699304	1.064032	-1.191011	-1.469474	0.041273	0.339102	-1.066214	1.634535	1.618107
7	0.445118	1.576147	0.536385	1.639405	-0.532903	-0.398114	0.221783	-1.035509	-1.280159	1.694987	1.677951
8	-0.535915	0.194434	-1.069211	0.183424	0.380527	-1.043929	-1.534709	-0.170212	0.400054	0.081753	0.080931
9	-1.135085	-0.600696	0.008076	0.275952	-1.856840	-1.486253	0.626945	-1.004554	-0.411630	-0.216884	-0.214704
10	-1.789107	-1.484121	-0.550499	-1.797233	-1.938543	-0.207475	0.995585	1.044672	0.270860	-1.943332	-1.923801
11	-0.562685	0.282256	-0.870142	1.242706	-0.849004	-1.537662	0.483220	0.171739	-0.218011	0.131529	0.130207
12	1.819299	1.798187	-0.472988	-0.303675	-1.211436	0.538658	-0.157367	-0.775693	-0.073313	1.395912	1.381882
13	0.323079	0.781727	0.938706	-1.455484	0.178760	1.426877	-1.340798	-0.456856	-0.382001	0.569496	0.563772
14	-0.157201	0.594958	-1.814552	1.358277	-0.848755	-0.468443	0.204179	1.623944	-1.600217	0.615228	0.609045
15	-1.122750	-1.170236	-0.427957	-1.149895	0.408924	-0.930045	0.857378	1.657995	1.255492	-2.040133	-2.019629
16	-0.753747	-0.940384	0.842002	1.406136	1.247625	1.724973	2.100454	-0.948216	-0.109832	-0.462004	-0.457361
17	-0.467153	-0.155433	-1.237768	-1.174710	-0.236480	-0.566975	-1.255930	-0.454173	0.140632	-0.223089	-0.220847
18	0.195792	1.114550	1.098897	0.860542	0.210395	1.925251	-0.567784	-0.672922	-0.511885	1.139696	1.128241
19	0.736698	1.042061	1.257365	-0.029637	0.984830	0.710732	-0.683658	0.286685	0.071384	0.770009	0.762270
20	1.526669	-0.746513	-1.407309	2.121895	-0.010552	-0.975028	0.301659	-0.646301	-0.206642	0.853191	0.844616
21	-1.489391	-0.781074	-0.402858	-0.238090	0.554146	-0.676218	0.729944	-0.283094	0.727346	-1.226084	-1.213762
22	-0.089752	-0.897302	0.094446	1.239515	0.779078	0.417991	-0.157892	-1.346092	-0.763038	0.507243	0.502145
23	-1.128786	0.475890	0.547950	-0.762059	0.616419	1.062379	-0.867499	1.206464	2.055118	-1.247761	-1.235220
24	1.379960	-0.105249	-0.494150	-0.991073	-0.071082	0.316960	0.324519	1.750447	-0.035416	-0.234457	-0.232100
25	1.762085	0.168159	1.263271	-0.817717	1.608065	1.376897	0.224936	-0.629378	1.156615	0.103724	0.102681
26	0.789976	-0.188336	-1.292395	-0.112947	-0.736662	0.230923	-0.101138	-1.534917	-1.272236	0.925218	0.915919
27	1.060298	0.827177	-0.110775	-0.507874	-0.149048	1.813866	-0.820814	-0.912308	-1.093431	1.264391	1.251684
28	-0.205492	-1.861447	0.373487	-1.572827	0.223348	-0.861858	0.430407	1.473503	1.974156	-1.210817	-2.099501
29	0.478711	-1.685330	-0.153837	-1.221506	-0.268115	0.613628	-0.749083	0.711595	-0.784743	-0.373844	-0.370087
30	0.868185	-0.256511	-1.662482	-0.162225	-0.621830	1.573247	0.781443	-1.624068	0.430716	0.175248	0.173487
31	0.467951	0.061636	0.169521	-0.309702	-1.868548	-0.837582	0.898105	-0.879908	0.827256	-0.088999	-0.088996
32	0.321767	-0.355695	0.234213	1.294819	-1.809014	-0.436670	-1.461927	0.237157	1.190034	0.321151	0.317923
33	-0.754272	-0.481864	0.160638	1.543332	0.345155	-1.558725	-0.678140	-0.132653	-1.432436	0.584273	0.578401
34	-1.426140	0.200352	-0.583964	-1.001709	-0.546354	0.023148	1.320083	-0.026374	2.147449	-1.656090	-1.639446
35	-0.611762	-1.557977	1.336662	-0.615645	-0.569520	-0.327427	1.516064	-0.324161	-0.511196	-0.654606	-0.648026
36	-1.247675	0.282493	-1.874593	0.967250	-0.488066	-0.364912	0.023931	-0.362339	0.211947	-0.142968	-0.141531
37	0.656389	0.832621	-0.322886	0.503549	0.596741	-0.931116	-1.486625	1.124329	-1.537170	1.163508	1.151814
38	1.310674	0.233010	-0.246359	0.166053	-0.006317	-0.719772	-1.696563	0.119528	-0.546681	1.647242	1.630686
39	1.275243	-0.831731	-0.237992	0.301477	-0.077309	-0.239871	-1.253845	-0.418865	-0.694305	0.643057	0.640357
40	-1.564713	-0.276395	1.074290	-0.901027	1.946086	0.827829	2.357950	0.115194	2.139869	-2.167739	-2.145952
41	-1.271033	-1.985013	0.170727	1.012273	2.132907	1.165195	-0.064353	1.170143	-0.539102	-0.958198	-0.948567
42	-0.648243	-0.587440	-0.773437	-1.304407	1.056320	-1.302755	-0.169191	1.575448	-0.108799	-1.085971	-1.085991
43	-0.430673	-0.100751	-0.073373	0.223130	1.268549	0.730010	-1.151880	-1.149630	0.663266	0.071111	0.070397
44	1.436386	1.791322	1.659441	-0.680166	1.583903	-0.297082	0.895740	-1.876661	-1.646382	1.563204	1.547493
45	1.476016	0.447010	-1.170591	0.492204	0.133923	0.056706	0.456157	0.216933	0.561897	0.343345	0.339895
46	-0.941135	0.602770	0.281704	-0.608555	1.553513	0.933858	-0.980566	1.644581	-0.720662	-0.211162	-0.209040
47	-0.955045	-0.583416	1.831688	-0.423500	-0.109691	-0.158923	0.078584	0.396473	0.648107	-0.845189	-0.836694
48	0.746147	0.400614	1.064693	-1.516105	-0.542119	1.143061	-0.373086	-0.204881	0.074485	0.202564	0.200528
49	0.033599	-0.762847	-1.677246	-0.360397	-0.803669	-0.912195	1.932031	-0.784773	1.278919	-1.061508	-1.050840
50	-0.345115	1.037618	1.192895	1.204064	-1.643865	0.547583	-0.099562	0.647621	0.211258	0.511356	0.506216

Table.B4. Transformed Crime Incidents in India – 2011 Dataset

State/UT	Murder (Sec. 302 IPC)	Attempt to Commit Murder (Sec. 307 IPC)	C.H. Not Amount- ing to Murder (Sec. 304, 308 IPC)	Rape (Sec. 376 IPC)	Kidnapping & Abduction (Sec. 363- 310, 371- 373 IPC)	Dacoity (Sec.395- 398 IPC)	Preparation & Assembly For Dacoity (Sec.399- 402 IPC)	BScore (Brown)	Score (Pearson)
1.Andhra Pr.	1.171231	0.482465	1.318533	-0.128862	0.270452	0.821229	1.039301	0.805917	0.794320
2.Arunachal Pr.	-0.750762	-0.661604	-0.846638	-0.656380	-0.738678	-0.693263	-0.336640	-0.788503	-0.777157
3.Assam	-0.264487	-0.817900	0.864876	1.071947	1.416584	0.552980	0.618254	0.537109	0.529381
4.Bihar	1.390559	1.820915	2.305963	2.036492	1.027998	0.313572	1.590260	1.737834	1.712828
5.Chhattisgarh	-0.104895	-0.274076	0.081390	-0.743013	0.090017	0.039608	0.036394	-0.156964	-0.154705
6.Goa	-0.749061	-0.656879	-0.891573	-0.702397	-0.776297	-0.686393	-0.346624	-0.809175	-0.797532
7.Gujarat	-0.077701	-0.435856	0.138155	0.276441	0.169928	-0.556590	0.214459	-0.056268	-0.055458
8.Haryana	0.224880	0.103735	0.097829	0.063892	0.185974	-0.132932	-0.431277	0.043342	0.042718
9.Himachal P.	-0.703013	-0.693452	-0.804186	-0.665783	-0.675543	-0.549860	-0.303938	-0.742205	-0.731526
10.J.&Kashmir	-0.729443	-0.527548	-0.473990	-0.255750	-0.708693	-0.191283	-0.119081	-0.516998	-0.509559
11.Jharkhand	0.354355	0.021602	0.449090	0.009123	0.761657	-0.652195	0.338407	0.211017	0.207980
12.Karnataka	1.308983	0.934878	0.726878	0.669966	0.027256	0.135052	-0.886119	0.560185	0.552124
13.Kerala	-0.110937	0.168662	-0.530231	-0.313733	0.385811	0.314335	-0.942805	-0.133840	-0.131914
14.Madhya Pr.	0.860182	0.844624	1.638571	-0.812338	1.826214	2.129052	0.328536	1.133055	1.116751
15.Maharashtra	1.182805	1.743989	2.159915	1.375505	2.690144	-0.534550	0.107552	1.506685	1.485005
16.Manipur	-0.372511	-0.372251	-0.823242	-0.390072	-0.795137	-0.465671	-0.883949	-0.658423	-0.648949
17.Meghalaya	-0.712146	-0.605090	-0.736752	-0.622008	-0.550537	-0.728277	-0.295741	-0.715907	-0.705606
18.Mizoram	-0.773320	-0.644402	-0.894496	-0.722637	-0.739072	-0.651214	-0.353582	-0.804297	-0.792724
19.Nagaland	-0.753809	-0.625552	-0.886253	-0.685710	-0.767385	-0.692372	-0.334633	-0.798627	-0.787135
20.Odisha	-0.028687	0.768130	1.152367	0.368328	0.984336	-0.376163	0.144444	0.510107	0.502767
21.Punjab	0.153344	0.163959	-0.293102	-0.292145	-0.479566	0.047504	-0.350439	-0.158125	-0.155850
22.Rajasthan	0.234362	-0.224500	0.822155	0.344045	0.318566	1.524207	0.371383	0.543231	0.535415
23.Sikkim	-0.771499	-0.656483	-0.920927	-0.706256	-0.786977	-0.692904	-0.355633	-0.822729	-0.810890
24.Tamil Nadu	0.382014	0.812126	1.555037	0.058794	-0.954678	0.512483	0.548781	0.472833	0.466029
25.Tripura	-0.696118	-0.666377	-0.752213	-0.687200	-0.626010	-0.563404	-0.309351	-0.725669	-0.715228
26.Uttar Pr.	3.240298	3.535300	1.345790	3.299056	2.201805	3.321114	4.754557	3.524207	3.473496
27.Uttarakhand	-0.650280	-0.507686	-0.813120	-0.573944	-0.662581	-0.518848	-0.185434	-0.662752	-0.653215
28.West Bengal	2.765764	2.027622	0.552783	2.587753	1.987720	2.679472	-1.751979	1.958995	1.930806

29.A&N Islands	-0.775454	-0.669730	-0.905792	-0.702744	-0.793419	-0.700878	-0.365087	-0.826292	-0.814402
30.Chandigarh	-0.770100	-0.639688	-0.880447	-0.672400	-0.773810	-0.682158	-0.354216	-0.802816	-0.791264
31.D&N Haveli	-0.780816	-0.666398	-0.896709	-0.691529	-0.782345	-0.725638	-0.367070	-0.825613	-0.813733
32.Daman&Diu	-0.784074	-0.669003	-0.905261	-0.698549	-0.794425	-0.720974	-0.371063	-0.831092	-0.819133
33.Delhi	-0.390283	-1.118165	-0.123583	0.946685	-0.356455	0.535399	0.304536	-0.066323	-0.065369
34.Lakshadweep	-0.785040	-0.673411	-0.913495	-0.707021	-0.808402	-0.711780	-0.374512	-0.836193	-0.824161
35.Puducherry	-0.734342	-0.621958	-0.917321	-0.677556	-0.774450	-0.698658	-0.377694	-0.805703	-0.794110
Source: National Crime Records Bureau, Ministry of Home Affairs, Govt. of India (http://ncrb.nic.in/CD-CII2011/cii-2011/Table%203.1.pdf)									

Table.B5. Transformed Dataset with Outliers

Sl	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	BScore (Brown)	Score (Pearson)
1	-1.688774	-1.122541	-1.540757	-1.289075	-1.225625	-1.173593	-1.194511	-1.611800	-1.588607
2	-1.610575	-1.069114	-1.471953	-1.236476	-1.168420	-1.115488	-1.135166	-1.537280	-1.515159
3	-1.515834	-1.008167	-1.393443	-1.165169	-1.106919	-1.056048	-1.080525	-1.452988	-1.432080
4	-1.422986	-0.979006	-1.312597	-1.103745	-1.043808	-1.008427	-1.023617	-1.377592	-1.357769
5	1.809941	-0.465374	-0.612031	0.038771	0.035582	0.072951	-0.044589	0.165426	0.163046
6	-1.251766	-0.872067	-1.176587	-0.970833	-0.934509	-0.882798	-0.910338	-1.221379	-1.203804
7	-1.161819	-0.827339	-1.100730	-0.917648	-0.861722	-0.832385	-0.850828	-1.143483	-1.127030
8	-1.073033	-0.774236	-1.012924	-0.837809	-0.804857	-0.756646	-0.793584	-1.056413	-1.041212
9	-0.985753	-0.715628	-0.932479	-0.772734	-0.733590	-0.707899	-0.728112	-0.973128	-0.959126
10	-0.916948	-1.261919	1.163508	1.604958	1.140223	1.172300	0.789530	0.573720	0.565465
11	-0.806870	-0.624372	-0.794515	-0.651084	-0.619182	-0.602533	-0.617285	-0.822758	-0.810919
12	-0.718021	-0.579947	-0.710454	-0.577530	-0.563189	-0.530311	-0.556799	-0.739277	-0.728639
13	-0.626614	-0.533294	-0.641331	-0.526752	-0.508703	-0.473786	-0.495207	-0.664116	-0.654560
14	-0.548210	-0.479475	-0.560875	-0.464142	-0.439409	-0.414386	-0.435494	-0.583371	-0.574977
15	0.600157	1.165771	2.762316	-1.477752	0.710969	1.099341	-1.026790	0.714097	0.703821
16	-0.361730	-0.383471	-0.412965	-0.340983	-0.326334	-0.306546	-0.321145	-0.427872	-0.421715
17	-0.268910	-0.325476	-0.319738	-0.272324	-0.248758	-0.237275	-0.259520	-0.336977	-0.332128
18	-0.183303	-0.276518	-0.253346	-0.214718	-0.182006	-0.181016	-0.210438	-0.261748	-0.257982
19	-0.100674	-0.222816	-0.169863	-0.149823	-0.126889	-0.122156	-0.140439	-0.180123	-0.177531
20	0.759513	1.458132	1.637894	1.100581	1.136662	-2.738609	2.296270	1.012868	0.998294
21	0.080238	-0.131592	-0.022215	-0.020083	-0.010659	-0.005317	-0.023628	-0.023157	-0.022824
22	0.172679	-0.076124	0.052815	0.054138	0.052195	0.061982	0.034189	0.061507	0.060622
23	0.259359	-0.035796	0.124775	0.108169	0.105516	0.111450	0.082854	0.132331	0.130427
24	0.347860	0.013966	0.193305	0.167496	0.170210	0.161807	0.145999	0.209840	0.206821
25	0.920426	1.764478	1.070762	0.633021	-1.498657	2.387700	3.436665	1.475291	1.454063
26	0.524033	0.114247	0.357951	0.300975	0.294147	0.282423	0.254287	0.371892	0.367000
27	0.607207	0.166146	0.425874	0.357644	0.357770	0.344600	0.320300	0.450367	0.443887
28	0.696707	0.213324	0.506012	0.429336	0.412011	0.390808	0.382717	0.529312	0.521695
29	0.800780	0.278733	0.591069	0.487639	0.489435	0.460725	0.448204	0.621161	0.612222
30	1.639476	2.876129	1.652693	3.795192	0.540994	1.494253	-0.940401	1.993110	1.964431
31	0.968842	0.359472	0.734928	0.615767	0.605556	0.573674	0.545529	0.769077	0.758011
32	1.060980	0.412190	0.813170	0.675688	0.669467	0.625270	0.608920	0.849806	0.837578
33	1.147662	0.461530	0.892581	0.743473	0.726089	0.684825	0.670400	0.930193	0.916808
34	1.235314	0.513308	0.977105	0.810634	0.787551	0.757005	0.737107	1.015669	1.001054
35	1.610647	2.966845	0.482047	1.065198	4.168860	2.464106	2.035446	2.537792	2.501275

Table.B6. Transformed Lévy Distributed Probability Density Function Dataset

Sl	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	BScore (Brown)	Score (Pearson)
1	5.039421	-0.284487	-1.161836	1.163818	1.906479	1.726509	1.854936	1.712594	1.695382
2	1.680129	4.134063	0.857482	-0.268956	0.876800	2.626207	3.335717	2.446571	2.421982
3	1.089211	2.960932	0.914043	-0.138148	0.766572	1.804884	1.607625	1.662829	1.646117

4	1.236474	1.212355	3.517854	1.903611	0.436460	1.354786	2.713416	2.321371	2.298040
5	1.174089	0.793373	3.432087	1.948767	0.442410	1.113964	2.186289	2.077965	2.057081
6	1.061846	0.786030	2.664295	1.467799	0.533211	0.963642	1.314515	1.637134	1.620680
7	-0.352129	0.888821	0.921728	2.957713	1.483718	1.025887	1.408309	1.619916	1.603635
8	-0.650148	0.908156	0.377945	3.100690	1.687553	0.995760	1.180566	1.492565	1.477564
9	-0.393314	0.886216	0.428653	2.580026	1.522593	0.921574	0.799538	1.313483	1.300282
10	-0.386042	0.599123	0.419127	1.879925	1.492526	0.859495	0.614628	1.072442	1.061663
11	-1.393503	-1.055033	0.159838	-0.329868	1.979628	0.795213	1.063380	0.328794	0.325490
12	-1.233555	-0.984529	0.181331	-0.461138	1.885682	0.764195	0.862123	0.279222	0.276416
13	-0.972202	-0.772709	0.254818	-0.395905	1.739906	0.739407	0.635143	0.303215	0.300167
14	-0.914036	-0.831759	0.023897	-0.392766	1.002193	0.795911	0.351516	0.071179	0.070464
15	-1.366729	-1.687586	-1.048101	-0.704372	-1.477401	1.069505	-0.169115	-0.935000	-0.925603
16	-1.196775	-1.517424	-0.960078	-0.626692	-1.460121	1.047262	-0.269923	-0.869626	-0.860886
17	-1.006889	-1.295181	-0.800600	-0.539577	-1.242000	1.001579	-0.332683	-0.735257	-0.727867
18	-0.952299	-1.216801	-0.984281	-0.731859	-1.377226	-0.007085	-0.310413	-1.008363	-0.998228
19	-1.030659	-1.241123	-1.528058	-1.164369	-1.823469	-2.031388	-0.197561	-1.678266	-1.661399
20	-0.889376	-1.085960	-1.372726	-1.078199	-1.636064	-1.919297	-0.248557	-1.535881	-1.520444
21	-0.754633	-0.942329	-1.212623	-0.983287	-1.460500	-1.739964	-0.304263	-1.383675	-1.369768
22	-0.646967	-0.813351	-1.078971	-0.887273	-1.284514	-1.575025	-0.346269	-1.242886	-1.230395
23	-0.540496	-0.695171	-0.945703	-0.806237	-1.138935	-1.406246	-0.388725	-1.112286	-1.101107
24	-0.443019	-0.583540	-0.828837	-0.720635	-1.008521	-1.267181	-0.425743	-0.994181	-0.984189
25	-0.362366	-0.495289	-0.723864	-0.661098	-0.884077	-1.140703	-0.455572	-0.892169	-0.883202
26	-0.285540	-0.412069	-0.626471	-0.613302	-0.768379	-1.041361	-0.483585	-0.802034	-0.793973
27	-0.228337	-0.355579	-0.548294	-0.561034	-0.696810	-0.915695	-0.508390	-0.724886	-0.717601
28	-0.162831	-0.275424	-0.469613	-0.512151	-0.588688	-0.854732	-0.526463	-0.647439	-0.640932
29	-0.108592	-0.213326	-0.409256	-0.463044	-0.521388	-0.760567	-0.554647	-0.581492	-0.575648
30	-0.054972	-0.156121	-0.338697	-0.419254	-0.442155	-0.673286	-0.575614	-0.513242	-0.508083
31	-0.007147	-0.104798	-0.279784	-0.385479	-0.377442	-0.605918	-0.592358	-0.456942	-0.452349
32	0.032517	-0.062028	-0.230334	-0.353849	-0.319952	-0.556855	-0.607298	-0.410244	-0.406121
33	0.074597	-0.013406	-0.178638	-0.323777	-0.261737	-0.487188	-0.622805	-0.357869	-0.354273
34	0.096152	0.013609	-0.143430	-0.305755	-0.208774	-0.435112	-0.631045	-0.320839	-0.317614
35	0.137855	0.057997	-0.100518	-0.275771	-0.164836	-0.393323	-0.649023	-0.279779	-0.276967
36	0.166822	0.091274	-0.063377	-0.246241	-0.127332	-0.344596	-0.660550	-0.242281	-0.239846
37	0.193319	0.125114	-0.030506	-0.222493	-0.077839	-0.296744	-0.672277	-0.204876	-0.202816
38	0.218155	0.149847	-0.001021	-0.205399	-0.049293	-0.265374	-0.681465	-0.178072	-0.176282
39	0.246806	0.180379	0.035846	-0.185272	-0.006696	-0.231084	-0.689071	-0.144103	-0.142655
40	0.264709	0.205444	0.060146	-0.166806	0.026898	-0.208239	-0.693081	-0.118658	-0.117466
41	0.285345	0.229309	0.086568	-0.146948	0.060120	-0.158315	-0.704936	-0.088586	-0.087695
42	0.304118	0.243048	0.107742	-0.141414	0.073867	-0.134556	-0.714228	-0.072798	-0.072067
43	0.327792	0.275208	0.137247	-0.114580	0.114495	-0.099438	-0.723197	-0.039843	-0.039443
44	0.342783	0.286151	0.156664	-0.106183	0.131071	-0.075109	-0.728121	-0.023371	-0.023136
45	0.360489	0.311772	0.179104	-0.089016	0.164891	-0.052317	-0.733644	0.001277	0.001264
46	0.374491	0.322663	0.194964	-0.079515	0.173626	-0.022723	-0.743109	0.015789	0.015630
47	0.391097	0.341010	0.215621	-0.073387	0.201658	-0.004097	-0.748062	0.034685	0.034336
48	0.403729	0.355024	0.225276	-0.064531	0.207755	0.003503	-0.754160	0.044022	0.043579
49	0.412929	0.364519	0.248109	-0.050149	0.237023	0.041019	-0.754389	0.067209	0.066534
50	0.417682	0.373587	0.265228	-0.006619	0.257013	0.053216	-0.727358	0.092682	0.091750