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Abstract

We examine the optimal consumption decisions of households in a micro-founded framework that assumes two financial frictions: heterogeneity between borrowing and saving households, and endogenous default. We study default in the context of two-period overlapping processes of consumer behavior, assuming that penalty costs are imposed on borrowers if they are delinquent in the first period and are subsequently refinanced by banks. The utility function of borrowing households is specified to include a term reflecting strategic decisions as regards non-payment of debt. From the solution of the household optimization problem, we derive an augmented Euler equation for consumption, which is a function *inter alia* of an expected default factor. We use this equation to calculate in a static equilibrium the optimal value of the percentage of debt repaid. We finally provide an ordering by size of the household discount factor: borrowers who do not repay all of their loans have the lowest discount factor, followed in turn by borrowers who fully repay them and by savers.

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1. Introduction

It is widely recognized that the recent financial market turmoil has been associated with the severest economic contractions since the Great Depression. In the years leading up to the global financial crisis, a combination of factors, including low interest rates and lax lending standards, fueled a rapid increase in household leverage, as measured by the ratio of debt to personal disposable income, which for US households exceeded 130% in 2007 (Glick and Lansing, 2009). Countries with very high increases in this ratio prior to the crisis, such as Denmark (with a ratio of 199%) and Ireland (191%), experienced the largest declines in real consumption (-6.3% and -6.7% from the second quarter of 2008 to the first quarter of 2009, respectively; see Glick and Lansing, 2010). In addition, other research (Mian and Sufi, 2010) has shown that the top 10% leverage growth counties in the US experienced an increase in the household default rate of 12 percentage points (from the second quarter of 2006 through the second quarter of 2009) and also the sharpest decline in durables consumption.

Against these stylized facts, in this paper we study the optimal consumption decisions of households in a micro-founded framework by setting up a dynamic equilibrium model in which we introduce two financial frictions: agent heterogeneity and endogenous default. As to agent heterogeneity, Tobin (1980) indicated that the population of households is not distributed randomly between debtors and creditors. Thus, debtors are frequently young families acquiring homes and consumer durables through borrowing; given the difficulty of borrowing against future income, they are liquidity-constrained and have a high marginal propensity to consume. Middle-aged families, on the other hand, are usually savers accumulating wealth. In our model, we incorporate household heterogeneity, as in Iacoviello (2005); Agenor et al. (2013); Gelain et al. (2013); Suh (2014); Kannan et al. (2012), where the household sector consists of two types of households: saving (unconstrained or patient) households and borrowing (constrained or impatient) households[1]; we assume that the former do not lend directly to the latter. Instead, as in Kannan et al. (2012), we consider that financial intermediaries take deposits from saving households, offering a deposit rate, and lend to borrowing households, charging a borrowing rate. Heterogeneity is also distinguished by the household discount factor, where the usual assumption is that the discount factor of borrowers is smaller than the respective discount factor of savers.

As to the second financial friction, the endogenous default, most models in the literature consider that default happens in a single step (Dubey et al., 2005; Chang, 2005; Goodhart et al., 2009). In Goodhart et al. (2009), households choose what percentage of outstanding debt they will repay, determining the level of the unpaid part. Einav et al. (2012) present a standard consumer theory

[1] Most dynamic stochastic general equilibrium (DSGE) models do not include agent heterogeneity, but assume that there is one representative household, the saving household.

framework where heterogeneous borrowers decide how much to borrow, taking decisions about whether to continue making loan payments or to default. They use a model to derive a set of linear estimating equations that capture purchasing of goods, borrowing, and repayment decisions. They put emphasis on the larger loans which cause higher monthly payments and also have a higher probability of default. Similarly, in Chatterjee et al. (2007), default risk will vary with the size of the loan and household's specific characteristics. Moreover, in Dubey et al. (2005), default can either occur for strategic reasons[2] or be due to ill fortune[3]. However, most of the literature presents endogenous default only for the business sector and not for the household sector in DSGE frameworks.

The existence of default affects banks' lending guidelines and induces them to follow a stricter path, especially for the cases of strategic default (Huber et al., 2012). To discourage borrowers from defaulting, financial intermediaries impose costs, either directly through penalty on the interest rate for defaulters or indirectly by raising the lending interest rates for borrowers with a past record of bankruptcy. As noted by Chatterjee et al. (2007), households for whom the period begins with a record of bankruptcy cannot get new loans. However, with an exogenous probability, their bad credit rating is expunged after a certain period, since an individual's credit history is kept only for a finite number of years. Borrowers enter again the credit market, being charged a pecuniary cost which affects their financial constraint. In de Walque et al. (2010), the cost of default affects both the utility function as a non-pecuniary burden (social stigma, reputation cost) and the budget constraint as a pecuniary burden, although these authors' analysis refers to default of firms. In a similar vein, Einav et al. (2012) mention that default affects utility of households due to potential costs associated with default, such as a constant per-loan indirect cost. It is argued (Dubey et al., 2005) that banks should charge penalties, irrespective of the cause of default, since it is difficult to study the strategic decision. Strategic defaulters have every incentive to disguise themselves as agents who are not able to repay their contractual obligations (Guiso et al., 2009).

In our model, we extend the analysis of loan default to two periods. In a given period, some of the unpaid loans of households come to default, while the rest is delinquent debt refinanced by banks in the following period and carrying a penalty cost. A percentage of delinquent debt is recovered in the second period but the other part also comes to default.

[2] Guiso et al. (2009) determine the reasons why households may strategically default on their debts, such as economic and moral incentives. Also, borrowers may have the incentive to use their borrowed money to finance consumption or simply to hide loan proceeds from their creditors, defaulting on their promise to repay (Manove et al., 2001).

[3] Baltensperger (1975) argues that the decision to default means that it is possible that the borrowing household's income is not sufficient to cover its debt.

Our work contains several novel features. First, unlike typical DSGE models, our household sector is split into two different sub-sectors, borrowing and saving households. Borrowers always borrow while savers always save. As a matter of fact, this dichotomy explains why optimal consumption decisions of these two groups are essentially independent of each other. For borrowing households we derive, by solving the household maximization problem, an augmented Euler equation, which determines the household optimal consumption, as a function *inter alia* of an expected default variable. Second, as mentioned above, default is studied in the context of an infinite number of overlapping processes of consumer behavior with periodicity two. Thus, borrowers do not repay all of their debt in a given period. From the unpaid debt, a part comes to default in the same period, while the other part is refinanced in the following period by banks, who charge a penalty premium on the interest rate; again, a proportion of refinanced debt is not repaid in that period but also comes to default. Third, we specify the borrowing household's utility function by including a term to reflect the household's strategic decisions regarding non-payment of the debt. Interestingly, this utility component is shown to be an indispensable part of the specification since, by including it, the Hessian matrix of the household's maximization problem is negative definite and therefore the solution is a maximum. Without it, the said Hessian matrix is non-definite and therefore does not guarantee the existence of a maximum. Fourth, the percentage of debt repaid is a basic decision variable in the household's optimization problem. The optimal value of this variable in a static equilibrium, in which there is optimal consumption smoothing, is shown to depend on a number of determinants, specifically the time preference rate, the borrowing interest rate, the penalty premium added on the interest rate and the percentage of unpaid debt which is refinanced by banks. Finally, an ordering by size is provided for the discount factor of the different types of households: borrowers who do not repay all of their loans have the lowest discount factor, followed in turn by borrowers who fully repay their loans and by savers.

The paper is organized as follows. In Section 2, the borrowing household's optimal consumption decisions are analyzed in the framework of an augmented Euler equation for consumption, which includes an expected default variable in addition to other variables already in the literature. Further, the determinants of the household decision to repay its debt obligations are analyzed in a static equilibrium context. In Section 3, the behavior of the saving household is briefly examined and the two types of households are combined in the same model. This section also provides an ordering by size of discount factors (or time preference rates) of households. Section 4 concludes and suggests some directions for further work.

2. Optimal decisions for borrowing households

In this section we develop a dynamic equilibrium model that determines the optimal consumption and default decisions for borrowing households[4,5]. In the following we assume that there is a population of infinitely-lived homogeneous[6] households that consume a consumption bundle. The representative household does not save but borrows from banks (the loan supply of which is perfectly elastic at the prevailing lending rate) to support consumption smoothing[7], i.e. it is a borrowing household. The household derives utility from consuming goods and services, disutility from supplying labor to producers of goods and services and also utility from the unpaid part of the debt which is a result of its choice (strategic decision). Further, we assume that the household can default on its loan[8]. The household can become delinquent (an earlier stage of default)[9] or come to default for reasons such as cash flow problems (due to negative income shocks or downturn of the business cycle) or failure to meet banks' minimum credit requirements. Also, according to Huber et al. (2012), if agents were allowed to borrow and the marginal utility of income was more than the marginal disutility of debt, this would be a good reason for them to borrow more and to default, applying strategic defaulting[10,11].

In the current period, the household buys consumer goods and services and works in the production process, earning income for labor services provided. At the end of each period, the household can obtain a loan from banks (one-period loan[12]). At the end of the current period, part of the previous period loan, including interest, is serviced, while the rest is not. The regular servicing of the loan is followed by an extension of a new one-period loan by the banks and the same process runs in the following period. In the case where part of the previous period's loan is not serviced, the household either obtains a new loan in the following period[13] (the proceeds of which are used to repay the

[4] The analysis is carried out in discrete time.
[5] The model is characterized by an infinite number of two-period overlapping processes.
[6] Homogeneity means that households have the same preferences and do not face any idiosyncratic shocks or frictions.
[7] The notion of consumption smoothing usually describes an attempt to keep consumption similar in each period.
[8] Not all households are assumed to default. Thus, default is measured by the default rate, i.e. the ratio of the value of debt in default to the value of total debt. It is in this sense that the default rate refers to the representative household.
[9] In the household decision not to repay, delinquency is preferable, at least initially, since surviving via delinquency may help to avoid any costs associated with default (Athreya et al., 2012).
[10] In Guiso et al. (2009), it is reported that survey data for the US indicate that about one out of four households that default applies strategic defaulting.
[11] As discussed in the literature (e.g. Guerrieri and Iacoviello, 2013), one way to discourage defaulters is to configure an appropriate macro-prudential framework with well-planned and effective tools, such as default penalties and strict indicators (e.g. loan-to-value ratio, debt-to-income ratio), which would factor into default decisions.
[12] Roszbach (2004) notes that in financial markets with perfect information any optimal multi-period financial contract can be obtained by a sequence of one-period loan agreements, while under asymmetric information things become more intricate. Also, Besanko and Thakor (1987) develop a single-period credit model under asymmetric information and assume that no borrower has an initial endowment and each must approach a lender for a one-period loan.
[13] We assume that banks refinance when they assess that households meet certain credit requirements set by them, e.g. debt-to-income threshold or credit history.

unpaid loan), being charged a penalty premium on the interest rate[14], or defaults. Again, only a proportion of households that renewed their loan for the next period are in a position to pay off their debt, while the rest also come to default. The above two groups of defaulters determine the household default rate. The debt in default is written off from banks' balance sheets.

The household is assumed to have a life-time utility function:

$$U_t = E_t \sum_{j=0}^{\infty} (\beta^b)^j \, u\left(C_{t+j}^b, N_{t+j}^b, \Omega_{t+j}\right) \tag{1}$$

where E_t denotes the expectation of the household, conditional on information available at time t, β^b is the subjective discount factor (capturing the idea of impatience)[15], which discounts future utility, with $0<\beta^b<1$, and u denotes utility which is related to real consumption (C_{t+j}^b), working hours (N_{t+j}^b, expressed as a ratio to total available time per day) and unpaid debt, reflecting strategic decisions (Ω_{t+j}) [16]. The utility function is assumed to be time separable and twice differentiable with respect to consumption, working hours and unpaid debt. The marginal utility of consumption and unpaid debt is positive and non-increasing, while that of working hours is negative and non-decreasing. Equation (1) simply states that the household is interested in life-time utility U_t obtained as the present discounted value of current and all future levels of expected utility. The functional form of the utility function is of the constant relative risk aversion type:

$$u(C_t^b, N_t^b, \Omega_t) = \frac{(C_t^b)^{1-\sigma}}{1-\sigma} - \frac{(N_t^b)^{1+\varphi}}{1+\varphi} + \frac{\Omega_t^{1-\psi}}{1-\psi} \tag{2}$$

where $1/\sigma$, $1/\varphi$, $1/\psi$ is the inter-temporal elasticity of substitution of consumption, working time and unpaid debt, respectively.

The household is also assumed to be subject to a sequence of budget constraints (in nominal terms), which describe its feasible choices through time. These constraints are of the form (for period t):

$$P_t C_t^b + \mu_t(1+i_{t-1}^L)[L_{t-1} - k_{t-1}(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2}] + \mu_t(1+i_{t-1}^L + f_{t-1})k_{t-1}(1-\mu_{t-1})$$

[14] The premium may be set by the financial regulator and its magnitude is determined by factors such as credit history and reputation of borrowers, search costs of banks and risk parameters.

[15] The impatient household values future utility less than present utility.

[16] At the end of each period, the household takes a decision to repay or not to repay its previous period debt. Regarding the second leg of the decision, there are two possible options: a) one in which households do not afford to repay, having already consumed all of their borrowed money (cash flow problems), and b) the strategic decision not to repay. In the latter case, the amount of money from the loan is neither consumed nor used for repaying banks. Further, the household cannot invest in a financial asset (deposit, bond), since in such an instance banks would immediately liquidate their claims. For this reason, the household's real money balances at the end of the period, after consumption goods have been purchased, increase current utility only by holding them (cf. Walsh, 2010).

$$(1 + i_{t-2}^L)L_{t-2} = W_t N_t^b + [L_t - k_t(1 - \mu_t)(1 + i_{t-1}^L)L_{t-1}] \tag{3}$$

Equation (3) displays the household's receipts (inflows) and payments (outflows) stemming from its income-generating and financing activities. The left-hand side contains payments. The first term is consumption expenditure, where P_t is the price index of consumer goods and services.

The second term involves the assumption that, concerning the loan contracted at the end of period *t-2*, i.e. L_{t-2}, the bank refinances only a part of the loan that has not been serviced at the end of period *t-1*, i.e. $k_{t-1}(1 - \mu_{t-1})(1 + i_{t-2}^L)L_{t-2}$, where i_{t-2}^L is the lending rate of period *t-2* in nominal terms, $1 - \mu_{t-1}$ is the percentage of the loan (plus interest), i.e. $(1 + i_{t-2}^L)L_{t-2}$, that has not been repaid at the end of period *t-1* ($0 < \mu \leq 1$) and k_{t-1} is the percentage of the unpaid loan at the end of *t-1* that is refinanced by the bank ($0 < k \leq 1$). The terms of that refinancing will be discussed below. Excluding this component, $L_{t-1} - k_{t-1}(1 - \mu_{t-1})(1 + i_{t-2}^L)L_{t-2}$ represents net borrowing of households at the end of period *t-1*, which is made for consumption purposes, and $\mu_t(1 + i_{t-1}^L)$ $[L_{t-1} - k_{t-1}(1 - \mu_{t-1})(1 + i_{t-2}^L)L_{t-2}]$ is the percentage of that borrowing (including interest) which is repaid at the end of period *t*, while the rest is not repaid.

Further, the third term focuses on the unpaid part of the loan at the end of period *t-1* that is assumed to be refinanced by banks. The interest rate charged on that loan is the lending rate i_{t-1}^L augmented by a penalty premium f_{t-1}[17] and the proceeds of the loan are used to repay the unpaid loan. Again, as already indicated, only a proportion of households that renewed their loan are in a position to repay their debt at the end of period *t* (the rest coming to default) so that the amount of the loan paid back is $\mu_t(1 + i_{t-1}^L + f_{t-1})[k_{t-1}(1 - \mu_{t-1})(1 + i_{t-2}^L)L_{t-2}]$.

The right-hand side includes receipts. The first term shows the income earned from labor, where W_t is the nominal wage rate, and the other term refers to net borrowing in period *t*, namely the loan obtained (L_t) less the amount used by the household to repay its unpaid but refinanced by the bank loan, which reduces the household default of the period.

Rearranging the terms of equation (3) and doing some simple algebra gives a simplified presentation of the budget constraint:

$$P_t C_t^b + [\mu_t + k_t(1 - \mu_t)](1 + i_{t-1}^L)L_{t-1} + \mu_t f_{t-1} k_{t-1}(1 - \mu_{t-1})(1 + i_{t-2}^L)L_{t-2} = W_t N_t^b + L_t \tag{4}$$

[17] In Huber et al. (2012), the imposition of default penalties, in addition to preventing strategic bankruptcy from borrowers, is shown to also resolve the multiplicity of equilibria that is known to exist in closed economies, and in Dubey et al. (2005), default penalties are imposed on agents who fail to repay their debts, irrespective of the cause of default.

This form of the budget constraint has an interesting reading. In particular, the second term is the proportion of capitalized debt that has been repaid by households or has not been repaid and has been refinanced by the banks. Also, the third term is the amount of the penalty paid on the previous period's capitalized debt that has been refinanced. The other terms are as in equation (3).

Given the analytical exposition of the budget constraint just presented, we can write an expression for the household default rate. The amount of household debt in default at the end of period *t* consists of two parts: a) the amount of capitalized debt, that has not been repaid by the household and has not been refinanced by the banks and b) the amount of the unpaid penalty. Thus, the household default rate (d_t) is the ratio of the sum of the above two components of default divided by the value of capitalized debt:

$$d_t = \frac{(1-k_t)(1-\mu_t)(1+i_{t-1}^L)L_{t-1} + (1-\mu_t)f_{t-1}k_{t-1}(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2}}{(1+i_{t-1}^L)L_{t-1}} \tag{5}$$

Now the problem facing the representative household is to choose its decision variables, i.e. consumption, working hours, loans and the percentage of debt repaid, to maximize utility given the state variables, i.e. wage rate, interest rate, price level, penalty premium on the interest rate and the percentage of unpaid debt which is refinanced. Thus, the problem is to maximize

$$U_t = E_t \sum_{j=0}^{\infty} (\beta^b)^j \left(\frac{(C_{t+j}^b)^{1-\sigma}}{1-\sigma} - \frac{(N_{t+j}^b)^{1+\varphi}}{1+\varphi} + \frac{\Omega_{t+j}^{1-\psi}}{1-\psi} \right), \tag{6}$$

where $\Omega_{t+j} = \omega_{t+j}(1-\mu_{t+j})(1+i_{t-1+j}^L)L_{t-1+j}$ (7)

and ω refers to the unpaid part of the debt in period *t,* which is a result of strategic decision, with $(0 < \omega < 1)$ [18],

under a set of flow budget constraints (like equation 4).

Combining equations (4), (6) and (7), we form the Lagrangian of this problem:

$$\mathcal{L} = \sum_{j=0}^{\infty} (\beta^b)^j \left\{ \begin{array}{c} \dfrac{(C_{t+j}^b)^{1-\sigma}}{1-\sigma} - \dfrac{(N_{t+j}^b)^{1+\varphi}}{1+\varphi} + \dfrac{[\omega_{t+j}(1-\mu_{t+j})(1+i_{t-1+j}^L)L_{t-1+j}]^{1-\psi}}{1-\psi} + \\ \lambda_{t+j} \left[\begin{array}{c} W_{t+j}N_{t+j}^b + L_{t+j} - P_{t+j}C_{t+j}^b - [\mu_{t+j} + k_{t+j}(1-\mu_{t+j})](1+i_{t-1+j}^L)L_{t-1+j} - \\ -\mu_{t+j}f_{t-1+j}k_{t-1+j}(1-\mu_{t-1+j})(1+i_{t-2+j}^L)L_{t-2+j} \end{array} \right] \end{array} \right\} \tag{8}$$

where λ is the Lagrange multiplier.

[18] Given the unpaid part of the debt, which is due to external economic factors, the percentage of debt repaid (μ) bears a constant relationship to the percentage of unpaid debt which is a result of strategic decision (ω); the latter variable therefore does not appear in the solution of the optimization problem.

We take first-order conditions for consumption, working hours, loans and the percentage of debt repaid, as shown in Appendix 1.

Combining equations (1.a) and (1.b) from this Appendix, yields the familiar result (see e.g. Gali, 2008):

$$-\frac{u_{N_t^b}}{u_{C_t^b}} = \frac{W_t}{P_t} \tag{9}$$

where $u_{N_t^b}$ is the marginal utility of working hours and $u_{C_t^b}$ is the marginal utility of consumption, describing the optimal consumption-leisure decision. According to this equation, the marginal rate of substitution between leisure and consumption equals the real wage, and households cannot gain utility by shifting from work to consumption.

Further, by combining equations (1.a) and (1.e) and writing equation (1.a) one period forward, we derive the Euler equation[19] for the household optimal consumption; it is essentially an equilibrium relation[20], where for given values of the state (exogenous) and other decision variables, there is no tendency for the path of consumption to change:

$$\frac{(C_t^b)^{-\sigma}}{P_t} = \beta^b \frac{E_t\{(C_{t+1}^b)^{-\sigma}\}}{E_t\{P_{t+1}\}}(1+i_t^L)\left[1+\frac{(1-E_t\{\mu_{t+1}\})f_t k_t(1-\mu_t)(1+i_{t-1}^L)L_{t-1}}{(1+i_t^L)L_t}\right] \tag{10}$$

The Euler equation determines the borrowing household's optimal consumption as a function *inter alia* of a component of the expected default rate, namely the expected amount of the unpaid penalty as a proportion of the value of capitalized debt (cf. eq. 5).

Using a log-linear approximation to equation (10) yields (cf. Gali, 2008):

$$c_t^b = E_t\{c_{t+1}^b\} - \frac{1}{\sigma}\left[i_t^L - E_t\{\pi_{t+1}\} - \rho^b + \frac{(1-E_t\{\mu_{t+1}\})f_t k_t(1-\mu_t)(1+i_{t-1}^L)L_{t-1}}{(1+i_t^L)L_t}\right] \tag{11}$$

where $c_t^b = \ln C_t^b$, $E_t\{c_{t+1}^b\} = \ln E_t\{C_{t+1}^b\}$, $E_t\{\pi_{t+1}\} = \ln\left(\frac{E_t\{P_{t+1}\}}{P_t}\right)$, $\rho^b = -\ln \beta^b$,

[19] This equation is the optimal solution to the representative household maximization problem, as proved in Appendix 2. Without the term Ω_t in the objective function (1), the Hessian matrix is non-definite and therefore does not guarantee the existence of a maximum.

[20] All endogenous (decision) variables are at their equilibrium values since all first-order conditions are satisfied and the Hessian matrix is negative definite implying that these values are necessarily maxima.

This equation is augmented relative to that of the literature (see, among others: Gali, 2008; Romer, 2012) by including a default premium over the real interest rate[21]. The equation presents the relation between the current level of consumption and the next period's expected level of consumption[22], the real interest rate[23], the time preference rate and an expected default factor. Consumption decisions of the current period are affected negatively by the default factor, since any rise in next period's expected default decreases current consumption, so that the future consequences of default, either pecuniary (default penalty, inability of future borrowing) or non-pecuniary (bad reputation), are limited. Further, any policy move either by the monetary authorities who adjust the short-term nominal interest rate thus affecting the corresponding real rate in the short run, or by the supervisory authorities who raise the penalty premium on the interest rate affecting upwards the expected default rate, or by the commercial banks in their refinancing policy, alters the optimal consumption path. Thus, the Euler equation includes the degree of control over the household available to the financial system (central bank, financial regulator, commercial banks) in the most basic model, i.e. that of a closed economy without a public sector.

Using the Euler equation, which as already indicated is a dynamic equilibrium relation, to deduce the long-run static equilibrium relation, i.e. one in which all variables are time-invariant[24], we obtain the following expression for the optimal value of the decision variable μ, i.e. the percentage of the debt repaid:

$$\mu = 1 - \sqrt{(\rho^b - i^L)/fk} \tag{12}$$

As can be seen from this equation, the variable μ depends on four determining factors: the penalty premium on the interest rate *(f)*, the percentage of unpaid debt which is refinanced *(k)*, the real interest rate (i^L) and the rate of time preference (ρ^b). The penalty premium has a positive effect on μ, since its rise induces households to repay more in order to avoid incurring higher premiums. The percentage of unpaid debt which is refinanced has similarly a positive effect, since the higher level of obligations arising from an increase in k is expected to lead to a higher μ. The interest rate has also a positive impact on μ, reflecting the household's willingness to repay more when i^L is higher

[21] As noted by Baltensperger (1975), it can be reasonably assumed that individuals expect that the rate of interest on a loan has to be adjusted upwards in the presence of default risk since in this case the interest rate should contain a risk premium.

[22] Most of the empirical literature on the Euler equation for consumption relies on a purely backward-looking specification of the optimal consumption decisions. Exceptions are Fuhrer and Rudebusch (2004) and Goodhart and Hofmann (2005), who allow for forward-looking elements.

[23] According to Etro (2009), optimal consumption is increasing over time if the real interest rate is larger than the rate of time preference. Consumption smoothing is optimal if they are equal.

[24] This relation corresponds to a situation of optimal consumption smoothing (see footnote 23 above).

so as to mitigate the higher risk of future default. Finally, an increase in the time preference rate implies a lower discount factor[25] and hence its effect on μ will be negative.

By rearranging the terms of equation (12), we arrive at the following expression for the real interest rate:

$$i^L = \rho^b - (1-\mu)^2 fk \tag{13}$$

whereby the default factor is negatively associated with the real interest rate.

In equation (12), when $\mu = 1$, namely households repay all of their debt and there is no household default in the economy, the interest rate (real) equals the default-free rate of time preference ρ^w, i.e.

$$i^L = \rho^w, \tag{14}$$

a standard result in models without consumer default (cf. Barro and Sala-i-Martin, 2004). When, on the other hand, $0 < \mu < 1$, which signifies the existence of default, the real interest rate can be shown to vary in the following interval:

$$\rho^b - fk < i^L < \rho^b \tag{15}$$

Thus, the household borrowing cost (real interest rate) is bounded between a minimum value equal to the benefit in terms of increase in future utility minus the extra cost of the refinanced debt due to the imposition of the penalty premium on the interest rate, and a maximum value equal to the above benefit.

3. Optimal decisions for borrowing and saving households

In this section, we assume that the household sector consists of two different types of households, the borrowing and the saving household. The former is characterized by a lower inter-temporal discount factor, which generates an incentive for it to borrow, i.e. this household is financially constrained. As in Agenor et al. (2013) and Suh (2014), savers can hold financial assets and trade in asset markets, while borrowers do not participate in asset markets[26]. They have also the same

[25] In the next section, we show that borrowers who do not repay all of their loans have a lower discount factor compared with either borrowers who repay fully their loans or savers.

[26] Saving households will always save and borrowing households will always borrow. The household in solving the optimization problem will never choose to engage in both saving and borrowing. This is ensured by the condition that the

inter-temporal elasticities of substitution. We assume a fraction ψ of households to be borrowers and the remaining fraction $1\text{-}\psi$ to be savers.

The maximization problem of the borrowing household has been analyzed in section 2, while the behavior of the saving household is going to be examined briefly in the following subsection. Finally, the combination of the two types of households in the same model is going to be analyzed in the second subsection.

3.1. The optimization problem of the saving household

The saver's utility maximization problem presents no novelties but we show it here for the sake of completeness. To ensure comparability of the results, our analysis uses the same general assumptions as in the case of the borrowing household. The only difference is that the saving household does not borrow but saves, investing its saving in deposits[27], the maturity of which is assumed to be one period. At the end of the current period, deposits including interest are withdrawn.

The saving household is further assumed to have a life-time utility function:

$$U_t = E_t \sum_{j=0}^{\infty} (\beta^s)^j \, u\big(C^s_{t+j}, N^s_{t+j}\big) \tag{16}$$

having as arguments only consumption and working hours; β^s is the discount factor of savers. As with borrowing households, the functional form of the utility function is of the constant relative risk aversion type:

$$u(C^s_t, N^s_t) = \frac{(C^s_t)^{1-\sigma}}{1-\sigma} - \frac{(N^s_t)^{1+\varphi}}{1+\varphi} \tag{17}$$

Also, the saving household is assumed to be subject to an infinite number of budget constraints, which have the following form (for period *t*):

$$P_t C^s_t + D_t = W_t N^s_t + (1 + i^D_{t-1}) D_{t-1} \tag{18}$$

where D_t are deposits at the end of period *t* and i^D_{t-1} is the deposit rate of the previous period.

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saving interest rate is lower than the borrowing rate. In fact, the borrowing rate is a markup over the saving rate since banks which intermediate between borrowers and savers need to cover the costs of intermediation. However, the relation between these two rates need not concern us further here.

[27] We assume that bonds that may be issued by the private sector (firms) are perfect substitutes for deposits as a means of saving, i.e. they have the same rate of return.

The household's maximization problem consists in choosing its decision variables, i.e. consumption, working hours and deposits, so as to maximize utility subject to an infinite number of inter-temporal budget constraints of the form of eq. (18). The Lagrangian of this problem is set up by combining equations (16), (17) and (18):

$$\mathcal{L} = \sum_{j=0}^{\infty} (\beta^s)^j \left\{ \frac{(C^s_{t+j})^{1-\sigma}}{1-\sigma} - \frac{(N^s_{t+j})^{1+\varphi}}{1+\varphi} + \lambda_{t+j}\left[W_{t+j}N^s_{t+j} + \left(1+i^D_{t-1+j}\right)D_{t-1+j} - P_{t+j}C^s_{t+j} - D_{t+j}\ \right]\right\} \quad (19)$$

In Appendix 3, we present the first-order conditions for consumption, working hours and deposits (eqs 3.a, 3.b and 3.d). From these equations the standard Euler equation for consumption can be derived, which is commonly encountered in the literature.

$$c^s_t = E_t\{c^s_{t+1}\} - \frac{1}{\sigma}\left[i^D_t - E_t\pi_{t+1} - \rho^s\right] \quad (20)$$

where ρ^s is the time preference rate of savers.

The long-run static equilibrium relation corresponding to this equation is:

$$i^D = \rho^s, \quad (21)$$

i.e. the (real) deposit rate equals the time preference rate of savers.

3.2. The optimization problem of borrowing and saving households

The Euler equations for consumption are next examined for both borrowing and saving households. The two equations are equations (11) and (20) of sections 2 and 3.1 above. By noting that the optimization problems of the two households are essentially independent of each other, we can combine them in a weighted average sense in one representative Euler equation for total consumption. Thus, by multiplying equation (4.a) of Appendix 4 by ψ[28] and equation (4.b) by $1-\psi$ and by adding outcomes we obtain the following result in terms of total household consumption:

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\psi\left[i^L_t - E_t\{\pi_{t+1}\} - \rho^b + \frac{(1-E_t\{\mu_{t+1}\})f_t k_t(1-\mu_t)(1+i^L_{t-1})L_{t-1}}{(1+i^L_t)L_t}\right]$$

[28] Assuming that households do not move between the two groups, which implies that ψ remains constant over time.

$$-\frac{1}{\sigma}(1-\psi)[i_t^D - E_t\pi_{t+1} - \rho^s]$$ [29] (22)

Overall consumption is a weighted average of consumption of both types of households (Wieland and Wolters, 2013), where the weights are the percentage of each type of households in the population. As noted by Suh (2014), the separation of the two Euler equations is useful to see how monetary policy and macro-prudential policy affect differently the inter-temporal consumption decisions of borrowing and saving households. From the analysis already presented above, it can be understood that monetary policy affects decisions of both households, while macro-prudential policy only affects decisions of borrowing households. This stems from the fact that macro-prudential policy tools, e.g. loan-to-value ratios and default penalties, affect borrowing households' behavior, in particular through loans, which are a basic determinant of the default rate (see equation 5 above).

A critical parameter in the household's optimization problem is the household's discount factor or, equivalently, its time preference rate[30]. The discount factor is a measure of how strongly consumer choices depend on expectations because it weighs current utility and expected utility from future choices. A change in expectations affects observable choices of consumers with higher discount factors more than choices of consumers with smaller discount factors. Therefore, an ordering by size of discount factors is required for all types of households, particularly in empirical calibrations of dynamic models of consumer behavior to make them relevant for optimal policy making.

The literature compares the discount factors of savers and borrowers who repay all of their debts. Because under optimal consumption smoothing, the time preference rate of each of these two groups is equal to the respective interest rate, and the borrowing rate is higher than the saving rate, the borrowers' time preference rate is also higher than that of savers ($\rho^w > \rho^s$); the reverse is true for the discount factor, i.e. $\beta^s > \beta^w$.

In our analysis we distinguish between borrowing households that repay all of their debts and those that do not repay the debts fully, and thus we admit the possibility of default in the latter's decisions. The parameter μ in the model developed, where μ lies between 0 and 1, characterizes this subgroup of borrowers. Equation (13) above shows that these borrowers' time preference rate is the borrowing rate (real) plus the default premium $(1-\mu)^2 fk$, which is a positive quantity, and,

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[29] With no capital, and therefore no investment, the economy's aggregate resource constraint is simply $y_t = c_t$, i.e. all output must be consumed, and equation (22) can be interpreted as an aggregate demand equation.

[30] The discount factor and the time preference rate are connected with the following relation: $\rho = -ln\beta$, see also eq. (11) above.

therefore, is higher than the default-free time preference rate, i.e. $\rho^b > \rho^w$. This provides a unique ordering by size of time preference rates as follows:

$$\rho^s < \rho^w < \rho^b \tag{23}$$

or a reverse ordering for discount factors

$$\beta^s > \beta^w > \beta^b \tag{24}$$

These calculations assess the discount factor/time preference rate when there is a possibility of non-payment, and consequently a default risk, associated with borrowing households.

4. Conclusions

This paper has examined the optimal consumption decisions of households in a micro-founded framework that assumes two financial frictions: agent heterogeneity and endogenous default. We set up a dynamic equilibrium model in which the household sector consists of two different sub-sectors: borrowing households and saving households. For the former, by solving the optimization problem, we derived an augmented Euler equation, which includes an expected default variable as an additional argument to those already in the literature.

In our model, default is studied in the context of an infinite number of overlapping processes of consumer behavior with periodicity two. In this respect, borrowers were assumed not to repay all of their debt in a given period. A percentage of unpaid debt comes to default in the same period, for reasons such as cash flow problems, while the other part is refinanced by banks in the following period and a penalty premium is added onto the interest rate of the loan. Again, a proportion of refinanced debt is not repaid in that period but also defaults.

We further obtained a static equilibrium relation for the proportion of debt repaid. In particular, according to our results, this decision variable was shown to depend on a number of determinants: the time preference rate, the borrowing interest rate, the penalty premium on the interest rate and the percentage of unpaid debt which is refinanced by banks. In addition, an ordering by size was provided for the discount factor, with borrowers who do not repay all of their loans having the lowest discount factor, followed in turn by borrowers who fully repay their loans and finally by savers.

Our work represents a promising line of research for incorporating macro-prudential tools in one of the basic components of DSGE models, making the latter more appropriate for analyzing monetary and macro-prudential policies. The results of our paper may generate interesting trade-offs in a typical DSGE model for the economy as a whole. This, however, remains an important task for future work.

References

Abraham-Frois, G., Berrebi, E., 1997. Prices, Profits and Rhythms of Accumulation. Cambridge: Cambridge University Press.

Agenor, P-R., Alper, K., da Silva, L. P., 2013. Capital regulation, monetary policy, and financial stability. International Journal of Central Banking 9 (3), 193-238.

Athreya, K., Sanchez, J. M., Tam, X. S., Young, E. R., 2012. Bankruptcy and delinquency in a model of unsecured debt. Federal Reserve Bank of St. Louis Working Paper 42.

Baltensperger, E., 1975. Uncertainty, risk of default and the savings-consumption decision. Zeitschrift für Nationalökonomie 35, 89-97.

Barro, R. J., Sala-i-Martin, X., 2004. Economic Growth, (2nd ed.). Cambridge MA: MIT Press.

Besanko, D., Thakor, A. V., 1987. Competitive equilibrium in the credit market under asymmetric information. Journal of Economic Theory 42, 167-182.

Chang, G., 2005. Asset prices and default-free term structure in an equilibrium model of default. Journal of Business 78, 997-1022.

Chatterjee, S., Corbae, D., Nakajima, M., Rios-Rull, J. V., 2007. A quantitative theory of unsecured consumer credit with risk of default. Econometrica 75, 1525-1589.

de Walque, G., Pierrard, O., Rouabah, A., 2010. Financial (in)stability, supervision and liquidity injections: A dynamic general equilibrium approach. Economic Journal 120, 1234-1261.

Dubey, P., Geanakoplos, J., Shubik, M., 2005. Default and punishment in general equilibrium. Econometrica 73, 1-37.

Einav, L., Jenkins, M., Levin, J., 2012. Contract pricing in consumer credit markets. Econometrica 80, 1387–1432.

Etro, F., 2009. Endogenous Market Structures and the Macroeconomy. New York and Berlin: Springer.

Fuhrer, J. C., Rudebusch, G. D., 2004. Estimating the Euler equation for output. Journal of Monetary Economics 51, 1133-1153.

Gali, J., 2008. Monetary Policy, Inflation, and the Business Cycle. Princeton: Princeton University Press.

Gelain, P., Lansing, K. J., Mendicino, C., 2013. House prices, credit growth, and excess volatility: Implications for monetary and macroprudential policy. International Journal of Central Banking 9 (June) 219-276.

Glick, R., Lansing, K. J., 2009. U.S. household deleveraging and future consumption growth. FRBSF Economic Letter. May 15.

Glick, R., Lansing, K. J., 2010. Global household leverage, house prices, and consumption. FRBSF Economic Letter. January 11.
Goodhart, C., Hofmann, B., 2005. The IS curve and the transmission of monetary policy: is there a puzzle? Applied Economics 37, 29-36.
Goodhart, C.A.E., Osorio, C., Tsomocos, D.P., 2009. Analysis of monetary policy and financial stability: A new paradigm. CESifo, Working Paper 2885.
Guiso, L., Sapienza, P., Zingales, L., 2009. Moral and social constraints to strategic default on mortgages. NBER Working Paper No. 15145.
Guerrieri, L., Iacoviello, M., 2013. Collateral constraints and macroeconomic asymmetries. Board of Governors of the Federal Reserve System, International Finance Discussion Paper No. 1082.
Huber, J., Shubik, M., Sunder, S., 2012. Default penalty as a selection mechanism among multiple equilibria. Cowles Foundation for Research in Economics, Yale University, Discussion Paper No. 1730R.
Iacoviello, M., 2005. House prices, borrowing constraints, and monetary policy in the business cycle. American Economic Review 95, 739-764.
Kannan, P., Rabanal, P., Scott, A., 2012. Monetary and macroprudential policy rules in a model with house price booms. The B.E. Journal of Macroeconomics 12 (1), Article 16, 1-44.
Manove, M., Padilla, A. J., Pagano, M., 2001. Collateral versus project screening: A model of lazy banks. RAND Journal of Economics 32, 726-744.
Mian, A. R., Sufi, A., 2010. Household leverage and the recession of 2007 to 2009. NBER Working Paper 15896.
Romer, D., 2012. Advanced Macroeconomics (4^{th} ed.). New York: McGraw-Hill.
Roszbach, K., 2004. Bank lending policy, credit scoring and the survival of loans. Review of Economics and Statistics 86, 946-958.
Suh, H., 2013. Macroprudential policy: its effects and relationship to monetary policy. Federal Reserve Bank of Philadelphia, Working Paper No. 12-28.
Suh, H., 2014. Dichotomy between macroprudential policy and monetary policy on credit and inflation. Economics Letters 122, 144-149.
Tobin, J., 1980. Asset Accumulation and Economic Activity: Reflections on Contemporary Macroeconomic Theory. Chicago: University of Chicago Press.
Wälde, K., 2011. Applied Intertemporal Optimization. Mainz: Gutenberg Press.
Walsh, C. E., 2010. Monetary Theory and Policy (3^{th} ed.). Cambridge MA: MIT Press.
Wieland, V., Wolters, M., (2013). Forecasting and Policy Making. In: Elliott, G., Timmermann, A., (Eds). Handbook of Economic Forecasting, Vol. 2A. Elsevier: Oxford, pp. 239-325.

Appendix 1

We take first-order conditions in equation (8) as follows:

$$\frac{\partial \mathcal{L}}{\partial C_t^b} = (C_t^b)^{-\sigma} - \lambda_t P_t = 0 \Rightarrow \boldsymbol{\lambda_t} = \frac{(C_t^b)^{-\sigma}}{P_t} \quad (1.\mathrm{a})$$

$$\frac{\partial \mathcal{L}}{\partial N_t^b} = -(N_t^b)^{\varphi} + \lambda_t W_t = 0 \Rightarrow \boldsymbol{\lambda_t} = \frac{(N_t^b)^{\varphi}}{W_t} \quad (1.b)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_t} = -\omega_t(1+i_{t-1}^L)L_{t-1}[\omega_t(1-\mu_t)(1+i_{t-1}^L)L_{t-1}]^{-\psi} - \lambda_t(1-k_t)(1+i_{t-1}^L)L_{t-1} -$$

$$-\lambda_t f_{t-1}k_{t-1}(1-\mu_{t-1})(1+i_{t-2}^L)L_{t-2} + \beta^b\lambda_{t+1}\mu_{t+1}f_t k_t(1+i_{t-1}^L)L_{t-1} = 0 \quad (1.c)$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = \lambda_t + \beta^b\omega_{t+1}(1-\mu_{t+1})(1+i_t^L)[\omega_{t+1}(1-\mu_{t+1})(1+i_t^L)L_t]^{-\psi} - \beta^b\lambda_{t+1}[\mu_{t+1}+k_{t+1}(1-\mu_{t+1})] -$$

$$-(1+i_t^L)(\beta^b)^2\lambda_{t+2}\mu_{t+2}f_{t+1}k_{t+1}(1-\mu_{t+1})(1+i_t^L) = 0 \quad (1.d)$$

By re-arranging equation (1.c), writing it one period forward and substituting in equation (1.*d*), we finally get:

$$\lambda_t = \beta^b\lambda_{t+1}(1+i_t^L)\left[1 + \frac{(1-E_t\mu_{t+1})f_t k_t(1-\mu_t)(1+i_{t-1}^L)L_{t-1}}{(1+i_t^L)L_t}\right] \quad (1.e)$$

Appendix 2

We take second-order conditions in equation (8) as follows:

$$H_b = \begin{bmatrix} L_{\lambda_t,\lambda_t} & L_{C_t,\lambda_t} & L_{N_t,\lambda_t} & L_{\mu_t,\lambda_t} & L_{L_t,\lambda_t} \\ L_{\lambda_t,C_t} & L_{C_t,C_t} & L_{N_t,C_t} & L_{\mu_t,C_t} & L_{L_t,C_t} \\ L_{\lambda_t,N_t} & L_{C_t,N_t} & L_{N_t,N_t} & L_{\mu_t,N_t} & L_{L_t,N_t} \\ L_{\lambda_t,\mu_t} & L_{C_t,\mu_t} & L_{N_t,\mu_t} & L_{\mu_t,\mu_t} & L_{L_t,\mu_t} \\ L_{\lambda_t,L_t} & L_{C_t,L_t} & L_{N_t,L_t} & L_{\mu_t,L_t} & L_{L_t,L_t} \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & -P_t & W_t & a & 1 \\ -P_t & -\sigma(C_t)^{-\sigma-1} & 0 & 0 & 0 \\ W_t & 0 & -\varphi(N_t)^{\varphi-1} & 0 & 0 \\ a & 0 & 0 & -\psi\gamma/(1-\mu_t)^2 & 0 \\ 1 & 0 & 0 & 0 & -\beta\psi\delta/(L_t)^2 \end{bmatrix}$$

where H_b is the Hessian matrix,

$\alpha = -(1-k_t)(1+i_{t-1}^L)L_{t-1} - f_{t-1}k_{t-1}\big(1-\mu_{t-1}\big)(1+i_{t-2}^L)L_{t-2}$

$\gamma = [\omega_t(1-\mu_t)(1+i_{t-1}^L)L_{t-1}]^{1-\psi} > 0$

$\delta = [\omega_{t+1}(1-\mu_{t+1})(1+i_t^L)L_t]^{1-\psi} > 0$

The leading principal minors of the above Hessian matrix are as follows:

$|H_1| = -(P_t)^2 < 0$

$|H_2| = (P_t)^2\varphi(N_t)^{\varphi-1} + (W_t)^2\sigma(C_t)^{-\sigma-1} > 0$

$|H_3| = -(P_t)^2\varphi(N_t)^{\varphi-1}\,\psi\gamma/(1-\mu_t)^2 - (W_t)^2\sigma(C_t)^{-\sigma-1}\,\psi\gamma/(1-\mu_t)^2 - a^2\sigma(C_t)^{-\sigma-1}\varphi(N_t)^{\varphi-1} < 0$

$$|H_4| = \psi\gamma/(1-\mu_t)^2\,\sigma(C_t)^{-\sigma-1}\varphi(N_t)^{\varphi-1} + \beta\psi\delta/(L_t)^2\,a^2\sigma(C_t)^{-\sigma-1}\varphi(N_t)^{\varphi-1} +$$
$$+\beta\psi^2\delta/(L_t)^2\,\gamma/(1-\mu_t)^2\,(W_t)^2\sigma(C_t)^{-\sigma-1} + \beta\psi^2\delta/(L_t)^2\,\gamma/(1-\mu_t)^2\,\varphi(N_t)^{\varphi-1}(P_t)^2 > 0$$

As a result, the Hessian matrix is negative definite.

Appendix 3

We take first-order conditions in equation (19) as follows:

$$\frac{\partial\mathcal{L}}{\partial C_t^s} = (C_t^s)^{-\sigma} - \lambda_t P_t = 0 \Rightarrow \boldsymbol{\lambda_t} = \frac{(C_t^s)^{-\sigma}}{P_t} \tag{3.a}$$

$$\frac{\partial\mathcal{L}}{\partial N_t^s} = -(N_t^s)^{\varphi} + \lambda_t W_t = 0 \Rightarrow \boldsymbol{\lambda_t} = \frac{(N_t^s)^{\varphi}}{W_t} \tag{3.b}$$

$$\frac{\partial\mathcal{L}}{\partial D_t} = -\lambda_t + \beta^s\lambda_{t+1}(1+i_t^D) = 0 \tag{3.c}$$

or

$$\lambda_t = \beta^s\lambda_{t+1}(1+i_t^D) \tag{3.d}$$

Appendix 4

From equation (11) we obtain :

$$lnC_t^b = ln(\psi C_t) = ln(\psi C_{t+1}) - \frac{1}{\sigma}\left[i_t^L - E_t\{\pi_{t+1}\} - \rho^b + \frac{(1-E_t\{\mu_{t+1}\})f_t k_t(1-\mu_t)(1+i_{t-1}^L)L_{t-1}}{(1+i_t^L)L_t}\right] \tag{4.a}$$

or

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left[i_t^L - E_t\{\pi_{t+1}\} - \rho^b + \frac{(1-E_t\{\mu_{t+1}\})f_t k_t(1-\mu_t)(1+i_{t-1}^L)L_{t-1}}{(1+i_t^L)L_t}\right] \tag{4.b}$$

From equation (20) we obtain:

$$ln\,C_t^s = ln((1-\psi)C_t) = ln((1-\psi)C_{t+1}) - \frac{1}{\sigma}\lfloor i_t^D - E_t\{\pi_{t+1}\} - \rho^s\rfloor \tag{4.c}$$

or

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\lfloor i_t^D - E_t\{\pi_{t+1}\} - \rho^s\rfloor \tag{4.d}$$