A Comparison Between Direct and Indirect Seasonal Adjustment of the Chilean GDP 1986-2009 with X-12-ARIMA

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A Comparison Between Direct and Indirect Seasonal Adjustment of the Chilean GDP 1986-2009 with X-12-ARIMA[*]

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Abstract

It is well known among practitioners that the seasonal adjustment applied to economic time series could involve several decisions to be made by the econometrician. In this paper, I assess which aggregation strategy delivers the best results for the case of the Chilean GDP 1986-2009 quarterly dataset (base year: 2003). This is done by performing an aggregate-by-disaggregate analysis under different schemes, as the fixed base year dataset allows this fair comparison. The analysis is based exclusively on seasonal adjustment diagnostics contained in X-12-ARIMA program. A detailed description of the program and its quality assessment are also provided. The results show that it is preferred, in terms of stability, to use the first block of supply-side disaggregation as well as the direct mode.

JEL-Codes: C14, C18, C49, C65, C87.

Keywords: *Seasonal adjustment, univariate time-series models, ARMA, X-12-ARIMA.*

Resumen

Es bien sabido que el ajuste estacional aplicado a series de tiempo económicas puede significar la toma de varias decisiones por parte de quién realiza el ajuste. Así, en este trabajo se analiza qué estrategia de agregación entrega los mejores resultados para el caso del PIB chileno 1986-2009 trimestral (año base: 2003). Lo anterior se realiza mediante comparaciones de las series agregadas a través de sus componentes bajo distintos esquemas de agregación, dado que los datos de año base fijo permiten una comparación justa. El análisis se basa exclusivamente en las herramientas de diagnóstico de estacionalidad contenidos en el programa X-12-ARIMA. También se describe detalladamente el programa y sus herramientas de análisis de la calidad del ajuste. Los resultados indican que es preferible, en términos de estabilidad, el uso del primer bloque de desagregación por el lado de la actividad, así como también del PIB ajustado como tal.

Códigos JEL: C14, C18, C49, C65, C87.

Palabras clave: *Ajuste estacional, modelos de series de tiempo univariados, ARMA, X-12-ARIMA.*

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Resumen no técnico

Es bien sabido que el ajuste de remoción de la estacionalidad–movimientos intraanuales sistemáticos–aplicado a series de tiempo económicas puede significar la toma de varias decisiones por parte de quién realiza el ajuste, sobre algunos parámetros que gobiernan el proceso. Así, en este trabajo se analiza qué estrategia de agregación entrega los mejores resultados para el caso del PIB chileno 1986-2009 de frecuencia trimestral (año base: 2003). De esta forma, se cuenta con una apreciación adecuada sobre qué conjunto de series, que componen el PIB chileno bajo la metodología de año base fijo, entregan un resultado más estable que otros conjuntos de series que también originan el PIB chileno.

El análisis se realiza mediante comparaciones de las series agregadas resultantes ajustadas por estacionalidad a través de sus componentes, bajo distintos esquemas de agregación tradicionalmente utilizados con este tipo de datos. Esta comparación es justa, en parte, debido a que los datos construidos bajo la metodología de año base fijo aseguran la aditividad exacta de las series originales. La respuesta se basa exclusivamente en las herramientas de diagnóstico de estacionalidad contenidos en el programa X-12-ARIMA, desarrollado por el *US Census Bureau*, y de amplia utilización en oficinas estadísticas internacionales. En el trabajo también se describe detalladamente el programa y sus herramientas de análisis de la calidad del ajuste.

Los resultados indican que es preferible en términos de estabilidad, es decir, los resultados de menor variación promedio mientras nuevas observaciones se agregan al análisis, el uso del primer bloque de desagregación por el lado de la actividad, así como también del PIB ajustado como tal sin desagregar.

1 Introduction

It is well known among practitioners that seasonal adjustment applied to economic time series could involve several decisions to be made by the econometrician. Many of these decisions concern parameters to be fixed prior to the adjustment process and are included in traditional programs such as X-12-ARIMA or TRAMO-SEATS.[1] In spite of these advances, there is no consensus about a particular method to obtain robust results. Specifically, the decisions to be made by the econometrician seem to be case dependent and based merely on empirics.[2] In the case of an aggregate series–that already is a weighted sum of disaggregates–, there are several strategies to perform a seasonal adjustment, for instance, by: (i) adjusting an aggregate series by itself, (ii) adjusting components of the aggregate with the same methodology and then aggregate to the original, and (iii) adjusting components with a different methodology and adding up to the original. These strategies could deliver results that strongly differ from each other. Some reasons for this difference are nonlinearities in the components, different seasonal patterns through themselves, and difficulties in identifying the trading day effect.

In this paper, I assess the question of which of these strategies performs the most stable results for the case of the Chilean Gross Domestic Product (GDP) 1986-2009 quarterly dataset (base year: 2003). I perform an aggregate-by-disaggregate analysis under different schemes, based exclusively on the diagnostics for seasonal adjustment contained in the X-12-ARIMA program (hereafter, X12). These capabilities include spectral plots, sliding-spans-based diagnostics and revision history diagnostics, all of them simple checks that cast for both quality and stability of results.[3] By performing this exercise, I will be able to provide an informed opinion about which scheme provides the most stable seasonal adjustment for the mentioned Chilean GDP vintage. Obviously, the same strategy can be applied to another dataset to investigate the same problem. However, notice that since 2009 the GDP dataset is released under the linked-chain methodology, losing its additive property. Hence, this exercise should be read as a benchmark for future research in this matter.

Note that as seasonal adjustment does not have an objective definition, the term "accurate" cannot operate under these circumstances. Instead, I considered the more stable as the best result. The exercise is carried out on the automatic program default mode, according to the suggestions presented in Maravall (2002). Then, if a component does not fulfill the statistical criteria of an acceptable result, minimal interventions are made one-by-one until all test are fulfilled.

The question about how we know if it is better to use direct or indirect adjustment is similar to that treated in Astolfi, Ladiray, and Mazzi (2001), Hood and Findley (2001), Otranto and Triacca (2002), among other papers. Nevertheless, to the author's knowledge, no similar study has been carried out with Chilean GDP data. It is worth mentioning that despite several testing procedures contained in X12, choosing between direct and indirect adjustment constitutes a different, separate question. This is because the challenge consists of choosing appropriate diagnostics to compare between several adjustments applied to the same variable. Some diagnostics would be replicated while others, as those of stability and quality, need more attention and careful treatment. This would lead to proceed without, for instance, the $M1$-$M11$ and Q statistics (described later) integrated in X12, or a deviation measure from direct approach. Moreover, as Hood and Findley (2001) suggest, the ratio of one adjustment to another is also not valid, because spurious seasonality emerges. These and many other examples remark the importance of a tailored quality assessment: different users would weigh different diagnostic outputs pursuing their own objectives. Take the case, for instance, of an adjuster that will make use of seasonally adjusted series in a turning point detection analysis. The smoothness in the resulting series will likely be a desirable outcome. For these and other reasons to be discussed later, I make use of diagnostics that allow a quality and stability assessment for indirect adjustment, namely, spectral analysis in the frequency domain, sliding spans and revision history. The key is to have always in mind the goal of the absence of *residual seasonality*; that is, absence of seasonality in series that theoretically should not have it.

[1] See Findley *et al.* (1998) and Gómez and Maravall (1997) for details.

[2] Astolfi, Ladiray, and Mazzi (2001).

[3] See Findley *et al.* (1998) and US Census Bureau (2011) for a full description and analysis of the new capabilities with respect to previous version of the program, the X-11-ARIMA (Dagum, 1980).

The results show that in terms of stability it is recommended to use the direct–the GDP by itself–as well as the first stage of disaggregation by supply-side. This result relies on the output of the three diagnostics tools used by X12, and exploiting the additive characteristic of the dataset. Therefore, I take into account just a stability criterion to discriminate between aggregations. Moreover, the results for the second and third stage of disaggregation by demand-side are very poor, according to the standard automatic setup described later. Particularly powerful tools to discriminate are spectral plots and sliding spans, both estimated to the final seasonally adjusted series.

The paper proceeds as follows. In section 2, I describe the program. Then, I review the diagnostics contained in X12. In section 3 I provide some intuition about why both kinds of adjustments may differ, along with some elements to consider to revert poor results.[4] In section 4 I apply these procedures to the Chilean GDP obtained by five different aggregation schemes–by supply- and demand-side–plus the GDP by itself. I conclude in section 5.

2 The X-12-ARIMA program

The X12 program is developed by the *US Census Bureau* (`http://www.census.gov/`) and is based in the well-known X-11 program introduced in 1965 (Shiskin, Young, and Musgrave, 1967), and the *Statistics Canada*'s X-11-ARIMA program (Dagum, 1980). X12 enhances its previous versions by including more filters and automatic modeling techniques that improve the quality of adjustment. It also offers a user-friendly customization of the process, especially useful for batch mode. Recall that the purpose of the program is to decompose a time series (y_t) into a trend-cycle component (y_t^τ) plus (or times, depending on the kind of seasonality) a seasonally adjusted component (y_t^{sa}), plus (or times) a residual irregular component (y_t^{ir}; then $y_t = y_t^\tau + y_t^{sa} + y_t^{ir}$, or $y_t = y_t^\tau \times y_t^{sa} \times y_t^{ir}$). Hence, the two latter components, y_t^{sa} and y_t^{ir}, should not exhibit a cyclical behavior. Furthermore, trends and seasonally adjusted series (the log-difference) tend to attract a lot of attention, which redound on the importance of stability as a measure of economic diagnostic reliability. As Granger (1979) pointed out, this decomposition is relevant because seasonality explains most of the variance of a series, being economically insignificant.

The description of the adjustment proposed by the X12 program lies in the description of the X-11 program, which is fully developed in Ladiray and Quenneville (2001). According to figure 1, the chronological sequence of a typical adjustment begins with preadjustments (the RegARIMA module) prior to entering the seasonal adjustment itself, made with X-11-based tools. Note that series of these blocks are tested with a battery of specific diagnostics, iterating until the required statistical characteristics are fulfilled.

The RegARIMA module, one of the major enhancements with respect to X-11,[5] works basically as follows. The objective is to prepare the series prior to the adjustment, by controlling for several undesirable effects. These corrections are made with a regression with ARIMA noise (which takes the name RegARIMA) that includes a control set $X = [\ x_1 \quad ... \quad x_I\ ]$:

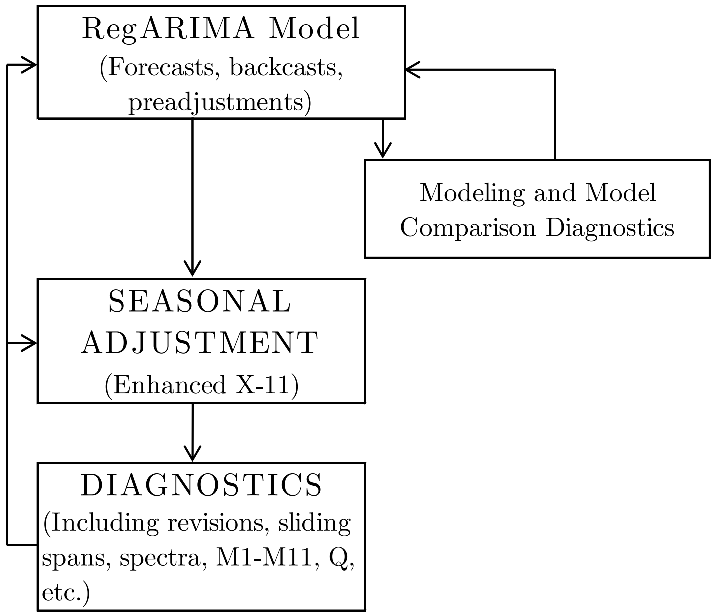
$$\phi(L)\Phi(L^s)(1-L)^d(1-L^s)^D(y_t - \sum_i \beta_i x_{it}) = \theta(L)\Theta(L^s)z_t,$$

where L is the backshift operator ($L^p y_t = y_{t-p}$), s is the seasonal period, $\phi(L)$ is the nonseasonal autoregressive (AR) operator, $\Phi(L^s)$ is the seasonal AR operator, $\theta(L)$ is the nonseasonal moving average (MA) operator, $\Theta(L^s)$ is the seasonal MA operator, and z_t is *iid* with zero mean and variance equal to σ_z^2 (white noise). Note that z_t is modeled with an ARIMA process as the series y_t after these controls commonly exhibits autocorrelation.

[4] Readers already familiarized with the program can skip sections 2 and 3 without losing information or structure.

[5] A complete list of these enhancements is: (i) the inclusion of sliding spans as a tool for testing stability, (ii) revision history diagnostics also for stability, (iii) new Henderson trend filter routine, (iv) new options for seasonal filters, (v) new outlier detection options, (vi) a new table to analyze pertinence of trading-day effects, and (vii) a pseudo-additive decomposition (US Census Bureau, 2011).

Figure 1: Flow diagram for seasonal adjustment with X12



Source: Findley *et al.* (1998).

This representation states for a time-varying mean for y_t while including controls; otherwise, y_t has a constant mean. In X12, x_i controls are optional, and some of them have options as well. These controls are the following (the option used by the original program is presented in [ ]; US Census Bureau, 2011, Table 4.1, pp. 36-38):

- **Trend constant** [`const`]:

$$(1-L)^{-d}(1-L^s)^{-D} I(t \geq 1), \text{ where } I(t \geq 1) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 1. \end{cases}$$

- **Fixed seasonal** [`seasonal`]:

$$M_{1,t} = \begin{cases} 1 & \text{in January} \\ -1 & \text{in December} \\ 0 & \text{otherwise} \end{cases}, ..., M_{11,t} = \begin{cases} 1 & \text{in November} \\ -1 & \text{in December} \\ 0 & \text{otherwise.} \end{cases}$$

  In series with frequency equal to s per year, it can be modeled with s variables. While the sum of those s variables adds to one, perfect collinearity problem arises with constant term–typically to control for measuring units. To overcome this shortcoming, X12 includes $s-1$ regressors with appropriate specification.

- **Fixed seasonal** [`sincos[ ]`]:

$$\sin(\omega_j t),\ \cos(\omega_j t), \text{ where } \omega_j = 2\pi j/12,\ 1 \leq j \leq 6 \text{ (drop } \sin(\omega_6 t) \equiv 0).$$

- **Trading day** (monthly or quarterly flow) [`tdnolpyear`, `td`]:

$$T_{1,t} = (\text{\# of Mondays}) - (\text{\# of Sundays}), ..., T_{6,t} = (\text{\# of Saturdays}) - (\text{\# of Sundays}).$$

- **One Coefficient Trading Day** (monthly or quarterly flow) [`td1nolpyear`, `td1coef`]:

$$(\text{\# of weekdays}) - \frac{5}{2}(\text{\# of Saturdays and Sundays}).$$

- **Length-of-Month** (monthly flow) [`lom`]:

  $m_t - \overline{m}$, where m_t = length of month t (in days) and $\overline{m} = 30.4375$ (average length of month).

- **Length-of-Quarter** (quarterly flow) [`loq`]:

  $q_t - \overline{q}$, where q_t = length of quarter t (in days) and $\overline{q} = 91.3125$ (average length of quarter).

- **Leap Year** (monthly or quarterly flow) [`lpyear`]:

$$LYt = \begin{cases} 0.75 & \text{in leap year February (first quarter)} \\ -0.25 & \text{in other Februaries (first quarter)} \\ 0 & \text{otherwise.} \end{cases}$$

- **Stock Trading Day** (monthly stock) [`tdstock[`w`]`]:

$$D_{1,t} = \begin{cases} 1 & \widetilde{w}^{th} \text{ day of the month } t \text{ is a Monday} \\ -1 & \widetilde{w}^{th} \text{ day of the month } t \text{ is a Sunday} \\ 0 & \text{otherwise} \end{cases}, \ldots,$$

$$D_{6,t} = \begin{cases} 1 & \widetilde{w}^{th} \text{ day of the month } t \text{ is a Saturday} \\ -1 & \widetilde{w}^{th} \text{ day of the month } t \text{ is a Sunday} \\ 0 & \text{otherwise.} \end{cases}$$

- **Statistics Canada Easter** (monthly or quarterly flow) [`sceaster[`w`]`]:

$$E(w,t) = \begin{cases} n_{E/w} & \text{in March} \\ -n_{E/w} & \text{in April} \\ 0 & \text{otherwise.} \end{cases}$$

- **Easter Holiday** (monthly or quarterly flow) [`easter[`w`]`]:

$$E(w,t) = \frac{1}{w} \times [\# \text{ of the } w \text{ days before Easter falling in month (or quarter) } t].$$

- **Labor Day** (monthly flow) [labor`[`w`]`]:

$$L(w,t) = \frac{1}{w} \times [\# \text{ of the } w \text{ days before Labor Day falling month } t].$$

- **Thanksgiving** (monthly flow) [`thank[`w`]`]:

$$ThC(w,t) = \text{proportion of days from } w \text{ days before Thanksgiving through December 24 that fall in month } t.$$

- **Additive Outlier at $\mathbf{t_0}$** [`ao`$date_0$]:

$$AO_t^{(t_0)} = \begin{cases} 1 & \text{for } t \neq t_0 \\ 0 & \text{for } t = t_0. \end{cases}$$

- **Level Shift at $\mathbf{t_0}$** [`ls`$date_0$]:

$$LS_t^{(t_0)} = \begin{cases} -1 & \text{for } t < t_0 \\ 0 & \text{for } t \geq t_0. \end{cases}$$

- **Temporary Change at $\mathbf{t_0}$** [`tc`$date_0$]:

$$TC_t^{(t_0)} = \begin{cases} 0 & \text{for } t < t_0 \\ \alpha^{t-t_0} & \text{for } t \geq t_0. \end{cases}$$

- **Ramp, $\mathbf{t_0}$ to $\mathbf{t_1}$** [`rp`$date_0$-$date_1$]:

$$RP_t^{(t_0,t_1)} = \begin{cases} -1 & \text{for } t \leq t_0 \\ \frac{t-t_0}{t_1-t_0} - 1 & \text{for } t_0 < t < t_1 \\ 0 & \text{for } t \geq t_1. \end{cases}$$

Note that some of these controls are stocks, while others are flows. Fuzzy special cases are those related to trading days, such as the moving (nonfixed) holidays. These are all those holidays defined as a rule, not falling on a specific calendar day, for instance, every first Thursday of every September. In the Chilean case, these cases would be all those affected by a bill passed some years ago that consists in moving particular holidays falling on a working day (different to Monday) to the nearest Monday of that week.

RegARIMA completes three substeps concerning *identification*, *estimation* and *diagnostic checking*. The identification of the ARIMA model is based on partial autocorrelation and sample autocorrelation techniques. It also includes an automatic algorithm of model selection similar to that proposed by Víctor Gómez and Agustín Maravall (the TRAMO procedure; 1997). For specific user-defined regressors, the program makes use of traditional information criteria to assess adequacy (AIC, AICC, BIC and Hannan-Quinn). However, traditional t-Statistics for individual parameter testing, and χ^2 for a group of variables, are also reported. The module also forecasts and backcasts the original series based on estimated ARIMA models with predefined (p, d, q) combinations. Forecasts are traditionally used in this context to reduce changes in the adjusted series as new observations become available, often referred to as *revisions* (Bobbitt and Otto, 1990). The criterion that casts best for a forecasting model used in X12 is the root mean squared error.

The estimation is made via maximum likelihood using the Iterated Generalized Least Squared method (IGLS; see Otto, Bell, and Burman, 1987). The first battery of diagnostics is applied next. These checks are used to assess the overall adequacy of controls and user-defined regressors as well as forecast and backcast accuracy. They are based mainly on the AR spectrum diagnostics similar to that contained in the BAYSEA program (*Bayesian Seasonal Adjustment*; Akaike and Ishiguro, 1980), by examining the RegARIMA resulting residuals in search for additive outliers, level shifts, among other anomalies. More details can be found in Findley *et al.* (1998) and US Census Bureau (2011).

The remaining seasonal adjustment process continues with the fitted value of the RegARIMA model. At the end of the process, the excluded controls are added to the irregular component to then sum up all the components to conform the original series.

Once all diagnostics are passed, the resultant series enters the seasonal adjustment module. This module is based on the X-11 iterative algorithm, which makes use of successive estimates of MAs. The goal of this module is to separate–with all diagnostics approved–the series into the following components: trend-cycle (y_t^{τ}), seasonal adjusted series (y_t^{sa}), trading-day component (y_t^{td}), Easter holiday (y_t^{E}), and an irregular component (y_t^{ir}). Note that X-11 does not distinguish between trend and cycle, basically because the series traditionally analyzed does not provide enough sample span to determine whether it is trend or cycle.[6] The seasonal component represents intra-year fluctuations, that is, regularities that are repeated more or less year after year. This implies that the method is built to deal exclusively with monthly or quarterly series. The trading day component measures the different composition of days of the week in a month or quarter. The Easter holiday measures this specific effect in the same spirit of the trading day effect. Finally, the irregular component combines all the remaining effects not covered by the abovementioned components, and exhibits an erratic behavior hard to associate with specific clearly identified intra-year fluctuations. So, X-11 proposes two decompositions:

1. **Additive**: $y_t = y_t^{\tau} + y_t^{sa} + y_t^{td} + y_t^{E} + y_t^{ir}$, and
2. **Multiplicative**: $y_t = y_t^{\tau} \times y_t^{sa} \times y_t^{td} \times y_t^{E} \times y_t^{ir}$.

[6] Note that this component, despite its name, does not necessarily match the concept of trend in the traditional economics usage.

In addition, X12 includes two more decompositions:

1. **Log-additive**: $\log(y_t) = \log(y_t^{\tau}) + \log(y_t^{sa}) + \log(y_t^{td}) + \log(y_t^{E}) + \log(y_t^{ir})$, and
2. **Pseudo-additive**: $y_t = y_t^{\tau} \times (y_t^{sa} + y_t^{td} + y_t^{E} + y_t^{ir} - 1)$.

The identification of each component is made one by one, based on MAs.[7] A simple algorithm of two stages–noted with super indices–as Ladiray and Quenneville (2001) show, is presented in table 1.

Table 1: The X-11 basic algorithm

1. Estimation of the trend-cycle by a 2×12 MA: $$y_t^{\tau(1)} = M_{2\times 12}(y_t).$$
2. Estimation of the seasonal plus irregular component: $$(y_t^{sa} + y_t^{ir})^{(1)} = y_t - y_t^{\tau(1)}.$$
3. Estimation of the seasonal component by 3×3 MA over each quarter: $$y_t^{sa(1)} = M_{3\times 3}[(y_t^{sa} + y_t^{ir})^{(1)}],$$ and normalization: $$\tilde{y}_t^{sa(1)} = y_t^{sa(1)} - M_{2\times 12}(y_t^{sa(1)}).$$
4. Estimations of the seasonally adjusted series: $$y_t^{sa(1)} = (y_t^{\tau} + y_t^{ir})^{(1)} = y_t - \tilde{y}_t^{sa(1)}.$$
5. Estimation of the trend-cycle by 13-term Henderson MA: $$y_t^{\tau(2)} = H_{13}(y_t^{sa(1)}).$$
6. Estimations of the seasonal plus irregular component: $$(y_t^{sa} + y_t^{ir})^{(2)} = y_t - y_t^{sa(2)}.$$
7. Estimation of the seasonal component by 3×5 MA over each quarter: $$y_t^{sa(2)} = M_{3\times 5}[(y_t^{sa} + y_t^{ir})^{(2)}],$$ and normalization: $$\tilde{y}_t^{sa(2)} = y_t^{sa(2)} - M_{2\times 12}(y_t^{sa(2)}).$$
8. Estimation of the seasonally adjusted series: $$y_t^{sa(2)} = (y_t^{\tau} + y_t^{ir})^{(2)} = y_t - \tilde{y}_t^{sa(2)}.$$

Source: Ladiray and Quenneville (2001).

Note that in the X-11 program the outliers, level shift, and other effects are identified after the application of the algorithm of table 1, to then iterate until all undesirable effects are disseminated off the adjusted series. In X12, this identification is made within the RegARIMA module, prior to the adjustment. The outlier detection-for-correction process is a key element for MA stability, along with forecasts to reduce revision changes as new data becomes available.

The final module of X12 is an overall diagnostics checking. It is based on several tables–for analyzing intra-annual movements of series with ease– provided by X-11-ARIMA as well as the $M1$-$M11$ quality-control statistics, summarized in annex A. These eleven M statistics, discussed in Lothian and Morry (1978), is a set of controls with an intuitive and easy interpretation: values above 1–ranging from 0 to 3–redound in a poor adjustment for a particular reason. All of them are constructed with the purpose of checking the contribution of trend-cycle, seasonally adjusted and irregular component to changes of the original series. A weighted average of all of them constitutes another couple of statistics, $Q1$ and $Q2$, both F distributed, pertaining to

[7] An MA of coefficients $\{\theta_i\}$ is defined as $M(y_t) = \sum_{i=-p}^{f} \theta_i y_{t+i}$, where p represents past values and f future values of y_t series. So, the problem is reduced to finding a set of coefficients $\{\theta_i\}$ with p and f fixed, that works for identification purposes. Several "known" distributions of θ_i are proposed. See, for instance, Koyck (1954) and Almon (1965). Note that a compact notations $m \times n$ or $M_{m\times n}$ are also used. For instance, a 3×4 moving average is the average of 4 consecutive γ_t terms (from t to $t+4$), where each γ_t is composed by 3 consecutive terms centered–equally weighted when MA is symmetric–at t: $\gamma_t = \frac{1}{3}y_{t-1} + \frac{1}{3}y_t + \frac{1}{3}y_{t+1}$. Thus, the sequence of θ_i terms corresponds to $\frac{1}{12}\{1, 2, 3, 3, 2, 1\}$. The Henderson MA-trend is also used because it has the ability to reproduce up to third-order polynomials, thereby capturing trend turning points. It consists of specific distributions of the θ_i values, that can be either symmetric ($\theta_i = \theta_{-i}$) or asymmetric ($\theta_i \neq \theta_{-i}$). See details in Henderson (1916).

irregular and seasonal component, respectively. Several improvements were made in order to merge both Q statistics into one (the Q-stat), keeping the quality signal anchored to 1. In the same spirit, X12 reports the "Table $D8$" as an output that contains the result of an ANOVA. X12 reports a test based on all statistics shown in Table $D8$ to test for stable and moving seasonality jointly, configuring a *combined test for the presence of identifiable seasonality*. As a set of individual significance tests in traditional econometrics–global significance–the joint statistic is F distributed, and receives the denomination "$D8$ F-test" for short.

3 Diagnostics and results handling

Instability of adjustment is a crucial problem recognized in X12. For that reason, user-defined regressors were included in X12, and major improvements were incorporated to assess the reliability of these interventions. Some of these are the use of spectral plots for detecting seasonality, and warnings to the user based on sliding spans and revision history diagnostics. All three will be detailed below and constitute adequate diagnostics to compare the adjustment between direct and indirect strategies–and perhaps, between a number of methods. It is common from a practitioner's point of view to see seasonal adjustment as a tool for smoothness–a filtering technique. But, as will be shown, the smoothness pursued by seasonal adjustment is in spectral plots of seasonally adjusted series and the irregular component, instead of the original series that generates those spectra.

An important remark on X12's diagnostics is provided by Hood (2007). As she explains, $M7$ and $D8$ F-tests are not designed to detect residual seasonality. Instead, they are for testing *stable seasonality* in the actual series–if the original series is seasonal. Note that this kind of testing is model-specific, implying that ultimately depends on user preferences of RegARIMA parameters. Rather, some model-free diagnostics as spectral plots applied to (log-differenced) original series produce better results in detecting seasonality.[8] Notice that in the case of indirect adjustment it is possible to finding residual seasonality despite the quality of its disaggregates, remarking that the answer relies ultimately on empirics (Hood and Findley, 2001).

3.1 Spectral plots

In the context of X12, the use of spectral plots is for detecting seasonality and trading-day effects in the original series as well as residual seasonality in seasonally adjusted series, plus the irregular component. Spectral analysis was introduced firstly for economic time series analysis in the 1960s by Cunnyngham (1963), Hatanaka (1963), Granger and Morgenstern (1963), Granger and Hatanaka (1964), and Nerlove (1964), among others.[9]

An economic time series can be analyzed from two points of view: *frequency domain* and *time domain*. The intuition behind frequency domain lies in the useful manner in which a variable can be plotted in terms of its cycles–measuring its strength in decibels–for any given frequency, without requiring new information. Thus, spectral analysis allows analyzing the relationships between the frequencies with ease. The graph of frequencies versus decibels is called the *periodogram*–a special case of the spectrum. In a spectral graph, the low frequencies to the left correspond to slowly changing components–like the trend–while higher frequencies correspond to rapidly changing components–like the irregular component. Peaks in the spectral plot at certain frequencies of actual data indicate the presence of seasonality and trading day effect. The sample spectrum for a size T for the series $\{y_t\}_{t=1}^{t=T}$, measured in decibels, corresponds to:

$$\widehat{s}(\lambda) = 10 \cdot \ln \left\{ \frac{\widehat{\sigma}_m^2}{2\pi \left| 1 - \sum_{j=1}^{m} \widehat{\phi}_j e^{i2\pi j\lambda} \right|^2} \right\},$$

[8]Findley *et al.* (1998) agree that the spectrum and sliding-span diagnostics can be used, as well as RegARIMA model diagnostics, as a better alternative to F-test, which tends to indicate false positive cases. Also, Hood (2007) found that X12's default spectrum–built in an AR(30) model–is a better indicator to find seasonality than $D8$ F-test, χ^2, and an AR(10) spectrum.

[9]For a comprehensive use of the spectrum for economic time series analysis, see Hamilton (1994).

where $0 \leq \lambda \leq \frac{1}{2}$ represents frequency measured in radians, $\lambda = \frac{k}{12}$, with $1 \leq k \leq 6$, being relevant the frequencies $\frac{1}{4}$ and $\frac{1}{2}$ cycles per quarter, for quarterly series. The $\widehat{\phi}_j$ coefficients are extracted from an estimated AR process of $y_t - \overline{y}$ regressed by $y_{t-m} - \overline{y}$, where $1 \leq j \leq m$, with $\overline{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$, and $\widehat{\sigma}_m^2$ is the sample variance of the resulting regression residuals. This AR process is of order 30–AR(30) or $m = 30$–in the default mode. The order 30 is chosen basically because captures most of the dynamic of the series. An order m chosen with information criteria, such as the AIC, tends to generate smoother spectral amplitudes. Since the $\widehat{\phi}_j$ parameters are variance-corrected covariances between y_t and y_{t-j}, the informational content of the spectrum with respect to the time domain remains fixed (Hamilton, 1994). Note that using the De Moivre's theorem, the term $e^{-i\lambda j}$, where i stands for $\sqrt{-1}$, is equal to $\cos(\lambda j) - i \cdot \sin(\lambda j)$, and using the known trigonometrical results $\cos(0) \equiv 1$, $\sin(0) \equiv 0$, $\sin(-\theta) \equiv -\sin(\theta)$, and $\cos(-\theta) \equiv \cos(\theta)$, the expression can be written in terms of the cosine function. Since $\cos(\lambda j) = \cos[(l + 2\pi k)j]$ for any integers k and j, the spectrum is a periodic function of λ. Hence, we need to know the values of $\widehat{s}(\lambda)$ between 0 and π to infer the periodogram value for any λ.

The decision about the quality of the adjustment in X12 consists of a "visual significance", similar to the BAYSEA program.[10] The series has seasonality if the spectrum exhibits a peak at frequencies of $\frac{1}{2}$ or $\frac{1}{4}$. The trading day effect is present if a peak is observed at the frequency of 0.34821 for monthly series, and at 0.04464 for quarterly series.[11] As the spectrum is used to test the quality of the adjustment, it is also estimated for the differenced final seasonal adjusted series and the final irregular component. Soukup and Findley (1999) also suggest to analyzing the resultanting RegARIMA residual spectra in cases where first two spectra do not discriminate. This can be done at the cost of hurting the rate of false discovery of seasonality as well as trading day effect.[12] A peak is called significant if it is above the median of $\widehat{s}(\lambda_k)$ values, and must be larger than its neighboring (not including $\lambda_{60} = \frac{1}{2}$) values $\widehat{s}(\lambda_{k-1})$ and $\widehat{s}(\lambda_{k+1})$ by at least $\frac{6}{52}$ times the range $\widehat{s}^{\max} - \widehat{s}^{\min}$, where $\widehat{s}^{\max} = \max_k \widehat{s}(\lambda_k)$ and $\widehat{s}^{\min} = \min_k \widehat{s}(\lambda_k)$. X12 plots spectra with 52 frequencies. So, the unit of measure is standardized to "stars"–equivalent to $\frac{1}{52}$ unit of frequency–so a peak is (six or more stars) easy to detect visually. As not all the series have trading day effect, it is more valuable to focus only on a single frequency: there of seasonality (0.348 cycles/month), instead of trading day effect (0.432).

A logarithmic transformation tends to found spurious seasonality, so it uses the differenced original series for testing (Soukup and Findley, 1999). Notice that the spectrum as a diagnostic is not as powerful as the likelihood-ratio is for detecting trading-day effects, but it is more versatile because it does not depend upon a correct model specification. Moreover, the F-test tends to find spurious seasonality since the irregular component exhibits a typical, but low, autocorrelation. This traditional significance test is inadequate also for a typical sample size. In X12, spectral diagnostics are computed with 96 observations in default–and desirable–mode, requiring a minimum of 60 observations to perform the routine.[13]

As spectral analysis allows seeing the relationships between the frequencies, I can quantify the importance of certain frequencies relative to the frequencies of other components. Thus, for a comparison between two or more adjustments for the same variable, the result is direct. A lower non-significant peak–or even better, the absence of it–in the seasonally adjusted and irregular component at specific frequencies reflects an adjustment of better quality. In annex B, I show the case of a simulated highly seasonal series. It is clear from the exercise how the spectral shape of both original and subsequent seasonally adjusted series should be.

[10] See Akaike and Ishiguro (1983) for a complete comparison between the two programs.

[11] These frequencies are obtained by considering the decimal part of the number of cycles per month. For instance, the average month of a year is $\frac{365.25}{12} = 30.4375$ days. Hence, a cycle that is repeated every seven days goes through $\frac{30.4375}{7}$ =4.**34821**. See Priestley (1981) and US Census Bureau (2011) for details. The trading day effect is relatively hard to find. Cleveland and Devlin (1980) suggest that it is present, but very weak, at the 0.432 frequency.

[12] The authors estimate a false discovery rate narrowly above 20% when using this criterion, by bootstrapping the irregular component of 42 macroeconomic series of the US. The acceptable rate of X12 is 10%. Both results equally obtained with the "six-stars criterion", are described next.

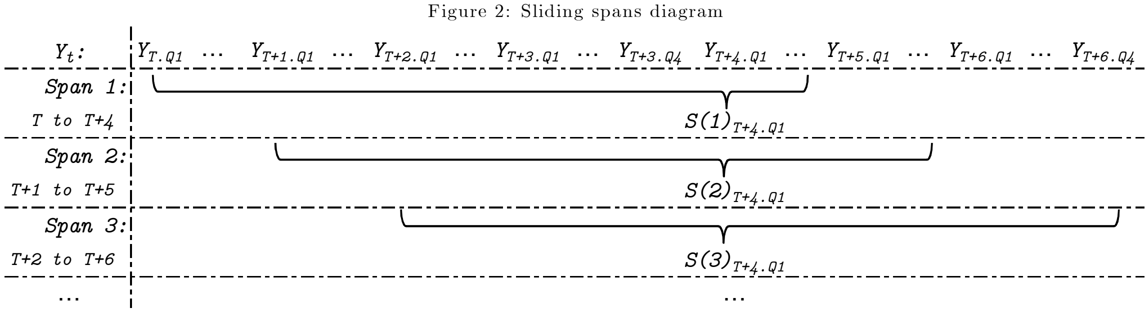
[13] Ladiray (2008) recommends having more than 80 observations for useful results with quarterly series.

3.2 Sliding spans

Sliding spans compare different estimates from seasonal adjustment from overlapping subspans of the time series data. They are fully presented and analyzed in Findley *et al.* (1990). As Findley *et al.* (1998) pointed out, sliding spans were included in X12 to enhance the diagnostics for: (i) determine whether a series is seasonally adjusted adequately, (ii) deciding between direct or indirect adjustment, and (iii) confirming option choice about filters' length.[14] In default mode, X12 analyzes four overlapping spans. Each one is delayed from the other with one-year-ahead difference, data availability.[15] The span begins in January of every year, by default. In cases where this is not possible, RegARIMA made forecast–or backcasts if needed– of the series to balance the sample. These forecast observations fulfill several accuracy tests to be included in the routine. Battipaglia and Focarelli (1995) show a number of scenarios where sliding spans result in a better stability control than Q statistic of X-11-ARIMA, a result used in this paper. These results are in line with Findley and Monsell (1986) and Findley *et al.* (1990).

As was mentioned, the sliding spans are computed as shown in figure 2, where spans are of four years' length. Then, the $S_t(k)$ values are compared–the values of observation t (fixed) obtained across different k (moving) spans–for all observations of the sample in which $k \geq 2$ and $k^{max} = 4$.

Figure 2: Sliding spans diagram



Source: Author's elaboration.

The determinacy of how well–stable–the seasonal adjustment is, is based on a double-threshold decision:

1. Compute the following quantity:

$$S_t(k) = \frac{\max_k y_t(k) - \min_k y_t(k)}{\min_k y_t(k)},$$

   where $y_t(k)$ is the y_t series adjusted at sliding span k–$S_t(k)$ is simply "the sliding span k of y_t". The seasonally adjusted series, y_t^{sa}, is *flagged as unstable* if $S_t(k) > 3\%$. For quarter-on-quarter changes of seasonally adjusted series, this is $QQ_t(k) = [y_t^{sa}(k) - y_{t-1}^{sa}(k)]/y_{t-1}^{sa}(k)$, they are flagged as unstable if:

$$S_t(k) = \max_k QQ_t(k) - \min_k QQ_t(k) > 3\%.$$

   Note that y_t^i is an observation belonging at least to two spans. X12 uses $k = \{2, 3, 4\}$, depending on data availability.

2. This leads to the second check (as X12 plots out):

[14] Findley *et al.* (1990) apply sliding-span-based diagnostics to determine empirically the choice of a seasonal filter to be used to adjust several US macroeconomic series.

[15] See Hood (2007) to know how long a series should be depending on filters selected. For instance, a 3×5 filter needs seven years of data, while a 3×3 filters needs just six years of observations. See also footnote 7.

Percentage of quarters flagged as unstable.

Seasonal Factors:	N_1 out of T (n_1=100×[N_1/T] %)
Quarter-to-Quarter Changes in SA Series:	N_2 out of T-1 (n_2=100×[N_2/T-1] %)

➢ An adjustment is considered acceptable if the:
- Percentage of unstable seasonal factors (n_1) is less than 15%,
- Percentage of unstable seasonal adjustment values is less than 15%, and
- Percentage of unstable quarter-on-quarter changes (n_2) is less than 35%.

➢ An adjustment is considered unacceptable if the:
- Percentage of unstable seasonal factors (n_1) is greater than 25%,
- Percentage of unstable seasonal adjustment values is greater than 25%, and
- Percentage of unstable quarter-on-quarter changes (n_2) is greater than 40%.

So, if the number of observations flagged as unstable exceeds the thresholds depending on the series considered, X12 warns the user on this finding. These estimates are calculated to seasonal factors, seasonally adjusted, and quarter-on-quarter changes in seasonally adjusted series. Notice that sliding spans estimates with year-on-year changes of both trend-cycle and seasonally adjusted series could be meaningless given that changes on its direction–a negative variation–may represent a genuine change in trend and so forth.

In the empirical application presented later, instead of using a threshold rule I focus on the mean of the $S_t(k)$ series to have a linear assessment of stability across the sample. Also because the original threshold used by X12 is set just by a particular specification.[16] I make use of this diagnostics to quarter-on-quarter changes in seasonally adjusted and trend-cycle series. Obviously, determinacy of stability is reached through a minor mean of $S_t(k)$ of each aggregation. Nevertheless, the analysis based on quarter-on-quarter series may present shortcomings compared with the sliding span analysis in levels. These are cases where several known problems arise in the seasonal adjustment of months or quarter with fixed calendar holidays, such as Independence Day in Chile (September 18^{th} and 19^{th} are fixed holidays). Thus, if the two days fall on Monday and Tuesday, it is more probable that the remaining week will be composed of working days instead of the case when the two days fall on Wednesday and Thursday. The latter case suggests a major probability that Friday will be a *de facto* holiday. Nevertheless, it is not considered as holiday by X12. For cases like this, it is worth deviating the attention–and statistical tolerance–for a while in sliding spans estimated with quarter-on-quarter changes series.

Regarding a comparison between M and Q statistics, Findley *et al.* (1990) show that Q statistics–a summary of $M1$-$M11$ statistics–is not as discriminator as a sliding span. This drawback is presented by examples, demonstrating that lower values of Q are not discriminating between adjustable series and series that that cannot be reliably adjusted. Moreover, Lothian and Morry (1978) show that the Q-statistic by itself is not a dominant option with respect to others such as the sliding span. Related to stability, it is uncommon that a large irregular component does not compromise the stability of the whole adjustment. Thus, an integral analysis should be made using Q-statistic instead of specific M-statistics, a shortcoming overriden by sliding spans. Another advantage is the interpretation of its outcome, given that the impact of outliers and other undesirable effects can be directly evaluated numerically on quarter-on-quarter changes.

The sliding span also serves to evaluate intermediate steps of the seasonal adjustment, which can be more fruitful with indirect adjustment. As it constitutes a parameter-free computation, it can be used as a decision rule for the model's goodness of fit. This is especially relevant in cases with a marked trading day effect, which is often detected with an F-test on the irregular series. The trading day effect tends to generate autocorrelated irregular components. A major problem for diagnostics is that the irregular becomes autocorrelated. Thus, F-tabulated values are no longer correct for statistical inference. As several alternatives emerge, the sliding span is a valid candidate to handle with stability in these contexts. Finally, sliding spans also deal with the so-called "raking" technique for reconciliation in aggregate time series. This means that when the sum of disaggregates does not fit with the aggregate, the difference is prorated to disaggregates based on its weights on aggregate.

[16] In fact, the sliding-spans methodology and their threshold-based stability decision has been set by "asking an expert from an economic statistics division at the Census Bureau how much variability is acceptable." Findley *et al.* (1990), p. 347.

3.3 Revision history

The basic revision calculated by the program is the difference between the earliest adjustment of a quarter's datum obtained when that quarter is the final quarter in the series and a later adjustment based on all future data available at the time of the diagnostic analysis (Findley *et al.*, 1998, pp. 137). Often, the informational content of last observations is more important for users, especially for those involved with conjunctural assessment. This measure complements previous diagnostics and focuses on the effect of new information on the historical record of seasonally adjusted series to basically determine the variance contribution of each component. Thus, it clearly constitutes a stability measure. As sliding spans, they may be constructed observation by observation, or by blocks of observations separated, say, by one year. As instability constitutes a recognized problem in X12, suggestions such as that of Bobbit and Otto (1990) help to fix part of the problem. The authors show that the use of a large number of forecasts prior to the adjustment can result in a smaller average revision between concurrent and final seasonal adjustments, a characteristic typically pursued by the adjuster. This finding is also nicely presented in Findley (2005).

The revision is calculated as follows. Suppose that the series to be seasonally adjusted is $\{y_t\}_{t=t_0}^{t=T}$, and $t_0 \leq t \leq T$. Hence, if y^{sa} is the seasonally adjusted series, then $y^{sa}_{t|t}$ is the concurrent seasonal adjustment of observation t, and $y^{sa}_{t|T}$ is the final adjustment of observation t–made with the full sample, T. The revision is:

$$R_t = \frac{y^{sa}_{t|T} - y^{sa}_{t|t}}{y^{sa}_{t|t}}.$$

Revisions are calculated to trend-cycle, seasonal factors, seasonally adjusted series, and optionally for out-of-sample forecast errors of RegARIMA. It can also be estimated in terms of quarter-on-quarter changes, $C_{t|t} = (y^{sa}_{t|t} - y^{sa}_{t-1|t})/y^{sa}_{t-1|t}$, in such case corresponding to: $R_t = C_{t|T} - C_{t|t}$. In general terms, the lag with which the revision can be estimated–the value of difference $(T - t)$–is a user-defined integer, where $T-t = \{4, 8\}$ are the defaults with quarterly data. As a measure of instability, an opposite sign between $C_{t|T}$ and $C_{t|t}$ represents the undesirable case where the series changes its direction as a result of methodological instability. A final remark on the motivation to analyze revisions is provided by US Census Bureau (2011). As the majority of users are concerned with last observations, it is worth mentioning that many of them are based on preliminary data. Hence, if final data are issued or revised three quarters after their first release, setting $T - t = 3$ (or sadjlags=3 and trendlags=3) would be appropriate to study the seasonal effect of that series.

3.4 Why may direct and indirect adjustment results differ?

There are several reasons why both kinds of adjustments may differ. I discuss some of these reasons next, supposing that the same methodology and sample span is applied for both adjustments.[17] I follow closely the discussion provided in Ladiray and Mazzi (2003), and also, but on a lesser extent, in Peronacci (2003). Firstly, note that several unrealistic conditions should fulfill the adjustments in order to deliver the same results. These include: when the combination and adjustment are linear, the series does not exhibit outliers, the same filter is applied to all series, and, if with multiplicative adjustment there is no irregular component, then it is more likely that both adjustments coincide. In the same way, similar results between both adjustments can be obtained if the statistical characteristics of the series are similar. This is a very uncommon fact with GDP datasets, but more likely with interest rates.

The adjustment made directly or by means of its disaggregation may differ because of:

- *The use of a multiplicative adjustment in disaggregates.* This is especially the case with an aggregate compound by an algebraic sum of components, such as GDP. Note that when components are adjusted in a multiplicative way, the original series are divided by a specific seasonal factor. Since the seasonal factors are not common across sectors, or even more, imperfectly correlated, the sum of those components should exhibit a different behavior to that of by, say, an additive adjustment of the aggregate.

[17] This assumption also refers to comparisons with same data vintages without revisions.

- *Presence of outliers and differences in the trading day effect of the disaggregation.* As in the previous case, idiosyncratic effects of components could cast for different parameters in the adjustment. A rough example consists of an aggregate of two equally weighted components. Suppose that both components exhibit the typical behavior of an economic series, and also related between them, except for its outliers. Also, the first series experience several outliers across sample span, followed by outliers in the second component in the same direction. Obviously, they will be corrected separately implying that they can still share the same filter characteristics. But, the aggregate will exhibit a very noisy behavior as it adds up both kinds of outliers–or shocks. Thus, it will be hard to find a stable filter for adjusting the aggregate, implying that different estimates would emerge.

- *Use of different filters.* Even in a series without outliers and with the same kind of adjustment, the results may diverge. This could be the case when different filters are used. Note that sensibility of MA to its order leads to dramatically different results in cases with volatile series. Recall that the order of an MA is closely related to the independence–lower correlation–of the series observations. So, the more independent the observations are, the more sensitive to its order the MA is. In the case of seasonal adjustment, it could occur that when just one component has a different filter than the rest, the indirect adjustment result moves away from that of the direct one. This point is exaggerated when the observations of the aggregate are less independent of those of its disaggregation.[18]

- *Different forecasting models.* If disaggregation is forecast with different ARIMA models, and consequently different from that of the aggregate, it is more likely that both adjustments will not coincide. This is because different forecasting models could induce major revisions due to forecasting accuracy. Furthermore, the forecasting model can account for different MA filters between disaggregations. Hence, redound into differences between direct and indirect adjustment.

- *Harshness of aggregation.* Obviously, as the number of series included in the aggregation increases, its weight in the aggregate decreases. This happens in any context such as, for instance, original and seasonally adjusted series. Thus, a series composed of n_1 subseries, it is more likely to deliver more unstable results than that one built with $n_2 \gg n_1$ subseries. This is not necessarily always the case as it still depends on how the aggregation is made. If the aggregation of components with erratic seasonality converges to a series with stable seasonality, it is desirable to aggregate just a little number of series in terms of stability. But, if the assumption $n_2 \gg n_1$ is computed with a large enough n_2 relative to n_1, the marginal contribution of a series with erratic seasonality will not affect the result of the aggregate.

- *Fading out of judgment.* As will be discussed later, a way to improve poor adjustment results consists in fixing some diagnostics by means of user's judgment. This is common with series that exhibit erratic seasonal components, requiring fine expertise rather than those that exhibit stable seasonality. It could be the case that the choice based on user judgment of a specific filter, forecasting model, specific fixed effect, or any other user intervention may help to correct some diagnostics. Nevertheless, as a result of this intervention, the disaggregation becomes adjusted under a different setup of that of the aggregate. This leads to almost surely divergent results between the direct and indirect adjustments.

3.5 How to revert poor results?

There are several strategies to evaluate when some diagnostics fail in order to extract seasonality. These corrections arise supposing that a more stable adjustment is preferable with most recent data. All these corrections require user intervention. Expert judgment is always desirable. Manipulations can be made at any stage of the process (described in figure 1) or even prior to the analysis without affecting results' validation. Some recommendations found in the literature include the following.

- Regarding seasonality detection, spectrum diagnostics tend to fail when changing seasonality is present. Spectra work well with medium-size series. With a long series, spectral plots show little change when

[18] See below, in the empirical application section, the cases of the supply sectors *Capture fishery* and *Electricity, gas and water*, to name a few, with respect to the case of GDP by itself.

more observations are added. This could hide most recent data seasonality (Hood and McDonald-Johnson, 2009). Hence, analysis of more recent data (latest 60 observations) leads directly to the detection of seasonality with the spectrum of original series.

- For the same abovementioned problem, a reduction of spectrum bandwidth increases the probability of detection of seasonality. As Hood (2007) finds, the use of an AR(10) model instead of an AR(30) casts for less smoother spectral plots. This change brings attached the risk of finding more false negative cases. Nevertheless, this represents a minor shortcoming when seasonality is hard to detect. This is especially the case when the AR(30) model cannot discriminate because of short peaks at seasonal or trading day frequencies.

- Another way to find seasonality relies on the suggestion proposed by Maravall (2005). This consists of a joint test of seasonal dummies contained in a regression of original series. The theoretical distribution is based on the nonparametric Kendall-Ord test (Kendall and Ord, 1990). As it constitutes a nonparametric test, its use with a short sample span, however, is not entirely recommended.

- The goal of seasonal adjustment is to achieve the absence of residual seasonality. This task could be eased with a series that has already been controlled for effects that facilitate residual seasonality–by removing outliers, for instance. With this purpose in mind, Soukup and Findley (1999) suggest testing residual RegARIMA series for seasonality through spectral plots. This check, as the authors suggest, improves the capacity of remaining diagnostics contained in the program to dissipate residual seasonality.

- Several rough interventions also facilitate both detection and quality improvement. This category includes fix data transformation–such as a logarithm–, change filters lengths, avoid sample span flagged as unstable–according to sliding span and revision history–, RegARIMA fixed effects manipulation and/or fix RegARIMA model, among others. Note that X12 has the option of deactivating specific stability diagnostics, such as sliding spans or RegARIMA residuals outliers' detection. An iterative process fixing one control at a time is recommended, basically to ensure consistency as new information arrives.

- A typical undesirable effect are outliers. RegARIMA specifically controls for additive outliers. Nevertheless, a second check is recommended specifically when it appears in recent observations. Taking the case of, for instance, an outlier at the last observation available. For sure, RegARIMA will replace this observation with one affine to the series prior to outlier arrival, and no further intervention will be required. But, if it is expected that few of next observations will exhibit the same value, then it is recommended to include the observation in the adjustment process. Keeping the outlier observation for processing can be managed in several ways inside the RegARIMA module. For instance, by using the `span` or `types=(ls tc)` options within `outlier` command. Of course, a different situation arises when the outlier is located near the half of series length. Obviously, the most ideal case with outliers is when they are at the beginning of the sample, and the sample is a long enough.

- For stability purposes, Bobbit and Otto (1990) suggest the use of long-horizon backcasts and forecasts. The authors conclude that revisions are smaller when a series is extended with enough long-horizon forecasts. This finding is achieved by comparing the stability of adjusted series with extended original series–using a symmetric filter–versus not extended or extended by just one year. It is also interesting, especially from a practitioners' point of view, that a simple automatic procedure is equally accurate as modeling individual ARIMA processes.

4 Empirical application: The case of Chile's GDP 1986-2009

In this section, I focus on applying the abovementioned diagnostics to determine whether is preferred a direct or indirect seasonal adjustment for the Chilean GDP 1986-2009 (base year: 2003). The diagnostics considered are all those that are robust to direct and indirect adjustment, namely spectral plots, sliding spans and revision history. The analysis is complemented with several statistics that resume part of the adjustment quality assessment as, for instance, bias in seasonally adjusted series.

Notice that the particular vintage used has several desirable characteristics for an economic-statistical analysis, such as: (i) it does not have induced breaks or level shifts due to methodological changes, (ii) it is prepared and released on a quarterly basis, (iii) it is compounded by a number of sectors with different seasonal patterns, and (iv) it does not have any missing values, mismatches, or any other shortcoming to deal with previous to the adjustment.

The dataset corresponds to the same used in Medel (2013). In that work, a complete out-of-sample exercise is provided with the original as well as seasonally adjusted series using the same aggregation schemes. Thus, it is clear and direct the impact on the forecast accuracy of seasonality remotion. It also accounts for the impact of trading day effect on out-of-sample performance.

4.1 Data: Descriptive statistics and aggregations

I use the first 96 observations of the Central Bank of Chile's Quarterly National Accounts (QNA), from 1986.I to 2009.IV, starting with GDP as the most aggregated, and with three levels of disaggregation on the demand and supply sides. The construction of the dataset is made as follows. First, it is used the dataset detailed in the official volume "National Accounts of Chile 2003-2009" (Banco Central de Chile, 2010). The original series are in levels, denominated in millions of 2003 Chilean pesos. Hence, the additivity–or subtraction–is ensured as the components are denominated in the same units and constructed with the same base year. This also implies a weight equal to 1 (or -1) for an added (or subtracted) component (Ladiray and Mazzi, 2003). Second, I calculate the quarterly variation of the dataset provided in Stanger (2007). This dataset contains quarterly data from the period 1986-2002 denominated–also–in millions of 2003 Chilean pesos. Finally, the quarterly overlapping method is used backwards by fixing the base year 2003 and building the series back to 1986 with the quarterly rates calculated previously. Nevertheless, I use only the first 96 observations–counting from 1986.I onwards–as they constitute the minimum length suggested by X12 developers to stress X12 capabilities, even though the minimum length of series to perform a routine is 60 observations.

The original as well as the adjusted series compound the Chilean GDP by demand side and supply side. A scheme of demand-side aggregations of all series with acronyms used in this paper is shown in annex C, and those of supply-side in annex D. There is a total of five different aggregations plus the GDP by itself ("Aggregation 6"). All five aggregation blocks as well as aggregation 6 deliver the GDP, calculated as a sum of every corresponding component.[19]

Since 2008 the Central Bank of Chile has adopted the linked-chain methodology to produce the GDP dataset. This methodology–used to improve representativeness–implies that the dataset lose its additive property. Under this scheme, the disaggregation added to an aggregate does not necessarily coincide with the direct aggregate. The indirect seasonal adjustment quality–and stability–depends in this context, among others factors, on the so-called non-additive term that emerges from the difference between both aggregates. A method to adjust indirectly a linked-chain variable is presented in Scheiblecker (2014), but it has not been thoroughly examined yet. So, indirect adjustment with linked-chain data is left to further research.

4.2 Setting up the exercise

The exercise consists of adjusting all the components by demand side and supply side to then add up according to the aggregation schemes. Note that some parameters governing the adjustment of each component may differ between them. The exercise is carried on the automatic program default mode, according to the suggestions presented in Maravall (2002). Then, if a component does not fulfill the statistical criteria of an acceptable result, minimal interventions are made one-by-one until all test are fulfilled. The choice of these interventions is made considering stability as a criterion. It is, trying to keep the adjustment parameters fixed along the evaluation sample span–from the 60^{th} to the 96^{th} observation (2000.IV to 2009.IV). Note that this dynamic scheme is made to compute the sliding span and revision history. Oppositely, spectral plots are computed with the whole sample.

[19] Annex E displays the share of every sector of real GDP at 2003 prices, while annex F displays some typical statistics of sectors with the full sample for the transformations used in the analysis. Annex G depicts all aggregation components in log levels to provide a general overview of their shape in the time domain.

The adjustment is made through *Eviews 7.2* interface for X-12-ARIMA version 0.2.10. Notice that *Eviews* makes use of its own notation for X12 options. These options used by *Eviews* are the following (while the script used for the adjustment is presented in annex H):

- **Mode**: For setting up the seasonal adjustment method: `"m"` for multiplicative, `"a"` for additive, `"p"` for pseudo-additive, and `"l"` for log-additive seasonal adjustment.
- **Filter**: For setting up the seasonal filter: `"msr"` for automatic, moving seasonality ratio, `"x11"` for X11 default, `"stable"` for stable, and `"s3xj"` for $3 \times j$ MA, $j \in \{1, 5, 9, 15\}$.
- **Trans.**: For setting up a transformation for RegARIMA: `"logit"` for logit, `"auto"` for automatically choose between no transformation and log transformation, `number` for Box-Cox power transformation using specified parameter, where `"0"` is for log transformation.
- **SSpan**: For analyzing the stability with sliding spans: `"sspan"`.
- **Check**: For checking the residuals of RegARIMA: `"check"`.
- **Outlier**: For analyzing the presence of outliers of RegARIMA: `"outlier"`.
- **ARIMA**: For setting up the ARIMA specification of RegARIMA: `"f"` for use forecasts from the chosen model, `"b"` for use forecasts and backcasts from the chosen model, `"(p,d,q)"` for manually entering specification. When this option is used, it could also be accompanied by `"oos"` for use out-of-sample forecasts for automatic model selection.

The adjustment options of each component are presented in table 2. Note that a logarithmic multiplicative adjustment with automatic filter selection is the preferred specification.[20] The sliding span for trend-cycle and seasonally adjusted, along with the diagnostics used for resultant RegARIMA residual series, are also selected. The trading day effect cannot be identified in these series because of sample length. All remaining parameters are left in *Eviews'* default mode.

Several adjustments deserve mentioning. The *Change in inventories* (*ci*) series does not show seasonality according to X12 diagnostics. When series containing this component are added up to its aggregation, *ci* is included on its original version. The *External demand* (*ed*) series corresponds to a series in level that allows for negative and zero values. Therefore, an additive adjustment was tried but with negative outcome. As a result, is used an indirect adjustment corresponding to its definition but with seasonally adjusted series: $x_t^{sa} - m_t^{sa}$. The seasonality of the *Exports of services* (xs) series is found with two RegARIMA specifications at two different sample spans. The first, with a (0,0,1) specification is obtained through 1986.I to 2003.I, while the second, for 2003.II to 2009.IV is found using a RegARIMA specification supported by an out-of-sample criterion. In general, all those series in which the RegARIMA specification is fixed–to the (0,0,1) model–or sliding span option deactivated, correspond to cases with an erratic behavior across time, with a relatively high variance (see annexes F and G), specially at the first stages of the sample.

[20] Common error messages in X12 by each kind of adjustment were:

Additive Adjustment:

`WARNING: The program will not generate backcasts for series longer than 15 years.`

`ERROR: Cannot calculate X-11 holiday adjustment for a quarterly series.`

`ERROR: An X-11 holiday adjustment cannot be performed unless the multiplicative seasonal adjustment option is chosen. No seasonal adjustment this run.`

Pseudo Additive Adjustment:

`ERROR: Cannot calculate X-11 holiday adjustment for a quarterly series.`

`ERROR: Pseudo-additive seasonal adjustment cannot be performed when preadjustment factors are derived from a REGARIMA model. No seasonal adjustment this run.`

Multiplicative Adjustment:

`ERROR: Cannot calculate X-11 holiday adjustment for a quarterly series. No seasonal adjustment this run.`

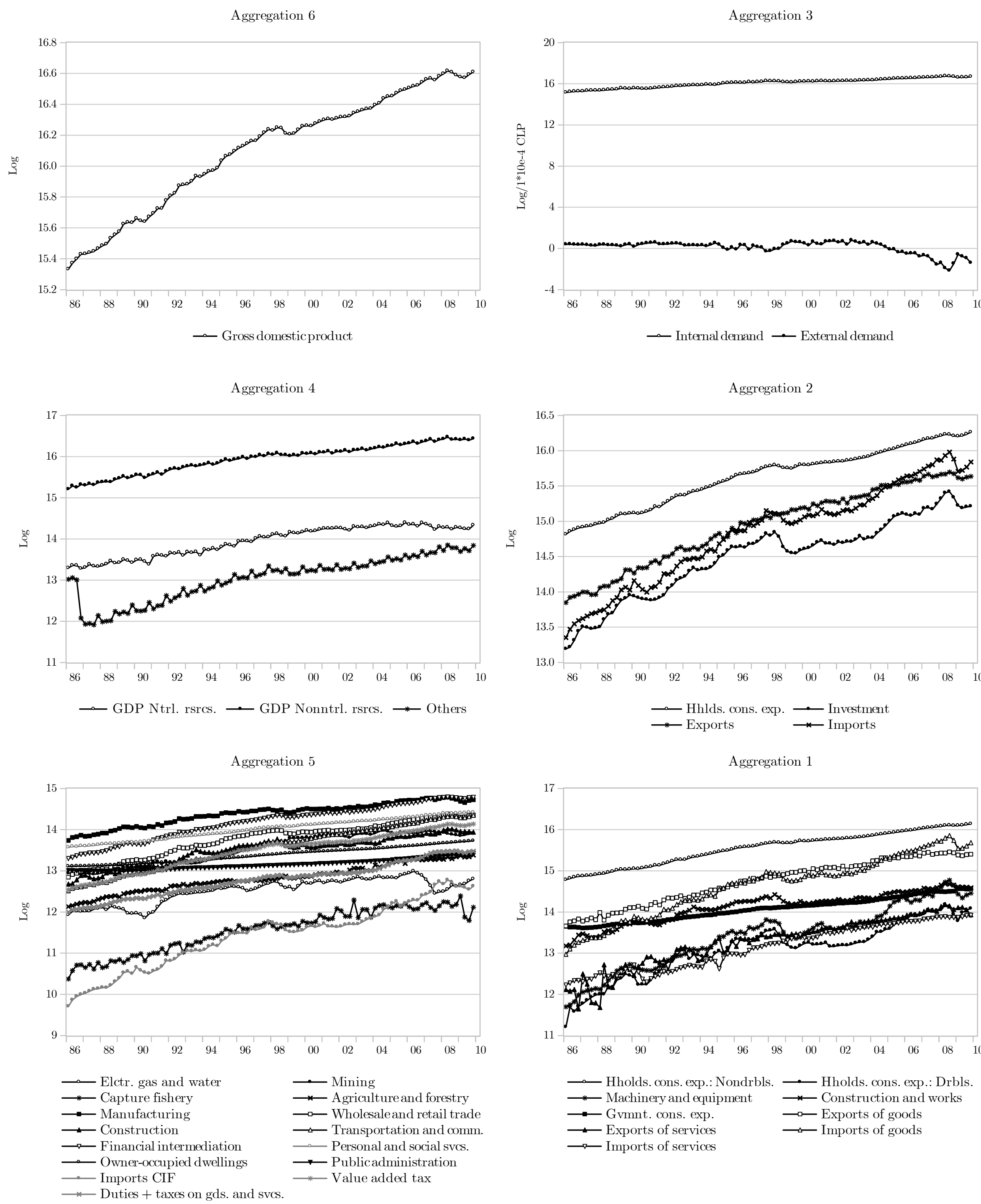
Table 2: Seasonal adjustment options of each sector

Sector	Agg.	Mode	Filter	Trans.	SSpan	Check	Outlier	ARIMA
Households cons. exp.: nondurables	*1*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Households cons. exp.: durables	*1*	*m*	*stable*	*auto*	✗	✓	✓	*F,oos*
Machinery and equipment	*1*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Construction and works	*1*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Changes in inventories (*)	*1*	✗	✗	✗	✗	✗	✗	✗
Government cons. expenditures	*1*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Exports of goods	*1*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Exports of services, first sample	*1*	*m*	*msr*	*auto*	✗	✓	✓	*(0,0,1)*
Exports of services, second sample	*1*	*m*	*msr*	*auto*	✗	✓	✓	*F,oos*
Imports of goods (**)	*1*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Imports of services (**)	*1*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Household cons. expenditures	*2*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Investment	*2*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Exports	*2*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Imports (**)	*2*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Internal demand	*3*	*m*	*msr*	*auto*	✗	✓	✓	*(0,0,1)*
External demand (***)	*3*	✗	✗	✗	✗	✗	✗	✗
Electricity, gas, and water	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Capture fishery	*5*	*m*	*msr*	*auto*	✓	✓	✓	*(0,0,1)*
Mining	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Wholesale and retail trade	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Manufacturing	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Construction	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Agriculture and forestry	*5*	*m*	*msr*	*auto*	✓	✓	✓	*F,oos*
Transport and communication	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Financial interm. and business services	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Personal and social services	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Owner-occupied dwellings	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Public administration	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Duties + taxes (**)	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Non-deductible VAT	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Imports CIF	*5*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
GDP Natural resources	*4*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
GDP Non-natural resources	*4*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
Other sectors	*4*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*
GDP	*6*	*m*	*msr*	*log*	✓	✓	✓	*F,oos*

(*) Not considered in analysis. (**) Subtracted. (***) Adjusted indirectly. Source: Author's elaboration.

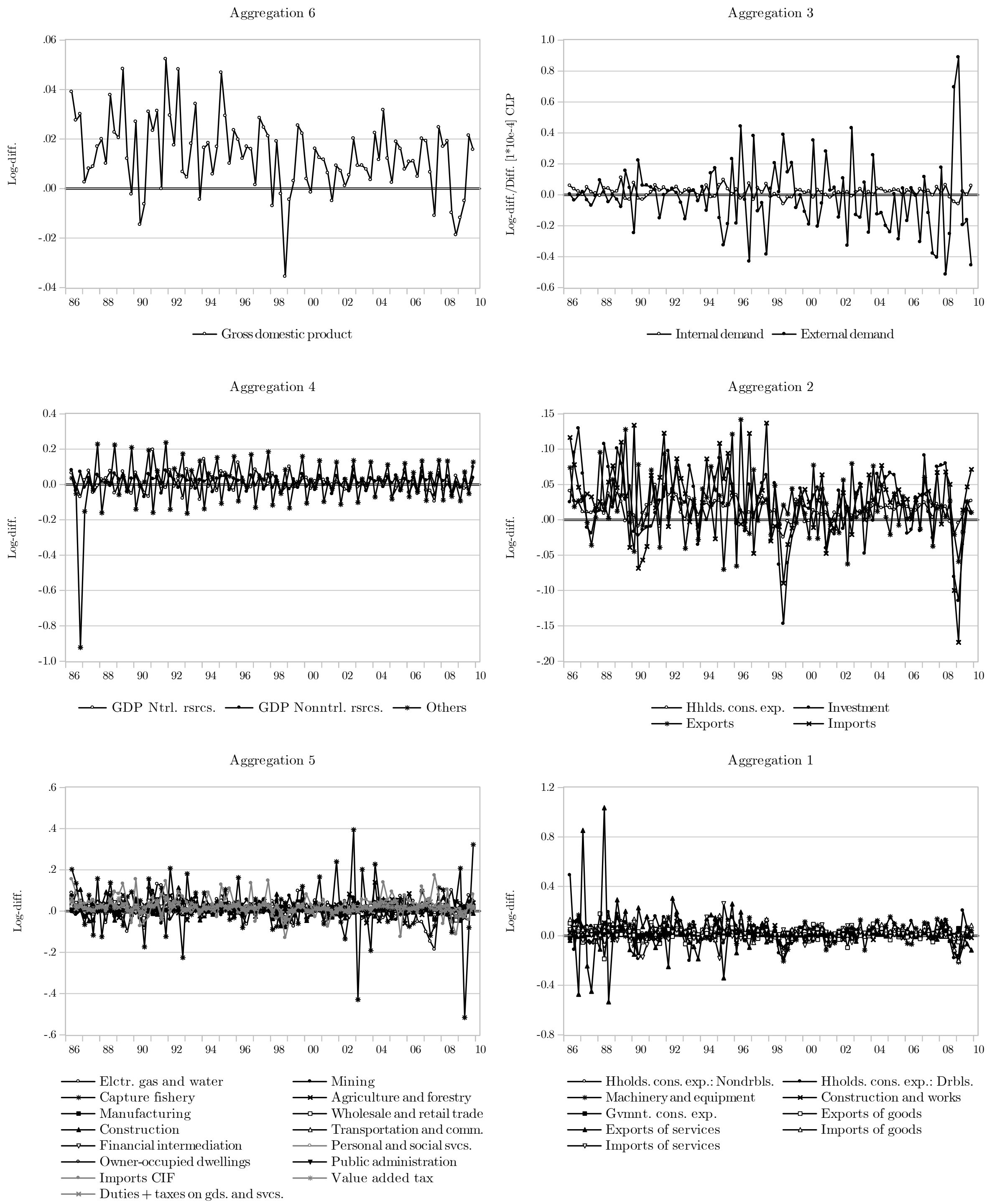
Figure 3: Chilean GDP - Seasonally adjusted and log-differenced series

A: Seasonally adjusted series



Source: Author's elaboration.

B: Log-differenced series



Source: Author's elaboration.

4.3 Results

In this subsection, I analyze the estimates of diagnostics checks for all six aggregations. Notice that, as aggregation 6 acts as a natural benchmark, deviations from seasonally adjusted series of that aggregation are considered a low quality adjustment. However, this result has to be considered just as a benchmark given the discussion of previous sections about the possibilities among which the different schemes may differ. Specifically, disaggregates can handle Easter effects and nonlinearities better, and when adding up to the original, different seasonal patterns do not cancel noisy effects at all. A corollary of this discussion is that an "actual seasonally adjusted" series does not exist, while many little changes in some parameters for the aggregate and all its diagnostics falling on the acceptance region. This implies that expert judgment plays a key role in determining which strategy is best.[21]

To have a general overview of the adjusted series, in figure 3 panel A, I plot the seasonally adjusted series in logarithmic levels (whole sample), while in panel B, its quarterly variation (quarter-on-quarter changes).

The first result concerns bias. As X12 adjusts the series within a year across all years of available sample, the mean of seasonally adjusted series should coincide with that of original series. Table 4 displays this result, showing a downward bias for the demand side aggregations (-1.0% on average) and an upward bias for those of supply side (1.2% on average). As displayed in the table, aggregations 4 and 6 are the least biased adjusted series. Regarding second moments, panel B shows the covariance of the first five aggregations with respect to adjusted GDP–aggregation 6–given that a full comparison with actual series is not possible. As in the case of mean, aggregation 4 is the closest to the benchmark, along with aggregation 3.[22]

Nevertheless, the original dataset described in Stanger (2007) and Banco Central de Chile (2010) recognize that the data construction method comprises bias, but with values surrounding 0.2 to 0.0% of total GDP. Thus, the results presented in table 4 are possibly due to the nonparametric characteristic of the program, being unable to detect special data effects without user intervention, namely trading day, at sectoral level. The implications for modeling–and forecasting–, however, can be easily controlled through the use of intercept corrections as is pointed out by, for instance, Clements and Hendry (1996).

Table 4: Bias and covariance analysis

Mean of original series: 10,474,000.			
Mean (% deviation to original) of aggregation:			
1	10,399,000 (-0.7%)	*4*	10,566,000 (0.9%)
2	10,349,000 (-1.2%)	*5*	10,633,000 (1.5%)
3	10,350,000 (-1.2%)	*6*	10,473,000 (0.0%)
Covariance with Aggregation 6 adjustment:			
1	0.9990	*4*	0.9998
2	0.9989	*5*	0.9996
3	0.9998	*6*	1.0000 (by definition)

Source: Author's elaboration.

4.3.1 Spectra-based diagnostics

Spectral graphics are computed with (free licensed) *gretl 1.9.12cvs* software, available at `http://gretl.sourceforge.net/`.[23] As mentioned, they are computed with the whole sample for the *final seasonally adjusted* and *irregular component*. The quality assessment should be easy considering pairwise comparisons of one aggregation to another. Anchoring to aggregation 6 as a benchmark is strongly recommended.

[21] Note that, however, the issue of robustness arises in many econometric problems with finite sample as a result of parameter uncertainty and/or misspecification.

[22] Nevertheless, notice that these results are not analyzed through a formal statistical test. Instead, they are provided as preliminary exploratory analysis.

[23] There are several (pay licensed) software alternatives to estimate the spectrum. For instance, using the command `pergram` in *Stata*, or the command `periodogram` in *Matlab*.

The results are presented in annex I. From the first panel of figures, concerned with seasonally adjusted series, it is easy to be aware of the inefficiencies of aggregations 1 and 2: they exhibit clearly a peak at $\frac{\pi}{2}$ frequency–thereof seasonality–but also in the irregular component (second panel). Regarding the four remaining cases, the three first are all visually congruent with aggregation 6. However, considering the irregular component, note that the spectrum of aggregation 4 is most alike to the benchmark. It is worth to mention that aggregation 6 positively overpass all X12 diagnostics, despite of a peak between frequencies $(\frac{\pi}{2}, \frac{3\pi}{4})$–associated with trading day effect; but no robust identification was possible due to sample size. As a result, aggregations 4 and 6 remain with promissory results.

4.3.2 Sliding-span-based diagnostics

The sliding spans are estimated with the minimum length of observations required by X12: 60 observations. As I dispose of the shortest length, the estimation of sliding spans is made in recursive scheme rather than a rolling-window alike, adding one year at a time. This treatment can be seen as a "lower bound quality" made by sliding spans, given two forces behind the dynamics that emerge while the span is increased. On the one hand, if more observations are available, they will be added while the initials will be dropped. The new data, similar to most recent dynamic, will redound in a better quality adjustment. Furthermore, with long enough series, it is possible to exclude observations that show different behavior from that of the more recent data. On the other hand, as Otto (1985) suggests, adding more observations to series–being actual as well as (back)forecasted–result in smaller revisions. To the matter of this empirical application, however, these both excluded possible improvements seem to have a minor impact. This, because the first years of data seem well behaved in most sectors.

The sliding spans are computed for final seasonally adjusted and trend-cycle series with two transformations: logarithmic and log-differenced levels. These four sets of variables are analyzed because those are the series that attract the attention of the users. The log-differentiation stands for quarter-on-quarter changes approximation. The results are provided in terms of descriptive statistics of $S_t(k)$ series. Recalling that for the series in logarithmic units $S_t(k) = [\max_k y_t(k) - \min_k y_t(k)] \,/ \min_k y_t(k)$ for span k, so the mean of $S_t(k)$ until K available spans corresponds to:

$$\overline{S} = \sum_{t=1}^{T} \frac{\max_k y_t(k) - \min_k y_t(k)}{\min_k y_t(k)}, \ \forall k \in K | K = \{1, ..., \frac{T}{s}\},$$

quantifying the average deviation from the minimum to maximum estimates of each observation. For log-differenced series, the analyzed statistic corresponds also to the mean of $S_t(k)$, but defined as:

$$\overline{S} = \sum_{t=1}^{T} \left[\max_k QQ_t(k) - \min_k QQ_t(k) \right], \ \forall k \in K | K = \{1, ..., \frac{T}{s}\},$$

where $QQ_t(k) = [y_t^{sa}(k) - y_{t-1}^{sa}(k)]/y_{t-1}^{sa}(k)$. Note that if $S_t(k) > \alpha\%$, then the observation t is flagged as unstable. The value of α used as a threshold is 3% for seasonally adjusted series and 33.3% for trend-cycle series. This last value is considered as an approximation to the 3% threshold applied to seasonal factors. Thus, it supposed a null expectation on irregular component movements. The adjustment is considered of poor quality if the number of observations flagged as unstable exceeds 25% with logarithmic and 40% with log-differenced series.

To make a more demanding comparison, I find the maximum and minimum values of $y_t(k)$ through 3 to 10 spans, recalling that X12 finds from 2 to 4 spans. Thus, including a higher number of spans gives more chances to consider more distant estimates. Given the recursive scheme, available data allows to estimate 10 sliding spans. The first is estimated until 2000.IV (1^{st} to 60^{th} observation) while the last, until 2009.IV (1^{st} to 96^{th} observation). Note that the last span that allows for a joint triple comparison includes observations until 2007.IV–this is, the last span where $\#k^{min} = 3$.

The results are reported for trend-cycle and seasonally adjusted series, for two transformations (log and log-differenced) in the following scheme:

	Log-differenced	Logarithms
Seasonally adjusted series:	Annex J	*Companion Tables*, Section 2
Trend-cycle series:	*Companion Tables*, Section 1	*Companion Tables*, Section 3

where *Companion Tables* refers to a separate document accompanying this work. As a summary, table 5 reports some descriptive statistics of $\overline{S}$ series.

As can be seen from table 5, most of the divergence between aggregations occurs with the log-differenced series. In the case of seasonally adjusted series, note that the benchmark achieves a 34% of unstable cases (threshold: 40%). Recalling that the adjustment made to aggregation 6 overpass all the diagnostics in the default mode. This 34% of unstable cases is achieved with the modifications made to the sliding-span basic scheme and threshold impositions. These modifications are made basically to stress the differences between aggregations. The results with this kind of data show that two aggregations exhibit better results than the benchmark, both belong to the supply side: aggregations 4 and 5, for median, mean and times flagged as unstable. A similar result is replicated qualitatively with trend-cycle series, but excluding aggregation 5. In this case, aggregation 4 also seems an alternative–especially given the stability of its point estimates. So, overall, aggregation 4 remains as the most stable along with aggregation 6 with both log and log-differenced data.

Table 5: Summary of sliding-span-based diagnostics

Trend-cycle series

	Log-differenced (threshold for % Unstable: 40%)						Logarithmic (threshold for % Unstable: 25%)					
Aggregation	*1*	*2*	*3*	*4*	*5*	*6*	*1*	*2*	*3*	*4*	*5*	*6*
Median	42.30	23.80	211.80	11.40	22.20	12.20	0.35	0.23	1.51	0.11	0.19	0.10
Mean	69.81	26.28	354.56	13.08	24.76	14.11	0.54	0.26	2.36	0.14	0.21	0.14
Maximum	316.00	80.40	1122.00	4610	83.70	52.70	3.05	0.70	7.20	0.56	1.08	0.52
Minimum	12.30	3.10	41.80	0.70	3.70	0.20	0.10	0.03	0.33	0.01	0.02	0.00
% Unstable	71%	27%	100%	5%	34%	11%	2%	0%	33%	0%	0%	0%

Seasonally adjusted series

	Log-differenced (threshold for % Unstable: 40%)						Logarithmic (threshold for % Unstable: 25%)					
Aggregation	*1*	*2*	*3*	*4*	*5*	*6*	*1*	*2*	*3*	*4*	*5*	*6*
Median	78.00	56.20	243.00	22.10	27.40	27.40	0.56	0.49	1.57	0.14	0.18	0.19
Mean	86.20	75.89	263.20	26.88	30.34	32.03	0.58	0.53	1.72	0.18	0.21	0.22
Maximum	215.10	220.80	608.80	100.40	98.90	96.90	1.81	1.50	3.54	0.51	0.54	0.54
Minimum	25.50	14.60	87.80	2.20	5.50	1.40	0.12	0.03	0.38	0.02	0.03	0.02
% Unstable	98%	86%	100%	30%	30%	34%	0%	0%	10%	0%	0%	0%

Shaded cells indicate the minimum median and mean of each transformation, and reliable ajdustments.

Source: Author's elaboration.

In the case of logarithmic series, just little differences can be adverted. However, aggregation 4 for seasonally adjusted data and aggregation 6 for trend-cycle, provide the most stable estimates, with aggregation 4 for trend-cycle being also strongly suggested to use. Among all possibilities, undoubtedly aggregation 3 has the worst performance, representing the case where adding up components with different seasonality results in a worst aggregate performance.

4.3.3 Revision history diagnostics

As defined in subsection 3.3, revision history is the distance (always nonnegative) between an observation of specific component of seasonal adjustment process, made with the sample available until that observation (*concurrent*), and the same observation when the adjustment is made with additional data (*most recent*). As concerns the stability of the overall process, this measure has the advantage of quantifying, and making comparable through different methods, the dynamic effect of adding new data. Hence, it clearly exposes the methodological instability of the X12 program. Notice that this instability arises mainly from the sensitivity of MA estimates to change in the dynamics of the series. Furthermore, the instability has to be observed through a set of observations and not by one observation alone. In that case, RegARIMA will treat it as an outlier and exclude it from the adjustment.

The results are presented graphically in annex K. The figures show, plotted for log-differenced trend-cycle (panel A) and seasonally adjusted series (panel B), the concurrent and the most recent (marked with ▾) estimates of each observation across time. Hence, two statistics are reported: (i) the absolute average change, and (ii) the number of times that the movement from concurrent to most recent (or vice versa) includes the zero. The absolute average change is the average of the distance between concurrent and most recent, across time. So, a shorter average results in better stability. The second statistic accounts for the undesirable case that is reporting firstly a decrease in the series to, quarters later, notice that the series indeed increases, or vice versa.

Table 6 reports a summary of both statistics analyzed. As shown, aggregation 4 exhibits better statistics than aggregation 6 for both trend-cycle and seasonally adjusted series jointly. In particular, the absolute change for trend-cycle reaches 0.2 with aggregation 6, while just 0.1 with aggregation 4. Notice that the best case of demand-side–aggregation 2–is not fully disposable. With seasonally adjusted series, the benchmark achieves an average of 0.3, in a tie with aggregations 4 and 5. With these series, however, demand-side aggregations exhibit a worse performance than in the previous case. Regarding the "Times equal to 0" measure, notice that the results are not generally scattered, except with aggregation 3. Undoubtedly, aggregation 4 represents the best case where never the estimates change their direction as new data are incorporated in the adjustment. Hence, aggregation 4 come out as the most stable aggregation.

Table 6: Revision history results summary

Trend-cycle series

Aggregation	*1*	*2*	*3*	*4*	*5*	*6*
Absolute average change	0.7	0.3	4.4	0.1	0.3	0.2
Times equal to 0	3	1	15	0	1	0

Seasonally adjusted series

Aggregation	*1*	*2*	*3*	*4*	*5*	*6*
Absolute average change	0.6	0.6	1.1	0.3	0.3	0.3
Times equal to 0	3	1	12	0	1	2

Shaded cells indicate the aggregation with minimum

Absolute Average Change and Times equal to 0.

Source: Author's elaboration.

5 Concluding remarks

This paper addresses the question of which aggregation scheme provides the best results for an overall seasonal adjusting process for the Chilean GDP 1986-2009 (base year: 2003) using the X-12-ARIMA program version 0.2.10. These are understood as the best results that achieve the most stability in resultanting series. This stability is tested with specific tools contained in the X12 program; specifically, spectral plots, sliding spans and revision history. This paper provides a detailed description of these tools as well as other capabilities contained in X12.

Spectral plots are used commonly to analyze a series through its frequency domain. It is used in this context as a model-free diagnostic to associate the cycles of series with their strengths. Therefore, spectral plots for final seasonally adjusted series as well as for irregular components are especially useful to detect residual seasonality. Hence, it evaluates the quality of the overall process.

Sliding-span diagnostics and revision history can be seen as pure stability measures. Sliding spans allow the user to detect unstable passages of the sample–indicating also if a user intervention is required. Revision history quantifies how reliable the adjustment applied to certain series is as the sample is increasing in length.

All the abovementioned diagnostics are analyzed for the case of Chilean GDP 1986-2009 (base year: 2003). The results show that in terms of stability, the use of the direct–the GDP by itself–is recommended as is the first stage of disaggregation by supply-side. Moreover, the results for the second and third stage of disaggregation by demand-side are strongly poor, according to the standard setup used for the adjustment. These results are not surprising at all, given that the aggregations made by supply side already groups components with more affine dynamics. These components are, at least, themselves more affine than the well-known demand-side National Accounts convention. Regarding methodological issues, particularly powerful tools to discriminate between stable aggregations are spectral plots and sliding spans, both estimated to the final seasonally adjusted series.

Finally, several topics emerge for future investigations. Especially relevant are the calibration of some thresholds used by diagnostics to certain–and relevant–cases as Chilean GDP, or if there is any difference in thresholds used for real or financial variables. Also, since 2008 the Central Bank of Chile produces the GDP dataset under a linked-chain methodology. As this method makes variables grow independently, a difference between the aggregates (may) emerge. Thus, the stability, ultimately, depends on the statistical properties of this non-additive term. Nonetheless, these and other topics are left to further research.

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A Quality measures in X-12-ARIMA

M1-M11 and Q Statistics

This annex, based on Lothian and Morry (1978), reviews the statistics used by X12 to assess the quality of an adjustment. These statistics constitute a set of eleven elements, which can be summarized into one–the Q statistic–when weighted by a specific constant factor. They are presented within "Table F.3" in the X12 log-file. The main purpose is to analyze the contribution of the trend-cycle, seasonally adjusted and irregular component to changes in original series. Notice that all estimations should be done with stationary series; otherwise, variance–as well as other moments–is not defined. The statistics are the following (its weight to conform the Q-stat is shown in [ ]).

- **M1** [13%] *Relative contribution of the irregular component to the variance of original series, over span* k. This measure is important because if the irregular variation is too high compared with the variation of the seasonal component, the two components cannot be separated successfully. Consider the following estimation for irregular series:

$$\overline{y}^{ir}(k)^2 = \frac{1}{T-1}\sum_{t=2}^{T}(y_t^{ir} - y_{t-1}^{ir})^2,$$

  and similarly $\overline{y}^{\tau}(k)^2$ and $\overline{y}^{f}(k)^2$ for trend-cycle and seasonal factor series, respectively; all for lag k. Then, the variance of original series with lag k is $\overline{\sigma}(k)^2 = \overline{y}^{\tau}(k)^2 + \overline{y}^{sa}(k)^2 + \overline{y}^{ir}(k)^2$. Hence, the relative variance contribution of irregular component, $R_{ir(k)}$, is defined as: $R_{ir(k)} = \left(\overline{y}^{ir}(k)^2/\overline{\sigma}(k)^2\right) \times 100$. Lothian and Morry (1978) suggest the use of a order lag 3 with monthly series ($k = 3$), consequently, equal to 1 with quarterly series ($k = 1$). This differencing also converts series into a stationary one. These lags allow a maximum contribution of the irregular series equal to 10% to be acceptable. So, a relative contribution $R_{ir(1)} > 10\%$ fails the test. This leads to the standardized statistic $M1$:

$$M1 = \frac{R_{ir(1)}}{10},$$

  which is not acceptable–the relative contribution of irregular is too high–if it exhibit values greater than 1.

- **M2** [13%] *Relative contribution of the irregular component to the variance of original series, over span* k, *with different transformation to achieve stationarity.* This measure is similar to $M1$, with the difference that the transformation to achieve stationarity is a linear trend remotion instead of differencing. Hence, if:

$$Contribution\ of\ \widetilde{y}^{ir} = \frac{variance\ of\ \widetilde{y}^{ir}}{variance\ of\ \widetilde{y}},$$

  where $\widetilde{y}$ stands for "after trend remotion", then the $M2$ statistic corresponds to:

$$M2 = \frac{Contribution\ of\ \widetilde{y}^{ir}}{10} \times 100,$$

  keeping the decision rule anchored to 1.

- **M3** [10%] *The amount of quarter-on-quarter change due to irregular component compared with quarter-on-quarter change due to trend-cycle.* In the same spirit of $M1$, this measure stands for analyzing whether it is possible to separate the series into the typical components of seasonal adjustment. If the movements of irregular series are dominant, it is difficult to identify each component, jeopardizing the overall adjustment process. The $M3$ statistic is:

$$M3 = \left(\frac{\overline{y}^{ir}}{\overline{y}^{\tau}} - \frac{1}{3}\right) \times \frac{2}{3},$$

  where $\overline{y}^{ir}$ and $\overline{y}^{\tau}$ are the mean of the quarter-on-quarter changes of series y^{ir} and y^{τ}, respectively. For monthly series, the values of $\frac{1}{3}$ and $\frac{2}{3}$ change to 1 and 2. Decision rule holds anchored to 1.

- **M4** [5%] *The autocorrelation of irregular component.* As its name indicates, the irregular component should have a purely random behavior, with mean equal to 0, a constant variance, and null covariance with other components. X12 makes use of the Average Duration of Run (ADR) to test randomness in the irregular component. This test, developed by Wallis and Moore (1941), is based on the number of turning points–sign changes in quarter-on-quarter series–to test against the alternative hypothesis that the process is an AR(1). A series of infinite length should exhibit an ADR statistic of 1.50. For a sample size of 120 observations, ADR's 99% interval of confidence is (1.30,1.75), and values on the left (right) indicate negative (positive) autocorrelation. The approximation of ADR to normal distribution of Bradley (1968) helps in the construction of the $M4$ statistic:

$$M4 = \frac{\left| \frac{T-1}{ADR} - \frac{2(T-1)}{3} \right|}{\sqrt{\frac{16T-29}{90}}} \times \frac{1}{2.58},$$

where T is the sample size, and 2.58 is the 1% limit value of the standard normal distribution in two-sided tests. If $M4$ is greater than 1, the irregular component exhibits significant autocorrelation, and the test fails.

- **M5** [11%] *Quarters for cyclical dominance statistic.* This statistic is built to cast for the number of quarters needed for the trend-cycle absolute changes to dominate those of the irregular component. It measures the relative size of changes in the irregular component respect to the trend-cycle, similarly to $M3$. As the ratio $\overline{y}^{ir}/\overline{y}^{\tau}$ is estimated with a lag varying from 1 to 4 for quarterly series, the same estimation is used to find k–the Quarters for Cyclical Dominance (QCD):

$$QCD = k \; if \; \begin{cases} \frac{\overline{y}^{ir}(k)}{\overline{y}^{\tau}(k)} \leq 1, \; and \\ \\ \frac{\overline{y}^{ir}(k-1)}{\overline{y}^{\tau}(k-1)} > 1. \end{cases}$$

For instance, if a successive estimation of $\frac{\overline{y}^{ir}(1)}{\overline{y}^{\tau}(1)},...,\frac{\overline{y}^{ir}(4)}{\overline{y}^{\tau}(4)}$ results in $\{1.22,0.73,0.90,1.54\}$, the QCD is $k = 3$, hence, implying that it takes 3 quarters for absolute changes in trend-cycle to become higher than changes in the irregular component. A QCD value lower than 2 in quarterly series has been considered acceptable. Hence, the $M5$ statistic corresponds to:

$$M5 = \frac{QCD - 0.17}{1.67},$$

which reflects that if $QCD > 2$, absolute changes in trend-cycle are not higher than those of the irregular.

- **M6** [10%] *The amount of year-on-year change due to irregular component compared with year-on-year change due to seasonal component.* This measure also tackles the issue of identification. One of the steps of the X-11 algorithm (step 7 in table 1) involves the application of $M_{3\times5}$ filter to (preliminary) seasonally adjusted and irregular components in order to separate both. According to Lothian and Morry (1978), when year-on-year changes in the irregular component are too small with respect to that of the seasonal component–a low $\overline{y}^{ir}/\overline{y}^{f}$ ratio–the $M_{3\times5}$ does not work well in accounting for seasonal movements. On the other hand, when the ratio $\overline{y}^{ir}/\overline{y}^{f}$ is too high, the filter does not fully permeate the seasonal component from irregular dynamics. Lothian (1978) pointed out–with actual seasonal series–that when the ratio $\overline{y}^{ir}/\overline{y}^{f}$ fell between 1.5 and 6.5, the $M_{3\times5}$ filter works well. If instead, the value of the ratio $\overline{y}^{ir}/\overline{y}^{f}$ fell at the left or the right of the interval, identification is not successful. The statistic is built using these cut offs:

$$M6 = \left| \frac{\overline{y}^{ir}/\overline{y}^{f} - 4.0}{2.5} \right|.$$

As in previous cases, values above 1 fail the test, and no successful identification can be done.

- **M7** [16%] *The amount of stable seasonality relative to the amount of moving seasonality.* As its label suggests, $M7$ statistic is compounded by two measures: stable and moving seasonality. Both tests, included in X-11, consist on a F-test applied to final $\overline{y}^{ir}/\overline{y}^{f}$ ratio to check for both kinds of seasonality. The seasonality is called identifiable if the absolute error of seasonal factors is low. Hence, the deviation, or *distortion*, depends on both F-values: F_S and F_M, for stable and moving. The $M7$ statistic, considering 50% distortion in the seasonal factor, is constructed as:

$$M7 = \left(\frac{7}{2F_S} + \frac{3F_M}{2F_S}\right)^{\frac{1}{2}},$$

  and the decision rule remains constant.

- **M8** [7%] *Changes in seasonal factor across all the sample.* The $M8$ statistic compound one of the four statistics–together with $M9$ to $M11$–that cast for year-on-year movements in the seasonal component. Note that if seasonality is not present, year-on-year movements of a detrended series should be a constant in the middle stages of the sample span. But, if the seasonality is present and also varying across the sample, seasonal factors are hard to estimate without error. This simply points out that at least two kinds of seasonality emerges: that with changes in the same direction–linear–and that with quasi random fluctuations. The former can be measured and analyzed with ease along with the $M9$ and $M11$ statistics, while the latter is tested with the $M8$ and $M10$ statistics. The difference between the pair ($M8$,$M9$) with respect to ($M10$,$M11$) is just the sample with which they are estimated. $M10$ and $M11$ are estimated considering only the last three years of observations. This is because the use of seasonal adjustment is often concerned with most recent data. Moreover, linear estimates at the end of the sample are almost surely distorted. To have a sensitive measure of this distortion, X12 normalize seasonal factors in the following way:

$$\widetilde{y}_t^f = \frac{y_t^f - \overline{y}_t^f}{\sigma(y_t^f)},$$

  where $\overline{y}_t^f$ stands for the mean of y_t^f, and $\sigma(\cdot)$ for its standard deviation. Thus, fluctuations of the seasonal factors of systematic–linear–changes are estimated with the average absolute change:

$$\left|\Delta\overline{\widetilde{y}}^f\right|_{M8} = \frac{1}{S(Ŧ-1)}\sum_{s=1}^{S}\sum_{t=2}^{Ŧ}\left|\widetilde{y}_{St+s}^f - \widetilde{y}_{S(t-1)+s}^f\right|,$$

  where $Ŧ$ is the length of the sample measured in years ($Ŧ = \{4 \times T, 12 \times T\}$,for quarterly and monthly series), and S is the seasonal frequency ($S = \{4, 12\}$). The maximum change acceptable by X12 is 10%. Hence, the $M8$ statistic is:

$$M8 = \frac{100}{10} \times \left|\Delta\overline{\widetilde{y}}_t^f\right|_{M8},$$

  indicating that values above 1 fail the test.

- **M9** [7%] *Average linear changes of seasonal factor across all the sample.* Notice that $M8$ statistic makes use of the absolute averaging measure because of systematic movements. Instead, when quasi random fluctuations of year-on-year variation is present, it is expected that its mean is close to zero. If the changes go in the same direction, the absolute average change will be very close to the average arithmetic change. By using the formula:

$$\sum_{t=1}^{Ŧ-1}\Delta\widetilde{y}_{St+S}^f = \widetilde{y}_{S(t-1)+s}^f - \widetilde{y}_s^f,$$

  and considering an acceptance limit of 10%, the $M9$ statistic corresponds to:

$$M9 = \frac{100}{10} \times \frac{\sum_{s=1}^{S}\left|\widetilde{y}_{S(Ŧ-1)+s}^f - \widetilde{y}_s^f\right|}{S(Ŧ-1)},$$

implying that values above 1 fail the test.

- **M10** [4%] *Changes in seasonal factor for last years of sample.* As is explained above, this measure has the same shape as $M8$, but estimated with the latest data. In particular, the years involved in calculations are: $\not T-2$, $\not T-3$, $\not T-4$, and $\not T-5$. Thus, the average absolute change becomes:

$$\left|\Delta\widetilde{\overline{y}}^f\right|_{M10} = \frac{1}{3S}\sum_{s=1}^{S}\sum_{\not t=\not T-4}^{\not T-2}\left|\widetilde{y}^f_{S\not t+s} - \widetilde{y}^f_{S(\not t-1)+s}\right|,$$

  and the statistic is given by:

$$M10 = \frac{100}{10} \times \left|\Delta\widetilde{\overline{y}}^f_t\right|_{M10},$$

  indicating that values above 1 fail the test.

- **M11** [4%] *Average linear changes of seasonal factor for last years of sample.* As is explained above, this measure has the same shape as $M9$, but estimated with the latest data. In particular, the years involved in calculations are: $\not T-2$, $\not T-3$, $\not T-4$, and $\not T-5$. Thus, $M11$ statistic corresponds to:

$$M11 = \frac{100}{10} \times \frac{\sum_{s=1}^{S}\left|\widetilde{y}^f_{S(\not T-2)+s} - \widetilde{y}^f_{S(\not T-5)+s}\right|}{3\not T},$$

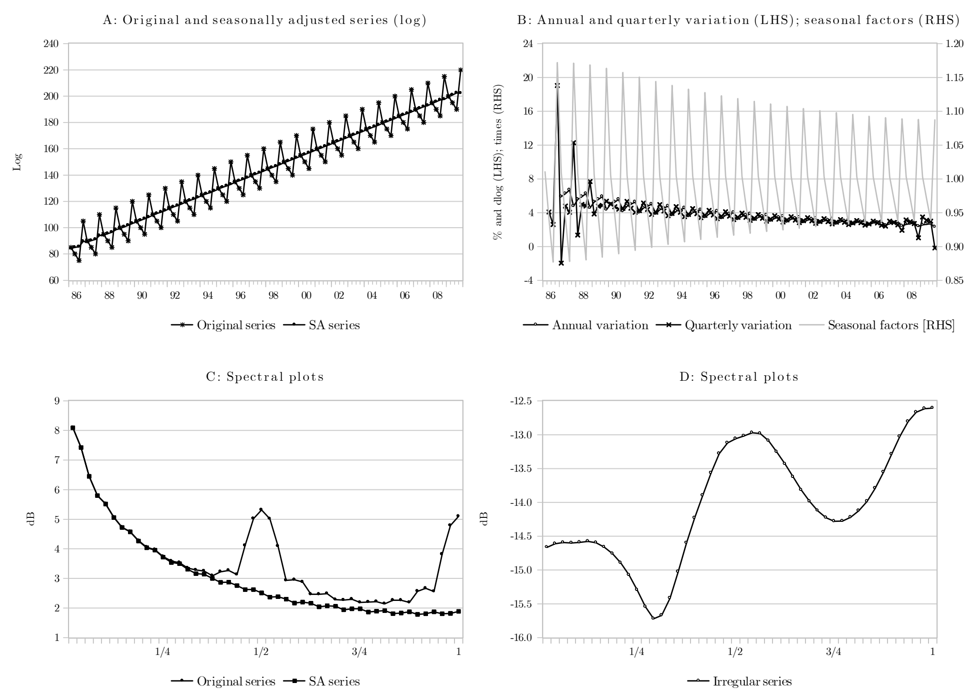
  indicating that values above 1 imply highly distorted seasonal factors in recent data due to the flattening effect of the end weights on linear movements.

- **Q** The Q statistic is a weighted sum of all M statistics that kept anchored to 1 the decision rule about adjustment quality. As each M-stat test for special potential problem, an overall quality assessment of the adjustment is required. One of the advantages of summarizing eleven M's output is that the interactions between them–what they are already measuring–are always considered. For instance, if the ratio $\overline{y}^{ir}/\overline{y}^f$ is low, it is more likely that $M1$ and $M2$ will fail, with *a priori* unknown spillovers effects onto remaining M-stats. Notice that a common strategy to improve the adjustment quality could be to keep some M-stats fixed and work in enhancing those that fail. Again, the Q-stat serves as a natural way to evaluate the effects of user interventions in the adjustment process. Nevertheless, the analysis should be done by considering always both M and Q statistics. This because, for instance, with certain trend-cycle shapes the proportion of the irregular to seasonal component can be large, failing the $M3$ test, but not representing an overall poor adjustment. All these shortcomings have to be avoided depending on the use of the adjustment.

B A simulated series for benchmarking

In this annex I make use of an extremely seasonal series to illustrate the seasonal adjustment process and the typical shape of their spectral plots. First, the illustrator series is built in "intrayear" cycles of 4 observations. Thus, it mimics a quarterly series. Second, it is composed of a (nonstochastic) trend divided by seasonal factors across "years" without outliers or trading day effect. These factors fluctuate by a little rate across years, leaving room for an irregular component. Hence, they correspond primarily to a multiplicative adjustment. Third, by construction, it contains a little variation of the irregular component relative to the variance of the series. Fourth, the series is an index with a base year (1989) equal to 100, spanning from 1986 to 2009 (96 observations).

Figure B1: An example with a highly seasonal series



Source: Author's elaboration.

The original and derivate series are presented in figure B1. Panel A shows the logarithmic level of the original and seasonally adjusted series. Notice that adjusted series should not exactly coincide with the original trend–the trend originally used to construct the series–because of the irregular component. Panel B shows the annual and quarterly variation of previous series, respectively. Obviously, the repeating pattern across the years of the quarterly variation of the seasonally adjusted series is given by the behavior of seasonal factors. Recall that the method is mean preserving. Hence, the yearly average of factors is always equal to 1. The seasonal factors are also depicted in panel B. Note that as they shrink as sample increases, the absolute value of quarterly variation decreases.

Panels C and D depict three different spectral plots. The first two, are those of original and seasonally adjusted series, while the third is of irregular component. Notice in panel C the effect on the spectrum caused by removing seasonality: a complete removal of peaks at seasonal frequencies. The resulting spectrum illustrates *the typical spectral shape of an economic variable* as Clive W.J. Granger shows on his influential *Econometrica* paper published in 1966.[24] As should be the case, panel D shows an erratic cyclical behavior of irregular series, typically associated to a successful seasonal adjustment.

[24] "The long-term fluctuations in economic variables, if decomposed into frequency components, are such that the amplitude of the components decreases smoothly with decreasing period." Granger (1966), p. 155.

C Chilean GDP in levels by demand side

Table C1: Chilean GDP in levels by demand side

$$gdp=id+ed=c+i+g+(x\text{-}m)=$$
$$(cn+cd)+(meq+cw+ci)+g+(xg+xs\text{-}mg\text{-}ms)$$

Aggregation 1		Aggregation 2		Aggregation 3		Aggregation 6	
cn	Household consumption expenditure: nondurables	*c*	Household consumption expenditure *(cn+cd)*	*id*	Internal demand *(c+i+g)*	*gdp*	Gross domestic product *(id+ed)*
cd	Household consumption expenditure: durables	*i*	Investment *(meq+cw)*	*ed*	External demand *(x-m)*		
meq	Machinery and equipment	*ci*	Changes in inventories (*)				
cw	Construction and works	*g*	Government consumption expenditure *(g)*				
ci	Changes in inventories (*)	*x*	Exports *(xg+xs)*				
g	Government consumption expenditure	*m*	Imports (**) *(mg+ms)*				
xg	Exports of goods						
xs	Exports of services						
mg	Imports of goods (**)						
ms	Imports of services (**)						

(*) Not considered in analysis. (**) Imports are subtracted. Source: Central Bank of Chile.

D Chilean GDP in levels by supply side

Table D1: Chilean GDP in levels by supply side

$$gdp=gdp\ nr+gdp\ nnr+others=(egw+caf+min)+$$
$$(com+man+con+agr+tra+fin+per+ood+pub)+(vat+cif\text{-}dut)$$

Aggregation 6		Aggregation 4		Aggregation 5	
gdp	Gross domestic product *(gdp nr+ gdp nnr+ others)*	*gdp nr*	GDP Natural resources *(egw+caf+min)*	*egw*	Electricity, gas and water
				caf	Capture fishery
		gdp nnr	GDP Non-natural resources *(com+man+con+ agr+tra+fin+ per+ood+pub)*	*min*	Mining
				com	Wholesale and retail trade, hotels and restaurants
				man	Manufacturing
		others	Other sectors *(-dut+vat+cif)*	*con*	Construction
				agr	Agriculture and forestry
				tra	Transportation and communications
				fin	Financial intermediation and business services
				per	Personal and social services
				ood	Owner-occupied dwellings
				pub	Public administration
				dut	Duties + taxes on goods and services (*)
				vat	Non-deductible VAT
				cif	Imports CIF

(*) DUT are subtracted. Source: Central Bank of Chile.

E Shares of sectorial components on real GDP

Table E1: Shares of sectorial components on real GDP

Aggregation 1		Aggregation 2		Aggregation 3		Aggregation 4		Aggregation 5	
58.2%	Household consumption	62.8%	Household consumption	95.9%	Internal demand	12.5%	GDP Natural	2.9%	Electricity, gas and water
	expenditure: nondurables		expenditure		*(c+i+g)*		resources	1.2%	Capture fishery
4.6%	Household consumption		*(cn+cd)*	4.1%	External demand		*(egw+caf+min)*	8.4%	Mining
	expenditure: durables	20.1%	Investment		*(x-m)*	82.5%	GDP Non-natural	9.7%	Wholesale and retail trade,
6.9%	Machinery and equipment		*(meq+cw+ci)*				resources		hotels and restaurants
13.3%	Construction and works	12.0%	Government consumption				*(com+man+con+*	16.4%	Manufacturing
1.0%	Changes in inventories (*)		expenditure *(g)*				*agr+tra+fin+*	6.9%	Construction
12.0%	Government consumption	36.5%	Exports				*per+ood+pub)*	3.6%	Agriculture and forestry
	expenditure		*(xg+xs)*			5.0%	Other sectors	9.2%	Transportation and communications
29.2%	Exports of goods	32.4%	Imports (**)				*(-dut+vat+cif)*	15.0%	Financial intermediation and
7.3%	Exports of services		*(mg+ms)*						business services
26.0%	Imports of goods (**)	1.0%	Changes in inventories (*)					11.6%	Personal and social services
6.4%	Imports of services (**)							5.8%	Owner-occupied dwellings
								4.3%	Public administration
								3.4%	Duties + taxes on goods
									and services (**)
								7.4%	Non-deductible VAT
								1.0%	Imports CIF

(*) Not considered in analysis. (**) Subtracted. Source: Central Bank of Chile.

F Typical statistics of series

Table F1: Typical statistics of demand side series, full sample

	Mean (Standard deviation)			Maximum (Minimum)		
	Level (*)	*%* (**)	*dlog (sa)* (***)	*Level* (*)	*%* (**)	*dlog (sa)* (***)
cn	6068013 (2241842)	5.811 (2.917)	0.014 (0.011)	10545773 (2604436)	15.747 (-1.763)	0.045 (-0.011)
cd	586936 (346364)	13.067 (20.681)	0.030 (0.094)	1433854 (66897)	76.864 (-36.143)	0.488 (-0.203)
meq	848577 (597871)	13.893 (20.247)	0.029 (0.075)	2621727 (114903)	57.374 (-31.887)	0.168 (-0.212)
cw	1451274 (493573)	6.345 (-21.837)	0.014 (0.040)	2435179 (571050)	32.680 (-21.837)	0.122 (-0.089)
g	1337538 (3982019)	4.143 (2.237)	0.009 (0.009)	2203213 (713499)	8.640 (-3.946)	0.059 (-0.018)
xg	2848950 (1373321)	7.567 (6.998)	0.018 (0.050)	5418178 (858705)	24.643 (-6.480)	0.178 (-0.187)
xs	685501 (363399)	13.086 (29.508)	0.019 (0.192)	1441582 (45949)	169.307 (-46.181)	1.034 (-0.535)
(mg)	2855276 (1868065)	12.112 (13.192)	0.028 (0.055)	7724684 (400759)	41.315 (-22.396)	0.161 (-0.199)
(ms)	580860 (296040)	7.741 (10.237)	0.017 (0.058)	1103390 (174309)	47.292 (-27.912)	0.261 (-0.181)
c	6652570 (2569375)	6.183 (3.857)	0.015 (0.013)	11965071 (2671333)	18.070 (-5.102)	0.049 (-0.025)
i	2256083 (1116785)	9.513 (13.269)	0.021 (0.046)	4940206 (545627)	36.790 (-25.618)	0.129 (-0.147)
x	3533607 (1722640)	7.901 (6.816)	0.018 (0.041)	6737383 (992725)	27.058 (-7.217)	0.141 (-0.070)
(m)	3438889 (2151185)	10.992 (11.427)	0.026 (0.050)	8774539 (575068)	34.234 (-19.163)	0.136 (-0.173)
id	10254896 (4120949)	6.591 (6.543)	0.016 (0.030)	18747826 (3591484)	21.216 (-10.031)	0.109 (-0.061)
ed (◇)	94717 (679884)	39.716 (414.744)	NA NA	1050973 (-2531999)	2046.979 (-1793.768)	NA NA
gdp	10473599 (3585032)	5.445 (3.725)	0.013 (0.014)	16874801 (4616475)	16.263 (-4.476)	0.052 (-0.035)

(*) Includes all observations from 1986.I to 2009.IV (96 observations). (**) Estimated from 1987.I to 2009.IV. (***) Estimated from 1986.II to 2009.IV. (◇) Log-differenced changes not calculated as with negative values. Source: Author's elaboration.

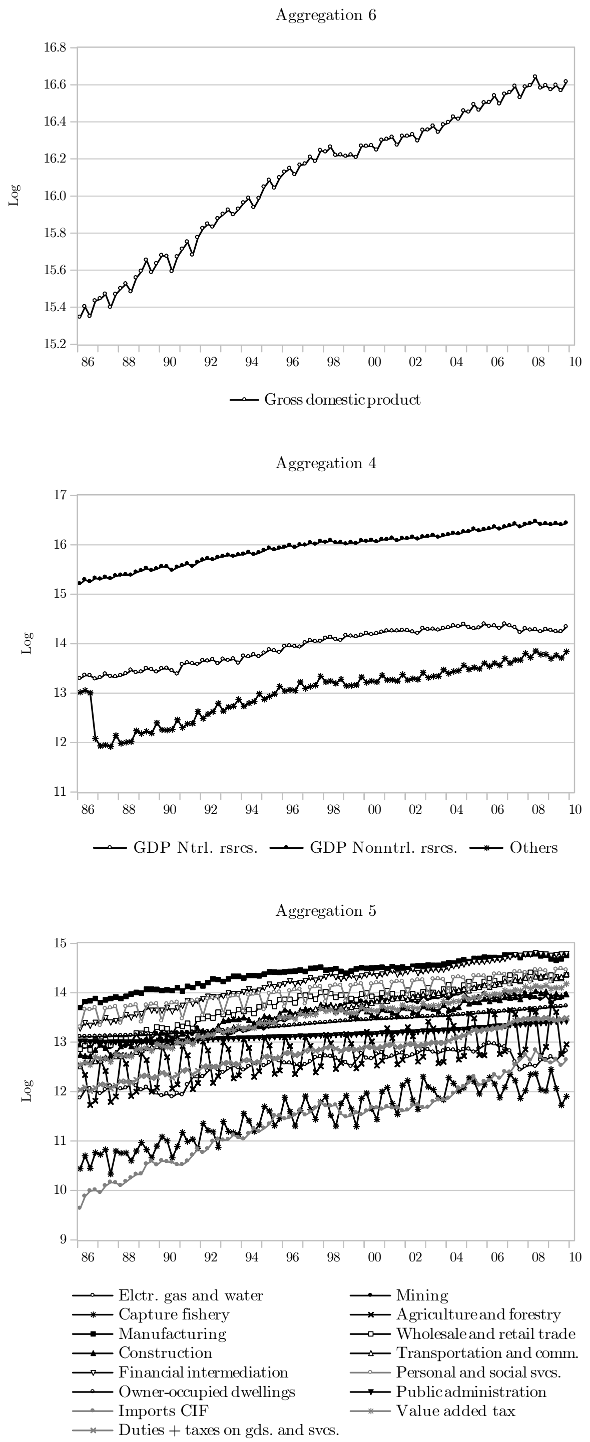
Table F2: Typical statistics of supply side series, full sample

	Mean (Standard deviation)			Maximum (Minimum)		
	Level (*)	% (**)	*dlog (sa)* (***)	*Level* (*)	% (**)	*dlog (sa)* (***)
egw	283364 (78522)	-0.001 (0.090)	0.009 (0.054)	430088 (142348)	0.246 (-0.215)	0.129 (-0.187)
min	804793 (276430)	0.000 (0.117)	0.010 (0.028)	1220424 (392020)	0.204 (-0.298)	0.097 (-0.060)
caf	117646 (58319)	-0.008 (0.434)	0.018 (0.128)	254936 (30590)	0.967 (-0.869)	0.394 (-0.515)
agr	388555 (192637)	-0.007 (0.729)	0.013 (0.031)	941539 (123887)	1.070 (-1.076)	0.139 (-0.059)
man	1832821 (476964)	-0.001 (0.066)	0.010 (0.026)	2634049 (889423)	0.163 (-0.197)	0.071 (-0.059)
com	1039792 (397285)	-0.001 (0.136)	0.015 (0.021)	1807611 (377328)	0.293 (-0.232)	0.066 (-0.062)
con	765692 (244934)	0.000 (0.113)	0.013 (0.036)	1239985 (330116)	0.210 (-0.245)	0.111 (-0.076)
tra	883142 (428532)	0.000 (0.055)	0.019 (0.018)	1743795 (270632)	0.165 (-0.130)	0.069 (-0.022)
fin	1572100 (618350)	-0.001 (0.073)	0.015 (0.017)	2703326 (585050)	0.136 (-0.197)	0.062 (-0.047)
per	1256098 (353191)	-0.003 (0.370)	0.001 (0.673)	1952350 (598307)	0.706 (-0.396)	1.102 (-1.036)
ood	660505 (125509)	0.000 (0.003)	0.008 (0.006)	916461 (492724)	0.025 (-0.021)	0.036 (-0.008)
pub	519036 (67809)	0.000 (0.009)	0.004 (0.005)	674628 (438530)	0.026 (-0.041)	0.015 (-0.025)
(dut)	388081 (160611)	0.000 (0.058)	0.015 (0.019)	723465 (165031)	0.173 (-0.116)	0.068 (-0.057)
vat	776675 (324558)	-0.001 (0.058)	0.016 (0.016)	1436665 (279024)	0.117 (-0.110)	0.060 (-0.034)
cif	120050 (88293)	-0.004 (0.128)	0.030 (0.062)	375036 (15246)	0.318 (-0.360)	0.172 (-0.130)
$gdp\ nr$	1203425 (393755)	-0.001 (0.086)	0.010 (0.027)	1782751 (586220)	0.265 (-0.169)	0.134 (-0.052)
$gdp\ nnr$	8844037 (2871629)	-0.001 (0.064)	0.012 (0.015)	14248987 (4015997)	0.129 (-0.116)	0.061 (-0.032)
$others$	518683 (242264)	-0.001 (0.057)	0.007 (0.121)	1035604 (149443)	0.112 (-0.110)	0.121 (-1.132)
gdp	10473599 (3585032)	-0.001 (0.069)	0.013 (0.014)	16874801 (4616475)	0.164 (-0.127)	0.052 (-0.035)

(*) Includes all observations from 1986.I to 2009.IV (96 observations). (**) Estimated from 1987.I to 2009.IV. (***) Estimated from 1986.II to 2009.IV. Source: Author's elaboration.

G GDP Components: Original series in levels

Figure G1: Chilean GDP - Original series in logarithmic levels



Source: Central Bank of Chile.

H An user-written *Eviews* script for X-12-ARIMA

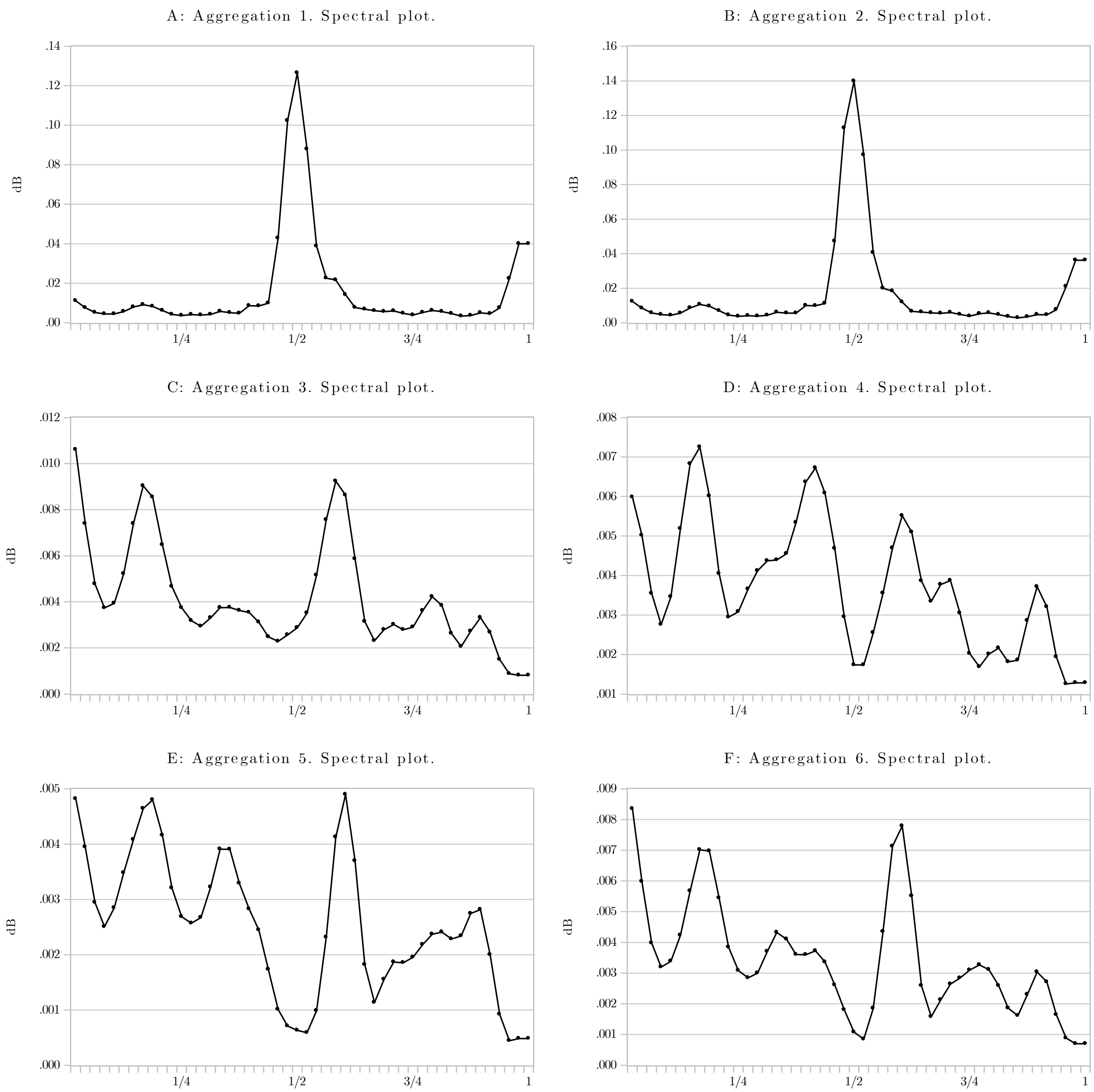
```
'Program code for "How to Seasonally Adjust the Chilean GDP with X-12-ARIMA?".
'Written by Carlos A. Medel (cnmedel@gmail.com) on 19 August 2013 (v.1.0).
'X-12-ARIMA is a product of US Census Bureau and is free available at http://www.census.gov/.

wfcreate(wf=chile_gdp,page=howtoseas) Q 1986 2010
read(t=xls,c3,s=pib)
M:\Howto_seas\Data\gdp_series.xls 36

for !d=61 to 96
smpl @first @first+{!d}
%kind="m" 'Seasonal adjustment method: [(default="m")] "m" (multiplicative adjustment; Series must take only non-negative values), "a" (additive adjustment), "p" (pseudo-additive adjustment), "l" (log-additive seasonal adjustment; Series must take only positive values).

%filter="msr" 'Seasonal filter: [(default="msr")] "msr" (automatic, moving seasonality ratio), "x11" (X11 default), "stable" (stable), "s3x1" (3x1 moving average), "s3x3" (3x3 moving average), "s3x5" (3x5 moving average), "s3x9" (3x9 moving average), "s3x15" (3x15 moving average seasonal filter; Series must have at least 20 years of data).

%trans="0" 'Transformation for regARIMA: "logit" (Logit transformation), "auto" (automatically choose between no transformation and log transformation), number (Box-Cox power transformation using specified parameter; use "tf=0" for log transformation).

%arspec="f, outsmpl" 'Automatically choose the ARIMA spec. f: Use forecasts from the chosen model in seasonal adjustment. Cannot be used together with the "arima=" option and must be used together with the "mfile=" option. b: Automatically choose the ARIMA spec. Use forecasts and backcasts from the chosen model in seasonal adjustment. Cannot be used together with the "arima=" option and must be used together with the "mfile=" option.

%easter=""'NO:",ea" 'ea: Nonparametric Easter holiday adjustment (x11easter). Cannot be used together with the "easter[w]" regressor in the "reg=" or "x11reg=" options.

'outsmpl: Use out-of-sample forecasts for automatic model selection. Default is in-sample forecasts. Must be used together with the "amdl=" option.
'sspan: Sliding spans stability analysis. Cannot be used along with the "h" option.
'history: Historical record of seasonal adjustment revisions. Cannot be used along with the "sspan" option.
'check: Check residuals of regARIMA.
'outlier: Outlier analysis of regARIMA.

'Adjusting process
series de=x-m
series ve_sa=ve
series ve_ir=ve
series ve_tc=ve

'Aggregation 1
for %k cn meq cw g xg mg ms
{%k}.x12(mode={%kind},filter={%filter},save="d10 d11 d12 d13",tf={%trans},sspan,check,outlier,
amdl={%arspec},plotspectra{%easter})
next
cd.x12(mode=m,filter=stable,save="d10 d11 d12 d13",tf=auto,check,outlier,amdl={%arspec},
plotspectra{%easter})
if !d>69 then
    xs.x12(mode=m,filter=msr,save="d10 d11 d12 d13",tf=auto,check,outlier,arima="(0 0 1)",plotspectra)
    else
    xs.x12(mode=m,filter={%filter},save="d10 d11 d12 d13",tf=auto,check,outlier,amdl={%arspec},plotspectra{%easter})
endif

'Aggregation 2
for %k c i g x m
{%k}.x12(mode={%kind},filter={%filter},save="d10 d11 d12 d13",tf={%trans},sspan,check,outlier,amdl={%arspec},
plotspectra{%easter})
next

'Aggregation 3
for %k id
{%k}.x12(mode=m,filter=msr,save="d10 d11 d12 d13",tf=auto,check,outlier,arima="(0 0 1)",plotspectra)
next

series ed_sa=x_sa-m_sa
series ed_ir=x_ir-m_ir
series ed_tc=x_tc-m_tc

'Aggregation 4
for %k nr nnr others
{%k}.x12(mode={%kind},filter={%filter},save="d10 d11 d12 d13",tf={%trans},sspan,check,outlier,amdl={%arspec},
plotspectra{%easter})
next

'Aggregation 5
for %k egw min com ind con tra fin per ood pub dut vat cif
```

```
{%k}.x12(mode={%kind},filter={%filter},save="d10 d11 d12
d13",tf={%trans},sspan,check,outlier,amdl={%arspec},
plotspectra{%easter})
next
agr.x12(mode=m,filter={%filter},save="d10 d11 d12
d13",tf=auto,sspan,check,outlier,amdl={%arspec},plotspectra{%easter})
caf.x12(mode=m,filter=msr,save="d10 d11 d12 d13",tf=auto,check,outlier,arima="(0
0 1)",plotspectra)

'Aggregation 6
gdp.x12(mode={%kind},filter={%filter},save="d10 d11 d12
d13",tf={%trans},sspan,check,outlier,amdl={%arspec},
plotspectra{%easter})

'Aggregations: Original series
genr aggr1=cn+cd+meq+cw+g+xg+xs-mg-ms+ci
genr aggr2=c+i+g+x-m+ci
genr aggr3=id+ed
genr aggr4=nr+nnr+others
genr aggr5=egw+cap+min+com+ind+con+agr+tra
+fin+per+ood+pub-dut+vat+cif
genr aggr6=gdp

'Aggregations: Trend-cycle series
genr aggr1_tc=cn_tc+cd_tc+meq_tc+cw_tc+g_tc
+xg_tc+xs_tc-mg_tc-ms_tc+ci_tc
genr aggr2_tc=c_tc+i_tc+g_tc+x_tc-m_tc+ci_tc
genr aggr3_tc=id_tc+ed_tc
genr aggr4_tc=nr_tc+nnr_tc+others_tc
genr aggr5_tc=egw_tc+caf_tc+min_tc+com_tc
+ind_tc+con_tc+agr_tc+tra_tc+fin_tc+per_tc
+ood_tc+pub_tc-dut_tc+vat_tc+cif_tc
genr aggr6_tc=gdp_tc

'Aggregations: Seasonally adjusted series
genr aggr1_sa=cn_sa+cd_sa+meq_sa+cw_sa+
g_sa+xg_sa+xs_sa-mg_sa-ms_sa+ci_sa
genr aggr2_sa=c_sa+i_sa+g_sa+x_sa-m_sa+ci_sa
genr aggr3_sa=id_sa+ed_sa
genr aggr4_sa=nr_sa+nnr_sa+others_sa
genr aggr5_sa=egw_sa+caf_sa+min_sa+com_sa+ind_sa
+con_sa+agr_sa+tra_sa+fin_sa+per_sa+ood_sa+pub_sa
-dut_sa+vat_sa+cif_sa
genr aggr6_sa=gdp_sa

'Aggregations: Irregular series
genr aggr1_ir=cn_ir+cd_ir+meq_ir+cw_ir+g_ir+xg_ir
+xs_ir+mg_ir+ms_ir+ci_ir
genr aggr2_ir=c_ir+i_ir+g_ir+x_ir+m_ir+ci_ir
genr aggr3_ir=id_ir+ed_ir
genr aggr4_ir=nr_ir+nnr_ir+others_ir
genr aggr5_ir=egw_ir+caf_ir+min_ir+com_ir+ind_ir+con_ir
+agr_ir+tra_ir+fin_ir+per_ir+ood_ir+pub_ir-dut_ir+vat_ir+cif_ir
genr aggr6_ir=gdp_ir

'Sorting results in terms of aggregations
'All three series
for !x=1 to 6
table res_tagg{!x}
table res_sagg{!x}
table res_iagg{!x}
    for !t=1 to 36
    !j={!t}+60
        for !r=1 to {!j}
        res_tagg{!x}({!r},{!t})=aatrend{!j}({!r}+1,{!x}+1)

        res_sagg{!x}({!r},{!t})=aaseas{!j}({!r}+1,{!x}+1)
        res_iagg{!x}({!r},{!t})=aairr{!j}({!r}+1,{!x}+1)
        next
    next
next
pagecreate(page=d12) Q 1986 2010
pagecreate(page=d11) Q 1986 2010
pagecreate(page=d13) Q 1986 2010
pageselect howtoseas
    for !a=1 to 6
        copy howtoseas\res_tagg{!a} d12\agg{!a}
        copy howtoseas\res_sagg{!a} d11\agg{!a}
        copy howtoseas\res_iagg{!a} d13\agg{!a}
    next
pageselect howtoseas
delete aa*
@uiprompt("Process completed")
```

I Diagnostic Result 1

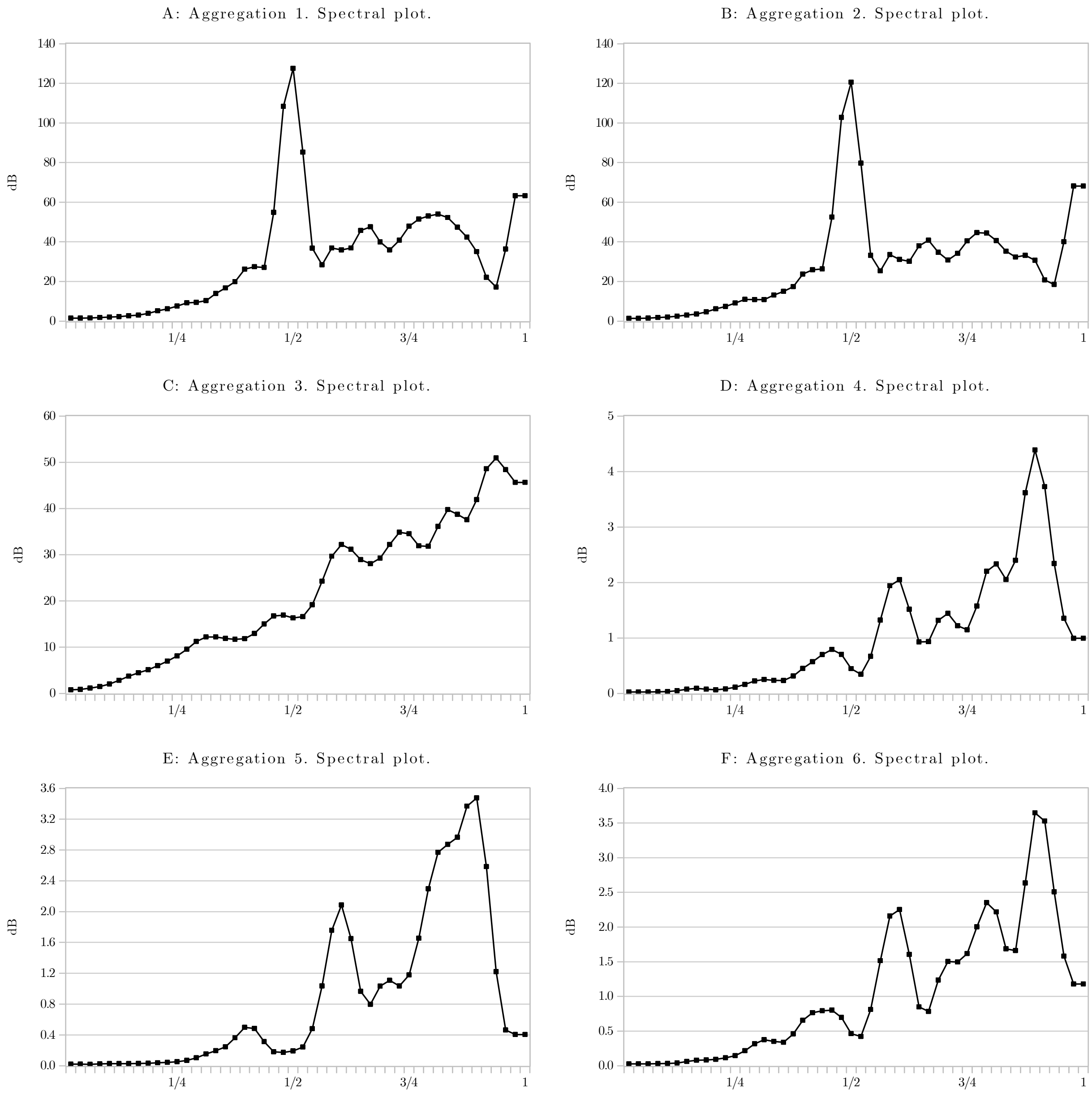
Spectral plots of final seasonally adjusted and irregular series

Figure I1: Spectral plots of final seasonally adjusted series (*)



(*) 10×(log-diff.). Bartlett window length=30. Source: Author's elaboration.

Figure I2: Spectral plots of irregular series (*)



(*) Bartlett window length=30. Source: Author's elaboration.

J Diagnostic Result 2

Sliding spans of (log-differenced) seasonally adjusted series

Table J1: Sliding spans of seasonally adjusted series (log-diff), Aggregation 1

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	-2.419	-2.424	-2.440	-2.560	-2.645	-2.620	-2.653	-2.950	-2.610	-2.449	53.1	✗
41	1996Q1	3.445	3.528	3.557	3.631	3.723	3.782	3.806	4.695	3.789	3.788	125.0	✗
42	1996Q2	4.243	4.113	4.114	4.030	4.004	3.937	3.868	3.669	3.996	4.036	57.4	✗
43	1996Q3	2.050	2.097	2.089	2.204	2.217	2.205	2.262	1.856	2.196	1.603	65.9	✗
44	1996Q4	-3.193	-3.174	-3.205	-3.414	-3.494	-3.475	-3.470	-3.669	-3.518	-2.647	102.2	✗
45	1997Q1	3.241	3.290	3.306	3.524	3.614	3.670	3.683	4.500	3.648	3.564	125.9	✗
46	1997Q2	3.665	3.543	3.602	3.527	3.530	3.467	3.393	3.067	3.453	3.157	59.8	✗
47	1997Q3	3.012	3.059	3.005	3.039	2.993	2.984	3.025	2.744	3.096	2.385	71.1	✗
48	1997Q4	-2.313	-2.255	-2.269	-2.466	-2.503	-2.484	-2.458	-2.477	-2.622	-1.223	139.9	✗
49	1998Q1	1.077	1.083	1.034	1.262	1.344	1.390	1.409	1.933	1.557	1.058	89.9	✗
50	1998Q2	3.790	3.675	3.852	3.861	3.888	3.832	3.745	3.374	3.622	3.373	51.5	✗
51	1998Q3	0.840	0.888	0.775	0.643	0.532	0.525	0.555	0.536	0.731	0.086	80.2	✗
52	1998Q4	-7.044	-6.994	-7.018	-7.102	-7.087	-7.073	-7.033	-7.024	-7.064	-6.071	103.1	✗
53	1999Q1	1.906	1.967	1.873	2.134	2.196	2.243	2.269	2.476	2.060	2.121	60.3	✗
54	1999Q2	1.296	1.098	1.377	1.296	1.387	1.332	1.230	1.002	1.290	0.900	48.7	✗
55	1999Q3	3.089	3.169	3.061	2.939	2.698	2.696	2.727	2.800	2.886	2.672	49.7	✗
56	1999Q4	-1.541	-1.482	-1.603	-1.647	-1.540	-1.537	-1.498	-1.433	-1.452	-1.116	53.1	✗
57	2000Q1	1.862	1.947	1.832	2.146	2.212	2.278	2.316	2.266	2.063	2.359	52.7	✗
58	2000Q2	1.290	1.083	1.482	1.263	1.330	1.250	1.141	1.052	1.218	0.738	74.4	✗
59	2000Q3	2.193	2.226	2.110	2.031	1.725	1.743	1.769	1.871	1.869	1.673	55.3	✗
60	2000Q4	-3.141	-3.047	-3.245	-3.213	-3.015	-3.024	-2.975	-2.888	-2.926	-2.586	65.9	✗
61	2001Q1	NA	3.110	2.978	3.363	3.419	3.506	3.542	3.374	3.329	3.631	65.3	✗
62	2001Q2	NA	2.660	3.137	2.598	2.710	2.607	2.493	2.510	2.591	2.188	94.9	✗
63	2001Q3	NA	-0.307	-0.434	-0.274	-0.754	-0.735	-0.716	-0.658	-0.682	-0.692	48.0	✗
64	2001Q4	NA	-2.665	-2.892	-2.769	-2.464	-2.467	-2.393	-2.306	-2.311	-2.387	58.6	✗
65	2002Q1	NA	NA	2.633	2.931	3.084	3.193	3.218	3.060	3.002	3.320	68.7	✗
66	2002Q2	NA	NA	2.268	1.480	1.478	1.333	1.196	1.216	1.322	1.049	121.9	✗
67	2002Q3	NA	NA	0.255	0.640	0.109	0.154	0.182	0.255	0.181	0.586	53.1	✗
68	2002Q4	NA	NA	-2.114	-1.786	-1.381	-1.417	-1.309	-1.309	-1.274	-1.787	84.0	✗
69	2003Q1	NA	NA	NA	3.116	3.330	3.523	3.520	3.471	3.415	3.921	80.5	✗
70	2003Q2	NA	NA	NA	2.083	1.960	1.712	1.594	1.562	1.682	1.149	93.4	✗
71	2003Q3	NA	NA	NA	1.291	0.625	0.692	0.614	0.730	0.591	0.949	70.0	✗
72	2003Q4	NA	NA	NA	-4.013	-3.401	-3.398	-3.064	-3.125	-3.109	-3.281	94.9	✗
73	2004Q1	NA	NA	NA	NA	4.769	5.012	4.832	4.798	4.900	5.445	67.6	✗
74	2004Q2	NA	NA	NA	NA	2.457	2.063	1.972	1.927	1.981	1.154	130.3	✗
75	2004Q3	NA	NA	NA	NA	3.428	3.553	3.408	3.644	3.389	3.502	25.5	✔
76	2004Q4	NA	NA	NA	NA	-2.768	-2.698	-2.109	-2.277	-2.323	-1.988	78.0	✗
77	2005Q1	NA	NA	NA	NA	NA	2.868	2.433	2.330	2.739	3.032	70.2	✗
78	2005Q2	NA	NA	NA	NA	NA	2.693	2.683	2.661	2.664	1.895	79.8	✗
79	2005Q3	NA	NA	NA	NA	NA	2.056	1.823	2.283	1.733	1.738	55.0	✗
80	2005Q4	NA	NA	NA	NA	NA	-3.159	-2.298	-2.567	-2.632	-1.776	138.3	✗
81	2006Q1	NA	NA	NA	NA	NA	NA	2.942	2.595	3.474	3.390	87.9	✗
82	2006Q2	NA	NA	NA	NA	NA	NA	2.035	2.055	1.961	1.112	94.3	✗
83	2006Q3	NA	NA	NA	NA	NA	NA	0.828	1.621	0.674	0.488	113.3	✗
84	2006Q4	NA	NA	NA	NA	NA	NA	-0.928	-1.192	-1.263	0.014	127.7	✗
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	2.629	3.977	3.812	134.8	✗
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	1.685	1.608	0.838	84.7	✗
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	0.545	-0.886	-1.606	215.1	✗
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	-0.679	-0.775	1.253	202.8	✗
	Median	-	-	-	-	-	-	-	-	-	-	78.00	-
	Mean	-	-	-	-	-	-	-	-	-	-	86.20	98%

Source: Author's elaboration.

Table J2: Sliding spans of seasonally adjusted series (log-diff), Aggregation 2

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	-2.763	-2.749	-2.757	-2.753	-2.796	-2.797	-2.819	-2.796	-2.817	-2.137	68.2	✗
41	1996Q1	3.017	2.991	3.009	3.011	3.067	3.067	3.043	3.067	3.042	2.587	48.0	✗
42	1996Q2	4.707	4.712	4.708	4.702	4.686	4.689	4.717	4.685	4.717	4.359	35.8	✗
43	1996Q3	1.548	1.553	1.550	1.545	1.545	1.543	1.546	1.545	1.546	1.407	14.6	✔
44	1996Q4	-3.932	-3.903	-3.919	-3.909	-3.948	-3.952	-3.953	-3.948	-3.952	-2.802	115.1	✗
45	1997Q1	4.072	4.022	4.028	4.035	4.094	4.095	4.078	4.094	4.078	3.752	34.3	✗
46	1997Q2	3.943	3.947	3.998	3.989	3.982	3.991	3.998	3.982	3.999	3.354	64.5	✗
47	1997Q3	2.918	2.942	2.884	2.857	2.810	2.810	2.808	2.811	2.808	2.364	57.8	✗
48	1997Q4	-3.453	-3.416	-3.414	-3.375	-3.361	-3.384	-3.361	-3.361	-3.361	-1.626	182.7	✗
49	1998Q1	2.079	1.984	1.959	1.969	2.012	2.033	2.006	2.013	2.007	1.168	91.1	✗
50	1998Q2	3.772	3.814	3.942	3.913	3.916	3.921	3.918	3.912	3.919	3.474	46.8	✗
51	1998Q3	0.975	1.000	0.881	0.842	0.741	0.741	0.743	0.743	0.742	0.325	67.5	✗
52	1998Q4	-7.767	-7.757	-7.745	-7.670	-7.587	-7.631	-7.587	-7.586	-7.589	-6.292	147.5	✗
53	1999Q1	2.604	2.569	2.502	2.515	2.522	2.574	2.528	2.529	2.528	2.042	56.2	✗
54	1999Q2	1.355	1.309	1.523	1.451	1.510	1.495	1.491	1.491	1.496	1.237	28.6	✔
55	1999Q3	3.173	3.236	3.136	3.100	2.872	2.886	2.884	2.882	2.880	2.746	49.0	✗
56	1999Q4	-2.205	-2.171	-2.296	-2.170	-1.975	-2.043	-1.973	-1.973	-1.976	-1.222	107.4	✗
57	2000Q1	2.449	2.403	2.400	2.420	2.412	2.498	2.432	2.434	2.430	2.085	41.3	✗
58	2000Q2	1.239	1.173	1.423	1.243	1.288	1.257	1.243	1.242	1.258	0.929	49.4	✗
59	2000Q3	2.348	2.423	2.356	2.366	2.063	2.058	2.071	2.070	2.058	1.957	46.6	✗
60	2000Q4	-3.451	-3.395	-3.644	-3.409	-3.126	-3.161	-3.092	-3.093	-3.095	-2.367	127.7	✗
61	2001Q1	NA	3.349	3.408	3.389	3.413	3.493	3.438	3.44	3.434	3.196	29.7	✔
62	2001Q2	NA	2.668	2.944	2.582	2.608	2.566	2.526	2.525	2.554	2.161	78.3	✗
63	2001Q3	NA	-0.135	-0.179	-0.040	-0.485	-0.511	-0.486	-0.484	-0.508	-0.386	47.1	✗
64	2001Q4	NA	-3.035	-3.392	-2.997	-2.595	-2.586	-2.517	-2.520	-2.522	-2.263	112.9	✗
65	2002Q1	NA	NA	3.011	2.860	3.010	3.074	3.041	3.038	3.035	2.847	22.7	✔
66	2002Q2	NA	NA	2.135	1.562	1.374	1.322	1.236	1.241	1.276	1.064	107.1	✗
67	2002Q3	NA	NA	0.614	0.968	0.529	0.489	0.540	0.548	0.512	0.985	49.6	✗
68	2002Q4	NA	NA	-2.465	-1.914	-1.395	-1.381	-1.308	-1.315	-1.317	-1.538	115.7	✗
69	2003Q1	NA	NA	NA	2.960	3.194	3.333	3.311	3.292	3.302	3.374	41.4	✗
70	2003Q2	NA	NA	NA	2.158	1.737	1.593	1.497	1.503	1.544	1.204	95.4	✗
71	2003Q3	NA	NA	NA	1.222	0.758	0.724	0.676	0.740	0.670	1.075	55.2	✗
72	2003Q4	NA	NA	NA	-4.188	-3.417	-3.371	-3.080	-3.151	-3.147	-3.225	110.8	✗
73	2004Q1	NA	NA	NA	NA	4.995	5.196	5.031	5.031	5.081	5.232	23.7	✔
74	2004Q2	NA	NA	NA	NA	2.079	1.828	1.723	1.684	1.733	1.281	79.8	✗
75	2004Q3	NA	NA	NA	NA	3.596	3.537	3.399	3.609	3.439	3.489	21.0	✔
76	2004Q4	NA	NA	NA	NA	-2.951	-2.821	-2.210	-2.429	-2.376	-1.988	96.3	✗
77	2005Q1	NA	NA	NA	NA	NA	3.318	2.888	2.947	3.052	2.986	43.0	✗
78	2005Q2	NA	NA	NA	NA	NA	2.458	2.398	2.263	2.381	2.021	43.7	✗
79	2005Q3	NA	NA	NA	NA	NA	2.026	1.757	2.186	1.772	1.644	54.2	✗
80	2005Q4	NA	NA	NA	NA	NA	-3.526	-2.509	-2.905	-2.828	-1.931	159.5	✗
81	2006Q1	NA	NA	NA	NA	NA	NA	3.573	3.683	4.050	3.640	47.7	✗
82	2006Q2	NA	NA	NA	NA	NA	NA	1.727	1.506	1.614	1.203	52.4	✗
83	2006Q3	NA	NA	NA	NA	NA	NA	0.634	1.240	0.454	0.203	103.7	✗
84	2006Q4	NA	NA	NA	NA	NA	NA	-1.105	-1.591	-1.418	-0.152	143.9	✗
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	4.239	4.965	4.325	72.6	✗
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	1.065	1.154	0.760	39.4	✗
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	-0.173	-1.441	-1.883	171.0	✗
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	-1.199	-0.936	1.009	220.8	✗
	Median	-	-	-	-	-	-	-	-	-	-	56.20	-
	Mean	-	-	-	-	-	-	-	-	-	-	75.89	86%

Source: Author's elaboration.

Table J3: Sliding spans of seasonally adjusted series (log-diff), Aggregation 3

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	2.668	4.046	0.836	0.452	0.401	0.412	0.400	1.006	0.459	2.187	364.6	✗
41	1996Q1	0.862	0.712	0.571	0.609	1.025	1.029	0.683	1.255	1.236	1.619	104.8	✗
42	1996Q2	3.404	1.325	4.161	5.869	4.719	4.702	5.848	3.541	4.349	1.176	469.3	✗
43	1996Q3	-1.045	-0.190	0.342	-1.099	-0.253	-0.253	-1.095	0.165	-0.144	1.025	212.4	✗
44	1996Q4	2.649	4.072	0.779	0.420	0.274	0.290	0.354	1.036	0.348	1.842	379.8	✗
45	1997Q1	0.061	-0.449	-0.489	0.046	0.079	0.091	0.104	-0.200	0.251	0.899	138.8	✗
46	1997Q2	3.890	2.450	5.265	6.067	5.830	5.780	6.132	4.983	5.442	2.383	374.9	✗
47	1997Q3	0.716	1.262	1.560	0.656	0.869	0.902	0.567	1.395	1.063	2.156	158.9	✗
48	1997Q4	3.056	4.344	1.405	1.090	1.021	1.032	1.041	1.610	1.051	2.082	332.3	✗
49	1998Q1	-1.001	-1.644	-1.575	-0.999	-1.058	-1.045	-0.946	-1.637	-0.934	-0.353	129.1	✗
50	1998Q2	2.533	1.596	4.136	4.485	4.680	4.604	4.637	4.335	4.307	1.901	308.4	✗
51	1998Q3	-1.111	-0.589	-0.795	-1.365	-1.573	-1.507	-1.546	-1.031	-1.275	-0.233	134.0	✗
52	1998Q4	-2.698	-1.937	-4.086	-4.360	-4.305	-4.294	-4.390	-4.036	-4.315	-3.734	245.3	✗
53	1999Q1	-0.626	-1.059	-0.835	-0.156	-0.272	-0.263	-0.218	-1.183	-0.380	0.079	126.2	✗
54	1999Q2	0.808	0.096	2.005	1.815	2.341	2.203	2.319	2.920	2.281	0.334	282.4	✗
55	1999Q3	2.016	2.613	2.189	2.064	1.429	1.545	1.569	1.631	1.635	2.834	140.5	✗
56	1999Q4	2.658	2.872	1.422	1.255	1.438	1.520	1.266	1.112	1.383	1.501	176.0	✗
57	2000Q1	-0.521	-0.641	-0.118	0.354	0.274	0.204	0.196	-0.084	0.116	0.744	138.5	✗
58	2000Q2	0.916	0.329	1.345	0.906	1.609	1.432	1.846	2.285	1.646	-0.260	254.5	✗
59	2000Q3	1.320	2.058	1.545	1.799	0.874	0.982	0.890	0.900	1.111	2.089	121.5	✗
60	2000Q4	1.398	1.064	0.269	0.087	0.312	0.554	0.147	-0.253	0.206	0.604	165.1	✗
61	2001Q1	NA	-0.149	0.498	0.890	0.921	0.859	0.594	0.847	0.561	1.368	151.7	✗
62	2001Q2	NA	2.382	2.804	2.120	2.935	2.509	3.710	3.681	3.359	0.978	273.2	✗
63	2001Q3	NA	-0.662	-1.035	-0.512	-1.627	-1.606	-1.981	-1.862	-1.597	-0.549	146.9	✗
64	2001Q4	NA	0.883	0.182	0.131	0.171	0.927	0.049	-0.277	0.076	0.722	120.4	✗
65	2002Q1	NA	NA	-0.557	-0.374	0.022	-0.116	-0.479	0.151	-0.526	0.490	104.7	✗
66	2002Q2	NA	NA	3.165	2.358	3.155	2.190	4.421	3.452	3.953	0.661	376.0	✗
67	2002Q3	NA	NA	-0.854	-0.145	-1.451	-1.280	-2.123	-1.315	-1.555	0.099	222.2	✗
68	2002Q4	NA	NA	1.614	1.690	1.658	2.796	1.415	1.129	1.390	2.223	166.7	✗
69	2003Q1	NA	NA	NA	-0.906	-0.195	-0.182	-0.617	-0.052	-0.695	0.665	157.1	✗
70	2003Q2	NA	NA	NA	3.886	4.502	2.813	6.090	4.521	5.467	1.086	500.4	✗
71	2003Q3	NA	NA	NA	0.759	-0.597	-0.293	-1.629	-0.039	-0.700	1.285	291.4	✗
72	2003Q4	NA	NA	NA	-0.973	-1.003	0.701	-1.296	-1.738	-1.511	-0.288	243.9	✗
73	2004Q1	NA	NA	NA	NA	0.764	0.683	0.741	1.009	0.496	2.235	173.9	✗
74	2004Q2	NA	NA	NA	NA	5.801	3.590	7.085	5.492	6.532	1.251	583.4	✗
75	2004Q3	NA	NA	NA	NA	1.441	1.893	0.342	2.376	1.673	3.684	334.2	✗
76	2004Q4	NA	NA	NA	NA	-0.094	1.878	-0.221	-0.924	-0.911	0.560	280.2	✗
77	2005Q1	NA	NA	NA	NA	NA	-1.784	-1.601	-1.615	-1.798	0.233	203.1	✗
78	2005Q2	NA	NA	NA	NA	NA	4.725	8.199	7.049	7.738	2.111	608.8	✗
79	2005Q3	NA	NA	NA	NA	NA	-0.233	-1.766	0.163	-0.237	2.030	379.6	✗
80	2005Q4	NA	NA	NA	NA	NA	2.043	-0.156	-0.905	-1.044	0.118	308.7	✗
81	2006Q1	NA	NA	NA	NA	NA	NA	-1.064	-1.352	-1.433	0.943	237.6	✗
82	2006Q2	NA	NA	NA	NA	NA	NA	7.542	6.862	7.363	1.519	602.3	✗
83	2006Q3	NA	NA	NA	NA	NA	NA	-2.963	-1.141	-1.547	0.923	388.6	✗
84	2006Q4	NA	NA	NA	NA	NA	NA	1.367	0.355	0.570	1.275	101.2	✗
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	-0.884	-1.116	1.644	276.0	✗
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	6.859	7.099	1.333	576.6	✗
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	-2.861	-3.229	-0.799	243.0	✗
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	0.962	1.614	1.840	87.8	✗
	Median	-	-	-	-	-	-	-	-	-	-	243.00	-
	Mean	-	-	-	-	-	-	-	-	-	-	263.20	100%

Source: Author's elaboration.

Table J4: Sliding spans of seasonally adjusted series (log-diff), Aggregation 4

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	0.824	0.812	0.816	0.816	0.815	0.834	0.834	0.834	0.834	0.828	2.2	✔
41	1996Q1	2.621	2.639	2.638	2.638	2.643	2.660	2.659	2.660	2.660	2.718	9.7	✔
42	1996Q2	1.257	1.244	1.243	1.242	1.243	1.185	1.185	1.184	1.184	1.195	7.3	✔
43	1996Q3	0.809	0.824	0.820	0.820	0.815	0.852	0.852	0.852	0.852	0.791	6.1	✔
44	1996Q4	2.213	2.189	2.196	2.198	2.194	2.189	2.190	2.190	2.190	2.168	4.5	✔
45	1997Q1	0.339	0.359	0.361	0.362	0.371	0.414	0.412	0.412	0.412	0.480	14.1	✔
46	1997Q2	2.743	2.722	2.715	2.711	2.714	2.613	2.613	2.613	2.613	2.678	13.0	✔
47	1997Q3	1.828	1.866	1.861	1.864	1.852	1.932	1.933	1.933	1.934	1.752	18.2	✔
48	1997Q4	2.447	2.414	2.425	2.427	2.423	2.411	2.413	2.412	2.412	2.508	9.7	✔
49	1998Q1	-0.573	-0.555	-0.554	-0.553	-0.537	-0.536	-0.539	-0.539	-0.539	-0.543	3.7	✔
50	1998Q2	1.793	1.728	1.725	1.716	1.723	1.683	1.680	1.681	1.681	1.779	11.3	✔
51	1998Q3	-0.650	-0.522	-0.547	-0.538	-0.570	-0.519	-0.515	-0.513	-0.514	-0.687	17.4	✔
52	1998Q4	-3.048	-3.120	-3.083	-3.078	-3.068	-3.089	-3.085	-3.088	-3.089	-3.066	7.2	✔
53	1999Q1	-0.328	-0.327	-0.339	-0.344	-0.324	-0.326	-0.333	-0.333	-0.332	-0.245	9.9	✔
54	1999Q2	0.265	0.148	0.145	0.128	0.137	0.139	0.135	0.136	0.136	0.080	18.5	✔
55	1999Q3	2.142	2.398	2.375	2.395	2.336	2.350	2.360	2.364	2.361	2.404	26.2	✔
56	1999Q4	2.487	2.355	2.412	2.424	2.453	2.430	2.435	2.430	2.431	2.397	13.2	✔
57	2000Q1	0.368	0.351	0.295	0.278	0.320	0.306	0.294	0.292	0.295	0.309	9.0	✔
58	2000Q2	-0.049	-0.236	-0.218	-0.242	-0.249	-0.179	-0.189	-0.186	-0.186	-0.205	20.0	✔
59	2000Q3	1.140	1.558	1.558	1.578	1.468	1.413	1.436	1.438	1.431	1.449	43.8	✖
60	2000Q4	1.443	1.231	1.291	1.352	1.432	1.424	1.430	1.435	1.433	1.452	22.1	✔
61	2001Q1	NA	1.085	0.977	0.903	0.974	0.976	0.958	0.946	0.964	0.941	18.2	✔
62	2001Q2	NA	0.932	0.955	0.922	0.908	0.962	0.929	0.929	0.920	0.903	5.9	✔
63	2001Q3	NA	-0.628	-0.561	-0.513	-0.738	-0.790	-0.728	-0.718	-0.730	-0.692	27.7	✔
64	2001Q4	NA	0.835	0.873	0.988	1.150	1.166	1.167	1.174	1.174	1.186	35.1	✖
65	2002Q1	NA	NA	0.406	0.250	0.410	0.390	0.355	0.341	0.371	0.345	16.0	✔
66	2002Q2	NA	NA	0.612	0.570	0.492	0.508	0.452	0.431	0.420	0.343	26.9	✔
67	2002Q3	NA	NA	0.495	0.583	0.222	0.224	0.339	0.371	0.345	0.468	36.1	✖
68	2002Q4	NA	NA	1.721	1.902	2.176	2.210	2.205	2.220	2.224	2.213	50.3	✖
69	2003Q1	NA	NA	NA	0.902	1.166	1.113	1.046	1.027	1.069	1.041	26.4	✔
70	2003Q2	NA	NA	NA	0.641	0.482	0.453	0.367	0.295	0.281	0.167	47.4	✖
71	2003Q3	NA	NA	NA	0.945	0.450	0.507	0.712	0.816	0.772	0.925	49.5	✖
72	2003Q4	NA	NA	NA	0.181	0.564	0.613	0.597	0.606	0.623	0.606	44.2	✖
73	2004Q1	NA	NA	NA	NA	2.634	2.572	2.422	2.390	2.430	2.415	24.4	✔
74	2004Q2	NA	NA	NA	NA	1.090	1.012	0.929	0.752	0.765	0.614	47.6	✖
75	2004Q3	NA	NA	NA	NA	2.597	2.691	3.013	3.289	3.189	3.313	71.6	✖
76	2004Q4	NA	NA	NA	NA	1.332	1.370	1.317	1.281	1.330	1.396	11.5	✔
77	2005Q1	NA	NA	NA	NA	NA	0.605	0.357	0.284	0.345	0.310	32.1	✔
78	2005Q2	NA	NA	NA	NA	NA	2.023	1.942	1.654	1.657	1.583	44.0	✖
79	2005Q3	NA	NA	NA	NA	NA	0.866	1.340	1.870	1.707	1.736	100.4	✖
80	2005Q4	NA	NA	NA	NA	NA	0.904	0.790	0.648	0.746	0.775	25.6	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	1.143	1.018	1.113	1.140	12.5	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	1.404	0.994	1.003	0.968	43.6	✖
83	2006Q3	NA	NA	NA	NA	NA	NA	0.127	0.953	0.669	0.629	82.6	✖
84	2006Q4	NA	NA	NA	NA	NA	NA	1.932	1.671	1.852	1.847	26.1	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	1.754	1.887	1.978	22.4	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	0.617	0.621	0.735	11.8	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	-0.453	-0.848	-1.099	64.6	✖
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	2.074	2.331	2.283	25.7	✔
	Median	-	-	-	-	-	-	-	-	-	-	22.10	-
	Mean	-	-	-	-	-	-	-	-	-	-	26.88	30%

Source: Author's elaboration.

Table J5: Sliding spans of seasonally adjusted series (log-diff), Aggregation 5

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	1.634	1.630	1.539	1.635	1.522	1.621	1.625	1.624	1.612	1.474	16.1	✔
41	1996Q1	2.650	2.649	2.659	2.604	2.660	2.649	2.644	2.644	2.650	2.562	9.8	✔
42	1996Q2	0.393	0.362	0.421	0.417	0.432	0.394	0.396	0.396	0.402	0.502	14.0	✔
43	1996Q3	0.972	1.011	1.017	0.994	1.018	0.991	0.991	0.991	0.991	1.112	14.0	✔
44	1996Q4	1.969	1.966	1.904	1.981	1.895	1.951	1.953	1.952	1.940	1.771	21.0	✔
45	1997Q1	1.205	1.198	1.186	1.137	1.183	1.198	1.195	1.195	1.201	1.109	9.6	✔
46	1997Q2	2.170	2.133	2.201	2.196	2.214	2.168	2.169	2.169	2.177	2.430	29.7	✔
47	1997Q3	1.830	1.874	1.891	1.867	1.885	1.873	1.874	1.874	1.873	1.812	7.9	✔
48	1997Q4	2.253	2.274	2.198	2.278	2.197	2.224	2.225	2.225	2.212	2.132	14.6	✔
49	1998Q1	-0.070	-0.106	-0.125	-0.172	-0.134	-0.101	-0.103	-0.103	-0.098	-0.159	10.2	✔
50	1998Q2	1.470	1.407	1.482	1.473	1.497	1.459	1.459	1.458	1.466	1.677	27.0	✔
51	1998Q3	-0.544	-0.443	-0.403	-0.437	-0.406	-0.426	-0.424	-0.423	-0.420	-0.549	14.6	✔
52	1998Q4	-3.115	-3.095	-3.190	-3.091	-3.190	-3.177	-3.177	-3.176	-3.192	-3.340	24.9	✔
53	1999Q1	0.113	0.050	-0.005	-0.058	-0.016	0.002	0.000	-0.001	0.004	0.150	20.8	✔
54	1999Q2	0.064	-0.040	0.062	0.054	0.071	0.107	0.105	0.103	0.110	0.203	24.3	✔
55	1999Q3	2.114	2.291	2.388	2.337	2.387	2.328	2.333	2.335	2.341	2.337	27.4	✔
56	1999Q4	2.305	2.336	2.193	2.328	2.221	2.185	2.185	2.188	2.170	2.022	31.4	✔
57	2000Q1	0.760	0.653	0.525	0.429	0.466	0.505	0.502	0.498	0.504	0.523	33.1	✔
58	2000Q2	-0.267	-0.439	-0.250	-0.248	-0.235	-0.114	-0.121	-0.127	-0.123	-0.045	39.4	✘
59	2000Q3	1.099	1.386	1.513	1.499	1.553	1.421	1.432	1.439	1.448	1.482	45.4	✘
60	2000Q4	1.458	1.475	1.312	1.443	1.350	1.281	1.288	1.293	1.276	1.192	28.3	✔
61	2001Q1	NA	1.128	0.892	0.720	0.769	0.833	0.815	0.813	0.819	0.802	40.8	✘
62	2001Q2	NA	0.735	1.000	1.056	1.022	1.238	1.230	1.209	1.211	1.235	50.3	✘
63	2001Q3	NA	-0.699	-0.517	-0.501	-0.424	-0.682	-0.660	-0.637	-0.631	-0.547	27.5	✔
64	2001Q4	NA	1.086	0.916	1.051	0.981	0.940	0.960	0.963	0.951	0.877	20.9	✔
65	2002Q1	NA	NA	0.319	0.071	0.129	0.203	0.161	0.162	0.165	0.141	24.8	✔
66	2002Q2	NA	NA	0.620	0.676	0.596	0.911	0.894	0.845	0.851	0.868	31.5	✔
67	2002Q3	NA	NA	0.542	0.657	0.759	0.304	0.355	0.410	0.407	0.516	45.5	✘
68	2002Q4	NA	NA	1.907	2.040	1.973	2.075	2.112	2.114	2.108	1.980	20.7	✔
69	2003Q1	NA	NA	NA	1.165	1.261	1.301	1.211	1.202	1.206	1.199	13.6	✔
70	2003Q2	NA	NA	NA	0.066	-0.081	0.270	0.236	0.153	0.159	0.170	35.1	✘
71	2003Q3	NA	NA	NA	0.682	0.789	0.171	0.296	0.423	0.412	0.559	61.8	✘
72	2003Q4	NA	NA	NA	0.251	0.193	0.482	0.518	0.496	0.501	0.292	32.5	✔
73	2004Q1	NA	NA	NA	NA	3.085	3.094	2.928	2.887	2.893	2.960	20.7	✔
74	2004Q2	NA	NA	NA	NA	0.625	0.937	0.882	0.755	0.751	0.765	31.2	✔
75	2004Q3	NA	NA	NA	NA	3.068	2.376	2.648	2.919	2.898	2.991	69.2	✘
76	2004Q4	NA	NA	NA	NA	0.979	1.405	1.397	1.331	1.360	1.278	42.6	✘
77	2005Q1	NA	NA	NA	NA	NA	0.928	0.656	0.524	0.539	0.482	44.6	✘
78	2005Q2	NA	NA	NA	NA	NA	1.883	1.830	1.653	1.612	1.662	27.1	✔
79	2005Q3	NA	NA	NA	NA	NA	0.813	1.224	1.765	1.722	1.801	98.8	✘
80	2005Q4	NA	NA	NA	NA	NA	0.787	0.750	0.536	0.644	0.538	25.1	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	1.378	1.163	1.166	1.142	23.6	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	1.244	1.011	0.922	0.977	32.2	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	0.123	0.962	0.878	0.985	86.2	✘
84	2006Q4	NA	NA	NA	NA	NA	NA	1.928	1.557	1.777	1.583	37.1	✘
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	1.951	1.961	1.906	5.5	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	0.544	0.354	0.644	29.0	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	-0.549	-0.647	-0.635	9.8	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	2.000	2.324	1.970	35.4	✘
	Median	-	-	-	-	-	-	-	-	-	-	27.40	-
	Mean	-	-	-	-	-	-	-	-	-	-	30.34	30%

Source: Author's elaboration.

Table J6: Sliding spans of seasonally adjusted series (log-diff), Aggregation 6

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	2.351	2.349	2.350	2.351	2.352	2.355	2.352	2.352	2.352	2.288	6.7	✔
41	1996Q1	1.969	1.975	1.974	1.970	1.969	1.968	1.970	1.970	1.970	2.212	24.4	✔
42	1996Q2	1.202	1.193	1.194	1.198	1.199	1.202	1.199	1.199	1.199	1.138	6.4	✔
43	1996Q3	1.690	1.697	1.696	1.693	1.692	1.685	1.692	1.692	1.691	1.516	18.1	✔
44	1996Q4	1.584	1.584	1.585	1.587	1.588	1.594	1.588	1.588	1.589	1.611	2.7	✔
45	1997Q1	0.149	0.146	0.145	0.141	0.141	0.142	0.142	0.142	0.142	0.324	18.3	✔
46	1997Q2	2.850	2.838	2.841	2.845	2.846	2.846	2.845	2.845	2.845	2.836	1.4	✔
47	1997Q3	2.450	2.470	2.469	2.465	2.462	2.449	2.461	2.462	2.461	2.196	27.4	✔
48	1997Q4	2.097	2.106	2.107	2.111	2.113	2.124	2.114	2.114	2.114	2.273	17.6	✔
49	1998Q1	-0.684	-0.708	-0.710	-0.713	-0.711	-0.704	-0.709	-0.711	-0.710	-0.613	10.0	✔
50	1998Q2	1.937	1.889	1.895	1.897	1.899	1.901	1.898	1.898	1.898	1.899	4.8	✔
51	1998Q3	-0.298	-0.193	-0.198	-0.205	-0.220	-0.257	-0.223	-0.219	-0.222	-0.436	24.3	✔
52	1998Q4	-3.573	-3.591	-3.589	-3.578	-3.564	-3.532	-3.560	-3.563	-3.560	-3.455	13.6	✔
53	1999Q1	-0.411	-0.460	-0.464	-0.466	-0.462	-0.452	-0.461	-0.463	-0.462	-0.427	5.5	✔
54	1999Q2	0.405	0.301	0.304	0.297	0.297	0.304	0.296	0.293	0.293	0.257	14.8	✔
55	1999Q3	2.333	2.572	2.583	2.574	2.543	2.462	2.538	2.546	2.541	2.581	25.0	✔
56	1999Q4	2.249	2.183	2.148	2.178	2.207	2.283	2.214	2.213	2.217	2.211	13.5	✔
57	2000Q1	0.408	0.329	0.372	0.372	0.387	0.392	0.388	0.384	0.385	0.389	7.9	✔
58	2000Q2	0.121	-0.062	-0.102	-0.136	-0.144	-0.135	-0.148	-0.157	-0.157	-0.190	31.1	✔
59	2000Q3	1.233	1.649	1.701	1.679	1.611	1.488	1.602	1.612	1.605	1.618	46.8	✗
60	2000Q4	1.266	1.120	1.043	1.132	1.205	1.333	1.225	1.233	1.240	1.272	29.0	✔
61	2001Q1	NA	1.055	1.149	1.150	1.158	1.150	1.149	1.158	1.159	1.151	10.4	✔
62	2001Q2	NA	0.874	0.766	0.654	0.680	0.682	0.671	0.616	0.615	0.578	29.6	✔
63	2001Q3	NA	-0.459	-0.335	-0.355	-0.532	-0.678	-0.536	-0.489	-0.500	-0.470	34.3	✗
64	2001Q4	NA	0.676	0.543	0.731	0.879	1.047	0.913	0.910	0.919	0.931	50.4	✗
65	2002Q1	NA	NA	0.638	0.618	0.662	0.651	0.648	0.695	0.702	0.704	8.6	✔
66	2002Q2	NA	NA	0.470	0.261	0.284	0.254	0.242	0.104	0.099	0.006	46.4	✗
67	2002Q3	NA	NA	0.753	0.753	0.432	0.288	0.452	0.554	0.541	0.655	46.5	✗
68	2002Q4	NA	NA	1.385	1.679	1.944	2.151	2.013	2.006	2.019	1.999	76.6	✗
69	2003Q1	NA	NA	NA	0.759	0.856	0.850	0.822	0.903	0.916	0.927	16.8	✔
70	2003Q2	NA	NA	NA	1.325	1.314	1.212	1.186	0.945	0.928	0.816	50.9	✗
71	2003Q3	NA	NA	NA	0.943	0.480	0.369	0.601	0.788	0.773	0.885	57.4	✗
72	2003Q4	NA	NA	NA	-0.134	0.245	0.486	0.325	0.318	0.342	0.314	62.0	✗
73	2004Q1	NA	NA	NA	NA	2.173	2.192	2.118	2.203	2.226	2.253	13.5	✔
74	2004Q2	NA	NA	NA	NA	1.788	1.575	1.538	1.189	1.154	1.056	73.2	✗
75	2004Q3	NA	NA	NA	NA	2.635	2.580	2.902	3.218	3.183	3.219	63.9	✗
76	2004Q4	NA	NA	NA	NA	1.074	1.350	1.169	1.147	1.209	1.251	27.6	✔
77	2005Q1	NA	NA	NA	NA	NA	0.239	0.102	0.170	0.210	0.209	13.7	✔
78	2005Q2	NA	NA	NA	NA	NA	2.487	2.426	1.947	1.868	1.891	61.9	✗
79	2005Q3	NA	NA	NA	NA	NA	0.764	1.205	1.733	1.651	1.561	96.9	✗
80	2005Q4	NA	NA	NA	NA	NA	0.927	0.709	0.624	0.779	0.801	30.3	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	0.954	0.978	1.032	1.045	9.1	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	1.740	1.158	1.018	1.208	72.2	✗
83	2006Q3	NA	NA	NA	NA	NA	NA	0.016	0.766	0.618	0.359	75.0	✗
84	2006Q4	NA	NA	NA	NA	NA	NA	1.916	1.761	2.037	2.021	27.6	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	1.749	1.814	1.884	13.5	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	0.705	0.490	0.888	39.8	✗
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	-0.627	-0.826	-1.356	72.9	✗
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	2.136	2.526	2.505	39.0	✗
	Median	-	-	-	-	-	-	-	-	-	-	27.40	-
	Mean	-	-	-	-	-	-	-	-	-	-	32.03	34%

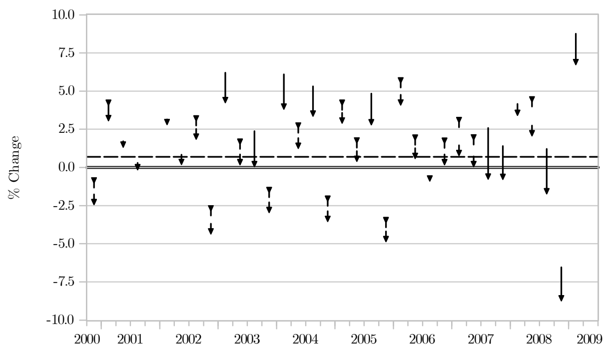
Source: Author's elaboration.

K Diagnostic Result 3

Revision history of trend-cycle and final seasonally adjusted series

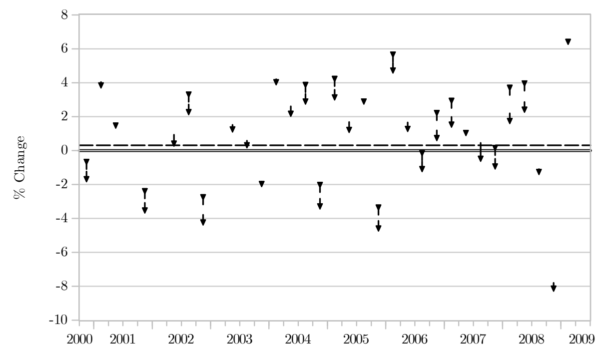
Figure K1: Revision history of trend-cycle series (log-differenced) (*)

A: Aggregation 1.



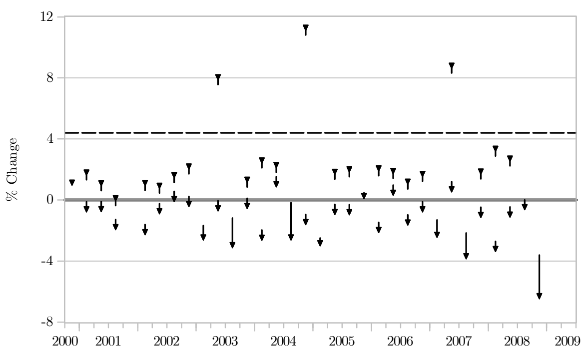
Times equal to 0: 3. Absolute average change: 0.7.

B: Aggregation 2.



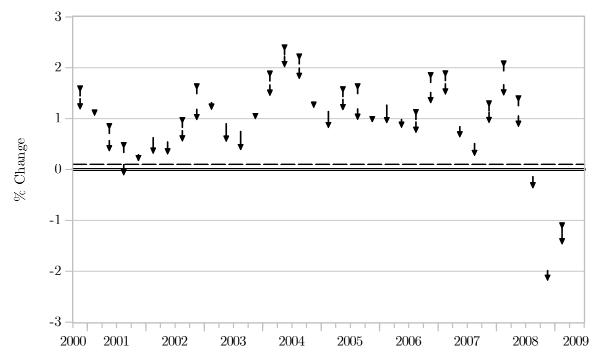
Times equal to 0: 1. Absolute average change: 0.3.

C: Aggregation 3.



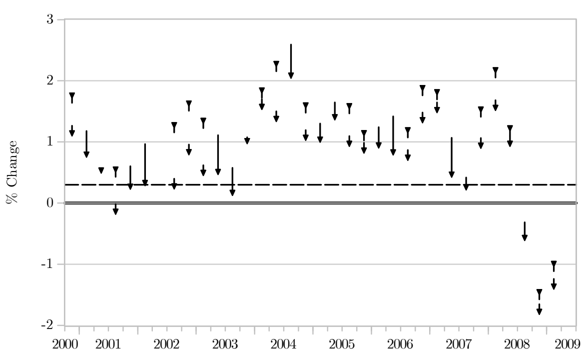
Times equal to 0: 15. Absolute average change: 4.4.

D: Aggregation 4



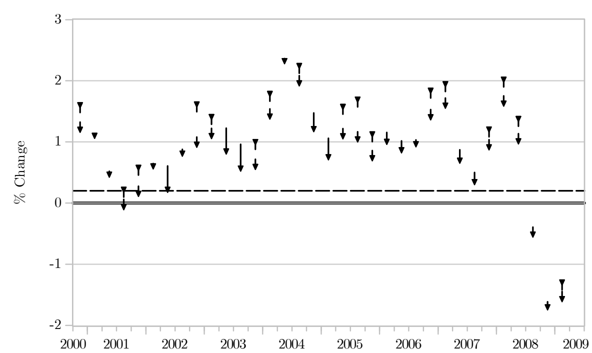
Times equal to 0: 0. Absolute average change: 0.1.

E: Aggregation 5.



Times equal to 0: 1. Absolute average change: 0.3.

F: Aggregation 6.

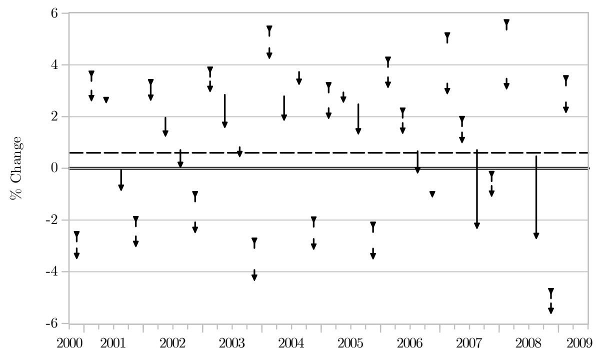


Times equal to 0: 0. Absolute average change: 0.2.

(*) ▾=Most recent. Source: Author's elaboration.

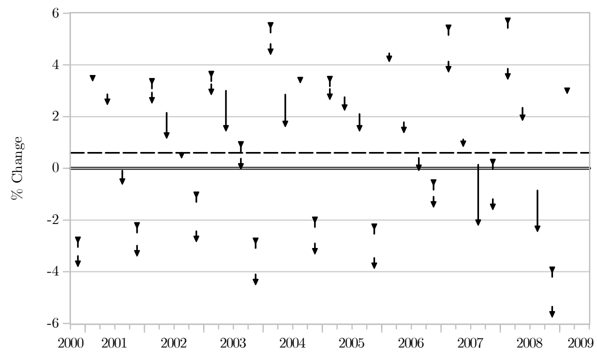
Figure K2: Revision history of seasonally adjusted series (log-differenced) (*)

A: Aggregation 1.



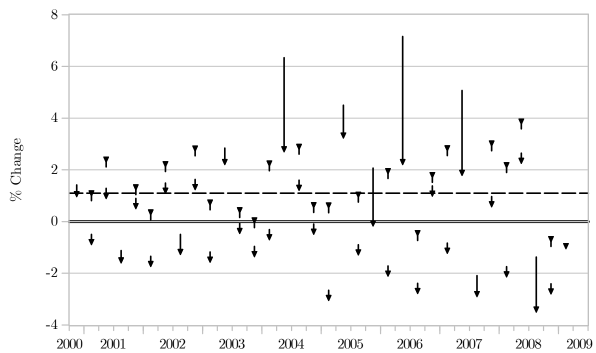
Times equal to 0: 3. Absolute average change: 0.6.

B: Aggregation 2.



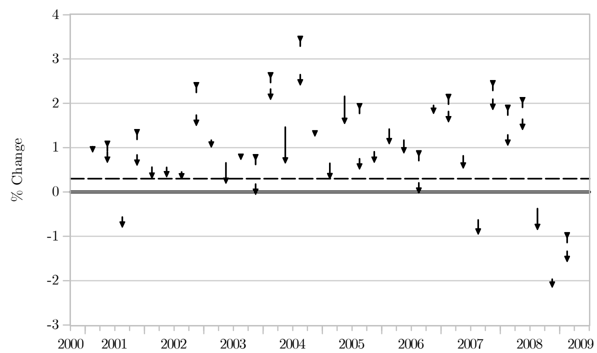
Times equal to 0: 1. Absolute average change: 0.6.

C: Aggregation 3.



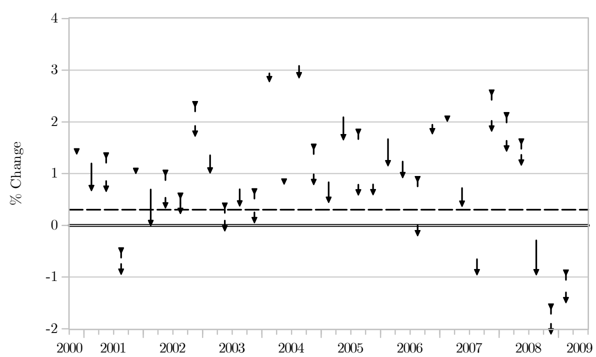
Times equal to 0: 12. Absolute average change: 1.1.

D: Aggregation 4



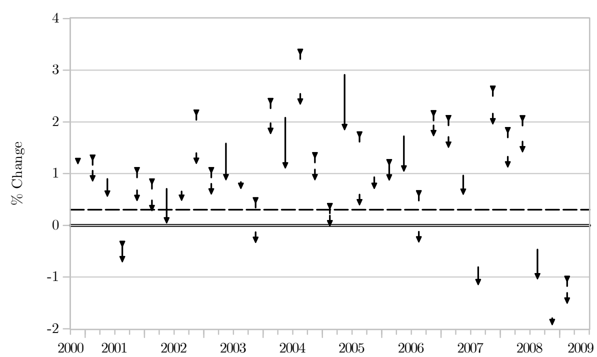
Times equal to 0: 0. Absolute average change: 0.3.

E: Aggregation 5.



Times equal to 0: 0. Absolute average change: 0.3.

F: Aggregation 6.



Times equal to 0: 1. Absolute average change: 0.3.

(*) Aggregation 3 contains an outlier at 2008.III: from -106.9 (concurrent) to -0.4 (most recent).
▾=Most recent. Source: Author's elaboration.

Companion Tables to
A Comparison Between Direct and Indirect Seasonal Adjustment of the Chilean GDP 1986-2009 with X-12-ARIMA

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May 1, 2014

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Sliding spans diagnostic results

1 *Sliding spans of (log-differenced) trend-cycle series*

Table 1: Sliding spans of trend-cycle series (log-diff), Aggregation 1

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	-5.059	-5.019	-5.026	-4.561	-4.630	-4.562	-4.664	-4.637	-4.628	-4.710	49.8	✘
41	1996Q1	7.081	7.077	7.089	6.695	6.724	6.737	6.803	7.146	6.791	6.804	45.1	✘
42	1996Q2	1.989	1.943	1.954	1.636	1.661	1.634	1.722	1.723	1.678	1.712	35.5	✘
43	1996Q3	2.482	2.443	2.434	2.486	2.461	2.433	2.424	2.063	2.431	2.448	42.3	✘
44	1996Q4	-1.517	-1.466	-1.459	-1.474	-1.471	-1.438	-1.405	-2.762	-1.374	-1.361	140.1	✘
45	1997Q1	0.694	0.712	0.743	0.760	0.786	0.806	0.785	3.591	0.573	0.503	308.8	✘
46	1997Q2	5.091	5.064	5.061	5.120	5.123	5.098	5.071	3.738	3.720	3.681	144.2	✘
47	1997Q3	2.076	2.072	2.053	1.972	1.953	1.921	1.914	1.674	4.727	4.834	316.0	✘
48	1997Q4	-1.680	-1.612	-1.658	-1.699	-1.836	-1.773	-1.672	-1.546	-3.055	-3.017	150.9	✘
49	1998Q1	2.713	2.675	2.693	2.804	2.666	2.672	2.793	3.020	2.586	2.504	51.6	✘
50	1998Q2	0.739	0.671	0.712	0.792	0.827	0.810	0.743	0.714	0.944	0.884	27.3	✔
51	1998Q3	2.449	2.464	2.466	2.320	2.895	2.884	2.309	2.154	2.528	2.587	74.1	✘
52	1998Q4	-7.554	-7.493	-7.600	-7.661	-8.283	-8.247	-7.571	-7.497	-7.578	-7.505	79.0	✘
53	1999Q1	2.405	2.400	2.427	2.558	2.963	2.926	2.650	2.684	2.469	2.420	56.3	✘
54	1999Q2	1.089	1.072	1.173	1.194	1.096	1.084	1.113	1.041	1.070	1.029	16.5	✔
55	1999Q3	3.309	3.296	3.317	3.162	3.083	3.065	3.059	3.051	3.222	3.232	26.6	✔
56	1999Q4	-3.346	-3.312	-3.418	-3.385	-3.355	-3.320	-3.263	-3.197	-3.273	-3.237	22.1	✔
57	2000Q1	3.274	3.417	3.122	3.214	3.163	3.175	3.172	3.123	3.043	3.051	37.4	✘
58	2000Q2	2.884	2.870	2.965	3.040	3.052	3.031	2.991	2.906	2.945	2.915	18.2	✔
59	2000Q3	-0.936	-1.107	-0.829	-0.751	-0.742	-0.809	-0.833	-0.714	-0.662	-0.678	44.5	✘
60	2000Q4	-1.783	-1.604	-1.428	-1.877	-1.977	-2.037	-2.063	-1.285	-1.339	-1.325	77.8	✘
61	2001Q1	NA	3.938	3.882	4.709	4.887	5.087	5.277	3.745	3.688	3.697	158.9	✘
62	2001Q2	NA	1.582	1.639	0.853	0.908	0.845	0.696	1.465	1.507	1.502	94.3	✘
63	2001Q3	NA	0.181	0.057	0.041	-0.081	-0.144	-0.201	-0.047	0.025	0.027	38.2	✘
64	2001Q4	NA	-2.939	-3.199	-2.966	-2.924	-2.922	-2.827	-2.924	-2.949	-2.951	37.2	✘
65	2002Q1	NA	NA	2.990	3.117	3.061	3.067	3.115	2.987	2.935	2.952	18.2	✔
66	2002Q2	NA	NA	0.848	0.732	0.530	0.466	0.396	0.360	0.375	0.371	48.8	✘
67	2002Q3	NA	NA	2.578	2.778	2.744	2.717	2.669	2.715	2.709	2.709	20.0	✔
68	2002Q4	NA	NA	-3.753	-3.615	-3.414	-3.333	-3.308	-3.218	-3.201	-3.199	55.4	✘
69	2003Q1	NA	NA	NA	3.919	4.158	4.250	4.273	4.250	4.336	4.340	42.1	✘
70	2003Q2	NA	NA	NA	1.436	1.259	1.265	1.267	1.225	1.202	1.202	23.4	✔
71	2003Q3	NA	NA	NA	0.663	0.343	0.318	0.366	0.371	0.265	0.244	41.9	✘
72	2003Q4	NA	NA	NA	-2.308	-1.952	-2.032	-1.987	-1.985	-1.963	-1.968	35.6	✘
73	2004Q1	NA	NA	NA	NA	4.058	3.963	3.905	3.821	3.885	3.923	23.7	✔
74	2004Q2	NA	NA	NA	NA	2.403	2.430	2.251	2.321	2.254	2.241	18.9	✔
75	2004Q3	NA	NA	NA	NA	3.430	3.487	3.615	3.693	3.560	3.491	26.3	✔
76	2004Q4	NA	NA	NA	NA	-2.910	-2.759	-2.586	-2.630	-2.593	-2.551	35.9	✘
77	2005Q1	NA	NA	NA	NA	NA	3.568	3.502	3.340	3.584	3.700	36.0	✘
78	2005Q2	NA	NA	NA	NA	NA	1.522	1.259	1.372	1.364	1.284	26.3	✔
79	2005Q3	NA	NA	NA	NA	NA	3.035	3.231	3.497	3.113	2.932	56.5	✘
80	2005Q4	NA	NA	NA	NA	NA	-4.277	-4.064	-4.193	-4.112	-4.011	26.6	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	4.685	4.260	4.796	5.094	83.4	✘
82	2006Q2	NA	NA	NA	NA	NA	NA	1.587	1.545	1.559	1.464	12.3	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	-0.275	0.183	-0.414	-0.708	89.1	✘
84	2006Q4	NA	NA	NA	NA	NA	NA	0.840	1.066	1.103	1.272	43.2	✘
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	1.454	2.087	2.606	115.2	✘
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	1.360	1.743	1.494	38.3	✘
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	1.520	0.361	-0.583	210.3	✘
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	1.399	-0.459	-0.605	200.4	✘
	Median	-	-	-	-	-	-	-	-	-	-	42.30	-
	Mean	-	-	-	-	-	-	-	-	-	-	69.81	71%

Source: Author's elaboration.

Table 2: Sliding spans of trend-cycle series (log-diff), Aggregation 2

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	-5.259	-5.253	-5.261	-5.261	-5.208	-5.208	-5.224	-5.208	-5.224	-5.208	5.3	✔
41	1996Q1	7.286	7.281	7.280	7.279	7.311	7.313	7.299	7.311	7.299	7.311	3.4	✔
42	1996Q2	2.104	2.096	2.106	2.104	2.047	2.048	2.049	2.047	2.048	2.047	5.9	✔
43	1996Q3	2.148	2.176	2.178	2.179	2.172	2.170	2.165	2.173	2.165	2.173	3.1	✔
44	1996Q4	-1.379	-1.367	-1.380	-1.374	-1.325	-1.329	-1.332	-1.325	-1.332	-1.325	5.5	✔
45	1997Q1	0.952	0.918	0.935	0.940	0.915	0.917	0.910	0.915	0.910	0.915	4.2	✔
46	1997Q2	4.984	4.958	4.969	4.954	4.896	4.903	4.901	4.896	4.901	4.896	8.8	✔
47	1997Q3	2.072	2.127	2.115	2.108	2.093	2.088	2.099	2.094	2.098	2.094	5.5	✔
48	1997Q4	-1.796	-1.773	-1.813	-1.791	-1.786	-1.794	-1.790	-1.785	-1.790	-1.786	4.0	✔
49	1998Q1	2.679	2.653	2.687	2.701	2.740	2.747	2.737	2.739	2.738	2.739	9.4	✔
50	1998Q2	0.839	0.845	0.876	0.843	0.820	0.833	0.820	0.818	0.821	0.819	5.8	✔
51	1998Q3	2.469	2.353	2.336	2.316	2.286	2.271	2.288	2.286	2.287	2.286	19.8	✔
52	1998Q4	-7.803	-7.917	-8.003	-7.983	-7.971	-7.977	-7.966	-7.964	-7.967	-7.966	20.0	✔
53	1999Q1	2.480	2.552	2.610	2.639	2.715	2.718	2.713	2.714	2.715	2.714	23.8	✔
54	1999Q2	1.225	1.337	1.439	1.408	1.367	1.368	1.362	1.361	1.364	1.363	21.4	✔
55	1999Q3	3.452	3.474	3.402	3.378	3.262	3.302	3.260	3.258	3.258	3.258	21.6	✔
56	1999Q4	-3.302	-3.270	-3.379	-3.320	-3.343	-3.294	-3.328	-3.328	-3.333	-3.332	10.9	✔
57	2000Q1	3.032	3.045	2.774	2.756	2.858	2.831	2.858	2.859	2.861	2.861	28.9	✔
58	2000Q2	2.632	2.636	2.646	2.682	2.901	2.840	2.868	2.870	2.876	2.875	26.9	✔
59	2000Q3	-1.007	-1.027	-0.799	-0.670	-0.565	-0.595	-0.576	-0.576	-0.576	-0.576	46.2	✘
60	2000Q4	-1.250	-1.162	-1.029	-1.012	-1.133	-1.124	-1.091	-1.093	-1.105	-1.104	23.8	✔
61	2001Q1	NA	4.168	4.110	3.891	3.764	3.796	3.772	3.771	3.776	3.775	40.4	✘
62	2001Q2	NA	1.552	1.748	1.615	1.501	1.498	1.456	1.458	1.466	1.465	29.2	✔
63	2001Q3	NA	-0.080	-0.155	-0.017	-0.125	-0.138	-0.121	-0.120	-0.125	-0.125	13.8	✔
64	2001Q4	NA	-3.131	-3.317	-3.095	-2.913	-2.955	-2.872	-2.881	-2.882	-2.877	44.5	✘
65	2002Q1	NA	NA	3.136	3.048	2.936	2.905	2.963	2.954	2.957	2.962	23.1	✔
66	2002Q2	NA	NA	1.020	0.816	0.503	0.457	0.438	0.438	0.429	0.421	59.9	✘
67	2002Q3	NA	NA	2.535	2.760	2.903	2.895	2.839	2.848	2.841	2.831	36.8	✘
68	2002Q4	NA	NA	-3.864	-3.744	-3.310	-3.226	-3.252	-3.254	-3.242	-3.238	63.8	✘
69	2003Q1	NA	NA	NA	3.890	4.048	4.159	4.206	4.196	4.175	4.194	31.6	✔
70	2003Q2	NA	NA	NA	1.307	1.067	1.189	1.241	1.264	1.232	1.242	24.0	✔
71	2003Q3	NA	NA	NA	0.735	0.274	0.277	0.312	0.333	0.329	0.317	46.1	✘
72	2003Q4	NA	NA	NA	-1.857	-1.932	-1.962	-1.995	-2.017	-1.980	-1.984	16.0	✔
73	2004Q1	NA	NA	NA	NA	4.109	4.030	3.938	3.881	3.941	3.958	22.8	✔
74	2004Q2	NA	NA	NA	NA	2.426	2.333	2.141	2.207	2.191	2.161	28.5	✔
75	2004Q3	NA	NA	NA	NA	3.436	3.325	3.449	3.488	3.413	3.379	16.3	✔
76	2004Q4	NA	NA	NA	NA	-2.879	-2.690	-2.505	-2.537	-2.539	-2.491	38.8	✘
77	2005Q1	NA	NA	NA	NA	NA	3.674	3.623	3.512	3.666	3.726	21.4	✔
78	2005Q2	NA	NA	NA	NA	NA	1.604	1.247	1.369	1.330	1.224	38.0	✘
79	2005Q3	NA	NA	NA	NA	NA	2.861	3.075	3.141	2.935	2.866	28.0	✔
80	2005Q4	NA	NA	NA	NA	NA	-4.234	-3.961	-4.004	-3.994	-3.861	37.3	✘
81	2006Q1	NA	NA	NA	NA	NA	NA	4.814	4.667	4.986	5.099	43.2	✘
82	2006Q2	NA	NA	NA	NA	NA	NA	1.467	1.327	1.393	1.272	19.5	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	-0.649	-0.430	-0.659	-0.596	22.9	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	1.173	1.523	1.540	1.760	58.7	✘
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	2.118	2.463	2.479	36.1	✘
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	1.253	1.354	1.036	31.8	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	0.322	-0.173	-0.482	80.4	✘
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	-0.491	-0.184	-0.270	30.7	✔
	Median	-	-	-	-	-	-	-	-	-	-	23.80	-
	Mean	-	-	-	-	-	-	-	-	-	-	26.28	27%

Source: Author's elaboration.

Table 3: Sliding spans of trend-cycle series (log-diff), Aggregation 3

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	2.105	2.678	0.665	0.951	-0.091	-0.079	0.993	0.395	0.009	1.790	276.9	✗
41	1996Q1	3.323	2.942	2.604	3.280	3.386	3.386	3.322	3.301	3.400	3.352	79.6	✗
42	1996Q2	2.343	1.205	2.521	3.107	2.782	2.772	3.056	2.225	2.667	2.032	190.2	✗
43	1996Q3	-2.298	-4.438	1.593	1.130	1.216	1.215	1.114	1.311	1.130	-2.949	603.1	✗
44	1996Q4	6.402	10.893	1.408	0.593	1.033	1.049	0.619	1.532	1.089	7.899	1030.0	✗
45	1997Q1	-1.821	-3.679	-3.051	-2.051	-2.742	-2.746	-2.056	-2.626	-2.350	-2.626	185.8	✗
46	1997Q2	1.807	1.513	11.251	7.758	8.363	8.346	7.729	7.395	7.716	1.683	973.8	✗
47	1997Q3	3.092	2.826	-3.172	-0.082	-0.584	-0.584	-0.116	0.049	-0.217	3.238	641.0	✗
48	1997Q4	1.628	1.879	-0.168	1.002	1.069	1.055	0.940	1.370	1.127	1.950	211.8	✗
49	1998Q1	0.744	0.903	1.097	0.723	0.825	0.791	0.799	0.450	0.715	0.698	64.7	✗
50	1998Q2	0.306	0.252	1.948	1.454	1.572	1.588	1.512	1.309	1.488	-0.077	202.5	✗
51	1998Q3	-0.453	-0.267	-0.279	-0.524	-0.601	-0.556	-0.582	-0.183	-0.511	-0.55	41.8	✗
52	1998Q4	-2.267	-1.690	-3.212	-3.353	-3.468	-3.429	-3.493	-3.437	-3.388	-1.997	180.3	✗
53	1999Q1	-0.983	-1.291	-1.205	-0.907	-0.735	-0.781	-0.749	-1.053	-0.855	-0.844	55.6	✗
54	1999Q2	0.868	0.444	1.731	1.818	1.870	1.824	1.936	1.996	1.873	0.659	155.2	✗
55	1999Q3	2.243	2.478	2.297	2.038	1.903	1.957	1.926	2.176	2.004	1.991	57.5	✗
56	1999Q4	1.735	2.068	1.252	1.208	1.165	1.216	0.982	0.720	1.044	2.040	134.8	✗
57	2000Q1	0.886	0.682	0.065	0.620	0.146	0.214	0.550	0.656	0.542	0.641	82.1	✗
58	2000Q2	0.117	0.029	0.255	0.693	0.488	0.469	0.926	1.265	0.906	0.706	123.6	✗
59	2000Q3	0.935	0.910	1.267	0.874	1.367	1.341	1.100	0.984	1.133	1.257	49.3	✗
60	2000Q4	1.207	1.426	1.695	1.263	1.552	1.602	0.869	0.513	0.912	1.150	118.2	✗
61	2001Q1	NA	1.720	1.805	1.685	1.750	1.685	1.872	1.913	1.732	1.318	59.5	✗
62	2001Q2	NA	0.541	0.967	0.924	1.098	0.780	1.353	1.497	1.256	0.604	95.6	✗
63	2001Q3	NA	0.279	-0.139	0.058	-0.447	-0.316	-0.368	-0.491	-0.254	-0.360	77.0	✗
64	2001Q4	NA	0.011	-0.677	-0.488	-0.670	-0.178	-0.650	-0.409	-0.402	0.018	69.5	✗
65	2002Q1	NA	NA	0.581	-1.786	0.751	0.599	-1.602	-1.700	-1.816	0.631	256.7	✗
66	2002Q2	NA	NA	1.233	5.579	1.218	0.491	5.305	5.197	5.109	0.449	513.0	✗
67	2002Q3	NA	NA	1.367	-0.846	1.314	1.330	-1.217	-1.363	-1.072	1.141	273.0	✗
68	2002Q4	NA	NA	0.234	0.024	0.085	1.453	0.127	0.413	0.204	1.707	168.3	✗
69	2003Q1	NA	NA	NA	1.060	-3.010	-2.397	0.953	1.590	1.539	-2.475	460.0	✗
70	2003Q2	NA	NA	NA	1.822	8.774	6.776	2.174	2.715	2.733	7.285	695.2	✗
71	2003Q3	NA	NA	NA	1.358	-2.717	-2.453	1.079	0.577	0.705	-3.001	435.9	✗
72	2003Q4	NA	NA	NA	0.112	0.238	1.077	-0.256	-0.557	-0.982	0.843	205.9	✗
73	2004Q1	NA	NA	NA	NA	2.588	2.132	-1.221	-2.373	-1.600	2.086	496.1	✗
74	2004Q2	NA	NA	NA	NA	3.398	1.978	9.224	11.578	9.348	1.793	978.5	✗
75	2004Q3	NA	NA	NA	NA	2.001	-2.170	-0.904	-2.229	-0.510	-2.470	447.1	✗
76	2004Q4	NA	NA	NA	NA	-0.953	6.942	0.079	-0.840	0.117	10.267	1122.0	✗
77	2005Q1	NA	NA	NA	NA	NA	-1.481	-2.037	0.902	-2.762	-2.862	376.4	✗
78	2005Q2	NA	NA	NA	NA	NA	2.408	8.958	3.661	9.344	1.360	798.4	✗
79	2005Q3	NA	NA	NA	NA	NA	2.097	-1.952	2.045	-1.534	1.508	404.9	✗
80	2005Q4	NA	NA	NA	NA	NA	0.477	-0.957	-0.766	-1.323	0.261	180.0	✗
81	2006Q1	NA	NA	NA	NA	NA	NA	0.610	-3.318	1.251	1.584	490.2	✗
82	2006Q2	NA	NA	NA	NA	NA	NA	2.629	9.603	3.270	1.408	819.5	✗
83	2006Q3	NA	NA	NA	NA	NA	NA	2.382	-2.632	1.626	0.716	501.4	✗
84	2006Q4	NA	NA	NA	NA	NA	NA	-0.104	-0.104	-0.405	1.220	162.5	✗
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	1.788	-2.383	-2.302	417.1	✗
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	1.907	8.833	7.986	692.6	✗
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	1.360	-3.001	-3.740	510.0	✗
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	-0.477	1.614	1.373	209.1	✗
	Median	-	-	-	-	-	-	-	-	-	-	211.80	-
	Mean	-	-	-	-	-	-	-	-	-	-	354.56	100%

Source: Author's elaboration.

Table 4: Sliding spans of trend-cycle series (log-diff), Aggregation 4

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	1.448	1.453	1.452	1.453	1.453	1.476	1.476	1.476	1.476	1.476	2.8	✔
41	1996Q1	1.893	1.892	1.893	1.893	1.896	1.889	1.889	1.889	1.889	1.889	0.7	✔
42	1996Q2	1.496	1.504	1.501	1.499	1.501	1.484	1.483	1.483	1.483	1.483	2.1	✔
43	1996Q3	1.259	1.260	1.261	1.259	1.255	1.253	1.253	1.252	1.252	1.252	0.9	✔
44	1996Q4	1.269	1.254	1.257	1.259	1.257	1.290	1.290	1.290	1.290	1.290	3.6	✔
45	1997Q1	1.455	1.436	1.436	1.438	1.443	1.426	1.427	1.427	1.427	1.427	2.9	✔
46	1997Q2	1.847	1.866	1.860	1.859	1.861	1.842	1.842	1.842	1.842	1.842	2.4	✔
47	1997Q3	2.437	2.451	2.452	2.453	2.442	2.450	2.451	2.452	2.452	2.452	1.6	✔
48	1997Q4	1.760	1.759	1.764	1.767	1.763	1.786	1.786	1.786	1.786	1.786	2.7	✔
49	1998Q1	1.151	1.129	1.131	1.127	1.139	1.113	1.111	1.110	1.110	1.111	4.1	✔
50	1998Q2	0.348	0.314	0.312	0.314	0.321	0.324	0.325	0.325	0.325	0.325	3.6	✔
51	1998Q3	-1.099	-1.053	-1.064	-1.054	-1.038	-1.021	-1.018	-1.018	-1.018	-1.018	8.1	✔
52	1998Q4	-2.049	-2.019	-2.034	-2.032	-2.020	-2.021	-2.019	-2.020	-2.019	-2.019	3.0	✔
53	1999Q1	-1.060	-1.156	-1.147	-1.162	-1.167	-1.187	-1.189	-1.190	-1.189	-1.189	13.0	✔
54	1999Q2	0.743	0.753	0.759	0.754	0.721	0.731	0.730	0.732	0.731	0.731	3.8	✔
55	1999Q3	2.065	2.183	2.193	2.206	2.185	2.191	2.194	2.196	2.194	2.194	14.1	✔
56	1999Q4	2.093	2.100	2.103	2.106	2.125	2.106	2.106	2.105	2.106	2.106	3.2	✔
57	2000Q1	0.677	0.526	0.519	0.528	0.549	0.559	0.558	0.552	0.553	0.553	15.8	✔
58	2000Q2	0.132	0.169	0.153	0.154	0.106	0.129	0.132	0.130	0.128	0.128	6.3	✔
59	2000Q3	0.930	1.065	1.100	1.108	1.076	1.063	1.069	1.077	1.072	1.072	17.8	✔
60	2000Q4	1.380	1.394	1.392	1.401	1.455	1.424	1.428	1.432	1.436	1.437	7.5	✔
61	2001Q1	NA	1.150	1.075	1.050	1.110	1.131	1.113	1.108	1.116	1.116	10.0	✔
62	2001Q2	NA	0.584	0.742	0.698	0.711	0.737	0.725	0.710	0.706	0.705	15.8	✔
63	2001Q3	NA	0.112	0.306	0.364	0.327	0.329	0.351	0.351	0.337	0.337	25.2	✔
64	2001Q4	NA	0.288	0.156	0.194	0.213	0.188	0.205	0.220	0.226	0.229	13.2	✔
65	2002Q1	NA	NA	0.419	0.339	0.413	0.404	0.362	0.358	0.374	0.371	8.0	✔
66	2002Q2	NA	NA	0.576	0.508	0.397	0.393	0.363	0.366	0.362	0.361	21.5	✔
67	2002Q3	NA	NA	0.763	0.884	0.749	0.784	0.814	0.842	0.822	0.825	13.5	✔
68	2002Q4	NA	NA	1.174	1.251	1.407	1.424	1.458	1.467	1.474	1.480	30.6	✔
69	2003Q1	NA	NA	NA	1.161	1.366	1.326	1.266	1.215	1.238	1.230	20.5	✔
70	2003Q2	NA	NA	NA	0.847	0.674	0.639	0.635	0.621	0.611	0.607	24.0	✔
71	2003Q3	NA	NA	NA	0.751	0.290	0.334	0.424	0.453	0.439	0.445	46.1	✘
72	2003Q4	NA	NA	NA	1.084	0.922	0.982	1.009	1.111	1.046	1.051	18.9	✔
73	2004Q1	NA	NA	NA	NA	1.714	1.680	1.742	1.767	1.710	1.725	8.7	✔
74	2004Q2	NA	NA	NA	NA	2.184	2.121	2.234	2.183	2.235	2.230	11.4	✔
75	2004Q3	NA	NA	NA	NA	1.969	2.019	2.010	1.997	2.075	2.055	10.6	✔
76	2004Q4	NA	NA	NA	NA	1.310	1.468	1.323	1.243	1.264	1.267	22.5	✔
77	2005Q1	NA	NA	NA	NA	NA	1.124	0.981	0.860	0.888	0.876	26.4	✔
78	2005Q2	NA	NA	NA	NA	NA	1.316	1.358	1.486	1.441	1.424	17.0	✔
79	2005Q3	NA	NA	NA	NA	NA	1.147	1.336	1.534	1.460	1.477	38.7	✘
80	2005Q4	NA	NA	NA	NA	NA	1.000	0.957	0.912	0.960	0.989	8.8	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	1.107	0.870	0.957	0.971	23.7	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	1.010	0.962	0.885	0.880	13.0	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	0.930	1.113	1.019	0.978	18.3	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	1.501	1.616	1.711	1.695	21.0	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	1.561	1.596	1.731	17.0	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	0.709	0.605	0.692	10.4	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	0.485	0.467	0.332	15.3	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	1.117	1.225	1.148	10.8	✔
	Median	-	-	-	-	-	-	-	-	-	-	11.40	-
	Mean	-	-	-	-	-	-	-	-	-	-	13.08	5%

Source: Author's elaboration.

Table 5: Sliding spans of trend-cycle series (log-diff), Aggregation 5

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	1.782	1.790	1.732	1.789	1.735	1.627	1.627	1.626	1.624	1.785	16.6	✔
41	1996Q1	1.813	1.792	1.801	1.821	1.805	2.165	2.164	2.163	2.163	1.781	38.4	✘
42	1996Q2	1.100	1.077	1.155	1.097	1.157	0.917	0.918	0.919	0.923	1.112	24.0	✔
43	1996Q3	1.030	1.041	1.042	1.031	1.036	1.006	1.008	1.008	1.006	1.063	5.7	✔
44	1996Q4	1.484	1.493	1.449	1.475	1.442	1.489	1.489	1.489	1.482	1.470	5.1	✔
45	1997Q1	1.683	1.680	1.662	1.657	1.661	1.674	1.673	1.673	1.675	1.646	3.7	✔
46	1997Q2	1.838	1.842	1.883	1.848	1.889	1.857	1.857	1.857	1.865	1.866	5.1	✔
47	1997Q3	2.211	2.220	2.234	2.249	2.236	2.221	2.222	2.222	2.220	2.238	3.8	✔
48	1997Q4	1.686	1.693	1.661	1.689	1.640	1.664	1.666	1.666	1.654	1.656	5.3	✔
49	1998Q1	1.011	0.973	0.943	0.965	0.942	0.978	0.978	0.977	0.970	0.967	6.9	✔
50	1998Q2	0.253	0.206	0.264	0.207	0.276	0.251	0.250	0.250	0.272	0.255	7.0	✔
51	1998Q3	-0.998	-0.989	-0.758	-1.001	-0.750	-1.002	-1.002	-0.999	-0.926	-0.923	25.2	✔
52	1998Q4	-1.508	-1.492	-2.016	-1.472	-2.003	-1.529	-1.528	-1.530	-1.673	-1.652	54.4	✘
53	1999Q1	-0.861	-0.894	-0.649	-0.881	-0.646	-0.867	-0.869	-0.869	-0.812	-0.801	24.8	✔
54	1999Q2	0.684	0.748	0.809	0.734	0.795	0.770	0.768	0.767	0.781	0.739	12.5	✔
55	1999Q3	1.758	1.857	1.860	1.883	1.878	1.857	1.860	1.861	1.857	1.859	12.5	✔
56	1999Q4	1.991	2.009	1.934	1.969	1.937	1.924	1.926	1.927	1.920	1.952	8.9	✔
57	2000Q1	0.855	0.729	0.711	0.645	0.661	0.693	0.690	0.687	0.687	0.702	21.0	✔
58	2000Q2	0.260	0.214	0.330	0.298	0.344	0.347	0.346	0.344	0.354	0.329	14.0	✔
59	2000Q3	0.801	0.935	1.005	0.978	1.075	1.046	1.053	1.057	1.055	0.889	27.4	✔
60	2000Q4	1.257	1.461	1.340	1.544	1.332	1.321	1.322	1.327	1.320	1.626	36.9	✘
61	2001Q1	NA	1.153	1.064	0.918	0.983	0.975	0.968	0.963	0.965	0.793	36.0	✘
62	2001Q2	NA	0.554	0.654	0.658	0.584	0.560	0.559	0.555	0.543	0.533	12.5	✔
63	2001Q3	NA	-0.008	0.324	0.409	0.261	0.376	0.393	0.398	0.385	0.429	43.7	✘
64	2001Q4	NA	0.601	0.230	0.173	0.451	0.232	0.233	0.246	0.251	0.273	42.8	✘
65	2002Q1	NA	NA	0.461	0.340	0.239	0.381	0.353	0.344	0.362	0.328	22.2	✔
66	2002Q2	NA	NA	0.522	0.543	0.595	0.616	0.605	0.590	0.597	0.555	9.4	✔
67	2002Q3	NA	NA	0.685	0.924	1.019	1.019	1.054	1.065	1.063	1.146	46.1	✘
68	2002Q4	NA	NA	0.954	1.295	1.365	1.726	1.756	1.791	1.791	1.499	83.7	✘
69	2003Q1	NA	NA	NA	1.268	1.394	1.094	1.058	1.024	1.017	1.219	37.7	✘
70	2003Q2	NA	NA	NA	0.717	0.702	0.471	0.440	0.424	0.426	0.509	29.3	✔
71	2003Q3	NA	NA	NA	0.527	0.212	0.203	0.209	0.213	0.212	0.167	36.0	✘
72	2003Q4	NA	NA	NA	1.068	1.043	1.131	1.111	1.076	1.077	1.011	12.0	✔
73	2004Q1	NA	NA	NA	NA	1.631	1.751	1.721	1.711	1.707	1.705	12.0	✔
74	2004Q2	NA	NA	NA	NA	1.573	1.927	2.003	2.087	2.075	2.131	55.8	✘
75	2004Q3	NA	NA	NA	NA	2.582	1.862	1.961	2.030	2.034	2.066	72.0	✘
76	2004Q4	NA	NA	NA	NA	1.186	1.610	1.558	1.465	1.486	1.465	42.4	✘
77	2005Q1	NA	NA	NA	NA	NA	1.382	1.232	1.075	1.045	1.033	34.9	✘
78	2005Q2	NA	NA	NA	NA	NA	1.282	1.380	1.419	1.376	1.396	13.7	✔
79	2005Q3	NA	NA	NA	NA	NA	1.059	1.218	1.415	1.440	1.453	39.4	✘
80	2005Q4	NA	NA	NA	NA	NA	0.981	0.950	0.957	1.032	1.018	8.2	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	1.164	0.895	0.902	0.942	26.9	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	0.921	0.942	0.838	0.829	11.3	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	0.909	1.120	1.131	1.065	22.2	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	1.470	1.583	1.749	1.746	27.9	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	1.575	1.601	1.678	10.3	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	0.625	0.458	0.466	16.7	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	0.407	0.326	0.259	14.8	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	1.060	1.403	1.400	34.3	✘
	Median	-	-	-	-	-	-	-	-	-	-	22.20	-
	Mean	-	-	-	-	-	-	-	-	-	-	24.76	34%

Source: Author's elaboration.

Table 6: Sliding spans of trend-cycle series (log-diff), Aggregation 6

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	1.895	1.897	1.897	1.896	1.896	1.896	1.896	1.896	1.896	1.896	0.2	✔
41	1996Q1	1.939	1.939	1.938	1.938	1.938	1.940	1.939	1.939	1.939	1.939	0.2	✔
42	1996Q2	1.459	1.458	1.458	1.458	1.458	1.456	1.457	1.457	1.457	1.457	0.3	✔
43	1996Q3	1.606	1.608	1.608	1.608	1.608	1.605	1.607	1.608	1.607	1.607	0.3	✔
44	1996Q4	1.157	1.152	1.154	1.158	1.160	1.167	1.160	1.160	1.160	1.160	1.5	✔
45	1997Q1	1.110	1.098	1.100	1.104	1.107	1.114	1.107	1.106	1.107	1.107	1.6	✔
46	1997Q2	2.071	2.075	2.074	2.070	2.067	2.059	2.067	2.067	2.067	2.067	1.6	✔
47	1997Q3	2.721	2.743	2.741	2.736	2.733	2.724	2.733	2.734	2.733	2.733	2.2	✔
48	1997Q4	1.846	1.840	1.839	1.842	1.845	1.850	1.844	1.844	1.844	1.844	1.1	✔
49	1998Q1	1.074	1.032	1.032	1.034	1.038	1.047	1.039	1.037	1.038	1.038	4.2	✔
50	1998Q2	0.201	0.204	0.207	0.209	0.206	0.211	0.206	0.207	0.207	0.207	1.0	✔
51	1998Q3	-1.301	-1.247	-1.245	-1.243	-1.247	-1.248	-1.247	-1.245	-1.245	-1.245	5.8	✔
52	1998Q4	-2.038	-2.020	-2.023	-2.024	-2.021	-2.023	-2.020	-2.020	-2.020	-2.020	1.8	✔
53	1999Q1	-0.983	-1.068	-1.071	-1.077	-1.069	-1.058	-1.067	-1.071	-1.070	-1.070	9.4	✔
54	1999Q2	0.827	0.831	0.838	0.829	0.818	0.796	0.815	0.816	0.815	0.814	4.2	✔
55	1999Q3	2.095	2.188	2.182	2.184	2.174	2.152	2.173	2.176	2.175	2.175	9.3	✔
56	1999Q4	2.014	2.043	2.042	2.059	2.072	2.091	2.074	2.076	2.078	2.078	7.7	✔
57	2000Q1	0.660	0.495	0.493	0.494	0.516	0.558	0.518	0.512	0.514	0.515	16.7	✔
58	2000Q2	0.294	0.307	0.322	0.287	0.259	0.214	0.253	0.249	0.246	0.246	10.8	✔
59	2000Q3	0.985	1.134	1.119	1.122	1.102	1.074	1.103	1.107	1.105	1.105	14.9	✔
60	2000Q4	1.315	1.358	1.369	1.422	1.445	1.478	1.452	1.467	1.469	1.469	16.3	✔
61	2001Q1	NA	1.090	1.075	1.069	1.106	1.134	1.102	1.093	1.095	1.096	6.5	✔
62	2001Q2	NA	0.531	0.549	0.478	0.495	0.553	0.498	0.461	0.467	0.468	9.2	✔
63	2001Q3	NA	0.086	0.066	0.091	0.089	0.148	0.107	0.096	0.099	0.099	8.2	✔
64	2001Q4	NA	0.273	0.303	0.408	0.399	0.360	0.405	0.461	0.458	0.457	18.8	✔
65	2002Q1	NA	NA	0.623	0.590	0.632	0.538	0.607	0.603	0.599	0.600	9.4	✔
66	2002Q2	NA	NA	0.529	0.374	0.293	0.363	0.262	0.225	0.220	0.220	30.9	✔
67	2002Q3	NA	NA	0.862	0.894	0.792	0.863	0.812	0.818	0.811	0.808	10.2	✔
68	2002Q4	NA	NA	1.075	1.265	1.345	1.329	1.396	1.474	1.483	1.483	40.8	✘
69	2003Q1	NA	NA	NA	1.180	1.361	1.274	1.318	1.269	1.277	1.280	18.1	✔
70	2003Q2	NA	NA	NA	1.084	0.962	0.871	0.912	0.857	0.844	0.843	24.1	✔
71	2003Q3	NA	NA	NA	0.903	0.535	0.478	0.561	0.581	0.571	0.566	42.5	✘
72	2003Q4	NA	NA	NA	0.715	0.717	0.824	0.784	0.852	0.871	0.873	15.8	✔
73	2004Q1	NA	NA	NA	NA	1.562	1.615	1.476	1.640	1.651	1.652	17.6	✔
74	2004Q2	NA	NA	NA	NA	2.270	2.148	2.218	2.310	2.302	2.299	16.2	✔
75	2004Q3	NA	NA	NA	NA	2.064	2.066	2.195	2.103	2.106	2.102	13.1	✔
76	2004Q4	NA	NA	NA	NA	1.463	1.583	1.512	1.215	1.213	1.207	37.6	✘
77	2005Q1	NA	NA	NA	NA	NA	1.200	1.023	0.754	0.739	0.751	46.1	✘
78	2005Q2	NA	NA	NA	NA	NA	1.277	1.408	1.482	1.435	1.445	20.5	✔
79	2005Q3	NA	NA	NA	NA	NA	1.055	1.244	1.582	1.581	1.560	52.7	✘
80	2005Q4	NA	NA	NA	NA	NA	0.855	0.806	0.927	1.016	0.998	21.0	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	1.034	0.890	0.941	0.998	14.4	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	0.966	0.982	0.844	0.863	13.8	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	0.977	1.086	1.036	0.961	12.5	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	1.515	1.592	1.751	1.704	23.6	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	1.630	1.684	1.806	17.6	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	0.724	0.611	0.695	11.3	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	0.461	0.466	0.344	12.2	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	1.031	1.188	1.079	15.7	✔
	Median	-	-	-	-	-	-	-	-	-	-	12.20	-
	Mean	-	-	-	-	-	-	-	-	-	-	14.11	11%

Source: Author's elaboration.

2 *Sliding spans of seasonally adjusted series (log)*

Table 7: Sliding spans of seasonally-adjusted series (log), Aggregation 1

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9464611	9463831	9462119	9459166	9453467	9452869	9452742	9407386	9450576	9455226	0.60	✔
41	1996Q1	9796389	9803656	9804715	9808929	9812097	9817197	9819453	9859593	9815492	9820290	0.64	✔
42	1996Q2	10221015	10215336	10216528	10212288	10212911	10211446	10206705	10228078	10215641	10224749	0.20	✔
43	1996Q3	10432711	10431819	10432208	10439907	10441859	10439095	10440192	10419642	10442445	10389966	0.50	✔
44	1996Q4	10104866	10105924	10103201	10089543	10083305	10082591	10084129	10044311	10081494	10118585	0.73	✔
45	1997Q1	10437712	10443927	10442760	10451466	10454414	10459447	10462400	10506647	10456093	10485714	0.66	✔
46	1997Q2	10827366	10820596	10825722	10826698	10830078	10828487	10823453	10833837	10823445	10821987	0.12	✔
47	1997Q3	11158442	11156742	11155964	11160793	11159124	11156498	11155878	11135269	11163762	11083196	0.72	✔
48	1997Q4	10903355	10907972	10905661	10888930	10883281	10882759	10885014	10862788	10874816	10948453	0.78	✔
49	1998Q1	11021469	11026746	11019043	11027244	11030499	11035096	11039446	11074863	11045492	11064854	0.50	✔
50	1998Q2	11447187	11439482	11451792	11461314	11467764	11466195	11460757	11454857	11452918	11444494	0.24	✔
51	1998Q3	11543757	11541488	11540877	11535241	11528917	11526555	11524570	11516439	11536969	11454292	0.78	✔
52	1998Q4	10758546	10761844	10758721	10744428	10740112	10739410	10741913	10735330	10750092	10779633	0.41	✔
53	1999Q1	10965604	10975658	10962078	10976217	10978519	10983003	10988467	11004470	10973786	11010736	0.44	✔
54	1999Q2	11108689	11096793	11114021	11119432	11131829	11130265	11124514	11115236	11116243	11110249	0.31	✔
55	1999Q3	11457188	11454126	11459477	11451100	11436256	11434396	11432061	11430857	11441739	11411074	0.42	✔
56	1999Q4	11281972	11285622	11277230	11264097	11261534	11259940	11262042	11268255	11276798	11284452	0.22	✔
57	2000Q1	11494059	11507451	11485750	11508383	11513379	11519414	11525910	11526563	11511805	11553853	0.59	✔
58	2000Q2	11643278	11632740	11657247	11654598	11667473	11664318	11658211	11648493	11652895	11639423	0.29	✔
59	2000Q3	11901457	11894544	11905863	11893691	11870504	11869387	11866308	11868459	11872708	11835771	0.59	✔
60	2000Q4	11533435	11537581	11525719	11517630	11517971	11515790	11518497	11530544	11530395	11533646	0.18	✔
61	2001Q1	NA	11902050	11874061	11911608	11918636	11926718	11933832	11926179	11920746	11960113	0.72	✔
62	2001Q2	NA	12222935	12252473	12225128	12246077	12241751	12235102	12229255	12233633	12224721	0.24	✔
63	2001Q3	NA	12185491	12199467	12191623	12154104	12152146	12147764	12149105	12150526	12140405	0.48	✔
64	2001Q4	NA	11865083	11851768	11858612	11858259	11856003	11860577	11872196	11872935	11854066	0.17	✔
65	2002Q1	NA	NA	12167992	12211279	12229678	12240718	12248468	12241111	12234750	12254241	0.70	✔
66	2002Q2	NA	NA	12447167	12393325	12411792	12404926	12395854	12390849	12397590	12383434	0.51	✔
67	2002Q3	NA	NA	12478997	12472944	12425289	12424040	12418489	12422518	12420045	12456229	0.48	✔
68	2002Q4	NA	NA	12218015	12252172	12254870	12249174	12257048	12260942	12262814	12235615	0.36	✔
69	2003Q1	NA	NA	NA	12640021	12669781	12688466	12696165	12693947	12688775	12724907	0.67	✔
70	2003Q2	NA	NA	NA	12906018	12920542	12907614	12900143	12893739	12903944	12871996	0.37	✔
71	2003Q3	NA	NA	NA	13073718	13001555	12997295	12979613	12988204	12980485	12994759	0.72	✔
72	2003Q4	NA	NA	NA	12559458	12566852	12563127	12587959	12588537	12583187	12575262	0.23	✔
73	2004Q1	NA	NA	NA	NA	13180697	13208899	13211170	13207209	13215132	13279014	0.74	✔
74	2004Q2	NA	NA	NA	NA	13508616	13484225	13474343	13464175	13479592	13433168	0.56	✔
75	2004Q3	NA	NA	NA	NA	13979696	13971866	13941394	13963890	13944186	13911983	0.48	✔
76	2004Q4	NA	NA	NA	NA	13598085	13599920	13650485	13649589	13624060	13638095	0.38	✔
77	2005Q1	NA	NA	NA	NA	NA	13995552	13986659	13971352	14002438	14057899	0.61	✔
78	2005Q2	NA	NA	NA	NA	NA	14377525	14367012	14348084	14380473	14326899	0.37	✔
79	2005Q3	NA	NA	NA	NA	NA	14676173	14631265	14679402	14631812	14578139	0.69	✔
80	2005Q4	NA	NA	NA	NA	NA	14219850	14298920	14307438	14251762	14321527	0.71	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14725780	14683613	14755523	14815346	0.89	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	15028461	14988515	15047779	14980962	0.44	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	15153350	15233455	15149521	15054287	1.19	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	15013410	15053024	14959445	15056328	0.64	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15453969	15566350	15641428	1.21	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	15716614	15818723	15773010	0.64	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15802547	15679224	15521658	1.80	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	15695663	15558195	15717386	1.02	✔
	Median	-	-	-	-	-	-	-	-	-	-	0.56	-
	Mean	-	-	-	-	-	-	-	-	-	-	0.58	0%

Source: Author's elaboration.

Table 8: Sliding spans of seasonally-adjusted series (log), Aggregation 2

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9475554	9476889	9475999	9476158	9472754	9472656	9472723	9472753	9472759	9517182	0.47	✔
41	1996Q1	9765788	9764614	9765421	9765786	9767795	9767697	9765366	9767799	9765395	9766632	0.03	✔
42	1996Q2	10236427	10235715	10236147	10235975	10236369	10236651	10237075	10236357	10237079	10201808	0.34	✔
43	1996Q3	10396111	10395913	10396085	10395318	10395735	10395780	10396563	10395744	10396525	10346313	0.48	✔
44	1996Q4	9995223	9998005	9996503	9996783	9993309	9992982	9993641	9993337	9993643	10060450	0.67	✔
45	1997Q1	10410582	10408336	10407353	10408399	10410925	10410697	10409623	10410922	10409632	10445133	0.36	✔
46	1997Q2	10829260	10827353	10831871	10831955	10833888	10834542	10834280	10833799	10834320	10801358	0.30	✔
47	1997Q3	11149930	11150680	11148814	11145931	11142616	11143263	11142862	11142670	11142831	11059753	0.82	✔
48	1997Q4	10771510	10776219	10774637	10776049	10774321	10772501	10774573	10774376	10774519	10881407	1.02	✔
49	1998Q1	10997838	10992104	10987848	10990343	10993244	10993771	10992908	10993424	10992907	11009201	0.19	✔
50	1998Q2	11420586	11419396	11429691	11428908	11432324	11433446	11432144	11432019	11432270	11398330	0.30	✔
51	1998Q3	11532500	11534203	11530875	11525545	11517339	11518454	11517428	11517264	11517434	11435480	0.86	✔
52	1998Q4	10670680	10673304	10671501	10674632	10675808	10672234	10675908	10675916	10675759	10738102	0.63	✔
53	1999Q1	10952200	10951020	10941846	10946460	10948512	10950542	10949281	10949359	10949127	10959591	0.16	✔
54	1999Q2	11101649	11095336	11109809	11106479	11115042	11115530	11113746	11113848	11114200	11095992	0.18	✔
55	1999Q3	11459593	11460240	11463787	11456223	11438890	11441001	11438971	11438838	11438952	11404930	0.51	✔
56	1999Q4	11209625	11214131	11203564	11210263	11215200	11209637	11215475	11215408	11215177	11266366	0.56	✔
57	2000Q1	11487581	11486896	11475697	11484879	11489008	11493157	11491606	11491753	11491043	11503778	0.24	✔
58	2000Q2	11630822	11622436	11640141	11628471	11637960	11638486	11635325	11635335	11636545	11611173	0.24	✔
59	2000Q3	11907153	11907443	11917590	11906908	11880567	11880544	11878748	11878728	11878545	11840647	0.64	✔
60	2000Q4	11503266	11509924	11491140	11507845	11514930	11510860	11517050	11516908	11516489	11563667	0.63	✔
61	2001Q1	NA	11901914	11889485	11904519	11914660	11919998	11919922	11920019	11918806	11939152	0.41	✔
62	2001Q2	NA	12223758	12244737	12215955	12229475	12229865	12224832	12224867	12227135	12200022	0.36	✔
63	2001Q3	NA	12207239	12222878	12211040	12170324	12167507	12165567	12165845	12165224	12153044	0.57	✔
64	2001Q4	NA	11842350	11815273	11850532	11858583	11856903	11863212	11863057	11862309	11881053	0.55	✔
65	2002Q1	NA	NA	12176483	12194316	12220976	12227042	12229532	12229037	12227864	12224165	0.43	✔
66	2002Q2	NA	NA	12439247	12386268	12390075	12389800	12381661	12381761	12384887	12354910	0.68	✔
67	2002Q3	NA	NA	12515852	12506705	12455771	12450509	12448648	12449818	12448511	12477216	0.54	✔
68	2002Q4	NA	NA	12211120	12269549	12283211	12279698	12286935	12287114	12285627	12286761	0.62	✔
69	2003Q1	NA	NA	NA	12638135	12681871	12695929	12700510	12698325	12698050	12708404	0.55	✔
70	2003Q2	NA	NA	NA	12913780	12904126	12899807	12892098	12890641	12895601	12862344	0.39	✔
71	2003Q3	NA	NA	NA	13072584	13002271	12993584	12979514	12986373	12982315	13001300	0.71	✔
72	2003Q4	NA	NA	NA	12536427	12565431	12562843	12585863	12583516	12580088	12588640	0.41	✔
73	2004Q1	NA	NA	NA	NA	13209068	13232857	13235291	13232766	13235783	13264790	0.42	✔
74	2004Q2	NA	NA	NA	NA	13486500	13477030	13465336	13457480	13467153	13435819	0.37	✔
75	2004Q3	NA	NA	NA	NA	13980237	13962295	13930883	13952074	13938404	13912813	0.48	✔
76	2004Q4	NA	NA	NA	NA	13573702	13573907	13626389	13617263	13611090	13638926	0.48	✔
77	2005Q1	NA	NA	NA	NA	NA	14031823	14025598	14024504	14032889	14052386	0.19	✔
78	2005Q2	NA	NA	NA	NA	NA	14380963	14366011	14345434	14371031	14339282	0.29	✔
79	2005Q3	NA	NA	NA	NA	NA	14675365	14620622	14662514	14627952	14576963	0.67	✔
80	2005Q4	NA	NA	NA	NA	NA	14166866	14258325	14242761	14220117	14298118	0.92	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14776980	14777059	14807831	14828222	0.34	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	15034462	15001334	15048827	15007670	0.31	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	15130083	15188490	15117338	15038128	0.99	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	14963876	14948684	14904522	15015321	0.74	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15596018	15663246	15678915	0.53	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	15763067	15845078	15798600	0.52	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15735813	15618342	15503884	1.49	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	15548266	15472891	15661162	1.21	✔
	Median	-	-	-	-	-	-	-	-	-	-	0.49	-
	Mean	-	-	-	-	-	-	-	-	-	-	0.53	0%

Source: Author's elaboration.

Table 9: Sliding spans of seasonally-adjusted series (log), Aggregation 3

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9784890	9877900	9737399	9684133	9692566	9692990	9679761	9721569	9692488	9788268	2.04	✔
41	1996Q1	9869648	9948451	9793201	9743273	9792409	9793267	9746080	9844388	9813035	9948036	2.10	✔
42	1996Q2	10211415	10081169	10209265	10332207	10265593	10264744	10333043	10199216	10249217	10065737	2.65	✔
43	1996Q3	10105288	10062047	10244235	10219277	10239635	10238773	10220468	10216085	10234505	10169449	1.81	✔
44	1996Q4	10376509	10480239	10324303	10262291	10267703	10268556	10256713	10322484	10270154	10358481	2.17	✔
45	1997Q1	10382834	10433268	10273890	10267005	10275859	10277925	10267395	10301878	10295921	10452047	1.80	✔
46	1997Q2	10794641	10692053	10829290	10909161	10892790	10889526	10916684	10828267	10871736	10704092	2.10	✔
47	1997Q3	10872165	10827857	10999595	10980942	10987906	10988148	10978752	10980380	10987937	10937375	1.57	✔
48	1997Q4	11209599	11308600	11155243	11101305	11100688	11102078	11093613	11158650	11104078	11167530	1.97	✔
49	1998Q1	11097971	11124261	10980956	10990970	10983896	10986629	10989137	10977499	11000832	11128135	1.37	✔
50	1998Q2	11382657	11303187	11444653	11495161	11510211	11504272	11510691	11463815	11484971	11341759	1.83	✔
51	1998Q3	11256911	11236851	11354070	11339368	11330578	11332193	11334051	11346204	11339495	11315403	1.04	✔
52	1998Q4	10957209	11021328	10899486	10855617	10853120	10855913	10847197	10897437	10860592	10900707	1.60	✔
53	1999Q1	10888882	10905184	10808824	10838643	10823603	10827410	10823555	10769271	10819444	10909304	1.30	✔
54	1999Q2	10977213	10915699	11027687	11037145	11079934	11068597	11077529	11088409	11069038	10945766	1.58	✔
55	1999Q3	11200723	11204638	11271776	11267340	11239370	11240925	11252721	11270711	11251520	11260384	0.63	✔
56	1999Q4	11502459	11531061	11433248	11409647	11402204	11413039	11396078	11396710	11408162	11430707	1.18	✔
57	2000Q1	11442744	11457424	11419737	11450122	11433480	11436362	11418398	11387104	11421447	11516074	1.13	✔
58	2000Q2	11548012	11495183	11574416	11554374	11618965	11601352	11631186	11650244	11610949	11486217	1.42	✔
59	2000Q3	11701448	11734193	11754573	11764166	11720929	11715850	11735169	11755594	11740683	11728734	0.53	✔
60	2000Q4	11866240	11859668	11786227	11774379	11757588	11780921	11752428	11725909	11764865	11799758	1.19	✔
61	2001Q1	NA	11842029	11845026	11879681	11866418	11882519	11822453	11825672	11830996	11962299	1.18	✔
62	2001Q2	NA	12127492	12181896	12134269	12219841	12184455	12269289	12269032	12235181	12079869	1.56	✔
63	2001Q3	NA	12047453	12056457	12072293	12022594	11990339	12028625	12042750	12041338	12013699	0.68	✔
64	2001Q4	NA	12154280	12078449	12088098	12043156	12101990	12034517	12009493	12050462	12100752	1.20	✔
65	2002Q1	NA	NA	12011396	12042959	12045760	12087933	11976962	12027590	11987188	12160176	1.52	✔
66	2002Q2	NA	NA	12397637	12330307	12431854	12355521	12518333	12450031	12470513	12240801	2.26	✔
67	2002Q3	NA	NA	12292195	12312439	12252823	12198415	12255389	12287392	12278142	12252966	0.93	✔
68	2002Q4	NA	NA	12492221	12522253	12457610	12544247	12429999	12426948	12449938	12528409	0.94	✔
69	2003Q1	NA	NA	NA	12409318	12433306	12521432	12353525	12420459	12363688	12611945	2.09	✔
70	2003Q2	NA	NA	NA	12900974	13005804	12878656	13129189	12994814	13058434	12749671	2.97	✔
71	2003Q3	NA	NA	NA	12999204	12928388	12841019	12917103	12989689	12967353	12914615	1.23	✔
72	2003Q4	NA	NA	NA	12873337	12799356	12931404	12750767	12765923	12772882	12877488	1.41	✔
73	2004Q1	NA	NA	NA	NA	12897513	13020054	12845560	12895423	12836338	13168487	2.58	✔
74	2004Q2	NA	NA	NA	NA	13667802	13495961	13788650	13623431	13702861	13334244	3.40	✗
75	2004Q3	NA	NA	NA	NA	13866179	13753868	13835907	13951049	13934051	13834652	1.43	✔
76	2004Q4	NA	NA	NA	NA	13853133	14014638	13805301	13822667	13807675	13912385	1.51	✔
77	2005Q1	NA	NA	NA	NA	NA	13766787	13586020	13601165	13561578	13944902	2.82	✔
78	2005Q2	NA	NA	NA	NA	NA	14432813	14746849	14594491	14652703	14242425	3.54	✗
79	2005Q3	NA	NA	NA	NA	NA	14399222	14488679	14618237	14618008	14534572	1.52	✔
80	2005Q4	NA	NA	NA	NA	NA	14696392	14466124	14486611	14466242	14551789	1.59	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14312980	14292111	14260415	14689733	3.01	✗
82	2006Q2	NA	NA	NA	NA	NA	NA	15434187	15307233	15350107	14914635	3.48	✗
83	2006Q3	NA	NA	NA	NA	NA	NA	14983619	15133510	15114511	15052860	1.00	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	15189791	15187331	15200839	15246085	0.38	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15053720	15032173	15498770	3.10	✗
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	16122422	16138103	15706764	2.74	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15667626	15625284	15581732	0.55	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	15819122	15879595	15871125	0.38	✔
	Median	-	-	-	-	-	-	-	-	-	-	1.57	-
	Mean	-	-	-	-	-	-	-	-	-	-	1.72	10%

Source: Author's elaboration.

Table 10: Sliding spans of seasonally-adjusted series (log), Aggregation 4

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9777058	9776110	9776307	9776346	9776062	9776869	9776912	9776888	9776887	9774155	0.02	✔
41	1996Q1	10036706	10037501	10037618	10037644	10037857	10040403	10040375	10040424	10040412	10043484	0.06	✔
42	1996Q2	10163658	10163197	10163137	10163085	10163396	10160089	10160044	10160019	10160021	10164242	0.04	✔
43	1996Q3	10246190	10247284	10246824	10246734	10246529	10246975	10246957	10246971	10246990	10244929	0.02	✔
44	1996Q4	10475446	10474030	10474347	10474450	10473848	10473737	10473883	10473855	10473844	10469493	0.05	✔
45	1997Q1	10510997	10511741	10512233	10512397	10512780	10517220	10517138	10517145	10517133	10519841	0.08	✔
46	1997Q2	10803348	10801840	10801599	10801272	10801997	10795625	10795538	10795534	10795538	10805365	0.09	✔
47	1997Q3	11002639	11005326	11004495	11004488	11003963	11006255	11006234	11006295	11006322	10996382	0.09	✔
48	1997Q4	11275247	11274182	11274667	11274869	11273812	11274805	11275074	11275038	11275012	11275626	0.01	✔
49	1998Q1	11210874	11211773	11212423	11212724	11213449	11214568	11214479	11214399	11214388	11214571	0.03	✔
50	1998Q2	11413668	11407171	11407571	11406754	11408378	11404853	11404508	11404507	11404546	11415914	0.10	✔
51	1998Q3	11339740	11347831	11345374	11345511	11343511	11345775	11345902	11346125	11346106	11337789	0.08	✔
52	1998Q4	10999366	10999279	11000940	11001620	11000814	11000680	11001181	11001083	11001024	10995453	0.05	✔
53	1999Q1	10963304	10963322	10963673	10963870	10965269	10964923	10964661	10964496	10964563	10968563	0.04	✔
54	1999Q2	10992442	10979609	10979600	10977965	10980305	10980130	10979453	10979418	10979524	10977364	0.13	✔
55	1999Q3	11230481	11246133	11243529	11244028	11239839	11241267	11241663	11242053	11241863	11244513	0.13	✔
56	1999Q4	11513310	11514127	11518066	11519866	11518960	11517750	11518708	11518633	11518501	11517344	0.05	✔
57	2000Q1	11555726	11554634	11552111	11551932	11555932	11553061	11552584	11552273	11552558	11552953	0.03	✔
58	2000Q2	11550057	11527424	11526907	11523975	11527204	11532415	11530819	11530749	11531055	11529308	0.22	✔
59	2000Q3	11682520	11708446	11707854	11707265	11697680	11696542	11697574	11697726	11697239	11697531	0.22	✔
60	2000Q4	11852297	11853488	11860035	11866563	11866373	11864291	11866108	11866857	11866066	11868570	0.13	✔
61	2001Q1	NA	11982821	11976426	11974161	11982560	11980678	11980275	11979594	11981014	11980773	0.07	✔
62	2001Q2	NA	12095004	12091394	12085030	12091828	12096495	12092110	12091366	12091721	12089501	0.09	✔
63	2001Q3	NA	12019232	12023701	12023211	12002907	12001359	12004448	12004898	12003718	12006125	0.18	✔
64	2001Q4	NA	12120022	12129136	12142580	12141689	12142132	12145346	12146714	12145420	12149308	0.24	✔
65	2002Q1	NA	NA	12178522	12172914	12191579	12189525	12188481	12188259	12190591	12191332	0.15	✔
66	2002Q2	NA	NA	12253287	12242466	12251761	12251643	12243749	12240947	12241921	12233205	0.16	✔
67	2002Q3	NA	NA	12314100	12314013	12278948	12279161	12285296	12286480	12284195	12290588	0.28	✔
68	2002Q4	NA	NA	12527802	12550418	12549059	12553579	12559183	12562296	12560421	12565550	0.30	✔
69	2003Q1	NA	NA	NA	12664117	12696287	12694105	12691289	12691956	12695423	12697096	0.26	✔
70	2003Q2	NA	NA	NA	12745533	12757606	12751795	12737982	12729398	12731124	12718332	0.30	✔
71	2003Q3	NA	NA	NA	12866501	12815173	12816647	12828950	12833655	12829742	12836518	0.40	✔
72	2003Q4	NA	NA	NA	12889762	12887644	12895486	12905794	12911698	12909868	12914517	0.20	✔
73	2004Q1	NA	NA	NA	NA	13231618	13231481	13222204	13224046	13227423	13230240	0.07	✔
74	2004Q2	NA	NA	NA	NA	13376573	13366098	13345618	13323920	13328958	13311750	0.48	✔
75	2004Q3	NA	NA	NA	NA	13728513	13730722	13753879	13769491	13760852	13760123	0.29	✔
76	2004Q4	NA	NA	NA	NA	13912587	13920092	13936227	13946991	13945087	13953524	0.29	✔
77	2005Q1	NA	NA	NA	NA	NA	14004528	13986080	13986613	13993327	13996904	0.13	✔
78	2005Q2	NA	NA	NA	NA	NA	14290673	14260388	14219903	14227152	14220303	0.49	✔
79	2005Q3	NA	NA	NA	NA	NA	14415009	14452726	14488249	14472074	14469320	0.50	✔
80	2005Q4	NA	NA	NA	NA	NA	14545915	14567411	14582379	14580403	14581960	0.25	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14734891	14731644	14743521	14749139	0.11	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	14943265	14878754	14892145	14892645	0.43	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	14962244	15021234	14992164	14986549	0.39	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	15254172	15274316	15272431	15265971	0.13	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15544610	15563413	15570972	0.16	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	15640815	15660432	15685862	0.28	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15570141	15528127	15514445	0.35	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	15896483	15894344	15872748	0.14	✔
	Median	-	-	-	-	-	-	-	-	-	-	0.14	-
	Mean	-	-	-	-	-	-	-	-	-	-	0.18	0%

Source: Author's elaboration.

Table 11: Sliding spans of seasonally-adjusted series (log), Aggregation 5

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9814324	9814865	9810916	9815831	9810214	9813908	9814163	9814172	9813451	9812328	0.18	✔
41	1996Q1	10077835	10078375	10075299	10074822	10074676	10077317	10077134	10077141	10077021	10066981	0.24	✔
42	1996Q2	10117524	10114961	10117812	10116905	10118298	10117146	10117114	10117081	10117577	10117660	0.03	✔
43	1996Q3	10216379	10217772	10221233	10218002	10221790	10217901	10217864	10217890	10218342	10230828	0.03	✔
44	1996Q4	10419564	10420681	10417732	10422389	10417337	10419166	10419331	10419336	10418547	10413606	0.02	✔
45	1997Q1	10545865	10546226	10542005	10541534	10541291	10544748	10544589	10544605	10544380	10529728	0.03	✔
46	1997Q2	10777193	10773646	10776592	10775552	10777330	10775865	10775844	10775811	10776426	10788733	0.02	✔
47	1997Q3	10976250	10977494	10982317	10978595	10982427	10979621	10979640	10979663	10980220	10986026	0.04	✔
48	1997Q4	11226322	11229955	11226398	11231584	11226373	11226591	11226688	11226711	11225830	11222795	0.03	✔
49	1998Q1	11218441	11218073	11212394	11212311	11211358	11215223	11215105	11215127	11214815	11205017	0.07	✔
50	1998Q2	11384524	11377026	11379807	11378682	11380504	11380038	11379958	11379875	11380405	11394535	0.05	✔
51	1998Q3	11322708	11326694	11334090	11329056	11334341	11331679	11331774	11331814	11332660	11332186	0.26	✔
52	1998Q4	10975439	10981492	10978219	10984188	10978489	10977364	10977456	10977562	10976688	10959965	0.31	✔
53	1999Q1	10987867	10986981	10977705	10977768	10976714	10977585	10977492	10977486	10977120	10976404	0.10	✔
54	1999Q2	10994901	10982571	10984511	10983700	10984564	10989296	10989033	10988819	10989166	10998738	0.03	✔
55	1999Q3	11229821	11237054	11250014	11243423	11249893	11248145	11248377	11248412	11249472	11258824	0.15	✔
56	1999Q4	11491622	11502623	11499463	11508208	11502590	11496572	11496860	11497246	11496272	11488844	0.11	✔
57	2000Q1	11579298	11577947	11559974	11557655	11556368	11554797	11554671	11554676	11554406	11549052	0.11	✔
58	2000Q2	11548404	11527261	11531162	11529031	11529251	11541615	11540754	11540061	11540174	11543845	0.06	✔
59	2000Q3	11676001	11688134	11707000	11703208	11709696	11706747	11707208	11707358	11708524	11716195	0.26	✔
60	2000Q4	11847457	11861756	11861600	11873280	11868852	11857724	11859016	11859758	11858825	11856670	0.44	✔
61	2001Q1	NA	11996274	11967904	11959120	11960531	11956949	11956110	11956522	11956376	11952128	0.19	✔
62	2001Q2	NA	12084772	12088243	12086058	12083395	12105912	12104074	12101985	12102071	12100601	0.23	✔
63	2001Q3	NA	12000603	12025943	12025619	12032217	12023658	12024426	12025097	12025951	12034597	0.40	✔
64	2001Q4	NA	12131601	12136603	12152706	12150800	12137183	12140416	12141490	12140848	12140629	0.18	✔
65	2002Q1	NA	NA	12175393	12161382	12166518	12161886	12159940	12161233	12160945	12157705	0.21	✔
66	2002Q2	NA	NA	12251056	12243817	12239266	12273197	12269103	12264373	12264919	12263736	0.18	✔
67	2002Q3	NA	NA	12317629	12324468	12332454	12310621	12312740	12314706	12314945	12327183	0.27	✔
68	2002Q4	NA	NA	12554778	12578430	12578127	12568752	12575548	12577840	12577282	12573706	1.08	✔
69	2003Q1	NA	NA	NA	12725795	12737786	12733354	12728768	12729916	12729895	12725358	0.34	✔
70	2003Q2	NA	NA	NA	12734254	12727449	12767820	12758833	12749457	12750181	12747059	0.28	✔
71	2003Q3	NA	NA	NA	12821432	12828276	12789664	12796636	12803499	12802843	12818507	0.27	✔
72	2003Q4	NA	NA	NA	12853617	12853051	12851461	12863143	12867203	12867096	12855971	0.33	✔
73	2004Q1	NA	NA	NA	NA	13255797	13255312	13245320	13244064	13244769	13242238	0.21	✔
74	2004Q2	NA	NA	NA	NA	13338937	13380105	13362618	13344409	13344585	13343889	0.39	✔
75	2004Q3	NA	NA	NA	NA	13754483	13701842	13721200	13739617	13736976	13749083	0.35	✔
76	2004Q4	NA	NA	NA	NA	13889853	13895691	13914261	13923719	13925120	13925911	0.13	✔
77	2005Q1	NA	NA	NA	NA	NA	14025257	14005787	13996826	14000316	13993198	0.30	✔
78	2005Q2	NA	NA	NA	NA	NA	14291918	14264493	14230050	14227822	14227656	0.19	✔
79	2005Q3	NA	NA	NA	NA	NA	14408544	14440207	14483451	14474916	14486179	0.24	✔
80	2005Q4	NA	NA	NA	NA	NA	14522385	14548898	14561311	14568453	14564301	0.25	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14750777	14731683	14739331	14731581	0.17	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	14935391	14881415	14875862	14876227	0.22	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	14953722	15025314	15007003	15023508	0.09	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	15244767	15261148	15276114	15263293	0.28	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15561812	15578619	15557005	0.17	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	15646733	15633892	15657516	0.03	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15561013	15533051	15558381	0.13	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	15875357	15898222	15867861	0.23	✔
	Median	-	-	-	-	-	-	-	-	-	-	0.18	-
	Mean	-	-	-	-	-	-	-	-	-	-	0.21	0%

Source: Author's elaboration.

Table 12: Sliding spans of seasonally-adjusted series (log), Aggregation 6

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9790734	9790568	9790610	9790727	9790775	9790882	9790758	9790757	9790766	9779914	0.11	✔
41	1996Q1	9985411	9985831	9985759	9985563	9985475	9985450	9985528	9985530	9985525	9998679	0.13	✔
42	1996Q2	10106112	10105669	10105725	10105877	10105945	10106182	10105939	10105934	10105951	10113108	0.07	✔
43	1996Q3	10278389	10278609	10278589	10278470	10278406	10277911	10278360	10278373	10278341	10267564	0.10	✔
44	1996Q4	10442456	10442716	10442752	10442914	10442953	10443083	10442930	10442938	10442949	10434278	0.08	✔
45	1997Q1	10458040	10458000	10457874	10457676	10457684	10457958	10457779	10457742	10457755	10468134	0.10	✔
46	1997Q2	10760346	10759094	10759238	10759445	10759537	10759877	10759527	10759523	10759547	10769302	0.09	✔
47	1997Q3	11027278	11028204	11028179	11027932	11027694	11026600	11027606	11027663	11027594	11008441	0.17	✔
48	1997Q4	11260979	11262967	11262988	11263217	11263209	11263260	11263178	11263215	11263222	11261551	0.02	✔
49	1998Q1	11184203	11183560	11183307	11183196	11183420	11184300	11183557	11183470	11183512	11192770	0.08	✔
50	1998Q2	11402917	11396828	11397201	11397375	11397834	11398924	11397898	11397732	11397818	11407378	0.09	✔
51	1998Q3	11368946	11374818	11374641	11374007	11372746	11369645	11372455	11372772	11372560	11357748	0.15	✔
52	1998Q4	10969956	10973618	10973690	10974288	10974612	10975126	10974691	10974735	10974788	10971999	0.04	✔
53	1999Q1	10924986	10923219	10922929	10923269	10924011	10925597	10924190	10924082	10924166	10925238	0.02	✔
54	1999Q2	10969287	10956132	10956237	10955788	10956512	10958908	10956565	10956120	10956256	10953372	0.14	✔
55	1999Q3	11228249	11241632	11242922	11241396	11238656	11232068	11238177	11238596	11238216	11239723	0.13	✔
56	1999Q4	11483595	11489693	11487078	11488928	11489434	11491425	11489708	11490098	11490201	11491030	0.06	✔
57	2000Q1	11530508	11527563	11529915	11531696	11534017	11536556	11534332	11534328	11534511	11535845	0.07	✔
58	2000Q2	11544508	11520422	11518218	11516052	11517401	11520971	11517290	11516254	11516446	11513992	0.26	✔
59	2000Q3	11687709	11711959	11715817	11710989	11704396	11693652	11703340	11703365	11702739	11701800	0.24	✔
60	2000Q4	11836657	11843901	11838701	11844261	11846323	11850601	11847612	11848611	11848757	11851572	0.12	✔
61	2001Q1	NA	11969503	11975521	11981305	11984294	11987722	11984497	11986581	11986932	11988830	0.16	✔
62	2001Q2	NA	12074532	12067546	12059967	12066074	12069775	12065191	12060604	12060915	12058268	0.13	✔
63	2001Q3	NA	12019234	12027192	12017283	12002089	11988266	12000687	12001717	12000715	12001768	0.32	✔
64	2001Q4	NA	12100815	12092677	12105446	12108047	12114500	12110775	12111478	12111529	12114044	0.18	✔
65	2002Q1	NA	NA	12170034	12180462	12188472	12193629	12189510	12195998	12196897	12199643	0.24	✔
66	2002Q2	NA	NA	12227379	12212246	12223126	12224608	12219043	12208748	12208933	12200348	0.22	✔
67	2002Q3	NA	NA	12319780	12304565	12276012	12259926	12274392	12276611	12275129	12280462	0.48	✔
68	2002Q4	NA	NA	12491573	12512893	12517005	12526515	12523977	12525394	12525524	12528362	0.29	✔
69	2003Q1	NA	NA	NA	12608177	12624622	12633461	12627373	12639027	12640826	12645064	0.29	✔
70	2003Q2	NA	NA	NA	12776309	12791603	12787565	12778046	12759075	12758695	12748638	0.33	✔
71	2003Q3	NA	NA	NA	12897399	12853107	12834804	12855136	12860076	12857650	12861924	0.48	✔
72	2003Q4	NA	NA	NA	12880074	12884632	12897340	12896979	12901065	12901651	12902339	0.17	✔
73	2004Q1	NA	NA	NA	NA	13167654	13183165	13172986	13188453	13192015	13196371	0.21	✔
74	2004Q2	NA	NA	NA	NA	13405248	13392377	13377167	13346182	13345111	13336448	0.51	✔
75	2004Q3	NA	NA	NA	NA	13763231	13742365	13771061	13782679	13776674	13772780	0.29	✔
76	2004Q4	NA	NA	NA	NA	13911820	13929184	13933014	13941614	13944233	13946173	0.24	✔
77	2005Q1	NA	NA	NA	NA	NA	13962511	13947192	13965393	13973583	13975284	0.20	✔
78	2005Q2	NA	NA	NA	NA	NA	14314096	14289692	14239972	14237003	14242110	0.54	✔
79	2005Q3	NA	NA	NA	NA	NA	14423901	14462975	14488836	14473999	14466139	0.45	✔
80	2005Q4	NA	NA	NA	NA	NA	14558284	14565936	14579552	14587131	14582511	0.19	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14705598	14722891	14738453	14735671	0.22	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	14963718	14894332	14889317	14914793	0.49	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	14966075	15008848	14981564	14968422	0.28	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	15255611	15275483	15289909	15273984	0.22	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15544979	15569765	15564517	0.15	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	15654921	15646264	15703282	0.36	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15557102	15517540	15491721	0.42	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	15892978	15914556	15884665	0.18	✔
	Median	-	-	-	-	-	-	-	-	-	-	0.19	-
	Mean	-	-	-	-	-	-	-	-	-	-	0.22	0%

Source: Author’s elaboration.

3 *Sliding spans of trend-cycle series (log)*

Table 13: Sliding spans of trend-cycle series (log), Aggregation 1

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9288531	9290559	9289505	9350400	9347611	9348911	9334794	9327336	9342823	9339026	0.66	✔
41	1996Q1	9970126	9971909	9971905	9997870	9997793	10000473	9991926	10018237	9999351	9996571	0.48	✔
42	1996Q2	10170395	10167577	10168640	10162805	10165280	10165260	10165514	10192332	10168527	10169146	0.29	✔
43	1996Q3	10425950	10418987	10419148	10418647	10418572	10415620	10414911	10404818	10418776	10421169	0.20	✔
44	1996Q4	10268932	10267375	10268214	10266224	10266418	10266872	10269619	10121325	10276570	10280295	1.57	✔
45	1997Q1	10340455	10340774	10344752	10344533	10347405	10349933	10350513	10491336	10335633	10332115	1.54	✔
46	1997Q2	10880507	10877914	10881788	10888009	10891358	10891308	10888918	10890888	10727365	10719479	1.60	✔
47	1997Q3	11108747	11105692	11107453	11104838	11106120	11102597	11099368	11074714	11246616	11250365	1.58	✔
48	1997Q4	10923672	10928091	10924862	10917767	10904048	10907495	10915337	10904827	10908208	10915988	0.22	✔
49	1998Q1	11224088	11224352	11223018	11228179	11198671	11202911	11224476	11239213	11194019	11192800	0.41	✔
50	1998Q2	11307330	11299886	11303213	11317511	11291637	11293997	11308135	11319743	11300236	11292136	0.24	✔
51	1998Q3	11587650	11581735	11585385	11583102	11623303	11624512	11572257	11566190	11589563	11588104	0.50	✔
52	1998Q4	10744519	10745667	10737493	10728893	10699349	10704301	10728416	10730804	10743735	10750265	0.47	✔
53	1999Q1	11006047	11006737	11001257	11006926	11021154	11022181	11016511	11022719	11012278	11013570	0.19	✔
54	1999Q2	11126593	11125344	11131097	11139137	11142559	11142306	11139788	11138026	11130692	11127464	0.15	✔
55	1999Q3	11500918	11498155	11506503	11496984	11491397	11489070	11485788	11483053	11495215	11492980	0.20	✔
56	1999Q4	11122490	11123618	11119832	11114315	11112250	11113893	11117101	11121720	11125078	11126897	0.13	✔
57	2000Q1	11492677	11510314	11472437	11477324	11469388	11472404	11475382	11474572	11468782	11471621	0.36	✔
58	2000Q2	11828979	11845401	11817736	11831593	11824882	11825462	11823760	11812875	11811548	11810954	0.29	✔
59	2000Q3	11718797	11714949	11720148	11743115	11737439	11730208	11725727	11728877	11733637	11731132	0.24	✔
60	2000Q4	11511738	11528510	11553947	11524695	11507686	11493662	11486302	11579183	11577627	11576735	0.80	✔
61	2001Q1	NA	11991596	12011279	12080364	12084027	12093492	12108716	12021071	12012606	12012796	0.97	✔
62	2001Q2	NA	12182826	12209762	12183819	12194216	12196063	12193323	12198471	12194956	12194600	0.22	✔
63	2001Q3	NA	12204925	12216720	12188849	12184331	12178509	12168860	12192773	12197982	12197942	0.39	✔
64	2001Q4	NA	11851468	11832072	11832622	11833277	11827761	11829719	11841410	11843470	11843191	0.20	✔
65	2002Q1	NA	NA	12191157	12207208	12201094	12196162	12203968	12200416	12196250	12198060	0.13	✔
66	2002Q2	NA	NA	12294971	12296850	12265923	12253151	12252433	12244356	12242119	12243424	0.44	✔
67	2002Q3	NA	NA	12616048	12643244	12607164	12590679	12583805	12581283	12578260	12579600	0.51	✔
68	2002Q4	NA	NA	12151344	12194377	12184048	12177935	12174308	12182889	12181966	12183491	0.35	✔
69	2003Q1	NA	NA	NA	12681809	12701399	12706705	12705750	12711815	12721782	12723957	0.33	✔
70	2003Q2	NA	NA	NA	12865235	12862374	12868528	12867775	12868481	12875576	12877767	0.11	✔
71	2003Q3	NA	NA	NA	12950839	12906618	12909468	12915018	12916350	12909763	12909221	0.34	✔
72	2003Q4	NA	NA	NA	12655320	12657110	12649855	12660924	12662496	12658810	12657589	0.09	✔
73	2004Q1	NA	NA	NA	NA	13181325	13161219	13165059	13155639	13160267	13164027	0.19	✔
74	2004Q2	NA	NA	NA	NA	13501964	13484970	13464702	13464500	13460296	13462298	0.30	✔
75	2004Q3	NA	NA	NA	NA	13973046	13963518	13960377	13971053	13948096	13940588	0.23	✔
76	2004Q4	NA	NA	NA	NA	13572269	13583517	13604043	13608389	13591061	13589526	0.26	✔
77	2005Q1	NA	NA	NA	NA	NA	14076865	14088961	14070531	14086982	14101732	0.22	✔
78	2005Q2	NA	NA	NA	NA	NA	14292796	14267498	14264894	14280399	14283984	0.19	✔
79	2005Q3	NA	NA	NA	NA	NA	14733266	14736035	14772601	14731986	14708959	0.43	✔
80	2005Q4	NA	NA	NA	NA	NA	14116378	14149126	14165925	14138561	14130710	0.35	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14827718	14782399	14833136	14869196	0.58	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	15064936	15012569	15066145	15088445	0.50	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	15023498	15040084	15003939	14982035	0.38	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	15150231	15201312	15170374	15173852	0.33	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15423995	15490249	15574409	0.97	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	15635196	15762642	15808790	1.11	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15874616	15819665	15716959	1.00	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	16098259	15747199	15622227	3.04	✘
	Median	-	-	-	-	-	-	-	-	-	-	0.35	-
	Mean	-	-	-	-	-	-	-	-	-	-	0.54	2%

Source: Author's elaboration.

Table 14: Sliding spans of trend-cycle series (log), Aggregation 2

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9266144	9266763	9265844	9265782	9271575	9271545	9273788	9271569	9273838	9271572	0.08	✔
41	1996Q1	9966433	9966643	9965592	9965421	9974833	9974958	9975997	9974810	9976034	9974812	0.10	✔
42	1996Q2	10178310	10177761	10177662	10177297	10181114	10181354	10182492	10181105	10182492	10181104	0.05	✔
43	1996Q3	10399289	10401684	10401806	10401539	10404693	10404714	10405384	10404722	10405352	10404717	0.05	✔
44	1996Q4	10256915	10260481	10259288	10259594	10267701	10267390	10267662	10267727	10267653	10267717	0.10	✔
45	1997Q1	10355064	10355140	10355688	10356514	10362105	10362010	10361504	10362082	10361541	10362081	0.06	✔
46	1997Q2	10884198	10881447	10883235	10882483	10882059	10882697	10881940	10882035	10881973	10882059	0.02	✔
47	1997Q3	11112062	11115402	11115844	11114345	11112262	11112284	11112731	11112292	11112703	11112297	0.03	✔
48	1997Q4	10914273	10920060	10916105	10917047	10915522	10914746	10915618	10915644	10915555	10915609	0.05	✔
49	1998Q1	11210608	11213638	11213434	11215896	11218745	11218713	11218547	11218758	11218563	11218748	0.07	✔
50	1998Q2	11305042	11308775	11312121	11310891	11311119	11312602	11310915	11310952	11310997	11311017	0.06	✔
51	1998Q3	11587597	11578026	11579535	11575881	11572705	11572481	11572678	11572527	11572700	11572571	0.13	✔
52	1998Q4	10717777	10696729	10688952	10687689	10686104	10685172	10686591	10686610	10686491	10686509	0.30	✔
53	1999Q1	10986860	10973228	10971599	10973491	10980171	10979591	10980512	10980596	10980604	10980560	0.13	✔
54	1999Q2	11122300	11120894	11130646	11129050	11131316	11130870	11131093	11131090	11131370	11131251	0.09	✔
55	1999Q3	11512978	11514069	11515794	11511462	11500365	11504588	11499915	11499763	11500006	11499919	0.13	✔
56	1999Q4	11139029	11143597	11133198	11135561	11122309	11131799	11123477	11123360	11123017	11123023	0.19	✔
57	2000Q1	11481949	11488099	11446381	11446765	11444801	11451410	11445973	11445975	11445876	11445880	0.37	✔
58	2000Q2	11788209	11794896	11753311	11757871	11781682	11781239	11779048	11779244	11779852	11779744	0.35	✔
59	2000Q3	11670122	11674381	11659812	11679308	11715303	11711356	11711368	11711555	11712137	11712031	0.47	✔
60	2000Q4	11525130	11539543	11540468	11561705	11583263	11580449	11584247	11584262	11583476	11583443	0.51	✔
61	2001Q1	NA	12030673	12024619	12020385	12027534	12028512	12029517	12029392	12029191	12029103	0.08	✔
62	2001Q2	NA	12218802	12236672	12216057	12209443	12210089	12205970	12206071	12206841	12206652	0.25	✔
63	2001Q3	NA	12209036	12217720	12213925	12194237	12193288	12191161	12191492	12191567	12191358	0.21	✔
64	2001Q4	NA	11832647	11819064	11841673	11844186	11838220	11846059	11845318	11845214	11845587	0.22	✔
65	2002Q1	NA	NA	12195535	12208186	12197034	12187132	12202361	12200496	12200673	12201667	0.17	✔
66	2002Q2	NA	NA	12320591	12308190	12258537	12242965	12255903	12254099	12253165	12253139	0.63	✔
67	2002Q3	NA	NA	12636861	12652596	12619668	12602553	12608836	12608051	12606229	12605039	0.39	✔
68	2002Q4	NA	NA	12157940	12187673	12208771	12202531	12205363	12204387	12204053	12203477	0.41	✔
69	2003Q1	NA	NA	NA	12671075	12713069	12720763	12729622	12727322	12724300	12726139	0.46	✔
70	2003Q2	NA	NA	NA	12837759	12849390	12872953	12888552	12889263	12882058	12885189	0.40	✔
71	2003Q3	NA	NA	NA	12932522	12884607	12908615	12928888	12932224	12924546	12926084	0.37	✔
72	2003Q4	NA	NA	NA	12694620	12638056	12657795	12673463	12674044	12671185	12672195	0.44	✔
73	2004Q1	NA	NA	NA	NA	13168173	13178357	13182556	13175562	13180519	13183768	0.11	✔
74	2004Q2	NA	NA	NA	NA	13491571	13489402	13467838	13469514	13472486	13471736	0.17	✔
75	2004Q3	NA	NA	NA	NA	13963232	13945534	13940461	13947658	13940205	13934720	0.20	✔
76	2004Q4	NA	NA	NA	NA	13567029	13575388	13595651	13598290	13590720	13591905	0.23	✔
77	2005Q1	NA	NA	NA	NA	NA	14083448	14097315	14084329	14098163	14107922	0.17	✔
78	2005Q2	NA	NA	NA	NA	NA	14311110	14274270	14278426	14286976	14281623	0.25	✔
79	2005Q3	NA	NA	NA	NA	NA	14726397	14720092	14733971	14712524	14696840	0.25	✔
80	2005Q4	NA	NA	NA	NA	NA	14115966	14148409	14155659	14136537	14140178	0.28	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14846235	14831999	14859180	14879855	0.32	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	15065562	15030171	15067600	15070337	0.26	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	14968065	14965708	14968558	14980739	0.10	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	15144620	15195366	15200801	15246665	0.67	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15520567	15579895	15629414	0.70	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	15716287	15792290	15792231	0.48	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15766940	15764934	15716290	0.32	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	15689749	15735883	15673893	0.39	✔
	Median	-	-	-	-	-	-	-	-	-	-	0.23	-
	Mean	-	-	-	-	-	-	-	-	-	-	0.26	0%

Source: Author's elaboration.

Table 15: Sliding spans of trend-cycle series (log), Aggregation 3

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9671298	9756410	9665693	9596191	9602736	9603241	9601605	9632222	9608678	9691974	1.66	✔
41	1996Q1	9998056	10047676	9920689	9916163	9933467	9933930	9925932	9955464	9940963	10022401	1.32	✔
42	1996Q2	10235053	10169433	10173931	10229141	10213728	10213159	10233974	10179458	10209705	10228188	0.64	✔
43	1996Q3	10002544	9727962	10337312	10345391	10338716	10338025	10348605	10313751	10325733	9930932	6.37	✘
44	1996Q4	10663900	10847486	10483862	10406874	10446017	10447088	10412901	10473022	10438822	10747159	4.23	✘
45	1997Q1	10471499	10455612	10168833	10195582	10163514	10164164	10201044	10201538	10196342	10468624	3.03	✘
46	1997Q2	10662432	10615029	11379799	11017997	11050068	11048870	11020806	10984540	11014289	10646328	7.20	✘
47	1997Q3	10997212	10919297	11024474	11008945	10985758	10984570	11008080	10989889	10990461	10996663	0.96	✔
48	1997Q4	11177670	11126395	11005951	11119843	11103834	11101035	11112011	11141439	11115001	11213202	1.88	✔
49	1998Q1	11261134	11227371	11127335	11200564	11195788	11189219	11201180	11191667	11194721	11291797	1.47	✔
50	1998Q2	11295593	11255721	11346196	11364640	11373207	11368302	11371785	11339079	11362541	11283089	1.04	✔
51	1998Q3	11244512	11225654	11314593	11305236	11305073	11305286	11305749	11318376	11304618	11221221	0.86	✔
52	1998Q4	10992457	11037577	10956929	10932424	10919781	10924205	10917615	10935990	10928012	10999379	1.09	✔
53	1999Q1	10884973	10896044	10825705	10833685	10839804	10839192	10836096	10821465	10834925	10906911	0.78	✔
54	1999Q2	10979818	10944534	11014730	11032431	11044466	11038698	11047913	11039604	11039732	10979010	0.94	✔
55	1999Q3	11228825	11219074	11270639	11259583	11256643	11256875	11262718	11282447	11263179	11199785	0.73	✔
56	1999Q4	11425315	11453555	11412645	11396463	11388512	11394600	11373903	11363957	11381386	11430659	0.78	✔
57	2000Q1	11527024	11531976	11420092	11467386	11405160	11418965	11436602	11438804	11443244	11504186	1.11	✔
58	2000Q2	11540510	11535318	11449276	11547162	11460952	11472615	11542980	11584438	11547361	11585674	1.19	✔
59	2000Q3	11648964	11640784	11595246	11648558	11618715	11627541	11670711	11699042	11678939	11732182	1.18	✔
60	2000Q4	11790464	11807993	11793425	11796620	11800428	11815314	11772583	11759193	11785919	11867909	0.92	✔
61	2001Q1	NA	12012900	12008190	11997020	12008803	12016107	11995024	11986295	11991847	12025407	0.32	✔
62	2001Q2	NA	12078116	12124860	12108374	12141397	12110184	12158403	12167087	12143386	12098316	0.73	✔
63	2001Q3	NA	12111868	12108067	12115418	12087238	12071961	12113685	12107545	12112533	12054819	0.50	✔
64	2001Q4	NA	12113236	12026339	12056467	12006479	12050550	12035185	12058109	12063885	12057046	0.88	✔
65	2002Q1	NA	NA	12096445	11843077	12096997	12123007	11843907	11854858	11846743	12133368	2.45	✔
66	2002Q2	NA	NA	12246474	12522584	12245260	12182728	12489210	12487220	12467664	12187914	2.78	✔
67	2002Q3	NA	NA	12415062	12417101	12407272	12345789	12338094	12318168	12334697	12327750	0.80	✔
68	2002Q4	NA	NA	12444199	12420053	12417811	12526491	12353810	12369179	12359836	12540003	1.50	✔
69	2003Q1	NA	NA	NA	12552442	12049562	12229809	12472055	12567377	12551554	12233460	4.29	✘
70	2003Q2	NA	NA	NA	12783233	13154610	13087201	12746133	12913301	12899324	13157883	3.23	✘
71	2003Q3	NA	NA	NA	12958069	12802069	12770046	12884368	12988051	12990559	12768926	1.73	✔
72	2003Q4	NA	NA	NA	12972644	12832633	12908260	12851482	12915938	12863632	12877043	1.09	✔
73	2004Q1	NA	NA	NA	NA	13169059	13186441	12695471	12613040	12659483	13148514	4.54	✘
74	2004Q2	NA	NA	NA	NA	13624201	13449928	13922201	14161269	13899949	13386416	5.78	✘
75	2004Q3	NA	NA	NA	NA	13899616	13161207	13796879	13849123	13829283	13059808	6.43	✘
76	2004Q4	NA	NA	NA	NA	13767786	14107270	13807782	13733217	13845454	14471962	5.37	✘
77	2005Q1	NA	NA	NA	NA	NA	13899893	13529314	13857597	13468266	14063632	4.42	✘
78	2005Q2	NA	NA	NA	NA	NA	14238683	14797252	14374313	14787460	14256239	3.92	✘
79	2005Q3	NA	NA	NA	NA	NA	14540479	14511252	14671229	14562362	14472876	1.37	✔
80	2005Q4	NA	NA	NA	NA	NA	14610021	14372986	14559261	14371039	14510670	1.66	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14460995	14084117	14551946	14742313	4.67	✘
82	2006Q2	NA	NA	NA	NA	NA	NA	14846278	15503639	15035647	14951384	4.42	✘
83	2006Q3	NA	NA	NA	NA	NA	NA	15204136	15100976	15282114	15058873	1.48	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	15188311	15085307	15220383	15243691	1.04	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15357434	14861978	14896794	3.33	✘
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	15653038	16234388	16135312	3.71	✘
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15867451	15754368	15543065	2.08	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	15791955	16010743	15757890	1.60	✔
	Median	-	-	-	-	-	-	-	-	-	-	1.51	-
	Mean	-	-	-	-	-	-	-	-	-	-	2.36	33%

Source: Author's elaboration.

Table 16: Sliding spans of trend-cycle series (log), Aggregation 4

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9819331	9819252	9819312	9819341	9819183	9820605	9820627	9820628	9820627	9820627	0.01	✔
41	1996Q1	10006963	10006830	10006967	10007021	10007148	10007883	10007879	10007892	10007882	10007884	0.01	✔
42	1996Q2	10157765	10158470	10158341	10158171	10158496	10157458	10157378	10157386	10157391	10157391	0.01	✔
43	1996Q3	10286493	10287316	10287228	10286820	10286786	10285530	10285407	10285393	10285406	10285401	0.01	✔
44	1996Q4	10417829	10417114	10417358	10417140	10416937	10419065	10418955	10418946	10418943	10418940	0.02	✔
45	1997Q1	10570504	10567780	10568060	10568028	10568344	10568752	10568670	10568659	10568647	10568647	0.02	✔
46	1997Q2	10767589	10766869	10766467	10766333	10766860	10765237	10765145	10765166	10765175	10765178	0.02	✔
47	1997Q3	11033265	11034040	11033763	11033687	11033046	11032246	11032302	11032339	11032347	11032348	0.01	✔
48	1997Q4	11229141	11229844	11230165	11230438	11229237	11231040	11231156	11231140	11231135	11231129	0.01	✔
49	1998Q1	11359183	11357350	11357903	11357747	11357859	11356757	11356593	11356523	11356548	11356548	0.02	✔
50	1998Q2	11398761	11393102	11393402	11393438	11394426	11393655	11393544	11393545	11393521	11393522	0.04	✔
51	1998Q3	11274174	11273737	11272831	11274023	11276733	11277918	11278106	11278177	11278074	11278077	0.04	✔
52	1998Q4	11045550	11048454	11045874	11047276	11051180	11052329	11052723	11052661	11052599	11052604	0.06	✔
53	1999Q1	10929135	10921467	10919949	10919607	10922979	10921934	10922135	10921928	10922012	10922021	0.08	✔
54	1999Q2	11010690	11004012	11003176	11002199	11002006	11002021	11002114	11002130	11002163	11002160	0.07	✔
55	1999Q3	11240461	11246839	11247163	11247592	11245091	11245738	11246175	11246398	11246261	11246262	0.06	✔
56	1999Q4	11478246	11485537	11486199	11487028	11486604	11485095	11485530	11485619	11485574	11485583	0.07	✔
57	2000Q1	11556174	11546103	11545931	11547872	11549791	11549528	11549803	11549153	11549288	11549254	0.08	✔
58	2000Q2	11571389	11565618	11563644	11565673	11562087	11564428	11565083	11564197	11564137	11564038	0.08	✔
59	2000Q3	11679509	11689407	11691522	11694491	11687196	11688019	11689420	11689379	11688721	11688676	0.12	✔
60	2000Q4	11841783	11853557	11855391	11859506	11858431	11855601	11857592	11858032	11857813	11857904	0.14	✔
61	2001Q1	NA	11990667	11983574	11984657	11990802	11990430	11990260	11990189	11990928	11990994	0.06	✔
62	2001Q2	NA	12060911	12072863	12068565	12076370	12079142	12077486	12075592	12075927	12075857	0.15	✔
63	2001Q3	NA	12074449	12109922	12112629	12115983	12118916	12119920	12117992	12116688	12116658	0.37	✔
64	2001Q4	NA	12109244	12128835	12136155	12141800	12141710	12144828	12144648	12144112	12144415	0.29	✔
65	2002Q1	NA	NA	12179708	12177353	12192037	12190896	12188886	12188213	12189629	12189547	0.12	✔
66	2002Q2	NA	NA	12250047	12239359	12240527	12238884	12233159	12232864	12233889	12233570	0.14	✔
67	2002Q3	NA	NA	12343827	12348047	12332606	12335168	12333108	12336271	12334827	12334946	0.12	✔
68	2002Q4	NA	NA	12489643	12503441	12507300	12512107	12514201	12518537	12517989	12518812	0.23	✔
69	2003Q1	NA	NA	NA	12649441	12679307	12679098	12673664	12671623	12673967	12673728	0.23	✔
70	2003Q2	NA	NA	NA	12756992	12765098	12760373	12754422	12750558	12751650	12750905	0.11	✔
71	2003Q3	NA	NA	NA	12853094	12802142	12803050	12808640	12808415	12807790	12807741	0.39	✔
72	2003Q4	NA	NA	NA	12993177	12920743	12929379	12938566	12951569	12942414	12943007	0.56	✔
73	2004Q1	NA	NA	NA	NA	13144119	13148419	13165885	13182427	13165674	13168181	0.29	✔
74	2004Q2	NA	NA	NA	NA	13434318	13430303	13463353	13473369	13463203	13465164	0.32	✔
75	2004Q3	NA	NA	NA	NA	13701433	13704231	13736645	13745107	13745479	13744780	0.32	✔
76	2004Q4	NA	NA	NA	NA	13882085	13906842	13919657	13917000	13920361	13919969	0.27	✔
77	2005Q1	NA	NA	NA	NA	NA	14064076	14056856	14037151	14044503	14042500	0.19	✔
78	2005Q2	NA	NA	NA	NA	NA	14250453	14248997	14247247	14248356	14243861	0.04	✔
79	2005Q3	NA	NA	NA	NA	NA	14414893	14440608	14467474	14457903	14455743	0.36	✔
80	2005Q4	NA	NA	NA	NA	NA	14559786	14579465	14600078	14597436	14599483	0.27	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14741733	14727694	14737843	14741885	0.09	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	14891341	14870111	14868909	14872149	0.15	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	15030415	15036552	15021195	15018311	0.12	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	15257763	15281505	15280486	15275079	0.15	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15521933	15526250	15541755	0.12	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	15632343	15620505	15649737	0.18	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15708419	15693603	15701783	0.09	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	15884912	15887098	15883123	0.02	✔
	Median	-	-	-	-	-	-	-	-	-	-	0.11	-
	Mean	-	-	-	-	-	-	-	-	-	-	0.14	0%

Source: Author's elaboration.

Table 17: Sliding spans of trend-cycle series (log), Aggregation 5

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9837548	9840904	9834768	9838670	9835301	9823056	9822867	9822870	9822954	9839089	0.05	✔
41	1996Q1	10017532	10018840	10013451	10019467	10014484	10038008	10037800	10037702	10037695	10015871	0.11	✔
42	1996Q2	10128307	10127346	10129760	10129953	10131056	10130491	10130399	10130372	10130792	10127914	0.03	✔
43	1996Q3	10233136	10233302	10235839	10234957	10236553	10232956	10233023	10233027	10233227	10236116	0.14	✔
44	1996Q4	10386163	10387235	10385202	10387086	10385261	10386432	10386497	10386504	10386058	10387686	0.08	✔
45	1997Q1	10562435	10563204	10559231	10560587	10559171	10561754	10561712	10561705	10561457	10560103	0.15	✔
46	1997Q2	10758399	10759566	10759971	10757514	10760543	10759759	10759674	10759671	10760230	10759041	0.14	✔
47	1997Q3	10998960	11001073	11003073	11002189	11003879	11001447	11001437	11001459	11001833	11002567	0.08	✔
48	1997Q4	11185951	11188912	11187347	11189544	11185800	11186088	11186242	11186224	11185319	11186248	0.07	✔
49	1998Q1	11299612	11298269	11293328	11298064	11291664	11295974	11296163	11296048	11294344	11294898	0.11	✔
50	1998Q2	11328256	11321607	11323184	11321464	11322850	11324365	11324421	11324285	11325103	11323684	0.15	✔
51	1998Q3	11215715	11210163	11237725	11208705	11238204	11211438	11211540	11211699	11220758	11219613	0.10	✔
52	1998Q4	11047900	11044187	11013487	11044935	11015370	11041335	11041508	11041517	11034633	11035765	0.22	✔
53	1999Q1	10953229	10945874	10942204	10948059	10944421	10946024	10945960	10945995	10945400	10947771	0.10	✔
54	1999Q2	11028428	11028058	11031033	11028735	11031798	11030614	11030401	11030275	11031244	11028949	0.14	✔
55	1999Q3	11224049	11234742	11238105	11238346	11240989	11237378	11237522	11237491	11237968	11235946	0.25	✔
56	1999Q4	11449704	11462755	11457552	11461808	11460854	11455711	11456050	11456176	11455784	11457440	0.16	✔
57	2000Q1	11547968	11546614	11539284	11536000	11536916	11535429	11535323	11535207	11534766	11538113	0.26	✔
58	2000Q2	11577984	11571366	11577405	11570443	11576713	11575500	11575248	11574971	11575684	11576118	0.18	✔
59	2000Q3	11671126	11680011	11694374	11684145	11701794	11697210	11697801	11697932	11698508	11679543	0.34	✔
60	2000Q4	11818756	11851856	11852143	11866005	11858723	11852734	11853521	11854218	11853937	11871041	0.21	✔
61	2001Q1	NA	11989245	11978979	11975444	11975812	11968916	11968862	11968903	11968871	11965599	0.36	✔
62	2001Q2	NA	12055874	12057616	12054457	12045941	12036133	12035965	12035543	12034082	12029506	0.18	✔
63	2001Q3	NA	12054859	12096754	12103825	12077364	12081519	12083305	12083583	12080483	12081242	0.28	✔
64	2001Q4	NA	12127540	12124625	12124764	12131933	12109595	12111520	12113298	12110882	12114232	0.17	✔
65	2002Q1	NA	NA	12180638	12166045	12160993	12155817	12154382	12155086	12154801	12154072	0.14	✔
66	2002Q2	NA	NA	12244343	12232286	12233543	12230950	12228187	12227017	12227606	12221744	0.27	✔
67	2002Q3	NA	NA	12328533	12345842	12358792	12356263	12357740	12357883	12358297	12362645	0.17	✔
68	2002Q4	NA	NA	12446732	12506818	12528653	12571390	12576720	12581207	12581632	12549321	0.18	✔
69	2003Q1	NA	NA	NA	12666423	12704555	12709739	12710530	12710710	12710291	12703200	0.09	✔
70	2003Q2	NA	NA	NA	12757565	12794008	12769747	12766639	12764688	12764608	12768019	0.31	✔
71	2003Q3	NA	NA	NA	12825018	12821115	12795697	12793343	12791879	12791710	12789393	0.30	✔
72	2003Q4	NA	NA	NA	12962752	12955517	12941283	12936296	12930248	12930217	12919302	0.12	✔
73	2004Q1	NA	NA	NA	NA	13168527	13169909	13160795	13153338	13152854	13141491	0.10	✔
74	2004Q2	NA	NA	NA	NA	13377335	13426147	13427044	13430797	13428618	13424487	0.30	✔
75	2004Q3	NA	NA	NA	NA	13727190	13678487	13692966	13706195	13704587	13704732	0.38	✔
76	2004Q4	NA	NA	NA	NA	13890924	13900538	13908022	13908415	13909794	13907038	0.25	✔
77	2005Q1	NA	NA	NA	NA	NA	14093957	14080431	14058786	14055918	14051420	0.22	✔
78	2005Q2	NA	NA	NA	NA	NA	14275744	14276099	14259702	14250728	14248954	0.45	✔
79	2005Q3	NA	NA	NA	NA	NA	14427738	14451102	14462847	14457486	14457559	0.53	✔
80	2005Q4	NA	NA	NA	NA	NA	14570041	14588974	14601944	14607443	14605560	0.31	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14759792	14733277	14739737	14743815	0.13	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	14896428	14872733	14863721	14866575	0.40	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	15032478	15040300	15032843	15025761	0.47	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	15255056	15280259	15298116	15290439	0.20	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15522820	15545012	15549212	0.13	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	15620085	15616434	15621838	0.15	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15683815	15667351	15662339	0.18	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	15850902	15888758	15883117	0.19	✔
	Median	-	-	-	-	-	-	-	-	-	-	0.19	-
	Mean	-	-	-	-	-	-	-	-	-	-	0.21	0%

Source: Author's elaboration.

Table 18: Sliding spans of trend-cycle series (log), Aggregation 6

No. Obs.	Date	S(1)	S(2)	S(3)	S(4)	S(5)	S(6)	S(7)	S(8)	S(9)	S(10)	S^{max}	Score
40	1995Q4	9781679	9781741	9781734	9781715	9781709	9781682	9781710	9781710	9781708	9781708	0.00	✔
41	1996Q1	9973168	9973210	9973201	9973182	9973170	9973308	9973199	9973197	9973206	9973206	0.00	✔
42	1996Q2	10119721	10119654	10119652	10119625	10119612	10119574	10119613	10119614	10119612	10119612	0.00	✔
43	1996Q3	10283549	10283741	10283720	10283677	10283630	10283342	10283593	10283612	10283594	10283593	0.00	✔
44	1996Q4	10403202	10402920	10403089	10403447	10403640	10404005	10403608	10403575	10403601	10403602	0.01	✔
45	1997Q1	10519363	10517814	10518193	10518954	10519398	10520566	10519385	10519301	10519381	10519385	0.02	✔
46	1997Q2	10739540	10738330	10738568	10738912	10739092	10739428	10739058	10739027	10739052	10739053	0.01	✔
47	1997Q3	11035736	11036967	11036983	11036837	11036628	11035942	11036581	11036636	11036593	11036589	0.01	✔
48	1997Q4	11241351	11241907	11241775	11242017	11242104	11241984	11242006	11242044	11242042	11242040	0.00	✔
49	1998Q1	11362724	11358517	11358411	11358850	11359438	11360252	11359363	11359274	11359339	11359348	0.03	✔
50	1998Q2	11385590	11381724	11381970	11382642	11382871	11384232	11382827	11382818	11382917	11382915	0.03	✔
51	1998Q3	11238409	11240686	11241135	11242085	11241815	11243084	11241808	11241978	11242069	11242053	0.04	✔
52	1998Q4	11011644	11015910	11015972	11016848	11016953	11017966	11017016	11017155	11017226	11017218	0.05	✔
53	1999Q1	10903976	10898887	10898646	10898886	10899862	10901959	10900043	10899809	10899931	10899952	0.04	✔
54	1999Q2	10994521	10989857	10990396	10989617	10989374	10989061	10989289	10989100	10989078	10989081	0.04	✔
55	1999Q3	11227261	11233021	11232854	11232256	11230860	11228160	11230651	11230878	11230716	11230690	0.05	✔
56	1999Q4	11455668	11464880	11464641	11465937	11465950	11465437	11466060	11466509	11466484	11466470	0.09	✔
57	2000Q1	11531493	11521798	11521321	11522711	11525219	11529546	11525556	11525332	11525594	11525636	0.08	✔
58	2000Q2	11565476	11557279	11558444	11555832	11555105	11554232	11554710	11554035	11553994	11554006	0.09	✔
59	2000Q3	11679952	11689084	11688453	11686214	11683118	11678974	11682835	11682685	11682395	11682352	0.08	✔
60	2000Q4	11834588	11848856	11849523	11853624	11853177	11852888	11853720	11855312	11855276	11855230	0.17	✔
61	2001Q1	NA	11978720	11977640	11980978	11984995	11988091	11985039	11985600	11985808	11985844	0.08	✔
62	2001Q2	NA	12042448	12043624	12038371	12044478	12054583	12044815	12040932	12041870	12042080	0.13	✔
63	2001Q3	NA	12052780	12051583	12049316	12055171	12072410	12057653	12052445	12053747	12053957	0.19	✔
64	2001Q4	NA	12085762	12088118	12098626	12103342	12115902	12106604	12108191	12109123	12109185	0.24	✔
65	2002Q1	NA	NA	12163649	12170248	12180129	12181260	12180281	12181381	12181894	12182010	0.15	✔
66	2002Q2	NA	NA	12228167	12215792	12215819	12225514	12212184	12208827	12208746	12208842	0.15	✔
67	2002Q3	NA	NA	12334077	12325457	12312981	12331416	12311755	12309056	12308122	12307926	0.21	✔
68	2002Q4	NA	NA	12467425	12482425	12479715	12496342	12484775	12491791	12491972	12491824	0.23	✔
69	2003Q1	NA	NA	NA	12630629	12650761	12656527	12650453	12651291	12652511	12652805	0.20	✔
70	2003Q2	NA	NA	NA	12768263	12773052	12767286	12766357	12760213	12759764	12759957	0.10	✔
71	2003Q3	NA	NA	NA	12884107	12841595	12828502	12838169	12834569	12832879	12832435	0.43	✔
72	2003Q4	NA	NA	NA	12976536	12933984	12934633	12939228	12944433	12945123	12944946	0.32	✔
73	2004Q1	NA	NA	NA	NA	13137649	13145182	13131604	13158445	13160667	13160617	0.22	✔
74	2004Q2	NA	NA	NA	NA	13439297	13430657	13426174	13465924	13467192	13466673	0.30	✔
75	2004Q3	NA	NA	NA	NA	13719558	13710981	13724159	13752172	13753792	13752730	0.31	✔
76	2004Q4	NA	NA	NA	NA	13921765	13929709	13933303	13920344	13921575	13919704	0.09	✔
77	2005Q1	NA	NA	NA	NA	NA	14097821	14076543	14025767	14024886	14024583	0.52	✔
78	2005Q2	NA	NA	NA	NA	NA	14279001	14276122	14235226	14227539	14228648	0.36	✔
79	2005Q3	NA	NA	NA	NA	NA	14430429	14454822	14462177	14454318	14452333	0.22	✔
80	2005Q4	NA	NA	NA	NA	NA	14554349	14571756	14596829	14601868	14597302	0.32	✔
81	2006Q1	NA	NA	NA	NA	NA	NA	14723221	14727334	14739885	14743673	0.13	✔
82	2006Q2	NA	NA	NA	NA	NA	NA	14866168	14872740	14864875	14871410	0.05	✔
83	2006Q3	NA	NA	NA	NA	NA	NA	15012129	15035157	15019695	15015018	0.15	✔
84	2006Q4	NA	NA	NA	NA	NA	NA	15241362	15276449	15284931	15273097	0.28	✔
85	2007Q1	NA	NA	NA	NA	NA	NA	NA	15527560	15544436	15551391	0.15	✔
86	2007Q2	NA	NA	NA	NA	NA	NA	NA	15640410	15639643	15659785	0.12	✔
87	2007Q3	NA	NA	NA	NA	NA	NA	NA	15712632	15712635	15713739	0.00	✔
88	2007Q4	NA	NA	NA	NA	NA	NA	NA	15875506	15900422	15884213	0.15	✔
	Median	-	-	-	-	-	-	-	-	-	-	0.10	-
	Mean	-	-	-	-	-	-	-	-	-	-	0.14	0%

Source: Author's elaboration.