Stock Price, Real Riskless Interest Rate and Learning

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3 July 2014

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July 2, 2014

*I am specially grateful to my advisor Albert Marcet for his continuous advise and encouragement. I would also like to thank Jordi Gali and Luca Gambetti for their helpful conversations and suggestions. I acknowledge financial support from La Caixa-Severo Ochoa International Doctoral Fellowships. Email address: tongbin.zhang@uab.es

Abstract

In this paper, I first discover how real riskless interest rate, the tool for conducting monetary policy, is empirically related to stock price. Then, consumption based asset pricing model with rational expectations has been shown to fail in generating the same relationship. However, allowing a small deviation from RE by introducing learning mechanism can quantitatively account for the weak relationship between stock price and the risk-free interest rate. Therefore, I claim that this model could be favorable workhorse for studying monetary policy and asset price.

"...my conclusion that the generally supportive stance of Alan Greenspan and other central bankers was only a contributing factor to the millennium stock market boom and to the real estate price boom that came on its heels." —Robert Shiller (2005)

1. Introduction

The recent financial crisis caused by the collapse of U.S. house price beginning in 2007 witnesses the great effect of asset price on the real economy. Hence, whether monetary policy should be set up to control the variation of asset price has been a heated topic. However, before directly going into this topic the relationship between riskless interest rate, the channel of conducting monetary policy, and asset price should be well-understood as a first step.

The traditional viewpoint claims that the riskless interest rate should be highly negatively correlated with stock price-dividend ratio and is an important source in driving the volatility of it. However, this paper first shows that these wisdoms contradict with my empirical findings. In the data, I find that the correlation between riskless real interest rate and the stock PD ratio is close to zero and the corresponding high p-value implies non-significance. Besides this, interest rate's correlation with the growth of PD ratio is also insignificant small. In addition to these unconditional correlations, I also adopt a variance decomposition analysis introduced by Campbell (1991) and Campbell and Ammer (1993) with updated data to study this relationship controlling other variables. The results suggest that the news about future riskless interest rate can only account for an extremely small percentage of the total variance of excess stock return.

Then, theoretically I build a standard Lucas asset pricing model with two variations: open economy and collateral constraint which allows me to introduce time-varying interest rate. As the benchmark one, the equilibrium with rational expectation can be derived with closed form. But both qualitative analysis and quantitative results from simulation demonstrate its failure in explaining my empirical evidences.

The rational expectation's unsuccess motivates me to depart from this assumption and introduce the learning mechanism. Similar to Adam, Marcet and Nicolini (2013), I assume agents are "internal rationality", that is they optimize their behaviors based on their subjective beliefs about exogenous variables and subjective ones are allowed to be different from objective ones. This assumption challenges the correspondence from fundamentals such as dividend and consumption to stock price. In such a setting, agents update their subjective expectations about stock price behaviors responsive to the realized one. Then, their expectations can influence current stock price, which should feed back into agents' expectations next period. This self-referential aspect of model establishes that high volatility of stock price (or PD ratio) is mostly driven from agents' expectations not from riskless interest rate. Thus, the presence of agents' subjective beliefs breaks the close relationship between riskless interest rate and stock price displayed in rational expectation.

As shown in section 7, the quantitative performance of model with learning can in the first place replicates several important behaviors in stock market such as high volatility of stock return, high persistence of PD ratio and predictability. Most importantly, both the simulated coefficients between riskless interest rate with the PD ratio and with PD ratio growth announce the obvious improvements of the model in matching data. Meanwhile, results from variance decomposition using simulated data can further confirm the favorable performance of my simple model with learning.

The paper is organized in the following manner. Section 2 discusses related literature. In section 3, I present my three empirical findings about the relationship between riskless interest rate and stock price. The theoretical model is outlined in the section 4. Section 5 derives explicit expression for rational expectation equilibrium. The dynamic analysis of the model with learning is conducted in section 6. In section 7, I compare simulated results of both rational model and learning model with ones from data. Section 8 talks about some implications from my model. Finanlly, section 9 concludes.

2. Literature Review

To my best knowledge, there is not too many papers studying the relationship between riskless interest rate and stock price. The most recent theoretical one is Gali (2014). In the paper, he challenges the traditional "lean against wind" monetary policy on asset price when allowing the existence of rational bubble. As there is no dividend paid for the bubble, the bubble in the equilibrium has to grow at the level of risk-free interest rate. Thus, contractionary monetary policy could rise up the bubble value instead of decreasing it. However, in his model the fact that bubbly component is highly positively correlated with the riskless interest rate mismatches my empirical evidences.

There are several empirical papers on this topic. Both Campbell and Ammer (1993) and Hollifield, Koop and Li (2003) based on the variance decomposition analysis arrive at the same conclusion that the news on future real riskless interest rate can be ignored in explaining stock market volatility. And recently, Gali and Gambetti (2014) use the impulse response functions from time-varying VAR model to explore the response of stock price to exogenous monetary policy shock. Their conclusions can support positive conditional correlation between real interest rate and stock price bubble, but lack of variance decomposition analysis leads to the ambiguity about the importance of interest rate on stock price.

Besides these explicit analyses on this relationship, models with rational expectation addressing stock market volatility should be cited as the potential explanations. In order to generate sufficient high volatility of stock price, Campbell and Cocharane (1999) introduce exogenous habit into agent's utility function. It has to be confessed that if time-varying riskless interest rate is allowed [1], low correlation between it and price-dividend ratio can be reproduced in this model because most of stock price variation is caused by time-varying risk premia instead of interest rate. Although results are satisfied, risk aversion coefficient in their model is controversial because it can range from 60 to several hundred with steady state value at 80.

[1] In their model, real interest rate is pinned down as a constant.

Being different from varying risk aversion, Bansal and Yaron (2004) justify stock market behavior by adopting Epstein and Zin preference and different dividend and consumption growth rate process. This model with the volatility of stock price driven by risk premium for long-run risk is also possible to match my empirical findings. Nevertheless, Constantinides and Ghosh (2011) estimate and test the Bansal and Yaron's model with latent state variables. The most notable finding is that one cannot support the hypothesis that the intertemporal elasticity of substitution is higher than one, which is a crucial assumption in detecting long-run risk. In contrast to these two models, my model assumes traditional CRRA preference and reasonable risk aversion coefficient.

At the same time, my paper, of course, is closely related to Adam, Marcet and Nicolini (2013), which targets at generating high volatility in the stock market with learning. But their model has a constant real riskless interest rate.

3. Stylized Facts

This section describes stylized facts regarding the relationship between US stock price and real riskless interest rate. The measurements considering the relationship here are correlations between interest rate with the level of price-dividend ratio and with the growth of price-dividend ratio, and variance decomposition analysis based on Vector Autoregression.[2]

According to the Lucas asset pricing model with rational expectation, as shown in the section 5, stock price-dividend ratio should be highly negatively correlated with risk-free interest rate. However, this correlation unfortunately cannot be observed in the data displayed in the table 1. The quarterly correlation coefficients are small positive numbers and insignificant regardless of contemporaneous one or one-period lag's. Even though the statistics using monthly data in the third column present significant negative correlation, their values still cannot support sufficient high correlation implied by the theoretical model with rational expectation.

[2] Details of data resources and the method of data analysis are provided in the Appendix

Not only the level of price-dividend ratio, but also the growth rate of it has almost no correlation with real interest rate. The results on this are reported in the table 2. Similar to the fact above, all of four correlations between interest rate and PD ratio growth are negligible small and insignificant. Thus, it is confident to claim that there is no correlation between these two variables.

Instead of conditional correlation coefficients before I just compute the unconditional ones, but it is important to further study the effect of riskless interest rate on stock price behavior when controlling other terms such as dividend and risk premia. Therefore, I will use the variance decomposition analysis developed by Campbell (1991) and Campbell and Ammer (1993). Though these analyses have been extensively documented in finance papers, I reproduce this in order to incorporate updated data and match its simulated theoretical counterpart.[3] The results from variance decomposition are summarized in table 3. The value in the second row, second column can be interpreted as following: the variance of news about future dividend can account for 32.6% variance of excess stock return. This value for risk-free interest rate is almost zero in the third row, second column, but more than half of excess return's variance can be explained by news on future excess return as value in the fourth row, second column. These values can vary a lot due to different sample periods (Campbell and Ammer, 1993) or different prior distributions with Bayesian estimation (Hollifield, Koop and Li, 2003), but the ordering is the same: $Var(\widetilde{e}_e) > Var(\widetilde{e}_d) > Var(\widetilde{e}_r)$. Hence, my variance decomposition can match the ones in the literature.

Conclusively, these empirical findings can be summarized into three facts: the low correlation between interest rate and price-dividend ratio as **Fact 1**, almost zero correlation between interest rate and growth rate of price-dividend ratio as **Fact 2** and the small percentage of variance of future's interest rate contributed to the variance of excess return as **Fact 3**.

[3] The Appendix talks about the specific procedures about how to implement this variance decomposition

Statistics	Quarterly	Monthly
$corr(R_t, \frac{P_t}{D_t})$	0.0216	-0.1510
	(0.7259)	$(0.0000)^{***}$
$corr(R_{t-1}, \frac{P_t}{D_t})$	0.0327	-0.1475
	(0.5960)	$(0.0000)^{***}$

Table 1: The Correlation between Real Interest Rate and Price-Dividend Ratio

Statistics	Quarterly	Monthly
$corr(R_t, \frac{P_t/D_t}{P_{t-1}/D_{t-1}})$	0.0462	-0.0328
	(0.4518)	(0.3531)
$corr(R_{t-1}, \frac{P_t/D_t}{P_{t-1}/D_{t-1}})$	0.0380	0.0130
p-value	(0.5372)	(0.7135)

Table 2: The Correlation between Real Interest Rate and Price-Dividend Ratio Growth Rate

4. The Model

This section presents a Lucas asset pricing model with two small variations: open economy and collateral constraint. The analytical equilibrium can be derived within rational expectation, which produces counterfactual results again empirical findings above. The presence of internal rationality, that is decision-making agents hold subjective beliefs about stock price behavior instead of knowing objective distribution, with belief updating rule has the ability to reconcile the Lucas asset pricing model with the three facts.

4.1 The Process for Exogenous Variables

Any unit of stock can be traded in the competitive stock market and pays dividend D_t. In addition to D_t, each agent receives an endowment Y_t of perishable consumption goods. Hence, the feasibility condition guarantees the equation of total consumption supply $C_t = Y_t + D_t$ to be held in every period. Following traditional setting in Campbell and

Statistics	Value
$Var(\widetilde{e}_d)$	32.6%
$Var(\widetilde{e}_r)$	0.02%
$Var(\widetilde{e}_e)$	57.6%

Table 3: Variance Decomposition of Excess Stock Return

Cochrane (1999) and Adam, Marcet and Nicolini (2013), in order to capture the property of consumption and dividend's volatilities and the weak correlation between them the processes of dividend and consumption are assumed to follow

$$\frac{D_t}{D_{t-1}} = a\epsilon_t^d, \log \epsilon_t^d \sim iiN(-\frac{s_d^2}{2}, s_d^2)$$

$$\frac{C_t}{C_{t-1}} = a\epsilon_t^c, \log \epsilon_t^c \sim iiN(-\frac{s_c^2}{2}, s_c^2)$$

where $a \geq 1$ is the averaged dividend or consumption growth rate and $(\log \epsilon_t^d, \log \epsilon_t^c)$ is joint normal distributed with correlation between them equaling to $\rho_{c,d} = 0.2$. Since consumption process is considerably less volatile than dividend process, the parameters' values of standard deviations are chosen as $s_c = \frac{1}{7}s_d$.

4.2 Preferences and Constraints

The economy is populated by a unit mass of infinite-horizon agents. Each agent $i \in [0, 1]$ is assumed to have the same time-seperable CRRA utility function. However, this fact is not a common knowledge among agents.[4]

The representative agent with identical preference and belief has his life-time utility in the form of

$$E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t \frac{C_t^{1-\gamma}}{1-\gamma} \tag{1}$$

where $C_t > 0$ is the consumption goods and δ is denoted as the time discount factor. Instead of objective probability measure, agent's expectations are computed using the subjective probability measure $\mathcal{P}$ that describes probability distributions for exogenous variables.

Agent's choices are subjected to budget constraint as following

[4] The lack of common knowldge about agents' preferences and beliefs provides microfoundation for the failure of present-value expression for stock price explained in Adam and Marcet (2011). Section 6 talks about this.

$$C_t + R_{t-1}b_{t-1} + P_t S_t = (P_t + D_t)S_{t-1} + b_t + Y_t \tag{2}$$

where b_t is agent's new loans, S_t the units of stock agent buys in period t and R_t as exogenous real riskless interest rate on maturing loans b_t. Hence, this constraint intuitively suggests that agent in every period spends his income coming from holding stock S_{t-1}, new loans b_t and endowment Y_t into the purchase of consumption goods C_t, the claim of new stock S_t and the repayment of old loans b_{t-1}.

However, in addition to budget constraint the collateral constraint is introduced here. I assume that consumer's borrowing in term of loans is subjected to a collateral constraint as Kiyotaki and Moore (1997) in the form of

$$b_t \leqq \theta \frac{E_t^{\mathcal{F}}\left(P_{t+1} + D_{t+1}\right)}{R_t} S_t \tag{3}$$

Besides transfering income across time, the stock S_t as important component of agent's wealth plays the role of collateral. This constraint implies that new loans b_t should be smaller than the fixed part of tomorrow's stock value discounted by R_t. The parameter θ governs the certain share of stock value that can be served as collateral. Section 4.4 shows that the introducing of collateral constraint allows us to have time-varying riskless interest rate R_t.

4.3 Probability Space

This subsection explicitly describes the probability space as $(\mathcal{F}, ß, \Omega)$, where ß is the corresponding $\sigma-$Algebra of Borel subsets of Ω and $\mathcal{F}$ is the agent's subjective probability measure over $(ß, \Omega)$. Representative agent considers the joint process of endowment, dividend and riskless interest rate sequence $\{Y_t, D_t, R_t\}_{t=0}^{\infty}$ as exogenous one. And the non-existence of common knowledge on agents' identical preferences and beliefs guarantees perfect exogeneity of stock price process $\{P_t\}_{t=0}^{\infty}$. Then, the state space of realized exogenous variables can be defined as

$$\Omega = \Omega_P \times \Omega_D \times \Omega_Y \times \Omega_R$$

where Ω_X is the space of all possible infinite sequences for the variable $X \in [P, D, Y, R]$. Hence, a specific element in the set Ω is an infinite sequences $\omega = \{P_t, Y_t, D_t, R_t\}_{t=0}^{\infty}$. Then, the expected utility with probability measure $\digamma$ is defined as

$$E_0^{\digamma} \sum_{t=0}^{\infty} \delta^t \frac{C_t^{1-\gamma}}{1-\gamma} \equiv \int_{\Omega} \sum_{t=0}^{\infty} \delta^t \frac{C_t(\omega)^{1-\gamma}}{1-\gamma} d\digamma(\omega) \tag{4}$$

Then subjected to the budget constraint and collateral constraint, policy function deciding the endogenous variables conditional on realizations should be the mapping in the following

$$(C_t, S_t, b_t) : \Omega^t \rightarrow R^3$$

where Ω^t represents the set of histories from period zero up to period t.

4.4 Optimality Conditions

In this subsection optimal conditions characterizing agent's behaviors are derived from his maximization problem. First order conditions are sufficient and necessary for agent's optimality because of the concavity of objective function and linearity of two constraints.

Representative agent should maximize his expected lifetime utility (1) subject to budget constraint (2) and collateral constraint (3). The Lagrangian of agent's problem can be explicitly written by

$$\max_{\{C_t, S_t, b_t\}} E_0^{\digamma} \sum_{t=0}^{\infty} \delta^t (\frac{C_t^{1-\gamma}}{1-\gamma} - \lambda_t (C_t + R_{t-1} b_{t-1} + P_t S_t - (P_t + D_t) S_{t-1} - b_t - y_t)$$
$$+ \gamma_t (\theta E_t^{\digamma} (P_{t+1} + D_{t+1}) S_t - R_t b_t))$$

where S_{-1}, b_{-1} are given initial conditions and agent is price-taker for P_t.

The agent's first order conditions can be expressed as

$$C_t : C_t^{-\gamma} - \lambda_t = 0 \tag{5}$$

$$S_t : -\lambda_t P_t + \delta E_t^{\mathcal{F}} (\lambda_{t+1}(P_{t+1} + D_{t+1})) + \gamma_t \theta E_t^{\mathcal{F}} (P_{t+1} + D_{t+1}) = 0 \tag{6}$$

$$b_t : \lambda_t = \delta R_t E_t^{\mathcal{F}} \lambda_{t+1} + \gamma_t R_t \ \& \ \gamma_t(\theta E_t^{\mathcal{F}} (P_{t+1} + D_{t+1}) S_t - R_t b_t) = 0 \tag{7}$$

After substituting λ_t in equation (7) using the expression in equation (5), I can have

$$C_t^{-\gamma} = \delta R_t E_t^{\mathcal{F}} (C_{t+1}^{-\gamma}) + \gamma_t R_t \tag{8}$$

To avoid the complicate problem of occasional binding, I assume the collateral constraint is binding in every period. As I mentioned before, if there is no collateral constraint (always non-binding), $\gamma_t = 0$ for every t and equation (8) should produce a constant R_t as Adam. Marcet and Nicolimi (2013). Therefore, the collateral constraint is important to allow me to introduce exogenous time-varying interest rate. Hereinafter, the Lagrangian multiplier γ_t can be explicitly expressed as following

$$\gamma_t = \frac{C_t^{-\gamma} - \delta R_t E_t^{\mathcal{F}} (C_{t+1}^{-\gamma})}{R_t} \tag{9}$$

Substitute equation(9) back into equation (6), I have

$$-C_t^{-\gamma} P_t + \delta E_t^{\mathcal{F}} (C_{t+1}^{-\gamma}(P_{t+1} + D_{t+1})) + \frac{C_t^{-\gamma} - \delta R_t E^{\mathcal{F}} (C_{t+1}^{-\gamma})}{R_t} \theta E_t^{\mathcal{F}} (P_{t+1} + D_{t+1}) = 0$$

Rearrange the term above to have the expression for stock price P_t

$$P_t = E_t^F \varphi_t (P_{t+1} + D_{t+1}) \tag{10}$$

where $\varphi_t \equiv (1-\theta)\delta \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} + \frac{\theta}{R_t}$.

In order to close the model, I assume the real interest rate to follow the process capturing its first two moments as[5]

$$R_t = \left\{ \begin{array}{ll} R + \epsilon_t^R & \text{if } R_t < \frac{1}{\delta} E_t^F (\frac{C_t}{C_{t+1}})^{-\gamma} \\ \frac{1}{\delta} E_t^F (\frac{C_t}{C_{t+1}})^{-\gamma} & \text{if else} \end{array} \right\} \tag{11}$$

where $\epsilon_t^R \sim N(0, \epsilon^r)$.

5. Rational Expectation Equilibrium

For comparison I assume rational expectation here, that is agent's subjective probability measure coincides with objective one ($E_t^F = E_t$). As is well known, under rational expectation stock price should equal with the present value of dividend stream. Hence, recursively deriving from equation (10) P_t here can be written as

$$P_t = (1-\theta) \frac{\delta a^{1-\gamma} \rho_\epsilon}{1 - \delta a^{1-\gamma} \rho_\epsilon} D_t + E_t \sum_{j=1}^{\infty} \frac{\theta^j a^j}{\prod_{k=0}^{j-1} R_{t+k}} D_t \tag{12}$$

where

$$\begin{aligned} \rho_\epsilon &= E[(\epsilon_{t+1}^c)^{-\gamma} \epsilon_{t+1}^d] \\ &= e^{\gamma(1+\gamma)\frac{s_c^2}{2}} e^{-\gamma \rho_{c,d} s_c s_d} \end{aligned}$$

Because interest rate process as equation (11) implies $E_t[R_{t+j}] = R$, I can approximate

[5] The threshold of riskless interest rate guarantees the binding of collateral constraint. But it doesn't influence the variance of interest rate since interest rate rarely hits the threshold.

the price P_t as equation (13) [6]

$$P_t \approx [(1-\theta)\frac{\delta a^{1-\gamma}\rho_\epsilon}{1-\delta a^{1-\gamma}\rho_\epsilon} + \frac{\theta a/R_t}{1-\theta a/R}]D_t \tag{13}$$

Except time-varying interest rate R_t, stock price P_t and dividend D_t, all of other variables in equation (13) are constant parameters. Thus, the property that the variation of stock price-dividend ratio $\frac{P_t}{D_t}$ is only driven by R_t demonstrates perfect negative correlation between these two variables contradicting Fact 1. Then, I can express the growth rate of price-dividend ratio as

$$\frac{P_t/D_t}{P_{t-1}/D_{t-1}} = \frac{(1-\theta)\frac{\delta a^{1-\gamma}\rho_\epsilon}{1-\delta a^{1-\gamma}\rho_\epsilon} + \frac{\theta a/R_t}{1-\theta a/R}}{(1-\theta)\frac{\delta a^{1-\gamma}\rho_\epsilon}{1-\delta a^{1-\gamma}\rho_\epsilon} + \frac{\theta a/R_{t-1}}{1-\theta a/R}} \tag{14}$$

Obviously, riskless interest rate R_t and growth rate of price-dividend ratio $\frac{P_t/D_t}{P_{t-1}/D_{t-1}}$ is contemporaneous negatively correlated. And one period lag correlation $corr(R_{t-1}, \frac{P_t/D_t}{P_{t-1}/D_{t-1}})$ should be significant positive. These miss Fact 2. In addition, Section 7 presents concrete simulated correlation and also shows the failure of rational model in matching Fact 3.

6. Equilibrium Analysis with Learning

6.1 Agent's Subjective Belief

Under rational expectation hypothesis, agents are assumed to know the true joint distribution of exogenous shocks and then stock price can be linked to the fundamentals. However, here I allow a small deviation from rational expectation such that agents with uncertainty formulate their own joint probability distribution $\mathcal{P}$ different from true one. And Adam and Marcet (2011) show that this joint distribution $\mathcal{P}$ could generate non-singularity delinking stock price to fundamentals. Hence, the presente-value expression of stock price P_t as equa-

[6] In principle $E_t[\frac{1}{R_{t+k}}] \neq \frac{1}{R}$. But because R_{t+k} is very close to one, $E_t[\frac{1}{R_{t+k}}] = \frac{1}{R}$can be good approximation.

tion (12) doesn't hold here. Without knowing how to map from the fundamentals to stock price, agents should have their own beliefs regarding the process of stock price based on subjective distribution $\mathcal{F}$. Thus, their beliefs are defined as the subjective expectations of risk-adjusted stock price growth

$$\beta_t \equiv E_t^{\mathcal{F}}[(\frac{C_{t+1}}{C_t})^{-\gamma}\frac{P_{t+1}}{P_t}] \tag{15}$$

subjective non-adjusted expectation of stock price growth

$$m_t \equiv E_t^{\mathcal{F}}[\frac{P_{t+1}}{P_t}] \tag{16}$$

Rearrange the terms in equation (10) and substitute the relevant ones with the two definitions of beliefs, then I can have the equation mapping from percieved stock price to realized one as[7]

$$P_t = \frac{(1-\theta)\delta a^{1-\gamma}\rho_\epsilon + \frac{\theta}{R_t}a}{1-(1-\theta)\delta\beta_t - \frac{\theta}{R_t}m_t}D_t \tag{17}$$

As shown in equation (17), the distinguishing between risk-adjusted stock price growth belief β_t and one for non-adjusted m_t implies that agent's belief doesn't incorporate the process of risk-free interest rate R_t [8] and provides the convenience for algebraic calculation. Being different from rational expectation equilibrium equation (12) or equation (13), in addition to R_t stock price P_t under learning mechanism can be varied from the change of two beliefs β_t and m_t. Hence, this model has potentials to reproduce Fact 1 to Fact 3.

6.2 Beliefs Updating Rule

Here, I specify the subjective probability distribution $\mathcal{F}$ and derive the optimal belief updating rule. Similar to the setting in Adam, Marcet and Nicolini (2013), the true process

[7] Following Adam, Marcet and Nicolini (2013), I assume that agents know the true process for dividend growth and consumption growth but not stock price growth.

[8] It should be interesting to study the case that agent learns the process of interest rate as future work..

for risk-adjusted stock price growth and non-adjusted one can be modeled as the sum of a persistent component and of a transitory component

$$\begin{aligned}(\frac{C_{t+1}}{Ct})^{-\gamma}\frac{P_{t+1}}{P_t} &= e_t^{\beta}+\epsilon_t^{\beta}, \epsilon_t^{\beta}\sim iiN(0,\sigma^2_{\epsilon,\beta})\\ e_t^{\beta} &= e_{t-1}^{\beta}+\xi_t^{\beta}, \xi_t^{\beta}\sim iiN(0,\sigma^2_{\xi,\beta})\end{aligned}$$

$$\begin{aligned}\frac{P_{t+1}}{P_t} &= e_t^{m}+\epsilon_t^{m}, \epsilon_t^{m}\sim iiN(0,\sigma^2_{\epsilon,m})\\ e_t^{m} &= e_{t-1}^{m}+\xi_t^{m}, \xi_t^{m}\sim iiN(0,\sigma^2_{\xi,m})\end{aligned}$$

Agents can just observe the realizations of risk-adjusted and non-adjusted price growth (the sum of two components), hence the requirement to filter out the persistent components e_t^{β} and e_t^{m} calls for a learning problem. The priors of agents' beliefs can be centered at their rational expectation values and given by

$$e_0^{\beta}\sim N(e^{\beta}_{RE,}\sigma^2_{0,\beta})$$

$$e_0^{m}\sim N(e^{m}_{RE,}\sigma^2_{0,m})$$

and the variance of prior distribution should be set up to equal with steady state Kalman filter as

$$\sigma^2_{0,\beta}=\frac{-\sigma^2_{\xi,\beta}+\sqrt{\sigma^4_{\xi,\beta}+4\sigma^2_{\xi,\beta}\sigma^2_{\epsilon,\beta}}}{2}$$

$$\sigma^2_{0,m}=\frac{-\sigma^2_{\xi,m}+\sqrt{\sigma^4_{\xi,m}+4\sigma^2_{\xi,m}\sigma^2_{\epsilon,m}}}{2}$$

Then agents' posterior beliefs will be

$$e_t^{\beta}\sim N(\beta_t,\sigma^2_{0,\beta})$$

$$e_t^m \sim N(m_t, \sigma_{0,m}^2)$$

Thus, the optimal updating rule implies that the evolution of β_t and m_t are taking the form of

$$\beta_t = \beta_{t-1} + \frac{1}{\alpha^\beta}((\frac{C_{t-1}}{C_{t-2}})^{-\gamma}\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}) \tag{18}$$

$$m_t = m_{t-1} + \frac{1}{\alpha^m}(\frac{P_{t-1}}{P_{t-2}} - m_{t-1}) \tag{19}$$

where $1/\alpha^\beta = (\sigma_{0,\beta}^2 + \sigma_{\xi,\beta}^2)/(\sigma_{0,\beta}^2 + \sigma_{\xi,\beta}^2 + \sigma_{\epsilon,\beta}^2)$ and $1/\alpha^m = (\sigma_{0,m}^2 + \sigma_{\xi,m}^2)/(\sigma_{0,m}^2 + \sigma_{\xi,m}^2 + \sigma_{\epsilon,m}^2)$ given by optimal (Kalman) gain. Since the mean of $(\frac{C_{t+1}}{Ct})^{-\gamma}$ is close to one and variance is very small compared to variance of $\frac{P_{t+1}}{P_t}$, values of variances in risk-adjusted stock price growth should be extremely close to their counterparts in non-adjusted price growth process. Then, I assume that $\alpha^\beta = \alpha^m = \alpha$.

In order to avoid the explosion of stock price P_t, some projection facilities should be imposed to bound agent's beliefs β_t and m_t.

$$\beta_t = \omega^\beta(\beta_{t-1} + \frac{1}{\alpha}((\frac{C_{t-1}}{C_{t-2}})^{-\gamma}\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1})) \tag{20}$$

$$m_t = \omega^m(m_{t-1} + \frac{1}{\alpha}(\frac{P_{t-1}}{P_{t-2}} - m_{t-1})) \tag{21}$$

where

$$\omega^\beta(x) = x,\ \omega^m(x) = x \text{ if } x \in (0, \beta^L)$$

As long as beliefs are smaller than the threshold β^L, they continue to evolve as equation (18) and (19). But if beliefs are larger than β^L, there exists a truncated value for beliefs, which guarantees the burst of stock price bubble and the property of mean reversion.[9]

[9] Details of the specific funtional forms and thresholds' value on projection facilities are presented in appendix.

7. Quantitative Performance

7.1 Calibration

To implement the quantitative analysis based on model's simulation, free parameters appearing in the model should be calibrated. Although the focus of this paper is to study the relationship between stock price and risk-free interest rate, the prerequisite of theoretical model must be to replicate several phenomena in stock market such as high volatility of stock return. Hence, I borrow some parameters' value directly from Adam, Marcet and Nicolini (2013) as time discount factor $\delta = 0.992$, constant gain coefficient $1/\alpha = 0.0073$, mean of dividend growth rate $a = 1.0003$, $s^d = 0.0216$ from the standard deviation of it and the risk-aversion coefficient $\gamma = 5$.

And the mean of riskless interest rate R and standard deviation σ_r are calibrated at 1.002 and 0.007 respectively using historical data. The collateral ratio θ means how much could increase in international borrowing in term of current account deficit responding to one dollar increase in the value of total stock market. Then, $\theta = 0.1$ is the 1988-2012 average of annual value current account deficit over changes in the U.S. stock market.

7.2 Simulation Results

Table 4 reports the stock market behaviors coming form data and model with learning in the second and third column respectively. The comparison about results in these two columns illustrates that adding collateral constraint and belief in non-adjusted stock price growth into Adam, Marcet and Nicolini (2013)'s model can still replicate asset pricing moments. As mentioned, this qualifies my learning model to be appropriate one in studying the relationship between riskless interest rate and stock price.

Table 5 displays the relevant coefficients describing the relationship between riskless interest rate and stock price as the ones in stylized facts. The second column contains the simulated results from rational expectation equilibrium and third column has them from

Statistics	Data	Model
E_{rs}	2.41	1.55
E_{rb}	0.18	0.20
E_{PD}	113.20	109.65
σ_{rs}	11.65	10.36
σ_{PD}	52.98	64.86
$\rho_{PD,-1}$	0.92	0.92
c^2	-0.0048	-0.0053
R^2	0.1986	0.1449

Table 4: Simulation Moments on Stock Market Behavior

Statistics	Data	RE	Learning
$corr(R_t, \frac{P_t}{D_t})$	0.0216	-1.0000	-0.0753
	(0.7259)	$(0.0000)^{***}$	(0.2220)
$corr(R_{t-1}, \frac{P_t}{D_t})$	0.0327	0.0043	-0.0720
	(0.5960)	(0.4864)	(0.2961)
$corr(R_t, \frac{P_t/D_t}{P_{t-1}/D_{t-1}})$	0.0462	-0.7087	-0.3419
	(0.4518)	$(0.0000)^{***}$	$(0.0001)^{***}$
$corr(R_{t-1}, \frac{P_t/D_t}{P_{t-1}/D_{t-1}})$	0.0380	0.7077	-0.0099
	(0.5372)	$(0.0000)^{***}$	(0.5101)

Table 5: Simulated Relationship between Riskless Interest Rate and Stock Price

model with learning. When we compare table 5 with table 1 and table 2, it is obvious that rational model fails in capturing Fact 1 and Fact 2, but learning one has the ability to perfectly match every coefficient except $corr(R_t, \frac{P_t/D_t}{P_{t-1}/D_{t-1}})$. Even though the coefficient between R_t and $\frac{P_t/D_t}{P_{t-1}/D_{t-1}}$ is significant negative, the fact that it is only half of the same coefficient in rational expectation suggests that the model with learning can improve model's ability a lot in matching data.

To check whether the rational model or learning model can reproduce Fact 3, simulated stock price P_t, dividend D_t and riskless interest rate R_t are used to implement variance decomposition analysis as the one in empirical section. The results are presented in table 6 with RE and learning in second and third column respectively. Comparing these with table 3, model with learning can match the Fact 3. Nevertheless, in rational model the variance of future news about dividend can account for more than 100% of variance of excess return,

Statistics	Data	RE	Learning
$Var(\widetilde{e}_d)$	32.60%	100.40%	41.13%
$Var(\widetilde{e}_r)$	0.02%	0.02%	0.01%
$Var(\widetilde{e}_{e,})$	57.62%	1.02%	74.51%

Table 6: Variance Decomposition

which seriously contradicts with empirical evidences.

8. Model Implication

Based on the empirical evidences and my simulation results, it is normal to the challenge the effect of monetary policy on stock price. If the correlation between two variables is low and the variation of interest rate cannot significantly change the volatility of stock price, monetary policy through the channel of interest rate should not be the powerful tool in controlling asset price. However, Adam, Kuang and Marcet (2012) shows that change of real interest rate could be an important factor in driving house price. One difference between their paper and mine is the process of real interest rate. In their model, real interest rate initially is constant and then suddenly decreases. But in the following it holds at the same low level, which implies actually the process there is very persistent. Thus, I am willing to theoretically explore whether the persistence of real interest rate could play an indispensable role in affecting stock price.

To simplify my analysis, in this part assume constant real interest rate R (full persistent). Deriving from the equation (17), I can express the growth rate of realized stock price as

$$\begin{aligned} \frac{P_t}{P_{t-1}} &= \frac{1-(1-\theta)\delta\beta_{t-1}-\frac{\theta}{R}m_{t-1}}{1-(1-\theta)\delta\beta_t-\frac{\theta}{R}m_t} \\ &= \frac{1-\lambda_1\beta_{t-1}-\lambda_2 m_{t-1}}{1-\lambda_1\beta_t-\lambda_2 m_t} \end{aligned} \tag{22}$$

Statistics	$\rho_r = 0.45$	$\rho_r = 0.7$	$\rho_r = 0.9$
E_{rs}	1.43	1.59	1.81
E_{rb}	0.2	0.2	0.2
E_{PD}	127.19	126.57	126.66
σ_{rs}	12.39	13.56	15.06
σ_{PD}	72.44	78.21	92.35
$\rho_{PD,-1}$	0.92	0.91	0.92
c^2	-0.0073	-0.0068	-0.0062
R^2	-0.2300	0.2067	0.1685
$corr(R_t, \frac{P_t}{D_t})$	-0.1634	-0.2656	-0.3831
p-value	0.0757	0.0110	0.0013
$corr(R_t, \frac{P_t/D_t}{P_{t-1}/D_{t-1}})$	-0.1378	-0.1686	-0.0908
p-value	0.1077	0.0760	0.2282
$Var(e_d^{\sim})$	36.32%	40.50%	39.80%
$Var(e_r^{\sim})$	0.09%	0.64%	5.84%
$Var(e_e^{\sim})$	85.48%	81.57%	74.75%

Table 7: Simulation Results with Different Autocorrelation Coefficient

where $\lambda_1 = (1-\theta)\delta$, $\lambda_2 = \frac{\theta}{R}$. Then take derivative on this expression, I have

$$\frac{d(\frac{P_t}{P_{t-1}})}{d\lambda_2} = \frac{m_t - m_{t-1} + \lambda_1(m_{t-1}\beta_t - m_t\beta_{t-1})}{(1-\lambda_1\beta_t - \lambda_2 m_t)^2} \tag{23}$$

Equation (23) implies that stock price increase is stronger in response to a decrease in real interest rate R when agents in period t is more optimistic (in which m_t, β_t are higher and equation (23) is larger than zero)[10]. Then this initial increase in stock price can feed back into belief updating rule equation (18) and (19). This leads to a sequence of futher increase in stock price. Coversely, when agents are more pessmisic (in which m_t, β_t are lower and equation (23) is smaller than zero), an increase in real interest rate could amplify the decrease in stock price. Thus, real interes rate could have power in influencing asset price when it is sufficient persistent.

To comfirm my qualitative analysis, instead of the process as equation (11) real interest rate is model as $R_t = R + \rho_r R_{t-1} + \epsilon_t, \epsilon_t \sim N(0, \sigma_r^2)$. Table 7 displays the simulation results when trying different autocorrelation ρ_r.

[10]Details are shown in the Appendix

The second column in table 7 presents the simulated results with autocorrelation $\rho_r = 0.45$ that is the value calibrated by data. We can find that introducing persistence in the process of real interest rate R_t doesn't significantly change my results above. The performance of model with learning is still much better than rational model in matching data. When I increase the autocorrelation from 0.45 to 0.9, the correlation between R_t and $\frac{P_t}{D_t}$ in absolute value rises from 0.16 to 0.38. Meanwhile, news about future riskless interest rate can now account for about 6% variance of excess return instead of 0.09% when $\rho_r = 0.45$. Therefore, I can claim that the more persistence of real interest rate, the more influence it has on stock price. However, this influence unfortunately is still limited.

9. Conclusion and Future Work

The present paper makes an effort to enhance our understanding of the relation between the real riskless interest rate and asset price as the first step before introducing monetary policy. The empirical studies confirm that stock price is not correlated with risk-free interest rate and the latter almost have no power in explaining the volatility of stock excess return. Then, theoretically Lucas asset pricing model with rational expectation cannot match my empirical evidences. A relaxation of the assumption of rational expectation by allowing "Internal Rationality" agents, however, is able to quantitatively replicate the empirical relationship between riskless interest rate and stock price. The intuition is that stock price here is mostly driven by agents' subjective beliefs, not by riskless interest rate which is the only source of stock price fluctuation in rational expectation equilibrium.

About future studies, there are two directions. The first one is that as shown in section 8 monetary policy is not such powerful in controlling asset price. The flutuation in my model is mostly driven by the high volatility of agents' subjective beliefs. However, perhaps it is not such realistic. Hence, it is important to further explore the importance of riskless interest rate on asset price. The second direction is to research how monetary policy should respond to asset price fluctuation through introducing the sticky price and Taylor rule into my learning model.

10. Appendix

10.1 Data Sources

The data about stock market behavior is downloaded from Robert Shiller's webpage (http://www.econ.yale.edu/~shiller/data.htm). Stock price is represented by "S&P 500 Composite Price Index". I directly take use of real stock index and real dividend calculated by Shiller and you can also find the details about calculation in the same webpage. The monthly data of stock index are transformed into quarterly by taking the value of the last month of the corresponding quarter. But quarterly dividend is computed as aggregating the dividends of three months of the considered quarter since the dividend is flow variable.

The real riskless interest rate is using 3-month Treasury Bill deflated by U.S. Consumer Price Index. The method of transforming monthly data into quarterly one is the same as stock index. These data is downloaded from the dataset of Federal Reserve Bank St. Louis. The sample period for me to compute coefficients and variance decomposition is from 1947 Q1 to 2013 Q4.

At the same time, in order to calibrate collateral ratio U.S. current account data is also downloaded from FRB St. Louis. And for the total value of U.S. stock market I use "Market capitalization of listed companies", which can be found in database of World Bank (http://data.worldbank.org/). Here I use the annual data and the sample is from 1988 to 2012.

10.2 Variance Decomposition

Introduce the method of variance decomposition adopted in Campbell (1991) and Campbell and Ammer (1993). Theoretically the excess return e_{t+1} of the stock hold from the end of period t to period $t+1$ relative to the return on short bond can be expressed as following

$$e_{t+1} - E_t e_{t+1} = (E_{t+1} - E_t)\left\{\sum_{j=0}^{\infty} \phi^j \Delta d_{t+1+j} - \sum_{j=0}^{\infty} \phi^j r_{t+1+j} - \sum_{j=0}^{\infty} \phi^j e_{t+1+j}\right\} \quad (24)$$

where e_t is excess return, d_t is dividend and r_t is short-term real interest rate.

To simplify the notation, equation (22) can be written as

$$e_{t+1}^{\sim} = e_{d,t+1}^{\sim} - e_{r,t+1}^{\sim} - e_{e,t+1}^{\sim} \quad (25)$$

where $e_{t+1}^{\sim}$ is the unexpected component of the excess return e_{t+1}, $e_{d,t+1}^{\sim}$ the news about future dividend, $e_{r,t+1}^{\sim}$ news about future real interest rate and $e_{e,t+1}^{\sim}$ to be the term representing news about future excess return.

Therefore, the variance of excess stock return can be decomposed as

$$\begin{aligned} Var(e_{t+1}^{\sim}) &= Var(e_{d,t+1}^{\sim}) + Var(e_{r,t+1}^{\sim}) + Var(e_{e,t+1}^{\sim}) && (26) \\ &\quad -2Cov(e_{d,t+1}^{\sim}, e_{r,t+1}^{\sim}) - 2Cov(e_{d,t+1}^{\sim}, e_{e,t+1}^{\sim}) + 2Cov(e_{r,t+1}^{\sim}, e_{e,t+1}^{\sim}) && (27) \end{aligned}$$

These variables are directly unobservable but can be discovered from Vector-Autoregression. Write z_t as the state vector containing excess return e_t, real interest rate r_t and price-dividend ratio $\frac{P_t}{D_t}$

$$z_t = [e_t, r_t, \frac{P_t}{D_t}]'$$

The first-order VAR model is

$$z_{t+1} = Az_t + w_{t+1} \quad (28)$$

With this VAR system $e^{\sim}_{t+1}$, $e^{\sim}_{r,t+1}$ and $e^{\sim}_{e,t+1}$ become

$$e^{\sim}_{t+1} \equiv e_{t+1} - E_t e_{t+1} = e1' w_{t+1} \tag{29}$$

$$e^{\sim}_{e,t+1} \equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} = e1' \sum_{j=1}^{\infty} \rho^j A^j \epsilon_{t+1} = e1' \rho A (I - \rho A)^{-1} \epsilon_{t+1} \tag{30}$$

$$e^{\sim}_{r,t+1} \equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r_{t+1+j} = e2' \sum_{j=1}^{\infty} \rho^j A^j \epsilon_{t+1} = e2' \rho A (I - \rho A)^{-1} \epsilon_{t+1} \tag{31}$$

where $e1$ and $e2$ are the first and second column of 3×3 identity matrix respectively. Then, $e^{\sim}_{d,t+1}$ can be treated as residual:

$$e^{\sim}_{d,t+1} = e^{\sim}_{t+1} + e^{\sim}_{r,t+1} + e^{\sim}_{e,t+1} \tag{32}$$

After recovering these unobservable variables, equation (26) is used to compute results on variance decomposition.

10.3 Projection Facilities

$$\omega^{\beta}(x) = \left\{ \begin{array}{cc} x & \text{if } x \leq \beta^L \\ \beta^L + \frac{x - \beta^L}{x + \beta^U - 2\beta^L} (\beta^U - \beta^L) & \text{if } \beta^L < x \leq \beta^U \\ \beta^L + \frac{1}{2}(\beta^U - \beta^L) \text{ if } x > \beta^U \end{array} \right\} \tag{33}$$

$$\omega^{m}(x) = \left\{ \begin{array}{cc} x & \text{if } x \leq m^L \\ m^L + \frac{x - m^L}{x + m^U - 2m^L} (m^U - m^L) & \text{if } m^L < x \leq m^U \\ m^L + \frac{1}{2}(m^U - m^L) \text{ if } x > m^U \end{array} \right\} \tag{34}$$

For simplification, I assume $\beta^L = m^L$ and $\beta^U = m^U$ because the only difference between two beliefs are term $(\frac{C_{t+1}}{Ct})^{-\gamma}$, which is sufficiently close to 1. And the values adopted in Adam, Marcet and Nicolini (2013) of these two thresholds are used here. However, being different from their paper the presence of time-varying interest rate R_t produces the problem that projection facilities above cannot surely guarantee the price-dividend ratio to locate in the interval between 0 and 500. Even though the event that price-dividend ratio jumps out the interval is rare in the sample (because of the projection facilities), it can produce significant errors in calculating second moments of stock market. Hence, constraints on simulated stock price are imposed here as

$$P_t = \left\{ \begin{array}{c} P_t \text{ if } 0 < \frac{P_t}{D_t} < 500 \\ P_{t-1} \quad \text{if } \frac{P_t}{D_t} \leq 0 \\ 500 * D^t \text{ if } \frac{P_t}{D_t} \geq 500 \end{array} \right\} \tag{35}$$

10.4 Simulation Method

I compute simulated moments and variance decomposition of theoretical model following Monte-Carlo procedure. The number of samples is set to $K = 1000$ and each sample has $N = 320$ periods matching stock market behavior in table 4 from 1925 Q4 to 2005 Q4. In each sample, I first simulate the model to generate artificial data and calculate considered moments, coefficients and variance decomposition. Then, final values of these are taking the average of K samples'.

10.5 Analysis of Real Interest Rate's Effect on Stock Price

I can write down the explicit expression for the second term of numerator in equation (23) as

$$\begin{aligned} & E^F(\frac{C_{t+1}}{C_t})^{-\gamma}E^F\frac{P_{t+1}}{P_t}E^F\frac{P_t}{P_{t-1}}+E^F\frac{P_t}{P_{t-1}}cov^F((\frac{C_{t+1}}{C_t})^{-\gamma},\frac{P_{t+1}}{P_t})- \\ & E^F(\frac{C_t}{C_{t-1}})^{-\gamma}E^F\frac{P_t}{P_{t-1}}E^F\frac{P_{t+1}}{P_t}-E^F\frac{P_{t+1}}{P_t}cov^F((\frac{C_t}{C_{t-1}})^{-\gamma},\frac{P_t}{P_{t-1}}) \\ = & E^F\frac{P_t}{P_{t-1}}cov^F((\frac{C_{t+1}}{C_t})^{-\gamma},\frac{P_{t+1}}{P_t})-E^F\frac{P_{t+1}}{P_t}cov^F((\frac{C_t}{C_{t-1}})^{-\gamma},\frac{P_t}{P_{t-1}}) \end{aligned} \tag{36}$$

If we assume risk-premia is time-invariant, equation (23) becomes

$$\frac{d(\frac{P_t}{P_{t-1}})}{d\lambda_2}=\frac{(1-\lambda_1 h)(m_t-m_{t-1})}{(1-\lambda_1\beta_t-\lambda_2 m_t)^2} \tag{37}$$

where $h=cov^F((\frac{C_{t+1}}{C_t})^{-\gamma},\frac{P_{t+1}}{P_t})<0\forall t$. Then it is easy to prove that

$$\frac{d(\frac{P_t}{P_{t-1}})}{d\lambda_2}\left\{\begin{array}{l} >0 \text{ if } m_t>m_{t-1} \\ <0 \text{ if } m_t<m_{t-1} \end{array}\right\}$$

Even though risk premia is not constant, the derivative expression is

$$\frac{d(\frac{P_t}{P_{t-1}})}{d\lambda_2}=\frac{(m_t-m_{t-1})+\lambda_1(m_{t-1}h_t-m_t h_{t-1})}{(1-\lambda_1\beta_t-\lambda_2 m_t)^2}$$

In order to make my results hold here, require $cov(h_t,m_t)>0$. Lettau and Ludvigson (2001) shows the positive correlation between stock price and time-varying risk prima using log consumption-wealth ratio as proxy for the latter.

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