A new financial metric for the art market

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23 September 2013

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(Working Paper, April 2014)

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Abstract

This paper introduces a new financial metric for the art market. The metric, which we call Artistic Power Value (APV), is based on the price per unit of area (dollars per square centimeter) and is applicable to two-dimensional art objects such as paintings. In addition to its intuitive appeal and ease of computation, this metric has several advantages from the investor's viewpoint. It makes it easy to: (i) estimate price ranges for different artists; (ii) perform comparisons among them; (iii) follow the evolution of the artists' creativity cycle overtime; and (iiii) compare, for a single artist, paintings with different subjects or different geometric properties. Additionally, the APV facilitates the process of estimating total returns. Finally, due to its transparency, the APV can be used to design derivatives-like instruments that can appeal to both, investors and speculators. Several examples validate this metric and demonstrate its usefulness.

Background

In the last thirty years, the art market −and more precisely, the market for paintings−has received an increasing amount of attention from economists, financial analysts, and investors. They have brought to this field many quantitative techniques already employed in more conventional markets. Not surprisingly, one topic that has received a great deal of attention is returns, specifically, how to compute returns for the art market. This is a challenging task not only because this market is still rather illiquid, at least compared with equities and bonds, but also because of its heterogeneity: every painting is essentially a unique object.

Several authors have employed hedonic pricing models (HPMs) to estimate returns (e.g., Chanel et al., 1994, 1996; de la Barre et al., 1994; Edwards 2004; Renneboog and Spaenjers 2013). Such models are suitable to manage product variety and can use all the available data. Their drawback, however, is that their application is limited by the explicatory power of the variables selected and sometimes it is difficult to fit a good model to the data (the academic literature frequently reports models with values of R^2 around 60% or below). Moreover, if the data are sparse (a common situation, especially for individual artists) the application of HPMs might not be possible (Galbraith and Hodgson 2012). An additional disadvantage of HPMs is the lack of stability that often affects the computation of the hedonic regression coefficients, coupled with the lack of reliability −not to mention the not-so-straightforward interpretation− of the time dummies (Collins et al., 2007). Finally, price indices based on the time-dummies do not satisfy the monotonicity condition --an essential requirement for any price index (Fisher, 1922; Melser, 2005). This is a critical issue for violation of this condition might lead to spurious returns, a fact that the cultural economics community has not yet acknowledged.

A second alternative to estimate returns is to rely on repeat sales regressions (e.g., Anderson 1974; Baumol 1986; and Goetzmann 1993). While this approach has the advantage of using price data referring to the same object it has two disadvantages: a potential selection bias and the fact that it only employs a small subset of the available information. Ginsburgh et al. (2006) provide an excellent discussion on the merits of each approach plus a fairly complete literature review. Mei and Moses (2002); Renneboog and Spaenjers (2011); Higgs and Worthington (2005); Agnello and Pierce (1996); and artnet Analytics (2012) have dealt with the construction of art indices based on the two above-mentioned techniques or hybrid combinations of them.

The question of which approach is better to estimate returns still remains open. This issue is far more vexing than it appears. Superficially, it might be interpreted as a choice between two methods that lead to the same answer based on computational ease. However, there is no assurance that this is indeed the case. In fact, they might lead to different answers and it is not always clear which answer is the right one. Ashenfelter and Graddy (2003) have stated this point more forcefully: 'The hedonic index gives a real return of about 4 percent, while the repeat-sales index results in a real return of about 9 percent! Which is correct?'

Previous researchers have also focused on other topics. Just to name a few: Galenson (1999); Galenson (2000); Galenson (2001); Galenson and Weinberg (2000); and Ginsburgh and Weyers (2006) have looked at the creativity cycle of several artists (that is, the age at which they produced their best work). Renneboog and Van Houte (2002); Worthington and Higgs (2004); Renneboog and Spaenjers (2011); and Pesando (1993) have compared the returns of certain segments of the art market vis-à-vis more conventional investments. Coate and Fry (2012) and Ekelund et al. (2000) have investigated the death-effect in the price of paintings. Edwards (2004) and Campos and Barbosa (2009) have looked at the performance of Latin American painters. Scorcu and Zanola (2011) used a hedonic model approach to study Picasso's paintings, while Higgs and Forster (2013) investigated whether paintings which conformed to the golden mean commanded a price premium. And, Sproule and Valsan (2006) questioned the accuracy of hedonic models compared with the appraisals of experts.

Other issues that have been investigated, some of them still with inconclusive answers, are: whether the lack of signature affects the auction price of a painting; the importance of the auction house (in essence, Sotheby's or Christie's versus lesser known auction houses); whether masterpieces tend to underperform when compared to less expensive paintings; the correlation between the art market and the major equity and fixed income indices; whether an artist can be described, based on its creativity-cycle curve, as conceptual (early bloomer) or experimentalist (late bloomer); as well as the relationship between, withdrawing a painting from an auction, and its future sale price. All these analyses have relied on statistical and modeling techniques commonly used in financial and economic analysis.

In summary, although a great deal has been learned about the financial aspects of the art market in recent years, much needs to be understood, especially, from the investor's perspective. Therefore, the purpose of this paper is to contribute to this effort by introducing a new financial metric that can facilitate the understanding of some of the issues already

mentioned. In addition, we want to shift the focus towards the investor's viewpoint and move away from the purely econometric models which, even though are interesting from an academic angle, offer little guidance to somebody concerned with pricing issues. Thus, our goal is twofold: (i) to provide a new tool to enrich the analysts' toolbox; and (ii) to facilitate the investors' decision-making process by making it easier to assess the merits of a painting using some simple quantitative analyses.

We should note that the application of HPMs and repeat sales regression models has so far focused, mainly, on estimating market returns aimed at building indices. Although these indices can be useful for performing econometric analyses and describing market tendencies, in general, they are less useful for investors. The chief reason is that investors are concerned with actual or realized returns (that is, total returns) instead of returns based on an ideal painting whose characteristics do not change over time (which is the case of time-dummies based returns). To put the point more forcefully: an investor has little use for an index that controls for quality and paintings' characteristics. In fact, the investor wants information that actually captures these features as well as supply-demand dynamics. The metric introduced herein (a point we discuss in more detail later) captures exactly that.

A New Financial Metric

Paintings, notwithstanding their artistic qualities, are essentially two-dimensional objects that can command −sometimes− hefty prices. Based on this consideration, it makes sense to express the value of a painting not using its price but rather a price per unit of area (in this study, dollars per square centimeter). We call this figure of merit Artistic Power Value or APV. By normalizing the price, the APV metric intends to offer the investor a financial yardstick that goes beyond the price, while not attempting to control for the specifics of the painting beyond its area.

The intuitive appeal of this metric is obvious: simplicity, ease of computation, transparency, and straightforwardness. In fact, there is already a well-established precedent for this approach. For example, prices of other two-dimensional assets, such as raw land, are frequently quoted this way (e.g. dollars per acre, or euros per hectare). The same approach is sometimes used to quote prices of antique rugs.

More recently, many artisans, print makers, digital printing firms, and poster designers have started to quote price estimates using this same concept. Moreover, considering that the cost of materials (an important component of the production cost) employed in creating these

two-dimensional objects is often estimated on a per-unit-of-area basis, the idea of extending the same notion to express the value of the final product is not far-fetched.

Finally, the rationale for using the APV metric is not to negate the individuality of each painting or to trivialize the artistic process. It is really an attempt to synthetize in one parameter the financial value of a painting (or artists or body of work) with the goal of making comparisons easier. Additionally, many APV-based computations (a point treated in more detail in the subsequent section) can offer useful guidance for pricing purposes.

Alternatively, we can think of the APV as an attempt to find a common factor to compare and contrast the economic value of otherwise dissimilar art objects. If we accept the thesis that two paintings −even if they are done by the same artist and depict the same theme− are not only different but also unique, it is not possible to make a straight price-wise comparison. However, the APV metric, by virtue of removing the size-dependency, helps to make this comparison possible: in a sense the APV plays the role of unitary price.

The Data

Three data sets are employed in this study:

a.   Data set A consists of 1,820 observations of Pierre-Auguste Renoir's paintings auction prices and their characteristics covering the period [March 1985; February 2013]. The database was built based on information provided by the artnet database (www.artnet.com).

b.   Data set B consists of 441 observations of Henri Matisse's paintings auction prices and their characteristics covering the period [May 1960; November 2012]. The database was built based on information provided by the artnet database (www.artnet.com) and was supplemented by additional auction data from the Blouin Artinfo website (www.artinfo.com).

c.   Finally, data set C consists of 2,115 observations of paintings covering the period [March 1985; February 2013]. This data set gathers information from six artists (Alfred Sisley, Camille Pissarro, Claude Monet, Odilon Redon, Paul Gauguin, and Paul Signac) and was based on auction information provided by the artnet database.

All prices were adjusted to January-2010 U.S. dollars (using the U.S. CPI index) and are expressed in terms of premium prices (when hammer prices were reported, they were modified and expressed in terms of equivalent premium prices). Observations where the

selling price was below US$ 10,000 or the APV was less than 1 US$/cm^2 were eliminated. Sotheby's and Christie's dominate the data sets, as together they account for 86% of the sales.

The selection of artists was somewhat arbitrary. The chief consideration was to effectively examine the merits of the APV metric without regard to the qualities of the painters selected. Renoir was an ideal choice because of the high number of observations available, which were distributed over a long period of time, and without time-gaps. This situation facilitates the comparison between the APV metric and the HPMs (which require many data points to be built). Matisse data had the advantage of being distributed over a longer time span, but included less observations, and had a few time-gaps. Data set C, despite its strong impressionist flavor, was not aimed at capturing in full the characteristics of the impressionist movement; it represents a group of painters who happened to live roughly at the same time and for which there were enough observations to make certain computations feasible. Nevertheless, and simply for convenience, in what follows we refer to this group as the Impressionists group. Renoir, despite his strong impressionist credentials was purposely left out of data set C. Otherwise, he would have dominated the group, making it highly correlated with data set A: an undesirable situation given the need to test the APV metric under different scenarios.

In summary, the selection of artists was not done with the idea of deriving any specific conclusion regarding these painters or the artistic tendencies they represented; the leading consideration was to showcase the attributes and benefits of the APV metric.

Table 1 summarizes the key features of the three data sets. **Table 2** describes in more detail the characteristics of the painters in the Impressionist group (data set C). Notice that the APV distribution is far from normal: the differences between the arithmetic mean (average) values and the medians are manifest, with the means always higher than the medians. Additionally, the values of the skewness and kurtosis reveal a strong positively skewed distribution with fat tails. The Jarque-Bera (JB) statistic and its corresponding p-value (close to 0.000 for each of the three data sets) indicate that the APV is not normally distributed. These facts should serve as a warning against APV-based projections based on normality assumptions. Finally, the relatively high values of the coefficient of variation for several artists (Renoir and Matisse exhibit the most variability) are somehow evidence of what experts already know: even masters are uneven producers and their paintings differ greatly in quality. Whether ranking artists by their average or median APV values is

consistent with the critics' assessment of their merits, it is a topic we leave for others to decide

Table 1. Description of the three data sets and key statistics

	Data Set: A	Data Set: B	Data Set: C
Artist	Pierre-Auguste Renoir	Henri Matisse	Impressionists group
Born–Died	1841–1919	1869–1954	NA
Number of Sales	1,820	441	2,121
Period of Sales	Mar 1985–Feb 2013	May 1960–Nov 2012	Mar 1985–Feb 2013
Geometric Mean APV (US$/cm^2)	399	356	312
Median APV (US$/cm^2)	377	308	311
Average APV (US$/cm^2)	646	803	537
Standard Deviation (US$/cm^2)	1,331	1,332	786
Coefficient of Variation	2.06	1.66	1.46
Skewness	15.56	3.87	4.86
Kurtosis	344.06	19.83	31.87
Jarque-Bera	9,040,581.38	8,328.44	97,801.30
JB p-value	0.000	0.000	0.000

Table 2. Detailed characteristics and key statistics of the artists included in data set C

Artist	Number of Sales	Born–Died	Average APV (US$/cm^2)	Standard Deviation (US$/cm^2)	Coeff. of Variation	Geometric Mean APV (US$/cm^2)	Median APV (US$/cm^2)
Alfred Sisley	343	1839–1899	389	282	0.73	311	313
Camille Pissarro	586	1839–1903	432	335	0.78	324	338
Claude Monet	586	1840–1926	760	999	1.31	422	411
Odilon Redon	193	1840–1916	167	156	0.93	109	118
Paul Gauguin	167	1848–1903	1,138	1,631	1.43	539	465
Paul Signac	247	1863–1935	353	454	1.28	217	202

Applications of the APV Metric

This section is intended to demonstrate the usefulness of the APV metric with the help of some examples.

Comparisons Among All Artists

The fact that the APV follows a highly non-normal distribution calls for comparisons to be based on the median rather than the average value. To this end we employ the median comparison test using the Price-Bonett variance estimation for medians (Price and Bonett 2001; Bonett and Price 2002), described in Wilcox's (2005) review of methods for comparing medians.

Table 3.   Comparisons among the APV medians for all artists (1985-2012 sales only)

Median APV (diagonal) Difference between medians (off-diagonal) (US$/cm²)	Henri Matisse[a]	Paul Gauguin	Claude Monet	Pierre-Auguste Renoir	Camille Pissarro	Alfred Sisley	Paul Signac	Odilon Redon
Henri Matisse[a]	513							
Paul Gauguin	NS	465						
Claude Monet	102**	NS	411					
Pierre-Auguste Renoir	136***	88*	34*	377				
Camille Pissarro	175***	127**	73***	39***	338			
Alfred Sisley	200***	152**	98***	64***	25*	313		
Paul Signac	311***	263***	209***	175***	136***	111***	202	
Odilon Redon	395***	347***	293***	259***	220***	195***	84***	118

NOTE: [a]: Median calculated from sales between 1985-2012 only;

NS: Not Significant; *p<.10; **p<0.05; ***p<0.01

Table 3 summarizes the results of such comparison. The median values for each artist are shown along the diagonal with the values decreasing from top-left to bottom-right: Matissse[1] has the highest value (513 US$/cm²) while Redon the lowest (118 US$/cm²). The remaining entries in the table can be interpreted, using matrix notation, as follows: the (i, j) entry represents the median APV value of artist j minus the median APV value of artist i. Hence, Pissarro's median APV exceeds that of Signac by 136 US$/cm² while there is no significant difference between Gauguin and Matisse's median APVs.

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[1] In order to have similar periods for all comparisons among artists, we only considered the sales between 1985 and 2012 for Matisse.

These calculations, trivial by all accounts, offer a convenient way to rank artists. They also offer useful guidance for pricing purposes.

Vertical versus Horizontal Orientation for a Given Artist

Table 4. Comparisons of APV medians: vertical versus horizontal oriented paintings for each artist

Artist	Vertical		Horizontal		Vertical versus Horizontal	
	Number of Sales	Median APV (US$/cm^2)	Number of Sales	Median APV (US$/cm^2)	Difference US$/cm^2	P-Value
Alfred Sisley	21	298	321	317	-19	NS
Camille Pissarro	132	327	450	346	-19	NS
Claude Monet	124	352	440	426	-74	<0.10
Henri Matisse	203	498	237	199	299	0.000
Odilon Redon	133	131	53	84	47	<0.01
Paul Gauguin	81	580	86	328	252	<0.05
Paul Signac	23	129	224	212	-83	<0.05
Pierre-Auguste Renoir	843	505	949	289	216	0.000

NOTE: Paintings with height=width are excluded from the table. NS: Not significant.

Certain painters, Modigliani for instance (not part of this study) decidedly preferred the vertical orientation. Sisley and Signac, on the contrary, favored the horizontal orientation. **Table 4** compares, for all the artists considered here, the median APV as a function of the orientation using the median-comparison algorithm already described. The results are interesting and far from obvious. In the case of Sisley and Pissarro, the painting orientation does not affect the APV in a significant way. In the case of Matisse and Renoir, the difference in median APV values is highly relevant. More interesting is the fact that even though both were much better at doing vertical-oriented paintings, they did not seem to favor this orientation. They both painted −according to these sets of observations− roughly the same number of vertical-oriented paintings and horizontal-oriented paintings (203 and 237 in the case of Matisse; 843 and 949 in the case of Renoir). Finally, Monet and Signac were better at doing horizontal-oriented paintings, at least as seen by the market.

In conclusion, the orientation of a painting, in most cases, has a definite influence on its market value.

Comparisons of Different Subjects for the Same Artist

Tables 5, 6, and 7 display the median APV value, for each artist, as a function of three dummy variables, namely: (i) Still life; (ii) Landscapes and (iii) People (whether the painting shows one or several human figures regardless of the amount of detail); 0 refers to the absence of the condition.

Clearly, certain artists are more appreciated for certain topics: Redon (see **Table 5**) is more valued when executing still lives while the opposite happens with Renoir. Landscapes painted by Matisse, Gauguin, and Renoir (**see Table 6**) are less desirable than other themes. And Gauguin, Renoir, and Matisse (see **Table** 7) commanded higher prices when their paintings included people. These considerations are useful when appraising paintings.

Table 5. Comparisons of APV medians: still-life versus no-still-life for each artist

Artist	Subject: Still-Life=Yes		Subject: Still-Life=No		Difference US$/cm^2	P-Value
	Number of Sales	Median APV (US$/cm^2)	Number of Sales	Median APV (US$/cm^2)		
Alfred Sisley	NA	NA	NA	NA	NA	NA
Camille Pissarro	NA	NA	NA	NA	NA	NA
Claude Monet	59	279	527	424	-145	<0.05
Henri Matisse	69	335	372	308	27	NS
Odilon Redon	58	214	135	86	129	0.000
Paul Gauguin	24	821	143	411	409	<0.05
Paul Signac	NA	NA	NA	NA	NA	NA
Pierre-Auguste Renoir	364	302	1456	396	-94	0.000

NA: Not enough sales for this artist in this subject (<10 sales). NS: Not significant.

Table 6. Comparisons of APV medians: Landscape versus no-landscape for each artist

Artist	Subject: Landscape=Yes		Subject: Landscape=No		Difference US$/cm^2	P-Value
	Number of Sales	Median APV (US$/cm^2)	Number of Sales	Median APV (US$/cm^2)		
Alfred Sisley	283	311	59	321	-10	NS
Camille Pissarro	325	342	261	340	2	NS
Claude Monet	413	424	173	355	69	<0.10
Henri Matisse	143	161	298	459	-298	0.000
Odilon Redon	42	61	151	135	-74	0.000
Paul Gauguin	58	288	109	649	-361	0.000
Paul Signac	103	218	144	200	18	NS
Pierre-Auguste Renoir	478	267	1342	429	-162	0.000

NS: Not significant.

Table 7. **Comparisons of APV medians: people (one or many persons) versus no-people for each artist**

| Artist | Subject: People=Yes | | Subject: People=No | | Difference US$/cm^2 | P-Value |
	Number of Sales	Median APV (US$/cm^2)	Number of Sales	Median APV (US$/cm^2)		
Alfred Sisley	NA	NA	NA	NA	NA	NA
Camille Pissarro	71	267	515	348	-82	<0.05
Claude Monet	12	338	574	415	-77	<0.10
Henri Matisse	190	586	251	206	381	0.000
Odilon Redon	25	56	168	124	-67	<0.01
Paul Gauguin	31	1,115	136	388	727	<0.01
Paul Signac	NA	NA	NA	NA	NA	NA
Pierre-Auguste Renoir	817	528	1003	285	243	0.000

NA: Not enough sales for this artist in this subject (<10 sales).

Life-Cycle Creativity Patterns

The idea behind this concept is to explore how the quality of an artist's paintings (using the APV metric as a proxy) evolves over time. That is, as a function of the age at which the painting was executed. Or more precisely, identify the period(s) at which the artist produced its most valuable work (financially speaking).

Figures 1, 2, 3, and **4** display the median APV values, as a function of the age-at-execution for Renoir, Matisse, Monet, and Pissarro; i.e., the artists for whom we had more than 400 observations.

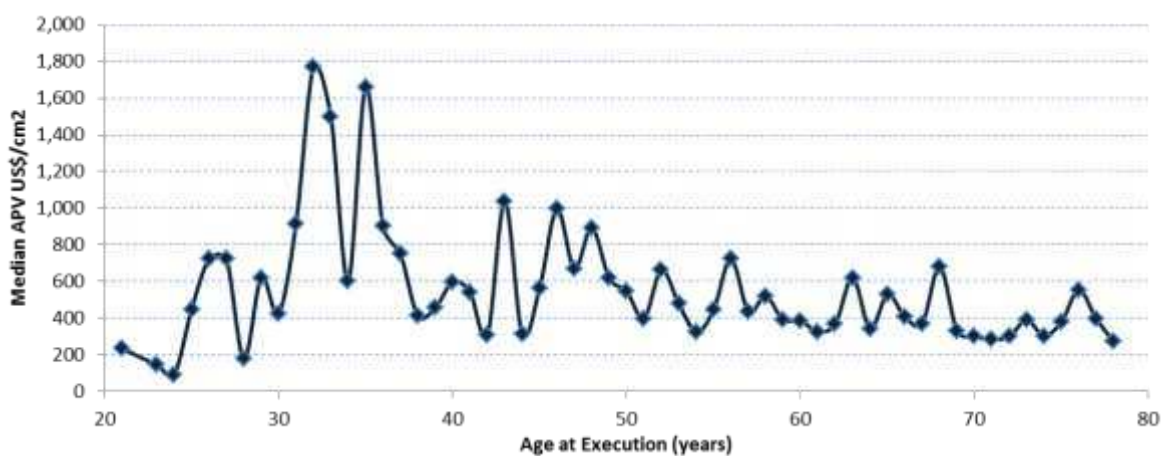**Figure 1.** **Pierre-Auguste Renoir Life-Cycle Creativity Curve**

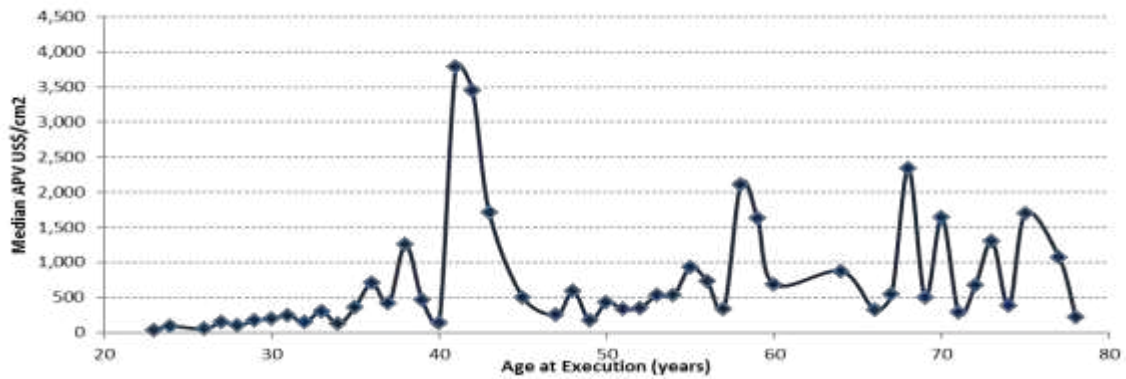**Figure 2.   Henri Matisse Life-Cycle Creativity Curve**

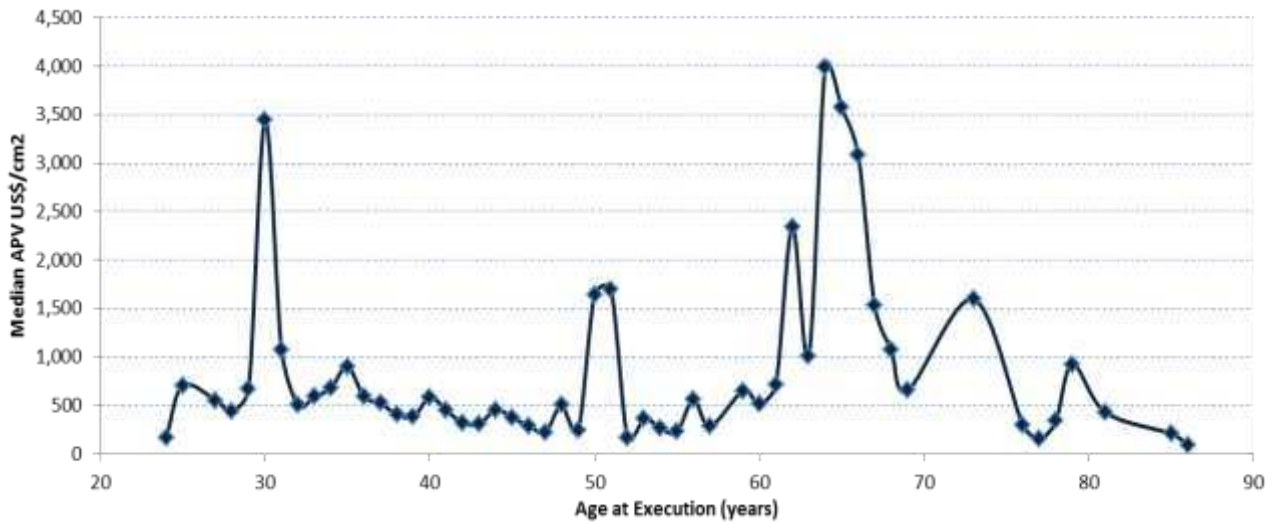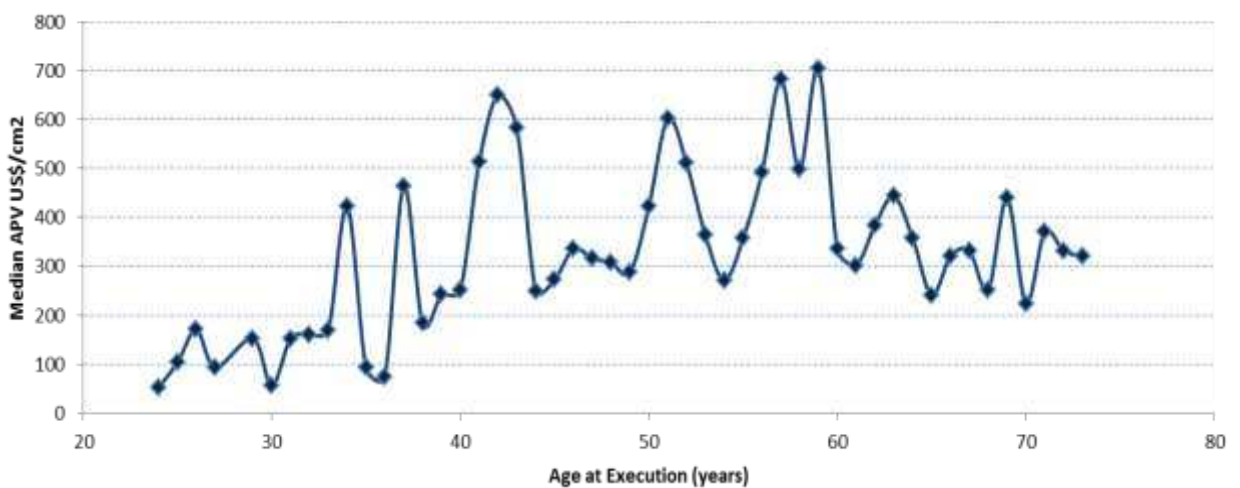**Figure 3.   Claude Monet Life-Cycle Creativity Curve**

Figure 4.   Camille Pissarro Life-Cycle Creativity Curve

The patterns shown are interesting as they reveal quite different tendencies. Renoir seems to have reached a peak around the mid-thirties and then experienced a slow decline. Matisse enjoyed a strong peak in his early forties, and a minor peak around his late fifties

followed by a sequence of peaks and valleys in his late years. Monet's career is marked by two salient peaks: an early one (when he was thirty) and a later one (in his mid-sixties) while Pissarro's life is characterized by a more jagged curve that exhibited no significant decline in his old age and is more regular than those of either Monet and Matisse. This situation is somewhat consistent with the fact that his coefficient of variation (0.78 from **Table 2**) is lower than that of Monet (1.31) and Matisse (1.66).

Returns for Different Artists or Group of Artists

Tables **8**, **9** and **10** present the year-to-year returns for Renoir, Matisse and the Impressionists (based on the information provided by data sets A, B and C respectively) along with other key values. Notice the salient peak APV values (at year 1989 and then around 2006) with their corresponding steep declines afterwards. They are consistent across the three data sets and are in agreement with trends already detected in the broader art market.

Return computations are straightforward. First, we compute for each year the geometric mean of the APV values (GM-APV). This is simply the nth root of the product of the APV-values associated with the n paintings sold during the year considered. Then, the year-to-year returns are computed based on the GM-APV values for two consecutive years. In short, the return between years i and $i+1$ is simply $[GM\text{-}APV_{i+1}/GM\text{-}APV_i] - 1$.

We have purposely carried out this calculation using the geometric mean and not the conventional arithmetic mean. We think that using the geometric mean is more reasonable since it is less sensitive to extreme values, something that becomes even more relevant when the distributions depart significantly from normality (which is the case with the APV).

Leaving aside the ease of computation (undoubtedly an attractive feature) a valid question needs to be answered: What does this return mean? The APV captures both, art market trends and supply-demand dynamics for the artist or artists considered, as it is based on actual sales. It does not intend to control the actual prices observed for any factor other than the area of the painting. Hence, the APV-based returns can be interpreted as total (actual or realized) returns for the artist or artists in question (inflation has been removed since prices are expressed in January 2010 dollars).

Table 8. Data set A: Pierre-Auguste Renoir, Key Statistics and Year-to-Year Returns

Year of Sale	Number of Obs.	Geometric Mean APV (US$/cm²)	Stand. Error Geom. Mean APV (US$/cm²)	95% Conf. Interval	Year-to-Year Return (APV)
1985	32	267	57	154 - 379	
1986	41	316	71	178 - 455	0.186
1987	83	436	94	252 - 620	0.377
1988	70	673	174	332 - 1015	0.546
1989	103	1043	258	537 - 1550	0.549
1990	93	919	217	493 - 1345	-0.119
1991	31	329	69	194 - 465	-0.642
1992	43	345	79	190 - 500	0.047
1993	56	338	90	162 - 514	-0.020
1994	45	309	51	208 - 410	-0.086
1995	75	279	60	162 - 396	-0.096
1996	69	244	53	140 - 348	-0.126
1997	75	351	90	174 - 528	0.438
1998	77	246	63	122 - 370	-0.299
1999	75	307	71	168 - 445	0.247
2000	75	340	79	187 - 494	0.110
2001	49	278	67	146 - 410	-0.183
2002	38	338	80	182 - 495	0.216
2003	44	328	64	202 - 454	-0.030
2004	63	309	68	177 - 442	-0.057
2005	79	368	55	261 - 475	0.190
2006	73	462	67	331 - 593	0.256
2007	94	516	96	328 - 704	0.117
2008	62	437	108	225 - 650	-0.153
2009	59	347	68	215 - 480	-0.205
2010	66	421	81	263 - 579	0.213
2011	66	382	81	222 - 542	-0.094
2012	84	384	88	211 - 557	0.006

*The 95% confidence interval was computed based on the standard error of the geometric mean for each year. We required a sample size of at least 10 observations to compute the confidence interval.

Table 9. Data set B: Henri Matisse, Key Statistics and Year-to-Year Returns

Year of Sale	Number of Obs.	Geometric Mean APV (US$/cm²)	Stand. Error Geom. Mean APV (US$/cm²)	95% Conf. Interval	Year-to-Year Return (APV)
1960	2	65	NA	NA	
1961	1	76	NA	NA	0.165
1962	4	88	NA	NA	0.156
1963	2	69	NA	NA	-0.214
1965	3	59	NA	NA	-0.574
1966	4	124	NA	NA	1.104
1968	4	123	NA	NA	-0.504
1970	7	206	NA	NA	-0.159
1971	5	41	NA	NA	-0.800
1972	10	242	409	NA	4.880
1973	7	344	NA	NA	0.418
1974	9	265	NA	NA	-0.227
1975	5	156	NA	NA	-0.413
1976	10	129	204	NA	-0.172
1977	9	198	NA	NA	0.536
1978	9	148	NA	NA	-0.251
1979	16	210	495	NA	0.413
1980	7	198	NA	NA	-0.056
1981	11	134	313	NA	-0.322
1982	10	130	263	NA	-0.033
1983	8	267	NA	NA	1.057
1984	8	215	NA	NA	-0.196
1985	11	233	542	NA	0.084
1986	10	212	889	NA	-0.091
1987	13	332	840	NA	0.569
1988	12	375	1,145	NA	0.130
1989	12	849	2,927	NA	1.264
1990	13	752	2,790	NA	-0.114
1991	5	511	NA	NA	-0.320
1992	9	712	NA	NA	0.392
1993	11	415	1,258	NA	-0.417
1994	6	246	NA	NA	-0.408

Table 9. Data set B: Henri Matisse, Key Statistics and Year-to-Year Returns (continued)

Year of Sale	Number of Obs.	Geometric Mean APV (US$/cm^2)	Stand. Error Geom. Mean APV (US$/cm^2)	95% Conf. Interval	Year-to-Year Return (APV)
1995	10	594	2,073	NA	1.420
1996	6	198	NA	NA	-0.668
1997	11	404	1,278	NA	1.044
1998	12	221	736	NA	-0.452
1999	12	503	1,193	NA	1.274
2000	7	769	NA	NA	0.528
2001	18	469	1,368	NA	-0.390
2002	8	756	NA	NA	0.613
2003	3	160	NA	NA	-0.788
2004	9	1,047	NA	NA	5.526
2005	7	450	NA	NA	-0.570
2006	8	1,143	NA	NA	1.540
2007	23	832	3,430	NA	-0.272
2008	20	813	2,470	NA	-0.023
2009	8	659	NA	NA	-0.189
2010	11	2,422	5,207	NA	2.673
2011	7	454	NA	NA	-0.813
2012	8	326	NA	NA	-0.281

*The 95% confidence interval was computed based on the standard error of the geometric mean for each year. We required a sample size of at least 10 observations and that standard error of the geometric mean less than geometric mean to compute the confidence interval.

Table 10. Data Set C: Impressionists Group, Key Statistics and Year-to-Year Returns

Year of Sale	Number of Obs.	Geometric Mean APV (US$/cm²)	Stand. Error Geom. Mean APV (US$/cm²)	95% Conf. Interval	Year-to-Year Return (APV)
1985	60	170	17	137 - 203	
1986	59	194	19	157 - 232	0.144
1987	85	286	24	239 - 333	0.471
1988	72	421	57	310 - 532	0.473
1989	149	719	51	618 - 820	0.707
1990	62	481	59	366 - 595	-0.332
1991	36	271	38	196 - 347	-0.436
1992	40	211	34	144 - 279	-0.221
1993	60	268	24	221 - 315	0.268
1994	61	206	25	156 - 255	-0.232
1995	81	221	26	169 - 272	0.072
1996	69	257	26	206 - 308	0.163
1997	87	277	29	221 - 333	0.079
1998	90	204	24	157 - 251	-0.264
1999	109	258	24	211 - 304	0.264
2000	80	306	40	229 - 384	0.189
2001	71	291	39	214 - 369	-0.049
2002	67	248	28	194 - 302	-0.150
2003	50	301	35	232 - 371	0.216
2004	75	293	31	232 - 355	-0.027
2005	86	290	28	236 - 344	-0.011
2006	89	376	38	302 - 450	0.296
2007	106	485	50	387 - 584	0.290
2008	89	415	49	319 - 511	-0.145
2009	62	276	40	198 - 354	-0.334
2010	73	329	37	257 - 402	0.193
2011	58	288	44	201 - 375	-0.125
2012	95	370	42	286 - 453	0.283

*The 95% confidence interval was computed based on the standard error of the geometric mean for each year. We required a sample size of at least 10 observations to compute the confidence interval.

Table 11. Year-to-year returns based on the APV plus other relevant metrics

APV	Data Set A: Renoir	Data Set B : Matisse	Data Set C: Impressionists
Average APV Return (per Year)	5.14%	34.11%	6.61%
Standard Deviation of Average Return	31.10%	118.26%	27.53%
Cumulative Return*	43.94%	400.5%	117.47%
Initial Year Geometric Mean APV (US\$/cm^2)**	267	65	170
Final Year Geometric Mean APV (US\$/cm^2)**	384	326	370

* Cumulative returns computed for 27 years for data sets A and C [1985-2012] and 52 years for data set B [1960-2012].

**Initial year-APV for data sets A and C is 1985 and for data set B is 1960. Final year-APV for all data sets is 2012

Table 11 summarizes the return results including both, average year-to-year returns, and cumulative returns for the relevant time-periods. The easiness with which one can compute these returns −contrasted, for example, with those estimated with HPMs (to be discussed later)− is striking.

Repeat Sales Vis-à-Vis the Entire (All-Sales) Data Set

Many analysts have estimated returns using only data from repeat sales. As pointed out before, a concern with this approach is that there could be a risk of selection bias. **Table 12** shows the median APV values for each of the artists considered using: (i) all the observations; and (ii) the repeat-sales subset. In two cases (Matisse and Renoir) the differences in medians are significant at the 5% level. And, in four of the remaining six cases the discrepancies are marginally significant (significant at the 10% level).

Finally, and somehow expectedly, the estimated returns (based, as before, on the geometric mean of the APV-values) are quite different for the two groups. The fact that in most cases the returns are higher when computed based on the repeat-sales set gives credibility to the hypothesis that paintings are more likely to be sold if they have increased in value.

These findings support the view that a selection bias cannot be ruled out when dealing with repeat-sales data. Thus, return estimates based on repeat-sales regressions (despite the claim that one has controlled for all the relevant factors) should be regarded with suspicion. The same goes for any other estimate based on repeat-sales information.

Table 12. Comparisons of APV medians and returns: all-sales versus repeat-sales for each artist

	All-sales			Repeat-sales		
Artist	Number of Sales	Median APV (US$/cm2)	Avg. Returns (per Year)	Number of Sales	Median APV (US$/cm2)	Avg. Returns (per Year)
Alfred Sisley	342	313	8.11%	118	327	19.84%
Camille Pissarro	586	338	8.01%	146	378	17.71%
Claude Monet	586	411	15.47%	176	476	27.54%
Henri Matisse	441	308	34.11%	160	249	24.00%
Odilon Redon	193	118	24.91%	36	91	35.06%
Paul Gauguin	167	465	42.93%	37	612	136.83%
Paul Signac	247	202	29.52%	90	180	23.79%
Pierre-Auguste Renoir	1,820	377	5.14%	426	425	10.11%

Validation of the APV Metric

A useful way to assess the validity of the new metric is to compare the results obtained with the APV and those obtained with the commonly used hedonic models.

Returns

First, we estimate individual HPMs for each of the three cases (Renoir, Matisse, and the Impressionists) using the entire corresponding data set. And second, we estimate the returns based on the time-dummies of the corresponding hedonic model.

The HPMs employ the natural logarithm of the painting selling price as the dependent variable. The independent variables (right-hand side of the regression equation) involve: (i) linear and higher-order polynomial expressions based on the age of the artist at the time the painting was executed; (ii) linear and higher-order polynomial expressions based on variables associated with the geometry of the painting; and (iii) dummy (binary) variables associated with the year the painting was sold, and, in the case of data set C, dummies to account for the identity of the painter. The return between two consecutive years, say $i+1$ and i, is estimated as $\exp(\beta_{i+1})/\exp(\beta_i)$ where the β's denote the time-dummy coefficients of the hedonic regression.

The corresponding adjusted R^2's (Renoir, Matisse, and Impressionists) are as follows: 0.75 (F= 137.47, $p<.0001$), 0.72 (F=18.78, $p<.0001$), and 0.72 (F= 99.87, $p <.0001$) respectively. In addition, we used White's (1980) test for heteroscedasticity and the null hypothesis of homoscedasticity in the least-squares residuals was not rejected in each of the three samples (results can be provided upon request).

Table 13. Year-to-year returns: averages, standard deviations, and correlations (APV and HPM)

	Data Set A: Renoir	Data Set B : Matisse	Data Set C: Impressionists
Average APV Return (per Year)	5.14%	34.11%	6.61%
Standard Deviation of Average APV Return	31.10%	118.26%	27.53%
Average HPM Return (per Year)	6.61%	17.00%	9.62%
Standard Deviation of Average HPM Return	26.59%	65.72%	31.13%
Correlation APV Return- HPM Return	0.90	0.85	0.92

Table 13 shows the comparison between the average year-to-year return estimated with (i) the APV metric; and (ii) the HPMs, as described before. In the case of Renoir and the Impressionists both returns are close. This fact is also consistent with the high correlation values reported, and the visual agreement displayed by the curves in **Figures 5** and **7**. In the case of Matisse, both return curves (**Figure 6**) show the same tendencies and trends.

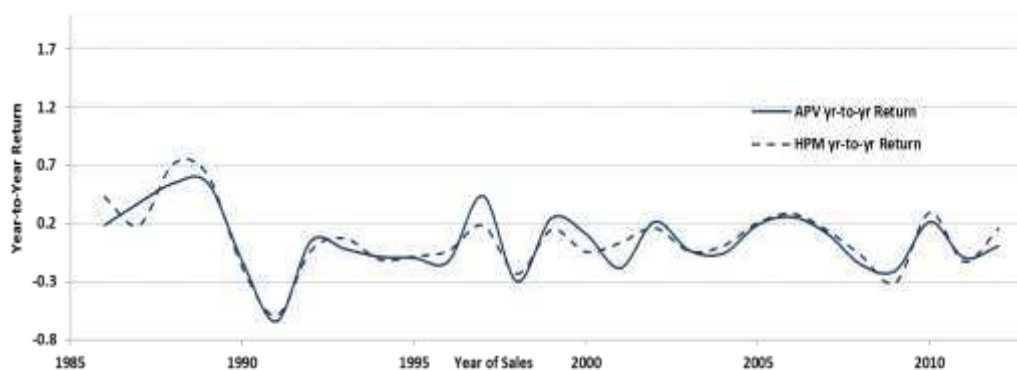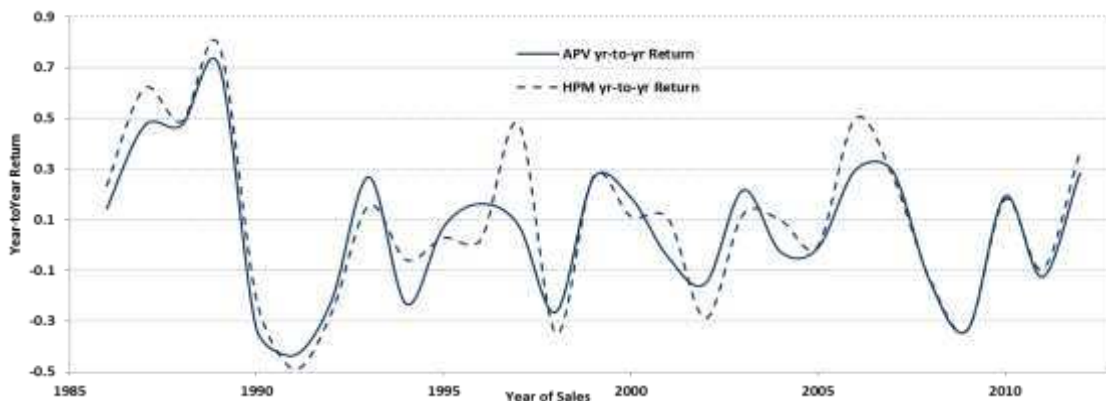**Figure 5.   Year-to-year (APV and HPM) returns for Pierre-Auguste Renoir sales.**

Figure 6.   **Year-to-year (APV and HPM) returns for Henri Matisse sales.**

Figure 7.   **Year-to-year (APV and HPM) returns for Impressionists group sales.**

However, the APV curve gives a better account of the peak values. This is in agreement with the high correlation value reported (85% from **Table 13**) and the well-known fact that time-dummies based-returns, since they take into consideration the entire dataset at once (52 years in this case), tend to mitigate the effect of peaks and valleys, and thus, render smoother curves. This explains, at least in part, the difference between the APV and the HPM returns.

The high degree of consistency might seem surprising. However, the following two observations can explain, appealing partly to intuition, the success of the APV: (1) regressing the logarithm of the price on just the logarithm of the area of the painting, for the case of Renoir, Matisse, and the Impressionists, we obtained adjusted R^2's values equal to 0.60, 0.35, and 0.51 respectively. Recall that the R^2's values of the corresponding hedonic models were 0.75, 0.72, and 0.72 respectively. Hence, the APV metric −for all its roughness and simplicity− is able to explain, just by itself, more than a half of what all the factors of the HPMs do; and (2) if we compute the correlation between the logarithm of the area of the paintings and the logarithm of the prices for all the artists considered (Sisley, Pissarro,

Monet, Matisse, Redon, Gauguin, Signac, and Renoir) we obtain the following (fairly high) values: 0.41; 0.73; 0.67; 0.59; 0.66; 0.64; 0.76, and 0.77 respectively. These observations provide some basis for making an argument that using the area of a painting as a normalization factor is not that eccentric or bizarre; it has some sound foundation.

Life-Cycle Creativity Patterns

Hedonic models have also been used in the past to investigate the age at which an artist produced its most valuable work. Typically, a HPM is fitted to the entire data available (which normally cover several years) and then the natural logarithm of the average price versus the artist's age-at-the-time-the-painting-was-executed, based on such model, is plotted. That is, the hedonic pricing equation is evaluated, for each age, using the average characteristics corresponding to that age.

Figure 8. **Life-Cycle Creativity Curve, Pierre-Auguste Renoir: Comparison between (i) Log of APV profile and (ii) Log of Price (from HPM) profile**
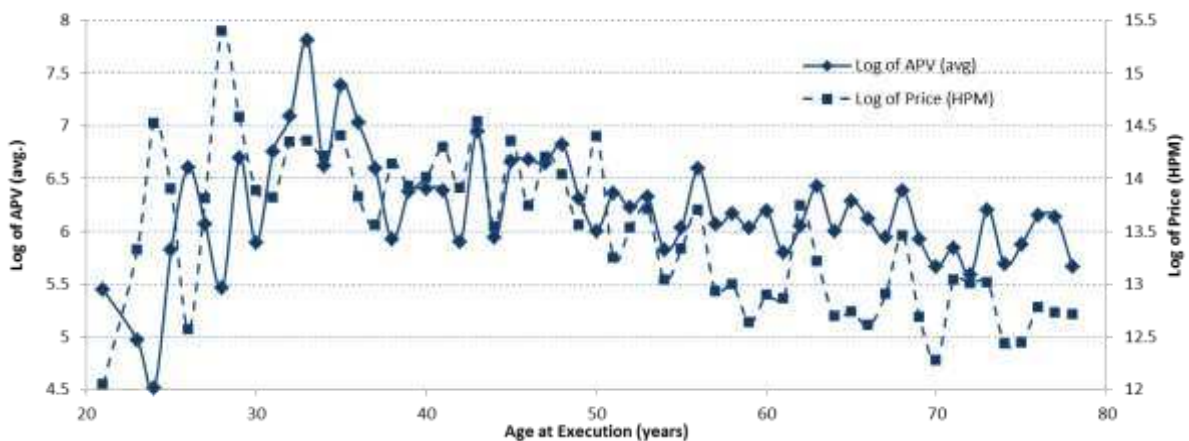
Figure 9. **Life-Cycle Creativity Curve, Henri Matisse: Comparison between (i) Log of APV profile and (ii) Log of Price (from HPM) profile**
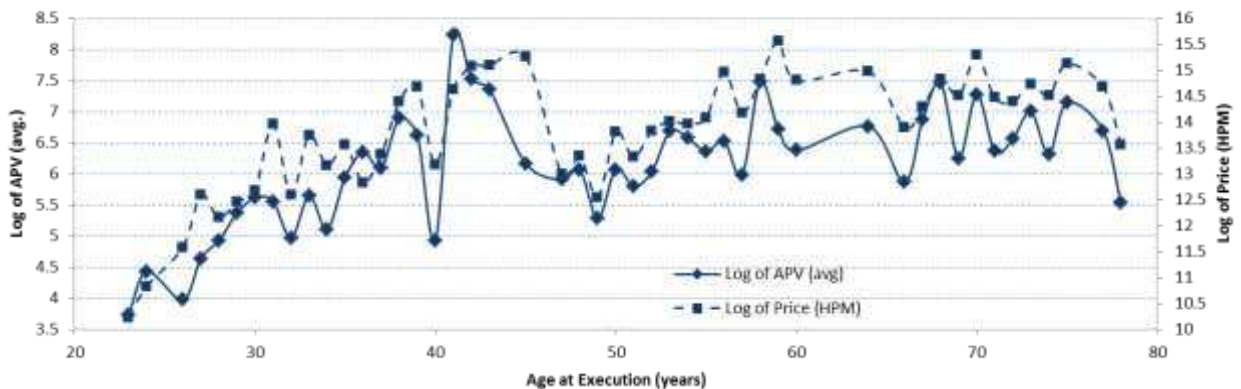
Figure 10. Life-Cycle Creativity Curve, Claude Monet: Comparison between (i) Log of APV profile and (ii) Log of Price (from HPM) profile
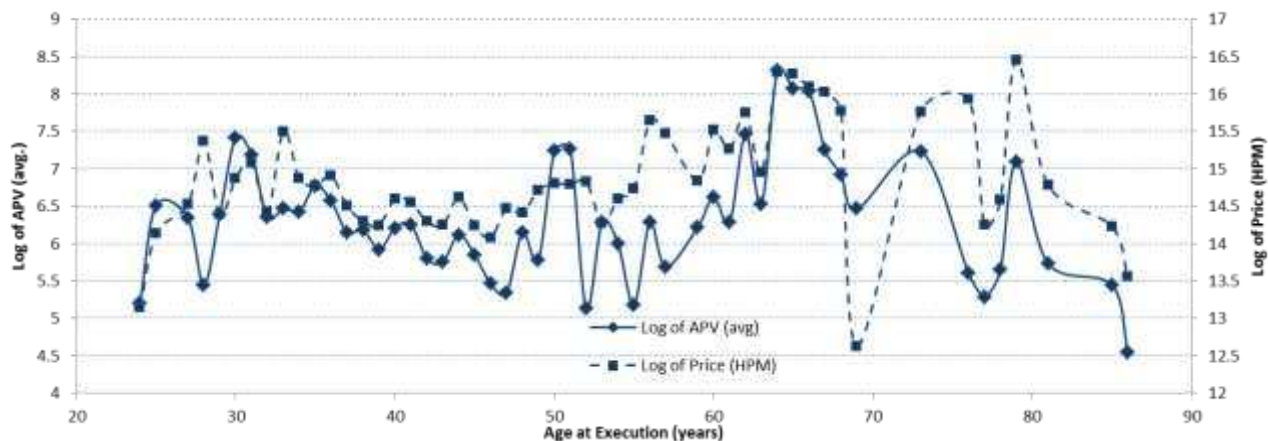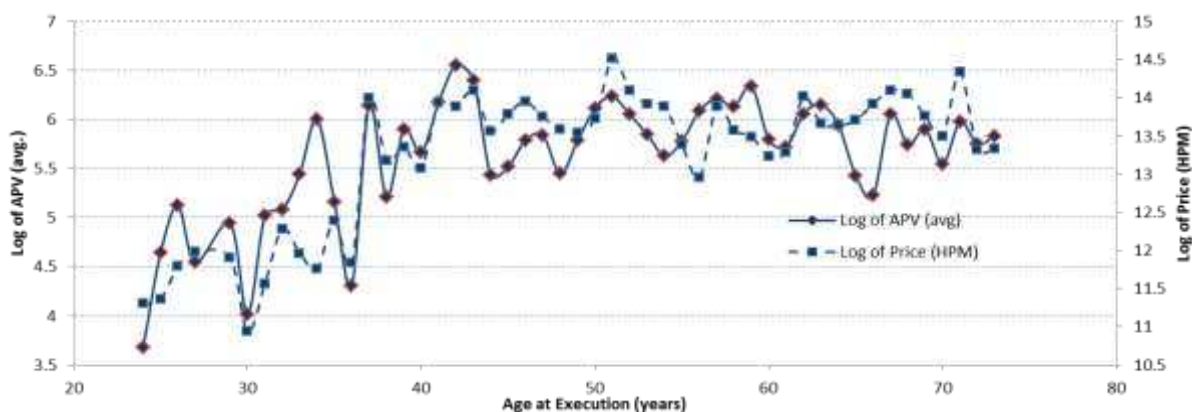

Figure 11. Life-Cycle Creativity Curve, Camille Pissarro: Comparison between (i) Log of APV profile and (ii) Log of Price (from HPM) profile

Figures 8, 9, 10, and **11** compare the curves obtained: (i) using the above-mentioned approach; and (ii) plotting the logarithm of the average APV versus age-at-execution. In this case we used the average APV rather than the median, since the HPM-based curves are normally done with the mean. The four artists considered were the only artists for whom we had more than 400 sales observations: Renoir, Matisse, Monet, and Pissarro. All four graphs show very consistent trends between the two curves. In essence, the HPM-curves do not seem to offer anything more than the simpler APV-based curves show.

A more interesting point becomes obvious when we compare these life-cycle curves with those displayed before in **Figs. 1, 2, 3,** and **4** which were obtained using the median APV instead of the log(average-APV) or log(average-price). Obviously, the first group of curves shows much more clearly the evolution of life cycle-patterns patterns. To some

extent, this is to be expected, as the log-function tends to mitigate the effect of peaks and valleys. Furthermore, this phenomenon calls into question the benefits of building these curves using the log-function (regardless of the underlying variable) instead of using the real thing, that is, the actual variable −for example the APV (with no log applied).

To sum up, the APV-based calculations, in all cases considered, yielded very similar results to those obtained with the hedonic models. This provides good evidence that the APV metric, despite its simplicity, offers results consistent with conventionally accepted methods.

Suggestions for Future Applications

APV-based Derivatives and Index Contracts

The market for paintings lacks a widely accepted index or indices that could be used to design derivatives contracts for hedging and/or speculative purposes. We reckon that the reason is that the most popular indices (Mei-Moses index, artnet.com family of indices, AMR indices, etc.) while effective for the purpose they were designed −namely, tracking broad market trends− are unsuitable for financial contracts. The reason is that they involve certain elements (proprietary databases, discretionary rules in terms of which sales should be included, ad hoc combinations of repeat sales techniques coupled with some undesirable features of HPMs) that make them opaque and −at least in theory− vulnerable to manipulation. In contrast, indices such as the S&P 500 or the Barclays Capital bond indices family −which are based on well-defined and transparent rules− are easy to reproduce and difficult to game. Not surprisingly, derivatives contracts based on these indices have enjoyed wide market acceptance.

We think that the APV metric provides a natural tool to create well-defined indices that could be the foundation for a derivatives art market. If one wishes to design an index to represent a specific market segment −for example, the Impressionists− the main point is to agree on the artists that should be part of the index. Once this issue is settled −a rule that must stay unaltered over time− what remains to agree upon is simply a mechanistic recipe to calculate the value of the index. For instance, it could be the average APV value of all the paintings sold in public auctions in the last twelve months as long as their values exceeded US$ 50,000.

A contract built around an index of this type could be used to gain exposure to this market or short it, in amounts much smaller than the typical price paid for a masterpiece. In

that sense, these types of contracts could help to expand the investor base, and contribute to improve market liquidity. The operational details are similar, for instance, to those encountered in the agricultural derivatives market or commodities markets. This topic is presently under investigation by the authors.

Testing the CAPM Validity in the Art Market

Several authors have investigated the validity of the CAPM model within the context of the art market. Although the results have been mixed we also think they have been irrelevant. The reason is that most authors —erroneously in our view— have placed on the left-hand side of the CAPM equation estimates of returns obtained, in general, via the time-dummy coefficients of a suitable hedonic model. We reckon that the correct approach is to place on the left-hand side of the CAPM equation estimates of total returns—not returns based on the time-dummies—which, at best, seem to capture (although this topic is still subject to debate) market returns. Total returns, of course, can be easily estimated with the APV metric.

This suggestion might sound strange until one realizes that, for instance, if we were to apply the CAPM model to, say, IBM's stock ,we would place on the left-hand side of the equation the return based on the price of IBM stock over some time period: in short, the total return. We would never place on the left-hand side the IBM stock return computed after controlling for whatever market factors might influence it (composition of revenue, number of employees, technology changes, etc.)

Moreover, most researchers never account for the fact that returns estimated via the time-dummies are just estimates, and therefore, subject to error. Obviously, this error translates itself into an additional error when estimating the CAPM's betas. Add to this the possibility of spurious return estimates as a result of the violation of the monotonicity condition, and inevitably one needs to wonder about the meaning or validity of such CAPM-related findings.

In summary, it is quite odd that the validity of the CAPM within the art market context has been carried out using returns that: (i) do not capture supply-demand changes from period-to-period; and (ii) could be contaminated by spurious effects due to the violation of the monotonicity condition.. At present, we are investigating this topic.

Conclusions

We have introduced an easy-to-compute financial metric suitable for two-dimensional art objects that is both intuitive and transparent. It has several appealing features: it is difficult to game since not much discretion comes into its evaluation (unlike hedonic models that are data intensive and often exhibit lack of stability); it can be applied to artists for whom there are few observations, albeit with all the caveats appropriate for small data sets; it facilitates comparisons between artists, between different types of paintings by the same artist, or, paintings done by the same artist at different life-periods; it is also appropriate to explore artists' consistency, by looking at its standard deviation or coefficient of variation; and, finally, it can be employed to construct well-defined total-return indices to create financial derivatives.

However, it must be emphasized that the main goal of this new metric is to offer an investor a useful yardstick that captures, after normalizing by the area, a representative price. It is not the aim of the APV to control prices for other characteristics or to build a market index based on a time-independent ideal painting. For these reasons the APV metric is ideally suited to compute actual returns.

In terms of estimating returns, the APV metric offers three attractive features: (i) unlike repeat-sales regression models, it uses all the available data; (ii) unlike HPMs, whose effectiveness can depend substantially on the variables chosen and the analyst's skill to select them, the APV gives a unique value: the actual total return; and (iii) APV-based returns can always be computed regardless of the number of observations. On the other hand, HPM-based returns can be computed only in the limited number of cases where one has enough data, with the caveat that the accuracy of such returns estimates is weakened by the explicatory power of the relevant model since the R^2 is never 1.

Although the topic of this paper has been to introduce a new tool to the analyst's toolkit, rather than questioning the virtues of the HPMs in the context of the art market, one thing is obvious: hedonic models, considering how data-intensive they are plus the additional limitations already mentioned, do not seem to offer a lot more insight than the simple APV metric −at least for the examples discussed in this study. Moreover, the high correlation observed between returns computed using the APV and those based on HPMs reinforces this point.

In summary, we hope investors, financial analysts, and future researchers will be able to explore −and exploit− the merits of the APV metric. Our goal has been simply to introduce the tool, showcase a few applications, and perform some validation tests.

Finally, the main advantage of the APV is that it is a financial metric and not a modeling technique; therefore, it is what it is, and it can always be computed. In short, it can be useful or useless, but never wrong.

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