Does one word fit all? The asymmetric effects of central banks’ communication policy

Hamza Bennani

University of Lille 1

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Abstract

This paper provides an extension of Morris and Shin’s (2002) model (Morris, S., Shin, H. S. (2002). Social value of public information. The American Economic Review, 92(5), 1521-1534.). It considers an “interpretation bias” of the public signal sent by central banks such as the ECB or the FED. It is shown that such a bias is detrimental and should be considered when central banks implement their communication policy.

Keywords: central bank communication, monetary policy, public information.

JEL classification: C71, C78, E52

1 Introduction

Through their words, central bankers try to influence expectations of financial markets, hence, central bank watchers, financial media and market participants pay considerable attention to central bankers’ statements. Communication has thus become an important tool for central banks, more and more described as the art of managing expectations (Woodford, 2001), since it can enhance the predictability of monetary policy decisions and helps achieve central bank’s macroeconomic objectives. According to Blinder et al. (2008), central bank communication is used nowadays to manage expectations by “creating news” (i.e., the central bank’s announcements that influence expectations and move asset prices in the desired way) and “reducing noise” (i.e., how a central bank talks increases the predictability of its actions).

Most of the empirical studies that focus on the predictability of central banks statements refer to central banks such as the ECB and the FED. There is a broad consensus that ECB and FED communication contains forward guidance and moves financial markets in the intended direction (Musard-Gies, 2006; Willhemsen and Zaghini, 2011; Ehrmann and Fratzscher, 2009). Carlson et al. (2006) find that the communication framework built by the FOMC improved the public’s ability to predict interest rate decisions. Ullrich (2008) investigates the influence of the ECB communication on the inflation expectations of experts. She finds that the ECB statements given at the press conferences following the interest rate decisions influence inflation expectations of experts. Rosa (2009) finds that the tone of central bank statements is an important explanatory variable of future changes in the ECB main refinancing rate.

However, in these currency areas, monetary policy is particular in the sense that it is conducted within a multi-cultural and multi-lingual context. As an illustration, half of the US dollar circulates abroad

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*Corresponding author: EQUIPPE - Universités de Lille, Cité Scientifique, Faculté d’Economie et de Sciences Sociales, Bâtiment SH2 - 59655 Villeneuve d’Ascq, France. Contact: hamza.bennani@ed.univ-lille1.fr, Tel: +33(0)3.20.43.66.12, Fax: +33(0)3.20.33.71.26.
(Judson, 2012), this currency is then also used by agents with different cultural backgrounds and belongings. Therefore, are agents from different (member\(^1\)) countries able to understand in the same way the communication of these central banks, i.e., have similar expectations about the future path of the policy rate? Or, are expectations about monetary policy influenced by different national backgrounds?

Berger et al. (2009, 2011), using a database of surveys of professional ECB and FED policy forecasters, find persistent differences in forecast accuracy. According to the authors, these differences are related not only to the skills of analysts, but also to geography and to national macroeconomic conditions (i.e., deviations of national inflation from the euro/US area average). For instance, they find that financial institutions that are based in Frankfurt perform better in predicting ECB policy decisions.

Given that forecasters rely heavily on central bank’s communication, these results might unveil the presence of an asymmetry in the transmission mechanism of ECB’s and FED’s communication policy, which takes the form of a different interpretation of these central banks’ public signals. Indeed, the assumption of common interpretation of public information has been put in question by the literature in many fields. Lahiri and Sheng (2008) argue that professional forecasters, while observing the same statistical data, persistently disagree on the future rates of inflation, unemployment and GDP growth. Psychological studies find that one reason of these persistent differences may be overconfidence\(^2\). Finally, Odean (1998) find empirical evidences that agents keep on following their convictions, even after learning that they disagree and that they may be wrong.

To the best of our knowledge, no study has yet assessed the consequences of the presence of an asymmetry in the transmission mechanism of the communication policy of central banks such as the ECB or the FED. Hence, in this paper, we extend the theoretical framework of Morris and Shin (2002) (henceforth MS, 2002) to include the “interpretation bias” that may emerge among agents located in different countries when considering ECB’s and FED’s public announcements. We show how their individual welfare is affected by this misinterpretation. Our results highlight the negative effects induced by the presence of an “interpretation bias” of the public signal.

The remainder of the paper is organized as follows: Section 2 outlines the model. Section 3 presents the results, while the last section concludes.

### 2 The model

We consider a central bank that has an inflation objective \(\pi^o\), as the ECB or the FED:

\[
L^M_t = \frac{1}{2} E[(1 + \theta) y_t^2 + (\beta - \theta)(\pi_t - \pi^o)^2]
\]  

(1)

where \(\pi_t\) denotes the inflation rate at time \(t\), \(\pi^o\) the inflation objective, \(E_t\) the expectations operator, \(y_t\) the output gap, and where uncertainty about the central bank’s preferences is represented by the random variable \(\theta\). It is assumed that \(\theta \in [-1, \beta]\) and that \(E(\theta) = 0\), \(E(\theta^2) = \sigma^2_\theta\). In other words, there is an informational asymmetry between the central bank and the general public about the weight of the arguments in the monetary authority’s objective function, as in, e.g., Chortareas and Miller (2003) or Ciccarone and Marchetti (2012).

The central bank acts under the constraint of a standard Lucas-supply function\(^3\):

\[ y_t = \pi_t - \pi^o_t + \xi_t \]  

(2)

\(^1\)Such as in the Euro area

\(^2\)Ben David et al. (2010) show that top financial executives are too confident with respect to their own knowledge and own understanding of the model of the world. They are persistently failing in learning how to make correct inferences from the data.

\(^3\)Fendel and Rülke (2012) and Abbott and Martínez (2008) provide empirical evidence on the Lucas Supply function for developed economies, they find that the inflation surprise positively correlates with the output gap.
where $\pi^e_t$ denotes private sector expectations about the relevant state of inflation, and $\xi_t$ is the supply shock with zero mean and constant variance, $\sigma^2_\xi$. We assume that the central bank’s instrument is $\pi_t$.

Standard resolution by minimizing the loss function with regard to inflation delivers the inflation rate under the non inflation targeting framework:

$$\pi_t = \frac{(\beta - \theta)^2\pi^o + \alpha^2(1 + \theta)^2(\pi^e_t - \xi_t)}{\alpha^2(1 + \theta)^2 + (\beta - \theta)^2}$$  \hspace{1cm} (3)

For simplification purposes, we drop $\xi_t$ as it does not change the qualitative nature of our results:

$$\pi_t = \frac{(\beta - \theta)^2}{\alpha^2(1 + \theta)^2 + (\beta - \theta)^2} \pi^o + \frac{\alpha^2(1 + \theta)^2}{\alpha^2(1 + \theta)^2 + (\beta - \theta)^2} \pi^e_t$$  \hspace{1cm} (4)

This specification suggests that the inflation outcome is a function of both the inflation objective the central bank pursues $\pi^o$, and the expectations of the private sector $\pi^e_t$.

### 2.1 The formation of expectations

We assume that private agents form expectations and aim at minimizing the expected error with regard to the actual inflation rate. Therefore, the loss function of agent $i$ takes the following form:

$$L_i(\pi^e_t, \pi^o) = \frac{1}{2} E_i((\pi^e_{i,t} - \pi_t)^2)$$  \hspace{1cm} (5)

where $\pi^e_{i,t}$ is agent $i$’s expectation of inflation at time $t$, and $\pi_t$ is the ex-post inflation outcome. Agent $i$ seeks to minimize her loss function, given her own information (see Appendix A).

Agent $i$ decides her inflation expectation $\pi^e_{i,t}$, based on the first-order condition of (5).

$$\arg \min L_i(\pi^e_{i,t}, \pi^o) = E_i(\pi_t)$$  \hspace{1cm} (6)

and from (3),

$$\pi^e_{i,t} = E_i(\pi_t)$$

$$\pi^e_{i,t} = \frac{(\beta - \theta)^2}{\alpha^2(1 + \theta)^2 + (\beta - \theta)^2} E_i(\pi^o) + \frac{\alpha^2(1 + \theta)^2}{\alpha^2(1 + \theta)^2 + (\beta - \theta)^2} E_i(\pi^e_t)$$  \hspace{1cm} (7)

where $E_i(\pi^o)$ is agent’s $i$ expectation of the inflation objective of the central bank, and $E_i(\pi^e_t)$ is the expectation of agent $i$ of private sector expectations.

We can rewrite (7) as follows:

$$\pi^e_{i,t} = (1 - r)E_i(\pi^o) + r E_i(\pi^e_t)$$  \hspace{1cm} (8)

This form is of the same type as in MS (2002), with the parameter $r$ representing the importance the agent attaches to the “beauty contest”, i.e., the strength with which the agent tries to second-guess the others’ expectations.

Therefore, the inflation expectation of agent $i$ is a function of two things: its expectation of the objective policy of the central bank, and the average expectation formed by all the agents.

Following the model of MS (2002), we suppose that information used by the agents is available in the form of a public signal. We add that this signal is common knowledge to all agents but can interpreted differently according to their respective cultural backgrounds, given that they are located in different countries (Berger et al., 2009, 2011). Agents’ information is also composed of a private signal that is
specific to each agent. Agent $i$ observes $p$ and $s$:

$$
Public\ signal: p_i = \pi^o + \eta + \lambda_i
$$

$$
Private\ signal: s_i = \pi^o + \varepsilon_i
$$

where $\eta$, $\lambda_i$, and $\varepsilon_i$ have a zero mean and constant variance, $\sigma^2_\eta$, $\sigma^2_\lambda$, and $\sigma^2_\varepsilon$, respectively. The three error terms are independent of $\pi^o$ and of each other, such that $E(\varepsilon_i, \varepsilon_j) = 0$ and $E(\lambda_i, \lambda_j) = 0$ for $i \neq j$. We consider that the public signal contains an error term $\eta$ that is common to all agents and an error term $\lambda_i$ that is idiosyncratic, in the same spirit as Cornand and Baeriswyl (2014).

One might argue that the “interpretation bias” of the public signal could end up in a different private signal for every agent, and thus, be included in the error term ($\varepsilon_i$). However, our focus here is on the subjective interpretation, which leads to a differently interpreted common (public) signal. This can be grounded, for instance, on results from the behavioral or psychological literatures (Grosjean and Oswald, 2004). It is then important to model the “interpretation bias” as an error term in the public signal ($\lambda_i$) to disentangle its specific impact on the agent’s loss function, with respect to the impacts of the other motives$^4$ that have already been raised in the existing literature (Amato and Shin, 2003; Demertzis and Viegi, 2009; James and Lawler, 2012).

### 2.2 Equilibrium

Following equation (8), in order to derive the Bayesian equilibrium expectation of agents, we express the first order expectation of agent $i$ about the inflation objective of the central bank and the average expectation of the public signal observed by the other agents$^5$.

$$
E_i(\pi^o \mid p_i, s_i) = \frac{\sigma^2_\eta + \sigma^2_\lambda}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} s_i + \frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} p_i 
$$

$$
E_i(p^o \mid p_i, s_i) = \frac{\sigma^2_\eta}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} s_i + \frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} p_i 
$$

Supposing that agent $j$ (with $i \neq j$) is following a linear strategy of the form:

$$
\pi_j = (1 - k)s_j + kp_j
$$

With

$$
\int_0^1 \pi_j^e d_j = \pi_i^e
$$

$$
E_i \int_0^1 s_j^e d_j = E_i(s^e) = E_i(\pi^o \mid p_i, s_i)
$$

$$
E_i \int_0^1 p_j^e d_j = E_i(p^e) = E_i(p^o \mid p_i, s_i)
$$

Then, agent’s $i$ estimate of the average expected inflation across all agents is:

$$
E_i(\pi_i^e) = (1 - k)E_i(\pi^o \mid p_i, s_i) + kE_i(p^o \mid p_i, s_i)
$$

$$
E_i(\pi_i^e) = (1 - k) \frac{(\sigma^2_\eta + \sigma^2_\lambda)s_i + \sigma^2_\varepsilon p_i}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} + k \frac{\sigma^2_\lambda s_i + (\sigma^2_\varepsilon + \sigma^2_\eta)p_i}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda}
$$

---

$^4$E.g., different information sets or different models.

$^5$Given the “interpretation bias”, public signal is no longer a fully common knowledge, i.e., agent $i$ has to make an expectation of the public signal observed by the other agents.
Thus, the inflation expectation made by agent \( i \) is an average of both his signals whose weighting depends upon their relative precision and the value of the “beauty contest” \( r \).

The weight attributed by each agent to the public signal is smaller than in the equilibrium of MS (2002). This indicates that a public signal with an “interpretation bias” has a lower weight than in MS (2002).

We decompose \( p_i \) and \( s_i \) from (9) and (10) to obtain the following form:

\[
\pi_{i,t}^e = \pi^o + \frac{\epsilon_i[(1-r)\sigma^2_{\eta} + \sigma^2_{\lambda}]}{\sigma^2_{e} + (1-r)\sigma^2_{\eta} + \sigma^2_{\lambda}}
\]

The average inflation expected by all agents yields\(^6\):

\[
\pi_t^e = \int_0^1 \pi_j d_j
\]

\[
\pi_t^e = \int_0^1 \pi^o + \frac{\epsilon_j[(1-r)\sigma^2_{\eta} + \sigma^2_{\lambda}]}{\sigma^2_{e} + (1-r)\sigma^2_{\eta} + \sigma^2_{\lambda}} d_j
\]

\[
\pi_t^e = \pi^o + \frac{\sigma^2_{\eta}}{\sigma^2_{e} + (1-r)\sigma^2_{\eta} + \sigma^2_{\lambda}}
\]

Equation (18) reveals that the average inflation expectation across all agents is distorted by the precision of the three terms and the preference attached to the “beauty contest” \( r \).

### 3 Expected welfare

Following (5), we determine agent’s \( i \) loss function (see Appendix C):

\[
L_i = \frac{\sigma^2_{\lambda}[x^2 + \sigma^2_{\lambda} + 2x\sigma^2_{\lambda}] + \sigma^2_{\eta}(\sigma^2_{\eta}(1-r)^2 + \sigma^2_{\lambda} + 2\eta(1-r)\lambda_i) + q[x + \sigma^2_{\eta}]}{2[\sigma^2_{e} + x + \sigma^2_{\lambda}]^2}
\]

In the next step, we differentiate the loss function with respect to \( \lambda_i \), \( \sigma^2_{\eta} \), and \( \sigma^2_{\lambda} \), to determine how agent’s \( i \) welfare is affected by the existence of an “interpretation bias” of the public signal, and the degree of precision of the public and private signals.

---

\(^6\)Following the set up made by MS (2002), when aggregating private errors (\( \lambda_i \) and \( \epsilon_i \)) across all agents, these errors are eliminated, i.e., \( \int_0^1 \lambda_i \, dt = 0 \) and \( \int_0^1 \epsilon_i \, ds = 0 \), while the public information always appears in the final solution with a non-zero error term. According to Demertzis (2012), this is due to the law of large numbers.
3.1 The effect of the interpretation bias

We start by estimating the impact of the “interpretation bias” on agent’s individual welfare:

\[
\frac{\partial L_i}{\partial \lambda_i} = \sigma_i^2 \epsilon \left( \sigma_i^2 \eta (1-r) + \sigma_i^4 \right) + \eta (1-r) \sigma_i^4 \epsilon \left( \sigma_i^2 \lambda_i (1-r) + \sigma_i^2 \eta \right) ^2 \tag{20}
\]

The result reveals that the relation between the term referring to the “interpretation bias” of the public signal, \( \lambda_i \), and the individual welfare loss, \( L_i \), is strictly positive. In other words, the more agent’s interprets differently the public signal sent by the central bank, the more detrimental is its effect on his welfare. This result seems intuitive given the recent findings about the negative effects of divergent expectations on the individual welfare (Richter and Throckmorton, 2013).

**PROPOSITION 1:** The presence of an “interpretation bias” of the public signal has a detrimental impact on the individual welfare of the agent.

3.2 Public Information precision

To determine the expected welfare of the agent with a variance of the precision of public information, we differentiate the expression (19) with respect to \( \sigma_i^2 \eta \):

\[
\frac{\partial L_i}{\partial \sigma_i^2} = (r-1) \frac{q(x + \sigma_i^2) + \sigma_i^2 (x^2 + 2x \sigma_i^2 + \sigma_i^2) + \sigma_i^2 (\sigma_i^2 (1-r)^2 + 2 \eta \lambda_i (1-r) + \sigma_i^2)}{(\sigma_i^2 + x + \sigma_i^2)^3} \tag{21}
\]

\[
+ \frac{q(1-r) + \sigma_i^2 (1-r)^2 + \sigma_i^2 (2 \sigma_i^2 (1-r)^2) + 2x}{2(\sigma_i^2 + x + \sigma_i^2)^2} \tag{22}
\]

Given that \( 0 < r < 1 \), \( a > 0 \) and \( b > 0 \), the impact of more precise public signal (corresponding to a decrease of \( \sigma_i^2 \)) on individual welfare is ambiguous when public information is interpreted differently. When the weight attached to the “beauty contest” \( r \) converges to 1, i.e., when agent \( i \) aims to align his expectations close to the expectations of the other agents, the negative effect of more precise public information on individual welfare decreases. This result seems intuitive as an increase of the weight attached to \( r (r \to 1) \) means that the effect of the “interpretation bias” on the signal announced by the ECB or the FED is diminishing. Therefore, given that inflation is also determined by private-sector expectations (eq.4), the welfare loss of the agent (eq.5) is decreasing when he sets his expectations close to the ones of the rest of the agents, and gives less weight to his own interpretation. However, when the value of the “beauty contest” decreases \( (r \to 0) \), i.e., when the agent is less concerned by the expectations of the other agents and tends to put more weight on his own interpretation of the public signal, the effect of more precise public information is more negative on his welfare, as a consequence, the latter decreases with the precision of public information.

This result confirms the findings of previous empirical studies, for which better public information is beneficial only in particular economic contexts (Woodford, 2005; Hellwig, 2005; Angeletos and Pavan, 2007; Roca, 2010).

**PROPOSITION 2:** The precision of public information has an ambiguous impact on the individual welfare of the agent, its potential negative effect decreases when the agent does not consider his own interpretation of the public signal and aims to align his expectations to the expectations of the other agents.
3.3 Private information precision

The impact of the precision of private information on expected welfare is:

\[
\frac{\partial L_i}{\partial \sigma^2} = -\left[ \frac{\sigma_i^2 (x^2 + \sigma_i^4 + 2x\sigma_i^2) + \sigma_i^2 (\sigma_i^2 (1 - r)^2 + \sigma_i^4 + 2q(1 - r)\lambda_i)^2 + q(x + \sigma_i^2)}{(\sigma_i^2 + x + \sigma_i^4)^3} \right] \tag{23}
\]

The sign of \( \frac{\partial L_i}{\partial \sigma^2} \) is strictly negative. The welfare of the agent is decreasing in the precision of the private signal (corresponding to a reduction of \( \sigma_i^2 \)), i.e., in equilibrium, greater precision of the agent’s private information is detrimental to individual welfare.

This finding comes in contrast with the results of MS (2002) and previous empirical studies, for which an increase in the precision of private information is always beneficial. But in the case of multiple interpretation of the public signal, this result seems rather intuitive. Indeed, given that agents do not interpret the information given by the central bank in a similar way, some of them are considered as having better information sets than others (for instance the forecasters located in Frankfurt, see Berger et al., 2009), thus digging the gap between agents’ inflation expectations. This leads to an increase of the difference between agent’s \( i \) expectation and the private sector expectation, and thus, between agent’s \( i \) inflation expectation and the actual inflation rate (eq.4). Given the specific form of the loss function (eq.5), the individual welfare declines necessarily. Therefore, the existence of an “interpretation bias” of the public signal makes the impact of a more precise private signal harmful for the welfare of the agents.

**PROPOSITION 3:** An Increase in the precision of the private signal has negative effects on individual welfare in the presence of an “interpretation bias” of the public signal.

**Conclusion**

Given that some central banks are implementing a monetary policy in a multi-lingual and multi-cultural context, it is a challenge for these institutions to be understood uniformly when communicating about their monetary policy. In this paper, we highlight the detrimental effects of the presence of an “interpretation bias” of the public information conveyed by the ECB or the FED among heterogeneous agents. This raises the question of whether the communication policy of these central banks is consistent enough to tackle the negative consequences of the “interpretation bias”, and opens further questions for upcoming researches.

**APPENDIX**

**Appendix A**

\[
L_i(\pi^e, \pi^o) = \frac{1}{2} E_i((\pi^e_{i,t} - \pi_t)^2)
\]

\[
L_i(\pi^e, \pi^o) = \frac{1}{2} E_i((\pi^e_{i,t} - 2\pi^e_{i,t}\pi_t + (\pi_t)^2)
\]

\[
\frac{\partial L_i}{\partial \pi_t}(\pi^e, \pi^o) = \frac{1}{2} E_i(2\pi^e_{i,t} - 2\pi_t) = 0
\]

\[
E_i(\pi^e_{i,t}) = E_i(\pi_t)
\]

**Appendix B**

\[
\pi_i = (1 - r) E_i(\pi^o) + r E_i(\pi^e_i)
\]
With

\[ E_i(\pi^0 | s_i, p_i) = \frac{\sigma^2_\eta + \sigma^2_\lambda}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} s_i + \frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} p_i \]

\[ E_i(\pi^e_i) = (1 - k)s^e_i + kp^e_i \]

\[ E_i(s^e_i) = E_i[\pi^0 | p_i, s_i] \]

\[ E_i(p^e_i) = E_i[p^e_i | p_i, s_i] = \frac{\sigma^2_\lambda}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} s_i + \frac{\sigma^2_\varepsilon + \sigma^2_\eta}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} p_i \]

\[ \pi^e_{i,t} = (1 - r)\left[ \frac{\sigma^2_\eta + \sigma^2_\lambda}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} s_i + \frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} p_i \right] + r\left[ (1 - k) \frac{\sigma^2_\eta + \sigma^2_\lambda}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} s_i + \frac{\sigma^2_\varepsilon + \sigma^2_\eta}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} p_i \right] + \frac{k}{(1-k)} \frac{\sigma^2_\lambda s_i + (\sigma^2_\eta + \sigma^2_\lambda) p_i}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\lambda} \]

With

\[ \pi^e_{i,t} = kp_i + (1 - k)s_i \]

Then

\[ k = \frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon + \sigma^2_\eta(1 - r) + \sigma^2_\lambda} \]

\[ 1 - k = \frac{\sigma^2_\eta(1 - r) + \sigma^2_\lambda}{\sigma^2_\varepsilon + \sigma^2_\eta(1 - r) + \sigma^2_\lambda} \]

**Appendix C**

\[ L_i(\pi^e_i, \pi^0_i) = \frac{1}{2} E_i(\pi^e_{i,t} - \pi_i)^2 \]

With

\[ \pi^e_{i,t} = \pi^0_i + \frac{\varepsilon_i[(1 - r)\sigma^2_\eta + \sigma^2_\lambda] + \sigma^2_\varepsilon(\eta + \lambda_i)}{\sigma^2_\varepsilon + (1 - r)\sigma^2_\eta + \sigma^2_\lambda} \]

And

\[ \pi_i = \frac{(\beta - \theta)^2}{\alpha^2(1 + \theta)^2 + (\beta - \theta)^2} \pi^0_i + \frac{\alpha^2(1 + \theta)^2}{\alpha^2(1 + \theta)^2 + (\beta - \theta)^2} \pi^e_i \]

\[ \pi^e_i = \pi^0_i + \frac{\sigma^2_\eta}{\sigma^2_\varepsilon + (1 - r)\sigma^2_\eta + \sigma^2_\lambda} \]

Following (7), we know that:

\[ \frac{(\beta - \theta)^2}{\alpha^2(1 + \theta)^2 + (\beta - \theta)^2} = (1 - r) \]

\[ \frac{\alpha^2(1 + \theta)^2}{\alpha^2(1 + \theta)^2 + (\beta - \theta)^2} = r \]
Therefore
\[ \pi_t = \pi^o + \frac{r \sigma^2 \eta}{[\sigma^2 + (1 - r) \sigma^2 + \sigma^2]} \]

Then
\[ L_i(\pi^o, \pi^o) = \frac{1}{2} E_i[\pi^o + \frac{\epsilon_i[(1 - r) \sigma^2 + \sigma^2(\eta + \lambda_i)]}{\sigma^2 + (1 - r) \sigma^2 + \sigma^2} \]
\[ - (\pi^o + r \sigma^2 \eta (\sigma^2 + (1 - r) \sigma^2 + \sigma^2))^2 \]
\[ L_i = \frac{1}{2} E_i[\epsilon_i[(1 - r) \sigma^2 + \sigma^2(\eta + \lambda_i)]}{\sigma^2 + (1 - r) \sigma^2 + \sigma^2} \]
\[ - \frac{r \sigma^2 \eta}{[\sigma^2 + (1 - r) \sigma^2 + \sigma^2]^2} \]
\[ L_i = \frac{1}{2} E_i[\epsilon_i[(1 - r) \sigma^2 + \sigma^2(\eta + \lambda_i)]}{\sigma^2 + (1 - r) \sigma^2 + \sigma^2} \]
\[ + [\sigma^2(\eta(1 - r) + \lambda_i)]^2 + 2 \epsilon_i[(1 - r) \sigma^2 + \sigma^2(\eta(1 - r) + \lambda_i)] \]
\[ \frac{[\sigma^2 + (1 - r) \sigma^2 + \sigma^2]^2}{[\sigma^2 + (1 - r) \sigma^2 + \sigma^2]^2} \]

With
\[ E_i(\epsilon^2) = \sigma^2 \]
\[ E_i(\eta^2) = \sigma^2 \]
\[ E_i(\lambda^2) = \sigma^2 \]

We obtain
\[ L_i = \frac{\sigma^2[(1 - r)^2 \sigma^4 + 2(1 - r) \sigma^2 + \sigma^2(1 - r)^2 + 2 \eta(1 - r) \lambda_i) + 2 \epsilon_i[(1 - r) \sigma^2 \eta + \sigma^2(\eta(1 - r) + \lambda_i)]}{2[\sigma^2 + (1 - r) \sigma^2 + \sigma^2]^2} \]

with
\[ q = 2 \sigma^2 \epsilon(\eta(1 - r) + \lambda) \]
\[ x = \sigma^2(1 - r) \]

Then
\[ L_i = \frac{\sigma^2[x^2 + \sigma^4 + 2x \sigma^2] + \sigma^4(1 - r)^2 + \sigma^2 + 2 \eta(1 - r) \lambda_i) + q[x + \sigma^2]}{2[\sigma^2 + x + \sigma^2]^2} \]

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