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Abstract

The objective of this paper is to provide a comprehensive efficiency measures to estimate the performances of OECD and non-OECD countries. A Russell directional distance function that appropriately credits the decision making unit not only for increase in desirable outputs but also for the decrease of undesirable outputs is derived from the proposed weighted Russell directional distance model. The method was applied to a panel of 99 countries over 1991 and 2003. This framework also decomposes the comprehensive efficiency measure into individual input/output components' inefficiency scores that are useful for policy making. The results reveal that the OECD countries perform better than the non-OECD countries in overall, goods, labor and capital efficiencies, but worse in bad and energy efficiencies.

Keywords: data envelopment analysis, directional distance function, undesirable output, Russell measure, slacks-based model

1. Introduction

It has long been recognized in the technical efficiency literature that data envelopment analysis (DEA) is particularly adept at computing multiple input and output production correspondences (Seiford and Zhu, 1999). Generally, there are two types of measure in DEA, namely radial and non-radial (Tone, 2010). Radial measures are represented by CCR (Charnes et al., 1978) and BCC (Banker et al., 1984) models. However, because radial measures of efficiency overestimate technical efficiency when there are non-zero slacks in the constraints defining the piece-wise linear technology (Fukuyama and Weber, 2009), recent research has sought to construct alternative non-radial efficiency measures that account for slacks.

In the literature of the non-radial models, the proposed methods can be roughly divided into the following three groups: (1) the Russell measure, which was first presented by Färe and Lovell (1978) with an input-oriented form. Nevertheless, it only accounts for all the slacks of inputs, but fails to consider the inefficiencies associated with outputs. It was later extended by Färe et al. (1985) in a nonlinear form that they refer to as the “Russell graph measure”, which combines the input and output Russell measures in an additive way and accounts for all the input slacks as well as the output slacks. Pastor et al. (1999) then further revised it to a new measure called the “Enhanced Russell graph measure” (ERGM), which in turn combines input and output Russell measures in a ratio form. (2) the additive model, which was developed by Charnes et al. (1985) and also accounts for all sources of inefficiency both in inputs and outputs. However, it does not directly provide an efficiency measure (Pastor et al., 1999). (3) the slacks-based model (SBM), which was proposed by Tone (2001) with the objective of maximizing all the input and output slacks in fractional programming form. Cooper et al. (2007) showed that SBM is equivalent to ERGM.

Recently, Fukuyama and Weber (2009) introduced the directional distance function technology into SBM to develop a generalized measure of technical inefficiency which also accounts for all slacks in input and output constraints. This new measure was referred to as the directional slacks-based inefficiency (SBI) measure. This is shown to yield the same information on performance as Tone’s SBM of efficiency when the directional vectors for inputs and outputs are chosen to equal the actual input and output vector, and can also be thought of as a generalization of the original Russell measure of efficiency. On the other hand, Färe and Grosskopf (2010) also proposed a generalization of the SBM measure based on the directional distance function. The optimization problem of this measure is based on the sum of directional distance function and can tell how much excess inputs have been employed and how much outputs short of an efficient level have been produced.

Admitting the contributions of these previous studies, however, the undesirable outputs are ignored in almost all these measures. In practice, there are some cases in which both outputs which are desirable (goods) and undesirable (bads; such as air pollution, waste or bad loans) are produced jointly[1]. It is therefore reasonable to consider not only all the inefficiency sources of inputs and desirable outputs but also all the inefficiency sources of undesirable outputs when we evaluate the performance of a decision making unit (DMU). To our knowledge, both Zhou et al. (2006) and Zhou et al. (2007) had extended the SBM and Russell measure to incorporate undesirable outputs. Nevertheless, the models established in the former one do not really account for all the inefficiency sources of undesirable outputs. The models constructed in the latter study only measure the performance of undesirable outputs (referred to as a Russell environmental performance index in Zhou et al., 2007) and thus ignore the inefficiency sources of inputs and desirable outputs.

The purpose of this study is to extend the directional Russell measure of inefficiency proposed by Fukuyama and Weber (2009) into the cases where undesirable outputs exist. We refer to the proposed model as the *weighted Russell directional distance model* (WRDDM).[2] Compared to the original SBM and ERGM, which combine input and output efficiency measures in a nonlinear fractional form, our directional distance function based measure is evaluated in linear form and hence possesses the attractive advantages of easy computation and easy extension of incorporating the additional undesirable outputs into the programming problems.

The remainder of this paper is organized as follows. The next section presents the model. For comparison purpose, we first illustrate the traditional directional distance function model (TDDFM) problem, which credits a producer for simultaneously increasing production of the good output, reducing production of bad outputs and contracting employment of inputs. Then, the extended models are followed. The SBM and Russell measure, which correspond to our WRDDM, are also presented. Section three demonstrates using a case study of panel data of 99 countries. Our measure obtained is easily decomposed into separate measures of input, desirable and undesirable output efficiencies, and can help us to shed more light on the sources of inefficiencies. In the case of the environmental issues are evaluated, it provides us an integrated measure of economic and environmental performance. The final section concludes.

2. Preliminary

2.1 The Traditional Directional Distance Function Model

Let inputs be denoted by $x \in R_+^N$, good outputs by $y \in R_+^M$, and bad or undesirable outputs by $b \in R_+^J$, where R_+^* represents the non-negative Euclidean *-orthant. The directional distance function seeking to increase the desirable outputs and decrease the undesirable outputs and inputs directionally can be defined by the following formulation:

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[1] As Smith (1990) pointed out, undesirable outputs may also appear in health care (e.g. complications of medical operations) and business (e.g. tax payments) applications.

[2] The concept of weighted Russell measure is not new. Ruggiero and Bretschneider (1998) had developed a weighted Russell measure to evaluate technical efficiency. However, their measure is evaluated in distance function rather than in directional distance function and did not consider undesirable outputs, so it is different from our measure.

$$\vec{D}(x, y, b; g) = \sup\{\rho : (x - \rho g_x, y + \rho g_y, b - \rho g_b) \in T\}, \tag{1}$$

where the non-zero vector $g = (-g_x, g_y, -g_b)$ determines the "directions" in which inputs, desirable outputs and undesirable outputs are scaled, and the technology reference set $T = \{(x, y, b) : x \textit{ can produce } (y, b)\}$ satisfies the assumptions of constant returns to scale, strong disposability of inputs, and weak disposability of both desirable outputs and undesirable outputs.

Suppose there are $k = 1, \cdots, K$ DMUs in the data set. Each DMU uses input $x^k = (x_1^k, x_2^k, \cdots, x_N^k) \in R_+^N$ to jointly produce desirable outputs $y^k = (y_1^k, y_2^k, \cdots, y_M^k) \in R_+^M$ and undesirable outputs $b^k = (b_1^k, b_2^k, \cdots, b_J^k) \in R_+^J$. The DEA piecewise reference technology can be constructed as follows:

$$
\begin{aligned}
T = \{(x, y, b) :\ & \sum_{k=1}^{K} z_k y_{mk} \geq y_m, && m = 1, \ldots\ldots, M, \\
& \sum_{k=1}^{K} z_k b_{jk} = b_j, && j = 1, \ldots\ldots, J, \\
& \sum_{k=1}^{K} z_k x_{nk} \leq x_n, && n = 1, \ldots N, \\
& z_k \geq 0, && k = 1, \ldots K. \},
\end{aligned} \tag{2}
$$

where z_k are the intensity variables to shrink or expand the individual observed activities of DMU k for the purpose of constructing convex combinations of the observed inputs and outputs.

Relative to the reference technology T constructed in (2), traditionally, for each DMU $k' = 1, \cdots, K$, the directional distance function can be obtained by solving the following linear programming problem[3]:

$$
\begin{aligned}
& \vec{D}(x^{k'}, y^{k'}, b^{k'}; g) = \max \rho^{k'} \\
\text{s.t.}\ & \sum_{k=1}^{K} z_k y_{mk} \geq y_{mk'} + \rho^{k'} g_{y_m}, && m = 1, \ldots M, \\
& \sum_{k=1}^{K} z_k b_{jk} = b_{jk'} - \rho^{k'} g_{b_j}, && j = 1, \ldots J, \\
& \sum_{k=1}^{K} z_k x_{nk} \leq x_{nk'} - \rho^{k'} g_{x_n}, && n = 1, \ldots N, \\
& z_k \geq 0, && k = 1, \ldots K.
\end{aligned} \tag{3}
$$

[3] A referee reminds that it is still debating about variable return to scale in weakly disposable technology. Thus, the efficiency is evaluated with variable returns to scale technology, it needs not only to impose the additional constraint of $\sum_{k=1}^{K} z_k = 1$ in the formulation of (3) (Kuosmanen and Podinovski, 2008).

where $\rho^{k'}$ measures the maximum expansion of desirable outputs and contraction of undesirable outputs and inputs that remain technically feasible and can serve as a measure of technical inefficiency. If $\rho^{k'} = 0$, then DMU k' operates on the frontier of *T* with technical efficiency. If $\rho^{k'} > 0$, then DMU k' operates inside the frontier of *T*.

Other than being the generalization of the Shephard's distance functions[4], one of the important characteristics of the directional distance function is that the direction in which performance is scaled can be specified flexibly to accommodate different analysis purposes. For example, if we set $g = (-g_x, g_y, -g_b) = (-x^{k'}, y^{k'}, -b^{k'})$, i.e., the direction is chosen based on the observed data, $\rho^{k'}$ represents the potential proportionate change in goods, bads and inputs. If instead we take $g = (-g_x, g_y, -g_b) = (-1, 1, -1)$, then we can interpret the solution value as the net improvement in performance in terms of feasible increase in goods outputs and feasible decreases in bad outputs and inputs (Färe and Grosskopf, 2004). On the other hand, setting $g = (0, g_y, -g_b)$, we get the directional output (including goods and bads) distance function or environmental directional output distance function as referred to by Färe et al. (2007) (cf. Färe et al. (2007) for more details).

However, as mentioned previously, the measure of this approach fails to consider the inefficiencies associated with non-zero slacks and would have the problem of incorrectly regarding some evaluated DMUs as efficient units.

2.2. The Weighted Directional Distance Model

The efficiency measurement constructed in (3) expands all desirable outputs and contracts all inputs and undesirable outputs by the same rate of ρ. However, there is no guarantee that the proportional contraction (expansion) rate of for input items (γ), desirable output items (α) and undesirable output items (β) must be the same (ρ) in practice. We believe this is a strong assumption of the model. Thus, the formulation of (3) can be generalized to accommodate different expansion and contraction scales as follows:

$$\vec{D}^{w}(x^{k'}, y^{k'}, b^{k'};\, g) = \max \omega_y \alpha^{k'} + \omega_b \beta^{k'} + \omega_x \gamma^{k'} = \rho_w^{k'}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} z_k y_{mk} \geq y_{mk'} + \alpha^{k'} g_{y_m}, \qquad m = 1, \ldots M,$$

$$\sum_{k=1}^{K} z_k b_{jk} = b_{jk'} - \beta^{k'} g_{b_j}, \qquad j = 1, \ldots J,$$

[4] Details of the relationship between directional distance functions and Shephard distance functions can be found in Chung et al. (1997) and Färe and Grosskopf (2000).

$$\sum_{k=1}^{K} z_k x_{nk} \le x_{nk'} - \gamma^{k'} g_{x_n}, \qquad n = 1, \ldots N, \tag{4}$$

$$z_k \ge 0, \qquad k = 1, \ldots K.$$

It is required that the directional vectors have same units of measurement as the vectors of the observed data, so that it allows the α, β, γ to be added. [5] The measure $\vec{D}^w(x^{k'}, y^{k'}, b^{k'}; -g_{x_n}, g_{y_m}, -g_{b_j})$ given in (4) is maximized hyperbolically $\rho_w^{k'} = \omega_y \alpha^{k'} + \omega_b \beta^{k'} + \omega_x \gamma^{k'}$ by comparing the observed $(x_n^{k'}, y_m^{k'}, b_j^{k'})$ with the frontier $((x_{nk'} - \gamma^{k'} g_{x_n}), (y_{mk'} + \alpha^{k'} g_{y_m}), (b_{jk'} - \beta^{k'} g_{b_j}))$. The weighted directional distance function gives the expansion in good outputs and contraction in bad outputs and inputs simultaneously. When $\vec{D}^w(x^{k'}, y^{k'}, b^{k'}; -g_{x_n}, g_{y_m}, -g_{b_j}) = 0$, DMU *k'* is technically efficient because no additional improvements in good outputs, bad outputs and inputs are feasible. $\vec{D}^w(x^{k'}, y^{k'}, b^{k'}; -g_{x_n}, g_{y_m}, -g_{b_j}) > 0$ indicates technical inefficiency. The coefficients ω_y, ω_b and ω_x are associated with the priorities or managerial preferences given to the outputs (goods and bads) and inputs and their sum is normalized to unity. The improvements for desirable outputs, undesirable outputs, and inputs can be measured by $\alpha^{k'}$, $\beta^{k'}$, and $\gamma^{k'}$, respectively, and then used to calculate the weighted inefficiency score $\rho_w^{k'}$.

Note that if we set $\alpha^{k'} = \beta^{k'} = \gamma^{k'}$, then model (4) degenerates to model (3) [6]. In addition, in the one input, one desirable output and one undesirable output case, formulation (4) is able to account for all the slacks. However, in the multiple inputs, desirable outputs and undesirable outputs case, it may still fail to identify all the non-zero slacks associated with the input and output constraints.

3. The Model

3.1. The Weighted Russell Directional Distance Model

Inspired by the equivalence of the ERGM and SBM, we follow an idea similar to that of the ERGM to generalize the traditional directional distance function and develop another similar measure called the weighted Russell directional distance model (WRDDM). It is important to note that the WRDDM is a closely related measure of ERGM, while the ERGM and SBM are special

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[5] This requirement is the same as what Fukuyama and Weber (2009, p276) point out, that the directional vectors have the same units of measurement as the vectors of the input and output slack, and when g =1, the role of the directional vectors is also similar to that of the e_l in Färe and Grosskopf (2010).

[6] One of the anonymous referees reminds that considering $\omega_y \alpha$ in the objective function together with αg_{y_m} among the constraints. The following can be done, assuming that ω_y is strictly positive ($\omega_y \alpha g_{y_m} / \omega_y = \alpha' \cdot g'_{y_m}$ where $\alpha' = \omega_y \alpha$, $g'_{y_m} = g_{y_m} / \omega_y$). Change the objective function in the same way, then (4) becomes the model of Färe and Grossskopf (2007) with undesirable outputs.

cases of the WRDDM measure[7]. The proposed programming model is:

$$\vec{D}^R(x^{k'}, y^{k'}, b^{k'};\ g) = \rho_R^{k'} = \max w_y(\sum_{m=1}^{M} \varpi_m^y \alpha_m^{k'}) + w_b(\sum_{j=1}^{J} \varpi_j^b \beta_j^{k'}) + w_x(\sum_{n=1}^{N} \varpi_n^x \gamma_n^{k'})$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{k=1}^{K} z_k y_{mk} \geq y_{mk'} + \alpha_m^{k'} g_{y_m}, && m = 1, \ldots M, \\
& \sum_{k=1}^{K} z_k b_{jk} = b_{jk'} - \beta_j^{k'} g_{b_j}, && j = 1, \ldots J, \\
& \sum_{k=1}^{K} z_k x_{nk} \leq x_{nk'} - \gamma_n^{k'} g_{x_n}, && n = 1, \ldots N., \qquad (5) \\
& z_k \geq 0, && k = 1, \ldots K.
\end{aligned}$$

where $\alpha_m^{k'}, \beta_j^{k'}, \gamma_n^{k'}$ are the individual inefficiency measure for each desirable output y_m, each undesirable output b_j and each input x_n. In other words, this specification allows for not only the technical inefficiency associated with desirable output, undesirable output and input to be different, but also allows the technical inefficiency among each of the desirable outputs, the undesirable outputs and the inputs to be different ($\alpha_m^{k'}(m = 1,\cdots,M) \neq \beta_j^{k'}(j = 1,\cdots,J) \neq \gamma_n^{k'}(n = 1,\cdots,N)$). This makes sense, because , for example, the inefficiency of the input uses of a firm could be more from the inefficient use of labor (use too many workers or there is labor congestion) but less from capital. On the other hand, a producer may produce several products at the same time (e.g. crops and livestock production of farmers or loans and securities investment production of banks), but with different production ability, and hence the production efficiency for different product would be different. Therefore, one of the advantages of this model is that it can help us to identify the source where we need to improve most.

Once again, the directional vectors are required to have the same units of measurement as the vectors of the observed data, so that it allows the $\alpha_m^{k'}, \beta_j^{k'}, \gamma_n^{k'}$ to be added. If the coefficients w_y, w_b and w_x denote the given priorities associated with the outputs (goods and bads) and inputs, and their sum is normalized to unity, and the inefficiencies of each corresponding input (output) is also specified to allow assigning different priorities to each of it and their sums are assumed to be one: $\sum_{m=1}^{M} \varpi_m^y = 1$, $\sum_{j=1}^{J} \varpi_j^b = 1$ and $\sum_{n=1}^{N} \varpi_n^x = 1$, then this is similar to what Liu and Tone (2008) did.[8]

7 The difference between the ERGM and the WRDDM exists in their objective functions and constraints. The objective function of the ERGM is specified for calculating efficiency measure, while those respective variables in the WRDDM are inefficiency measures. In addition, WRDDM has additive form objective function, while ERGM is ratio form and ERGM does not consider undesirable outputs (cf. Pastor et al. (1999) for more details).

8 That is, if we alternatively specify the objective function in (5) as $\frac{1}{3}(\sum_{m=1}^{M} \varpi_m^y \alpha_m^{k'} + \sum_{j=1}^{J} \varpi_j^b \beta_j^{k'} + \sum_{n=1}^{N} \varpi_n^x \gamma_n^{k'})$, the corresponding weighted SBM for the WRDDM will be close to the weighted slacks-based measure presented by Liu and Tone (2008) in which only inputs and outputs are considered. Therefore, Liu and Tone's formulation is the special

However, it is noted that if the direction vectors do not have the same units of measurement as the vectors of the observed data, we can alternatively set the weights ϖ_m^y, ϖ_j^b and ϖ_n^x as values which can normalized the direction vectors, such as the sample standard deviations of the inputs and outputs. We will say more about this below when we discuss the unit invariant property of WRDDM.

By means of the following change of variables[9], the Russell directional type inefficiency measures can be changed to the slacks-based ones.

$$\alpha_m^{k'} = \frac{s_{mk'}^+}{g_{y_m}}, \quad \beta_j^{k'} = \frac{s_{jk'}^-}{g_{b_j}}, \quad \gamma_n^{k'} = \frac{s_{nk'}^-}{g_{x_n}}$$

where $s_{mk'}^+$, $s_{jk'}^-$, $s_{nk'}^-$ are the desirable and undesirable outputs, and inputs slacks respectively which cause the inefficiency for the evaluated unit k'. Then, the model can be re-expressed as follows:

$$\vec{D}^R(x^{k'}, y^{k'}, b^{k'};\, g) = \rho_R^{k'} = \max\ w_y\Big(\sum_{m=1}^{M} \frac{\varpi_m^y s_{mk'}^+}{g_{y_m}}\Big) + w_b\Big(\sum_{j=1}^{J} \frac{\varpi_j^b s_{jk'}^-}{g_{b_j}}\Big) + w_x\Big(\sum_{n=1}^{N} \frac{\varpi_n^x s_{nk'}^-}{g_{x_n}}\Big)$$

$$\begin{aligned}
\text{s.t.}\quad & \sum_{k=1}^{K} z_k y_{mk} = y_{mk'} + s_{mk'}^+, && m = 1,\ldots\ldots,M, \\
& \sum_{k=1}^{K} z_k b_{jk} = b_{jk'} - s_{jk'}^-, && j = 1,\ldots\ldots,J, \\
& \sum_{k=1}^{K} z_k x_{nk} = x_{nk'} - s_{nk'}^-, && n = 1,\ .\ .\ .\ N, \qquad (6) \\
& s_{mk'}^+,\ s_{jk'}^-,\ s_{nk'}^- \geq 0 \quad \forall m, j, n \\
& z_k \geq 0, && k = 1,\ldots\ldots,K
\end{aligned}$$

Because the slacks for each variable are allowed to be different, the objective in (6) can help us to reflect all inefficiencies by calculating the maximum expansion of all desirable outputs and contraction of all undesirable outputs and inputs that the model can identify. Thus, $\rho_R^{k'}$ provides us an aggregate or overall inefficiency measure of performance in a non-radial manner in which the component of $\sum_{m=1}^{M} \varpi_m^y \alpha_m^{k'} = \sum_{m=1}^{M} \frac{\varpi_m^y s_{mk'}^+}{g_{y_m}} = \bar{\alpha}^{k'}$ in the objective of (5) or (6) corresponds to the average desirable output mix inefficiencies and similarly, the $\sum_{j=1}^{J} \varpi_j^b \beta_j^{k'} = \sum_{j=1}^{J} \frac{\varpi_j^b s_{jk'}^-}{g_{b_j}} = \bar{\beta}^{k'}$ and

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case of our corresponding weighted SBM which will be introduced in section 3.2.

[9] In ERGM, if considering bads, $\alpha_m^{k'} = \frac{s_{mk'}^+}{y_m^{k'}}$, $\beta_j^{k'} = \frac{s_{jk'}^-}{b_j^{k'}}$, $\gamma_n^{k'} = \frac{s_{nk'}^-}{x_n^{k'}}$.

$\sum_{n=1}^{N}\varpi_n^x\gamma_n^{k'}=\sum_{n=1}^{N}\frac{\varpi_n^x s_{nk'}^-}{g_{x_n}}=\bar{\gamma}^{k'}$ correspond to the average undesirable output and input mix inefficiencies respectively.

Let an optimal solution of model (6) be $z_{k'}^*$, $s_{mk'}^{+*}$, $s_{jk'}^{-*}$ and $s_{nk'}^{-*}$, the WRDDM overall inefficiency is the weighted average of the desirable output inefficiency, $\bar{\alpha}^{k'*}$, undesirable output inefficiency, $\bar{\beta}^{k'*}$, and input inefficiency, $\bar{\gamma}^{k'*}$. We define the WRDDM overall inefficiency measure $\rho_R^{k'*}$ by

$$\rho_R^{k'*}=w_y\bar{\alpha}^{k'*}+w_b\bar{\beta}^{k'*}+w_x\bar{\gamma}^{k'*}\,(k=1,\cdots,K), \tag{7}$$

where

$$\bar{\alpha}^{k'*}=\sum_{m=1}^{M}\frac{\varpi_m^y s_{mk'}^{+*}}{g_{y_m}},$$

$$\bar{\beta}^{k'*}=\sum_{j=1}^{J}\frac{\varpi_j^b s_{jk'}^{-*}}{g_{b_j}},$$

$$\bar{\gamma}^{k'*}=\sum_{n=1}^{N}\frac{\varpi_n^x s_{nk'}^{-*}}{g_{x_n}}.$$

The optimal solution to this linear program (6) is an inefficiency score which measures the largest rectilinear distance from the observation being evaluated to the efficient production frontier. Therefore, we have the following definitions:

Definition 1. *WRDDM desirable output efficient.*
If all optimal solutions of (6) satisfy $\bar{\alpha}^{k'*}=0$, DMU *k'* is called desirable output efficient. This implies that the optimal slacks for the desirable outputs in (6) are all zero, i.e. $s_{mk'}^{+*}=0(\forall m)$.

Definition 2. *WRDDM undesirable output efficient.*
If all optimal solutions of (6) satisfy $\bar{\beta}^{k'*}=0$, DMU *k'* is called undesirable output efficient. This implies that the optimal slacks for the undesirable outputs in (6) are all zero, i.e. $s_{jk'}^{-*}=0(\forall j)$.

Definition 3. *WRDDM input efficient.*
If all optimal solutions of (6) satisfy $\bar{\gamma}^{k'*}=0$, DMU *k'* is called input efficient. This implies that the optimal slacks for the inputs in (6) are all zero, i.e. $s_{nk'}^{-*}=0(\forall n)$.

Definition 4. *WRDDM overall efficient.*
If all optimal solutions of (6) simultaneously satisfy $\bar{\alpha}^{k'*}=0$, $\bar{\beta}^{k'*}=0$ and $\bar{\gamma}^{k'*}=0$, DMU *k'* is called WRDDM overall efficient. This implies that the optimal slacks for the desirable outputs,

undesirable outputs and inputs in (6) are all zero, i.e. $s_{mk'}^{+*} = s_{jk'}^{-*} = s_{nk'}^{-*} = 0(\forall m, j, n)$.

If the WRDDM inefficiency measure is zero, then the DMU is fully efficient. The inefficiency measure of WRDDM has the following properties:

Theorem 1. *DMU k' is WRDDM overall efficient, if and only if it is WRDDM efficient for all the desirable output efficient, undesirable output efficient and input efficient.*
Proof. From the equality (7), this theorem holds.□

Theorem 2. *The projected DMU o is WRDDM overall efficient.*
Proof.
Let an optimal solution of model (6) be z_o^*, s_{mo}^{+*}, s_{jo}^{-*} and s_{no}^{-*}. We define the projection of DMU o as follows:

$$\tilde{y}_{mo} = \sum_{k=1}^{K} z_k^* y_{mk} = y_{mo} + s_{mo}^{+*} \quad (m = 1, \ldots\ldots, M),$$

$$\tilde{b}_{jo} = \sum_{k=1}^{K} z_k^* b_{jk} = b_{jo} - s_{jo}^{-*} (j = 1, \ldots\ldots, J),$$

$$\tilde{x}_{no} = \sum_{k=1}^{K} z_k^* x_{nk} = x_{no} - s_{no}^{-*} (n = 1, \ldots\ldots, N)$$

Then, re-estimate the overall-efficiency of the projected DMU. Let an optimal solution of the projected DMU be $\widehat{z}_o^*$, $\widehat{s}_{mo}^{+*}$, $\widehat{s}_{jo}^{-*}$ and $\widehat{s}_{no}^{-*}$. We have:

$$\tilde{y}_{mo} = \sum_{k=1}^{K} \widehat{z}_k^* y_{mk} - \widehat{s}_{mo}^{+*} \quad (m = 1, \ldots\ldots, M),$$

$$\tilde{b}_{jo} = \sum_{k=1}^{K} \widehat{z}_k^* b_{jk} + \widehat{s}_{jo}^{-*} (j = 1, \ldots\ldots, J),$$

$$\tilde{x}_{no} = \sum_{k=1}^{K} \widehat{z}_k^* x_{nk} + \widehat{s}_{no}^{-*} (n = 1, \ldots\ldots, N)$$

Replacing $\tilde{y}_{mo} = y_{mo} + s_{mo}^{+*}$ ($m = 1, \ldots\ldots, M$), $\tilde{b}_{jo} = b_{jo} - s_{jo}^{-*}$ ($j = 1, \ldots\ldots, J$), and $\tilde{x}_{no} = x_{no} - s_{no}^{-*} (n = 1, \ldots\ldots, N)$, we have:

$$y_{mo} = \sum_{k=1}^{K} \widehat{z}_k^* y_{mk} - \widehat{s}_{mo}^{+*} - s_{mo}^{+*} \quad (m = 1, \ldots\ldots, M),$$

$$b_{jo} = \sum_{k=1}^{K} \widehat{z}_k^* b_{jk} + \widehat{s}_{jo}^{-*} + s_{jo}^{-*} (j = 1, \ldots\ldots, J),$$

$$x_{no} = \sum_{k=1}^{K} \widehat{z}_k^* x_{nk} + \widehat{s}_{no}^{-*} + s_{no}^{-*} \, (n = 1,......, N)$$

Corresponding to this expression we have the overall-inefficiency,

$$\tilde{\rho}_R^{k*} = w_y \tilde{\alpha}^{o*} + w_b \tilde{\beta}^{o*} + w_x \tilde{\gamma}^{o*} \, (k = 1,\cdots, K),$$

Where

$$\tilde{\alpha}^{o*} = \sum_{m=1}^{M} \frac{\varpi_m^y (\widehat{s}_{mo}^{+*} + s_{mo}^{+*})}{g_{y_m}}$$

$$\tilde{\beta}^{o*} = \sum_{j=1}^{J} \frac{\varpi_j^b (\widehat{s}_{jo}^{-*} + s_{jo}^{-*})}{g_{b_j}}$$

$$\tilde{\gamma}^{o*} = \sum_{n=1}^{N} \frac{\varpi_n^x (\widehat{s}_{no}^{-*} + s_{no}^{-*})}{g_{x_n}}$$

If any element of $\left\{\widehat{s}_{no}^{-*}\right\}, \left\{\widehat{s}_{jo}^{-*}\right\}, \left\{\widehat{s}_{yo}^{+*}\right\}$ is positive, then it holds that $\tilde{\rho}_R^{o*} > \bar{\rho}_R^{o*}$. This contradicts the optimality of $\bar{\rho}_R^{o*}$. Thus, we have $\widehat{s}_{no}^{-*} = 0, \widehat{s}_{jo}^{-*} = 0, \widehat{s}_{mo}^{+*} = 0 (\forall n, j, m)$. Hence, the projected DMU is overall-efficient. □

In addition, we can verify that WRDDM has the following properties:

Theorem 3. *If, for any two DMUs k' and k", their inefficiencies simultaneously satisfy all of the three inequalities* $\bar{\alpha}^{k'*} \geq \bar{\alpha}^{k''*}$, $\bar{\beta}^{k'*} \geq \bar{\beta}^{k''*}$, $\bar{\gamma}^{k'*} \geq \bar{\gamma}^{k''*}$ *and then it holds that* $\rho_R^{k'*} \geq \rho_R^{k''*}$.

Proof. From the equality (7), this theorem holds. □

Theorem 4. *The WRDDM is translation invariant if and only if the convexity constraints imposed on the production possibility set.*

Proof.

Let us translate the data set $x^k = (x_1^k, x_2^k, \cdots, x_N^k)$, $y^k = (y_1^k, y_2^k, \cdots, y_M^k)$, $b^k = (b_1^k, b_2^k, \cdots, b_J^k)$ by introducing arbitrary constants $\varepsilon_n (n = 1,\cdots, N)$, $\tau_m (m = 1,\cdots, M)$, $\psi_j (j = 1,\cdots, J)$ to obtain new data

$$x_n'^k = x_n^k + \varepsilon_n, (n = 1,\cdots, N : k = 1,\cdots, K)$$

$$y_m'^k = y_m^k + \tau_m, (m = 1,\cdots, M : k = 1,\cdots, K)$$

$$b_j'^k = b_j^k + \psi_j, (j = 1,\cdots, J : k = 1,\cdots, K)$$

Due to $\sum_{k=1}^{K} z_k = 1$, we have

$\sum_{k=1}^{K} z_k \varepsilon_n = \varepsilon_n (n=1,\cdots,N)$, $\sum_{k=1}^{K} z_k \tau_m = \tau_m (m=1,\cdots,M)$, and $\sum_{k=1}^{K} z_k \psi_j = \psi_j (j=1,\cdots,J)$,

We observe that the first set of constraints in model (6) become

$$\sum_{k=1}^{K} z_k (y'_{mk} - \tau_m) - s^{+}_{mk'} = \sum_{k=1}^{K} z_k y'_{mk} - s^{+}_{mk'} - \tau_m = y'_{mk} - \tau_m, \qquad m = 1, \ldots M,$$

So that

$$\sum_{k=1}^{K} z_k y'_{mk} - s^{+}_{mk'} = y'_{mk},$$

The same relationships are applicable to the second and third sets of constraints. Thus, the original problem is translation invariant. The proof relies on the convexity constraint $\sum_{k=1}^{K} z_k = 1$.□

Definition 5. *If the directional vector* $g = (-g_x, g_y, -g_b)$ *is set to be* $g = (-x, y, -b)$, *then the WRDDM is called the observation directional WRDDM.* □

Theorem 5. *The observation directional WRDDM is units invariant.*

Proof.

Consider rescale desirable output y_m, undesirable output b_j and input x_n by multiplying by the scalar μ_m, σ_j and v_n, respectively. Then the corresponding slacks of each output and input will also be rescaled by the same scalar. The objective function in model (6) will be as follows:

$$(w_y \sum_{m=1}^{M} \frac{\varpi^{y}_{m} s^{+}_{mk'}}{y_m}) + (w_b \sum_{j=1}^{J} \frac{\varpi^{b}_{j} s^{-}_{jk'}}{b_j}) + (w_x \sum_{n=1}^{N} \frac{\varpi^{x}_{n} s^{-}_{nk'}}{x_n})$$

$$= (w_y \sum_{m=1}^{M} \frac{\varpi^{y}_{m} \mu_m s^{+}_{mk'}}{\mu_m y_m}) + (w_b \sum_{j=1}^{J} \frac{\varpi^{b}_{j} \sigma_m s^{-}_{jk'}}{\sigma_m b_j}) + (w_x \sum_{n=1}^{N} \frac{\varpi^{x}_{n} v_n s^{-}_{nk'}}{v_n x_n})$$

$$= (w_y \sum_{m=1}^{M} \frac{\varpi^{y}_{m} s^{+}_{mk'}}{y_m}) + (w_b \sum_{j=1}^{J} \frac{\varpi^{b}_{j} s^{-}_{jk'}}{b_j}) + (w_x \sum_{n=1}^{N} \frac{\varpi^{x}_{n} s^{-}_{nk'}}{x_n})$$

In addition, the translated restrictions are also equivalent to the original restrictions. The value of the objective is thus not affected, there, the observation directional WRDDM is units invariant. □

Corollary 1. *The observation directional WRDDM is units invariant and translation invariant if and only if the convexity constraints imposed on the problem.*

Proof. From Theorems 4 and 5, this Corollary holds. □

Definition 6. *If the directional vector* $g = (-g_x, g_y, -g_b)$ *is set to be unit direction, then the*

WRDDM is called the unit directional WRDDM.

It is noted in this case, the $\alpha_m^{k'}, \beta_j^{k'}, \gamma_n^{k'}$ in the objective function in model (5) are equivalent to $s_{mk'}^{+*}$, $s_{jk'}^{-*}$ and $s_{nk'}^{-*}$ without units, respectively, and $\rho_R^{k'}$ is the weighted average of those weighted sum of respective slacks. When all the individual weights, $\varpi_m^y, \varpi_j^b, \varpi_i^x$, are set to be one, then model (5) degenerates to Färe and Grosskopf's (2010) model (5).

Theorem 6. *The unit directional WRDDM is units invariant.*

Proof.

The proof is similar to that of the proof of Theorem 4. □

Corollary 2. *The unit directional WRDDM is units invariant and translation invariant if and only if the convexity constraints imposed on the problem.*

Proof. From Theorems 4 and 6, this Corollary holds. □

Definition 7. *If the directional vectors do not have the same units of measurement as the vectors of the observed data and the weights corresponding to each of the inputs, desirable and undesirable outputs are the reciprocal of sample standard deviations, the WRDDM is called the inverse variance WRDDM.*[10]

Theorem 7. *The inverse variance WRDDM is units invariant.*

Proof.

Let the sample standard deviations of the desirable output y_m, undesirable output b_j and input x_n be ζ_m^y, ζ_j^b and ζ_n^x, respectively and the corresponding weights of their slacks be $\frac{1}{\zeta_m^y}$, $\frac{1}{\zeta_j^b}$ and $\frac{1}{\zeta_n^x}$.

Consider rescale desirable output y_m, undesirable output b_j and input x_n by multiplying by the scalar μ_m, σ_j and ν_n, respectively. Then the rescale sample standard deviations of the desirable output $\mu_m y_m$, undesirable output $\sigma_j b_j$ and input $\nu_n x_n$ become $\mu_m \zeta_m^y$, $\sigma_j \zeta_j^b$ and $\nu_n \zeta_n^x$ and the weight, ϖ_m^y, ϖ_j^b and ϖ_n^x in the objective function in model (6) becomes $\frac{1}{\mu_m \zeta_m^y}$, $\frac{1}{\sigma_m \zeta_j^b}$ and $\frac{1}{\nu_n \zeta_n^x}$, respectively. This implies that those rescale slacks corresponding to desirable

[10] Lovell and Pastor (1995) used the same way to propose a unit invariant weighted additive DEA model.

output y_m, undesirable output b_j and input x_n are normalized by their standard deviations. Thus, the value of the objective function (6) is indifferent from the original one[11]. Furthermore, the translated restrictions are equivalent to the original restrictions. The value of the objective is also not affected, thus, the standard deviation WRDDM is units invariant. □

Corollary 3. *The inverse variance WRDDM is units invariant and translation invariant if and only if the convexity constraints imposed on the problem.*

Proof. From Theorems 4 and 7, this Corollary holds. □

3.2. Relationship between WRDDM, SBM and ERGM

Fukuyama and Weber (2009) have shown that their directional slacks-based inefficiency (SBI) measure yields the same information on performance as Tone's SBM of efficiency when the directional vectors for inputs and outputs are chosen to equal the actual input and output vector. That is, SBM is a special case of SBI. Besides, as mentioned, Cooper et al. (2007) showed that SBM is equivalent to ERGM. Therefore, it is natural for us to develop the corresponding weighted SBM and ERGM models to extend this relationship to the situation where undesirable outputs exist.

Proposition 1. *By setting* $g=(-g_x, g_y, -g_b)=(-x^{k'}, y^{k'}, -b^{k'})$*, the corresponding weighted SBM and ERGM yields the same information on performance as the WRDDM.*

Proof.

When $g=(-g_x, g_y, -g_b)=(-x^{k'}, y^{k'}, -b^{k'})$, the programming problem of (6) can be re-expressed as follows:

$$\vec{D}^R(x^{k'}, y^{k'}, b^{k'};\ g)=\rho_R^{k'}=\max(w_x\sum_{n=1}^{N}\frac{\varpi_n^x s_{nk'}^-}{x_{nk'}}+w_x\sum_{m=1}^{M}\frac{\varpi_m^y s_{mk'}^+}{y_{mk'}}+w_b\sum_{j=1}^{J}\frac{\varpi_j^b s_{jk'}^-}{b_{jk'}})$$

$$\text{s.t.}\quad \sum_{k=1}^{K} z_k y_{mk}=y_{mk'}+s_{mk'}^+,\qquad m=1,\ldots M,$$

[11] That is

$$w_y\sum_{m=1}^{M}\frac{\varpi_m^y s_{mk'}^+}{g_{y_m}}+w_b\sum_{j=1}^{J}\frac{\varpi_j^b s_{jk'}^-}{g_{b_j}}+w_x\sum_{n=1}^{N}\frac{\varpi_n^x s_{nk'}^-}{g_{x_n}}$$
$$=w_y\sum_{m=1}^{M}\frac{1}{\mu_m\zeta_m^y}\frac{\mu_m s_{mk'}^+}{g_{y_m}}+w_b\sum_{j=1}^{J}\frac{1}{\sigma_m\zeta_j^b}\frac{\sigma_m s_{jk'}^-}{g_{b_j}}+w_x\sum_{n=1}^{N}\frac{1}{\nu_n\zeta_n^x}\frac{\nu_n s_{nk'}^-}{g_{x_n}}$$
$$=w_y\sum_{m=1}^{M}\frac{\varpi_m^y s_{mk'}^+}{g_{y_m}}+w_b\sum_{j=1}^{J}\frac{\varpi_j^b s_{jk'}^-}{g_{b_j}}+w_x\sum_{n=1}^{N}\frac{\varpi_n^x s_{nk'}^-}{g_{x_n}}$$

$$\sum_{k=1}^{K} z_k b_{jk} = b_{jk'} - s^{-}_{jk'}, \qquad j = 1, \ldots J,$$

$$\sum_{k=1}^{K} z_k x_{nk} = x_{nk'} - s^{-}_{nk'}, \qquad n = 1, \ldots N, \tag{8}$$

$$s^{+}_{mk'}, s^{-}_{jk'}, s^{-}_{nk'} \geq 0 \quad \forall m, j, n$$

$$z_k \geq 0, \qquad k = 1, \ldots\ldots, K$$

Since the value of the objective in (8) is greater than or equal to zero, we have

$$w_x \sum_{n=1}^{N} \varpi_n^x \frac{s^{-}_{nk'}}{x_{nk'}} + w_y \sum_{m=1}^{M} \varpi_m^y \frac{s^{+}_{mk'}}{y_{mk'}} + w_b \sum_{j=1}^{J} \varpi_j^b \frac{s^{-}_{jk'}}{b_{jk'}} \geq 0 \quad \Leftrightarrow$$

$$2 + w_y \sum_{m=1}^{M} \varpi_m^y \frac{s^{+}_{mk'}}{y_{mk'}} \geq (1 - w_x \sum_{n=1}^{N} \varpi_n^x \frac{s^{-}_{nk'}}{x_{nk'}}) + (1 - w_b \sum_{j=1}^{J} \varpi_j^b \frac{s^{-}_{jk'}}{b_{jk'}}) \Leftrightarrow \tag{9}$$

$$\frac{(1 - w_x \sum_{n=1}^{N} \varpi_n^x \frac{s^{-}_{nk'}}{x_{nk'}}) + (1 - w_b \sum_{j=1}^{J} \varpi_j^b \frac{s^{-}_{jk'}}{b_{jk'}})}{2 + w_y \sum_{m=1}^{M} \varpi_m^y \frac{s^{+}_{mk'}}{y_{mk'}}} \leq 1$$

Therefore, the corresponding weighted SBM which yields the same information on performance as our WRDDM would be

$$\min \theta^{k'} = \frac{(1 - w_x \sum_{n=1}^{N} \varpi_n^x \frac{s^{-}_{nk'}}{x_{nk'}}) + (1 - w_b \sum_{j=1}^{J} \varpi_j^b \frac{s^{-}_{jk'}}{b_{jk'}})}{2 + w_y \sum_{m=1}^{M} \varpi_m^y \frac{s^{+}_{mk'}}{y_{mk'}}}$$

s.t. $$\sum_{k=1}^{K} z_k y_{mk} = y_{mk'} + s^{+}_{mk'}, \qquad m = 1, \ldots M,$$

$$\sum_{k=1}^{K} z_k b_{jk} = b_{jk'} - s^{-}_{jk'}, \qquad j = 1, \ldots J,$$

$$\sum_{k=1}^{K} z_k x_{nk} = x_{nk'} - s^{-}_{nk'}, \qquad n = 1, \ldots N, \tag{10}$$

$$s^{+}_{mk'}, s^{-}_{jk'}, s^{-}_{nk'} \geq 0 \quad \forall m, j, n$$

$$z_k \geq 0 \qquad k = 1, \cdots, K$$

As states in equation (9), the value of $\theta^{k'}$ in the objective function of formulation (10) is less than or equal to one, and it provides us an aggregate measure of efficiency performance in which the numerator of $\theta^{k'}$ corresponds to the sum of the mean reduction rate of inputs and undesirable output; and similarly, the denominator corresponds to the mean expansion rate of desirable output plus one. In addition, a larger value of $\theta^{k'}$ indicates that k' performs better. If there are no slacks in all outputs, then $\theta^{k'}$ will attain the value of unity, indicating that k' is a technically efficient firm.

Furthermore, if we set each weight to be equal (i.e. $w_y = w_b = w_x$) in (8), the corresponding objective function in (9) will become

$$w_x \sum_{n=1}^{N} \varpi_n^x \frac{s_{nk'}^-}{x_{nk'}} + w_y \sum_{m=1}^{M} \varpi_m^y \frac{s_{mk'}^+}{y_{mk'}} + w_b \sum_{j=1}^{J} \varpi_j^b \frac{s_{jk'}^-}{b_{jk'}} \ge 0 \quad \Leftrightarrow \sum_{n=1}^{N} \varpi_n^x \frac{s_{nk'}^-}{x_{nk'}} + \sum_{m=1}^{M} \varpi_m^y \frac{s_{mk'}^+}{y_{mk'}} + \sum_{j=1}^{J} \varpi_j^b \frac{s_{jk'}^-}{b_{jk'}} \ge 0$$

$$2 + \sum_{m=1}^{M} \varpi_m^y \frac{s_{mk'}^+}{y_{mk'}} \ge (1 - \sum_{n=1}^{N} \varpi_n^x \frac{s_{nk'}^-}{x_{nk'}}) + (1 - \sum_{j=1}^{J} \varpi_j^b \frac{s_{jk'}^-}{b_{jk'}}) \Leftrightarrow \frac{(1 - \sum_{n=1}^{N} \varpi_n^x \frac{s_{nk'}^-}{x_{nk'}}) + (1 - \sum_{j=1}^{J} \varpi_j^b \frac{s_{jk'}^-}{b_{jk'}})}{2 + \sum_{m=1}^{M} \varpi_m^y \frac{s_{mk'}^+}{y_{mk'}}} \le 1$$

The objective function in (10) becomes

$$\min \theta^{k'} = \frac{(1 - \sum_{n=1}^{N} \varpi_n^x \frac{s_{nk'}^-}{x_{nk'}}) + (1 - \sum_{j=1}^{J} \varpi_j^b \frac{s_{jk'}^-}{b_{jk'}})}{2 + \sum_{m=1}^{M} \varpi_m^y \frac{s_{mk'}^+}{y_{mk'}}} \tag{11}$$

Then, the weighted SBM can further be shown to be equivalent to the following enhanced Russell graph measure by means of the following change of variables:

$$\begin{aligned} \theta_{mk'}^y &= \frac{y_{mk'} + s_{mk'}^+}{y_{mk'}} = 1 + \frac{s_{mk'}^+}{y_{mk'}} \\ \theta_{jk'}^b &= \frac{b_{jk'} - s_{jk'}^-}{b_{jk'}} = 1 - \frac{s_{jk'}^-}{b_{jk'}} \\ \theta_{nk'}^x &= \frac{x_{nk'} - s_{nk'}^-}{b_{nk'}} = 1 - \frac{s_{nk'}^-}{x_{nk'}} \end{aligned} \tag{12}$$

We derive the alternative expression. Since $\sum_{m=1}^{M} \varpi_m^y = 1$, $\sum_{j=1}^{J} \varpi_j^b = 1$ and $\sum_{n=1}^{N} \varpi_n^x = 1$, the formulation (12) can be easily re-expressed as follows:

$$\min \theta^{k'} = \frac{\sum_{n=1}^{N} \varpi_n^x \theta_{nk'}^x + \sum_{j=1}^{J} \varpi_j^b \theta_{jk'}^b}{1 + \sum_{m=1}^{M} \varpi_m^y \theta_{mk'}^y}$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{k=1}^{K} z_k y_{mk} \geq \theta_{mk'}^y y_{mk'} && m = 1, \cdots, M \\
& \sum_{k=1}^{K} z_k b_{jk} = \theta_{jk'}^b b_{jk'} && j = 1, \cdots, J \\
& \sum_{k=1}^{K} z_k x_{nk} \leq \theta_{nk'}^x x_{nk'} && n = 1, \cdots, N \\
& \theta_{mk'}^y \geq 1, \quad \theta_{jk'}^b \leq 1, \quad \theta_{nk'}^x \leq 1 \quad \forall m, j, n \\
& z_k \geq 0 && k = 1, \cdots, K
\end{aligned} \tag{13}$$

The objective function in (13) gives

$$\begin{aligned}
0 \leq \frac{\sum_{n=1}^{N} \varpi_n^x \theta_{nk'}^x + \sum_{j=1}^{J} \varpi_j^b \theta_{jk'}^b}{1 + \sum_{m=1}^{M} \varpi_m^y \theta_{mk'}^y} &= \left(\sum_{n=1}^{N} \varpi_n^x \theta_{nk'}^x + \sum_{j=1}^{J} \varpi_j^b \theta_{jk'}^b\right)\left(1 + \sum_{m=1}^{M} \varpi_m^y \theta_{mk'}^y\right)^{-1} \\
&= (\overline{\theta}_{k'}^x + \overline{\theta}_{k'}^b)(1 + \overline{\theta}_{k'}^y)^{-1} \leq 1
\end{aligned} \tag{14}$$

This expression (14) is interpreted as the ratios of the weighted average of input and undesirable output efficiency scores to the average desirable output efficiency scores that represent the efficiency measure for each of the DMU k (k=1,…,n). This measure is also able to provide us an overall efficiency measure which simultaneously considers resource utilization, good output production and bad output reduction. Moreover, because $\theta^{k'}$ contains the performance information about each y_m, b_j and x_n of DMU k', we can easily find the percentage by which each y_m, b_j and x_n ought to improve from this programming problem, and hence it allows us to give a more comprehensive picture on the performance of the DMU. Nevertheless, the formulation (13) is nonlinear and its solution is not easily obtained. We may follow the methods developed by Pastor et al. (1999) or Ray and Jeon (2008) to transform it into a linear programming problem when we execute the solution calculation process.

3.3. Uniqueness Problems

Unique determination of the optimum value is one of the desirable properties for efficiency comparison. However, as Sueyoshi and Sekitani (2009) pointed out, many DEA models suffer from an occurrence of multiple projections, especially for non-radial measures. Mathematically, although the overall inefficiency ρ_R calculated form WRDDM is uniquely determined, the corresponding slacks s_m^+ and/or s_j^- and/or s_n^- may have multiple optimum and hence solutions of $\overline{\alpha}_y$, $\overline{\beta}_b$ and $\overline{\gamma}_x$ may also be multiple. If the obtained slacks have multiple optima, it means that the projections to the efficient status described in Theorem 2 cannot be uniquely determined. We therefore have the following proposition:

Proposition 2. *The WRDDM violates the property of unique projection for efficiency comparison.*

We give the following example to show the WRDDM inefficiency scores may not be uniquely determined for efficiency comparison:

~Insert Table 1 here~

In this example, we generate the data for 5 DMUs with one good output, one bad output and 2 inputs. We use model (5) to calculate each DMU's overall inefficiency score, and the inefficiency scores of goods, bads and inputs. Because there are two inputs, we also have the inefficiency scores for each input. In order to find its range of variation, we follow Tone and Tsutsui's (2010) method to adopt two-step procedures to solve max (min) $\overline{\alpha}_y$, $\overline{\beta}_b$ and $\overline{\gamma}_x$, while keeping ρ_R at the optimum respectively in the first step; then in turn to solve max (min) of each α_m, β_j and γ_n, while keeping ρ_R as well as $\overline{\alpha}_y$, $\overline{\beta}_b$ or $\overline{\gamma}_x$ (depending on we solve the bounds for α_m, β_j or γ_n, respectively) at the optimum in the second step. That is, according to the above example, we need to carry out six times of solving in the first step and four times of solving in the second step.

Accordingly, it can be found the projected desirable output and undesirable output are uniquely determined, while the projected inputs are not because their max and min values are not equal, respectively. Tone and Tsutsui's (2010) indicated that if the relative gap (max–min)/max is comparatively small (e.g. <0.5%), we can practically neglect the multiple optima problem. In this example, it is shown the relative gaps for the two inputs (100% and 19.9%) are too large to be neglected.

4. Empirical Application

Increasing environmental performance has long been a policy objective in most countries. As concern over global warming mounts, there have been several studies carrying out cross-country performance comparisons or time-series productivity growth analysis taking into account carbon dioxide (CO_2) emission by using the method of DEA, such as Zaim and Taskin (2000), Zaim and

Taskin (2001), Färe et al. (2004), Jeon and Sickles (2004), Zhou et al. (2006), Zhou et al. (2007) and Kumar and Managi (2010). However, no matter which model these papers adopted, few of them gauge performance in terms of increased good outputs and decreased bad outputs and inputs simultaneously and account for all the slacks which the model can identify. We apply a dataset consisting of 99 countries from 1991 to 2003 to illustrate our WRDDM.

4.1. Data and Variables Description

Our measure of aggregate outputs is gross domestic product (GDP) as the desirable output and CO_2 emission as the undesirable output in this study. Regarding inputs, other than capital and employment, energy use is included to reflect the fact that CO_2 emissions are directly related to the use of energy.[12] That is, we regard each county employ labor, capital and energy to produce aggregate outputs including goods and services associating the undesirable by-product, CO_2.

As for the sources of these data, energy use and CO_2 are compiled from World Development Indicators (WDI, World Bank), GDP, employment and capital are calculated from data provided by Penn World Tables (PWT 6.2). GDP expressed in 2000 international prices is calculated from real GDP per capita and population, and employment is obtained from real GDP and real GDP per worker.[13] To the best of our knowledge, data of recent capital stock (after the year 2000) are not available from any statistical yearbook or database. Due to data constraint, the capital stock (K_t) in the year t is hence estimated by the following perpetual inventory method formula presented in the file of "How to Use the Data Files for EPWT2.1" on the EPWT Home Page:[14]

$$K_t = \sum_{i}^{T} (1-0.075)^{(T-i)} I_{T-i}, \qquad i = 1, \cdots 14 \tag{15}$$

where I is the investment series calculated from the variables of real investment share (ki) of GDP, real GDP per capita in constant dollars (chain index), and population in the PWT 6.2. T is the asset life, which is considered to be 14 years, and the depreciation rate is 7.5 percent. Please see the file of "How to Use the Data Files for EPWT2.1" for more details of the description of the estimation problems of capital stock.

Table 2 displays the summary statistics of the growth rates of all the input and output variables used in the study as well as the CO2 intensities in terms of CO2 emissions per GDP and CO2 factor in terms of CO2 emissions per energy consumption according to groups classified by income levels following the World Bank classification and OECD/non-OECD countries. It can be seen that the economies of the lower middle income countries grew the fastest (3.379%) with the highest growth rates of labor and energy usages as well as the bad output by-product in the sample period on average. However, both of the CO2 intensities for the higher income countries (including high and

[12] This variable specification is the same as Jeon and Sickles (2004) and Kumar and Managi (2010).

[13] PWT 6.2 is the same as PWT 6.1 (data provided up to 2000) in that it does not provide the data of GDP and employment directly. Therefore, there is an extended database referred to as EPWT 2.1 which provides some of the data such as GDP and employment not obtained directly in the PWT 6.1. We follow the procedure by which EPWT 2.1 computed GDP and employment to calculate our GDP and employment. Put more concretely, by using the variable names specified by PWT 6.2, our computation of GDP and employment can be expressed as: GDP = real GDP per capita (=rgdpch) * population (=pop), employment = GDP / real GDP per worker (=rgdpwok).

[14] The web site address is: http://homepage.newschool.edu/~foleyd/epwt/.

upper middle income groups) are higher than in the lower income countries. As for OECD/non-OECD countries, the input/ouput trends for non-OECD countries are similar to those of lower middle income countries. Nevertheless, their CO2 intensities in terms of CO2 emissions per GDP are in general higher than those in OECD countries.

~Insert Table 2 here~

4.2. Differences between the TDDFM and WRDDM models

Table 3 reports the results of average overall inefficiency scores of all countries calculated by the TDDFM, the WRDDM, and the differences (BIAS) between these two models. Note that models are solved by setting $g = (-g_x, g_y, -g_b)$ = $(-x^{k'}, y^{k'}, -b^{k'})$ because we would like to observe how much the percentage needed to be improved and all the weights in WRDDM are specified to be equal (i.e. $w_y = w_b = w_x$ =1/3, ϖ_m^y =1/M, ϖ_j^b =1/J and ϖ_n^x =1/N). [15] In addition, the aggregate overall technical inefficiency computed from the models of the traditional directional distance function and weighted Russell directional distance function are evaluated at the same input bundles transformed to outputs of 99 countries in the same periods. The uniqueness of WRDDM results are examined using the method aforementioned and it is good to find that all the efficiency scores are uniquely determined and do not have any multiple solutions problem for this dataset.

In 1991-2003, the TDDFM shows slightly larger differences than it does in 2000-2003 (0.149 versus 0.126). The same difference pattern is also found in the respective periods of the WRDDM (0.293 in 1991-2003, and 0.261 in 2000-2003). The average overall inefficiency score estimated by the TDDFM is lower than those by the WRDDM, thus the BIAS scores during the study period are all positive. It is an explicit relationship derived from the TDDFM and WRDDM models that inefficiency scores estimated by the latter are no less than the former due to the lack of accounting for non-zero slacks in the former one. It is also found that all inefficiency scores are broadly smooth over the period 1991-2003. This may imply that these sample countries did not have large efficiency changes during this 1991-2003 period, even though the average inefficiency scores of these countries showed slight improvement in 2000-2003.

A Wilcoxon test is used to test the hypothesis that the two vectors of means of overall inefficiency score were equal. We find there are highly significant differences (z-score=-3.180, p-value= 0.001) between the results of these two models, implying that the null hypothesis of equal overall inefficiency score has to be rejected. Therefore, this confirms that the TDDFM and the

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15 It is important to observe that requiring $g = (-g_x, g_y, -g_b)$ to have a value of one for each type of input and output implies that the directional vector is in the units of the observed data. the resulting value of the inefficiency measures are interpreted as the number of units of each type of input (output) that can be decreased (increased) for each observation (Färe and Grosskopf, 2010).

WRDDM model are unequally able to measure comprehensive efficiencies for production of the sample data.

~Insert Table 3 here~

4.3. Comparison of input with output efficiencies

We now turn to the computed overall inefficiency and its components separated by OECD and non-OECD countries (see summary in Table 4). Our efficiency modeling not only describes the input side of the production technology, but also includes desirable and undesirable output effects, which are present on the output side of production. Especially for the bad outputs, which are common in production, they are expected to influence the provision of a country's desirable output. Thus, these undesirable by-products can have an evident effect on the output efficiency for countries at a point where joint-production appears in production. Table 4 also presents the decomposition of input inefficiency scores of OECD and non-OECD countries into labor, capital and energy parts. For instance, the 0.269 value (the value of overall inefficiency, which is the weighted sum of corresponding figures of inputs, goods and bads) computed for the OECD countries in 1991 on average, shows the factor by which the good output can be expanded by more 17.4% times compared to its current level while simultaneously cutting 43.4% of the pollution emissions and 19.9% of resource use (1.9% labor, 15.2% capital and 42.6% energy) and still remaining in the feasible production set. A similar interpretation applies to the values of the non-OECD countries in each component.

~Insert Table 4 here

The degree of OECD's overall inefficiency was generally lower than non-OECD throughout the period, which is confirmed by the Mann-Whitney U test with p-value= 0.001. This implies that OECD's comprehensive efficiency is generally better than non-OECD countries. It is hence interesting to investigate whether all components of overall inefficiency have the same pattern. Therefore, the rank order correlation coefficients for both OECD and non-OECD countries are also computed for each component. The results are displayed in Table 5.

On the output side, the means of the rank order correlation coefficients for both categories of countries are significantly unequal for the goods inefficiency (OECD 7.0 and non-OECD 20.0 with p-value=0.001). Therefore, we accept that the OECD countries' goods efficiencies are better than non-OECD countries'. However, this is not the case for bads inefficiency (OECD 14.650 and non-OECD 12.350 with p-value=0.442). The non-OECD countries are better than the OECD countries in bads efficiency.

On the input side, a similar result is found for the labor component; the mean of the rank order correlation coefficients of the OECD countries is 7.0, and that of the non-OECD countries 20.0. Again, the labor efficiency of OECD countries is significantly better than the non-OECD countries' (p-value<0.001). As for the capital component, the performance of the OECD countries is slightly better than that of the non-OECD countries, although the Mann–Whitney test indicates that their differences are not significant. This means that we can accept the hypothesis that capital inefficiencies of the OECD and the non-OECD countries are not different. However, a different result is found for the energy efficiency for the OECD and the non-OECD countries; the mean of the rank order correlation coefficients of the OECD countries is 18.850, and that of the non-OECD countries is 8.150. This implies that the non-OECD countries are significantly better than the OECD countries in energy efficiency. This result coincides with the results of bads inefficiency. Therefore, we accept the hypothesis that the OECD and the non-OECD countries are significantly different in energy inefficiency, while this is not the case in input inefficiency when all of the input items are put together.

~Insert Table 5 here~

One interesting discrepancy in inefficiency between the OECD and non-OECD countries is the annual trend pattern of efficiency variation. Figure 1 shows the relative difference in levels and the relative comparability in temporal patterns for the overall inefficiency scores and their components during the sample period for the OECD and non-OECD countries. The average level of overall inefficiency scores was calculated to be 0.248 and 0.311 for OECD and non-OECD countries, respectively. Inefficiency was at its lowest in 2003 for OECD countries and in both 2001 and 2002 for non-OECD countries when the level of inefficiency was 0.201 and 0.271, respectively. It is followed by 0.207 in 2001 for OECD countries and 0.270 in 1996 for non-OECD countries. In figure 1(a), there is a relatively wide gap in the technical efficiencies between the OECD and non-OECD countries of about 10% during 1991-94. The overall inefficiency converges to nearly no difference during 1995-97. Then, it diverges to about 10% during 1998-2003. The overall inefficiency averages to 0.248 over the 13-year period for the OECD countries compared to 0.311 for the non-OECD countries.

Figures 1(b)-(g) show the temporal patterns of the overall inefficiency's decompositions. On average, figures 1(b) and 1(e) indicate that the goods inefficiency and labor inefficiency components for OECD countries are considerably below those for their non-OECD counterparts at the beginning of the study period and that this pattern was rather stable over the sample period. The goods (labor) inefficiency averages to 0.113 (0.033) over the 13-year period for the OECD countries, which is smaller than 0.285 (0.224) for the non-OECD countries. Moreover, figure 1(g) shows OECD countries had an annual energy inefficiency which was above that for the non-OECD countries, 0.392 versus 0.242. Figures 1(c) and 1(f), however, indicate that the differences of the annual inefficiencies of bads and capital between the OECD countries and non-OECD countries do not show consistent differences over time. For example, the OECD countries perform better in bads abatement than non-OECD countries over 1991-94, while it is not the case during 1995-98. In 1999, the OECD countries and non-OECD countries have almost the same performance in bads efficiency. During 2000-02, the OECD countries become slightly more inefficient in bads than non-OECD countries.

Overall, energy and bads efficiencies of non-OECD countries are better than OECD countries' during the whole study period and for the 1995-2003 period. The results show that the bads and the energy inefficiency for the OECD countries are generally larger than those of the non-OECD countries. The other results indicate opposite results, i.e., the OECD countries generally perform better than the non-OECD countries in overall, goods and two (labor and capital) of the three input items efficiencies. In particular, the labor inefficiency of the OECD countries is much smaller than that of the non-OECD countries. The high inefficiency scores of bads and energy inefficiencies for the OECD countries show the importance of policies that focus on improving pollution abatement and the energy use of the OECD countries is not efficient. The results imply that the advantage of the OECD countries' higher technology level results in lower labor and capital inefficiencies, while the OECD countries' people consume too much energy in their daily lives. They might waste more energy resources than those people in non-OECD countries, probably because energy prices have been decreasing in our study period. Therefore, the relative price of energy is becoming less important in OECD countries. To the extent that there is an energy disadvantage associated with lifestyle, the OECD countries' input inefficiency is affected by the increase in energy inefficiencies in the input utilization.

~Insert Figure 1 here~

Based on this discussion, we conclude that membership in the OECD is one of the most important factors influencing the components' inefficiencies. A possible explanation for this variability is that the income levels for these two groups of countries are different. Therefore, in order to capture more insights, we further separate all of the countries into four income levels and investigate various components' inefficiencies in all income level countries.

4.4. Differences in income levels

Table 6 shows that the overall inefficiency in the high income countries is the lowest. This implies the high income countries are in general more efficient than those at the other income levels and have limited room for improvement in their efficiencies. To some extent, this may be because higher income countries are more likely to employ advanced technology which is a potential source of their competitive advantage.

As can be seen in Table 6, the effect of outputs on overall inefficiency is generally more serious than that of inputs (except energy) of each income level country. Also, the bads impact on inefficiency is more demanding than that of the goods, except in low income countries. This indicates that improving efficiency of bads is more effective than improving efficiency of goods and inputs for increasing overall efficiency in the world. In addition, we can find that the inefficiency scores of bads increase from 0.424 and 0.340 on average for the low income countries, to 0.464 and 0.514 for the upper middle income countries, then decrease to 0.436 and 0.382 for the high income countries over 1991-2003 and 2000-2003, respectively. In low income countries, improving efficiency of goods is more effective for increasing overall efficiency. If we regard the inefficiency score of bads as the environmental performance measure, this tendency to some extent suggests that there exists a inverse-U-shaped environmental Kuznets curve relationship in our sample data. This is because the performance deteriorates somewhat along with the increase of the income, but later improves for the highest income level of countries. More evidence is of course needed to support the existence of this relationship in our case. However, that is beyond the scope of our study.

In contrast to the impact of outputs on overall inefficiency, the relative magnitude of the impact of each input item on overall inefficiency between each income level is shown to have dissimilar patterns. For example, the labor inefficiency of the low income countries is greater than that of the other income level countries, while the low income countries perform the best in capital efficiency. When we compare energy efficiency, apart from the low income countries, there seems to be a tendency for lower income countries to have lower energy inefficiency. However, the trend reverses for the low income countries, which have the second highest energy inefficiency. Moreover, if we focus on the performance of the lowest income countries, we find that even though they have the lowest capital inefficiency, its positive impact on the overall inefficiency is offset by both the labor and energy inefficiency. This result indicates that improving the labor and energy efficiency of countries in this income level is important. Overall, the energy inefficiency scores are relatively high in the input inefficiency scores on average. The capital inefficiency scores are marginally lower than the labor inefficiency scores, and thus capital efficiency contributes to a very large extent to the input utilization performance.

~Insert Table 6 here~

The WRDDM analysis offers both the aggregate inefficiency measure and its components. Therefore our basic hypothesis is that the aggregate inefficiency measure and its components themselves are not different among these groups of countries by income level. The first analysis concentrates on testing whether the overall inefficiency of the different income level countries is not different from others. The second hypothesis addresses whether the influence of each input/output inefficiency on overall inefficiency is not different at a given income level countries

HYPOTHESIS 1: The overall inefficiencies of different income level countries are not different from each other.

This hypothesis assumes differences of overall inefficiency between different income level countries are not obvious, whereas we expect the overall inefficiencies vary distinctly between different income level countries. A Kruskal Wallis test was used to test this hypothesis, and the results displayed in Table 7 show a significant difference among different income level countries ($p\text{-value}<0.001$), implying that the null hypothesis of equal overall inefficiency has to be rejected. This finding confirms our expectation that the overall inefficiency was significantly different among different income level countries and implies that the overall performance of a country would be affected by its income level.

~Insert Table 7 here~

HYPOTHESIS 2: The influence of all components' inefficiencies on overall inefficiency is the same in a given income level group of countries.

This hypothesis concerns the impacts of the various input and output items on the overall inefficiency within each of the income levels of countries. A Kendall's W statistic is used to measure the extent to which the ranking of all the input and output items' inefficiencies resemble the ranking of each other items'. The results for each of income levels' countries are displayed in Table 8. As can be seen in the first row of Table 8, the rank order correlation coefficients of each component inefficiency score are significantly different from each other (goods 2.385, bads 5.293, labor 1.154, capital 2.692 and energy 5.000), resulting in a very small p-value of the Kendall's W test. This implies that the null hypothesis of equal inefficiency scores for the high income countries has to be rejected. For further exploration, the same outcome can be found for the remainder of the three income levels. These results provide evidence to show that different components' inefficiencies lead to different degrees of impact on the overall inefficiency and, hence, indicate that separating overall inefficiency into different parts is important to provide management insight for the policy decision making of each country.

~Insert Table 8 here~

To elaborate more details on each component of the production performance of the countries, we plot in Figure 2 the mean value of the goods, bads and input inefficiency indices computed over the 99 countries for the period 1991–2003. The mean bads inefficiency index shows a dramatic improvement in environmental efficiency in terms of CO2 emissions in 1995, and stable environmental performance since then. Despite the differences in overall means, we observe that the changes in the goods and input inefficiency are relatively stable during the study period. The decomposition of overall inefficiency index into the goods, bads and input inefficiency indices are also successful in capturing each component of the overall inefficiency of countries. The result shows that the goods and input efficiency are better than bads efficiency globally.

~Insert Figure 2 here~

5. Conclusions

As environmental concerns become increasingly pronounced in relation to global commons, environmental issues are being treated more and more as international matters. The accurate assessment of environmental conditions is essential. In this study, the WRDDM model is proposed and applied to a 99-country dataset from 1991 to 2003 to examine the input, desirable output, and undesirable output items' efficiencies in order to comprehensively gauge overall production efficiency.

From the performance evaluation aspect, one of the advantages of the WRDDM over the traditional directional distance function model is that it directly incorporates weights to consider the appropriate relationship among input and output items, while the traditional model weights them equally. In addition, the proposed model accounts for all the slacks for the inputs, desirable outputs and undesirable outputs, and thus is able to provide more accurate performance evaluation results. From the application aspect, this model allows a DMU to be able to adjust all the amounts of inputs, desirable outputs and undesirable outputs (subject only to the constraints imposed by the production technology) and hence the technical efficiencies for each input, desirable output and undesirable output may be rated differently and can help us to identify which resource uses or production of outputs (including goods and bads) need to be improved most.

The empirical results reveal that the separation of overall inefficiency into different parts is important to provide management insight for the policy decision making of each country. For example, it is shown that the OECD countries perform better than the non-OECD countries in overall, goods and two out of three input items' efficiencies. On the other hand, the bads and the energy inefficiency for the OECD countries are larger than those of the non-OECD countries. These results imply that an improvement in the bads and the energy efficiency of OECD countries would be more important than those of the non-OECD countries. While this result does not deny the importance of efforts to improve efficiency in the other components, the most effective way to improve the overall production efficiency of the non-OECD countries is to focus on the improvement in the goods and bads efficiencies.

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Table 1. Numerical test of uniqueness.

DMU	Good	Bad	Input 1	Input 2	ρ_R	$\overline{\alpha}_y$	$\overline{\beta}_b$	$\overline{\gamma}_x$	γ_1	γ_2	max $\overline{\alpha}_y$	min $\overline{\alpha}_y$	max $\overline{\beta}_b$	min $\overline{\beta}_b$	max $\overline{\gamma}_x$	min $\overline{\gamma}_x$	max γ_1	min γ_1	max γ_2	min γ_2
1	2	1	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	2	1	2	2	0.083	0	0	0.250	0	0.500	0	0	0	0	0.250	0.250	**0.500**	**0**	**0.500**	**0**
4	2	2	4	4	0.417	1	0	0.250	0	0.500	1	1	0	0	0.250	0.250	**0.500**	**0**	**0.500**	**0**
5	2	1	6	6	0.250	0	0	0.750	0.667	0.833	0	0	0	0	0.750	0.750	**0.833**	**0.667**	**0.833**	**0.667**

Table 2. Summary Statistic (1991-2003)

	GDP growth rate (%)	labor growth rate (%)	capital growth rate (%)	energy growth rate (%)	CO2 growth rate (%)	CO2/energy (kt/kt of oil equivalent)	CO2/GDP (kt/million)
99 countries							
mean	3.169	2.035	2.785	2.673	3.388	2.080	0.523
SD	2.026	1.115	3.346	1.916	6.085	1.050	0.619
max	9.872	5.470	10.981	6.552	52.543	7.688	5.381
min	-2.846	0.039	-10.444	-3.699	-9.329	0.163	0.067
High income countries (36)							
mean	3.204	1.320	3.102	2.415	1.841	2.347	0.604
SD	1.588	1.059	2.126	1.663	2.329	0.611	0.420
max	7.413	4.069	8.552	6.206	8.170	3.238	1.786
min	0.859	0.039	-2.038	-0.051	-1.144	0.774	0.193
Upper middle income countries (17)							
mean	2.987	1.914	3.336	2.504	2.186	2.374	0.510
SD	1.842	0.994	3.453	2.446	3.999	0.490	0.313
max	6.592	3.971	8.568	6.552	9.496	3.416	1.176
min	-0.284	0.183	-5.718	-2.285	-6.844	1.628	0.149
Lower middle income countries (27)							
mean	3.379	2.732	1.911	3.269	6.392	2.178	0.441
SD	2.113	0.955	4.278	1.768	9.556	0.877	0.390
max	9.872	5.470	10.981	5.720	52.543	4.420	2.023
min	-2.177	1.027	-10.444	-2.175	-1.075	0.576	0.104
Low income countries (19)							
mean	2.966	2.506	2.934	2.468	3.125	1.174	0.497
SD	2.798	0.630	3.673	2.010	5.019	1.687	1.189
max	7.599	3.298	8.292	4.995	9.771	7.688	5.381
min	-2.846	0.792	-5.959	-3.699	-9.329	0.163	0.067
OECD countries (28)							
mean	2.872	0.903	3.161	1.701	1.154	2.236	0.471
SD	1.354	0.713	1.720	1.444	1.602	0.636	0.193
max	7.413	2.761	7.216	6.206	4.624	3.416	1.176
min	0.859	0.039	-0.405	-0.651	-1.144	0.774	0.210
Non-OECD countries (71)							
mean	3.286	2.481	2.637	3.057	4.269	2.019	0.543
SD	2.234	0.912	3.802	1.951	6.933	1.172	0.722
max	9.872	5.470	10.981	6.552	52.543	7.688	5.381
min	-2.846	0.226	-10.444	-3.699	-9.329	0.163	0.067

Note: 1. We use the classification drawn from the World Bank in which the Economies are divided according to 2006 GNI per capita, calculated using the World Bank Atlas method. The groups are: low income, \$905 or less; lower middle income, \$906 - \$3,595; upper middle income, \$3,596 - \$11,115; and high income, \$11,116 or more.

2. The value inside the parentheses after each category is the number of countries.

Table 3. Aggregate (overall) technical inefficiency

	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	Average 91-03	Average 00-03
TDDFM	0.179	0.167	0.164	0.181	0.157	0.157	0.151	0.142	0.132	0.128	0.128	0.126	0.121	0.149	0.126
WRDDM	0.345	0.325	0.314	0.344	0.287	0.284	0.286	0.307	0.278	0.267	0.253	0.254	0.269	0.293	0.261
BIAS	0.166	0.158	0.150	0.163	0.130	0.127	0.135	0.165	0.146	0.139	0.125	0.128	0.148	0.145	0.135

Notes: TDDFM: traditional directional distance function results, formulations (3)
WRDDM: weighted Russell directional distance function results, formulations (6) in which weights are set equally as 1/3.
BIAS= WRDDM – TDDFM.

Table 4. Components of the overall inefficiency by OECD and non-OECD countries

	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	Average 91-03	Average 00-03
OECD countries															
overall	0.269	0.259	0.260	0.296	0.281	0.293	0.280	0.243	0.206	0.216	0.207	0.211	0.201	0.248	0.209
goods	0.174	0.149	0.117	0.165	0.171	0.178	0.137	0.075	0.054	0.060	0.079	0.061	0.045	0.113	0.061
bads	0.434	0.419	0.446	0.504	0.460	0.503	0.512	0.499	0.427	0.429	0.406	0.413	0.403	0.450	0.413
inputs	0.199	0.208	0.217	0.220	0.213	0.199	0.191	0.153	0.138	0.160	0.136	0.159	0.154	0.181	0.152
labor	0.019	0.020	0.035	0.044	0.083	0.082	0.068	0.008	0.008	0.009	0.007	0.032	0.012	0.033	0.015
capital	0.152	0.207	0.157	0.113	0.086	0.053	0.029	0.246	0.184	0.113	0.034	0.060	0.090	0.117	0.074
energy	0.426	0.397	0.458	0.505	0.471	0.462	0.477	0.206	0.221	0.358	0.368	0.385	0.359	0.392	0.367
Non-OECD countries															
overall	0.375	0.351	0.335	0.362	0.289	0.280	0.288	0.332	0.306	0.287	0.271	0.271	0.296	0.311	0.281
goods	0.347	0.297	0.259	0.339	0.289	0.269	0.253	0.332	0.263	0.261	0.251	0.268	0.282	0.285	0.266
bads	0.589	0.547	0.560	0.557	0.354	0.348	0.390	0.426	0.433	0.403	0.380	0.384	0.435	0.447	0.400
inputs	0.189	0.209	0.187	0.192	0.223	0.224	0.221	0.238	0.222	0.196	0.182	0.162	0.170	0.201	0.177
labor	0.158	0.207	0.188	0.208	0.255	0.259	0.265	0.300	0.256	0.222	0.212	0.176	0.203	0.224	0.203
capital	0.123	0.120	0.078	0.081	0.088	0.100	0.085	0.250	0.240	0.187	0.113	0.155	0.164	0.137	0.155
energy	0.286	0.300	0.295	0.287	0.327	0.313	0.314	0.163	0.169	0.179	0.220	0.155	0.143	0.242	0.174

Table 5. Rank Order Correlation Coefficient of the OECD and non-OECD countries: Mann-Whitney Test

Component	Rank Order Correlation Coefficient		U	p-value
	OECD	non-OECD		
Overall	8.380	18.620	18.000	0.001*
Goods	7.000	20.000	0.000	<0.001*
Bads	14.650	12.350	69.500	0.442
Inputs	10.85	16.150	50.000	0.077
labor	7.000	20.000	0.000	<0.001*
capital	12.230	14.770	68.000	0.397
energy	18.850	8.150	15.000	<0.001*

*Significant at 0.01 level.

Table 6. Components of the overall inefficiency by income levels

	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	Average 91-03	Average 00-03
99 countries															
overall	0.345	0.325	0.314	0.344	0.287	0.284	0.286	0.307	0.278	0.267	0.253	0.254	0.269	0.293	0.261
goods	0.298	0.255	0.219	0.290	0.256	0.243	0.220	0.259	0.204	0.205	0.202	0.209	0.215	0.237	0.208
bads	0.545	0.511	0.527	0.542	0.384	0.392	0.425	0.446	0.431	0.410	0.388	0.392	0.426	0.448	0.404
inputs	0.192	0.209	0.195	0.200	0.220	0.217	0.213	0.214	0.198	0.186	0.169	0.161	0.166	0.195	0.170
labor	0.118	0.154	0.145	0.162	0.206	0.209	0.209	0.217	0.186	0.162	0.154	0.135	0.149	0.170	0.150
capital	0.131	0.145	0.100	0.090	0.088	0.087	0.069	0.249	0.224	0.166	0.090	0.128	0.144	0.132	0.132
energy	0.326	0.327	0.341	0.349	0.368	0.355	0.360	0.175	0.184	0.229	0.262	0.220	0.204	0.285	0.229
High Income countries															
overall	0.246	0.238	0.246	0.274	0.261	0.274	0.266	0.221	0.181	0.192	0.191	0.200	0.184	0.229	0.192
goods	0.099	0.098	0.109	0.115	0.095	0.103	0.091	0.080	0.036	0.044	0.046	0.055	0.059	0.079	0.051
bads	0.442	0.417	0.428	0.496	0.487	0.523	0.516	0.451	0.384	0.394	0.386	0.392	0.358	0.436	0.382
inputs	0.198	0.199	0.199	0.211	0.201	0.195	0.192	0.131	0.123	0.139	0.141	0.154	0.136	0.171	0.142
labor	0.014	0.016	0.030	0.037	0.064	0.065	0.053	0.000	0.000	0.003	0.012	0.020	0.006	0.025	0.010
capital	0.153	0.184	0.143	0.116	0.065	0.045	0.045	0.216	0.143	0.047	0.045	0.042	0.060	0.100	0.049
energy	0.426	0.395	0.425	0.481	0.474	0.475	0.476	0.178	0.226	0.366	0.368	0.399	0.339	0.387	0.368
Upper Middle Income countries															
overall	0.340	0.331	0.301	0.343	0.306	0.303	0.295	0.354	0.322	0.300	0.276	0.270	0.289	0.310	0.284
goods	0.402	0.350	0.289	0.380	0.434	0.420	0.347	0.311	0.232	0.184	0.180	0.168	0.120	0.294	0.163
bads	0.444	0.460	0.461	0.493	0.326	0.335	0.381	0.538	0.532	0.521	0.488	0.480	0.569	0.464	0.514
inputs	0.175	0.181	0.153	0.154	0.159	0.154	0.156	0.213	0.203	0.197	0.159	0.163	0.180	0.173	0.175
labor	0.002	0.000	0.000	0.001	0.063	0.076	0.075	0.071	0.055	0.041	0.054	0.046	0.039	0.040	0.045
capital	0.273	0.287	0.157	0.142	0.128	0.126	0.095	0.392	0.378	0.345	0.104	0.258	0.307	0.230	0.254

energy	0.251	0.256	0.303	0.319	0.285	0.259	0.297	0.175	0.177	0.205	0.319	0.184	0.194	0.248	0.225
Lower Middle Income countries															
overall	0.435	0.412	0.383	0.414	0.328	0.301	0.311	0.372	0.344	0.320	0.275	0.268	0.305	0.344	0.292
goods	0.513	0.457	0.381	0.451	0.397	0.342	0.309	0.407	0.325	0.336	0.323	0.264	0.278	0.368	0.300
bads	0.644	0.611	0.618	0.630	0.333	0.317	0.371	0.459	0.478	0.423	0.348	0.383	0.478	0.469	0.408
inputs	0.148	0.168	0.149	0.159	0.255	0.243	0.254	0.249	0.230	0.200	0.153	0.156	0.159	0.194	0.167
labor	0.123	0.185	0.170	0.204	0.373	0.347	0.393	0.376	0.320	0.311	0.269	0.249	0.268	0.276	0.274
capital	0.103	0.105	0.077	0.086	0.112	0.127	0.074	0.300	0.317	0.235	0.092	0.190	0.179	0.154	0.174
energy	0.218	0.214	0.200	0.188	0.280	0.256	0.294	0.072	0.052	0.054	0.097	0.030	0.028	0.153	0.052
Low Income countries															
overall	0.408	0.361	0.357	0.378	0.259	0.263	0.278	0.333	0.325	0.302	0.318	0.323	0.360	0.328	0.326
goods	0.277	0.180	0.134	0.311	0.201	0.211	0.222	0.342	0.323	0.340	0.345	0.460	0.507	0.296	0.413
bads	0.691	0.592	0.647	0.545	0.312	0.301	0.366	0.336	0.363	0.323	0.357	0.327	0.354	0.424	0.340
inputs	0.257	0.310	0.291	0.278	0.264	0.277	0.246	0.321	0.290	0.244	0.252	0.181	0.220	0.264	0.224
labor	0.414	0.509	0.456	0.481	0.367	0.405	0.364	0.534	0.463	0.357	0.347	0.273	0.350	0.409	0.332
capital	0.000	0.000	0.000	0.000	0.061	0.074	0.082	0.111	0.109	0.133	0.162	0.087	0.103	0.071	0.121
energy	0.356	0.422	0.418	0.352	0.365	0.354	0.292	0.317	0.297	0.242	0.245	0.184	0.206	0.311	0.219

Table 7. Kruskal Wallis test: Hypothesis 1

Income level	Rank Order Correlation Coefficient	p-value
High income level	7.846	<0.001
Upper middle income level	28.577	
Lower middle income level	36.192	
Low income level	33.385	

Table 8. Kendal's W test: Hypothesis 2

Income level	Rank Order Correlation Coefficient						p-value
	Goods	Bads	Inputs	Labor	Capital	Energy	
High Income level	2.385	5.923	3.846	1.154	2.692	5.000	<0.001
Upper Middle Income level	4.231	5.846	2.692	1.000	3.538	3.692	<0.001
Lower Middle Income level	5.000	5.615	2.154	4.077	1.769	2.385	<0.001
Low Income level	3.538	5.154	2.692	5.231	1.000	3.385	<0.001

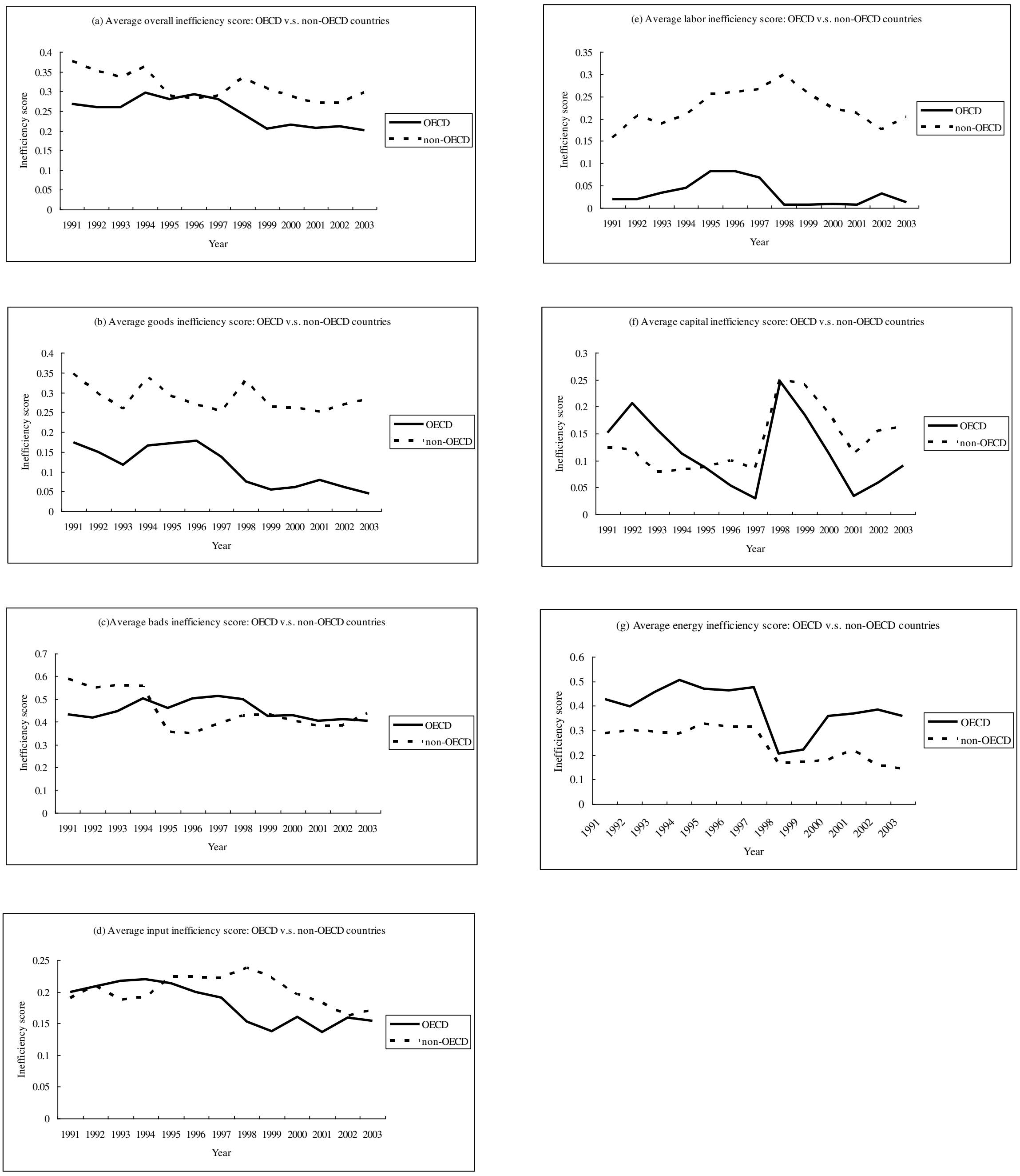

Figure 1. Temporal patterns of the overall inefficiency and its decompositions during 1991- 2003, OECD and non OECD countries

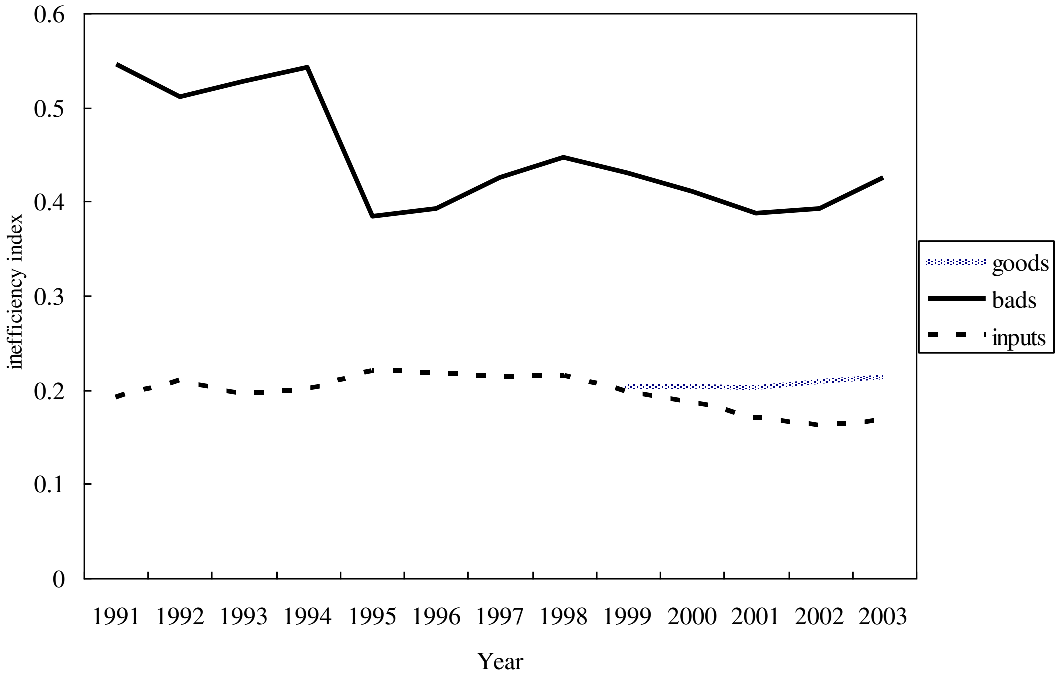

Figure 2. Comparison of average goods, bads and input inefficiencies during 1991- 2003