Natural Rate of Interest with Endogenous Growth, Financial Frictions and Trend Inflation

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Natural Rate of Interest, Monetary Policy and Growth with Trend Inflation and Financial Frictions*

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Abstract

Given the unobservable quality of the natural rate of interest, the consequences of central banks using an incorrect value in the monetary policy rule are analyzed in a New Keynesian DSGE model with endogenous growth, financial frictions and trend inflation. Our results confirm the financial structure plays a key role in the determination of the natural rate of interest and show that the mismeasurements affect the long-run growth rate by modifying the actual inflation rate trend, which is different from the target. Finally, we develop a mechanism to monitor the accuracy of the natural rate estimate.

JEL code: E43, E44, E58, O40

Keywords: natural rate of interest, New Keynesian DSGE models, endogenous growth, financial frictions, trend inflation

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1 Introduction

The long-run real rate of interest, also known as the natural rate of interest, has been studied by academics since Knut Wicksell introduced it in the late nineteenth century. Although there are various definitions of this concept, the most extended is the rate that ensures aggregate price stability and the reaching of the potential output in the absence of exogenous shocks or, as defined by Woodford (2003), the equilibrium real rate of return when prices are fully flexible. Nevertheless, its definition and importance have been modified over time due to the progress in the understanding of the economy and the changes in the tools used by the monetary authorities.

The way that monetary policy is currently conducted by central banks, leaving aside the monetary aggregates and using the nominal interest rate as the main instrument, recovers a key role for the natural rate of interest. The link between monetary policy and the natural rate of interest stems from the rules through which the macroeconomic mainstream, the New Keynesian models, considers that monetary policy is conducted. In these models, the nominal interest rate is set by rules in which the natural rate is the intercept and somehow characterizes the stance of the monetary policy. In this context, it is assumed that this rate is known exactly. However, the natural rate of interest is unobservable, so central banks must estimate it. During this process, monetary authorities could over or infraestimate the true value. A proof of this lack of accuracy is the gap between the observed interest rates and those that would result from the rules, as noted by Judd and Rudebusch (1998) and Orphanides (2003), who relate these gaps to the uncertainty associated with the estimation of the unobservable variables. Therefore, a deeper analysis of the fundamentals of the natural rate and the consequences of its incorrect estimation becomes convenient.

The academic literature has made progress in the understanding of the natural rate of interest in the context described above. Woodford (2001) discusses the fluctuations of this rate and its implications for the monetary policy. He argues that the natural rate of interest varies across time in response to real disturbances, but is always supposed to be accurately known. Woodford does not inquire about the possible implications of the use of a rate different to the endogenous one. By contrast, Orphanides and Williams (2002) consider that the estimated value introduced into the policy rule may differ from the endogenous rate. They show that the cost of underestimating the natural rate of interest is larger than the cost of overestimating it, considering the consequences in terms of stabilizing the economy. Meanwhile, Tristani (2009) analyzes the determinants of the natural rate of interest and also raises the possibility that the value estimated by the monetary authority may differ from the correct value. This author concludes that, if there were such a differential, the inflation target would never be reached[1]. Moreover, even if monetary authorities were able to know the exact law of motion of the natural rate of interest, delays in obtaining reliable information could lead central banks to carry out inaccurate policies in real time, as pointed out earlier in Levin, Wieland, and Williams (1999) and recently in Arestis and Sawyer (2008) and Neri and Ropele (2012). Accordingly, the unobservability of the natural rate of interest, the lags in obtaining precise information and other drawbacks have led many authors to question its usefulness for the accuracy of the monetary policy, as asserted by[2] Clark and Kozicki (2005) and Weber, Lemke and Worms (2008).

Nevertheless, and despite the difficulty of estimating the natural rate of interest, its importance is decisive for policymakers since this variable is the element integrated into the widely-used monetary rules which reflects the information about changes in the forces that guide the economy. Taylor and Williams (2010) analyze the optimal

[1]Other works focus on connected issues such as the relationship between money and the natural rate of interest (Andres, Lopez-Salido and Nelson, 2009) and the impact of misunderstandings in this rate on the zero lower bound of the nominal interest rates (Williams, 2009).

[2]Alternative but related research studies the effects of mismeasurements of the output gap on the optimal monetary policy, such as Orphanides et al. (2000), Rudebusch (2001), McCallum (2001), Smets (2002) and Orphanides (2003), among others.

monetary policy rules and conclude that the optimal coefficients assigned to variables such as the deviations of inflation or the output gap change depending on the mis-measurements in the natural rate of interest. Cúrdia et al. (2011) assert that the optimal monetary policy rules have to integrate this rate and Canzoneri, Cumby and Diba (2012) show how the rules perform better if the monetary authority can track the natural rate of interest[3].

Besides, some relevant information may be overlooked in the estimation process or, as Arestis and Sawyer (2008) pointed out, the theoretical assumptions imposed may not be valid. Amato (2005) indicates that some features of financial markets, such as the existence of external finance premiums or agency problems, may affect the natural rate of interest by generating a wedge between the actual long-term interest rates and the long-term natural rate. In addition, as outlined in that work, the literature has always assumed that the steady state inflation is zero. However, many studies such as Ascari (2004) and Cogley and Sbordone (2008), among others, have proposed that trend inflation is positive, in general, so the relaxation of this assumption could shift the equilibrium and, thus, the natural rate of interest. Hence, the definition of the natural rate should be modified. We can no longer understand it as the rate that ensures price stability, but the rate which provides inflation stability around a long-term level.

The need to know the value of the natural rate of interest has led the empirical literature to estimate it and to analyze the gap between this rate and the short-term rate. These estimates go beyond simple historical averages as they are not precise, particularly in periods of a high variability of output gap and inflation. One of the first articles that address this issue is Bomfim (1997), who derives the natural rate of interest of the U.S. by assessing the IS and the potential output curves. Laubach and Williams (2003) estimate the time-varying natural rate of interest for the U.S.,

[3]However, this finding depends on the framework employed because other approaches as Laxton and Pesenti (2003) argue that inflation-forecast-based rules perform better in small open economies.

concluding that its underestimation worsens macro stabilization. Moreover, as also noted by Tristani (2009), they suggest that, if the natural rate included in the Taylor rule is not correctly estimated, the inflation target is never achieved. But they do not delve into the determination of the fundamentals of this rate nor go beyond the mechanism which can be applied to verify whether the monetary authority is deviated from the correct value. Other applied works are Mésonnier and Renne (2007), who estimate the time-varying natural rate of interest for the euro zone, and Manrique and Marques (2004), who do the same for the U.S. and Germany, both papers employing the methodology of Laubach and Williams (2003).

The present paper carries out an analysis of the determination of the natural rate of interest as well as of the impact of the wrong identification of this rate, emphasizing the long-term perspective. The theoretical context is a modified New Keynesian model with endogenous growth, financial frictions and trend inflation developed in Olmos and Sanso (2014). Firstly, we obtain the steady state of the economy and the expression for the natural rate of interest, identified as the long-run real interest rate, which depends on the long-run growth rate and on the structure of the financial system. We conduct a sensitivity analysis of the natural interest rate to changes in the parameters of the financial sector, concluding how are the responses to shifts in the financial structure.

Furthermore, when the central bank inaccurately estimates the natural rate of interest, its value depends on both the wrong value and the inflation target. Under this assumption, it follows that the value estimated for the natural rate of interest is non-neutral to monetary policy. The reason is found not only in the gap generated between the targeted and the actual long-run inflation when the central bank misunderstands the natural rate, as Tristani (2009) and Laubach and Williams (2003) stated, but also in its influence on the long-run growth rate.

Delving into the relationship between the long-run growth rate and the potential error in the natural rate estimate, we prove that it depends on the inflation target. As our model predicts that the long-term growth rate is maximized when the central bank approaches a certain inflation rate in the long run, this relationship will depend on the differential between this optimal inflation rate and the inflation target. We show that the natural interest rate target can serve as a policy instrument. Specifically, we demonstrate that, in order to stimulate the long-term growth rate, if the inflation target is below the optimal one, the natural interest rate target should be lower than the endogenous rate.

Finally, we develop a procedure to monitor the accuracy of the estimate of the natural rate based on our conclusions to verify whether the central bank is wrong and how to follow a path towards a more convenient or the true value.

This paper is organized as follows. In Section 2, the theoretical model used is briefly presented. Section 3 is devoted to displaying the steady state equations and to introducing our definition of the natural rate of interest, submitting this rate to a sensitivity analysis. Section 4 presents the impact on the economy's trend that mismeasurements of the natural rate of interest would trigger. In Section 5, we design a procedure to verify and correct the deviations from the true value of this rate. Finally, Section 6 summarizes the main findings.

2 Theoretical Model

The theoretical setup used to derive the natural rate of interest and its characteristics is fully developed in Olmos and Sanso (2014) so, here, we briefly present it in order to detail the notation and the basic structure. It is an adaptation of the benchmark New Keynesian DGSE model which incorporates spill-over effects as the source of economic growth. Following Gertler and Karadi (2011), financial frictions have been included and, finally, trend inflation is allowed. The model considers the presence of six types of agents in the economy: households, capital producers, intermediate goods firms, financial intermediaries, retail firms and the central bank.

Households Households consume and use part of their savings to create deposits in the financial intermediaries. Each household has a fraction σ of its members who are bankers, who manage one financial institution each and transfer the profits to their households, and a fraction $(1-\sigma)$ who are workers, who produce goods and earn the competitive wages. Their preferences are given by:

$$E_t \sum_{t=0}^{\infty} \beta^i \left[\log C_{t+i} - \chi \frac{N_{t+i}^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where $\beta \in (0,1)$ is the discount rate, N_t the labor supply, $\chi > 0$ the relative utility weight of labor, φ the intertemporal estasticity of labor supply and C_t the consumption of final good. The households' budget constraint is:

$$C_t + \frac{D_t}{R_t} = D_{t-1} + \Gamma_t + W_t N_t - T_t \tag{2}$$

where D_t are one-period life deposits and public debt, R_t is the real gross interest rate, Γ_t are firms' profits, W_t is the real wage and T_t are lump sum taxes. The labor supply and the Euler equation are:

$$W_t = \chi C_t N_t^{\varphi} \tag{3}$$

$$E_t \Lambda_{t,t+1} R_t = 1 \tag{4}$$

with

$$\Lambda_{t,T} = \beta^{T-t} \frac{C_t}{C_T} \qquad \text{where} \qquad T = t+1 \tag{5}$$

Intermediate goods firms Intermediate goods producers obtain their output by using the capital acquired and the labor force hired from the households. These firms have a common production function that yields economic growth following Romer (1986):

$$Y_{jt}^i = e^{z_t} K_{jt}^{\alpha} \left(K_t N_{jt} \right)^{1-\alpha} \tag{6}$$

Y_{jt}^{i} being the production obtained by firm j with j pertaining to the interval [0,1], K_{jt} the capital stock and N_{jt} the labor used. The index $K_t = \int_0^1 K_{jt}dj$ represents the stock of knowledge generated by capital accumulation by all the intemediate producers, taken as given by firms. z_t is the AR(1) productivity shock common to all firms. Aggregating the firms' production functions and assuming that they are all identical:

$$Y_t^i = e^{z_t} K_t N_t^{1-\alpha} \tag{7}$$

Furthermore, goods producers fund capital purchases by issuing financial claims (S_t) at a price Q_t:

$$Q_t S_t = Q_t K_{t+1} \tag{8}$$

Denoting by P_t^i the price of the intermediate good and δ the depreciation rate, the wage that minimizes costs and the capital return (R_t^q) are:

$$W_t = P_t^i \left(1-\alpha\right) \frac{Y_t^i}{N_t} \tag{9}$$

$$E_t \left\{ R_{t+1}^q \right\} = \frac{\frac{P_{t+1}^i \alpha Y_{t+1}^i}{K_{t+1}} + Q_{t+1} - \delta}{Q_t} \tag{10}$$

Capital producers We define the capital accumulation, affected by adjustment costs, as follows:

$$K_{t+1} = K_t + I_t^n \tag{11}$$

where $I_t^n = I_t - \delta K_t$ is the net investment and I_t the gross investment. Capital producers refurbish depreciated capital adquired from intermediate goods producers and resell it, along with newly-created capital, to goods producers. The solution to the investment problem, common to all capital producers, leads to the relative price:

$$Q_t = 1 + f + \frac{I_t^{n,k} + I^k}{I_{t-1}^{n,k} + I^k} f' - E_t \Lambda_{t,t+1} \left(\frac{I_{t+1}^n + I^k}{I_t^n + I^k} \right)^2 f' \tag{12}$$

where $I_t^{n,k} = \frac{I_t^n}{K_t}$, $I^k = \frac{I}{K}$ is the value of gross investment relative to capital in the state steady and $\Lambda_{t,t+1}$ is the discount rate derived from the households' decision.

Retail firms These firms make use of intermediate goods, transforming them into differentiated final goods following the Dixit-Stiglitz technology. In order to include nominal rigidities, we follow the model of Calvo (1983). If we abandon the zero trend inflation assumption and define $X_t = \frac{P_t^*}{P_t}$ and $\frac{P_t}{P_{t+i}} = \frac{1}{\prod_{k=1}^{i} \Pi_{t+k}}$, P_t being the final good price index, P_t^* the optimal price in t and Π_t the gross inflation rate, the equations related to fixing prices are:

$$X_t = \mu \frac{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\prod_{k=1}^{i} \Pi_{t+k} \right)^{\epsilon} Y_{t+i} P_{t+i}^i}{E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left(\prod_{k=1}^{i} \Pi_{t+k} \right)^{\epsilon-1} Y_{t+i}} \tag{13}$$

$$X_t = \left[\frac{1-\theta}{1-\theta \Pi_t^{\epsilon-1}} \right]^{\frac{1}{\epsilon-1}} \tag{14}$$

where θ is the probability of not changing the price, $\mu = \frac{\epsilon}{\epsilon-1}$ is the mark-up of the retailers and ϵ is the parameter of the aggregate Dixit-Stiglitz index.

Financial intermediaries The structure of our financial market follows the approach of Gertler and Karadi (2011). Financial intermediaries raise funds from households remunerated at R_{t+1} and lend them to intermediate goods firms yielding a rate R_{t+1}^q. Their objective function maximizes their expected wealth (V_{ft}):

$$V_{ft} = \max E_t \sum_{i=0}^{\infty} (1-\gamma) \gamma^i \Lambda_{t,t+1+i} \left[\left(R_{t+1+i}^q - R_{t+1+i} \right) Q_{t+i} S_{ft+i} + R_{t+1+i} F_{ft+i} \right] \tag{15}$$

where γ is the probability of survival of the bankers and F_{ft} is the net wealth held by intermediary f at the end of period t. Financial intermediaries provide funds if they do not obtain losses or, analogously, the discounted external finance premium is greater than zero. Furthermore, the bankers have the chance to divert a fraction λ of the disposable funds to their households. But, if this happens, the depositors could force the bankruptcy of the financial intermediary and rescue the proportion $(1-\lambda)$ of the available funds. Thus, the depositors are willing to lend their funds to the bankers whenever the following equation holds:

$$h_t F_{ft} + v_t Q_t S_{ft} \geq \lambda Q_t S_{ft} \tag{16}$$

where v_t is the marginal gain of bankers derived from expanding their assets $Q_t S_{ft}$ but maintaining the net wealth fixed and h_t is the expected value of having an additional unit of F_{ft}, supposing that S_{ft} remains constant:

$$v_t = E_t \left\{ (1-\gamma) \Lambda_{t,t+1} \left(R^q_{t+1} - R_{t+1} \right) + \Lambda_{t,t+1} \theta x_{t,t+1} v_{t+1} \right\} \tag{17}$$

$$h_t = E_t \left\{ (1-\gamma) + \Lambda_{t,t+1} \theta t_{t,t+1} h_{t+1} \right\} \tag{18}$$

where $x_{t,t+i} = \frac{Q_{t+i} S_{ft+i}}{Q_t S_{ft}}$ and $t_{t,t+i} = \frac{F_{ft+i}}{F_{ft}}$ are the growth rates of assets and wealth, respectively. Hence, the total funding that banks can obtain depends on their wealth:

$$Q_t S_{ft} = \frac{h_t}{\lambda - v_t} F_{ft} = \phi^p_t F_{ft} \tag{19}$$

with ϕ^p_t being the private leverage ratio. We define the banker's wealth as follows:

$$F_{ft+1} = \left[\left(R^q_{t+1} - R_{t+1} \right) \phi^p_t + R_{t+1} \right] F_{ft} \tag{20}$$

Given that the banks' total wealth is independent of firm-specific factors, we can aggregate it to obtain:

$$Q_t S_t = \phi^p_t F_t \tag{21}$$

Distinguishing between the wealth of the new (F_t^n) and the old (F_t^o) bankers, and given that the initial funds of the new bankers are defined as the ratio $\frac{\omega}{1-\gamma}$ of the old bankers' wealth $(1-\gamma)\, Q_t S_{t-1}$, the total wealth can be stated as:

$$F_t = F_t^o + F_t^n \tag{22}$$

where

$$F_t^o = \gamma \left[\left(R_t^q - R_t \right) \phi_{t-1}^p + R_t \right] F_{t-1} \tag{23}$$

$$F_t^n = \omega Q_t S_{t-1} \tag{24}$$

Central Bank The central bank implements monetary policy by setting the short-term nominal interest rate (R_t^n) following a Taylor rule:

$$R_t^n = R_t^* \Pi_t^e \left(\frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \left(\frac{Y_t}{Y^*} \right)^{\phi_y} \exp\left(\eta_t\right) \tag{25}$$

where R_t^* is the time-varying natural rate of interest estimated by the central bank, Π_t^e is the expected inflation rate, Π^* is the steady state gross inflation target, Y^* is the natural level of output consistent with Π^*, and ϕ_π, ϕ_y are the parameters that measure the central bank's reaction to inflation and output deviations from its target levels, respectively. The monetary policy shock η_t is an AR(1) process. Moreover, the relationship between real and nominal interest rates is defined by the Fisher equation $R_t^n = R_t E_t \Pi_{t+1}$.

We also contemplate the possibility that the central bank intermediates a small part of the total claims (S_t^{cb}) at price Q_t carrying efficiency costs τ. In order to simplify the model dynamics, but without ignoring the effects that this kind of intermediation could generate on the natural rate of interest, we consider that this proportion (ψ) is fixed and corresponds to the steady state value. This lending mechanism is supported by taxes and by issuing riskless debt acquired by households which is remunerated at the rate R_t, and yields R_t^q. Hence, the structure of the financial intermediation of the economy is fully described by the equation:

$$Q_t S_t = \phi_t^p F_t + \psi Q_t S_t = \phi_t^T F_t \tag{26}$$

with $\phi_t^T = \frac{1}{1-\psi}\phi_t^p$ being the leverage ratio of the economy.

Equilibrium The aggregate equilibrium of the economy is given by:

$$Y_t = C_t + I_t + f\left(\frac{I_t^{n,k} + I^k}{I_{t-1}^{n,k} + I^k}\right)(I_t^n + I) + \tau\psi Q_t K_{t+1} \tag{27}$$

Finally, the total output of the economy is equivalent to the intermediate goods firms' output weighted by the inverse of the price dispersion of the retailers (Δ_t):

$$Y_t = Y_t^i \Delta_t^{-1} \tag{28}$$

$$\Delta_{t+1} = \theta \Pi_{t+1}^{\epsilon} \Delta_t + (1-\theta) X_{t+1}^{-\epsilon} \tag{29}$$

3 The Natural Rate of Interest

In this section we introduce the equations that characterize the steady state and we display the values of the parameters used in the calibration of the economy. Then, we present the definition of the natural rate of interest that emerges from the model developed, carrying out a sensitivity analysis in order to evaluate the shifts in this rate induced by changes in the financial parameters.

3.1 Steady State Equations

First of all, in order to find a well-defined endogenous and stable steady state, we have to normalize the growing variables. We choose the capital stock as the normalization variable because it is the source of economic growth, whose gross growth rate is defined as $G_t = \frac{K_t}{K_{t-1}}$. All the normalized variables has the superscript k, whilst the variables evaluated in the steady state have no subscript. Below we show the main equations distinguishing among the different blocks of the model.

From households, intermediate goods producers, capital producers and retail firms, we can obtain the following expressions:

$$\chi C^k N^{\varphi} = (1-\alpha)\frac{X}{\mu}\frac{1-\theta\beta\Pi^{\epsilon}}{1-\theta\beta\Pi^{\epsilon-1}}N^{-\alpha} \tag{30}$$

$$\frac{1}{R} = \frac{\beta}{G} \tag{31}$$

$$R^q = P^i \alpha N^{1-\alpha} + 1 - \delta \tag{32}$$

$$X = \left[\frac{1-\theta}{1-\theta\Pi^{\epsilon-1}}\right]^{\frac{1}{\epsilon-1}} \tag{33}$$

$$P^i = \frac{X\left(1-\theta\beta\Pi^{\epsilon}\right)}{\mu\left(1-\theta\beta\Pi^{\epsilon-1}\right)} \tag{34}$$

From financial intermediaries:

$$v = (1-\gamma)\,\Lambda\left(R^q - R\right) + \gamma\Lambda t v \tag{35}$$

$$h = (1-\gamma) + \gamma\Lambda t h \tag{36}$$

$$\phi^p = \frac{h}{\lambda - v} \tag{37}$$

$$t = \left(R^q - R\right)\phi^p + R \tag{38}$$

$$G = \frac{1}{1-\psi}\phi^p F^k \tag{39}$$

$$F^k = \gamma \left[\left(R^q - R \right) \phi^p + R \right] F^k + (1-\gamma)\,\omega \tag{40}$$

And, from the equilibrium of the economy:

$$\frac{N^{1-\alpha}\left(1-\theta \Pi^{\epsilon}\right)}{(1-\theta)\, X^{-\epsilon}} = C^k + G - (1-\delta) + \tau\psi G \tag{41}$$

In order to analyze the consequences of the incorrect estimate of the natural rate of interest by the central bank, we contemplate the possibility of $R \neq R^*$, where R is the endogenous natural rate of interest. Thus, the Taylor rule in the long run is defined as follows:

$$R^n = R^* \Pi \left(\frac{\Pi}{\Pi^*} \right)^{\phi_\pi} \tag{42}$$

To derive this expression, we have made use of the equivalence in the steady state $\Pi^e = \Pi$. Also, we have assumed $Y = Y^*$ but, for reasons given below, we can no longer assume that $\Pi = \Pi^*$. We should note that, if $R = R^*$, this expression is reduced to the standard relationship at the steady state between real and nominal interest rate, i.e. $R^n = R\Pi$.

Substituting the left-hand side of (42) by the Fisher equation:

$$R = R^* \left(\frac{\Pi}{\Pi^*} \right)^{\phi_\pi} \tag{43}$$

Hence, the natural rate of interest in the steady state depends on the effective inflation rate, but also on the targets (R^*, Π^*). Rearranging terms:

$$\Pi = \Pi^* \left(\frac{R}{R^*} \right)^{\frac{1}{\phi_\pi}} \tag{44}$$

Thus, we have endogeneized the effective inflation rate in the steady state, which is now related to the inflation target and the real interest rate deviation weighted by the parameter of reaction to deviations in the inflation rate included in the Taylor rule. This expression, in logs, coincides with that indicated by Laubach and Williams (2003):

$$\pi = \pi^{*} + \frac{1}{\phi_{\pi}}\left(r - r^{*}\right) \tag{45}$$

Therefore, the long-run endogenous inflation rate and the target rate will match if and only if the central bank correctly estimates the natural rate of interest. This finding is in accordance with the conclusion of Tristani (2009). From (44) and (31), we obtain the expression:

$$\Pi = \Pi^{*}\left(\frac{G}{\beta R^{*}}\right)^{\frac{1}{\phi_{\pi}}} \tag{46}$$

So the effective steady state inflation rate depends on the target of both inflation and the natural rate of interest, as well as on the long-term growth.

If we draw on (30) and (41), we can solve for G:

$$G = \frac{1}{1+\tau\psi}\left[\frac{1-\theta\Pi^{\epsilon}}{\left(1-\theta\right)X^{-\epsilon}}N^{1-\alpha} - \frac{X\left(1-\alpha\right)\Psi}{\chi\mu\Upsilon}N^{-\varphi-\alpha} + \left(1-\delta\right)\right] \tag{47}$$

where $\Upsilon = \left(1-\theta\beta\Pi^{\epsilon-1}\right)$ and $\Psi = \left(1-\theta\beta\Pi^{\epsilon}\right)$.

Furthermore, from (35)-(38) we obtain that:

$$\left(R^{q} - R\right) = \frac{\lambda\left(1-\gamma\right)-v}{\lambda - v}\frac{Rv}{\left(1-\gamma\right)} \tag{48}$$

Now, relying on (31), (37), (39) and (40):

$$G = \frac{\left(1-\gamma\right)\omega v R}{\left(1-\psi\right)\left(R^{q}-R\right)\left(\lambda-v\right)\left[1-\gamma\frac{\lambda R}{\lambda - v}\right]} \tag{49}$$

Replacing (48) in (49):

$$G = \frac{\left(1-\gamma\right)^{2}\omega}{\left(1-\psi\right)\left[\lambda\left(1-\gamma\right)-v\right]\left[1-\frac{G}{\beta}\frac{\gamma\lambda}{\lambda-v}\right]} \tag{50}$$

From (31) and (32):

$$(R^q - R) = \frac{\alpha X \Psi}{\mu \Upsilon} N^{1-\alpha} + (1-\delta) - R \tag{51}$$

Equating (48) and (51) taking into account (31):

$$N = \left\{ \frac{\mu \Upsilon}{\alpha X \Psi} \left[\left(1 + \frac{[\lambda (1-\gamma) - v]\, v}{(\lambda - v)} \right) \frac{G}{\beta} - (1-\delta) \right] \right\}^{\frac{1}{1-\alpha}} \tag{52}$$

And, introducing the last expression into (47):

$$G = \frac{1}{1+\tau\psi} \left\{ \frac{\mu \Upsilon \Theta}{\alpha X \Psi} \left[\frac{1 - \theta \Pi^{\epsilon}}{(1-\theta) X^{-\epsilon}} - \frac{(1-\alpha) X \Psi}{\chi \mu \Upsilon} \left(\frac{\mu \Upsilon \Theta}{\alpha \Psi X} \right)^{\frac{-(1+\varphi)}{1-\alpha}} \right] + (1-\delta) \right\} \tag{53}$$

where $\Theta = \left[\left(1 + \frac{[\lambda(1-\gamma) - v] v}{(1-\gamma)(\lambda - v)} \right) \frac{G}{\beta} - (1-\delta) \right]$.

Summing up, the steady state is determined by (50) and (53), along with the expressions of the relative optimal price (33) and the effective inflation rate in the steady state (46). These equations indicate that the equilibrium is determined by the effective steady state inflation rate, which, in turn, coincides with its target if the central bank correctly estimates of the natural rate. However, if $R \neq R^*$, the long-term inflation rate is determined not only by its target in the monetary policy rule but also by the error in the real interest rate estimate through relationship (44). So, if this constraint holds, the deviation in the estimation of the natural rate of interest will be non-neutral in the long run, a key outcome in our analysis. In Section 5, we delve into this question.

3.2 Calibration

The parameter values used in the calibration of the model are reported in Table 1 and correspond to the baseline model in Olmos and Sanso (2014). The values of most conventional parameters are standard in the literature, the policy and financial parameter values are taken from Gertler and Karadi (2011), whilst the value of χ

is set so that the values of the long-run annual growth rate (2.5%), the annualized external finance premium (1.17%) and the leverage ratio (4.8) are admissible for an annual inflation rate of 2.5%. Using these values, we find the solution of equations (50) and (53) in the plane $\{v, G\}$ once the expressions of X and Π are substituted from (33) and (46).

Table 1: Parameters values

Parameter	Interpretation	Value
β	Discount Rate	0.99
α	Participation of capital	0.332
δ	Depreciation Rate	0.03
χ	Relative utility weight of labor	14.1
θ	Probability of keeping prices fixed	0.779
ς	Adjustment capital costs	5.85
γ	Survival rate of bankers	0.97
λ	Fraction of capital that can be diverted	0.382
ω	Proportional transfer to new bankers	0.002
φ	Elasticity of labor supply	0.276
ϵ	Elasticity of substitution	4.167
ϕ_π	Coefficient of inflation in the Taylor rule	2.05
ϕ_y	Coefficient of output gap in the Taylor rule	0.5/4
ρ_z	Technology shock persistence	0.9
ρ_e	Monetary shock persistence	0.5

3.3 Definition of the Natural Rate of Interest

According to (31), the endogenous natural rate of interest depends on the long-run growth rate of the economy[4], which, in turn, depends on the financial parameters from (50) and on the effective steady state inflation rate from (53). From these three expressions we can write:

$$R = \frac{(1-\gamma)^2 \omega}{(1-\psi)\left[\lambda(1-\gamma)-\upsilon\right]\left[\beta - \frac{G\gamma\lambda}{\lambda-\upsilon}\right]} =$$

$$= \frac{1}{\beta(1-\tau\psi)}\left\{\frac{\mu\Upsilon\Theta}{\alpha X\Psi}\left[\frac{1-\theta\Pi^{\epsilon}}{(1-\theta)X^{-\epsilon}} - \frac{(1-\alpha)X\Psi}{\chi\mu\Upsilon}\left(\frac{\mu\Upsilon\Theta}{\alpha\Psi X}\right)^{\frac{-(1+\varphi)}{1-\alpha}}\right] + (1-\delta)\right\} \quad (54)$$

Hence, in this type of models with financial frictions, the natural rate of interest depends on the structure of the financial sector[5] as in the definition proposed by De Fiore and Tristani (2011), even assuming that the central bank correctly estimates it. Moreover, if the central bank does not match the long-term interest rate estimate with the endogenous one, this rate will also depend on the monetary policy parameters (Π^*, R^*, ϕ_π) because the efective long-run inflation is a function of these values as has been shown in (46). This finding leads us to maintain that, assuming $R \neq R^*$, the deviations affect the value of the effective natural rate and the steady state inflation rate.

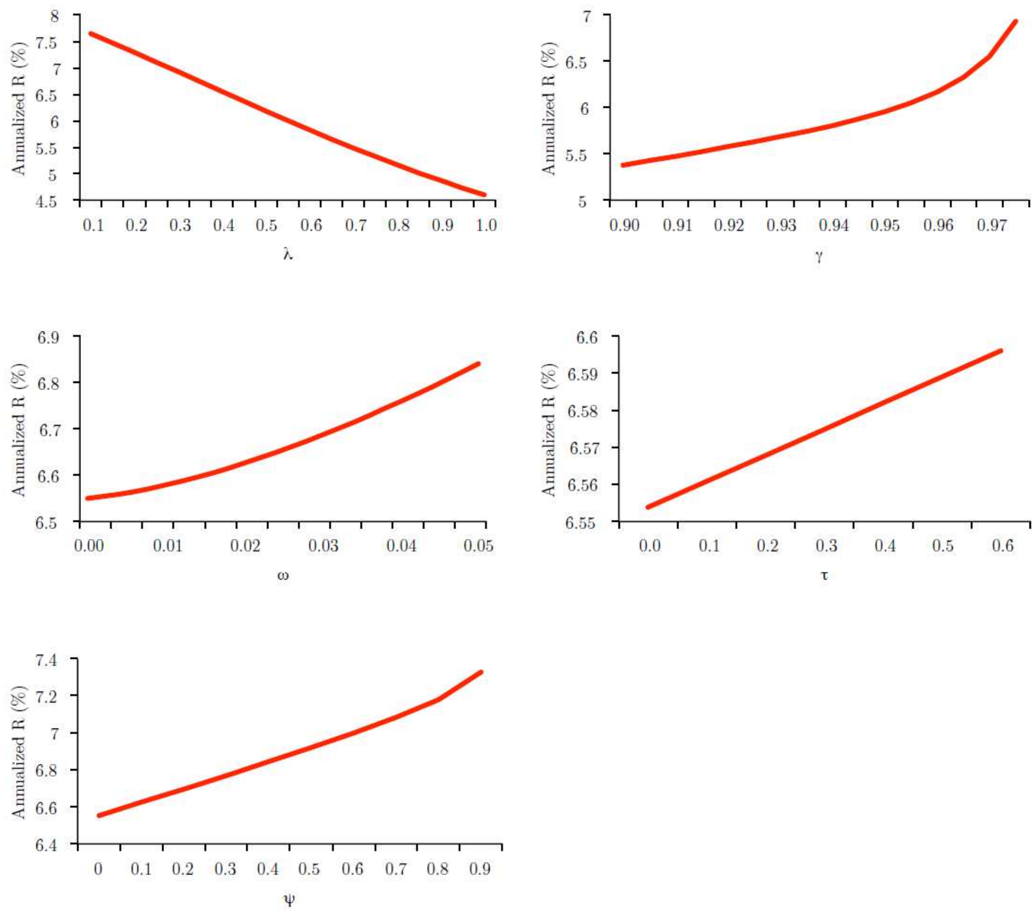
We now explore the relationships among the financial parameters and the natural rate of interest assuming $R = R^*$ since differences among alternative scenarios in terms of magnitude are mild. The natural rate of interest and the steady state

[4]This result contradicts the finding of Weber, Lemke and Worms (2008), who argue that there is no a desirable level for the natural rate. Clark and Kozicki (2005) also empirically analyze the link between this rate and the long-run growth rate and conclude that it is weak.

[5]In line with this result, previous works such as Borio, Disyatat and Juselius (2013) support the inclusion of financial variables in the definition of long-run references such as the potential output.

growth rate are positively related, so all parameter changes that increase the natural rate of interest also stimulate the long-term growth rate.

Figure 1: Endogenous natural rate of interest and financial parameters if $\Pi = 2.5\%$



First, we analyze the effects of shifts in the financial parameters λ, γ, ω and, then, the effects of variations in the steady-state unconventional monetary policy parameters τ, ψ. Figure 1 displays the results for the annualized natural rate of interest. The first plot shows how the natural rate drops when the funds that bankers can divert increase. This means that, when the financial system profits can be diverted away and not reinvested, the trend of the deposits' yield offered by financial intermediaries is smaller. Meanwhile, the effects of increments in the survival rate of the bankers and in the funds with which the new bankers start on the natural rate of interest are positive. The lower the rotation of the intermediaries in the financial system and the more funds the new financial intermediaries own, the higher the risk-free rate of return in the steady state. The most prominent result is perhaps the effect of a change in γ, given that it indicates that the renovation of the intermediaries is not positive for the natural interest rate and, consequently, for the long-run economic growth.

As regards the unconventional monetary policy parameters, both efficiency costs and the proportion of the credit policy in the steady state increase the endogenous natural rate of interest. The first parameter weakly affects the natural rate of interest, while the more funds the central bank intermediates in the steady state, the greater the natural rate of interest.

3.4 Stabilization

As discussed above, some previous work, such as Orphanides and Williams (2002), reveals that the cost of underestimating the natural rate of interest in terms of economic stabilization is greater than the cost of overestimating it. In order to assess the stabilization costs of potential estimation errors of the natural rate of interest, we have submitted the model to the standard types of perturbations, a monetary policy shock and a technological shock. We have considered three possible scenarios corresponding to $R < R^{*}$, $R = R^{*}$ and $R > R^{*}$. Under the two kind of shocks, the responses of the variable do not depend on the accuracy of the estimation of the natural rate[6]. Therefore, our results do not support the finding of, among others, Orphanides and Williams (2002), because the effects of the errors are undifferentiated, at least for this sort of perturbations. Nevertheless, this does not mean that these effects are not significant on the trend, since the values around which these deviations are obtained change depending on the different assumptions about the errors. We delve into this issue in the next section.

[6]We do not display the plots because the response variable is standard and indistinguishable among scenarios.

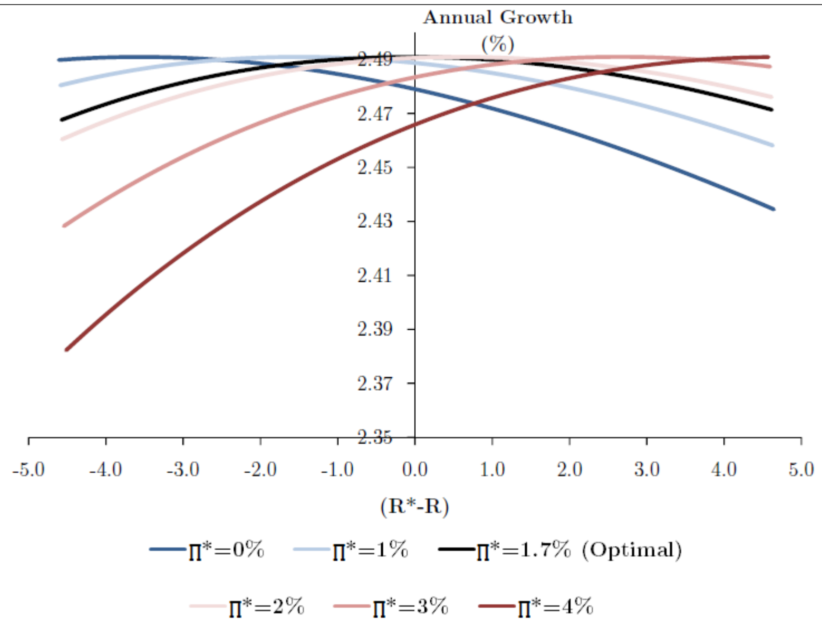
4 Steady State Effects

In this section we evaluate the impact of errors in the estimation of the natural rate of interest on the trend of the economy, focusing on the effects on economic growth and examining their impact on the marginal gain of banks and on the gap between the effective and the targeted inflation. To obtain a complete perspective, we will asses the equilibrium value of our key variables for a wide range of the differential $(R - R^*)$ and the inflation rate target. We should note that the value of R is endogenously fixed by (31), so we modify the value of the interest rate differential by adjusting the interest rate targeted by the central bank.

Firstly, we analyze the effect of the differential $(R^* - R)$ on the long-run economic growth rate. As shown in Figure 2, where all variables are expressed in annualized rates, the behavior depends on Π^*. We can distinguish two differentiated types of effects. The first for values of Π^* below 1.7% (the rate which provides the maximum steady state growth, denoted as Π^{op} and represented by the black line) and the second for higher rates. The cause of the existence of this specific threshold is evidenced in Olmos and Sanso (2014), since $\Pi^* = \Pi^{op} = 1.7\%$ is the rate at which the long-run growth rate is maximized in the absence of estimation errors, as shown in the vertical axis points (when $R^* = R$). In this particular case, satisfying $\Pi^* = \Pi^{op}$, the behavior is symmetrical around the maximum when $R^* = R$ for the two scenarios $R^* < R$ and $R^* > R$.

When the inflation rate target is below Π^{op}, economic growth decreases when $R^* > R$, so the overestimation of the natural interest rate (right-hand side of the graph) discourages the long-run growth rate. For example, if $\Pi^* = 0\%$ and R^* exceeds the natural rate by 200 basis points, equilibrium growth decreases more than 0.015 percentage points with respect to the scenario $R^* = R$. This long-term conclusion under the assumption $\Pi^* = 0\%$ is in line with the short-term finding of Laubach and Williams (2003), who defined the monetary policy as contractionary when the real rate of interest is above the natural rate.

Figure 2: Annual growth and interest rate differential

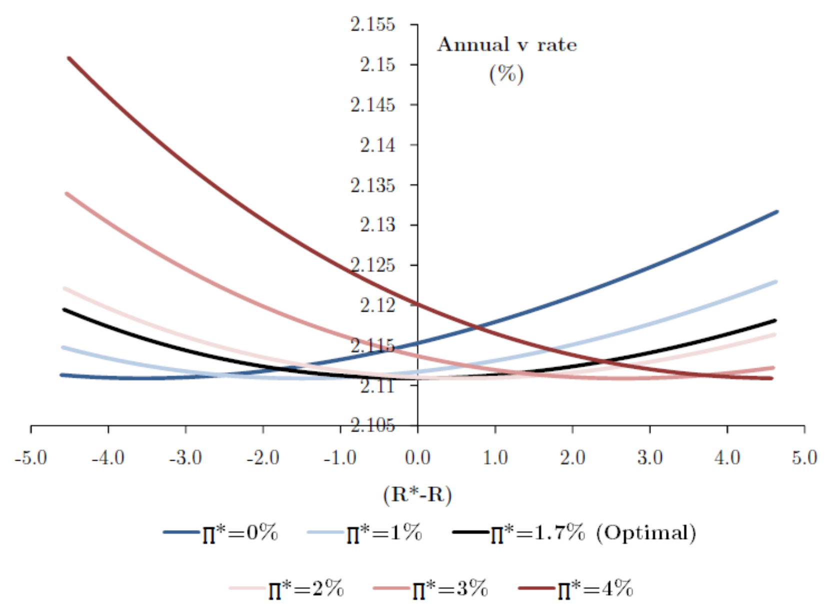


On the other hand, if $\Pi^* > \Pi^{op}$, the underestimation of the natural rate of interest has a negative effect on the long-term growth rate. If $\Pi^* = 4\%$, the growth rate decreases 0.03 percentage points when R^* is 200 basis points below R. It is noteworthy that such differences are accentuated the farther away the central bank is from the optimal inflation.

However, we underline that, when the natural rate of interest is underestimated and $\Pi^* < \Pi^{op}$ or is overestimated and $\Pi^* > \Pi^{op}$, the conclusions are not univocal. In these scenarios the relationship between $(R^* - R)$ and G is hump-shaped. This can be explained by the fact that, for any inflation target, when $R^* \neq R$ there is an attainable maximum value for the long-run growth rate, which is almost identical to that corresponding to the case of $R^* = R$ and $\Pi^* = \Pi^{op}$ but slightly lower. These maximum steady-state growth points are located on the left-hand side where $R^* < R$ for $\Pi^* < \Pi^{op}$ and on the right-hand side where $R^* > R$ for $\Pi^* > \Pi^{op}$. Moreover, the relationship between $(R^* - R)$ and G approximates the scenario $R^* = R$ the closer the inflation target is to the optimal rate.

Secondly, in Figure 3, we explore the relationship between the marginal gain of the financial intermediaries from expanding their assets and the interest rate differential $(R^* - R)$. As in the previous exercise, the results depend on the inflation target although, in this case, the direction is the opposite due to the negative relationship[7] between G and v. When the inflation target does not overcome threshold Π^{op}, the expected earnings of the financial system, which depend on the steady state external finance premium $(R^q - R)$, rise if $R^* > R$. Contrarily, when $\Pi^* > \Pi^{op}$, the financial gain increases if $R^* < R$ and, for the particular case of $\Pi^* = \Pi^{op}$, the behavior is symmetric around the minimum when $R^* = R$. Analogously to the case of G, for the scenarios $(\Pi^* < \Pi^{op}, R^* < R)$ and $(R^* > R, \Pi^* > \Pi^{op})$, the relationship has a mixed behavior because it is U-shaped.

Figure 3: Marginal gain and interest rate differential

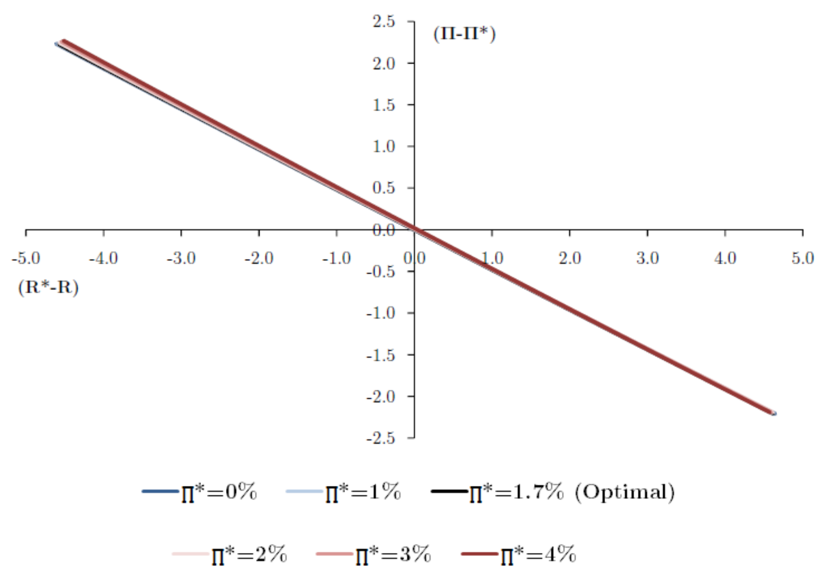


Finally, the inflation rate differential is studied in Figure 4. In this case, Π^* is fixed and set by the monetary authority whilst Π is endogenously determined by (46). When $R^* = R$, the differential is zero, when $R^* < R$, it is positive and, when $R^* > R$,

[7]This relationship is non-linear although, for reasonable levels of trend inflation, it is negative.

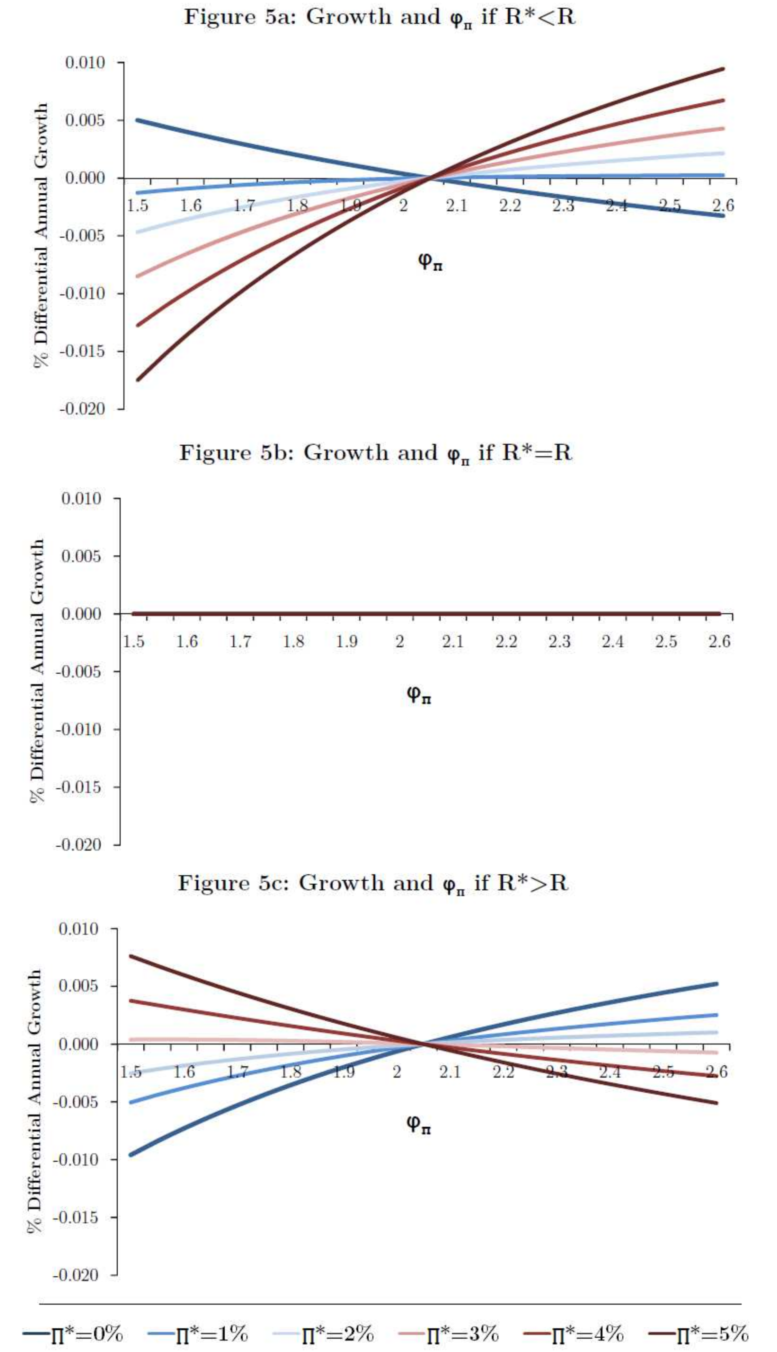
it is negative. Thus, the differentials $(\Pi - \Pi^*)$ and $(R^* - R)$ have a negative linear relationship across the whole range, disclosing a very slight difference for any Π^*.

Figure 4: Inflation differential and interest rate differential



The interpretation of these results suggests that, when the inflation target is below the optimal rate, a lax monetary policy that reduces R^* stimulates the long-run growth rate because the effective inflation rate in the steady state approaches the optimal inflation rate up to a threshold value. By contrast, for high levels of the inflation target $(\Pi^* > \Pi^{op})$, a contractionary monetary policy $(R^* > R)$ leads to an actual long-term inflation closer to the optimum, again up to a limit, so the growth rate increases. Therefore, if the central bank wants to stimulate the long-run growth, monetary policy must always reduce the long-term inflation gap $|\Pi^{op} - \Pi^*|$. This can be implemented in two different ways. The first, as we have seen, by modifying the estimate of the natural rate of interest. This policy will not be useful when $R^* < R$ and $\Pi^* < \Pi^{op}$ or when $R^* > R$ and $\Pi^* > \Pi^{op}$.

Figure 5: Growth and ϕ_π



But the objective of increasing the long-run growth rate can also be reached through the parameter of the Taylor rule that reacts to deviations of inflation, ϕ_π. Although the central bank interprets this parameter as a regulator of the short and medium-term nominal interest rate, if the estimated natural rate of interest does not match the endogenous one, it will also influence the long-run growth rate according to (46), which is a prominent result of our work. In order to analyze this relationship, we now again assume three scenarios, $R^* < R$, $R^* = R$ and $R^* > R$, setting the difference between interest rates $(R^* - R)$ at 200 basis points above and below 0 and changing the value of the parameter ϕ_π to observe the effects. As shown in Figure 5, the effects of shifts in ϕ_π depend on both the inflation target and the differential $(R^* - R)$. If $R^* < R$, an increase in ϕ_π triggers a reduction of the long-run growth rate if and only if the inflation target is below 1%. However, if the inflation target is higher than 1%, the relationship is positive. If, instead, the central bank sets a long-run interest rate equal to the endogenous value, there are no differences in the growth rate for any variation of ϕ_π regardless of the value of Π^*. But, if $R^* > R$, the long-run growth rate decreases with ϕ_π for higher inflation targets than 3% and increases with lower ones.

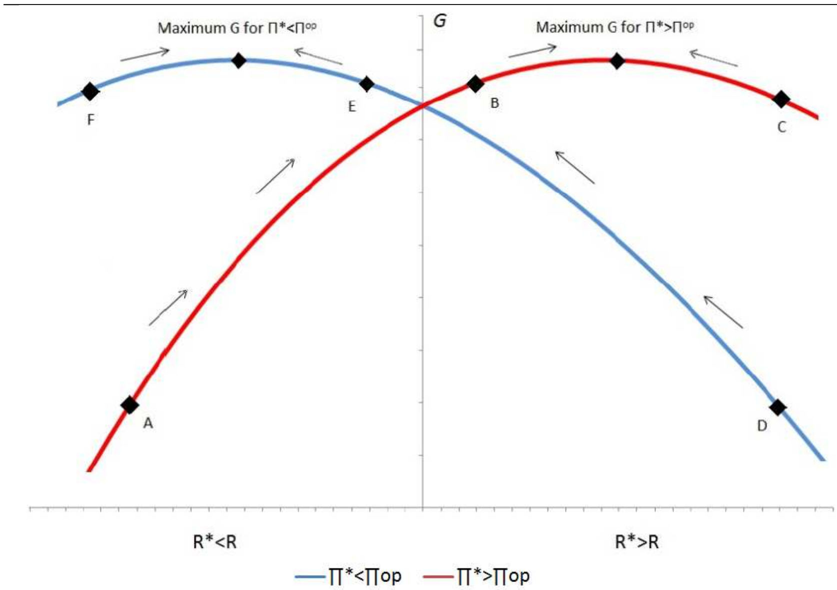
This section contains a wealthy set of conclusions about the many different situations that could be taking place in the long run. The necessary information to know where an economy is situated will be of a great importance for reaching the objectives of the monetary policy. In the following section we use these results in order to identify where is the economy in a given point of time.

5 Verification

Although the exact knowledge of the natural rate of interest in real time is a challenging task, some of the conclusions drawn from our analysis could assist the monetary authorities in this objective. With the exercise we propose in this section, the central bank would be able to find out both its monetary policy stance regarding the inflation rate that maximizes long-run growth and the accuracy of the estimate of the natural rate of interest. In this way, monetary authorities could monitor the performance of monetary policy in relation to long-term growth.

To clarify the iterative mechanism we have designed, in Figure 6 we propose a stylized outline of Figure 2.

Figure 6: Verification process. The long run



Let us suppose that the central bank is located at an unknown point of Figure 6 in which the only known long-run variables are the targets included in the monetary policy rule (Π^*, R^*). There are two possible strategies; the first is the aim of maximizing the long-term growth rate and the second widens this goal by including the correct estimate of the natural rate of interest. Below, we describe the steps required to achieve both goals. We study the changes in the steady state that would generate the modification of some policy tools and, then, we provide some guidelines for interpreting the transition path in the short term.

Maximizing G The success of the first strategy is very straightforward to achieve. The central bank only has to move the estimate of the natural rate of interest until the long-run growth rate stops increasing. Assuming[8] that $\Pi^* \neq \Pi^{op}$, there are six

[8]Otherwise the procedure would be the same.

possible types of initial points (A, B, C, D, E, F), marked on Figure 6, and two optimal final points depending on whether $\Pi^* < \Pi^{op}$ or $\Pi^* > \Pi^{op}$. The direction of the correct shifts of R^* depends on the position of the initial point with respect to the global maximum located at a specific value of $(R^* - R)$, which differs for each of the displayed inflation targets. As the central bank does not possess this information, the first movement tests the proper adjustment of R^* and the subsequent changes iteratively modify R^* up to the maximum long-run growth rate in the direction indicated by the arrows. To illustrate this method, let us assume that the initial point is $(\Pi^* = 0\%; R^* = 4.5\%)$. The aim of the central bank is to maximize G, but it does not know either Π^{op} or R. Keeping Π^* constant, the first random move of the policymaker might be to increase R^*, but this would bring down G. Hence, the central bank has ascertained that it has to reduce R^* in order to increase G, though it still does not know if the starting point was C, D or E. This process will make G reach its maximum at the point $(\Pi^* = 0\%; R^* = 3\%)$.

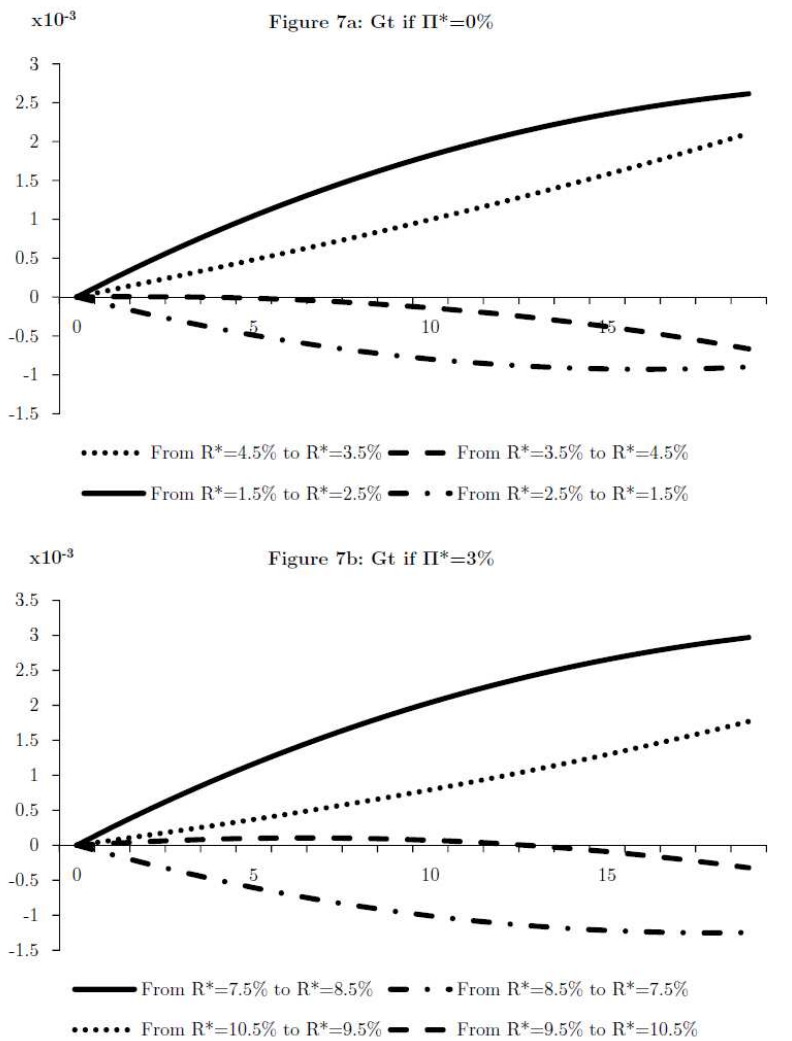
This exercise would be simple if G were known. Unfortunately, monetary authorities do not know the long-term growth rate exactly, but they perform estimates based on provisional information. Therefore, we now relax the assumption of the correct knowing[9] of G and we suppose that the central bank can only perceive the direction of changes in G_t, that is, the short-run growth rate. Although some authors such as Orphanides and Van Norden (2002) have criticized the confidence of real-time calculations, we consider that this assumption is not unreasonable.

Above, we have computed the steady state values of the endogenous variables for the different levels of $(R^* - R)$, so the exercise now builds on the evaluation of the transition path between scenarios in the absence of other shocks. In order to do this,

[9] Much academic literature has empirically addressed related issues with different perspectives, such as the estimation of the potential output and its trend as in Edge *et al.* (2007). There are some earlier works, such as Kuttner (1994), that suppose this rate is constant, but later works suppose a time-varying rate as in Laubach and Williams (2003), who jointly estimate the natural rate of interest and the trend growth rate.

we propose a simulation in which a permanent change in the estimate of the natural rate of interest is imposed. Thus, our model loses its stochastic quality, which is explained by the anticipated nature of this kind of alteration of the model.

Figure 7: Verification process. The short run I



As in the previous example, we suppose that the initial target is $\Pi^* = 0\%$, the case of the blue line in Figure 6. Figure 7a displays the response of G_t across 20 quarters to a change in R^*. As expected, when the central bank reduces the intercept in the Taylor rule from 4.5% to 3.5%, the short-term growth rate increases. Conversely, if the central bank raises the natural rate estimate from 3.5% to 4.5%, the growth rate diminishes after some periods. However, this consequence is valid if and only if the estimate of the natural rate is above the rate that ensures the maximum long-run growth rate (points[10] C, D and E in Figure 6), which corresponds to $R^* = 3\%$ when $\Pi^* = 0\%$. If, for instance, the central bank raises the estimate from $R^* = 1.5\%$ to $R^* = 2.5\%$ (point F), G_t does not decrease but, as can be seen in the solid line in Figure 7a, increases. Analogously, a change from $R^* = 2.5\%$ to $R^* = 1.5\%$ causes a drop in G_t.

The conclusions are the same if the target inflation rate exceeds the optimal rate. Figure 7b shows the transition path of the short-term growth rate between steady states if $\Pi^* = 3\%$. As in the previous simulation, G_t increases when the central bank approximates to the estimate of the natural rate that maximizes G (in this case $R^* = 9\%$) and decreases otherwise.

Therefore, even if both the optimal inflation rate and the natural rate of interest were unknown, central banks would have a tool that allows them to maximize the long-term growth rate through the modification of the intercept in the Taylor rule.

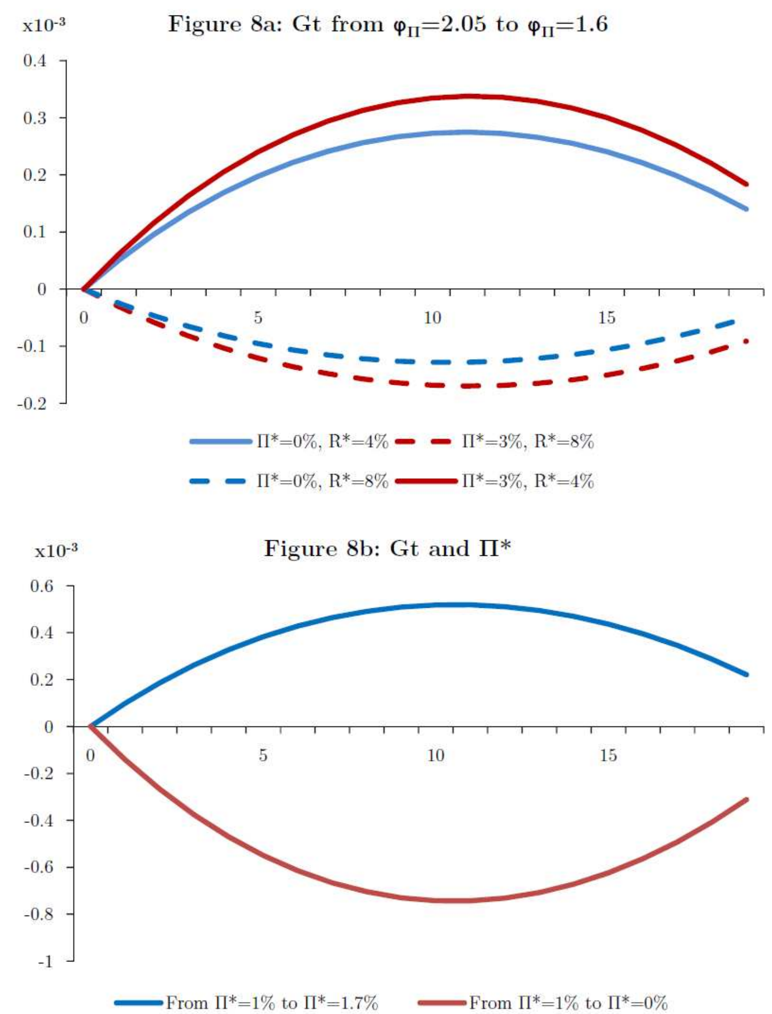
Maximizing G and reaching $R^* = R$ The second strategy, which includes the reaching of the maximum long-term growth and the correct estimate of R, requires a multiple-step process. Firstly, the policymaker has to move ϕ_π to assess the sign of the differential $(R^* - R)$. Figure 8a plots the responses of the short-term growth rate to a decrease in ϕ_π from $\phi_\pi = 2.05$ to $\phi_\pi = 1.6$ for several combinations of Π^* and R^*. Figure 5 showed that the shifts in the long-run growth rate from a modification in ϕ_π depend on both $(R^* - R)$ and $(\Pi^* - \Pi^{op})$. However, in the short-term horizon, G_t rises when ϕ_π decreases if and only if $R^* < R$ whatever the value of Π^* and vice versa. Given that R approaches 6.5% for both levels of Π^* considered, the state $R^* < R$ is represented in Figure 8a by the solid lines and $R^* > R$ by dashed lines[11]. The

[10] We include point C because the central bank does not know Π^{op}.

[11] For the sake of clarity, we have shown only these cases, although the behavior described holds for all other scenarios.

justification is based on that, as long as ϕ_π penalizes deviations in Π_t from Π^*, its reduction means a more expansive policy which enhances the short-term growth rate if and only if the intercept set in the Taylor rule does not constrain the monetary policy or, equivalently, if $R^* < R$. Thus, if G_t rises when ϕ_π increases, the central bank can discard, following Figure 6, points B, C and D.

Figure 7: Verification process. The short run II



At this point, the central bank has the information derived from the first step of this mechanism about the sign of $(R^* - R)$. Once this information is known, policymakers have to move R^* until arriving to the $R^* = R$ scenario, that is, when

G_t does not change when ϕ_π varies. In our example, where $\Pi^* = 0\%$, this happens when $R^* = R = 6.5\%$.

If the central bank additionally wants to maximize G, shifts in Π^* will reveal the sign of $(\Pi^* - \Pi^{op})$. As shown in Figure 8b, and supposing that the starting inflation target is 1%, if the central bank lowers Π^* to zero and G_t decreases, this means that $\Pi^* < \Pi^{op}$, so the achieving of the double goal involves the increase of Π^* until G_t stops growing, that is, when $\Pi^* = 1.7\%$.

Hence, policymakers have all the information required to approach the desirable long-run equilibrium. The possibilities shown in this exercise can, therefore, provide the central banks with enough information to achieve their goals and to adjust the precision of its estimate of the natural rate of interest.

6 Conclusions

In this paper we have analyzed, from a monetary policy point of view, the role of the natural rate of interest in a New Keynesian model with endogenous growth, financial frictions and trend inflation. First, we have introduced the DSGE model and we have delved into the steady state obtaining a definition of the natural interest rate. We have shown that this rate depends on the financial structure and have carried out a sensitivity analysis to determine the nature of this dependence. Furthermore, if the central bank does not use the correct natural rate of interest in the monetary policy rule, the long-term inflation rate does not match the target. As a result, the natural rate of interest becomes dependent on the monetary policy in two ways, since it is sensitive to its own estimate error and to the inflation target.

As regards the short-term economic stabilization, we have analyzed a monetary and a technology shock and have found that the errors in the natural rate used in the monetary policy rule do no affect the responses of the variables and that there is no asymmetry between under- and over-estimation.

Moreover, we have determined the relationship between the potential errors in the estimation of the natural rate and the long-run growth rate. We have proved that this relationship depends on the inflation target because, if it is below (above) the optimal long-run inflation rate, long-term growth decreases when the estimation of the natural rate is above (below) the endogenous one. The opposite happens with the relationship between the marginal gain for expanding the assets of the banks and the error in the estimation. However, in the other possible scenarios, the relationship is hump-shaped, so the conclusions are not univocal. Furthermore, the parameter included in the Taylor rule that reacts to deviations of inflation in the short term will also generate shifts in the long-run growth if the natural rate estimate is not the endogenous one. Therefore, we can conclude that, according to these results, the errors in the estimation of the natural rate of interest are not neutral in the long run.

Finally, on the basis of our results, we have developed a mechanism to verify the stance of the monetary policy with respect to the optimal inflation from the growth perspective and the accuracy of the natural rate estimation, which can lead to maximizing the long-run growth. The central bank has to find out the sign of the differential between the estimated and the endogenous natural rate of interest as well as the position of the inflation target with respect to the rate that maximize the long-run growth. The monetary authority first makes use of the parameter that reacts to inflation deviations in the monetary policy rule. Then, by modifying the intercept of the Taylor rule and the inflation target, the central bank will be able to determine the position of both the inflation target with respect to the optimal rate and the estimated natural rate of interest with respect to the endogenous one.

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