Multiple-bidding in auctions as bidders become confident of their private valuations.

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Abstract

A bidder may increase his bid over the course of an auction when (1) he becomes more certain about his private valuation over time (as he has more time to consider using the item), and (2) there is a positive probability he is unable to return to the auction to submit a bid in a later period.

(*JEL*: D44, D82 *Keywords*: auction, eBay, multiple bidding, value discovery)

Introduction

In a standard second-price auction, it is optimal for a potential buyer to simply bid his private valuation for the item at any point during the auction (Vickrey 1961). Such a strategy means the bidder always wins the item whenever it is sold for less than his valuation, and he never pay more than his valuation for it. However, in online second-price auctions, bidders often submit bids less than their valuations, and increase their bids over the course of the auction.

Why do bidders increase their bids over the course of an online auction? I show how such behavior can arise when a bidder becomes more certain about his valuation over the course of the auction, and when there is a positive probability a bidder is unable to return to the auction in a later period to submit a bid. Unlike other explanations of multiple-bidding, I do not require that bidders are naive about the second-price auction mechanism (e.g., Roth and Ockenfels 2002, Ockenfels and Roth 2003), that learning one's true valuation is costly (e.g., Rasmusen 2006), that

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the item has a common value (e.g., Bajari and Hortacsu 2003), or even that bidders observe other players' bids (e.g., Rasmusen 2006).[1]

This paper presents a simple model in which an auction lasts for two periods. In the first period, when a potential buyer first observes the item up for auction, he has expectations about his valuation for the item; however, he knows his expected valuation may change as he has more time to consider the item. Additionally, the potential bidder recognizes something may come up that prevents him from returning to the auction in the second period, after he has had time to fully consider the item and learn his valuation with certainty. I show that in equilibrium the buyer does bid in the first period of the auction, and his bid is less than his expected valuation. If he is able to return to the auction for the second period, and his realized valuation is higher than his previous bid, he will increase his bid, thus engaging in multiple bidding.

Model

There are $N+1$ bidders in a two-period, sealed-bid, second price auction. Each bidder $i \in \{1, ..., N+1\}$ has private valuation V_i for the auctioned item, which is the independent realization of a random variable uniformly distributed on $[0, 1]$. To simplify the model, I assume N of the bidders are certain regarding their valuations from the beginning of the game. The one other bidder is uncertain about his own valuation in the first period of the auction, but learns his valuation before the second period of the auction.[2] (One could imagine that the bidder spends time considering the benefit he will get from the item and his outside options between the time he first discovers the auction for the good, and the period in which the auction ends.)

With probability $\alpha \in (0, 1)$ a bidder is unable to participate in the second period of the auction. This could be because something comes up that results in the opportunity cost of returning to the auction being higher than the expected benefit of returning, or the bidder could have technical issues connecting to the internet or submitting a bid.

The game takes place as follows:

[1] Rasmusen (2006, page 6) argues that multiple bidding occurs in any auction in which the bidders become more certain of their valuations over time. This is not entirely correct. If players are certain they can submit a bid in a future period of the auction, they should wait to submit a bid until after they learn their valuations. If they submit a bid before they are most certain of their valuation, they risk paying more for the item than their valuation.

[2] The results should not change if more than one bidder does not realize his valuation until after the first period of the auction.

1. Players $2, ..., N+1$ realize their valuations, and each player $i \in \{1, ..., N+1\}$ can submit a first-period bid $B1_i$.

2. Player 1 realizes his valuation V_1. With probability $1-\alpha$, a player can submit a second-period bid $B2_i > B1_i$.

3. All players i such that $\max\{B1_i, B2_i\} \in \max\{B1_j, B2_j\}_{j=1}^{N+1}$ win the item with equal probability.

Equilibrium and Results

First, consider the strategies of players $2, ..., N+1$ who know their valuations in the first period. In equilibrium these players bid their valuations in period one. If they bid more than their valuation, they risk winning the item at a higher price than it is worth to them. If they bid less than their valuation, or do not submit a bit, they risk not winning the good when they would benefit from doing so.

Given the equilibrium strategies of the other players, I consider the strategy of player 1. With probability α, player 1 is unable to submit a bid in the second period. With probability $1-\alpha$, player 1 is able to submit a bid in the second period. It is straightforward to see that if $B1_1 < V_1$, then $B2_1 = V_1$, and if $B1_1 \geq V_1$, then the player will not submit a bid in the second period, or $B2_1 = \emptyset$.

Now, consider player 1's first-period bid. Without loss of generality, suppose player 2 has the highest realized valuation of the N other bidders, and let F define the distribution of V_2 (with density f) given that it is the highest non-player 1 valuation. Since all valuations are drawn from a uniform distribution on the unit interval, it follows that $F(V) = V^N$, or the valuation of player 2 is less than V iff the valuations of all bidders except player 1 are less than V. Therefore, $f(V) = NV^{N-1}$.

Player 1 chooses his bid $B1_1$ to maximize the following expression with respect to b

$$\alpha \int_0^1 \int_0^b NV_2^{N-1}(V_1 - V_2)dV_2dV_1 +$$

$$(1-\alpha)\left(\int_0^b \int_0^b NV_2^{N-1}(V_1 - V_2)dV_2dV_1 + \int_b^1 \int_0^{V_1} NV_2^{N-1}(V_1 - V_2)dV_2dV_1\right).$$

The first part of this expression is the expected payoff to player 1 when he is unable to submit a bid in the second period. The second part of the expression is his expected payoff when he is able to bid his valuation in the second period. The expression simplifies to

$$\alpha \left(\frac{b^N}{2} - \frac{N b^{N+1}}{1+N} \right) + (1-\alpha) \left(\frac{b^{N+2}(1-N)}{2(N+1)} + \frac{1-b^{N+2}}{(N+2)(N+1)} \right). \tag{1}$$

The first order conditions for equation 1 are

$$\frac{1}{2} N b^{N-1} \left(\alpha - 2b\alpha - b^2(1-\alpha) \right) = 0. \tag{2}$$

Solving equation 2 for b, we find player 1's first period contribution. This gives us the first lemma.

Lemma 1 *In equilibrium,*

$$B1_1 = \frac{\sqrt{\alpha}}{1+\sqrt{\alpha}},$$

and, conditional of being able to bid in the second period, $B2_1 = V_1$ *when* $V_1 > B1_1$*, and* $B2_1 = \emptyset$ *when* $V_1 \leq B1_1$*.*

The lemma follows immediately from the above analysis, and the fact that $B1_1 \in (0, 1/2)$ is always feasible. Given player 1's strategy, and the distribution of his valuation, we can determine the probability the player increases his bid in the second period.

Proposition 1 *Player 1 engages in multiple bidding with ex ante probability* $1 - \sqrt{\alpha}$*.*

For the player to engage in multiple bidding, he must be able to submit a bid in the second period, and his valuation must be greater than his first-period bid. These conditions are met with probability $(1-\alpha)\left(1 - \frac{\sqrt{\alpha}}{1+\sqrt{\alpha}}\right)$ which simplifies to $1 - \sqrt{\alpha}$.

When player 1 is able to submit a bid in the second auction (which happens with probability $1-\alpha$), he is more likely than not to increase his bid. This is because his first period bid is strictly less than his expected valuation, and therefore his realized valuation is greater than $B1_1$ more than half the time. Furthermore, when a player becomes more certain about his own, private valuation over time, increasing his bid over the course of the auction can be rational, even if his realized valuation by the end of the auction is not greater than his expected valuation when he first bid. In the above model, player 1 had first-period expectations 1/2; however, so long as V_1 is greater than $\frac{\sqrt{\alpha}}{1+\sqrt{\alpha}}$ (which is less than 1/2), he will increase his bid in the second period, if he is able to.

Closing Comments

Although the paper only considers a very simple auction model, similar results should hold in a more complex setting. For example, more than one bidder may be uncertain about his valuation at the beginning of the auction; the game may involve more than two discrete periods; and valuations may be drawn from a more complicated distribution. None of these alterations should change the results.

Furthermore, I have said nothing about the phenomenon of sniping, or how bidders in online auctions often wait to submit a bid until the final moments of the auction. If a bidder becomes more certain of his valuation over time, he benefits from bidding immediately before the auction closes. However, it may not be reasonable to believe that a bidder learns a significant amount about his valuation during the final few minutes of an auction. Yet, bidders often wait until the closing seconds of an auction to bid. Therefore, other explanations of sniping may be more appropriate. Future research could incorporate this paper's model into a more complicated framework to simultaneously explain multiple-bidding and sniping. For example, I assume the bidder realizes his true valuation over time. However, a refinement of the model could allow players to experience a quasi-endowment effect, or for their perceived valuation to increase if they are the high bidder early in the auction. In this case, some players may bid in the closing moments of the auction to avoid this behavioral issue. In such a setting, both sniping and multiple-bidding may be present. Another option may be to incorporate the model into a multiple-auction framework.

References

[1] Bajari, Patrick and Ali Hortaçsu (2003), "Winner's Curse, Reserve Prices and Endogenous Entry: Empirical Insights from eBay Auctions," *RAND Journal of Economics*, Summer 2003, pp. 329-355.

[2] Ockenfels, Axel and Alvin E. Roth (2006), "Late and Multiple Bidding in Second Price Internet Auctions: Theory and Evidence Concerning Different Rules for Ending an Auction." *Games and Economic Behavior*, 55, pp 297-320.

[3] Rasmusen, Eric Bennett (2006), "Strategic Implications of Uncertainty over One's Own Private Value in Auctions." *Advances in Theoretical Economics*, 6(1), Article 7.

[4] Roth, Alvin E. and Axel Ockenfels (2002), "Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet," *The American Economic Review*, 92, pp 1093-1103.

[5] Vickrey, William (1961), "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, 16, pp 8-37.