Interest Rate Rigidity and the Fisher Equation

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Interest Rate Rigidity and the Fisher Equation

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Abstract

I create a model where private banks face adjustment costs in nominal interest rates. The model’s inflation responds to interest rate changes (both nominal and real) by moving in the opposite direction. That response justifies the Taylor rule and explains, through credit conditions, the procyclicality of inflation. The model permits the analysis of different types of monetary policy using a variable inflation target. I use this feature to simulate different policies and compare them to interest rate data from the last century. The interest rate rigidity model leads to credit-conditions-driven inflation, which I believe is more realistic than competing models of inflation.

Keywords: Interest Rate Rigidity, Inflation, Monetary Policy, Fisher Effect.

JEL: E31, E43, E52.

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1 Introduction

On the onset of the crisis, the debate at the Fed about where inflation was going and more recently at the Sveriges Riksbank highlight radical divergence of views. What we know about inflation mostly consists of two stylized facts: its inverse relation with interest rates and procyclicality. This paper offers an alternative model concentrating on these two stylized facts, but also on credit conditions. Specifically, it examines whether rigidity in nominal interest rates creates a simple and credible narrative for the behavior of inflation.

Interest rate rigidity has been discussed since Keynes, but the literature on interest rate rigidity does not fully address its macroeconomic implications. Models rarely isolate the effects of interest rate rigidity (or even credit frictions) from other factor affecting inflation, keeping us from knowing what interest rate rigidity actually does to inflation. How nominal interest rate rigidity interacts with the Fisher equation is simple, yet the implications are interesting. If nominal rates cannot catch up to real rates, the Fisher effect becomes inverted in the short term: central banks must lower interest rates to stimulate inflation, and credit crunches lower inflation.

In the model, a private bank, facing adjustment costs, cannot fully adjust nominal rates to account for changes in real rates. The other agent, a central bank, affects real rates while monitoring their indirect effect on nominal rates (i.e. internalizing the reaction of the private bank). The central bank does so to achieve its inflation target. How the central bank uses the target provides a way to analyze monetary policy; for example, a variable target simulates a peg to gold or to a foreign currency.

To visualize nominal interest rate rigidity, imagine a fictional case of constant nominal interest rates: when the real interest rate goes up, inflation necessarily goes down. This is the point made by Carmichael and Stebbing (1983), but it also happens when nominal interest rate hit the zero lower bound.¹

¹The recent brush with the zero lower bound reactivated the debate on the Fisher effect. Two strands of
Research about interest rate setting points to sluggish pass-through from a central bank’s key rate to market rates (interest rate pass-through), and from market rates to lending rates (bank rate rigidity). Kobayashi (2008, Section 2) surveys the literature on interest rate pass-through and de Bondt, Mojon and Valla (2005), on bank rate rigidity (see also Illes and Lombardi (2013) for recent issues related to the financial crisis). Still, most of this research focuses on empirical test for nominal interest rate rigidity, theoretical justifications for it, or its microeconomic effects. Little research concentrates on the macroeconomic effects of this rigidity. Exceptions include Ravenna and Walsh (2006) and Kobayashi (2008) who investigate the implications of rate rigidity by incorporating them to standard models with price rigidity (DSGEs).

Price rigidity is not a fact, but a way to explain its persistence, which is the actual statistical fact. In this paper, I ignore price rigidity. First, modeling both price and nominal interest rate rigidities makes it hard to discern which has what effect. Second, for the sake of parsimony, the model presented here does not need to be built on top of both the NKPC postulates and the ones I propose. Third, as shown by Craig and Dinger (2014), nominal interest rates display lower volatility in micro data than do prices. Fourth, if we take the view that both prices and interest rates behave sluggishly, interest rate rigidity may dominate. Interest rates stir profits more than prices or even wages because of the interest rates use in discounting future profits, while only relative prices and real wages affect profits. I partially compensate for this (as some might feel) oversight by comparing results with annual data; as shown by Klenow and Malin (2010), most prices reset within a year. Annual data also permits me to stretch the sample back to a time when, as shown by Benati (2008), persistence was absent.

As stated earlier, this paper focuses on rigidity coming from the market, not the central arguments appear. The first relates to Benhabib, Schmitt-Grohé and Uribe (2001), who investigate the possible incompatibility between the Fisher equation and the Taylor rule. The Fed’s Bullard (2013) provides a good example. The second relates to Cochrane (2011), who argues that without rigidity, the Taylor rule leads to indeterminacy. The obvious consequence of his arguments is that monetary authorities should increase nominal interest rates to generate inflation. See Williamson (2014) for this side of the debate.
bank. Macroeconomists may have ignored this type of interest rate rigidity because the control of nominal rates by monetary authorities seems perfect as indicated by Figure 1.1 and 1.2 of Woodford (2003a). From that perspective, interest rate rigidity stems from the central bank itself. For example, Woodford (2003b) considers the benefits for the central bank of interest rate smoothing, while DSGE models commonly add the lag of interest rate to the Taylor rule, as in Christiano, Eichenbaum and Evans (2005). But if nominal rigidity stems from the market, the central bank can just push real rates by a lot to get the change it wants in nominal rates. Perfect control does not imply direct control.

Section 2 presents a model that yields rigidity through adjustment costs of nominal interest rates of a private bank, given the real ones. The private bank wants to have the real interest rate at its equilibrium value, but cannot immediately do so because of the cost of adjusting the nominal rate. The equilibrium real interest rate appears when inflation reaches the central bank’s target or when there is no monetary policy.

Section 3 presents numerical illustrations in the form of impulse response functions (IRFs) and simulations. The IRFs show how inflation targeting reduces real interest rates volatility. Anticipated shocks suggest that the preemptive nature of nominal adjustments explains the price puzzle from Sims (1992). Then, I simulate three different monetary policies with the model using the central bank’s inflation target process: a volatile inflation target simulates the Gold Standard; a constant target simulates inflation targeting; and a variable, but persistent target simulates the period characterized by gradually rising inflation in the seventies and declining inflation in the eighties. The simulations show we can model different episodes of inflation and interest rate movements of the last hundred years simply through monetary policy shocks, that is to say without changing the nature of the other shocks. Simulations also suggest inflation targeting reduces real interest rates volatility, while a gold standard increases it, giving a sense for the causes of the Great Moderation.
2 The model

The model leads to three equations that determine inflation, real and nominal interest rates. The rigidity equation translates real interest rates into nominal interest rates. The Fisher equation translates real and nominal interest rates into inflation. Finally, the real interest rates equation comes from the real economy, but with feedback from monetary policy. A larger, more complete, model would be superfluous: there is no point in defending the postulates necessary to build it and it would make this paper overly long and complicated.

Agents in the model consist of a private bank, a central bank and an exogenous real economy. The private bank sets nominal interest rates. The real economy sets a real interest rate (according to intertemporal preferences, the marginal productivity of capital, from abroad or whatever source). Finally, the central bank affects real interest rates (through open market operations and the like) in order to achieve an inflation target. Inflation influences the real economy only because of the central bank’s actions.

The private bank acts as the stylized representation of the financial market. It minimizes the gap between the real interest rate and what the rate would be under a binding inflation target, but faces quadratic adjustment costs of changing nominal interest rates,

$$\min_{i_t} \left\{ E_t \left[ \sum_{s=0}^{\infty} \beta^s \left( (r_{t+1+s} - r_{t+1+s}^*)^2 + \theta (i_{t+s} - i_{t-1+s})^2 \right) \right] \right\}, \quad (1)$$

where $i$ represents the nominal interest rate, $r^*$, the real interest rate in the absence of monetary policy and $r$, the real interest rate. For future reference, $\pi^*$ represents the target inflation rate and $\pi$, inflation. In order to model uncertainty easily and coherently, $i$ at time $t$ corresponds to $r$ and $\pi$ at time $t+1$ by convention. Furthermore, all rates appear in logarithm form.\(^2\) Parameters $\beta$ represents the private bank’s discount factor and $\theta$, the cost of adjust-

\(^2\)Inflation is the log difference of price, $\pi_t = \ln(P_t) - \ln(P_{t-1})$, and interest rates, the log of one plus the actual rates, so a deposit $D$, left untouched, has grown according to $\ln(D_t) = i_{t-1} + \ln(D_{t-1})$, or $\ln(D_t) = r_t + \pi_t + \ln(D_{t-1})$.\]
The choice of quadratic adjustment costs is straightforward. Quadratic adjustment costs occur frequently in macroeconomics, for example in Rotemberg (1982) for price adjustment costs or in Christiano, Eichenbaum and Evans (2005) for investment adjustment costs. As shown later, posing Calvo pricing for nominal interest rates would lead to same result without any useful gains or insight. Furthermore, here I just assume interest rate rigidity without delving into its justifications, see de Bondt (2005) for the possible causes of interest rate rigidity.

The long-term part of the objective function also has a simple intuition. The private bank eventually needs to relent to monetary authorities in order for expected real interest rates to equal the ones offered. The central bank’s actions affect real interest rates in a predictable way when inflation differs from its target. Consequently, the model assumes that the private bank’s business as an intermediary needs, at least for long maturities, unbiased expected real rates. The private bank does not want the central bank messing up the rates it offers.

The second equation of the model,\[ i_{t-1} = r_t + \pi_t, \quad (2) \]
is the familiar ex post definition of real interest rates (in log form) that determines inflation in the model. The equation implies the ex-ante version or Fisher equation.

An issue is whether the equation’s interest rates represent the interest a strong-credit entity, like a government, or a representative agent pays, since headline inflation does not represent what governments or strong-credit entities buy, but what everyone buys. I ignore this issue, as this is not an empirical paper.

The third equation, the real interest rates equation, consists of a monetary policy part and an exogenous part. The monetary policy part acts as an error correction term between
inflation and its target,

\[ r_t = r_t^* + \phi (\pi_t - \pi_t^*), \]

for a positive \( \phi \). The shock term, \( r^* \), represents equilibrium real interest rates in the absence of monetary policy intervention; in other words, it represents the real economy. The monetary policy part means the central bank influences nominal interest rates through real interest rates. The buying and selling of bonds involved in setting interest rates move real interest rates directly, but nominal interest rates indirectly through the rigidity equation.

The only-partially-exogenous real interest rate stems from the existence of a Taylor rule. As showed by Cochrane (2011), no Taylor rule can exist if the movements of \( i \) do not influence \( r \). Imagine \( r \) independent of \( i \): after an unexpected rise in \( \pi \), the central bank raises \( i \); according to the Fisher equation, \( \pi \) goes up again so the central bank keep raising \( i \) endlessly. All solution paths are explosive. The possibility of a Taylor rule, coherent with practice and estimations (Taylor (1993), Clarida, Galí and Gertler (1998), plus I cannot find a single instance of hyperinflation caused by a central bank stubbornly following a Taylor rule), means the central bank cannot alter \( i \) without affecting \( r \). In fact, the central bank’s action needs to move \( r \) more than \( i \) for it to generate the inverse effect on \( \pi \). So the central bank, if it affects inflation, must also affect real rates of interest. As such, the institution is not neutral.\(^3\)

When part of a more complete model, the exogenous variable, \( r^* \), would instead come from the model itself. This would involve a central bank inserting itself in the equilibrium by buying or selling bonds. However, as stated earlier, such an approach lays well beyond the scope of this paper.

Finally, isolating \( \pi_t \) from equation (2) and inserting it into equation (3), than isolating \( r_t \) and subtracting \( r_t^* \) transforms \( r_{t+1+s} - r_{t+1+s}^* \) from equation (1) to \( \phi (1 + \phi)^{-1}(\mu_s - r_{t+1+s}^*) \)

\(^3\)Expressed in deviation from the steady state, the Taylor rule becomes \( \hat{r}_t = \alpha_p \hat{\pi}_t + \alpha_y \hat{y}_t \), where \( \hat{y} \) represents the output gap. Inserted into Fisher equation, it yields \( E_t \hat{\pi}_{t+1} + E_t \hat{\pi}_{t+1} = \alpha_p \hat{\pi}_t + \alpha_y \hat{y}_t \), which is explosive if both \( r \) and \( y \) are exogenous since \( \alpha_p > 1 \). If instead we write the Taylor rule as \( \hat{r}_t = \phi E_t \hat{\pi}_{t+1} + \alpha_y \hat{y}_t \), insertion in the Fisher equation will yield \( (\alpha_p - 1)E_t \hat{\pi}_{t+1} = E_t \hat{\pi}_{t+1} - \alpha_y \hat{y}_t \) which is not explosive, but in which reactions of inflation to both real interest rates and the output gap are counterintuitive.
\(\pi^*_{t+1+s}\), therefore transforming the objective function to

\[
\min_{i_t} \left\{ E_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \left( \frac{\phi}{1+\phi} \right)^2 \left( i_{t+s} - r^*_{t+1+s} - \pi^*_{t+1+s} \right)^2 + \theta (i_{t+s} - i_{t-1+s})^2 \right) \right] \right\}.
\]

So the minimization of the objective function yields

\[
i_t = \psi \theta i_{t-1} + \psi \beta \theta E_t [i_{t+1}] + \psi \left( \frac{\phi}{1+\phi} \right)^2 E_t [r^*_{t+1} + \pi^*_{t+1}],
\]

where

\[
\psi = \left( \frac{\phi}{1+\phi} \right)^2 + \theta + \beta \theta \right)^{-1}.
\]

Equation (4) constitutes the rigidity equation. While equations (2) and (3) are conventional, the rigidity equation defines the model. The equation directs the movements of \(i_t\) as it heads toward its equilibrium value.

Incidentally, the equation could also had been written as

\[
\Delta i_t = \beta E_t \Delta i_{t+1} + \frac{1}{\theta} \left( \frac{\phi}{1+\phi} \right)^2 \left( E_t [r^*_{t+1} + \pi^*_{t+1}] - i_t \right),
\]

which, for adequate values of \(\theta\) and \(\phi\), is the solution to a monopolistic banking sector with Calvo-type rigidity on nominal interest rates. See Kobayashi (2008) for proof.

Restrictions on the model are needed to make it useful. The next section proposes some and presents results.

### 3 Results

This section builds on the model by making assumptions on parameters and postulating stochastic processes for real interest rates. Specifically, \(r^*\) consists of the sum of two exogenous shocks. Illustration purposes govern parameter calibration so as to yield reasonable
results. These assumptions make it possible to derive IRFs and conduct simulations.\footnote{I used Dynare and Matlab\textsuperscript{tm} to create the IRFs, simulations and figures, using the default seed so not to influence results and make them replicable.} The next two subsections present the results.

For illustration purposes, $r^*_t$ of equation (3) takes the form of two exogenous shocks,

$$r^*_t = \bar{r}^* + \epsilon_{a,t-4} + \epsilon^*_t,$$

(5)

where $\epsilon^a$ represents an anticipated shock (four periods in advance), $\epsilon^s$, a surprise shock and $\bar{r}^*$, the steady state real interest rate. Both shocks follow first-order autoregressive processes.

Equation (4) implies a steady state of $\bar{i} = \bar{r}^* + \bar{\pi}^*$ that, inserted to equation (2) and (3), implies $\bar{\pi} = \bar{\pi}^*$ and $\bar{r} = \bar{r}^*$.

IRFs and simulations share the same non-shock parameters. The adjustment cost parameter, $\theta$, equals 1. It represents the weight given to each part of equation (1), but also determines persistence. There exists no prior for the parameter; 1 means both part of the objective function carries the same weight. The discount factor, $\beta$, equals 0.995 and $\bar{r}^*$ equals $1 - \beta$. For monetary policy, the steady state inflation target, $\bar{\pi}^*$, translates a 3 percent per year inflation target, at 0.75 percent per quarter. The inflation target does not alter the results; a 3 percent target simply makes Figures 3 and 4 easier to read than the more common 2 percent target.

Furthermore, the parameter $\phi$ shows what happens when the central bank reacts ($\phi = 0.5$) or not ($\phi = 0$) to shocks. The choice of 0.5 for $\phi$ comes from inserting equation (3) into equation (2), $i_t = E_t[r^*_{t+1} + (1 + \phi)\pi_{t+1} - \phi \pi^*_{t+1}]$, yielding an influence of inflation on nominal interest rate of 1.5, as in Taylor (1993). If $\phi = 0$, the long term part of equation (4), $E_t[r^*_{t+1} + \pi^*_{t+1}]$, disappears and the private bank forever choose not to change nominal interest rates. Interestingly, that looks like the ancient equilibrium practice, where nominal interest rates came more or less from tradition, so the market achieved changes in real inter-
Table 1: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state inflation target</td>
<td>$\bar{\pi}^*$</td>
<td>0.0075</td>
</tr>
<tr>
<td>Steady state real interest rate</td>
<td>$r^*$</td>
<td>0.005</td>
</tr>
<tr>
<td>Monetary policy tightness</td>
<td>$\phi$</td>
<td>0.5</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.995</td>
</tr>
<tr>
<td>Adjustment cost</td>
<td>$\theta$</td>
<td>1</td>
</tr>
</tbody>
</table>

est rates through inflation.

Table 1 lists the parameter values.

### 3.1 Impulse response functions

This subsection presents the implications of the model in terms of impact analysis. Results from IRFs include: (1) a constant inflation target lowers real interest rate volatility compared to no monetary policy intervention, (2) a moving inflation target raises volatility even more, (3) persistent shocks have more effect on nominal interest rates, temporary shocks on inflation and (4) anticipated real interest rate shocks lead to immediate response thereby simulating the price puzzle. Furthermore, the results reinforce the intuition behind nominal interest rate rigidity, as central banks need to move interest rates in the opposite direction of where they want to move inflation (except when the inflation target changes) and credit conditions cause inflation to be procyclical.

For IRFs, the persistence parameters for the shocks switch from $\rho = 0.75$ for temporary shocks to $\rho = 0.999999$ for permanent shocks (not exactly 1 to secure a numerical solution).

Figures 1 and 2 illustrate three possible outcomes for respectively temporary and permanent positive one percent shocks. The three shocks of each figure consist of a surprise real interest rate shock, an anticipated real interest rate shock and a surprise shock to the inflation target. For an anticipated shock to the inflation target, see Figure 5 in the Appendix. The shaded area represents the inflation implied by the difference between nominal and real...
interest rates. The shocks appear in deviations from control levels.

In terms of interpretation, real interest shocks originate either from the real economy or from the central bank trying to attain its target inflation rate. They illustrate shocks to $r^*$, but also show the effects of monetary policy when inflation differs from its target.

The figures uncover three results. First, the inflation target shock means that real interest rates increase in volatility if a central bank adopts a volatile inflation target. The next subsection provides further results for the monetary policy influence on real interest rates volatility. Second, persistent shocks affect nominal interest rates more, while temporary shocks mostly affect inflation. Third, anticipated shocks simulate the price puzzle. As we can see in the figures for anticipated real interest rate shocks, rises in nominal interest rate and inflation

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For more information on the price puzzle, see Sims (1992) and Eichenbaum (1992).

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Sims (1992) discovered the price puzzle, and described it as an empirical anomaly. It states that an increase (decrease) in interest rates will increase (decrease) inflation in the short term before having the expected effect some quarters later. Eichenbaum (1992) quoted the name, price puzzle.
Figure 2: Permanent one percent shock responses

Note: the vertical axis represents the inflation rate ($\pi_t$), the ex post real interest rate ($r_t$) and the nominal interest rate ($i_{t-1}$) in percent deviation from control state; the horizontal axis indicates quarters to or from the shock.

appear simultaneously some time before deflation sets in.

These results reinforce the intuition behind nominal interest rate rigidity. First, it shows that, in the presence of rate rigidity, the Fisher effect is inverted in the short term. The argument simply applies the Fisher equation ($i = r + \pi$). A rise in real interest rates can either translate to a one to one rise in nominal interest rates, an inverse one to one drop in inflation or a mix of the two. In the presence of rigidity, a rise in real interest rates cannot lead to an equal rise in nominal interest rates, therefore leads to disinflation. Consequently, the central bank’s operations do not directly lower nominal interest rates, but, in reality, yanks down the real ones. In doing so, it creates inflation.

Second, the results imply that the relationship between economic fluctuations and inflation supports a relationship between credit conditions and fluctuations. The relationship between economic fluctuations and prices, between inflation and the output gap or the pres-
ence of deflationary pressures in (say) ordinary recessions needs to come from an inverse relation between real interest rates and production within the cyclical time frame.

The existence of a link between credit conditions, as a fuller characterization of recessions, and of the 2008 recession in particular, emerged as a crucial subject of research. Such is the case in other periods as shown, for example, by Eckstein and Sinai (1986) or Ng and Wright (2013). To quote Bernanke, Gertler and Gilchrist (1999, p. 1343): “First, it appears that introducing credit-market frictions into the standard models can help improve their ability to explain even ‘garden-variety’ cyclical fluctuations.”

Furthermore, the predicting power of an inversion of the yield curve on recessions (see Stock and Watson (1989), Adrian, Estrella and Hyun (2010) and Rudebusch and Williams (2009)) also suggests credit crunches are a critical aspect of the business cycle. The yield curve inverts when markets expect temporarily tight credit in the near future.

Of course, such a link does not predict a causal relationship, whether from production to credit conditions in a financial accelerator type of argument, as in Bernanke, Gertler and Gilchrist (1996, 1999) or Kiyotaki and Moore (1997), or from credit conditions to production in a credit cycle type of argument. Without clarifying the issue, this paper makes a strong case for future research into the relationship between fluctuations and credit conditions. In fact, the literature pertaining to credit conditions, whether of financial accelerator or credit cycle flavors, is developing at a healthy pace.

### 3.2 Monetary regime simulations

Four different monetary regimes approximate interest rates data: a gold standard, a liquidity trap, a non-target fiat and an inflation target regime. A volatile inflation target simulates the gold standard and replicates the regime’s high volatility in ex post real interest rates. I skipped the liquidity trap. A very persistent inflation target process simulates the non-target fiat regime, replicating the non-stationarity seen in nominal interest rates and the more
Table 2: Shock parameter values

<table>
<thead>
<tr>
<th></th>
<th>standard deviation</th>
<th>persistence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surprise real interest rate shock</td>
<td>0.005</td>
<td>0.6</td>
</tr>
<tr>
<td>Anticipated real interest rate shock</td>
<td>0.005</td>
<td>0.6</td>
</tr>
<tr>
<td>Inflation target shocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>... under gold standard regime</td>
<td>0.13</td>
<td>-0.05</td>
</tr>
<tr>
<td>... under non-target fiat regime</td>
<td>0.004</td>
<td>0.975</td>
</tr>
<tr>
<td>... under inflation target regime</td>
<td>0</td>
<td>$10^{-8}$</td>
</tr>
</tbody>
</table>

persistent ex post real rates. For the inflation target regime, the target obviously does not move. Overall, simulations show that inflation target schemes exist that can simulate how most real and nominal interest rates behavior looks.

For all simulations, surprise and anticipated real-interest-rate shocks respectively follow processes each with 0.6 persistence and standard deviations of 0.005. Table 2 lists the shock parameter values applied to the regimes.

Table 3 compares the standard errors and correlation of nominal and real interest rates for the data and the three regimes simulated. The small size of the annual sample for each regime makes these results unreliable. Thus, the calibration efforts mostly concentrate on showing what the model can do. A more thorough empirical investigation lies outside the scope of an exposition paper.

The first part of Figure 3 shows data from Shiller (1989, Chapter 26) and updated by him for one year nominal and ex post real interest rates. The other part of Figure 3 and Figure 4 show simulations of the different monetary regimes for one year nominal and ex post real interest rates. The figures present annualized simulation output in order to simulate the averaging effect that yearly data produces. Figure 6 in the Appendix shows these simulations with the simulated inflation target, and Figure 7 shows the original quarterly results.
### Table 3: Standard deviations and correlation

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<th></th>
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</thead>
<tbody>
<tr>
<td><strong>Gold Standard</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>8.07</td>
<td>8.34</td>
<td>2.30</td>
<td>3.00</td>
</tr>
<tr>
<td>Persistence</td>
<td>0.09</td>
<td>0.15</td>
<td>0.32</td>
<td>0.68</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.39</td>
<td>0.29</td>
<td>0.20</td>
<td>0.62</td>
</tr>
<tr>
<td><strong>Non-target Fiat</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2.30</td>
<td>3.00</td>
<td>2.11</td>
<td>2.94</td>
</tr>
<tr>
<td>Persistence</td>
<td>0.32</td>
<td>0.68</td>
<td>0.95</td>
<td>0.59</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.20</td>
<td>0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Inflation Target</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.99</td>
<td>1.92</td>
<td>0.96</td>
<td>1.43</td>
</tr>
<tr>
<td>Persistence</td>
<td>0.32</td>
<td>0.47</td>
<td>0.59</td>
<td>0.74</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.83</td>
<td>0.65</td>
<td></td>
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</tr>
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*Note: standard deviations in percent.*

#### 3.2.1 Gold standard regime

The first regime, before the Great Depression, corresponds to gold standard periods. It represents one regime. Different implementations do not show in rates data though monetary authorities implemented the gold standard differently at different time in practice. The regime displays a high volatility in ex post real interest rates and inflation, but a lower volatility in nominal interest rates. The focus of the central bank on inflation, that started much later, may explain the absence of the recent inflation persistence in this period. Inflation persistence emerged as a stylized fact after World War II.

A volatile inflation target simulates the regime. This inflation target needs a large variance to make the figures look right. Consequently, it follows a first-order autoregressive process with a standard deviation of 0.13 and a persistence of -0.05. This supposes a large variance in gold prices, coherent with gold prices still seen today.

Inflation target volatility creates an interesting effect. This volatility greatly increases real
interest rate volatility and puts the Great Moderation in perspective. The model predicts standard deviation of real interest rates under the gold standards at about four times that of real interest rates under the inflation target regime.

### 3.2.2 Liquidity trap regime

The second regime, from the Great Depression to before the Bretton Woods system, saw the economy going in and out of either a liquidity trap or severe financial repression. That regime displays often negative ex post real interest rates and nominal interest rates near the zero bound. Since the simulation model cannot deal with asymmetries caused by the zero lower bound of nominal interest rates, I did not simulate the regime.

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Still, the model predicts a liquidity trap. If nominal interest rates hit the zero lower bound, agents doubt the inflation target because the central bank cannot lower interest rates to increase inflation. The private bank then chooses not to react to changes in real interest rates or in the inflation target. It results in a long period of zero interest rates that only ends when real interest rates descend under minus target inflation. In terms of policy, the model therefore predicts that the economy can only get out of a liquidity trap through financial repression or luck.

3.2.3 Non-target fiat regime

The third regime, the non-target fiat, starts with the Bretton Woods Agreement and ends sometimes in the late eighties or early nineties. Though Bretton Woods officially implements a gold standard regime, it acts more like a fiat regime. Fixed exchange rates meant that central
banks managed the inflation target globally. This regime seems plagued by non-stationarity. Furthermore, unlike under the other regimes, sometimes real and nominal interest rates move in visibly opposite directions. From the model’s perspective, as the IRFs showed, these opposite moves signal changes in real rates prompted by changes in the inflation target.

For the postwar period until around the eighties, I determined that a persistent inflation target process, of 0.975, best describes what was happening as inflation went gradually higher until Volker, and then went gradually down until the early nineties. The apparent non-stationarity in the data pleads for the use of persistent shocks. Thus, the inflation target follows a first-order autoregressive process with a standard deviation of 0.004 and a persistence of 0.975.

The results from simulation of this regime show how persistence in inflation leads to high persistence in nominal rates. Furthermore, real and nominal interest rates move sometimes in opposite directions as found in the data.

Still, the simulation lacks a Volker moment, namely the moment when a sharp drop in the inflation target lead to a sharp rise in real interest rates. In the simulation, changes in the inflation target translate to almost equivalent changes in the nominal interest rate. Abrupt changes in the inflation target only translate to abrupt changes in the real interest rate when temporary, as in the Gold standard simulation. This may signal a failure not of the model, but of the Fed’s credibility, as agents may have doubted the Fed’s resolve to keep inflation low.

3.2.4 Inflation target regime

The last regime saw some form of inflation targeting scheme. Consistent with a persistent inflation rate, both real and nominal interest rates seem to want to move in tandem. In this regime, the inflation target stays constant.

The inflation target process could have included a small non-persistent error term to ac-
count for errors in predictions made by the central bank. Though a useful addition to a projection model preoccupied with curve fitting, a small shock serves no purpose here.

As expected, the results show the annualized nominal and real rates tend to move in tandem except when the real rate moves in a spike. In such a spike move, the nominal rate will not completely follow.

### 3.2.5 Summary

Different inflation target processes can describe all three regimes: a volatile inflation target for the gold standards, a less volatile, but more persistent target for the non-target fiat regime, and a fixed target for the more recent regime. In terms of results, the variable inflation targets increase real interest rate volatility, persistent inflation targets lead to persistent nominal interest rates, while a constant target leads to more stable real interest rates.

We can envision that inflation target schemes exist that can simulate how real and nominal interest rates behavior looks. For lack of a thorough empirical investigation, we can say, at the very least, that the model shows promise. The interest rate processes that emerge from simulations appear realistic.

### 4 Conclusion

A large literature supports the central postulate that interest rate are rigid, as well as the procyclicality of inflation and how we believe monetary authorities use interest rates to manipulate inflation. This paper looked at the macroeconomic implications of rigidity in nominal interest rates.

In terms of results, an interest rate rigidity model of inflation emerges as a satisfactory implementation of price evolution that could be used inside a larger macroeconomic model. The interest rate rigidity model explains the two major stylized facts about inflation: how
central banks move nominal interest rates in opposite direction to control inflation and why credit conditions amid recessions lead to deflationary pressures. Side contributions include a possible explanation of the price puzzle, from the preemptive aspect of nominal interest rate setting, and of the Great Moderation, from the destabilizing effect of variable inflation target policies prior to the mid-eighties.

Although the model displays realistic simulation results, the paper offers no estimation of the model’s parameters or test of the theory. Issues related to identification and the choice of data force estimation to be left for future research.

References


# A Supplemental material: For Online Publication

**Figure 5: One percent inflation target shock responses**

**Note:** the vertical axis represents the inflation rate ($\pi_t$), the ex post real interest rate ($r_t$) and the nominal interest rate ($i_{t-1}$) in percent deviation from control state; the horizontal axis indicates quarters to or from the shock.

Figure 5 shows nominal and real interest rate, as well as the inflation response to anticipated temporary and permanent shocks to the inflation target.
Figure 6: Simulated regimes showing inflation targets

Gold standard simulations

Non-target fiat simulations

Inflation target simulations

Note: the vertical axis represents the inflation target (π_t^*), the ex post real interest rate (r_t) and the nominal interest rate (i_{t-1}) in percent; the horizontal axis indicates years.

Figure 6 shows annualized nominal and real interest rates, but also the inflation targets for the three regimes simulated.
Figure 7: Original 568 quarterly simulated periods

Note: the vertical axis represents the inflation target ($\pi^*_t$), the ex post real interest rate ($r_t$) and the nominal interest rate ($i_{t-1}$) in percent; the horizontal axis indicates quarters.

Figure 7 shows the original quarterly nominal and real interest rates, as well as the inflation targets for the three regimes simulated.