An unnoted fair bet in german state run lotteries, a short notice

Frank, Sascha and Rehm, Jan

15 November 2007
An unnoted fair bet in german state run lotteries
a short notice

Sascha Frank∗
and
Jan Rehm
SLS No. 07017†
15. November 2007

This paper is according to the work of Krautmann
and Ciecka [1] who made this analysis for state-run
lotteries in the U.S.1 It’s common knowledge that
gambling markets don’t provide fair bets. In fact
state-run lotteries in Germany are some of the worst
gamblings, because the ToTo-Lotto Block as host of
the lotteries keeps 50 percent of the stakes. In other
words expected return would be 50 cents on the euro.

As Krautmann and Ciecka [1] have shown it, the
chance for a fair bet depends on the size of the pot. Unli-
ce the state-run lotteries in the U.S., lotteries in Germa-
y pay out the winning in a lump sum and lottery win-
ings are free of tax. Size of the pot depends first on the
share of the stakes which is take away by host, second
the share of the first prize level in payout, because this
money would be sent to the pot. We only take the pot
for the first prize level in account, the value of lower-
tier prize is uncared in this examination; the reason for
doing so is gamblers play to win the first prize and not
the second. For a further discussion about the motiva-
tion about players interests see also Garrett and Sobel
[2]. If, according to the methode described by Kraut-
mann and Ciecka [1] a fair bet was given, it should be
possible to apply the same to a game with higher sum
of winnings. By this reasons we have to change their
analysis in these points.

Pot Size
If \( B_t \) is the number of outstanding tickets and \( q \) the price
of one ticket, the whole sum of bets is \( S_t \). And with
\( g \in (0, 1) \) as the part which is take away by host, the

\[ PO_t = (1 - g)S_t \quad (1) \]

So the pot is equal to the share \( p \) in payout \( PO_t \), with
\( p \in (0, 1) \).

\[ Pot_t = p(1 - g)S_t \quad (2) \]

Like most lotteries, lotteries in Germany roll over the
pot whenever there is no winner. So pot size is sum of
the last pot and the part of new bets which went to first
level prize.

\[ Pot_t = Pot_{t-1} + p(1 - g)S_t \quad (3) \]

Upon substitution, we get the sum of bets \( S_t \).

\[ S_t = \frac{Pot_t - Pot_{t-1}}{p(1 - g)} \quad (4) \]

winner or multiple winners
The expected number of winners, \( EN_t \) is the probability
of winning, \( \omega \in (0, 1) \), times the number of ticketes,
i.e. \( EN_t = \omega B_t \). The expected return\(^2\) is the pot di-
vided by the number of winners,

\[ ER_t = \frac{\omega Pot_t}{EN_t} = \frac{Pot_t(pq)(1 - g)}{Pot_t - Pot_{t-1}} \quad (5) \]

Like Matheson [3] noted, this equation didn’t show the
expected return; the expected return depends on the
number of gamblers who win, as well. Hence we can
use this equation as condition for multiple winners\(^3\).

Fair bet conditions
To provide a fair bet two conditions must be satisfied.
The first one is the pot growth condition; the pot must

\(^∗\)Sascha Frank: E-mail:frank@faw.uni-freiburg.de
\(^†\)All papers in the SL-Series should be considered draft versions
subject to future revision. Comments are welcome.
grow slow between drawings. If not the stakes increase and thus the number of tickets, and by this chance for multiple winners will rise.

\[ Pot_{t-1} > Pot_t (1 + pg - p) \]  \hspace{1cm} (6)

The second condition, is the 'buy all' condition, the value of the pot is to be at least as big as the cost of playing all combination of the lottery.

\[ \omega Pot_t \geq q \]  \hspace{1cm} (7)

If the condition of inequality (7) was met once it holds through the pot lifetime, but inequality (6) has been proven at each drawing.

**German Lotteries**

Now we check the conditions to find a fair bet in one type of state-run lotteries which operates in Germany, “6 aus 45 Auswahlwette”. Until first of June 1985 there was an upper limit for winning sum of 3,000,000 DM for this lottery, so survey started on this date.

This lottery is an unpopular game in Germany, gamblers have to choose 6 out of 45 soccer games which end in a tie\(^4\). It also can be played by randomly picking numbers without knowing that this is not a normal lottery. According to data, part taken by the host is \( g = 0.5 \), part of the pot is \( p = 0.4 \), probability \( w = 1/8, 145, 060 \) and price \( q = 0.65 \) €, so we can estimate the size of \( Pot_t \) and \( Pot_{t-1} \).

\[
\begin{align*}
Pot_{t-1} & > 4,235,431 \\
Pot_t & \geq 5,294,289
\end{align*}
\]

As an example, we take a look at the 2007 super jackpot of Euro 6.3 million. Was this lottery a fair bet? For \( g = 0.5, p = 0.4, w = 1/8, 145, 060 \) and \( q = 0.65 \), equations (6) and (7) yield to

\[
\begin{align*}
Pot_{t-1} & > 6,333,002 (1 + 0.2 - 0.4) = 5,066,401 \\
\frac{1}{8,145,060} & 6,333,002 = 0.77 > 0.65
\end{align*}
\]

According to the glüXmagazin 09/04/2007 No.36 [4], the jackpot that turned over from the previous drawing was 5,839,683 € so both inequalities (6) and (7) were satisfied. And so, this was a fair bet.

**Conclusion**

The result presented in this paper suggest that it is not only theoretically but also practical possible for a fair bet in german state run lotteries.

**Notes**

1. When are State Lotteries a Good Bet?

2. If all possible combinations are played with same probability.

3. We tested the equation for the jackpots of 6 out 49 Lottery during 1985 to 1991, all of the pots failed this condition and ended up with multiple winners. See alt-hought [5]

4. The rules of the game are described in detail in [6].

**References**


URL www.lotto-bw.de

URL www.lotto-bw.de

URL www.lotto-bw.de

2