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Abstract

Most of the assets on the balance sheet of typical banks are illiquid. This exposes banks to liquidity risk, which is one of the key risks for banks. Since the value of assets is determined by their risks, liquidity risk should be included in valuation. This paper develops a valuation framework for liquidity risk. It is argued that the liquidity risk that illiquid assets on bank balance sheets are exposed to, is the risk of being liquidated at a discount in a liquidity stress event (LSE). This may be contrasted to extensions of CAPM including liquidity risk, where liquidity risk is considered to be the risk of changes in liquidity of a security. The main result is that the discount rate used for valuation includes a liquidity spread that is composed of three factors: 1. the probability of an LSE, 2. the severity of an LSE, and 3. the liquidation value of the asset.

The liquidity risk of a bank is determined mainly by its funding composition. An interesting connection to funds transfer pricing and funding valuation adjustment exists for a special bank balance sheet where the income from liquidity spreads and the costs of funding are equal. In particular, in this case the valuation adjustment due to liquidity risk is proportional to liquidity of the asset and the funding valuation adjustment.

*Earlier versions of this paper were titled "Discounting Cashflows of Illiquid Assets on Bank Balance Sheet" .

1 Introduction

One of the main risks of a bank is liquidity risk. This is reflected by, for instance, the inclusion of liquidity risk measures in the Basel 3 framework [1]. Already before Basel 3 the BIS issued the paper "Principles for Sound Liquidity Risk Management and Supervision" [2], aimed at strengthening liquidity risk management in banks. This paper stresses the importance of liquidity risk as follows: "Liquidity is the ability of a bank to fund increases in assets and meet obligations as they come due, without incurring unacceptable losses. The fundamental role of banks in the maturity transformation of short-term deposits into long-term loans makes banks inherently vulnerable to liquidity risk, both of an institution-specific nature and those affecting markets as a whole."

Since liquidity risk may result in actual losses, this paper argues it should be included in the valuation of balance sheet items. This paper assumes that the liabilities are liquid and as such are valued consistently with market prices. Instead the impact of liquidity risk on the valuation of assets is considered. The aim is to develop a valuation framework for liquidity risk that can be applied consistently across the different assets on a bank balance sheet. In particular the aim is to include derivatives, other traded assets, but also banking book assets. Banking book assets are held at historical cost and therefore their valuation is not required for financial reporting. Nevertheless valuation is important to calculate sensitivities such as duration and PV01's. Valuation of banking book assets is also important to determine profitability of assets. Therefore, although for accounting purposes valuation of banking book items is not relevant, these are included in the valuation framework developed here.

The research in this paper is motivated by a number of questions regarding the valuation of assets:

1. What is the impact of liquidity risk on the valuation of assets?

2. Liquidity risk events typically involve some complex dynamics. Can assets be valued without modelling the full complicated dynamics?

3. It is well know from research in recent years that investors do expect a discount in the price for illiquid assets. But how do individual investors, in this paper specifically banks, determine at what discount they are willing to buy or sell?

4. How are liquidity discounts of different assets related?

5. How does the funding composition affect the valuation including liquidity risk? In particular, how does the inclusion of liquidity risk in the valuation of assets relate to recent proposals to include funding costs in the valuation of derivatives?

To address these questions this paper considers the impact of a bank's liquidity risk on the value of its assets. The purpose of this paper is to value this risk consistently across all assets on a bank's balance sheet, such as securities, mortgages, loans, and derivatives.

The approach focuses on the discounting of cashflows generated by the different assets. It is recognized that the discounting of cashflows of assets is determined by their liquidity through the possibility that the bank has to liquidate (a fraction of) the asset in the event of liquidity stress. As a consequence the discount rate includes a liquidity spread. The main result of this paper is that the liquidity spread is composed of the probability of a liquidity stress event (an event in which the bank is forced to sell some of its assets), the severity of the liquidity stress event, and the liquidation value of the asset.

The outline of this paper is as follows: Firstly, section 2 develops a liquidity risk valuation framework and discusses some consequences. Section 3 extends the model to include credit risk and optionality. Section 4 considers the impact of the funding composition. In section 5 a paradox is discussed and the value of the assets on a stylized bank balance sheet is calculated. Lastly the conclusions are summarized.

2 Liquidity Risk Valuation Framework

2.1 First pass: Liquidity risk and valuation

In recent years the impact of liquidity risk on pricing of assets has been studied. In particular, research has been done to extend the CAPM model to include liquidity risk, such as the work of Acharya and Pedersen [3]. It is useful to recall these extensions to clarify the differences with the approach in this paper.

Acharya and Pedersen define a stochastic illiquidity cost C_i for security i that follows a normal process in discrete time. The illiquidity cost is interpreted as the cost of selling the security. Furthermore it is assumed that an investor buying a security at time t will sell the security at time $t+1$. Liquidity risk in this model comes from the uncertainty of the cost of selling the security. With this set-up Acharya and Pedersen derive a liquidity-adjusted CAPM with three additional betas.

Although the extension of CAPM including liquidity risk is useful to understand prices of traded assets, such as securities, it does not help to understand the valuation of most of the assets on a bank's balance sheet. The reason is that most of these assets are not traded. Loans, mortgage, and other assets in the banking book are intended to be held to maturity. Hence assuming the asset will be sold and assuming a stochastic cost is not appropriate for these assets. Even assets in the trading book may not be traded. For instance OTC derivatives, whose market risks will be hedged through trading hedge instruments, may well be held to maturity. Hence the CAPM approach, which assumes that an asset needs to be sold and model liquidity risk by stochastic liquidity costs, is not appropriate for most assets on a bank balance sheet.

The question is how these assets that are intended to be held to maturity are sensitive to liquidity risk. Whatever the changes in liquidity cost, as long as these assets are held to maturity as intended, their pay-off is not affected by liquidity risk. Hence it seems that these assets are not sensitive to liquidity risk, which would imply that liquid and illiquid assets with the same pay-off should have the same value.

The resolution this paper proposes is that, although the assets may be intended to be held to maturity, when the bank is experiencing a liquidity stress event, the bank may be forced to liquidate some of its assets at a discount. Therefore the pay-off generated by the asset may be lower than the contractual pay-off when a bank is exposed to liquidity risk. This discount should be reflected in the value of the asset. It is clear that an illiquid asset, which has a larger discount in a forced liquidation than a liquid asset, will have a lower value (when they have the same contractual pay-off).

These considerations lead to the following definition of liquidity risk is used in this paper:

Definition: Liquidity risk is the risk for an event to occur, that would force a bank to liquidate some of its assets.

Such an event can therefore be termed a liquidity stress event (LSE).
In the next section, a simple model for such events is proposed.

2.2 Liquidity Risk Model

In this paper LSEs are modelled as random events. The model consists of three components:

- The probability that an LSE occurs: $PL(t_1, t_2)$ will denote the probability of such an event between t_1 and t_2.
- The severity of an LSE. The severity will be indicated by the fraction of the assets that a bank needs to liquidate f. By definition $0 \leq f \leq 1$. For simplicity f will be assumed to be a fixed (non-random) number.
- The dependence structure of LSEs and other events. The model assumes that LSEs are assumed to be independent from each other and from other events such as credit risk or market risk events.

In particular, the model assumes that LSEs follow a Poisson process with a constant intensity $p \geq 0$, which implies for an infinitesimal time interval dt

$$PL(t, t + dt) = pdt. \tag{2.1}$$

This set-up simplifies the complicated dynamics of an LSE to the probability that the event occurs and the fraction of assets that the bank needs to liquidate in such an event. This simplification is justified since, the return of the asset to the bank is only affected by whether or not it needs to be liquidated. Hence the value of an asset depends on above effective parameters.

Of course, more insight in the liquidity risk of a bank is obtained by considering all potential contributors, such as retail deposits run-off, wholesale funding risk, collateral outflows, intraday risks etc. However for the valuation of an asset it only matters if and when it gets liquidated, not if the liquidation is a result of retail deposits or wholesale funding withdrawal.

The interpretation of above model is that the bank gets hit at random times by an LSE. In particular, the bank has at any time the same risk of being hit by an LSE, there is no notion of increased risk. An extension of the model that would support multiple states, such as "high risk" and "low risk" with different probabilities of an LSE and some probabilities to migrate from one state to the other, might be more realistic, but would also have many more parameters to calibrate. As discussed later, the lack of traded instruments to hedge liquidity risk make it difficult to calibrate the parameters to traded market instruments. Because of the inherent difficulties to calibrate parameters for liquidity risk, this paper chooses the above set-up with a minimum of parameters that need to be assessed.

2.3 Valuation with liquidity risk

In an LSE a bank will liquidate some of its assets. These assets will be sold at a discount depending on the liquidity of the asset. This discount in case of an LSE may be recognized by defining an effective pay-off.

$$\text{Effective pay-off} = \begin{cases} \text{contractual pay-off} & \text{if no LSE occurs} \\ \text{stressed value} & \text{if LSE occurs} \end{cases} \tag{2.2}$$

The contractual pay-off includes all cashflows of the asset, for example optionality, cashflows in case of default, cashflows if triggers are hit etc.

The stressed value includes the discount for liquidating part of the position in the LSE. In case of a single LSE at time τ the stressed value may be expressed as

$$\text{stressed value} = f_A V(\tau) LV + (1 - f_A)V(\tau), \tag{2.3}$$

where $V(\tau)$ is the value of the asset at time τ, f_A is the fraction of the asset that the bank will liquidate, and LV is the liquidation value as a fraction of the value of the asset. It is assumed here that assets are divisible and any part of the assets can be liquidated.

The fraction f_A of the asset that the bank will liquidate will be determined by a liquidation strategy. In the next section the liquidation strategy that should be used in valuation is derived.

Definition: The value of an asset under liquidity risk is defined as the present value of the effective pay-off

$$V = PV[\text{Effective pay-off}] \tag{2.4}$$

Consider a cashflow of an illiquid asset at some future time T. In absence of default risk the value at time t of the cashflow is related to the value at time $t+dt$ through

$$V(t) = e^{-rdt}V(t+dt)(1-pdt) + e^{-rdt}[f_A V(t+dt)LV + (1-f_A)V(t+dt)]pdt \tag{2.5}$$

The first term on the r.h.s. is the contribution from the scenario that no LSE occurs between t and $t+dt$, the second term is based on (2.3) and is the contribution from the scenario that an LSE occurs. The contribution from multiple LSEs between t and $t+dt$ may be neglected as long as p is finite, since this contribution is of order $(pdt)^2$ and dt is infinitesimal small.

Equation (2.5) may be rewritten as

$$V(t) = e^{-rdt}V(t+dt)[1 - p(1-LV)fdt]\,. \tag{2.6}$$

By introducing a liquidity spread

$$l = p(1-LV)f_A\,, \tag{2.7}$$

this becomes

$$V(t) = e^{-rdt}V(t+dt)(1-ldt). \tag{2.8}$$

The value of a cashflow at a future time T of notional 1 in absence of default risk is derived by iterating (2.8)

$$V = e^{-(r+l)T}, \tag{2.9}$$

since $\lim_{dt\downarrow 0}(1-ldt)^{T/dt} = e^{-lT}$.

The liquidity spread (2.7) used in discounting depends on the fraction of the asset f_A that a bank liquidates, this fraction will be determined in the next section.

2.4 Liquidation strategy

Consider a balance sheet with a set of assets A_i with $i = 1, 2, ..., N$, where A_i denotes the market value and each asset has a unique liquidation value LV_i. Without loss of generality an ordering of the assets can be assumed: $LV_i > LV_j$ if $i < j$.

Definition: A liquidation strategy for a set of assets A_i is a set of fractions s_i of assets to sell such that

$$\sum_{i=1}^{N} s_i A_i = f \sum_{i=1}^{N} A_i. \tag{2.10}$$

with $0 \leq s_i \leq 1$ and the sum over i covers all assets on the balance sheet. Here A_i denote the market values of the assets.

Such a strategy could be, for instance, to sell the most liquid assets until sufficient assets have been liquidated to reach $f \sum_i A_i$. Note that the strategy is allowed to depend on the order of the assets, but not on the liquidation values LV_i. The motivation is that a bank's liquidation strategy will be, more likely, of the type to liquidate assets based on their relative liquidity (e.g. most liquid assets first) instead of on their exact liquidation values.

Definition: An admissible liquidation strategy is a strategy s_i^* such that the liquidity spreads implied by the strategy

$$l_i = p(1 - LV_i)s_i^* \,, \tag{2.11}$$

satisfy the condition that for any set LV_i

$$LV_i < LV_j \Rightarrow l_i > l_j \,. \tag{2.12}$$

Definition: An optimal admissible liquidation strategy is an admissible liquidation strategy with the lowest loss in an LSE. This loss is defined as

$$\text{loss} = \sum_i s_i A_i (1 - LV_i) \,. \tag{2.13}$$

To demonstrate that the optimal admissible liquidation strategy is given by $s_i^* = s_j^*$ for all i, j, it first needs to be noted that a strategy with $s_i > s_j$ for $i < j$ is not an admissible strategy. Consider e.g. $s_1 > s_2$. Then the choice $LV_1 = LV_2 + \frac{s_1 - s_2}{2s_1}(1 - LV_2)$ implies $l_1 > l_2$. (It can be checked that this expression for LV_1 is a valid choice in the sense that $LV_1 > LV_2$ and $LV_1 < 1$.) Therefore $s_1 > s_2$ violates the requirement (2.12). Note that the same reasoning can be applied to any i, j with $i < j$, and that it is sufficient to have one choice of LV's that violates (2.12), since definition (2.12) should hold for any set LV's.

It can be concluded that the set of admissible liquidation strategies may be characterized by: $s_1 \leq s_2 \leq s_3 \leq ... \leq s_N$, where N denotes the last asset. Within this set the optimal choice is $s_1 = s_2 = s_3 = ... = s_N$, since it will lead to the lowest loss for the bank in an LSE. The conclusion is that the optimal admissible strategy is specified by $s_1 = s_2 = s_3 = ... = s_N = f$.

The final step in the completion of the valuation framework is the determination what fraction of an asset f in (2.7) a bank will liquidate in an LSE. The optimal admissible liquidation strategy has been defined to determine this fraction. It is the natural choice for valuation of possible liquidation strategies, since it preserves the relation between liquidation values and liquidity spreads (2.12) and within this admissible set minimizes the loss of the liquidation of assets.

2.5 Summary of the model

Putting the above liquidity risk model, valuation approach and optimal admissible liquidation strategy together the result is the following.

A cashflow at time T of an asset A_i without default risk should be discounted with the discount factor

$$DF = e^{-(r+l_i)T}, \tag{2.14}$$

where the liquidity spread is given by

$$l_i = p(1 - LV_i)f. \tag{2.15}$$

Note that the discount factor of the cashflow depends on the liquidity of the asset that generates the cashflow through LV_i. The other two factors, the probability of an LSE p and the severity of an LSE f, are not asset specific, but are determined by the balance sheet of the bank.

2.6 Some consequences of the model

A consequence of (2.15) is that liquidity spreads of different assets (on the same balance sheet) are related. Since in (2.15) the probability of an LSE and the fraction of assets that need to be liquidated are the same for all assets, it follows immediately that

$$\frac{l_i}{l_j} = \frac{1 - LV_i}{1 - LV_j}. \tag{2.16}$$

The liquidity spread of asset i and asset j are related through their liquidation values.

A nice feature of the model is that it allows to explain a different discount rate for a bond and a loan. Consider, for example, a zero-coupon bond and a loan with the same issuer/obligor, same maturity, notional, and seniority. The zero-coupon bond and loan therefore have exactly the same pay-off (even in case of default). Nevertheless if the zero-coupon bond is liquidly traded, a difference in valuation is expected. The model developed here, can provide an explanation for this difference. The above relation (2.16) shows that the liquidity spreads are related through the liquidation values of the zero-coupon and the loan. For example, if the probability of an LSE for a bank is estimated at 5% per year, and the severity of the event is that 20% of the assets need to be sold, and the liquidation value for the ZC-bond is estimated at 80% and for the loan at 0% (since the loan cannot be sold or securitized quickly enough) then the liquidity spreads for the bond and loan are:

$$l_{\text{bond}} = 20\text{bp}, \tag{2.17}$$

$$l_{\text{loan}} = 100\text{bp}. \tag{2.18}$$

These spreads are based on above example, and may differ significantly between banks. Nevertheless, they clarify that it is natural in this framework that a different discount rate is used for loans and bonds.

In this framework also the position size will affect the discount rate. Empirical studies find a linear relation between the size of the sale and the price impact [4, 5]. In the context of this paper this translates into a linear relation between the position size and the liquidation value:

$$LV_i = cx_i \tag{2.19}$$

where x_i is the size of position in asset i, e.g. the number of bonds, and c a constant. Consider a different position x_j in the same asset. From (2.16) it immediately follows that

$$\frac{l_i}{l_j} = \frac{x_i}{x_j}. \tag{2.20}$$

Given a linear relation between the size of a sale and the price impact, the framework derived here implies a linear relation between liquidity spread and position size.

2.7 Replication and Parameter Estimation

One of the important concepts in finance is the valuation of derivatives through determining the price of a (dynamic) replication strategy. Unfortunately, liquidity risk is a risk that cannot be replicated or hedged. In principle it is conceivable that products will be developed that guarantee a certain price for a large sale; e.g. for a certain period the buyer of the guarantee can sell N shares for a value $N \times S$, where S denotes the value of a single share. Such products would help in determining market implied liquidation values, but it is difficult to imagine that such products will be developed that apply to large parts of the balance sheet.

In any case, currently liquidity risk cannot be hedged. Nevertheless the risk should be valued. Therefore it seems appropriate to use the physical probability of an LSE and liquidation value to determine the liquidity spread in (2.15) as opposed to an imaginary risk neutral probability and liquidation value. Clearly, if it would be possible to hedge this risk then the risk neutral values implied by market prices should be used.

The physical probability of LSEs and the severity of the events are required to estimate the liquidity spread, see (2.15). These may be difficult to estimate. Perhaps more importantly, in the absence of hedge instruments and associated implied parameters, estimates may be less objective than desired.

On the other hand a bank should already have a good insight in the liquidity risk it is exposed to. E.g. through stress testing a bank has insight in the impact of different liquidity stress events. The BIS paper "Principles for Sound Liquidity Risk Management and Supervision" [2] gives guidance to banks how to perform stress tests. Such stress tests should provide some provide insight in bank-specific risks, that in combination with market perception of liquidity risk through e.g. liquidity spreads on traded instruments should provide estimates for p and f.

3 Extensions of the model

3.1 Including Credit Risk

This section adds credit risk to the framework. Recall (2.6) with (2.7). The inclusion of default risk is straightforward under the assumption that default events are independent from LSEs. The result is

$$V(t) = e^{-rdt} V(t+dt)[1 - ldt - pd \times \mathrm{LGD}dt]\,, \tag{3.1}$$

where pd is the instantaneous probability of default and LGD the Loss Given Default. By introducing a credit spread

$$s_{\text{credit}} = pd \times LGD \tag{3.2}$$

and solving (3.1) in a similar way as (2.6) gives the following value of a cashflow of nominal 1

$$V = e^{-(r+l+s_{\text{credit}})T}. \tag{3.3}$$

The discount rate consists of a risk-free rate, a liquidity spread and a credit spread.

3.2 Liquidity Risk for Derivatives

Liquidity risk also affects the value of derivatives. In a Black-Scholes framework liquidity risk results in an extra term in the PDE [6].

A brief derivation starts from a delta-hedged derivative's position. Demanding that the value of riskless portfolio of derivative's position and delta-hedge grows at the risk-free rate gives

$$dV - \Delta dS = r(V - \Delta S)dt, \tag{3.4}$$

where V denotes not the value of the derivative, but the value of the derivative's position, as indicated above. The Delta has the usual definition: $\Delta = \partial_S V$, and S denotes the underlying that follows a geometric Brownian motion. Including liquidity risk

$$dV = \partial_t V dt + \partial_S V dS + \frac{1}{2}\sigma^2 S^2 \partial_S^2 V - f(1 - LV_V)\max(V, 0)dN, \tag{3.5}$$

The last term on the r.h.s. is the extra term coming from liquidity risk, here N follows a Poisson process with intensity p. LV_V denotes the liquidation value of the derivative. The max-function reflects that the value of the derivative can be both positive and negative (depending on the type of derivative) and that only positions with a positive value will be potentially liquidated in an LSE.

Taking the expectation of the Poisson process dN, under the assumption of independence with dS gives

$$\partial_t V + rS\partial_S V + \frac{1}{2}\sigma^2 S^2 \partial_S^2 V = rV + l_V \max(V, 0). \tag{3.6}$$

Here V denotes the value of the derivative's position, S the underlying stock, σ the volatility, and l_V the liquidity spread of the derivative's position. The last term on the r.h.s. is the extra term coming from liquidity risk and is in fact equivalent to the last term on the r.h.s. of (2.8). Note that it is assumed that the underlying is perfectly liquid (in the sense that its liquidation value $LV = 1$).

In [6] also extensions of (3.6) are discussed that include credit risk.

A remarkable feature of (3.6) is that it is similar to models that some authors have proposed for inclusion of funding costs in the valuation of derivatives. In particular the extra term $l_V \max(V, 0)$ has the exact same form as the term for inclusion of funding costs derived by e.g. [7], with funding spread replaced by liquidity spread. The model above is more complex than the model including funding costs since the liquidity spread may be dependent on, for example, position size.

4 Funding costs and liquidity risk

The probability and severity of an LSE for a bank is largely determined by its funding composition. In the previous sections we treated the funding of a bank simply as a given, which resulted in some liquidity risk that should be included in the valuation of assets. Here the funding is considered more explicitly, through two examples:

1. adding an asset to the balance sheet that is term funded,
2. considering a special balance sheet where the income from the liquidity spreads exactly compensates the funding spread costs.

4.1 Adding an asset that is term funded

Consider the following simple balance sheet

A_i	L_j
	E

where all assets A_i have the same maturity T, without optionality or coupon payments. These could be thought of as a combination of zero coupon bonds and bullet loans. The liabilities have varying maturities and may include for instance non-maturity demand deposits.

Define the impact of liquidity risk on the total value of the assets as the Liquidity Risk Adjustment (LRA)

$$LRA = \sum_i A_i - \sum_i A_i^0 \tag{4.1}$$

where A_i^0 is the value of the asset without liquidity risk

$$A_i^0 = A_i(l_i = 0) = A_i e^{l_i T} \tag{4.2}$$

Now consider adding an asset A_{new} with the same maturity T that is term funded. The question is what is the impact on the LRA. The new LRA is

$$LRA_{\text{new}} = \sum_i A_i^{\text{new}} - \sum_i A_i^0 + A_{\text{new}} - A_{\text{new}}^0 \tag{4.3}$$

where A_i^{new} is the value with the new liquidity spread after adding the new asset and its term funding.

To estimate the impact on LRA the first step is to determine the new liquidity spread. Clearly the liquidation values LV_i of the assets do not change. Also the probability of an LSE is not expected to change, since the funding composition has not changed for the exception of adding a liability with the same maturity as the assets, which therefore does not contribute to the probability of an LSE. The only change is in the fraction of assets that need to be liquidated. Since the funding withdrawn in an LSE is the same before or after adding the asset when the asset is term-funded, the following relation holds:

$$[\sum_i A_i + A^{\text{new}}] f^{\text{new}} = [\sum_i A_i] f^{\text{old}} \tag{4.4}$$

Hence the new fraction is

$$f^{\text{new}} = \frac{\sum_i A_i}{\sum_i A_i + A^{\text{new}}} f^{\text{old}} \tag{4.5}$$

The old and new liquidity spreads are given by

$$l_i^{\text{old}} = p(1 - LV_i) f^{\text{old}} \tag{4.6}$$

$$l_i^{\text{new}} = p(1 - LV_i) f^{\text{new}} \tag{4.7}$$

The impact of adding the term-funded asset on the LRA is

$$\begin{aligned} LRA_{\text{new}} - LRA &= \sum_i (A_i^{\text{new}} - A_i) + A_{\text{new}} - A_{\text{new}}^0 && (4.8) \\ &= \sum_i (A_i e^{-(l_i^{\text{new}} - l_i^{\text{old}} T} - A_i) + A_{\text{new}} - A_{\text{new}} e^{-l_{\text{new}} T} && (4.9) \end{aligned}$$

Expanding this expression to first order in $A_{\mathrm{new}}/(\sum_i A_i)$ gives

$$LRA_{\mathrm{new}} - LRA = A_{\mathrm{new}}(l_{\mathrm{av}} - l_{\mathrm{new}})T, \tag{4.10}$$

where $l_{\mathrm{av}} = (\sum_i l_i A_i)/(\sum_i A_i)$. Hence, even though the new asset is term-funded the liquidity risk adjustment does change. The reason is that the new asset and its term funding is not isolated from the rest of the balance sheet. In an LSE the new asset may also (partly) be liquidated. And indeed, in the liquidation strategy derived in section 2.4 for valuation, it will be pro rata liquidated.

Equation (4.10) shows that the LRA increases when the new asset added is more liquid than the other assets on average.

4.2 A special balance sheet that balances funding costs and liquidity spread income

Up to now only the valuation of assets has been considered. However a bank also manages the income generated from these assets. From an income perspective a bank would want that the liquidity spread it earns on its assets is (at least) equal to the funding spreads it pays on its liabilities and equity:

$$\sum_i l_i A_i = \sum_j s_j^F L_j + s^E E \tag{4.11}$$

where s_j^F is defined as the spread on liability L_j relative to the risk-free rate r and s^E the spread paid on equity.

Define the average funding rate as

$$s_F = \frac{\sum_j r_j^F L_j + r^E E}{\sum_j L_j + E} \tag{4.12}$$

Then it is clear that (4.11) implies that the average liquidity spread equals the average funding spread

$$s_F = l_{\mathrm{av}} \tag{4.13}$$

Hence the liquidity spread for asset A_i in this special case is related to the average funding spread by

$$l_i = \frac{(1 - LV_i)}{(1 - LV_{\mathrm{av}})} r_F \tag{4.14}$$

This suggests that in this special case a bank can charge for liquidity risk through its funding costs when it corrects for the liquidity of the asset. In particular

- In the FTP framework of such a bank, the funding costs can be charged for the assets, but would differentiate between funding of liquid and illiquid assets through the factor $\frac{(1-LV_i)}{(1-LV_{\mathrm{av}})}$. E.g. the FTP for a mortgage portfolio would go down when a bank has securitized these (but have kept them on the balance sheet), since liquidation value LV of securitized mortgages is higher.

- Similarly the liquidity risk adjustment, introduced in the previous section, of a derivative is related to the Funding Valuation Adjustment that some authors have proposed. The LRA would however distinguish between liquid and less liquid derivatives, such as an OTC and exchange traded option that are otherwise the same. An example is given in [6].

Remains the question how "special" this special case is. Many banks would recognize (4.11) as something they apply ignoring the commercial margins on both sides of the balance sheet. However, most banks base their liquidity spreads on their funding costs, although (4.11) may be satisfied, its the liquidity spreads do not accurately price the liquidity risk of the bank. Nevertheless, adjusting for the liquidity of an asset according to (4.14) may improve pricing to account for the liquidity of the asset.

5 A paradox and an example

5.1 A paradox

As discussed in section 2 the liquidity spread is determined by the loss from a forced sale of part of the assets in a liquidity stress event. The applied sell strategy is to sell the same fraction of each asset. In practice however one would sell the most liquid assets as this results in a smaller loss. Since a larger loss is accounted for in the valuation, it seems that a risk-free profit can be obtained by holding an appropriate amount of liquid assets or cash as a buffer for a liquidity stress event.

To analyze the paradox, consider a bank with a simple balance sheet, as shown below

$A = 80$	$L = 80$
$C = 20$	$E = 20$

This bank has 80 illiquid assets, 20 cash, and its funding consists of 80 liabilities, and 20 equity. It is exposed to an LSE where 20% of the funding is instantaneously removed.

If the stress event occurs the resulting balance sheet used in the valuation is

$A = 64$	$L = 60$
$C = 16$	$E = 20$

The sale of the assets will result in a loss $= (1 - LV_A)16$. This loss is born by the equity holders, who in this setup, provide the amount $(1 - LV_A)16$. This amount combined with the result from the sale of the assets $LV_A 16$, and a cash amount of 4 covers the withdrawal of funding. Note that this can be viewed as a two-step approach whereby the funding withdrawal is covered by the cash and immediately supplemented by the sale of the assets and the cash provided by the equity holders.

In practice a bank will use its cash buffer to compensate the loss of funding. In contrast to the strategy of the pro rata sale of assets used for valuation, this strategy will not lead to a loss. The resulting balance sheet is

$A = 80$	$L = 60$
$C = 0$	$E = 20$

The paradox is that the value of the assets includes the possibility of a loss (through the liquidity spread), whereas in reality this loss seems to be avoided by using the cash as a buffer.

However, the bank is now vulnerable to a next LSE, whereby 20% of its funding is withdrawn. To be able to withstand such an event a cash buffer of 16 is required. To avoid any liquidity risk this buffer should be realized immediately, which can be achieved by the same sale of assets as in the strategy for valuation, resulting in the same loss. Therefore, to avoid any liquidity risk the same loss is born by the equity holders, which resolves the paradox.

In practice the assets may be sold over a larger period of time, thereby the bank chooses to accept some liquidity risk to avoid the full loss by an immediate sale. The optimal strategy in practice is the result of risk reward considerations.

5.2 Example for a stylized balance sheet

Consider the following stylized balance sheet of a bank.

retail loans	10	deposits	60
corporate loans	20	wholesale funding	30
mortgages	40	equity	10
central bank eligible bonds	10		
corporate bonds$> AA^-$	10		
cash	10		

The bank has considered its vulnerability to liquidity stress events, and it concludes that in a stress event its deposits can reduce by 15 and its wholesale funding also by an amount 15 within 3 months. The probability of such an event is estimated at 5% per year. The translates into the parameters

$$p = 5\% \qquad FL = 30\% \tag{5.1}$$

The bank decides to base the liquidation values of its assets on the Basel 3 Required Stable Funding (RSF) factors. The basel document [1] states: "The RSF factors assigned to various types of assets are parameters intended to approximate the amount of a particular asset that could not be monetised through sale or use as collateral in a secured borrowing on an extended basis during a liquidity event lasting one year". Although this does not exactly match the definition of the liquidity stress event identified by the bank, since the bank's stress event only lasts 3 months, the bank chooses to identify

$$1 - LV = RSF \tag{5.2}$$

for each asset.

The result for the liquidity spread for the different assets is given in the table below.

Asset	RSF	liquidity spread (in bp)
retail loans	85%	127.5
corporate loans	65%	97.5
mortgages	65%	97.5
central bank eligible bonds	50%	75
corporate bonds$> AA^{-}$	20%	30
cash	0%	0

The above liquidity spreads are the result of the assumptions of the bank in the above example. For a specific bank the liquidity spreads depend on bank-specific features, such as the fraction of stable, less-stable, and non-stable deposits, which affect FL and p. Nevertheless the stylized bank above illustrates how different assets get different liquidity spreads.

A slightly extended version of this example is included in [6], where also the valuation of call options is considered.

6 Conclusions

This paper develops a liquidity risk valuation framework. It is shown that liquidity risk of a bank affects the economic value of its assets. The starting observation is that under an LSE the bank needs to liquidate some of its assets, which means these will be sold at a discount. To develop the valuation framework a liquidation strategy of the bank needs to be determined. It is shown that the optimal liquidation strategy suitable for valuation is a strategy where of each asset the same fraction is liquidated. The result is that cashflows are discounted including a liquidity spread. This liquidity spread consists of three factors: the probability of an LSE, the severity of an LSE, and the asset-specific discount in case of liquidation in an LSE.

The answers to the questions posed in the introduction have been addressed in the main text. Here the answers are summarized:

1. Liquidity risk has an impact on the valuation of assets. This research suggests that the impact on the valuation is determined by the above mentioned three factors.

2. The valuation framework in this paper does not involve modelling the complex dynamics of LSEs. Determination of the probability and severity of LSEs in combination with the liquidity of the assets is sufficient.

3. In this framework the discount that banks should use to value illiquid assets is determined by the liquidity spread derived.

4. The framework implies that the liquidity spread of two assets on the same balance sheet is related through a simple relation involving only the liquidation values of the assets (2.16). This suggests the same relation should hold for traded prices of liquid and less liquid assets (at least if a sufficient number of investors trades both assets). This allows for an empirical test of the model.

5. Liquidity risk enters the valuation of assets in a very similar way as funding costs do in some recent proposals to include funding costs in the valuation of derivatives.

A few other noteworthy consequences of the valuation framework developed here:

- The value of a position is not independent of the rest of the balance sheet, since the balance sheet determines probability of an LSE and the severity of an LSE. In particular the same position on two different balance sheets may be valued differently.

- Two pay-offs that are exactly the same, but have a different liquidity may be valued differently. For example, a bullet loan and a zero-coupon bond of the same obligor/issuer with the exact same pay-off will have different liquidity spreads if the zero-coupon bond is liquidly traded (and the bullet loan is not).

- The size of a position affects the valuation. E.g. if a position in bonds is large compared to the turnover in an LSE, the liquidation value of the position may be lower than the liquidation value of a single bond. Therefore a large position will have a higher liquidity spread than a small position.

- The securitization of illiquid assets, such as loans and mortgages, into more liquid securities enhances the value of the assets. Within the liquidity risk valuation framework developed here, it is possible to estimate this value.

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