Human Knowledge and a Commonsensical Measure of Human Capital: A Proposal

Amavilah, Voxi Heinrich

30 July 2014

(((((REEPS))))) **Resource & Engineering Economics Publications Services**

The Shameless Knowledge Peddlers ©

Human Knowledge and a Commonsensical Measure of Human Capital: A Proposal*

REEPS Thinking Paper No. 12014, Draft5, Version 3

Voxi Heinrich Amavilah**
REEPS
PO Box 38061
Phoenix, AZ 85069-8061 USA
vhsamavilah@gmail.com

*

Cautions: Caution 1: I call this paper a "Thinking Paper" to suggest that an update may or may not be forthcoming. The idea for it started in 2011 when I wanted to use data on scholarships and other academic awards as instruments for human capital. First, I looked at Nobel Prize Awards data. I came away convinced that the money value of the Nobel Prize understates the Prize's full contribution to human capital building. I attempted to devise a practical (theory-less) way to estimating the full value of the Nobel Prize, and wrote up the results as "The Full Value of the Nobel Prize – Part 1: Mining 'Data Without Theory.' Part 1, because my intention was to use the data to assess the effect of the Nobel Prize on economic performance via its effect on human capital. However, as I was setting up an estimation model, I realized I have a slightly different working measure, not definition, of human capital, and for that reason I went back to existing literature for clarification. This paper is the outcome, but I am still thinking. **Caution 2**: The math of this paper is not real math; rather, it is a sign language resorted to only to fix a commonsensical and natural idea that knowledge is a multidimensional "solid" best measured as a volume, not as an area.

**

Amavilah is an independent resource and engineering economist for Resource and Engineering Economics Publications Services (REEPS), and an adjunct professor of economics at Glendale College, Glendale, Arizona, USA.

Abstract

Existing literature demonstrates clearly that knowledge is the sum of common knowledge and uncommon knowledge. Common knowledge is mostly inherited and it may or may not have scientific bases. Uncommon knowledge is mainly a product of the motions of science and technology. Scientific and technological motions depend on human capital, so that world knowledge is human capital by implication. From here analysis is not so unusual as the concept of human capital is not new. Through out history people have been interested in valuing human life. What prevented rapid progress in the beginning was inhibitions to likening humans to machines. As soon as economists overcame their inhibitions, human capital theory developed quickly along the familiar logistic curve, picking up speed after Mincer devised a practical formula for it. However, the Mincerian equation formalized a misconception in three ways. First, it based human capital only on labor, thereby overstating the production role and disregarding the importance of human capital in innovation and knowledge creation. Second, it measured human capital as an area, ignoring common language and understanding that as knowledge human capital is at least 3D "solid", with depth, width, and the *time over and in* which it accumulates. Finally, it neglected key interactions between the quantity and quality indicators of human capital. These misconceptions are what this paper tries to shed light upon by proposing a commonsensical measure of human capital as a volume. Analysis finds that disregarding interactions our commonsensical measure of human capital is larger than conventional Mincerian measures of human capital. Taking interactions into account, it is possible for our measure to be larger, smaller, or equal to conventional measures.

Key phrases: 3D human capital, Mincerian human capital, scientific knowledge, technological knowledge, common knowledge, wide knowledge, deep knowledge, solid knowledge, intimate knowledge, acquired knowledge, inherited knowledge

JEL Code: J24, I29, O15, Z00, D83

1. Introduction

Economists and allied scientists have long recognized that human knowledge is fundamental to human existence. An exhaustive review of the vast literature on this subject is literally impossible to conduct. Selectively, and for a more recent example, Stephen Parente and Edward Prescott (1994) used firm-level data and found that technology is a key factor in long-run economic growth and development (cf. Nelson, 2005, Foray, 2004). However, they also noted that technological change depends on investment in technology, and "the amount of investment required by a firm to go from one technology level to a higher technology level depends on two key factors: the level of general and scientific knowledge in the world and the size of barriers to adoption in the firm's country" (p. 299, cf. Romer, 1990). They have dubbed this knowledge stock "*world knowledge,*" and it is the sum of "*general knowledge*" and "*scientific knowledge,*" both assumed to be given and "to grow exogenously."

In this paper I generalize the Parente-Prescott insight to countries. I begin the generalization by defining human knowledge as *technology plus human capital.* Second, I characterize world knowledge by reviewing some present-day Mincerian measures of human capital. Finally, I propose a commonsensical measure of human capital as a volume rather than as an area.

2. Characterizing Human Knowledge (A)

Let us assume an economy with a Parente-Prescott *world [human] knowledge* (A), consisting of *general knowledge* (A_g) and *scientific knowledge* (A_s). This designation is not without precedent. Hayek (1937, 1945, 1974) divides A into scientific knowledge and common (ordinary) knowledge, where common knowledge can either drive or constrain scientific knowledge.[1] From such background support I claim that A_s consists of technological knowledge (A_τ) and human capital (H), such that $A=A_g+A_s$ by definition. General knowledge (A_g) depends on the world population (N), which is a function of both past (t-1) and present (t) time, i.e., $A_g=f(N(t-1,t))$. Now if we suppose A_g is at least weakly independent of, or strongly separable from H, and that A_s and A_τ are strongly dependent on, or weakly separable from H,[2] then

$$A=A_g+A_s=A_g+(1+A_\tau)H. \qquad (1)$$

Eq. (1) assumes that A_g and A_s are additive, which is a very strong assumption. Let us assume them to be multiplicative in the raw and additive in the natural logarithms so that

[1] I have seen somewhere Hayek's common knowledge described as "day-to-day knowledge", but I am sorry I am unable to trace the source.

[2] All variables are time-indexed, although for convenience often I have ignored the time subscript.

$$A = A_g[(1 + A_\tau)H], \quad (1')$$

which is consistent with Hayek. From here since $A_g = f(N(t-1,t))$, it can be assumed to grow at the rate n equal to the rate of growth of N. The implication is that A_g is a socioeconomic inheritance. Microeconomic applications to of such a perspective would include Becker, et. al. (1981), Becker (1992), Becker, Glaeser and Murphy (1999), and Zimmerman (1992). At the macroeconmic level the connection between inheritance and economic performance is not as straightforward, because A_g is passed on trans-genetically and intergenerationally through complex institutional channels and mechanisms (Becker, et. al., 1981). Even so, there are enough papers that one can go to, to explore that connection. For example, Agarwal, Echambadi, and Sarker (2004) examined how knowledge inherited from one business organization influences the performance of another receiving it (cf. Dun, 2003). They found that *inherited knowledge* augments *self-acquired knowledge* and thereby increases the probability of firm survival.

Bowles and Gintis (2000, 2002) developed an innovative model for measuring the intergenerational inheritance of income/wealth inequality, which is easy to understand and apply to describe the transfer of A_g. In their model,

$$A_g = \bar{A}_g + \psi(\Delta A_g) + e_{A_g} \quad (2)$$

where $\bar{A}_g$ is the mean stock of A_g available in the initial period, ψ is the intergenerational coefficient of A_g, $1-\psi$ is the coefficient of the distance, $A_g - \bar{A}_g = \Delta A_g$, or the coefficient of inheritance progress across generations, and e is the random error term, of which the precise structure is unknown *a priori*. Bowles and Gintis conclude that $1-\psi$ is getting smaller (bigger) the more (less) inherited A_g, such that

$$A = [\bar{A}_g + \psi\Delta A_g + e_A][(1 + A_\tau)H]. \quad (3)$$

Conceptually, given $(1 + A_\tau)H$, and because A_g growth rate is n, (3) can be written as

$$A = [(\bar{A}_g + \psi\Delta A_g + e_A)N_0 e^{nt}][(1 + A_\tau)H] = [(A_g N_0)e^{nt}][(1 + A_\tau)H], \quad (4)$$

where $(A_g N_0)$ is common knowledge available to N at $t = 0$ and inherited by next generations in $t > 0$. And we know from Arrow (1969) that $n \neq 0$, so that the growth of A, $\dot{A}$, is

$$\dot{A}=\varphi\Delta\bar{n}+(1+A_{\tau})\dot{H},\ \Delta\bar{n}=n^{*}-n<0, \tag{5}$$

where n is actual growth rate of N, $\bar{n}$ is average growth rate of N, and n^* is desired (target) growth rate of N. Hence, according to (5) the dynamics of A are mainly due to H. Thus, (4) is similar, though not identical, to the so-called Verdoorn-Kaldor hypothesis by which the rate of change of A is endogenously determined by learning from experience (cf. Kaldor, 1961, 1962, 1966, and also Choi, 1983, 148-180), i.e. $A_t=\exp(f(A_{t-1},b,\tilde{Y}_{t-1}))$, where b is the technical efficiency of the production process as experienced from previous times (cf. Kalecki, 1971), and the functional form of the dynamics is not as important as its determinants. Alwyn Young (1991, p. 371, cf. Young, 1928) has called b the "progress elasticity," where $\tilde{Y}(t)=\int_0^t Qdt$ is cumulative production up to and including the tth year (cf. Amavilah, 1997, Harris, 1992). A key difference is that the Verdoorn-Kaldor effect is assumed to be quadratic, whereas (4) may, or may not, be quadratic.

Because $A(t-1)$ and b are both likely unknown *a priori*, an alternative, easy, and natural conception of A at the aggregate level is to observe that an economic activity like Y ultimately depends on the economy's entire technical capability (A), which we can approximate as the product of A at t = 0 (A_0) and the cumulative human development index (HDI), i.e.,

$$A\ =\ A_0 e^{\int mdt}=A_0 e^{x};\ x=\int mdt\ \equiv\ HDI. \tag{6}$$

Then from Amartya Sen (1997) and his UNDP colleagues, see, e.g., Anand and Sen (1994), let $x\equiv HDI=H^{c}W^{d}Y^{e}$, where H is the human capital index of the population, W is the wellness (health) indicator of the population often measured by life expectancy at birth, and Y is the material conditions of the population measured as GDP, and $c=d=e=1/3$ are weights (UNDP, 2007/2008). Algebraically, $H=(x/(W^{d}Y^{e}))^{1/c}$. This means

$$A=[(\bar{A}_g+\psi\Delta A_g+e_A)N_0e^{nt}][(1+A_{\tau})x^{c_1}W^{c_2}Y^{c_3}],\ c_1=1/c,\ c_2=-d/c,\ c_3=-e/c. \tag{6'}$$

Solving (6) for Y we get the following nonlinear production function of A and H:

$$Y=\frac{A}{[(\bar{A}_g+\psi\Delta A_g+e_A)N_0e^{nt}][(1+A_{\tau})x^{c_1}W^{c_2}}]^{c_3}=f(A_i,H). \tag{6''}$$

Once again, the essence of (6") is consistent with Hayek (1937, 1945, 1974) in that Y depends on knowledge, but knowledge is a balance between A_g and $(1+A_\tau)H$. In fact, A_g determines the uses of $(1+A_\tau)H$ to the extent it influences the adoption and spread rates of $(1+A_\tau)H$, and because it depreciates faster than it appreciates. One can further follow an old idea summarized by Morton I. Kamien and Nancy L. Schwartz (1977) by which A is both Arrow-learning to capture technological change, as well as Hicks neutral to reflect technical progress (cf. Solow, 1997, 1960, 1957, 1956, Swan, 1956, 2002, Hicks, 1963[1932]), such that A evolves as

$$A = A_0 e^{\upsilon\chi + \omega t}, \tag{7.1}$$

where χ is a learning function of cumulative Y(t) in response to inputs, especially H (Amavilah, 1997, Amavilah and Newcomb, 2004, Newcomb, 1976). In other words, for $\chi = \varphi \tilde{Y}^{\xi}(t)$, so that

$$A = A_0 e^{\upsilon\varphi\tilde{Y}^{\xi}(t) + \omega t} \Rightarrow \dot{A} = \upsilon\varphi\xi\tilde{Y}^{\xi-1} + \omega, \tag{7.2}$$

where $\xi = b$ is Young's elasticity of technical progress (Arrow's learning effect), and ω is the conventional Hicks neutral effect (cf. Morishima and Saito, 1968, Equations 7 and 8, pp. 422-423, Allen, 1967, Chapter 13). This is in line with the historical "idea of progress" (Bury, 1960[1932]).

The basis for (7.2) is that in "Technology: More or less?," for example, Kamien and Schwartz (1977) tell us that "technology is application of known physical, social and behavioral principles to production of existing goods and development of new ones. *Advances in technology involve both deepening the knowledge base and expanding its area of application*" (p. 502, italics added). This much economists have known for long now (see Marx), although they "have been slower to realize that the pace and direction of technological advance are" endogenous (p. 501). Way back then, Adam Smith thought that technological change came from small improvements in labor productivity resulting from the division of labor. In this sense technology is exogenous like "manna from heaven". Indeed, re-examining Smith, Sherwin Rosen (1983) concludes that the rate of return from H depends on its utilization rate, which depends on factor specialization. Hence, "technical change in the development of new knowledge increases the amount, complexity, and productivity of skills available to be acquired and increases fixed elements of investment costs that are independent of utilization. This is one reason why the rate of return to education does not fall with economic development and why education is a more desirable investment in advanced economies than in undeveloped ones" (p. 48).

It is understandable why the high demand for education means high demand for investment in education in advanced economies. It is unclear and counterintuitive to suppose that education is a desirable good in some countries and an undesirable bad in others. However, Kamien and Schwartz also point out that Smith meant that technological revolutions happened only because of "men of independent means, who had the time to observe and study fundamental regularities of nature" (p. 502). Thus, some interpretations understate the fact that Smith did understand that technological change was a function of both resources and incentives, and hence it was endogenous, although it took years before JR Hicks (1963[1932]) hypothesis of "induced technological advance", and J.A. Schumpeter's (1939, 1942) extension of both entered standard theory.

3. Characterizing Human Capital (H)

This section briefly describes (a) the evolution of the H concept, and (b) the early Mincerian measures of H. The next section proposes a commonsensical measure of H as a volume vis-a-vis previous measures which calculate H as an area. The implications for policy and further research of this commonsensical measure are considerable.

Evolution of the concept of H

What philosophers Bertrand Russell (1948, 1956), and Nicholas Capaldi (1969) independently have called 'human knowledge' has always been valuable to human activities from the beginning of humanity (Amavilah, 2009). However, for much of human history, it has been A_τ that has been openly and frequently expressed in human activities ranging from tool-making, war-making, hunting and food gathering to hi-tech goods and services today (Varian, Farrell, and Shapiro, 2004). Without a systematic alphabet, reading, and writing, formal education and training were nearly impossible, so that all H came from experience. Moreover, experience as a key determinant of H was limited by the short life-expectancy of the inheritors. Consequently, it was hard to accumulate and spread, scientific knowledge (Ede and Cormack, 2004).

Despite the above, the importance of A in economic, as in other human, endeavors has long been acknowledged, see, e.g., Marshall (1961), Smith (1952 [1937]), cf. Stigler, 1957), Marx (1859, 1906; cf. Blaug, 1976), Hayek (1937, 1974, 1977), Sowell (1996), and so on. It is the precise measurement of A and hence H that remains an active challenge to-date, but not for lack of effort. E. Cohn's (1979, p. 13) attempt to define and measure education (an aspect of H) took him back to Leviticus in the Bible where the value of a person was understood to depend on the person's gender and age. Male persons were more valuable than female persons; and persons younger than 20 years and older than 60 years were less valuable than persons between the ages of 20 and 60 years old . This means that H was primarily a function of gender and experience, where gender is exogenous (determined by biology) and experience by age. Looking back we now know that such an understanding of H influenced policy-making including the exclusion of women from the formal educational systems and the mandating of children 16 years old and younger to be in school, both still current policies around many parts of the world.

Further along the way, we find Adam Smith (1952) essentially equating improved labor productivity to H. As he said it, "a man educated [to acquire] extraordinary dexterity and skill, may be compared to ... expensive machines. The work which he learns to perform, it must be expected, over and above the usual wages of common labor, will replace to him the whole expense of his education, with at least the ordinary profits of an equally valuable capital" (pp. 42-43; cf. Stigler, 1957). Following Smith, Nassau Senior's calculation finds H more important than K in Great Britain. Later J.S. Mill (1909) estimated H as a person's expenditure on raising a child up to a certain age. In fact, it is clear even from "a primitive theory of production" as described by Sir William Petty's *The Political Anatomy of Ireland* (1971) that

> by the same way we must make a par and equation between art and simple labour; for if by such labour I could dig and prepare for seed a hundred acres in a thousand days; suppose then I spend a hundred days in studying a more compendious way, and in contriving tools for the same purpose; but in all that hundred days did nothing, but in the remaining nine hundred days I did two hundred acres of ground; then I say, that the said art which cost but one hundred days' invention is worth one man's labour for ever; as much as two men could have done without it" (quoted by E.A.J. Johnson, 1965, p. 257).[3]

The preceding quote shows that the story of H is not new. Karl Marx (cited above) thought of H as 'labor power', quite distinct from the act of working. According to that view labor has a larger economic rent than land, but the problem in Marx's mind, and hence "the emancipation question," was that the economic rent of H was being appropriated by the *bougoisie* ("squanders of labor power"), and made a part of exploitative physical K. Wolfson, Ozech, and Hanna (1986) have dubbed such rent appropriation phenomenon "the depletion of human capital as an open-access resource" (cf. Hotelling, 1931). This of Marx's influences continues as evident from Bowles and Gintis's (1975) argument that "neoclassical economists have long treated labor as a commodity. ... Human capital theory is the most recent, and perhaps ultimate, step in the elimination of class as a central economic concept. ... Every worker, the human capital theorists are fond of observing, is now a capitalist" (p. 74). Bowles and Gintis lament further that by failing to recognize that production is social reproduction, H theorists underestimate the value of human resources, and by doing so fail to appreciate fully Julian Simon's (1981) insight that "the main fuel to speed our progress is our stock of knowledge, and the brake is our lack of imagination, [for] the ultimate resource is people – skilled, spirited, and hopeful people who will exert their wills and imaginations for their own benefit, and so, inevitably, for the benefit of us all" (p. 348). Nor, it appears, does the Bowles-Gintis argument account for Wesley Mitchell's proposition that "incomparably greatest among human resources is knowledge. It is greatest because it is the mother of other resources" (Zimmermann, 1951, De Gregori, 1987).[4] In spite of that wisdom, Bowles and Gintis question the end use of H by arguing that schooling, for example, does not enhance human welfare; it simply perpetuates "sexism, racism, and elitism." Instead of educating "good workers," schooling promotes worker oppression and wage

---

[3]This quote is from Johnson's book and the page number refers to that book; I have not read Petty's book.

[4]I first heard the quotes by Julian Simon and Wesley Mitchell from my Professor DeVerle P. Harris of Department of Mining and Geological Engineering, and Deaprtment of GeoSciences both at the University of Arizona, but have since read Simon's *The Ultimate Resource* (1981), and Zimmermann's *World Resources and Industries* (1951, Chapters 1 and 9), and De Gregori's (1987) review article "Resources are Not, They Become: An Institutional Theory."

inequality, and hence "it provides, in short, a good ideology for the defense of the status quo" (p. 82). Thus, it is *inefficient* because it makes someone better-off at the expense of another; it is *unfair* because it justifies such inefficiency. Other criticism of the Chicago-school theory of H has come from Michael Spence's (1973, 2002) and Joe Stiglitz (1975) models of uncertainty, risk, and asymmetric information in general, and/or "signaling" and "screening" theories in particular (cf. Akerlof, 1970, Arrow, 1974). This essay does not pursue these criticisms.

While the story of H is old, the term 'human capital' was not coined until people like W. A. Lewis, T.W. Schultz, and others created the field of Development Economics. We now understand that the term was not coined, not because people did not see the similarities between H and K, but, mainly because of the fear to liken human beings to machines. The apprehension led H.J. von Thunen (1968) to wonder why there was such reluctance to value H and K the same way even though in a war, "... with the hundred persons, a capital at least twenty times as large is lost as would result from the loss of one cannon. ... [The death of a soldier is the loss of] compensation to his family who lose in him perhaps their only source of support. ... Hence, [it is incomprehensible that] we regard [physical] capital more valuable than human beings" (p. 394). Given all this inhibitions, it is understandable that Marshall (1961) avoided the debate by simply equating H to "personal wealth," "personal capital," "the energies, facilities, and habits which directly contribute to making people industrially efficient" (pp. 204-205).

These old ideas must have filtered into Irving Fisher's (1897) "Senses of 'capital'", and John R. Walsh's (1935) "Capital concept applied to man." For comprehensive reviews of this literature I strongly recommend Cohn (1979) and B.F. Kiker (1966, 1967,1968a, b, 1969, 1971, 1974).[5] However, even for those unfamiliar with Kiker and Cohn, I was one until recently, it is still clear from Blaug (1979, 1976), Schultz (1961, 1981, 1979), and Becker (1993) that investment in H increases productivity, and therefore affects earnings (income). In that limited sense, James Coleman (1990, p. 304) is correct that

> probably the most important and most original development in the economics of education in the past thirty years has been the idea that the concept of physical capital, as embodied in tools, machines and other productive equipment, can be extended to include human capital as well (see Schultz, 1961, Becker, 1964). Just as physical capital is created by making changes in materials so as to form tools that facilitate production, human capital is created by changing persons so as to give them skills and capabilities that make them able to act in new ways.

Coleman is incorrect that the development of the H theory came only in the past thirty years; he is

---

[5]I am deeply indebted to the works of Professors Kiker and Cohn. I did not know anything about the history of human capital in the years before A.C. Pigou (1928) via W.A. Lewis. I guess JM Keynes (1936) was referring to me when he wrote,"It is astonishing what foolish things one can temporarily believe if one thinks too long alone, particularly in economics, (...), where it is often impossible to bring one's ideas to a conclusive test either formal or experimental" (*The General Theory of Employment, Interest, and Money*. New York, Harcourt, Brace and Company, pp. vii-viii).

however accurate in appreciating the role of H. The question is: How do we measure H accurately? Let us take a look.

Mincerian measures of H

The preceding section shows that many people worked on the definition, and on how to measure the importance, of H. However, Jacob Mincer (1958, 1974, 1981) was among the first to develop a practical formula for assessing H that has become the standard for empirical estimations. Reading Mincer's 1981 NBER paper alone it is hard not to appreciate the thought and care that he brought to the matter. First, he defines H as "... *acquired capabilities which are developed through formal and informal education at school and at home, and through training, experience, and mobility in the labor market,*" whether at the individual or national level. Second, "*at the national level, human capital can be viewed as a factor of production coordinate with physical capital, ... [implying that] its contributions to growth is greater the larger the volume of physical capital, and vice versa.*" Third, in that sense, "***human capital activities involve not merely the transmission and embodiment in people of available knowledge, but also in the production of new knowledge which is the source of innovation and of technical change which propels all factors of production***" (Emphasis added). Hence, finally, "... human capital generates worldwide economic growth regardless of its initial geographic locus, [and that is why] economic growth has not been eliminated by population growth" (see the Abstract).

Adolf Stroombergen, Dennis Rose, and Ganesh Nana (2002) demonstrate the powerful influence of Mincer on assessing H using the case of New Zealand. They outline three types of measurements of H. First is H as a reference to future earnings, i.e., $H=\sum_{t=p}^{N}[E_t^*/(1+r)^{t-p}]$, where E^* is market valued and nonmarket valued benefits of H accumulation, r is interest rate, t is any time, and p is the present time. This measurement of H underestimates the externalities of H. Hence, the second measurement takes H as referring to the stream of past expenditures on H building activities, such that $H=\sum_{t=0}^{p}[C_t(1+r-\delta)^{p-t}$, C_t being the economic cost of investment in H, and δ is H depreciation rate. The second measurement has the advantage of being easily quantifiable, but it overlooks nonmarket contributions to H accumulation and arbitrarily assumes C_t is fixed. The third measurement takes H as the capital stock consisting of both market and nonmarket attributes as well as capabilities of individuals, i.e., $H=\sum_{i}^{p}\lambda_i m_i+\sum_{j}^{w}\lambda_j O_j$, $i\neq j$, for $\lambda_i m_i$ the market value of individual attributes and capabilities, and $\lambda_j O_j$ is the nonmarket value of individual attributes and capabilities. While this third measure of H is more appealing than the first two, it raises even hard questions about quantifying O_j.

The three measurements are reasonable, but also problematic empirically. First, they all end up over-emphasizing the formal education, training, and experience of L, invariably stressing years of schooling (S) of L, and consequently the standard Mincerian formula has become

$$H = e^{\varphi S} L = e^{\varphi S} \int_0^t L_0 e^{nt} dt. \tag{8}$$

Again, (8) is fundamentally flawed: it over-estimates the value of H in production and under-states its value in the creation, accumulation, and transfer of knowledge (see Kamien and Schwartz, cited above). Not only is L too narrow a foundation for H, (8) also neglects essential interactions and intra-actions between A and H, and among all factors of production as Mincer has correctly observed (cf. Pedro Teixeira, 2008, Teixeira, 2004).[6]

A second problem has been that these measures are nearly all two-dimensional, at best. For example, Eric Hanushek and Dennis Kimko (2000) associate economic growth with the quality of L made possible by schooling (education). The association reveals that a rise in math and science scores increases the quality of L, which in turn raises economic growth. However, like Bowles and Gintis, but from a different ideological perspective, Bils and Klenow (2000) question whether schooling causes growth, while Hanushek and Woessmann (2008) found the positive effects of the cognitive skills of workers on economic growth, individual earnings, and income distribution to be greater than those of educational attainment in general (cf. Woessmann, 2003). However, along the direct effects of H, resource interactions are also important to both growth and technological change (cf. Amavilah, 2005, Eicher, 1996, Grier, 2005, 2002, Graca, Jafarey and Philippopoulos, 1995). This is not denying equations like
$E^* = a_0 + a_1 Education + a_2 Experience + a_3(Education * Experience) + a_4 Education^2 + a_5 Experience^2$.
Moreover, some economists link wealth with earnings, inheritance, and other influences. Even so, it is no exaggeration that many economists continue to measure H as an area, i.e., it is either cumulative quantity, or cumulative quality, of the skills of L. This, too, is a mistake.

Thirdly, saying $H = f(\textit{skills of L alone}) \Rightarrow L \to 0,\ H \to 0$. This is intuitively correct, and tuitively incorrect; H is knowledge, but a special kind of knowledge. Think of it this way: Paul Samuelson was a very knowledgeable (human capitalized) scientist – a genius. However, in this sense alone he was just a smart object ("human machine"). This type of H depreciated to zero with Professor Samuelson passing away (death). The outcome of his death is not different from what one would expect when an equipment reaches the end of its use life-cycle and it is beyond repair and it has zero salvage value. The good news is that there *is* a second cumulative Samuelson (H) that coexisted and has survived (most likely in perpetuity) the first Samuelson H. This is the idea Samuelson H. Again the first Samuelson H was the seed for the second Samuelson H, but the latter grew to be independent of the former. It is a common resource, whose value appreciates more than it depreciates as other scientists add to it. Samuelson the object may not even have liked Samuelson the cumulative idea, or how it is

---

[6]See varieous entries in Pedro Teixeira's (2014) *Human Capital (Critical Concepts in Economics)*, Routledge/Taylor and Francis..

used, but there is nothing he could do about it.[7]

I mention Professor Samuelson only because I am writing with economists and allied scientists in mind. In reality all H is like that: the first kind is a *quasi-private resource* built through private and/or public investment in education, health, training, experience, job mobility, inheritance, and it depreciates more than it appreciates. The second type of H grew out of the first, but it is a *public resource*, cumulatively built via social investment constrained by social capabilities (Temple and Johnson, 1998, Samuelson, 2004, Sequeira and Ferreira-Lopes, 2011). This H appreciates more and longer than it depreciates, and it underpins the relationships observed by many between population and economic performance, see, e.g., Banerjee (2011), Easterlin (1977), Galor and Weil (2000), Jones (2004), Kosobud and O'Neil (1974), Okita, Kuroda, Yasukawa, Yoichi, and Iio (1979), Peretto (1998), Pitchford (1972), Prettner (2012),

4. A Commonsensical Measure of H

The preceding section suggests that the 'true' measure of H must have a base larger than L in order to capture the production, innovation, and other aspects of *all* knowledge as opposed to just production knowledge. This claim is clear from the work of the originators of the H theory. For W.A. Lewis (1965), "the three proximate causes of economic growth ... are economic activity, *increasing knowledge*, and increasing capital" (p. 23). Lewis then stresses that "economic growth depends both upon *technological knowledge* about things and living creatures, and also upon *social knowledge* about man and his relations with his fellowmen. The former is often emphasized in this context, but the latter is just as important since growth depends as much upon such matters as learning how to administer large scale organizations, or creating institutions which favour economizing effort, as it does upon breeding new seeds or learning how to build bigger dams" (p. 164). Whereas the growth of knowledge comes from the inquisitive and experimental nature of human beings, its accumulation is a function of socio-historical inheritances, among these the alphabet, reading and writing, and the Scientific Method. These inheritances in turn explain the differences in the rates of growth and change of pre-scientific vis-a-vis scientific societies. Without reading, writing, and the Scientific Method the rates of H-accumulation and diffusion were very low in pre-scientific economies. Purposeful experimentation and similar inventive activities, as well as knowledge dissemination structures like educational institutions, agricultural experimental stations, and even industrial attitudes and business administration were challenges.

Lewis's characterization of H agrees fully with that of his Nobel Prize co-winner, T.W. Schultz (1961, 1979, 1981) who defines H as 'direct expenditures on education, health, and internal migration to take advantage of better job opportunities ..." as well as all manners of enhancing the quality of N – not

[7]Once built, knowledge becomes independent of its builders, like a genie that jumped out of the bottle and refuses to go back into the bottle. I remember reading somewhere that most scientists who work on the atomic bomb in the USA became anti-nuclear weapons and war pacifists after they realized the destructive power of what they accomplished. By then, however, the knowledge they built has acquired it own independent life.

just of L! (cf. Blaug, 1979, Birdsall, Fei, Kuznets, Ranis, and Schultz, 1979). Thus, H is the cumulative quality of N resulting from the time and effort spent on improving its productivity (cf. Teixeira, 2004). Just like mining land for a valuable mineral, human development is mining population for the cumulative quality attributes, which are collectively called H (cf. Amavilah, 2008). This shows that L alone, no matter how improved, is too narrow a base for H. Instead H depends at least on the quality (q) of the working-age population (N^*) – in many countries 15 to 65 years old (and rising), developed through investment in education, training, experience, health as well as socioeconomic inheritances, among many possibilities. It is a separate issue that investment itself may be of domestic source, FDI, foreign aid, and/or other transfers. If transfers are of a foreign source, a variety of H and A transfer models exist such as Javanovic and Nyarko (1997), Arrow (1969), and Leontief (1966). Thus, compared to (8),

$$H_t = e^{\varphi q} N_t^* = e^{\varphi q + nt} N_0^*, \tag{9}$$

where N_0^* is N_t^* at t = 0, L < N^* < N, and N^* is growing at n rate *over* and *in* time. Hence,

$$A = [(A_g e^{nt} N_0)][(1 + A_\tau) e^{\varphi q + nt} N_0^*], \tag{10}$$

whereas $A_g(N,n,t)$, $A_s(N^*,q,n,t)$, so that according to (8) $H=f(A_g,L,t)$, $H=f(A_s,A_\tau,q,N^*(N,t))$ in (9). This means that further study of (10) would show that A_g is a declining function of time, whereas both A_s and A_τ depend on N^*, q, t, and their common rate of growth. Again, A_g is the sum of existing knowledge and whatever efficiency gains obtain. But since A_g depreciates fast (I think at a rate reciprocal to the rate of modernization and technical progress), its level is falling even with positive learning and improved efficiency, i.e., the effect of the rate of depreciation is larger than, and of opposite sign from, the effect of the rate of learning.[8] On the other hand, $(1 + A_\tau)H$ appreciates more than it depreciates, even as we accept Arrow's (1969) contention that its expansion cannot be infinitely exponential.

The true H is Interactive and Intra-active

Both (8) and (9) do not reveal that H is inter- and intra-active although that is clearly implied by Mincer *originale*. For example, Becker's earnings Equations (76, p. 104, and 2, p. 261) indicate interactions as

---

[8]My hunch is that the rate of depreciation of folk knowledge is higher in developing economies than in already advanced economies, which partly explains the continued technological catch-up gap between the two sets of economies.

$$E^{*}=a+aE^{*}_{t-1}+\sum_{j}\bar{r}_j^{\prime}S_j+(a+v)E^{*}_{t-1}, \tag{11}$$

where E^* is earnings, $\bar{r}_j^{\prime}$ is "the adjusted average rate of return" on "total formal schooling years" (S_j), a is a technological constant interacting with E^*, experience, age, and a combination of contemporaneous random error and serial error term, $(a+v)E^{*}_{t-1}$. In addition, we know that Lucas (1988) interchanges A and H, and proxies H with years of schooling, but finds the effects of lower educational levels stronger than those of higher educational levels, which suggests an inverted-U relationship between economic growth and H. Lucas's "mechanics of development" raised more questions than it solved, and it led Lucas himself to the "making of miracle" model, which concentrated on the proportion μH of H actually spent on production as opposed to H measured as years of schooling. Thus, in the Lucas production function (LPF) framework of per worker growth rate, μH had a larger and statistically stronger impact on the economic growth of East Asian countries than conventional H, which convinced Lucas to conclude that the economic growth observed in East Asia was not a true miracle, it was a man-made miracle.

A second version of the new growth theory assumes that not only is A = H, both A and H are driven by innovations. This version is based on Paul Romer's production function (RPF), itself built upon the well-known Dixit-Stiglitz (1977, cf. Foltyn, 2012, Dingel, 2009) model by including as a key element innovations, represented by product varieties (Messinis and Ahmed, 2008). In this way A can enrich all other factors of production. This element, while clearly realistic, may be responsible for the observed instability of the RPF. For instance, whereas Dinopoulos and Thompson's (1996) implementation of the RPF confirmed that divergence is possible, and may actually be the only outcome of the endogenous growth process, remodeling the RPF Mankiw, Romer, and Weil (1992), and Barro and Sala-i-Martin (1992) found convergence, not divergence, and they concluded that the new growth theory does not work, or does not work as well as claimed.

The RPF model assumes homogenous and unchanging product varieties. Grossman and Helpman (1991), and Aghion and Howitt (1992) dropped that assumption by allowing A to improve, and to lead to the "creative destruction" among product varieties. The modification added clarity and flexibility to the RPF to capture both convergence and divergence, but it also suggested that economies are just sets of independent innovation monopolies made possible by the existence of scale effects. In an attempt to explain this situation Grossman and Helpman (1991) argued that convergence happened because one economy has access to a larger A_i than another that has A_j, i.e., $A_i > A_j$. However, Jones (1995, 1996) was not able to confirm such convergence using data for the OECD countries relative to the US data, arguing that unequal A's are unlikely because competition equalizes A's. Aghion and Howitt (2006) picked up from Jones by allowing $A_i = A_j$ transfer via trade, finance, mergers and acquitions, or other relations. Instead of convergence they found parallel long-run growth. Thus, looking at all this literature Stephen Parente (2001) is justifiably unimpressed by the performance record of the endogenous new growth theory, and concludes that the endogenous theory "... has not proven useful for understanding the most important question why the whole world is not rich" (p. 1). However, he admits that "...endognous growth may prove useful for understanding growth in world knowledge over time" (p. 1). We return to this insight later. For now, Table 1 below contrasts how the Lucas and Romer versions of the endogenous new growth theory treat A and H, and I recommend Chapters 1 and 2 of Grossman and Helpman (1991) for an excellent background discussion of these things. Other good reviews are in Islam (2004), McCallum (1996), M. Rogers (2003), and Parente (2001).[9]

In both the LPF and RPF, A and H are clearly endogenous, and the H formulations in Table 1 and elsewhere are consistent with Bils and Klenow's (2000) paper, see e.g., their Equation 4, p. 1163 (cf. Basu and Mehra, 2011). However, note that while the modeling efforts are novel and groundbreaking, the ideas about the endogeneity of knowledge are not so new. For example, according to Arrow (1969) "knowledge arises from deliberate seeking, but it also arises from observations incidental on other activities" (p. 30). In the latter case "production and investment may lead to increases in productivity without any identifiable allocation of resources to that end" (ib.) – "learning-by-doing," as Arrow (1962), Kaldor (1961, 1966), Haalvamo (1960), Fellner (1969), and many other have shown. In such a case the production or investment activity generates an output (Y) as well as knowledge (A) that influences future activities. Arrow interprets that as meaning that "deliberate ... expenditures [on A] are actual steps in the [Y] production process" (p. 30). In other words, knowledge and physical output are simultaneously created. This insight is clearly consistent with the Parente-Prescott characterization that $A = A_g[(1 + A_\tau)H]$.

Just as clearly, H interacts with other productive factors. For Sub-Saharan African countries Robin Grier (2005) estimated the marginal impact of K on H to be about 0.48 and that of H on K to be about 0.21. Earlier Grier (2002) extended the Nelson-Phelps (1966) model to Latin American data and found that educated people learn, innovate, and assimilate new knowledge easier than uneducated people. These results should not be surprising. In his piece on the implications for growth of "object gaps" and "idea gaps", Paul Romer (1993) asserts that resource interactions characterize the flow of ideas from industrialized countries to developing countries through multinational corporations.[10] There is some research on resource interactions by Philippe Aghion (2006), Chad Jones (2006), and Jones and Romer (2010). However, few of these studies explicitly incorporate interactions in their models. Among the few Graca, Jafarey, and Philippopoulos (1995), and Benhabib and Spiegel (1994, 2002) stand out. The former three modified Romer (1990) to account for the endogeneity of K and H and they found the effects of H on Y to be smaller at the low level of economic development than at the high level of development, i.e., they show that although insignificant, especially for the developing countries, H determines total factor productivity (TFP), which in turn affects economic growth significantly, and hence the continued debate over whether $\Delta H \rightarrow \Delta A \rightarrow \Delta TFP \rightarrow \Delta Y$, or $\Delta A \rightarrow \Delta H \rightarrow \Delta TFP \rightarrow \Delta Y$.

---

[9]I further recommend that the more interested reader in this area begin with Sydney Weintraub's (1977) *Modern Economic Thought*. Part VI, pp. 329-387, and follow up with all articles by growth theorists in *The Journal of Economic Perspectives*, 8(1), 1994.

[10]There is another excellent illustration in one of Paul Romer's writing where he supposes building H only through education, and asks where that process requires good students, good teachers, or good schools. Quite obviously, the interactions among the above are inevitable.

In a follow up paper these same authors demonstrate that the growth of A is a function of the growth rate of H and the diffusion rate of A from A-rich countries to A-poor countries (see Benhabib and Spiegel, 2002, Equations 2.1 - 2.4, pp. 7 - 9). The specific form of the function is logistic and they describe its microfoundations with Equation 5 (pp. 14 -19).

In all of this the interaction of H and K is defined as A, where H is generated by the time spent on accumulating knowledge and the efficiency of both H and K, see, e.g., Equation (7, p. 97) and Equation (14a, p. 100), and compare that to Caballe and Santos (1993). This understanding is consistent with Romer in that $A=f(\dot{A},k,h)$, see also M. Rogers (2003, Equation 6, p. 116).

Theo Eicher (1996) associates A and H by arguing that new technologies are skill intensive, while unskilled effective L is comfortable with past A. Norman Gemmell (1995) reformulates Solow (1960), see, e.g., Equations (15 - 19, pp. 100 -101) and finds that $A=f(A(t-1),I,L)=f(Y)$, a finding that is consistent with M. Desai's (1995, Equation 4, p.88).

Following Harris and Pan (2000, pp. 134-137), Amavilah (2005) has attempted to demonstrate that technological change can be represented as a g-variable correlation coefficient $\rho_{KHAN}=K\cap H\cap A\cap N=K\times H\times A\times N$, signifying resource interactions and intra-actions. To calculate the coefficient, Harris and Pan would let $U_K=K-\bar{K},\ U_H=H-\bar{H},\ U_A=A-\bar{A},\ U_N=N-\bar{N}$. Then on a sample of T size,

$$\rho_{KHAN}=[\sum_i^T |U|]/[(\sum_i^T |U|^g)^{1/g}],\ U=(U_K,U_H,U_A,U_N). \tag{13}$$

This means that over time $A=e^{\int\rho_{KHAN}dt}=e^{\rho_{KHAN}}+Constant$. And since we said that $A=A_g+(1+A_\tau)H$, we can set $A_g=Constant$, $\exp(\rho_{KHAN})=(1+A_\tau)H$ so that

$$A=Constant+(1+A_\tau)H=Constant+\exp(\rho_{KHAN}), \tag{14}$$

where 'Constant' may be a Lucas technological constant, or a Romer technological constant, depending on whether it is associated with LPF or RPF. The disadvantage of (14) is that we have bunched together A_s and A_τ, and have also lost the inheritance aspects of A_g. What I am intimating is not out of place; in a recent NBER paper Lucas and Moll (2011) follow-up on Lucas (2009) to propose a model similar in spirit to Romer (1993) and Lewis (1965). It envisions an economy that uses its old knowledge (including A_g) to produce goods while at the same time it interacts with other economies as it searches for productivity-enhancing knowledge, $(1+A_\tau)H$. One potential concern about this model is that either A_g does not depreciates, depreciates very slowly, or 'productivity ideas' are found and brought to fruition rapidly to prevent a similarly rapid decline in production.

The true H is a Volume - Not an Area

Currently Mincerian H is measured predominantly as an area under the curve of either the quantity of the skills or the qualit of the skills of L. For instance, representing H by years of schooling is an example of the quantity measure of H, and using cognitive skills is an example of the quality dimension of H. Often such estimations are justified on the basis of the lack of data. However, they fall short of precission, and they are not consistent with the common understanding of knowledge. In real life knowledge is a 3D "solid"– at least. For example, knowledge has depth, and we frequently

Table 1 - Lucas versus Romer Versions of Endogenous New Growth Model

Model	Production Function	Knowledge ($A=A_g+A_s$)	Human Capital (H)
Lucas	$Y=L^{\alpha}K^{\beta}(\mu H)^{1-\alpha-\beta}$	$A = \frac{Y}{L^{\alpha}K^{\beta}(\mu H)^{1-\alpha-\beta}}$	$H = \frac{1}{\mu}(\frac{Y}{AL^{\alpha}K^{\beta}})^{1/1-\alpha-\beta}$
Romer	$Y=L^{\alpha}H^{\beta}\eta\bar{x}^{1-\alpha-\beta}$	$A = \frac{Y}{L^{\alpha}H^{\beta}\eta\bar{x}^{1-\alpha-\beta}}$	$H = (\frac{Y}{AL^{\alpha}\eta\bar{x}^{1-\alpha-\beta}})^{1/\beta}$

speak of deep or shallow knowledge. We also speak of broad, wide, narrow, as well as of intimate knowledge.[11] *A person has wide/broad knowledge if he or she knows many subjects/topics. A person has deep knowledge if he or she understands well what he or she knows.* The latter is best achieved if one specializes, but specialization is only a necessary, not enough, requirement for understanding as in real life we often hear of the existence of polymaths. M.M. Lehman (1978) has put what I have in mind here cogently that for there to be progress, "each quantum of knowledge that is gained becomes the basis for further learning or other activities as required by society. [But] knowledge alone will not suffice to ensure the survival of humanity, [because] new ideas must be produced faster than the older ones may be forgotten. Understanding ... ensures that there will always exist [knowledge]"

[11]Intimate knowledge is likely of limited economic value, because "... knowledge does not grow rapidly if it is kept as secret for the few" (W. Arthur Lewis, 1965, p. 167).

(pp. 65 and 68).[12] In other words, understanding is knowing; knowing is not necessarily understanding.

These distinctions are no mere talk. The quantity and quality of knowledge evolve *over* and *in* temporal and causal time. Over time the quantity and quality of knowledge evolve semi-independently. *In time* there exists a negative correlation between the quantity and quality dimensions of H. *Estimating H as an area over- states the importance of H in production over time and under-states its role in other related spheres in time.*

Let me put one more stake in the ground in the way of recapping my claim: H is a volume, and the idea of H as a volume is natural, commonsensical, and conceptually easy to illustrate. First, we can integrate either the quantity, or quality of N^* over time to get H, that is normal practice (cf. Arrow, 1962), even though Nordhaus (2009) has cautioned against the "perils" of cumulative variables. Second, we acknowledge the important relationship between the quantity and quality of H both *in* and *over* time. Becker (1981) and others, for instance, have observed a negative correlation between the quantity and quality of the population. Fundamentally, such an observation is not surprising and can be explained easily as follows. Building H is like developing land (e.g., mineral). One explores for the mineral, finds it, grades it, mines it up, processes it, and finally uses it. Each step has both marginal benefits and marginal costs, with the best choice achieved when the marginal costs and marginal benefits are in balance. Developing H is the same only instead of land one has a human population. Also in both cases one starts with a stock of inherited knowledge. We have knowledge inheritance of varying degrees of quantity and quality. Thus, H building is really mining the fraction N^* of N for quality (q), and that quality accumulates into H – not unlike cumulating gold dust into gold bars. This perspective is consistent with early theories of H like those of Schultz (1961), Blaug (1979), Becker (1993), and others, which suggest that the basis for H is population – not L. Again, H as a function of L overestimates the importance of H in production *over time*, and underestimates its role in H creation and diffusion *in time* (cf. Barro and Lee, 2013, Barro, 1991). In fact, this may be the reason why H is found to be low in developing countries where L is a tiny fraction of N^*. Obviously, just because a large part of L in developing countries is unemployed, should notnecessarily mean H is low. It is possible, indeed likely, that H accumulation proceeds even under conditions of high unemployment, although its rate may slow down under such conditions.

Another advantage of the analogy between mining population and mining land is that it makes it possible for us to utilize quantity-quality models familiar to mining and geological engineering as well as mineral economics. The best reference for these types of models which I am personally familiar with is DeVerle P. Harris (1976, 1984, 1985, 1992a, b, 1993). Among this group of models, the Lasky (1950) model suggests that over time the log of cumulative H depends on the average grade (quality) of the economically active population (N^*), i.e.,

$$\log H = \varphi q + nt + \log N_0^* \Rightarrow q = \frac{1}{\varphi}\log H - \frac{n}{\varphi}t - \frac{1}{\varphi}\log N^*. \qquad (15)$$

---

[12][] added.

Eq. (15) = (9). "So, what?" – one might ask. Here is the answer. Let us visualize knowledge in its essential 3Ds. On the y-axis we have the quantity of H, call it X. On the x-axis we have time, *t*, so that $X = f(N(N^*(t))) = f(t) = X(t)$. On the third dimension we have the quality of H, designate it as Z, which is also a function of time, i.e., $Z = f(N(N^*(t))) = f(t)= Z(t)$. *X(t) and Z(t) are related via $N(N^*(t)$ and evolve together over a common time.* Now, also let us assume that a unit (or any other) circle limits the probable universe of knowledge according to Arrow's idea that the growth of H is not eternal. This assumption clarifies the conventional information theory which says that four key elements explain the diffusion process: the innovation, the channels of communication, the social system involved, and the time over and in which the diffusion process takes place (Rogers, 2003). Young (2004, 2005, 2007), and many others illustrate that the four elements are both constraints and prospects.[13] The channels of communication, for example, depend on the capacity of the human mind for which memory is limited and learning is constrained by technical as well as natural processes like forgetting – the Ebbinghaus effect (Amavilah, 2003). As Arrow (1969) articulates "natural memory can of course be supplemented [nowadays enormously] with articial aids – books, files, computer memories" (p. 32). However, these aids too have different capacities in terms of speed and bandwith as well as associated costs and benefits (Jovanovic and Nyarko, 1995). Even so, we can still state that how the world economy has fared is a function of the stock and spread (transmission) of knowledge across the world population. Amavilah (2007, 2008) argues that the rate of knowledge transfer is a random phenomenon, but according to Arrow (ib) it is determined by what adopters (the sink) expect to gain from it (diffusion) and the reliability of both the source, and message. This suggests that "eternal exponential technological growth is just as unreasonable as eternal exponential population growth" (Arrow, 1969, p. 34, cf. Andersen and Jensen, 2013). The circle itself is expandable and within it an economy accumulates X(t) and Z(t) over time, but the circle still constrains the logistic growth of X(t) and Z(t).

Arrow's perspective is consistent with Mansfield (1971) who identifies four determinants of diffusion rate: (a) economic superiority of new versus old innovations; (b) low uncertainty costs of the new innovation; (c) level and intensity of commitment to the innovation (irreversibility of investment); and (d) the productivity of the new innovation relative to the old (see, p. 88). In this understanding Mansfield distinguishes technology from technological change. Whereas technology is a "pool of knowledge regarding the industrial arts" (p.10), technological change is mainly a function of resources, and the demand and supply of output those resources produce – in the economist language, the demand for resources is derived demand (cf. Kamien and Schwartz, 1977, Berdt and Wood, 1975). Market structures, legal setting, and social attitudes towards, technology, technology workers, management, all these influence the rate of technological change as both a spill-over and a "spill-in" (Buchanan, 1979)

Not only does H grow logistically and subject to the capacity of the circle *over time*, *in time* there also exists an inverse relationship between X(t) and Z(t). Again, using the mining analogy, the richer the deposit, the smaller the quantity of ore is needed; the lower the quality of the population, the more people it takes to ascertain a target quality of the population, and hence H. These notions are not brand

[13] I read the working paper versions of Young's papers, and I advise the reader to find out if newer, perhaps cleaner, versions of the same papers were published since then.

new; at the aggregate level economists have long accepted the idea that the "general ability is something like normally distributed in the total population" (G.L. Bach, 1960, p. 527). However, that is different from saying that the higher the average quality (q) of N^*, the smaller the cumulative quantity of N^* is needed for economic activity. The functional form of the inverse relationship can be either quadratic, linear (Lasky), or power. For now let us say it is of the general form $Z(t)=f(X(t),t)\equiv X(t)=f(Z(t),t)$.

If the details above are accepted, then H is a volume rather than an area. To be able to compute that volume, we assume that X(t) and Z(t) follow logistic growth curves, each with an upper limit $K_x = K_z = 1$, imposed by the circle. Some fraction of H would be inherited (folk = ordinary) quantity of A_g, some would be inherited quality of A_g. Conventional theory of H would only measure H either as an area AX(t), or an area AZ(t), i.e.,[14]

$$\begin{aligned} H=AX(t)=\pi X(t)^2=\pi[f(t)]^2=\frac{\pi L_x^2 e^{2c_1 t}}{(a_1 e^{c_1 t}+b_1)^2} \quad (a) \\ H=AZ(t)=\pi Z(t)^2=\pi[f(t)]^2=\frac{\pi L_z^2 e^{2c_2 t}}{(a_2 e^{c_2 t}+b_2)^2}, \quad (b) \end{aligned} \qquad (16)$$

where $0<a_i<1$, if L = 1 (unit circle), and $a_i=1$ if L > 1, and $b_i\neq 0$.

Again, according to (16) H is just an area either under X(t) or Z(t), *which ignores that these two dimensions of H are inseparable; you can have more of one or the other, but not separately.* In other words, the stance of (16) is incorrect. As a volume H:

$$\begin{aligned} H=VX(t)=\int_t \pi X(t)^2 dt=\int_t \pi[f(t)]^2 dt=\int_t \frac{\pi L_x^2 e^{2c_1 t}}{(a_1 e^{c_1 t}+b_1)^2}dt \quad (a) \\ H=VZ(t)=\int_t \pi Z(t)^2 dt=\int_t \pi[f(t)]^2 dt=\int_t \frac{\pi L_z^2 e^{2c_2 t}}{(a_2 e^{c_2 t}+b_2)^2}dt \quad (b) \end{aligned} \qquad (17)$$

While (17) is a correct representation of H, it remains *incomplete*, because the true H is the sum of volumes V(X(t)) and V(Z(t)) plus/minus the volume of the area of intersection between the two, if any exists, i.e.,

---

[14] The specific functional forms themselves may not be identical, i.e., one may be a simple logistic such as the Pearl-Reed, others may be Gompertz, or any other in the same family of exponential functions, see, e.g., D.W. Thompson (1992[1942]).

$$H=\int_t \pi\left(\frac{L_x}{(a_1+b_1e^{-c_1t}}\right)^2 dt+\int_t \pi\left(\frac{L_z}{(a_2+b_2e^{-c_2t}}\right)^2 dt \pm \int_t \pi\left[\frac{L_x(a_2+b_2e^{-c_2t})-L_z(a_1+b_1e^{-ct})}{(a_1+b_1e^{-ct})(a_2+b_2e^{-c_2t})}\right]^2 dt \tag{18}$$

$$=VX(t)+VZ(t)\pm[VX(t)\cap VZ(t)],\ L_x=L_z=1.$$

Eq. (18) expresses fully the quantity and quality dimensions of H *over time*. Still missing from (18) are the interactions between X(t) and Z(t) *in time*. These are crucial considerations because the *quantity and quality of knowledge evolve semi-independently over time, but interdependently in time*. Using the Lasky set-up, in which $Z(X(t))=1-kX(t)=1-k/(a_1+b_1e^{-c_1t})=(a_1+b_1e^{-c_1t}-k)/(a_1+b_1e^{-c_1t})$, and assuming the interaction between X(t) and Z(t), $XZ=ZX$ exists, we can find the volume of the area under Z(X(t)) = 1- kX(t) as $VZ(X(t))=\int_t \frac{\pi}{3}(a_1+b_1e^{-ct}-k)/(a_1+b_1e^{c_1t}))^2dt$, and (18) becomes

$$H=\int_t \pi\left(\frac{L_x}{a_1+b_1e^{-c_1t}}\right)^2 dt+\int_t \pi\left(\frac{L_z}{a_2+b_2e^{-c_2t}}\right)^2 dt+\int_t \frac{\pi}{3}\left(\frac{a_1+b_1e^{-ct}-k}{a_1+b_1e^{-c_1t}}\right)^2 dt$$

$$\pm\int_t \pi\left[\frac{L_x(a_2+b_2e^{-c_2t})-L_z(a_1+b_1e^{-c_1t})}{(a_1+b_1e^{-c_1t})(a_2+b_2e^{-c_2t})}\right]^2 dt \tag{19}$$

$$=VX(t)+VZ(t)+VZ(X(t))\pm[VX(t)\cap VZ(t)].$$

However, (19) assumes that there is no intersection between the V(X(t)), V(Z(t)), and V(Z(X(t)). In fact, that cannot be given $XZ=ZX>0$! If so, then (19) is really

$$H=\int_t \pi\left(\frac{L_x}{(a_1+b_1e^{-c_1t}}\right)^2 dt+\int_t \pi\left(\frac{L_z}{a_2+b_2e^{-c_2t}}\right)^2 dt+\int_t \frac{\pi}{3}\left(\frac{a_1+b_1e^{-ct}-k}{a_1+b_1e^{-c_1t}}\right)^2 dt$$

$$\pm\int_t 3\pi\left[\frac{L_x(a_2+b_2e^{-c_2t})+L_z(a_1+b_1e^{-c_1t})+(a_1+b_1e^{-c_1t}-k)(a_2+b_2e^{-c_2t})}{3(a_1+b_1e^{-c_1t})(a_2+b_2e^{-c_2t})}\right]^2 dt \tag{20}$$

$$=VX(t)+VZ(t)+VZ(X(t))\pm[VX(t)\cap VZ(t)\cap VZ(X(t))].$$

In other words, the precise nature and magnitude of H depend on the volumes of its components, their interactions and intersections (overlaps), functional forms, and whether such forms are independent, identical, and/or symmetrical. Such things are conceptually easier to imagine than to operationalize both mathematically and economically. At least mathematically thinking, accounting for the interactions and intersections is simply subtracting the sum of the volumes of parts to obtain the net volume. Economically thinking, it is a little harder than that because interactions and intersections are important and it is quite reasonable for the whole volume to be greater or smaller than the sum of its parts. However, assuming $L_x = L_z = 1$, we are on a firmer and more commonsensical ground than before to estimate H as the following approximate solution to (20):

$$H=A^*+[\frac{\pi((a_1e^{c_1t}+b_1)\log(a_1e^{c_1t}+b_1)+b_1)}{a_1^2c_1(a_1e^{c_1t}+b_1)}] + [\frac{\pi((a_2e^{c_2t}+b_2)\log(a_2e^{c_2t}+b_2)+b_2)}{a_2^2c_2(a_2e^{c_2t}+b_2)}] + [\frac{\pi}{3}(\frac{b_1k^2}{a_1^2c_1(a_1e^{c_1t}+b_1)}+\frac{k(k-2a_1)\log(a_1e^{c_1t}+b_1)+t)}{a_1^2c_1})]. \quad (21)$$

"Ha! - where is the answer to the last term of (20)?" – one may ask. The answer is that, whereas X(t) and Z(t) evolve *over time*, the relationship between X(t) and Z(t) takes place *in time*. In that case the last two terms of (20) are constants and as such parts of A^*. So, A^* has three components: (i) the inherited quantity of H, (ii) the inherited quality of H, and (iii) the relevant interactions and intersections between the two.

Concluding remarks

Traditional theory measures H as either cumulative quantity of skills (16a), or as cumulative quality of skills (16b), assuming that X(t) and Z(t) evolve independently *over time*, i.e., H is the area either under the X(t) or Z(t) curve. However, it is more natural and commonsensical to think of H as knowledge. As knowledge H is at least 3D, and it must be measured as a volume, with depth (quality = Z(t)) and width (quantity = X(t)), both evolving *over time*, and interacting as (Z(t)X((t) *in time*. Conventional theory mis-measures H, and hence its effects on both production and innovation activities. Again, this is no brand new idea: in common speech we speak of wide or broad knowledge; we talk of deep knowledge, and of intimate knowledge.

This paper first examines some papers on knowledge using Parente-Prescott "world knowledge' as a template. From this examination it is clear that knowledge has two parts: ordinary knowledge, which depreciates with time, and "extraordinary" knowledge, generated from the laws of motion of science and technology. Both science and technology depend on H. In the final analysis functional world knowledge is H.

I found that the concept of H is not new – indeed, it is old, very old. People have always been interested in valuing human life, and initially they used variables like social status, race, gender, and age to estimate it. Many of these variables are still being used today. What did not exist for a long time, was the term H itself, and this was mainly because of socio-political inhibitions not to liken human beings to machines (man is an image of God), and, in Marxian philosophy, likening workers to exploitative capital for that matter was just unacceptable – capitalists are "squanders of labor-power". Sr W. Arthur Lewis (1954) reveals that A.C. Pigou (1928) was first to openly pronounce that: "There is such a thing as investment in H as well as investment in material capital. So soon as this is recognized, the distinction between economy in consumption and economy in investment becomes blurred. For, up to a point, consumption is investment in personal productive capacity. This is especially important in connection with children: to reduce unduly expenditure on their consumption may greatly lower their efficiency in after-life. Even for adults, after we have descended a certain distance along the scale of wealth, so that we are beyond the region of luxuries and 'unnecessary' comforts, a check to personal consumption is also a check to investment" (p. 29). And so, once economists made peace with the term, they took and ran with it.

However, soon after Jacob Mincer developed a practical formula for measuring H, practitioners separated Mincer from the originators of the H theory. The benefit was that a huge volume of work was carried out under the banner of Mincerian H; the cost is that such work formalized a misconception on three fronts. The first has based human capital only on labor. The second, is measuring H as an area, neglecting that common understanding as clearly evident from common language that knowledge is 3D, with depth, width, and the *time over and in* which it accumulates. The third relates to neglected factor interactions (and intra-actions). Analysis finds that disregarding interactions our commonsensical measure of H is larger than conventional Mincerian measures of H. Taking interactions into account, it is possible for our measure to be larger, smaller, or equal to conventional measures. If it falls short and accomplished nothing else, I hope this paper re-ignites a new look at how economists measure H.

References[15]

Agarwal, R, R Echambadi, AM Franco, and MB Sarkar 2004 Knowledge transfer through inheritance spin-out generation, development, and survival. *Academy of Management Journal* 47(4), 501-522.

Aghion, P. and P. Howitt 2006 Joseph Schumpeter lecture: appropriate growth policy: a unifying framework, *Journal of European Economic Association* 4(2-3): 269-314.

[15]This list of references is no particular style or format. It may also be incomplete in some areas or over-complete in others, because it is culled from my larger working literature. If anyone's idea is excluded that must be included, or included that must be excluded, my sincere apologies along with all due acknowledgment and the usual caveat.

Aghion, P. and P. Howitt 1992 A model of growth through creative destruction, *Econometrica* 60 (2): 323-351.

Akerlof, G. 1970 The market for "lemons": Quality uncertainty and the mechanism, *Quarterly Journal of Economics*, 84(3): 488-500.

Allen, RGD 1968 *Macroeconomic Theory: A Mathematical Treatment*. New York: St Martin's Press.. Chapter 13.

Amavilah, VH 2003 The Ebbinghaus effect and the implications of net learning for the performance of production systems. http://128.118.178.162/eps/exp/papers/0307/0307002.pdf.

Amavilah, VH 2008 The inhibited (exhibited) spread of innovations. http://mpra.ub.uni-muenchen.de/8993/1/MPRA_paper_8993.pdf.

Amavilah, VH 2007 Innovations spread more like wildfires than like infections. http://mpra.ub.uni-muenchen.de/3958/1/MPRA_paper_3958.pdf.

Amavilah, VH 2008 *Modeling Determinants of Embedded Economies*. New York: Nova Science Publishers.

Amavilah, VH 2005 Resource intra-actions and interactions: Implications for technological change and economic growth. http://129.3.20.41/eps/ge/papers/0508/0508004.pdf.

Amavilah, VH 2009 Knowledge of African countries: Production and value of doctoral dissertations, *Applied Economics* 41(7-9): 977-989.

Amavilah, Voxi Heinrich and Newcomb, Richard T. 2004 Economic Growth and the Financial Economics of Capital Accumulation under Shifting Technological Change: http://ideas.repec.org/p/wpa/wuwpge/0404001.html.

Amavilah, VH 1998 Resource Inefficiency and Poor Aggregate Economic Performance in African Countries: The Case of Namibia, 1968-1992. Working Paper No. 12. . http://econowpa.wustl.edu/eps/ge/papers/0307/0307001.pdf.

Amavilah, VH 1996 *Resources, Technology, and Mineral Trade in the Economic Growth* *of Namibia.* Ann Arbor: UMI (PhD Dissertation).

Andersen, TB and PS Jensen 2013 Institutions and growth acceleration. Discussion Papers on Business and Economics No. 7/2013.

Anand, S. and AK Sen 1994 Human Development Index: methodology and Measurement. UNDP.

Arrow, KJ 1962 The economic implications of learning by doing, *Review of Economic Studies* 29(June):155-173.

Arrow, KJ 1969 Classificatory notes on the production and transmission of technological knowledge, *American Economic Review* 59(2): 29-35.

Arrow, KJ 1974 Higher education as filter, *Journal of Public Economics* 2: 193-216.

Bach, GL 1960 *Economics: An Introduction to Analysis and Policy*, Third Edition. Englewood Cliffs (NJ): Prentice-Hall, Inc. Chapter 10.

Banerjee, R. 2011 Population growth and endogenous technological change: Australian economic growth in the long run. MPRA Paper No. 30892 (March). http://mpra.ub.uni-muenchen.de/30892/.

Barro, R. 1991 Economic growth in a cross-section of countries, *Quarterly Journal of Economics*, CVI(2): 407-443.

Barro, R and J. Lee 2013 A new data set of educational attainment in the world, 1950-2010. NBER Working Paper 15902 (April). http://www.nber.org/papers/w15902

Barro, R and Sala-i-Martin 1992 Convergence, *Journal of Political Economy* 100(2): 223-51.

Basu, S. and MK Mehra 2011 Endogenous human capital formation, distance to frontier and growth (July 14). http://mpra.ub.uni-muenchen.de/37701/1/MPRA_paper_37701.pdf.

Becker, GS., EL. Glaeser and KM. Murphy 1999 Population and economic growth, *American Economic Review, Papers and Proceedings* (May):145-149.

Becker, GS 1993 *Human Capital: A Theoretical and Empirical Analysis with Special Reference to*

Education, Third Edition. Chicago: University of Chicago Press.

Becker, G. 1992. Habits, addictions and traditions, *Kyklos* 45: 327-345.

Becker, GS 1981 *A Treatise on the Family*. Cambridge: Havard Univesity Press. Chapter 6.

Benhabib, J and M. Spiegel 2002 Human capital and technology diffusion. FRBSF Working Paper #2003-02. http://www.frbsf.org/economic-research/files/wp03-02bk.pdf.

Benhabib, J. and M. Spiegel 1994 The role of human capital in economic development: Evidence from aggregate cross country data, *Journal of Monetary Economics* 34(2): 143-173.

Berndt, ER., and DO. Wood 1975 Technology, prices, and the derived demand for energy, *Review of Economics and Statistics*, LVII (3): 259-268.

Bils, M and PJ Klenow 2000 Does schooling cause growth? *American Economic Review* 90(5): 1160-1183.

Birdsall,N., J. Fei, S. Kuznets, G. Ranis, TW Schultz 1979 Demography and development in the 1980s, In *World population and Development: Challenges and Prospects*. Philip M. Hauser (Editor). New York: Syracuse University Press, pp. 211-295.

Blaug, M 1976 The empirical status of human capital theory: A slightly jaundiced survey, *Journal of Economic Literature* 14(3): 827-855.

Blaug, M 1979 The quality population in developing countries, with particular referecne to education and training, In *World population and Development: Challenges and Prospects*. Philip M. Hauser (Editor). New York: Syracuse University Press, pp. 361-402

Bowles, S and H. Gintis 2002 The inheritance of inequality, *Journal of Economic perspectives* 16(3): 3-30.

Bowles, S. and H Gintis 2000 The inheritance of economic status: Education, class and genetics. http://www.santafe.edu/media/workingpapers/01-01-005.pdf.

Bowles, S and H Gintis 1975 The problem of human capital theory – A Marxian crique, *American Economic Review* 65(2): 74-82.

Buchanan, JM 1979 *What Should Economists Do?* Indianapolis: Liberty Fund.

Bury, JB (1960[1932])*The Idea of Progress: An Inquiry into Its Growth and Origin.* New York: Dover Publications, Inc.

Caballe, J. And MS Santos 1993 Endogenous growth with physical capital and human capital, Journal of Political Economy 101(6): 1042-1067.

Capaldi, N. 1969 *Human Knowledge: A philosophical analysis of its meaning and scope.* New York: Pegasus.

Choi, K 1983 *Theories of Comparative Economic Growth.* Ames: Iowa State University. Chapters 3 and, pp. 62-119.

Cohn, E 1979 *The Economics of Education*, Revised Edition. Cambridge: Ballinger Publishing Company.

Coleman, J 1990 *Foundations of Social Theory.* Cambridge: Belknap of Harvard University Press.

De Gregori, TR 1987 Resources are not, they become, *Journal of Economic Issues* 21(3): 1241ff.

Desai, M 1995 An endogenous growth cycle with vintage capital, *Economics of Planning* 28(2-3): 87-91.

Dinopoulos, E., and P Thompson 1996 A contribution to the empirics of endogenous growth, *Eastern Economic Journal* 22(4): 389-400.

Dingel, JI 2009 The basics of "Dixit-Stiglitz lite" . http://www.columbia.edu/~jid2106/td/dixitstiglitzbasics.pdf.

Dixit, A and JE Stiglitz 1977 Monopolistic competition and optimum product variety, *American Economic Review* 67(3): 297-308.

Dun, C (2003) International earningd mobility in Brazil and its determinants. http://www.econ.yale.edu/conference/neudc03/papers/4d-dunn.pdf.

Ede, A and LB Cormack, 2004 *A History of Science in Society: From Philosophy to Utility*, Volume 1. New York: Broadview Press.

Eicher, T 1996 Interaction between endogenous human capital and technological change, *Review of Economic Studies* 63: 127-144.

Easterlin, RA 1977 Population: The economics of the long run, in *Modern Economic Thought*, Sidney Weintraub (ed). University of Pennsylvania Press. pp. 485-499.

Fellner, W 1969 Specific interpretations of learning by doing, *Journal of Economic Theory* 1(2): 119-140.

Fisher, I 1897 Senses of "capital," *Economic Journal* (26): 199-213.

Foltyn, R 2012 Results from the Dixit/Stiglitz monopolistic competition model. http://www.foltyn.net/wp-content/uploads/2009/12/dixitstiglitz.pdf. .

Foray, D 2004 *The Economics of knowledge*. Cambridge: MIT Press.

Galor, O., and D. Weil 2000 Population, technology, and growth: From the Malthusian regime to the demographic transition and beyond, *American Economic Review* 90(4): 806-828.

Gemmell, N. 1995 Endogenous growth, the Solow model and human capital, *Economics of Planning* 28(2-3): 169-183.

Graca, J., S. Jafaray, and A. Philippopoulos, 1995 Interaction of human and physical capital in a model of endogenous growth, *Economics Planning* 28(2-3): 93.-118.

Grier, R 2005 The interaction of human and physical capital accumulation: Evidence from Sub-Saharan Africa, *Kyklos* 58(2): 195-211.

Grier, R 2002 On the interaction of human and physical capital in Latin America, *Economic Development and Cultural Change* 50(4): 891-913.

Grossman, G and E Helpman 1991 *Innovation and growth in global economy.* Cambridge (MA): MIT Press.

Haavelmo, T 1960 *A study in the theory of investment.* Chicago: University of Chicago Press.

Hall, RE and CI Jones 1999 Why do some countries produce so much more output per worker than others? *Quarterly Journal of Economics* 114(1): 83-116.

Hanushek, EA and DD. Kimko 2000 Schooling, labor-force quality, and the growth of Nations, *American Economic Review* 90(5): 1184-1208.

Hanushek, E and L. Woessmann 2008 The role of cognitive skills in economic development, *Journal of Economic Literature* 46(3): 607-668.

Harris, DP 1992a Long-term forecasting for mineral industries: Innovative methods and case studies, Unpublished Lecture Notes, Mineral Economics Program, Department of Mining and Geological Engineering, University of Arizona (January).

Harris, DP 1984 *Mineral Resource Appraisal.* Oxford: Clarendon press.

Harris, DP 1992b Technology, Productivity, and Learning. Unpublished Lectures. Tucson AZ:University of Arizona.

Harris, DP 1993 Mineral resource stocks information in Allen V. Kneese and Jones L. Sweeny (Eds) *Handbook of Natural Resource and Energy Economics*, Volume III, North Holland.

Harris, DP 1976 Quantitative Economic Analysis Applied to the Mineral Industries: Computer Applications. In *Economics of the Mineral Industries*, 3rd Edition, William A. Vogel (Ed.): New York: American Institute of Mining, Metallurgical, and Petroleum Engineers, Inc.: 345-360.

Harris, DP 1985 Mineral Resource Information, Supply, and Policy Analysis. In *Economics of the Mineral Industries*, 4rd Edition, William A. Vogel (Ed.). New York: American Institute of Mining, Metallurgical, and Petroleum Engineers, Inc.: 181-224.

Harris, DP and G Pan 2000 *Information Synthesis for Mineral Exploration*. New York: Oxford University Press.

Hayek, FA 1945 The use of knowledge in society, *American Economic Review*, 35, 4, pp. 519-530.

Hayek, FA1974) The pretense of knowledge. Nobel Prize Lecture (December 11). http://www.nobelprize.org/nobel_prizes/economics/laureates/1974/hayek-lecture.html.

Hayek, FA 1937 Economics and Knowledge, *Economica*, 4(February): 386-405.

Hicks, JR 1963[1932] *The Theory of Wages*, Second Edition. New York: St. Martin's Press. Chapter 13.

Hotelling, H 1931 The economics of exhaustible resources, *Journal of Political Economy* 39 (?): 137-175.

Islam, N. 2004 New growth theories: What is in there for developing countries? *Journal of Developing Areas* 38(1): 171-210.

Javanovic, B. and Y. Nyarko 1997 The transfer of human capital, *Journal of Economic Dynamics and Control* 19: 1033-1064.

Johnson, EAJ 1965 *Predecers of Adam Smith: The Growth of British Economic Thought*. Reprints of Economic Classics. New York: Augustus M. Kelley. Chapters XII, XIII, XIV, and XV.

Jones, CI 2006 Knowledge and the theory of economic development, Version 0.25. http://www.chicagobooth.edu/research/workshops/AppliedEcon/archive/WebArchive20052006/ChadJonesPaperApril19Spring06.pdf.

Jones, CI 2004 Population and ideas: A theory of endogenous growth. NBER Working Paper No. 6285. http://www.nber.org/papers/w6285population. See a recent version at http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.61.4939&rep=rep1&type=pdf.

Jones, CI and PM Romer 2010 The new Kaldor facts: Ideas, institutions, population, and human capital, *American Economics Journal: Macroeconomics* 2(1): 224-245.

Jones, CI 1997 Convergence revisited, *Journal of Economic Growth* 2(June): 131-153.

Jones, CI 1995 R&D-based models of economic growth, *Journal of Political Economy*, 103(4): 759-84.

Kaldor, N. and J. Mirrlees 1962 A new model of economic growth, *Review of Economic Studies*, 39(2):174-192.

Kaldor, N.1961 Capital accumulation and economic growth. In *The Theory of Capital*, F.A. Lutz and D.C. Hague (Eds.) New York: St. Martin's Press.

Kaldor, N. 1966 *Causes of the Slow Rate of Growth in the United Kingdom: An Inaugural Lecture*. London: Cambridge University Press.

Kalecki, M 1971 *Selected Essays on the Dynamics of the Capitalist Economy, 1933-1970*.Cambridge: Cambridge University Press. Chapter 10.

Kamien, MI and NL Schwartz 1977 Technology: More or less in *Modern Economic Thought*, Sidney Weintraub (ed). University of Pennsylvania Press. pp. 501-515.

Keynes, JM 1936 *The general theory of employment, interest, and money.* New York: Harcourt, Brace and Company.

Kiker, BF 1966 Historical roots of the concept of human capital, *Journal of Political Economy* 74(October): 481-499.

Kiker, BF 1967 The concept of human capital in the history of economic thought, *Indian Economic Journal* 14 (March): 481-499.

Kiker, BF 1968a *Human Capital: In Retrospect*. Columbia: Bureau of Business and Economic Research, University of South Carolina.

Kiker, BF 1968b Marshall on human capital: Comment, *Journal of Political Economy* 76 (Septmber-October): 1088-1090.

Kiker, BF 1969 von Thunen on human capital, *Oxford Economic Papers* 21 (November): 341-343.

Kiker, BF (ed) 1971 *Investment in Human Capital.* Columbia: University of South Carolina Press.

Kiker, BF 1974 Nicholson on human capital, *Scottish Journal of Political Economy* 21 (June): 171-176.

Kosobud, RF. and WD. O'Neil 1974 A Growth Model with Population endogenous, *American Economic Review*, (May): 27-32.

Lasky, SG 1950 How Tonnage and Grade Relations Help Predict Ore Reserves, *Engineering and Mining Journal*, 151, 81-85.

Lehman, MM 1978 Programs, cities, students – Limits to Growt? In David Gries (ed) *Programming Methodology*. New York: Springer Verlag. Chapter 5 (especially pp. 50-69).

Leontief, W. 1966 *Essays in Economics: Theories and Theorizing*. New York: Oxford University Press. Chapter XVII: 200-222.

Lewis, WA 1979 The slowing down of the engine of growth. http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/1979/lewis-lecture.html.

Lewis, WA 1965 *The Theory of Economic Growth*. New York: Harper Torchbooks.

Lewis, W. Arthur (1954) Economic development with unlimited surplus of labor, *Manchester School Economic and Social Studies* 22(2): 139-192.

Lucas Jr., R 1993 Making a Miracle, *Econometrica* 61(2) 251-272

Lucas Jr., R 1988 On the mechanics of economic development, *Journal of Monetary Economics* 22(1) 2-42.

Lucas, RE 2009 Ideas and growth, *Economica* 76(301): 1-19.

Lucas, RE and .B. Moll 2011 Knowledge growth and the allocation of time. NBER Working Paper No. 17495 (October).

Mankiw, NG, D Romer and D Weil 1992 A contribution to the emperics of economic growth, *Quarterly Journal of Economics* 107(2): 407-437.

Mansfield, E 1971 *Technological change*. New York: W.W. Norton & Company Inc.

Marshall, A 1961 *Principles of Economics*, 9th Edition. E.W. Guillebaund (ed). London: Macmillan.

Marx, K 1859 The emancipation question. *New York Daily Tribune*, July 17 and July 22.

Marx, and F. Engels 1848 *The Communist Manifesto.* New York: Vintage (Classics) Books.

Marx, K 1906 *Capital: A Critique of Political Economy.* New York: Modern Library.

McCallum, BT 1996 Neoclassical vs. endogenous growth analysis: An overview, *Federal Reserve Bank of Richmond Economic Quarterly*, 82($): 41-71.

Messinis, G., and AD Ahmed 2008 Valuable skills, human capital and technology diffusion. Working Paper No. 38. Centre for Strategic Economic Studies, Victoria University, Melbourne (April). http://www.cfses.com/documents/wp38.pdf.

Mill, JS 1909 *Principles of Political Economy.* W.J. Ashley (ed). New York: Longmans, Green, and Company.

Mincer, J 1981 Human capital and economic growth. NBER Working Paper No. 803. Cambridge: National Bureau of Economic Research.

Mincer, J 1974 *Schooling, Experience, and Earnings*. New York: Columbia University Press.

Mincer, J 1958 Investment in human capital and personal income distribution, *Journal of Political Economy* 66(4): 281-302.

Morishima, M and M Saito 1968 An economic test of Sir John Hicks's theory of biased induced inventions, In Value, Capital and Growth, Papers in Honour Sir John Hicks, JN Wolfe (ed).Chicago: Aldine Publishing Company for Edinburgh University Press. Chapter 17.

Nelson, R and ES Phelps 1966 Investments in humans, technological diffusion, and economic growth, *American Economic Review* 56 (1/2): 69-75.

Nelson, R 2005 *Technology, Institutions and Economic Growth.* Cambridge: Harvard University Press.

Newcomb, RT 1976 Modeling growth and change in the American coal industry, Growth and Change, 10(1):111-127.

Nordhaus, W 2009 The perils of the learning model for modeling endogenous technological change. Cowles Foundation Discussion Paper No. 1685 (January). http://cowles.econ.yale.edu/P/cd/d16b/d1685.pdf.

Okita, S., T. Kuroda, M. Yasukawa, Y. Yoichi, and K. Iio 1979 Population and development: The Japanese experience, In *World population and Development: Challenges and Prospects*. Philip M. Hauser (Editor). New York: Syracuse University Press, pp. 296-338.

Parente, S. and E. Prescott 1994 Barriers to technology adoption and development, *Journal of Political Economy* 102: 298-321.

Parente, S. 2001 The failure of endogenous growth, *Knowledge, Technology, and Policy* 13(4): 49-58.

Peretto, P. 1998 Technological change and population growth, *Journal of Economic Growth* 3(4): 283-311.

Petty, W 1971 *The Political Economy of Ireland.* Irish University Press.

Phelps, E. 1964 Models of technical progress and the golden rule of research. Cowles Foundation Discussion Paper No. 176. Yale University. https://www.researchgate.net/publication/4743289_Models_of_Technical_Progress_and_the_Golden_Rule_of_Research.

Pigou, AC 1928 *A Study in Public Finance*. London: Macmillan.

Pitchford, J. 1972 Population and optimal growth. *Econometrica* 40(1): 109-136.

Prettner, 2012 Population and aging endogenous growth. *Journal of Population Economics*: DOI: 10.1007/s00148-012-0441-9. http://link.springer.com/article/10.1007/s00148-012-0441-9#page-1

Rogers, M 2003 A survey of economic growth, *Economic Record*, 79(244):112-135).

Rogers, E 2003 *Diffusion of Innovations*, 5th Edition. London: Free Press.

Romer, PM 1993 Idea gaps and object gaps in economic development, *Journal of Monetary Economics* 32(3), 543-573.

Romer, PM 1990 Endogenous technological change, *Journal of Political Economy* 98(5), S71-S102.

Romer, PM 1994 The origins of endogenous growth, *Journal of Economic Perspectives* 8(1): 3-22.

Rosen, S 1983 Specialization and human capital, *Journal of Labor Economics* 1(1): 43-49.

Russell, B 1948 *Human Knowledge: Its Scope and Limits*. New York: Simon and Schuster.

Russell, B 1956 *Logic and Knowledge: Essays 1901-1950.* RC Marsh (ed). New York: Capricorn Books.

Samuelson, L. 2004 Modeling Knowledge in economic analysis, *Journal of Economic Literature* XLII: 367-403.

Schultz, TW 1981 *Investing in People: The Economics of Population Quality*. Berkeley: University of California press.

Schultz, TW 1961 Investment in human capital, *American Economic Review* LI(1): 1-17.

Schultz, Theodore W. (1979) "Investing in population quality throughout low-income countries," In *World population and Development: Challenges and Prospects*. Philip M. Hauser (Editor). New York: Syracuse University Press, pp. 339-360

Schumpeter, JH 1939 *Business Cycle*. New York: McGraw-Hill.

Schumpeter, JH 1942 *Capitalism, Socialism and Democracy*. New York: Harper & Row.

Sen, A. 1997 Human capital and human capability, *World Development* 25(12):1959-1961.

Sequeira, TN, and A. Ferreira-Lopes 2011 An endogenous growth model with human and social capital interactions, *Review of Social Economy* 4(December): 465-493.

Simon, J 1981 *The Ultimate Resource*. Princeton: Princeton University Press.

Smith, A 1952[1937] *An Inquiry into the Nature and Causes of the Wealth of Nations* (ed. Edwin Cannan; introducer Max Lerner). New York: Modern library. Book II, pp. 259-269.

Solow, RM 1957 Technical change and the aggregate production function, *Review of Economics and Statistics*, 39() 312-320.

Solow, RM 1956 A contribution to the theory of economic growth, *Quarterly Journal of Economics* 70 (1): 65-94.

Solow, RM 1960 Investment and Technical Progress, in Mathematical Methods in the Social Sciences, (Eds.) K. J. Arrow, S. Karlin and P. Suppes, Stanford University Press, pp.89-104.

Solow, RM 1997 *Learning from 'Learning by Doing': Lessons for economic growth*. Paolo Alto: Stanford University Press.

Solow, RM 1994 Perspectives on growth theory, *Journal of Economic Perspectives* 8(1): 45-54.

Sowell, T 1996 *Knowledge and Decisions*. New York: Basic Books.

Spence, M 1970 Job market signaling, *Quarterly Journal of Economics* 87(3): 355-374.

Spence, M 2002 Signaling in retrospect and the international shift of markets. *American Economic Review* 92(3): 434-459.

Stigler, GJ 1957 Selections from The Wealth of Nations Smith. New York: Wiley-Blackwell.

Stiglitz, JE 1975 The theory of "screening," education, and distribution of income, *American Economic Review*, 65(3): 283-300.

Stroombergen, Adolf, Rose, Dennis, and Nana, Gansh (2002) Review of the Statistical Measurement of Human Capital. Statistics New Zealand Infometrics Consulting Ltd (November).

Swan, T 2002 Economic growth, *Economic Record* 78: 375-80.

Swan, T 1956 Economic growth and capital accumulation, *Economic Record* 32: 334-61.

Teixeira, P 2008 Jacob mincer and the centrality of human capital for contemporary labor economics. CEMPRE- U. of Porto and CIPES. http://economix.fr/pdf/colloques/2007_HISRECO/5_Teixeira.pdf.

Teixeira, ACC 2004 Measuring aggregate human capital in Portugal: An update to 2001. Working Paper Research-work in progress, No. 152. http://wps.fep.up.pt/wps/wp152.pdf (August).

Temple, J. And PA Johnson 1998 Social capability and economic growth, *Quarterly Journal of Economics* 113(3): 965-990.

Thompson, DW 1992[1942] *On Growth and Form*. The Complete Revised Edition. New York: Dover Publications, Inc. Chapter III.

UNDP 2007/2008 Technical Note 1: Calculating the human development indices:

http://hdr.undp.org/en/media/HDR_20072008_Tech_Note_1.pdf. .Amavilah, Voxi Heinrich (2005) Resource intra-actions and interactions: Implications for technological change and economic growth. http://129.3.20.41/eps/ge/papers/0508/0508004.pdf.

Varian, HR, Farrell, J and Shapiro, C 2004 *The Economics of Information Technology: An Introduction*. Cambridge: Cambridge University Press.

von Thunen, HJ 1968 Costs of education as formation of productive capital, in Bowman, MJ et al. (Eds) *Readings in the economics of education* . Paris: UNESCO: 393-94.

Walsh, JR 1935 Capital concept applied to man, *Quarterly Journal of Economics,* 49(2): 255-285.

Weintraub, S 1977 *Modern Economic Thought.* University of Pennsylvania Press.

Woessmann, L. 2003 Specifying human capital, *Journal of Economic Surveys* 17(3): 239-270.

Wolfson, M., ZB Ozech, SS Hanna 1986 Karl Marx and the deletion of human capital as an open-access resource, *History of Political Economy* 18() 497-513.

Young, A 1991 Learning by doing and the dynamic effects of international trade, *Quarterly Journal of Economics*, CVI(2): 369-405.

Young, .HP 2005 The spread of innovations by social learning, Working Paper (February 17, 2006 Version). Available at: http://www.econ.jhu.edu/People/Young/Spread28march.pdf

Young, HP 2004 Innovation diffusion and population heterogeneity, Preliminary Draft, Johns Hopkins University (October 20, 2006 Version). Available at: http://www.econ.princeton.edu/seminars/SEMINARS/Fall06Seminars/young.pdf

Young, HP 2007 Innovation diffusions in heterogeneous populations. Discussion Paper Seris, No. 303. Department of Economics, University of Oxford. http://www.economics.ox.ac.uk/Research/wp/pdf/paper303.pdf

Young, Allyn A 1928 Increasing returns and economic progress, *Economic Journal*, 38: 527-542.

Zimmerman, D 1992 Regression Toward Mediocrity in Economic Stature, *American Economic Review* 82(3): 409-29.

Zimmermann, EW 1951 *World Resources and Industries*, Revised Edition. New York: Harper and Brothers, Publichers. Chapters 1 and 9.