Do Portfolio Diversification Opportunities exist across the Euro Zone Islamic Equity Markets? MGARCH-DCC and Wavelet Correlation Analysis

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Do Portfolio Diversification Opportunities exist across the Euro Zone Islamic Equity Markets? *MGARCH-DCC and Wavelet Correlation Analysis*

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ABSTRACT

The economic process that Euro Zone countries have gone through created contradictory claims about whether those particular financial markets have converged, which has implications for the portfolio diversification issue for the investors and policy makers alike. Thus, this paper aims to examine whether there are opportunities for creating optimally diversified portfolios from the perspectives of the Islamic investors which is a missing point in the existing literature for the above-mentioned markets. The focus of the study is the correlations among the selected Shariah Indices' daily returns covering the period from April 2008 to March 2014, to illustrate not only the static picture, but the dynamic comovements which can be tested through the recent econometric methodologies (M-GARCH/DCC, MODWT, and CWT/WTC). Hence, the unique contribution of this research is the enhancement of the existing literature by empirically testing the 'time-varying' and 'scale dependent' volatilities and correlations of the particular markets by relying on the Shariah (Islamic) indices. By incorporating scale dependence, the paper is able to identify unique portfolio diversification opportunities for different set of investors bearing different investment horizons/holding periods of stock (e.g. weekly, monthly, quarterly, etc.) The overall result driven is that since there appears to be only continuous portfolio diversification opportunities between the French and Danish Islamic equity markets for up to six-month investment holding periods, in general, the possibility of portfolio diversification benefits for the short-, medium- and long-run does not seem to be available among these markets. This result implies that the Euro Zone financial markets are highly correlated/converged yielding minimal portfolio diversification benefits for all the heterogeneous investment horizons.

Keywords: International Portfolio Diversification, Euro Zone Islamic Stock Market Investments, M-GARCH/DCC, Wavelet analysis, CWT, MODWT.

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1. Introduction

European integration is a long process of economic cooperation on progressive stages; a customs union, the single market, macroeconomic convergence, exchange rate coordination and ultimately a common currency have been realized respectively. Exchange rates have been fixed since the euro was introduced as the common currency for the 17 European countries. The European Central Bank is operating and implementing the common monetary policy, and all Euro Area government bills and bonds have been denominated in euro. Money and bond market integration was an immediate consequence of the economic and Monetary Union. The elimination of multiple currencies represented by the creation of the European Monetary Union implies the fall of an important barrier against financial integration. As a result, portfolio diversification issue as one of the main challenges for each Euro Zone financial market is getting crucial. This is because the enhanced European stock market integration implies reduced gains from the intra-Euro Zone portfolio diversification after risk adjustment (EUN and Lee, 2010; Beine et al., 2010).

Since the prominent study of Grubel (1968), the idea of international portfolio diversification resulting in lower portfolio risk is widely accepted in the literature. International stock market connections are of crucial importance to international investors to identify a mix of international stocks that creates optimally diversified portfolios with the lowest possible risks (Dajcman et al, 2012). As the fact, the essence of lower risks from international diversification is critically dependent upon low correlations across markets (Grubel and Fadner, 1971). Thus, an increase in co-movements between asset returns of international stock markets can diminish the advantage of internationally diversified investment portfolios (Ling and Dhesi, 2010). On the other hand, Dwyer and Hafer (1988), Longin and Solnik (1995), Errunza et al. (1999), Driessen and Laeven (2007), Eun and Shim (1989), Von Furstenberg and Jeon (1989), Bertera and Mayer (1990) examined various stock price indices around at the time of stock market crash of 1987 and found a substantial amount of interdependence among national stock markets as contradictory to the previous studies. It has been further studied in the literature that correlations amongst markets are evolving through time due to changes in interdependence across markets (Engle, 2002). Therefore, market returns are not only time varying, but may also be dependent on time scales highlighting the importance of investments horizons (Gencay et al, 2001)

The global financial markets over the past decade have witnessed that Islamic Financial Sector is rapidly expanding. As stated by DeLorenzo (2000), the Islamic financial system is based on the fundamentals of Shariah (Islamic Law) that requires gains from investments to be earned in an ethical and socially responsible manner that comply with teachings of Islam. Equities traded under Shariah indices undergo a screening process to ensure that they are free from prohibitive elements as dictated by Shariah. The common elements screened for are riba (interest), gharar (uncertainty), maysir (gambling), prohibited commodities (liquour, pork, etc.) and fulfillment of contractual requirements as required in the Islamic Law of Contracts (Rosly, 2005). As a matter of fact, the growing importance of the Islamic financial sector with an average annual growth rate ranging from 15% to 20% per annum over the past decade (IIFM, 2010) attracts researchers and investors to study the volatilities and co-movements of asset returns amongst Shariah compliant indices across countries to provide especially for the Islamic investors and fund managers an idea on riskiness and potential international portfolio diversification benefits.

Although there are myriad of studies concentrating on the convergence and portfolio diversification issues specifically on the Euro Zone markets, they are held only by relying on the conventional financial indicators such as bonds, Shariah non-compliant stock indices etc. Therefore, this paper attempts to study intra-Euro Zone portfolio diversification issue from the perspectives of the Euro Zone Islamic investors, since it is crucially important to draw some conclusions for the policy makers and investors alike. To this effect, we concentrate our investigation on five Euro Zone equity markets; those are France, Germany, Belgium, Denmark, and Italy due to the fact that these are the largest and most developed markets in terms of GDP, and the industrial backbone of the Euro Zone economy. The following are the stated objectives of this paper :

1. Are there any portfolio diversification benefits for the Euro Zone, in general, and French, in particular, Islamic stock market investors to invest in the Euro Zone markets to gain international portfolio diversification benefits?
2. Does the answer to the above question vary if we consider different holding periods (such as 2-4 days, 4-8 days, 8-16 days, etc.) for the Islamic investors since there are a variety of investors holding different expectations from the financial markets?

France is the focused financial market of this study since it recently sought to become a hub for Islamic banking in Europe and 1.5 million clients are willing to use Islamic banking tools and products which equal USD 18.2 billion. Besides the fact that a high number of Muslims lives there seeking Islamic banking services, French authorities have also offered strong support to develop Islamic banking services.[1]

The results from each of the research questions are expected to have significant implications for the Euro Zone, specifically for the French Islamic equity investors and fund managers in their decisions concerning portfolio allocations and investment horizons. The unique contribution of this paper is to enhance the existing literature by empirically testing the 'time-varying' and 'scale dependent' volatilities and correlations of the sample financial markets by relying on Sharih (Islamic) indices. By incorporating scale dependence, the paper is particularly able to identify unique portfolio diversification opportunities for the different sets of investors bearing different investment horizons or holding periods of stock (e.g. weekly, monthly, quarterly, etc.). The ability to do this is achieved by applying three recent empirical research techniques: Multivariate Generalized Autoregressive Conditional Heteroscedastic – Dynamic Conditional Correlations (MGARCH-DCC) model of Engle (2002), Maximal Overlap Discrete Wavelet Transform (MODWT), and Continuous Wavelet Transform (CWT) analysis of volatility and correlations.

Our findings tend to suggest that the possibility of creating optimally diversified portfolios for the Euro Zone, particularly French, Islamic investors through the other Euro Zone Islamic equity markets appears quite weak. This is because the results suggest that the French Islamic investors are only limited to the Danish Islamic stocks to create effective portfolios for only very short-, short-, and medium to 6 month-time duration. Furthermore, there are portfolio diversification possibilities within the Euro Zone Islamic stock markets on and off, not constant, between a few pairs of Islamic equity markets, not all pair-combinations of them, depending on time and investment holding period. Therefore, in the Euro Zone in general, and French in particular, the Islamic investors are highly recommended to explore the other Islamic financial markets around the globe for having effective portfolio diversification benefits.

The following sections of the paper are organized as follows. Section 2 reviews the relevant literature related to international portfolio diversification; time varying and scale dependence of international stock market correlations; and Islamic stock markets theory and performance. Section 3 briefly reviews the theoretical foundations being assumed in this paper. Section 4

[1] Please refer to http://www.zawya.com/story/Paris_seeks_to_become_Islamic_finance_hub_in_Europe-ZAWYA20120418054458/

details out the methodologies to be employed to achieve the research objectives of this paper. Section 5 contains the comprehensive data analysis, empirical results. Section 6 discusses the mechanical results obtained from the previous section using plausible explanations and past findings in the literature. Finally, Sections 7 and 8 provide the limitations/suggestions for future research and closing remarks to the paper. References make up the end of this paper.

2. Literature Review

2.1 International Portfolio Diversification

The first outstanding studies regarding the international stock market diversification issue started in the 1970s, some of the prominent ones as such Levy and Sarnat (1970), Solnik (1974, 1982), Black and Litterman 1992 and Jankus 1998, since globalization and international investing topics became very vital. The reason behind those studies was simply to determine the constant correlations in regard to the creation of international portfolios to gain diversification benefits. While Levy and Sarnat (1970), for instance, show how the correlations between the developed and developing countries provide a significant risk-reduction benefit, Solnik (1974) provides evidence that combining stocks from US and European countries generates portfolios that are only half as risky as domestically well-diversified portfolios of US stocks. However, most of the past research focuses on constant correlations and the benefits of diversification to US investors, hence other characteristics of international stock portfolios and the diversification benefits to foreign investors are typically ignored.

The stock markets crash of 1987, however, provided new insights into the literature of international financial markets interdependence, which contradicted past theories and empirical results. Dwyer and Hafer (1988), using daily data for seven months before and after the October 1987 crash report statistical evidence that changes in the stock price indices in Germany, Japan, US and USA are generally related. In addition, other studies such as Longin and Solnik (1995), Errunza et al. (1999), Driessen and Laeven (2007) Eun and Shim (1989), Von Furstenberg and Jeon (1989), Bertera and Mayer (1990) also examined various stock price indices around the stock market crash of 1987 and found a substantial amount of interdependence among national stock markets.

The appeared inconsistencies in the literature continued to be seen after then. More recently, the studies such as Becker et al. (1990), Hamao et al. (1990), and Kasa (1992) studies the possible linkages between US and Japanese markets finding strong correlation between them with an asymmetric spill-over effects from the US to Japanese market. Other studies, however, have posited opposite results for the same markets, for instance, Smith et al. (1993) and Park (1994) suggesting that the US equity prices do not lead Japanese equity prices. Besides, Li et al (2003), Rezayat and Yavas (2006), Flavin et al (2008), Middleton et al. (2008), and Mansourfar et al. (2010) are the most recent studies looking at the same topic. Briefly, the literature studying stock market linkages and its resulting impact for international portfolio diversification strategies has remained inconclusive with the results reporting contradictory evidence. Hence this subject needs further investigation.

2.2 Time-varying and scale dependent correlations across countries

Even though the studies given above focused on the correlations between the markets from constancy perspective, many recent studies have empirically tested and provided evidence that the correlations across national markets may not be constant and are evolving through time. For example Longin and Solnik (1995) study the correlation and covariance of monthly excess returns for seven major countries over the period 1960-1990 and find that both correlations and co-variances are unstable over time. Besides, Yang (2005) examines the international stock market correlations between Japan and the Asian Four Tigers (Taiwan, Singapore, Hong Kong, and South Korea) and finds that stock market correlations fluctuate widely over time and volatilities appear to be contagious across markets. In addition, correlations increase during periods of high market volatilities when risk diversification is needed most, which is bad news for international diversification. Most recently, Dacjman et al (2012) find that co-movement dynamics between the developed European stock markets of the United Kingdom, Germany, France and Austria are not constant and returns are time varying. Hence, when modeling volatilities and correlations, it is more appropriate to use time varying conditional correlations models as compared to constant correlations model.

Similarly, studies have also found that investment holding periods (for e.g. 2 days, 6 days, 30 days, etc.) also have an impact on the volatilities and correlations dynamics of stock market returns. This type of research is relatively new and there are only a few empirical papers that incorporate time scaling in examining volatilities and correlations. Gencay et al (2001) were one of the earliest proponents of the time scaled dependence of returns and correlations in financial markets. In and Kim (2013) have combined a cluster of their papers using wavelet time-scaling in finance to produce a book just published called 'An introduction to Wavelet Theory in Finance'. Dacjman et al (2012), in their recent study on co-movement dynamics between the developed European stock markets of the United Kingdom, Germany, France and Austria also find evidence in favor of scale dependence for stock market returns. Hence, future studies are recommended to consider the time scale properties in modeling volatilities and correlations.

2.3. Islamic stocks and portfolio diversification

As a fact that Islamic stock markets are quite recent, studies on stock market integration among Islamic stock markets worldwide are still relatively scarce (Moeljadi, 2012). Hakim and Rashidian (2004), for instance, attempt to examine the returns performances of the Dow Jones Islamic Market Index (DJIM), Dow Jones World Index (DJW) and Dow Jones Sustainability World Index (DJS) by using the capital asset pricing model (CAPM) framework. The study finds that DJIM has done relatively well compared to the DJW, but has underperformed the DJS. Hussein (2005), as another study, analyses the DJIM returns for the period 1996 to 2003 and posits that Islamic indexes provide investors with positive abnormal returns throughout the entire bull period, but they under-perform their non-Islamic index counterparts during the bear market period. The study held by Abderrezak (2008) scrutinizes 46 Islamic Equity funds (IEFs) during January 1997 to August 2002 by using Fama's 3 factor model. He finds that Islamic funds performed poorly against their respective indices. As the results of the study, the co-movement of IEFs returns with the market, measured by the betas, is low, poor evidence for selectivity does exist, small-cap firms and growth preference stocks significantly affect iEFs, and finally IEFs do suffer from lower diversification.

Achsani et al. (2007) finds that the interdependence of the Islamic stock markets tends to be asymmetric across a wide geographical area. While there are strong correlations between the Islamic stock indices of Indonesia and Malaysia, the US and Canada, and Japan and Asia Pacific, this is not true for across region basis. Besides, the study finds that while the Islamic stock market in the US has a strong influence on the other Islamic stock markets, the reverse is not the case. On the other hand, Aziz and Kurniawan (2007) contest that there are potential diversification benefits for investors considering the Islamic stock markets in Indonesia and Malaysia. In particular, the Jakarta Islamic Index and the Kuala Lumpur Shari'ah Index have significant leverage and asymmetric effects. Other recent studies such as Majid and Kassim (2012), Kamil et al (2012) etc. also find mixed results in their analysis. As evident from the review above, there are inconsistencies in empirical literature analysing the portfolio diversification properties of the Islamic indices across the globe. While some studies find evidence of strong correlations across Islamic stock indices, others have found Islamic stocks to be weakly correlated allowing investors to gain international diversification opportunities.

3. Theoretical Framework

The theoretical foundations assumed in this paper draw upon from the seminal works of Markowitz (1959) 'Modern Portfolio Theory' and Grubel (1968) 'Internationally Diversified Portfolios'. The Nobel Prize winning contributions of Markowitz shaped the modern portfolio theory where the volatility of a portfolio is less than the weighted average of the volatilities of the securities only if the portfolio consists of assets that are not perfectly correlated in returns. The variance of the expected return on a portfolio can be calculated as:

$$\sigma_p^2 = (\Sigma W_i^2\sigma_i^2 + \Sigma\Sigma W_i W_j Cov_{ij})$$

Where the sums are over all the securities in the portfolio, W_i is the proportion of the portfolio in security i, σ_i is the standard deviation of expected returns of security i, and Cov_{ij} is the covariance of expected returns of securities of i and j. Assuming that the covariance is less than one, this will be less than the weighted average of the standard deviation of the expected returns of the securities. This is why diversification reduces risk.

Drawing from Markowitz's model, Grubel (1968) applied the modern portfolio theory to explore the potential benefits of holding long-term international assets. Grubel modelled international portfolio diversification benefits between two counties A and B as follows:

$$E(r_{a,b}) = W_a R_a + W_b R_b$$
$$V(r_{a,b}) = W_a^2\sigma_a^2 + W_b^2\sigma_b^2 + 2W_a W_b Cov_{ab}$$

Where $E(r_{a,b})$ is the expected returns on portfolio invested in Country A and B with investment weights of W_a and W_b and $V(r_{a,b})$ measures the variance on the portfolio. The crucial factor here is the Cov_{ab} and the lower the covariance between countries A and B, the greater would be diversification benefits. Grubel found that if US investors allocate a part of capital to foreign stock markets, they could achieve a significant reduction in portfolio risk and better portfolio return opportunities.

Subsequent to these seminal papers, various writers have attempted to empirically test the covariance of asset returns amongst cross-border stock markets in order to identify international portfolio diversification opportunities. A higher covariance between asset returns can therefore diminish the advantage of internationally diversified investment portfolios (Ling and Dhesi, 2010). This paper drawing upon such theoretical foundations also studies the volatilities and cross-correlations amongst sample stock indices to answer the research questions albeit using recent empirical methodologies.

One of the criticisms of the earlier models of modern portfolio theory is the assumption that the portfolio variances are normally distributed. Markowitz himself thought normally distributed variance is inadequate measure of risk. However, subsequent models have been developed that using asymmetric and fat tailed distributions that are closer to real world data. The methodology to be adopted in this paper M-GARCH-DCC has the ability to adopt a student-*t* distribution of variances, which is more appropriate in capturing the fat-tailed nature of the distribution of index returns (Pesaran and Pesaran, 2009). Furthermore, the use of wavelet transform methodologies makes no assumptions and is tantamount to produce more realistic results (In and Kim, 2013). The paper elaborates the methodologies to be adopted in achieving the research objectives in the following section.

In addition, it is argued that Shariah compliant equity investments would be more related to other similar Islamic equity markets, as opposed to conventional ones. This is due to the fact that the asset composition of Shariah compliant equities is different to that of conventional equity markets. In particular, Shariah stock screening norms typically exclude a number of sectors, namely, the financial sector (including conventional insurance), gambling and gaming, alcohol sale and production, tobacco-related industries, pork and non-halal food sale and production, as well as the entertainment industry. Even common sense would suggest that Shariah compliant equities would be more related to comparable markets, especially Shariah compliant markets in the same geographical location, than conventional equity markets. However, we would like to utilize empirical data and scientific tools to observe the similarities and differences with rhetorical argumentation and intuition.

4. Methodology

4.1. Multivariate GARCH – Dynamic Conditional Correlation

Time-varying relationships and volatility are two methodological challenges that are particular to the field of time series. Among time-varying parameter models, the Dynamic Conditional Correlation (DCC) model is a creative and useful approach that deals effectively with over-time variation in both the mean and variance of time series. Further, it allows us to study the evolution of relationships over time in a multivariate setting by relaxing model assumptions. The DCC model does so by calculating a current correlation between variables of interest as a function of past realizations of both the volatility within the variables and the correlations between them. The association between variables can thus be seen to evolve over time in a manner that not only depends upon whether and to what degree the variables are moving in the same direction, but also takes account of the history of variance that each series has undergone.

With its attention to volatility, the DCC model is based within the family of Generalized ARCH (GARCH; see Bollerslev 1990; Engle 1982) models, which have flourished in recent years in the literatures on finance and econometrics. One of the earliest volatility models, autoregressive conditional heteroscedastic (ARCH), was proposed by Engle (1982), which captured the time-varying conditional variances of time series based on past information. This model was then enhanced by Bollerslev (1986) who proposed a generalized ARCH (GARCH) which took into account both past error terms and conditional variances into its variance equation simultaneously to avoid the problem that the number of parameters to be estimated becomes too large as the number of lagging periods to be considered increases in the ARCH model. Bollerslev (1990) further extended the GARCH model in a multivariate sense to propose a Multivariate GARCH – Constant Conditional Correlation (M- GARCH-CCC) model where the conditional correlation amongst different variables were assumed to be constant. The MGARCH-CCC model only allows the variances of each variable to be time varying while keeping the correlation coefficient among them constant. However, while the CCC assumption makes estimation simple, it may be inconsistent with reality (Longin and Solnik, 1995, 2001). Therefore, Engle (2002) finally proposed an M-GARCH-DCC model where the conditional correlations amongst variables were allowed to be dynamic and this paper makes use of this model in answering parts of the research questions. It can be stated as follows:

$$\mathbf{r}_t = \boldsymbol{\beta}_0 + \sum_{i=1}^{k} \boldsymbol{\beta}_i \mathbf{r}_{t-i} + \mathbf{u}_t = \boldsymbol{\mu}_t + \mathbf{u}_t$$
$$\boldsymbol{\mu}_t = E[\mathbf{r}_t|\Omega_{t-1}]$$
$$\mathbf{u}_t|\Omega_{t-1} \sim N(0, \mathbf{H}_t)$$
$$\mathbf{H}_t = \mathbf{G}_t \mathbf{R}_t \mathbf{G}_t$$
$$\mathbf{G}_t = \text{diag}\left\{\sqrt{h_{ii,t}}\right\}$$
$$\mathbf{z}_t = \mathbf{G}_t^{-1} \mathbf{u}_t$$

Source: Ku (2008)

Where $h_{ii,t}$ is the estimated conditional variance from the individual univariate GARCH models, $\mathbf{G_t}$ is the diagonal matric of conditional standard deviations, $\mathbf{R_t}$ is the time-varying conditional correlation coefficient matrix of returns, and $\mathbf{z_t}$ is the standardized residuals vector with mean zero and variance one.

After the above basic construction, the dynamic correlation coefficient matrix of the DCC model can be specified further:

$$\mathbf{R}_t = (\text{diag}(\mathbf{Q}_t))^{-1/2} \mathbf{Q}_t (\text{diag}(\mathbf{Q}_t))^{-1/2}$$
$$\mathbf{Q}_t = (q_{ij,t})$$
$$(\text{diag}(\mathbf{Q}_t))^{-1/2} = \text{diag}\left(\frac{1}{\sqrt{q_{11,t}}}, \ldots, \frac{1}{\sqrt{q_{nn,t}}}\right)$$
$$q_{ij,t} = \overline{\rho}_{ij} + \alpha(z_{i,t-1} z_{j,t-1} - \overline{\rho}_{ij}) + \beta(q_{ij,t-1} - \overline{\rho}_{ij})$$

Source: Ku (2008)

Where $\overline{\rho}_{ij}$ is the unconditional correlation coefficient and the new time-varying conditional correlation coefficient is $\rho_{i,j,t} = q_{i,j,t} / \sqrt{q_{ii,t}\, q_{jj,t}}$. Meanwhile, the returns on financial assets have often been documented to be fat tailed or leptokurtic where a normal distribution assumption is not appropriate. One possible remedy for such is to use a Student-t distribution setting. That is, the conditional distribution $\mathbf{u}_t|\Omega_{t-1} \sim N(0, \mathbf{H}_t)$ is replaced by $\mathbf{u}_t|\Omega_{t-1} \sim f_{\text{Student}-t}(\mathbf{u}_t; v)$, where v is the degree of freedom parameter.

4.2. Maximum Overlap Discrete Wavelet Transformation (MODWT)

Co-movements between stock market returns may not only be time varying, but also scale dependent (Gencay et al., 2001), and can be analysed with wavelet tools. A MODWT-based estimator has been shown to be superior to the earlier DWT-based estimators (Percival, 1995). The MODWT is a variant of the discrete wavelet transform (DWT) that, unlike the classical DWT, can handle any sample size and not just those that are multiples of 2^x. The MODWT is highly redundant, non-orthogonal transform – this enables alignment of the decomposed wavelet and scaling coefficients at each level with the original time series, thus allowing a ready comparison between the series and its decompositions, which is not captured in DWT. The MODWT variance estimator is also asymptotically more efficient than the same estimator based on the DWT. MODWT is generally known as stationary wavelet transform, shift or translation invariant DWT, time invariant DWT, and non-decimated DWT. Hence, this research makes use of the MODWT method, which can be described as follows.

Let X be an N-dimensional vector whose elements represent the real-valued time series $\{X_t: t = 0, \ldots, N-1\}$. For any positive integer, J_0, the level J_0 MODWT of X is a transform consisting of the $J_0 + 1$ vectors $\widetilde{\boldsymbol{W}}_1, \ldots, \widetilde{\boldsymbol{W}}_{J_0}$ and $\widetilde{V}_{J_0}$, all of which have dimension N. The vector $\widetilde{\boldsymbol{W}}_J$ contains the MODWT wavelet coefficients associated with changes on scale $\tau_j = 2^{j-1}$ (for $j = 1,\ldots J_0$) while $\widetilde{\boldsymbol{V}}_{J_0}$ contains MODWT scaling coefficients association with averages on scale $\lambda_{J_0} = 2^{J_0}$. Based on the definition of MODWT coefficients we can write (Percival and Walden, 2000, p. 200):

$\widetilde{\boldsymbol{W}}_J = \widetilde{W}_J \boldsymbol{X}$ and $\widetilde{\boldsymbol{V}}_J^0 = \widetilde{V}_J^0 \boldsymbol{X}$

Where $\widetilde{W}_J$ and $\widetilde{V}_{J_0}$ are N x N matrices. Vectors are denoted by bold italics.

By definition, elements of $\widetilde{W}_J$ and $\widetilde{V}_{J_0}$ are outputs obtained by filtering X, namely:

$$\widetilde{W}_{j,t} = \sum_{l=0}^{L_{j-1}} \widetilde{h}_{j,l} \, X_{t-l \bmod N}$$

and

$$\widetilde{V}_{j,t} = \sum_{l=0}^{L_{j=1}} \widetilde{g}_{j,l} \, X_{t-l \bmod N}$$

For $t = 0, \ldots, N-1$, where $\widetilde{h}_{j,l}$ and $\widetilde{g}_{j,l}$ are jth MODWT wavelet and scaling filters. The MODWT treats the series as if it were periodic, whereby the unobserved samples of the real-valued time series $X_{-1}, X_{-2}, \ldots, X_{-N}$ are assigned the observed values at $X_{N-1}, X_{N-2}, \ldots, X_0$.

The MODWT coefficients are thus given by:

$$\widetilde{W}_{j,t} = \sum_{l=0}^{N-1} \widetilde{h}_{j,l}^{\circ} X_{t-l \bmod N}$$

and

$$\widetilde{V}_{j,t} = \sum_{l=0}^{N-1} \widetilde{g}_{j,l}^{\circ} X_{t-l \bmod N} (\text{for } t = 0, \ldots, N-1;$$
$\widetilde{h}_{j,l}^{\circ}$ and $\widetilde{g}_{j,l}^{\circ}$ are periodization of $\widetilde{h}_{j,l}^{\circ}$ and $\widetilde{g}_{j,l}^{\circ}$ to circular filters of length N)

Wavelet variance is defined for stationary and non-stationary processes with stationary backward differences. Considering only the non-boundary wavelet coefficient, obtained by filtering stationary series with MODWT, the wavelet variance $v_X^2(\tau_j)$ is defined as the expected value of $\widetilde{W}_{j,t}^2$. In this case $v_X^2(\tau_j)$ represents the contribution to the (possibly infinite) variance of $\{X_t\}$ at the scale $\tau_j = 2^{j-1}$ and can be estimated by the unbiased estimator (Percival and Walden, 2000, p. 306):

$$\hat{v}_X^2(\tau_j) = \frac{1}{M_j} \sum_{t=L_j-1}^{N-1} \widetilde{W}_{j,t}^2$$

where $M_j \equiv N - L_j + 1 > 0$ is the number of non-boundary coefficients at the *j*th level. The MODWT correlation estimator for scale τ_j is obtained by making use of the wavelet cross-covariance and the square root of wavelet variances:

$$\hat{\rho}_{X,Y}(\tau_j) = \frac{\hat{v}_{X,Y}(\tau_j)}{\hat{v}_X(\tau_j)\hat{v}_Y(\tau_j)}$$

Where $|\hat{\rho}_{X,Y}(\tau_j)| \leq 1$. The wavelet correlation is analogous to its Fourier equivalent, the complex coherency (Gencay et al., 2002, p. 258).

4.3. Continuous Wavelet Transform (CWT)

There were two important criticisms against Fourier analysis, which are, firstly, total loss of time information leading to difficulty to discriminate ephemeral relations or to identify structural changes; secondly, the technique being appropriate only when time series is stationary. However, the time series of macro-economic variables are mostly noisy, complex and rarely stationary. To overcome such situation, Gabor (1946) introduced the short time Fourier transformation. Nevertheless, the technique was also criticized on the basis of its efficiency as it takes equal frequency resolution across all dissimilar frequencies (see, for details, Raihan et al., 2005).

Wavelet transform took birth to produce solutions to the above-mentioned problems. It offers a major advantage of having the ability to perform "natural local analysis of a time-series in the sense that the length of wavelets varies endogenously: it stretches into a long wavelet function to measure the low-frequency movements; and it compresses into a short wavelet function to measure the high-frequency movements" (Aguiar-Conraria and Soares, 2011, p. 646). Wavelet shows the evolution of change in the time series over time and at different periodic components i.e., frequency bands. However, the application of wavelet analysis in the economics and finance is mostly limited to the use of one or other variants of discrete wavelet transformation. One major benefit CWT has over DWT/MODWT is that we need not define the number of wavelets (time-scales) in CWT, which generates itself according to the length of data. Second of all, the CWT maps the series correlations in a two-dimensional figure that allows us to easily identify and interpret patterns or hidden information. For both MODWT and CWT, we use the Daubechies (1992) least asymmetric wavelet filter of length L=8 denoted by LA (8) based on eight non-zero coefficients. Previous studies on high-frequency data have shown that a moderate-length filter such as L = 8 is adequate to deal with the characteristic features of time- series data (see Gencay et al., 2001, 2002, In and Kim 2013, etc.). The continuous wavelet transform (CWT) $W_X(u,s)$ is obtained by projecting a mother wavelet ψ onto the examined time series $x(t) \in L^2(\mathbb{R})$, that is:

$$W_X(u,s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s}\right) dt$$

The position of the wavelet in the time domain is given by u, while its position in the frequency domain is given by s. Therefore, the wavelet transform, by mapping the original series into a function of u and s, gives us information simultaneously on time and frequency. To be able to study the interaction between two time series, how closely X and Y are related by a linear transformation; we need to apply a bivariate framework, which is called wavelet coherence. The wavelet coherence of two time series is defined as:

$$R_n^2(s) = \frac{\left|S(s^{-1}W_n^{xy}(s))\right|^2}{S(s^{-1}|W_n^x(s)|^2 . S(s^{-1}\left|W_n^y(s)\right|^2}$$

Where S is a smoothing operator, s is a wavelet scale, $W_n^x(s)$ is the continuous wavelet transform of the time series X, $W_n^y(s)$ is the continuous wavelet transform of the time series Y, $W_n^{xy}(s)$ is a cross wavelet transform of the two time series X and Y (Madaleno and Pinho, 2012). For brevity, we omit further detailed mathematical equations and interested readers may refer to Gencay et al (2001; 2002) and In and Kim (2013) for full methodological models.

5. Data Analysis and Empirical Results & Discussions

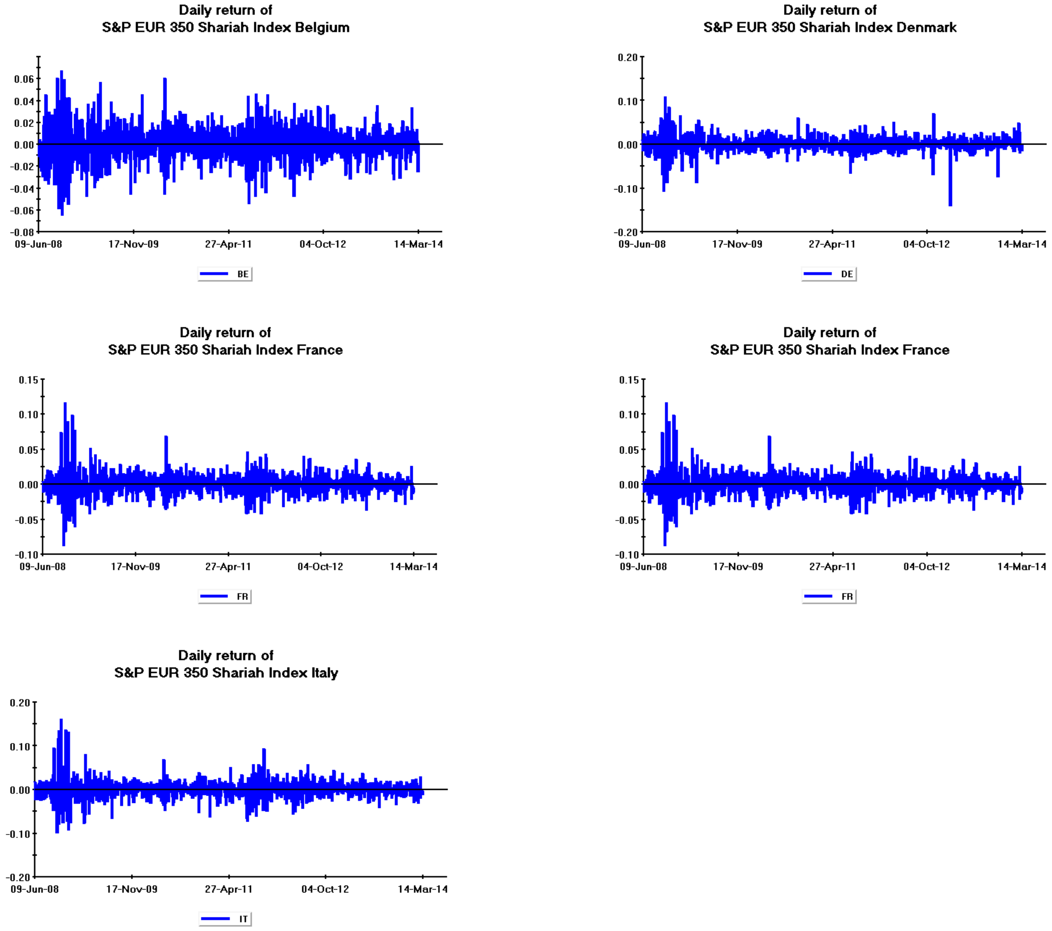
5.1. Data Summary Statistics

The data used in this research are the daily closing of the selected stock market indices in the period of 9th June 2008 to 14th March 2014 to see the possible effects of 2008 financial crisis and Euro Zone originated 2010 sovereign debt crisis. The total of 1505 observations was obtained, and the source of data is Data Stream database available from the Knowledge Management Centre of INCEIF University. S&P EUR 350 Shariah Index Germany, S&P EUR 350 Shariah Index Belgium, S&P EUR 350 Shariah Index Denmark, S&P EUR 350 Shariah Index France and S&P EUR 350 Shariah Index Italy are the variables used in this paper as proxies for these Islamic equity markets. The stock indices returns were calculated as differences of the logarithmic daily closing prices of indices $\{ln(p_t) - ln(p_{t-1})\}$ where p is an index value. This conversion is necessary to achieve stationarity in variance Engle's (2002).

Table 1: Selected Euro Zone Islamic Stock Indices for Study

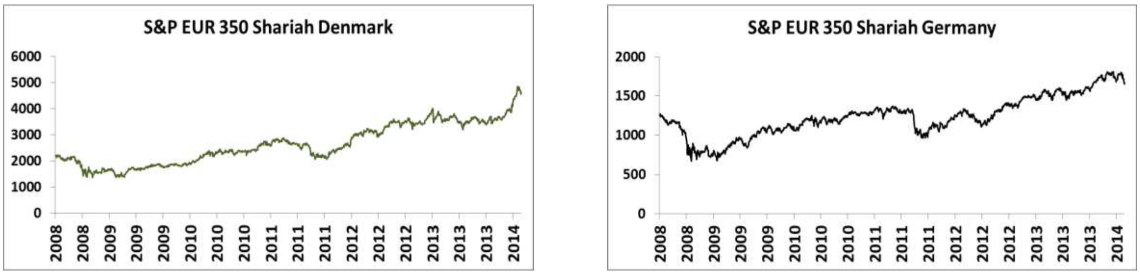
Symbol	Definition
GE	S&P EUR 350 Shariah Index Germany
BE	S&P EUR 350 Shariah Index Belgium
FR	S&P EUR 350 Shariah Index France
DE	S&P EUR 350 Shariah Index Denmark
IT	S&P EUR 350 Shariah Index Italy

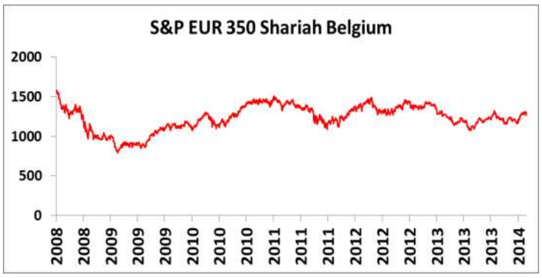
Figure 1: Daily Returns' Volatility Diagrams of Euro Zone Shariah Indices

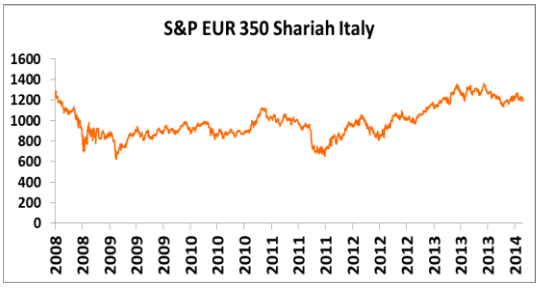


First glance driven from **Figure 1** shows that the 2008 global financial crisis has affected all markets obviously since the scattered diagrams for all fluctuated the most during the period covered. On the other hand, the second common unordinary fluctuation time zone appears about in the mid of 2011 which could be the effect of 2010/2011 sovereign debt crisis.

Figure 2: Euro Zone Shariah Indices' Trends







The sudden decrease is observed in **Figure 2** for all indices due to 2008 global financial crisis. However, the upward trend has been obvious afterwards for all markets. Even though all markets happened to experience a sudden drop around in the mid of 2011 due to 2010/2011 sovereign debt crisis, the upward correction can be seen in all markets. Adding more detail, **Table 2** below provides descriptive statistics indicating that Italy having the highest standard deviation is the most-, and Belgium having the lowest standard deviation is the least-volatile Islamic stock market among the five. On the other hand, Germany having the highest mean value is the most-, and Belgium having the lowest mean value is the least-return yielding Islamic stock markets among the five.

Table 2: Descriptive Statistics of Euro Zone Shariah Indices Returns Series

	Mean	Median	Maximum	Minimum	Std.Dev.	Kurtosis	Skewness	Obs
Germany	0.00019	0.00032	0.15908	-0.10094	0.01699	14.48821	0.74541	1505
Belgium	-0.00015	0.00000	-0.06538	0.06776	0.01494	1.87809	-0.05843	1505
France	0.00007	0.00028	0.11669	-0.08816	0.01506	7.72972	0.41409	1505
Denmark	0.00048	0.00006	0.10830	-0.14196	0.01721	8.15740	-0.50638	1505
Italy	-0.00004	0.00020	0.16143	-0.10120	0.02048	9.26839	0.51259	1505

5.2. Empirical Results and Discussion

5.2.1. Based on M-GARCH/DCC Method

First of all, we ran a MGARCH/DCC analysis on the S&P EUR 350 Shariah Index Germany returns, S&P EUR 350 Shariah Index Belgium returns, S&P EUR 350 Shariah Index

Denmark returns, S&P EUR 350 Shariah Index France returns and S&P EUR 350 Shariah Index Italy returns. First we look at the unconditional volatilities and correlations and the results are illustrated in **Table 3** below[2].

Table 3: Unconditional Volatility and Correlations - BE, DE, FR, GE, and IT

	BE	DE	FR	GE	IT
BE	*0.01496*	0.41958	0.68654	0.66597	0.62845
DE	**0.41958**	*0.01725*	0.50585	**0.48037**	**0.46361**
FR	0.68654	0.50585	*0.01513*	0.84460	0.80547
GE	0.66597	0.48037	0.84460	*0.01706*	0.76009
IT	0.62845	0.46361	0.80547	0.76009	*0.02055*

On-diagonal elements in italics represent the volatilities of the returns while off-diagonal elements illustrate the unconditional correlations between returns. Unconditional results show that Belgian Shari'ah index returns are the least volatile (0.01496) and Italy Shariah index returns are the most volatile (0.02055), supporting to the results driven in descriptive statistics above. When we do consider the correlations among the markets, Denmark has the lowest correlations with other Islamic stock markets such as 0.41958 with Belgian, 0.46361 with Italian, 0.48037 with Germany, and 0.50585 with French. This result suggests that Denmark is the only Islamic stock market with which other Euro Zone Islamic stock markets may create effective portfolios. On the other hand, even though French Islamic stock market has the lowest correlation with Danish Islamic stock market, it does not suggest an effective diversification opportunity, since it is higher than 0.5.[3] We have to bear in mind that these results are time-invariant, meaning to say that the stated correlations above are constant or static, which is not the fact in financial markets.[4] Therefore, from now onwards, we proceed to examining the dynamic conditional correlations, which capture the time-varying properties in volatilities and correlations. **Figure 3** and **Figure 4** illustrate the results.

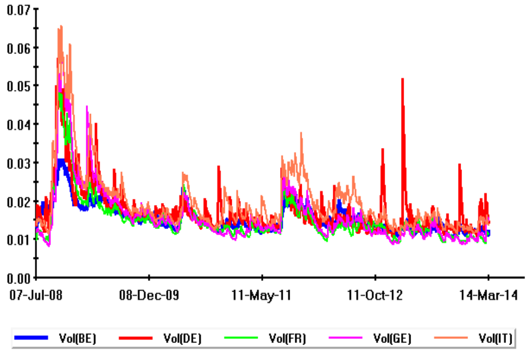
Figure 3: Conditional Volatilities - BE, DE, FR, GE, and IT

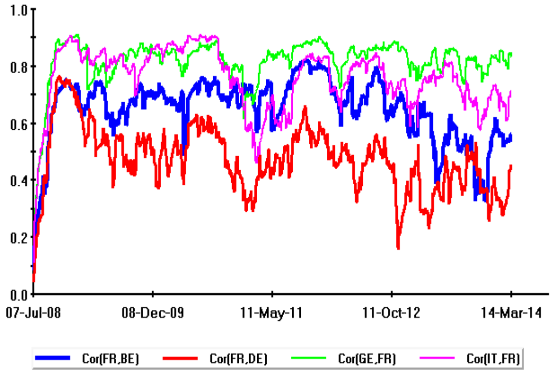
Figure 4: Conditional Correlations - BE, DE, FR, GE, and IT

[2] Please refer to Appendix 1 – 1.6 to see the underlying data driven from the Micro-fit upon which the results are shown in this part of the study and the necessary tests applied regarding MGARCH-DCC methodology.

[3] Correlations exceeding 0.5 levels are arbitrarily considered to be strong and hence not effective for portfolio diversification.

[4] For more elaboration, please refer to Multivariate GARCH – Dynamic Conditional Correlation part under Methodology section of this paper.





Both **Figure 3** and **Figure 4** confirm the time-varying properties of volatilities and correlations. Starting with **Figure 3**, the conditional volatilities of all Shariah indices' returns move closely during the observation period. As in line with the previous results driven by unconditional volatility and correlations matrix table, the Italian and Danish Islamic stock markets returns are the most volatile. In addition, it shows that while the 2008 global financial crisis hit all the particular markets of this study, Italian and Danish Islamic stock markets were the most affected ones respectively. In fact, the Italian and Danish Islamic stock markets kept being the most volatile even after 2008 global financial crisis until July 2011. Meanwhile, the effects of 2010/2011 sovereign debt crisis can be seen as the fluctuation experienced by all particular markets in the mid of 2011. Interestingly, Danish Islamic equity returns have been the most volatile since 2012 and onwards.

Second of all, **Figure 4** illustrates that the French Islamic stock market is consistently less correlated with the Danish Islamic stock market as opposed to Belgian, Italian and German Islamic stock markets; hence the French Islamic investors are better off investing in the Danish Islamic stock market to gain portfolio diversification benefits throughout the time period. This result is, again, in line with the results driven before.

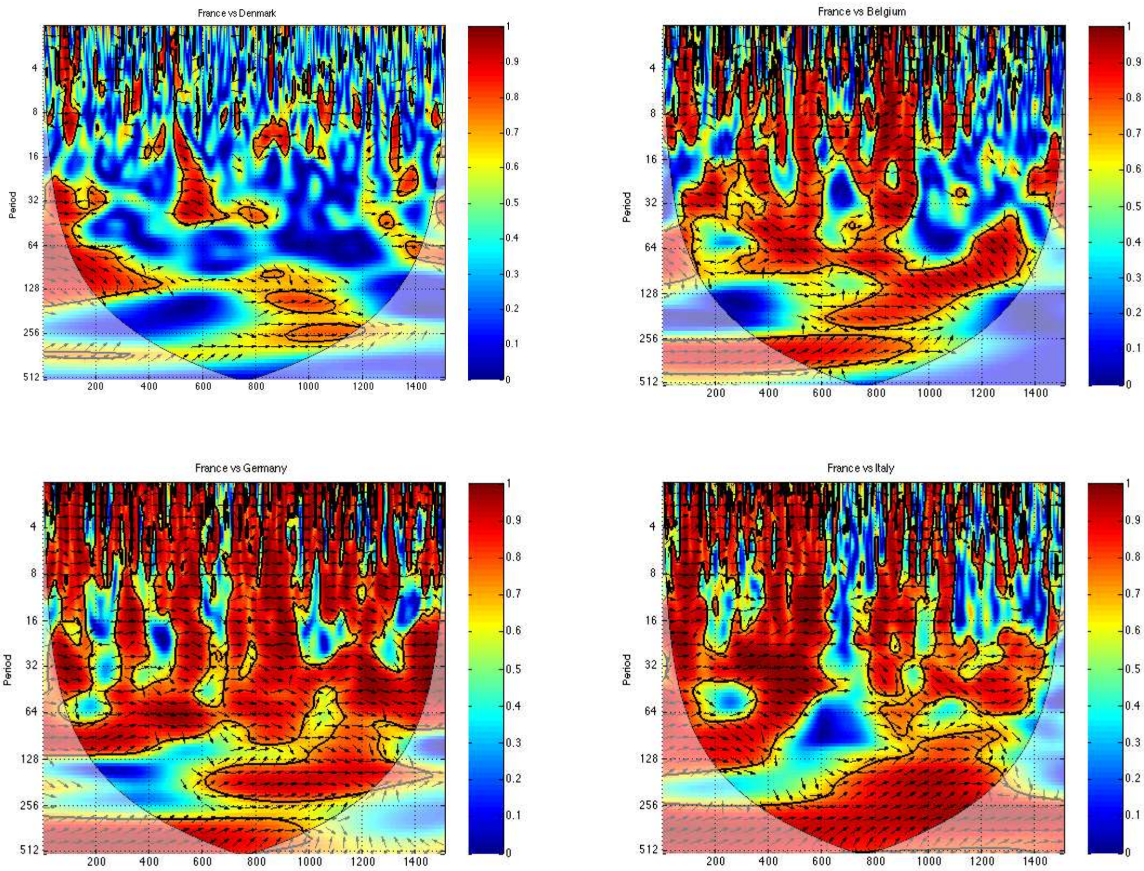
5.2.2. Based on Continuous Wavelet Transform (CWT)

Figure 5 below presents the estimated continuous wavelet transform and phase difference of French Islamic stock market returns with Danish, Belgian, Italian, and lastly German Islamic stock market returns from scale 1 (one day) up to scale of 9 (approximately two market years, 512 days). Time is shown on the horizontal axis in terms of number of days, while the vertical axis refers to the investment horizon. The curved line below shows the 5% significance level, which is estimated using Monte Carlo simulations. The figure follows a color code as illustrated on the right with power ranges from blue (low correlations) to red (high correlations). First of all, a first layman glance instantly suggests the higher correlations of the German and Italian Islamic stock markets with the French Islamic stock market as evident by the greater number of red spots on the coherence diagram. In contrast, France vs. Denmark diagrams is dominated most by the blue color emphasizing that the correlation between French and Danish Islamic stock markets is the least correlated. Therefore, it can be straightforwardly claimed that there is potential to create effective portfolios for French Islamic investors by including Danish Islamic stocks.

When we look at the diagrams in detail, some portfolio diversification opportunities appear even between French vs. German, and French vs. Italian Islamic stock markets on and off. For six months to one-year investment holding period, portfolio diversification opportunities appear between French and German Islamic stock markets up to first 600 observations, which amounts to the end of 2011. At this point it may be stated that 2010/2011 sovereign debt crisis caused convergence between French and German Islamic stock markets, which eliminated portfolio diversification benefits for French Islamic investors to include the German Islamic equity market stocks for six months to one-year investment holding period. Interestingly, however, the correlation between French and Italian Islamic stock markets declined for almost one financial year starting from 2011 onwards for three to six month investment holding period as a positive signal in exploiting a possible portfolio diversification benefit, which could be the result of 2010-2011 sovereign debt crisis. Therefore, we may state that even though 2010/2011 sovereign debt crisis increased the correlation between French and German Islamic stock markets for six months to one-year investment holding period, the effect of it on the correlation between French and Italian Islamic stock markets was in favor of the portfolio diversification benefit for three to six months investment holding periods. Besides, it is worth to mention that most recently up to 16 days investment holding periods, the portfolio diversification benefit appears between French and Italian Islamic stock markets.

If we look at the correlations between French and Belgian Islamic stock markets closer, we can see the different type of correlations based on the time and investment holding period. It is seen that even for medium term investment holding periods (i.e. up to 64 days) portfolio diversification benefit has been possible for almost last two financial years. In contrast, this statement cannot be expressed for other times and investment holding periods. On the other hand, we find that for very short, short, and medium investment holding periods consisting of 2-4 days, 4-8 days, 8-16 days, 16-32 days, and 32-64 days, the Danish Islamic stock market is consistently weakly correlated with French Islamic stock market over past 7 years, thus offering effective portfolio diversification opportunities.

Figure 5: Continuous Wavelet Transform Charts



5.2.3. Robustness and Validation of results-MODWT

To further reassure ourselves regarding the results from the previous methodologies, we applied Maximum Overlap Discrete Wavelet Transform (MODWT) to our original returns series. Unlike CWT, the MODWT requires the researcher to specify time-scales for the returns and we specified a total of seven scales (2-4 days, 4-8 days, 8-16 days, 32-64 days, 64-128 days, and 128-256 days). Using the newly generated MODWT returns series, we examined the correlations between the French Islamic equity market returns and the other Islamic equity market returns and the results are presented below.[5]

[5] Please refer to Appendix 2 – 2.4 to see the underlying correlation tables and charts driven from the Matlab upon which the results are shown in this part of the study regarding MODWT methodology.

Table 4: Correlations of French Islamic Equity Market Returns vis a vis Danish, Belgian, Italian, and German Islamic Equity Market Returns

MODWT/Scaling	DANISH	BELGIAN	ITALIAN	GERMAN
2-4 Days	0.46	0.66	0.80	0.85
4-8 Days	0.58	0.71	0.82	0.86
8-16 Days	0.50	0.74	0.79	0.82
16-32 Days	0.46	0.68	0.81	0.79
32-64 Days	0.40	0.67	0.72	0.81
64-128 Days	0.44	0.74	0.68	0.83
128-256 Days	0.65	0.88	0.83	0.88

The results are remarkably consistent with the results obtained from the earlier CWT analysis. Among four markets, only Danish Islamic equity market stocks offer effective portfolio diversification opportunities to the French Islamic investors from very short until 6-month holding periods. However, other Islamic equity markets are quite highly correlated with the French Islamic equity market for all investment holding periods, thus eliminating any potential portfolio diversification opportunities for the French Islamic investors.

5.2.4. Summary of the Empirical Results

Table 5 below summarizes the empirical results from the study concerning the intra Euro Zone portfolio diversification opportunities for French Islamic investors. The table clearly highlights the answers to the objectives of this study as to whether any possible intra Euro Zone portfolio diversification opportunities for French Islamic investors exist or not, with consideration the dimension of different investment horizons of various investors. This contribution is most probably the first of its kind for French Islamic investors.

Table 5: The summary of the overall study in a categorized form

	DENMARK	BELGIUM	ITALY	GERMANY
MGARCH/DCC	Possible	Not Possible	Not Possible	Not Possible
MODWT	Possible (up to 6 moths)	Not Possible	Not Possible	Not Possible
CWT/WC	Possible (up to 6 moths)	Not Possible	Not Possible	Not Possible

6. Discussion of the Results

The studies such as Hardouvelis et al. (2006) find full integration among the Euro Area stock markets by the end of the 1990s. Morana and Beltratti (2002), Fratzscher (2002), Kim et al. (2005) and Bartram et al. (2007) report the evidences in favor of integration around the introduction of the single currency. Besides, Baele (2005) demonstrates the existence of volatility linkages among European stock markets surged by the early 1990s suggesting that economic integration boosted by the 1986 Single European Act and European capital markets liberalization played a more prominent role in stock markets convergence than the introduction of the Euro. Bley (2009) posits the implementation of the Maastricht criteria may have led to a period of real convergence of European economies promoted by the anticipation and subsequently the formation of the currency union renders Euro markets more integrated between 1998 and 2003 etc. all implying that high convergence among the Euro Zone markets led to difficulty to find intra-Euro Zone portfolio diversification opportunities. In line with these studies, this paper illustrates that there are portfolio diversification possibilities within Euro Zone Islamic stock markets on and off, not constantly, between few pair of Islamic financial markets, not all pair-combinations of Islamic financial markets, depending on time and investment holding period.

France, as the main interest of this study, has been ranked only Denmark's 8th largest supplier behind Germany, Sweden, China, the Netherlands, the UK, Norway and Italy; and its 7th largest customer behind Germany, Sweden, the UK, the United States, Norway, and the Netherlands. The French market share dropped from 4.8% in 2005 to 3.8% in 2010, down nearly 20%. Furthermore, France is ranked 6th among countries receiving Danish direct investment. When we look at other particular countries', which are the interests of this study, financial and economic ties with Denmark, France appears to be ranked at low levels. These are the conceivable reasons contributed France Islamic market returns having lowest correlations with Danish Islamic market returns in different time horizons and scales among four particular Euro Zone Islamic markets. These facts imply that France occupies quite weak economic and financial relationship with Denmark in comparison to other Euro Zone countries. In supporting to this claim, a sharp decrease in trade between France and Denmark in 2009 which was around -18.5% was observed[6].

7. Limitations and Suggestions for Future Research

The followings are some conceivable limitations of this study and hence present opportunities for future research. First of all, this study is limited to understand the relationships between the selected stock markets to those selected countries; they are quite commonly preferred by other studies in the field though. Therefore, the choice of indices is somewhat arbitrary. Since the all Euro Zone is the interest of this study, it would be better to include the other Euro Zone countries' Shariah indices as well. As the second limitation, we do not consider the performances of the returns, but only the volatilities and correlations of the selected Islamic

[6] For further information, please refer to http://www.diplomatie.gouv.fr/en/country-files/denmark/france-and-denmark/economic-relations-7270/

indices. To draw better conclusions, we need to consider the relationship between returns and volatilities, but this study only does consider the movements of the volatilities, ignoring the returns. Most importantly, this study does not attempt a comparative analysis between the group of Shariah Indices and Shariah non-compliant indices for the same markets to see either there are any differences or not.

8. Conclusion

Since the prominent study of Grubel (1968), the idea of international diversification resulting in lower portfolio risks is widely accepted in the literature. On the other hand, conflicting results of the succeeding studies (i.e., Dwyer and Hafer, 1988; Longin and Solnik, 1995; Errunza et al., 1999; Driessen and Laeven, 2007; Eun and Shim, 1989; Von Furstenberg and Jeon, 1989; Bertera and Mayer, 1990) in the same area render the topic not having a consistent implication for financial markets all around the world. Even though the studies given above focused on the correlations between the markets from constancy perspective, many recent studies have empirically tested and provided evidence that the correlations across national markets may not be constant and are evolving through time. Gencay et al (2001), for instance, was one of the earliest studies of the time scaled dependence of returns and correlations in financial markets.

The global financial markets over the past decade have witnessed that Islamic Financial Sector is rapidly expanding. Therefore, the growing importance of the Islamic financial sector attracts researchers and investors to study the volatilities and co-movements of asset returns amongst Shariah compliant indices across countries to provide especially for Islamic investors and fund managers idea on riskiness and potential international portfolio diversification benefits. Nevertheless, since Islamic stock markets are quite recent, studies on stock market integration among Islamic stock markets worldwide are still relatively scarce (Moeljadi, 2012).

Euro Zone financial markets have been one of the focal point in this research area since the claim is enhanced European financial markets integration led to reduction in intra-Euro Zone portfolio diversification benefits. Considering the fact that even though there are plenty of researches on the convergence and portfolio diversification issues centered on Euro Zone, the literature is lagging in regards to examining the topic from Islamic investors' interest. Having strong support from authorities to become a hub for Islamic financial services in Europe with 1.5 million clients, France is chosen as the focus Islamic financial market of this study.

Therefore, by utilizing three recent methodologies (M-GARCH/DCC, CWT, and MODWT), this paper has made an attempt to provide Euro Zone, especially for French Islamic investors some unique intra-Euro Zone portfolio diversification strategies depending on their stock holding periods (for e.g. weekly, monthly, quarterly, etc.). Even though the study does not include all markets under the Euro Zone, the most substantial of them have been taken by relying on the literature.

Our findings suggest that tight opportunity for portfolio diversification benefit appears in general for Euro Zone, especially for French Islamic investors only if they are restricted to Euro Zone Islamic equity markets. First step of our research indicates that Denmark is the only Islamic stock market with which other Euro Zone Islamic investors may create effective portfolios. Since these results are time-invariant, we proceeded to examining the dynamic conditional correlations, which capture the time-varying properties in volatilities and correlations. The results driven from Continuous Wavelet Transform (CWT) illustrate that even though some portfolio diversification opportunities appear between French vs. German, and French vs. Italian Islamic stock markets on and off with the consideration of time scale and investment holding period, French Islamic investors are better off including Danish Islamic stocks to diversify their portfolio for very short-run, short-run, medium to 6 month-time duration. For robustness, Maximum Overlap Discrete Wavelet Transform (MODWT) is applied to our original returns series and the results again suggest that only Danish Islamic equity market stocks, among four markets, offer effective portfolio diversification benefits to the French Islamic investors from very short until 6-month holding periods. However, other Islamic equity markets are quite highly correlated with the French Islamic equity market for all investment holding periods, thus eliminating any potential portfolio diversification opportunities for the French Islamic investors. Therefore, overall, Euro Zone Islamic equity market investors are highly suggested to get exposure to other global Islamic equity markets to create effective portfolios.

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APPENDIX

1. Applied Tests & Results on the M-GARCH/DCC Method

1.1 MGARCH Estimates of the Gaussian DCC model on selected Euro Zone Islamic Indices Daily Returns for Normal Distribution

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    Multivariate GARCH with underlying multivariate Normal distribution
                      Converged after 48 iterations
******************************************************************************
Based on 1485 observations from 07-Jul-08 to 14-Mar-14.
The variables (asset returns) in the multivariate GARCH model are:
BE DE FR GE IT
Volatility decay factors unrestricted, different for each variable.
Correlation decay factors unrestricted, same for all variables.
******************************************************************************
Parameter                Estimate       Standard Error         T-Ratio [Prob]
lambda1_BE                .90298            .024348           37.0871[.000]
lambda1_DE                .81381            .033289           24.4468[.000]
lambda1_FR                .88692            .024004           36.9484[.000]
lambda1_GE                .90337            .025901           34.8782[.000]
lambda1_IT                .90031            .018539           48.5615[.000]
lambda2_BE               .069444            .015417            4.5043[.000]
lambda2_DE                .11761            .019282            6.0996[.000]
lambda2_FR               .091276            .017948            5.0856[.000]
lambda2_GE               .084260            .021172            3.9798[.000]
lambda2_IT               .083797            .014366            5.8331[.000]
delta1                    .95812           .0047431          202.0058[.000]
delta2                   .022321           .0021154           10.5516[.000]
******************************************************************************
Maximized Log-Likelihood =    22956.3
******************************************************************************

                  Estimated Unconditional Volatility Matrix
     1485 observations used for estimation from 07-Jul-08 to 14-Mar-14
   Unconditional Volatilities (Standard Errors) on the Diagonal Elements
          Unconditional Correlations on the Off-Diagonal Elements
******************************************************************************
                BE         DE         FR         GE         IT
BE          .014958     .41958     .68654     .66597     .62845

DE           .41958    .017253     .50585     .48037     .46361

FR           .68654     .50585    .015125     .84460     .80547

GE           .66597     .48037     .84460    .017059     .76009

IT           .62845     .46361     .80547     .76009    .020553

******************************************************************************
For the time-varying conditional volatilities and correlations see the Post Estimation Menu.
```

1.2 MGARCH Estimates of the Gaussian DCC model on selected Euro Zone Islamic Indices Daily Returns for t-Distribution

```
3/21/2014                                                        5:37:15 PM

       Multivariate GARCH with underlying multivariate t-distribution
                     Converged after 131 iterations
*******************************************************************************
 Based on 1485 observations from 07-Jul-08 to 14-Mar-14.
 The variables (asset returns) in the multivariate GARCH model are:
 BE DE FR GE IT
 Volatility decay factors unrestricted, different for each variable.
 Correlation decay factors unrestricted, same for all variables.
*******************************************************************************
```

Parameter	Estimate	Standard Error	T-Ratio [Prob]
lambda1_BE	.92281	.025713	35.8894[.000]
lambda1_DE	.81306	.082554	9.8489[.000]
lambda1_FR	.91130	.020484	44.4883[.000]
lambda1_GE	.91353	.017981	50.8054[.000]
lambda1_IT	.91139	.018607	48.9800[.000]
lambda2_BE	.050336	.014217	3.5407[.000]
lambda2_DE	.12606	.049322	2.5559[.011]
lambda2_FR	.066964	.013960	4.7969[.000]
lambda2_GE	.072328	.013991	5.1695[.000]
lambda2_IT	.072857	.014034	5.1914[.000]
delta1	.96219	.0042441	226.7153[.000]
delta2	.021736	.0021663	10.0339[.000]
df	5.1629	.30107	17.1487[.000]

```
*******************************************************************************
 Maximized Log-Likelihood =    23379.0
*******************************************************************************
df is the degrees of freedom of the multivariate t distribution
```

Estimated Unconditional Volatility Matrix

1485 observations used for estimation from 07-Jul-08 to 14-Mar-14

Unconditional Volatilities (Standard Errors) on the Diagonal Elements

Unconditional Correlations on the Off-Diagonal Elements

****

	BE	DE	FR	GE	IT
BE	.014958	.41958	.68654	.66597	.62845
DE	.41958	.017253	.50585	.48037	.46361
FR	.68654	.50585	.015125	.84460	.80547
GE	.66597	.48037	.84460	.017059	.76009
IT	.62845	.46361	.80547	.76009	.020553

****

For the time-varying conditional volatilities and correlations see the Post Estimation Menu.

1.3 MGARCH - Testing for Linear Restrictions

```
 3/24/2014                                                              4:03:36 PM

                  Analysis of Function(s) of Parameter(s)
*******************************************************************************
 The variables (asset returns) in the multivariate GARCH model are:
 BE DE FR GE IT
 Volatility decay factors unrestricted, different for each variable.
 Correlation decay factors unrestricted, same for all variables.
 1485 observations used for estimation from 07-Jul-08 to 14-Mar-14
*******************************************************************************

 List of specified functional relationship(s):
 ZEROS=1- LAMBDA1_BE - LAMBDA2_BE
*******************************************************************************
 Function                  Estimate       Standard Error         T-Ratio[Prob]
 ZEROS                     .026850            .012625             2.1267[.034]
*******************************************************************************

        Estimated Variance Matrix of the Function(s) of the Parameters
      1485 observations used for estimation from 07-Jul-08 to 14-Mar-14
*******************************************************************************
              ZEROS
 ZEROS       .1594E-3
*******************************************************************************
--------------------------------------------------------------------------------
```

```
 3/24/2014                                                              4:06:21 PM

                  Analysis of Function(s) of Parameter(s)
*******************************************************************************
 The variables (asset returns) in the multivariate GARCH model are:
 BE DE FR GE IT
 Volatility decay factors unrestricted, different for each variable.
 Correlation decay factors unrestricted, same for all variables.
 1485 observations used for estimation from 07-Jul-08 to 14-Mar-14
*******************************************************************************

 List of specified functional relationship(s):
 ZEROS=1- LAMBDA1_DE - LAMBDA2_DE
*******************************************************************************
 Function                  Estimate       Standard Error         T-Ratio[Prob]
 ZEROS                     .060877            .034430             2.7681[.047]
*******************************************************************************
        Estimated Variance Matrix of the Function(s) of the Parameters
      1485 observations used for estimation from 07-Jul-08 to 14-Mar-14
*******************************************************************************
              ZEROS
 ZEROS       .0011855

******************************************************************************
```

```
 3/24/2014                                                     4:07:26 PM

                  Analysis of Function(s) of Parameter(s)
******************************************************************************
 The variables (asset returns) in the multivariate GARCH model are:
 BE DE FR GE IT
 Volatility decay factors unrestricted, different for each variable.
 Correlation decay factors unrestricted, same for all variables.
 1485 observations used for estimation from 07-Jul-08 to 14-Mar-14
******************************************************************************

 List of specified functional relationship(s):
 ZEROS=1- LAMBDA1_FR - LAMBDA2_FR
******************************************************************************
 Function                  Estimate      Standard Error        T-Ratio[Prob]
 ZEROS                     .021739          .0072052            3.0172[.003]
******************************************************************************

       Estimated Variance Matrix of the Function(s) of the Parameters
     1485 observations used for estimation from 07-Jul-08 to 14-Mar-14
******************************************************************************
              ZEROS
 ZEROS       .5191E-4

******************************************************************************
------------------------------------------------------------------------------
```

```
 3/24/2014                                                     4:08:12 PM

                  Analysis of Function(s) of Parameter(s)
******************************************************************************
 The variables (asset returns) in the multivariate GARCH model are:
 BE DE FR GE IT
 Volatility decay factors unrestricted, different for each variable.
 Correlation decay factors unrestricted, same for all variables.
 1485 observations used for estimation from 07-Jul-08 to 14-Mar-14
******************************************************************************

 List of specified functional relationship(s):
 ZEROS=1- LAMBDA1_GE - LAMBDA2_GE
******************************************************************************
 Function                  Estimate      Standard Error        T-Ratio[Prob]
 ZEROS                     .014146          .0044926            3.1488[.002]
******************************************************************************

       Estimated Variance Matrix of the Function(s) of the Parameters
     1485 observations used for estimation from 07-Jul-08 to 14-Mar-14
******************************************************************************
              ZEROS
 ZEROS       .2018E-4

******************************************************************************
```

```
 3/24/2014                                                           4:08:57 PM

                   Analysis of Function(s) of Parameter(s)
*******************************************************************************
 The variables (asset returns) in the multivariate GARCH model are:
 BE DE FR GE IT
 Volatility decay factors unrestricted, different for each variable.
 Correlation decay factors unrestricted, same for all variables.
 1485 observations used for estimation from 07-Jul-08 to 14-Mar-14
*******************************************************************************

 List of specified functional relationship(s):
 ZEROS=1- LAMBDA1_IT - LAMBDA2_IT
*******************************************************************************
 Function                  Estimate       Standard Error         T-Ratio[Prob]
 ZEROS                      .015758          .0052075             3.0260[.003]
*******************************************************************************


        Estimated Variance Matrix of the Function(s) of the Parameters
      1485 observations used for estimation from 07-Jul-08 to 14-Mar-14
*******************************************************************************
              ZEROS
 ZEROS       .2712E-4

*******************************************************************************
```

Indices	1-Lamda_1 -Lamda_2	Std.Errors	t-ratio
Belgium (BE)	.026850	.012625	2.1267
Denmark (DE)	.060877	.034430	2.7681
France (FR)	.021739	.0072052	3.0172
Germany (GE)	.014146	.0044926	3.1488
Italy (IT)	.015758	.0052075	3.0260

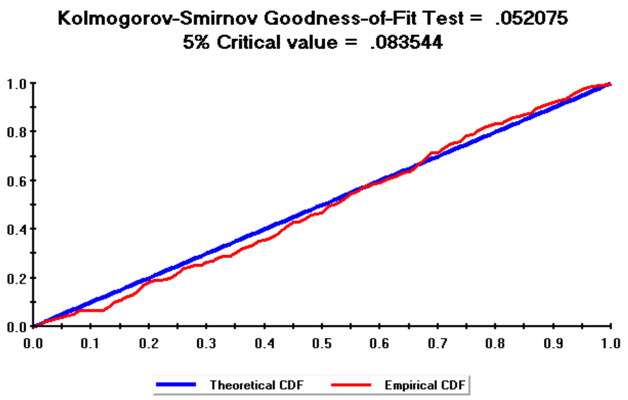
The above results suggest very slow but statistically significant mean reverting volatility for all Islamic stock indices.

1.4 M-GARCH- Testing the validity of the t-DCC model

```
                Test of Serial Correlation of Residuals (OLS case)
*******************************************************************************
 Dependent variable is U-Hat
 List of variables in OLS regression:
 Intercept
 265 observations used for estimation from 11-Mar-13 to 14-Mar-14
*******************************************************************************
 Regressor               Coefficient        Standard Error          T-Ratio[Prob]
 OLS RES(-1)               .016619             .062838               .26447[.792]
 OLS RES(-2)               .013568             .063039               .21522[.830]
 OLS RES(-3)             -.0065197             .063222              -.10312[.918]
 OLS RES(-4)              -.069795             .063208              -1.1042[.271]
 OLS RES(-5)               .027665             .063196               .43775[.662]
 OLS RES(-6)               .059895             .063447               .94400[.346]
 OLS RES(-7)               .042127             .063447               .66397[.507]
 OLS RES(-8)              -.074910             .063524              -1.1792[.239]
 OLS RES(-9)               .027671             .063904               .43302[.665]
 OLS RES(-10)             -.036982             .064468              -.57365[.567]
 OLS RES(-11)             -.094034             .064578              -1.4561[.147]
 OLS RES(-12)             -.073775             .064930              -1.1362[.257]
*******************************************************************************
 Lagrange Multiplier Statistic    CHSQ(12)=    8.2336[.767]
 F Statistic                      F(12,252)=    .67340[.776]
*******************************************************************************
 U-Hat denotes the probability integral transform.
 Under the null hypothesis, U-Hat should not display any serial correlation.
*******************************************************************************
```

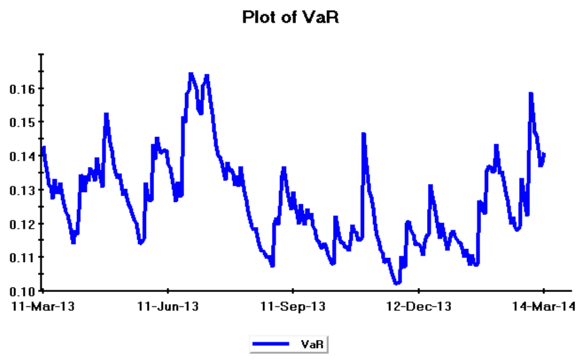
The LM test is equal to 8.2336 (P value = 0.767), which is not statistically significant; hence t-DCC model is correctly specified.

1.5 M-GARCH- Kolmogrov-Smirnov Goodness-of-Fit



The graph at the left side compares the empirical cumulative distribution function of the probability integral transform variable with that of a uniform. We can see that the Kolmogorov-Simimov test statistic is 0.052075, which is lower than 5% critical value of 0.083544. Therefore, we can claim that the probability integral transform is uniformly distributed.

1.6 M-GARCH- Plot of VaR



The table at the left side illustrates hat the mean hit rate (0.99623) is very close to the expected value (0.99000), and the test statistic is not significant, both supporting the validity of the t-DCC model.

```
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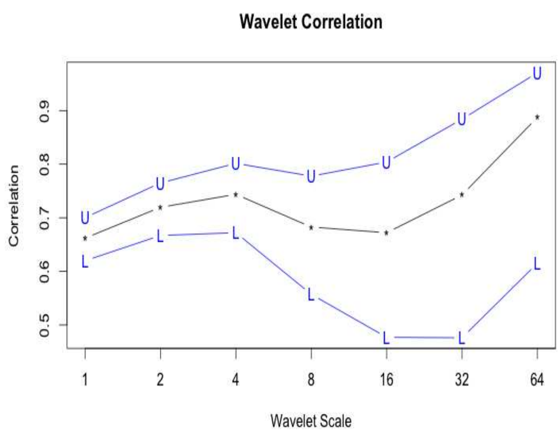
     Mean VaR Exceptions and the Associated Diagnostic Test Statistics
*******************************************************************************
 Mean Hit Rate (pihat statistic) =    .99623 with expected value of    .99000
 Standard Normal Test Statistic=   1.0187[.308]
*******************************************************************************
```

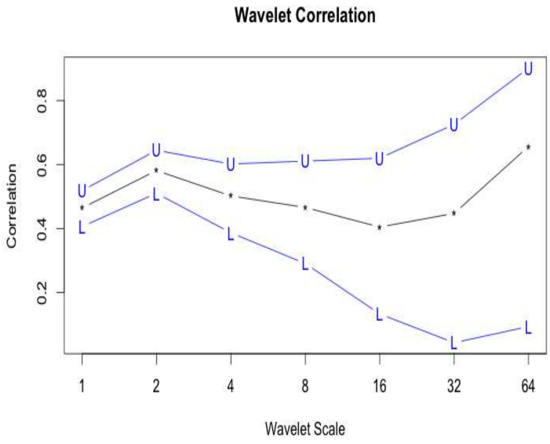
2. MODWT-Wavelet Correlation Charts & Underlying Numbers

2.1 MODWT-Wavelet Correlation France-Belgium



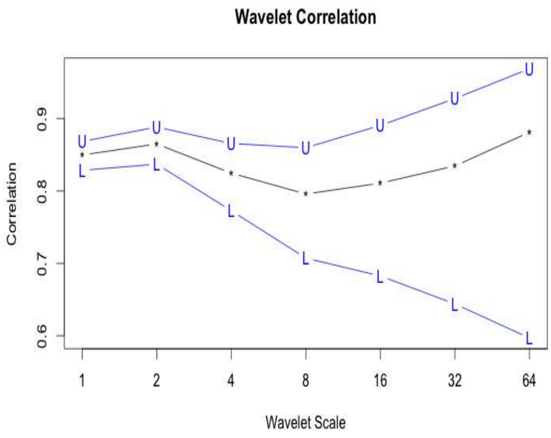
	wavecor	lower	upper
d1	0.662034	0.61988	0.700378
d2	0.719531	0.666928	0.764998
d3	0.743733	0.672165	0.801531
d4	0.682663	0.557104	0.777708
d5	0.672052	0.476927	0.804045
d6	0.887523	0.475955	0.884105
d7	0.887523	0.615163	0.970631
s7	0.77543	-0.33824	0.984301

2.2 MODWT-Wavelet Correlation France-Denmark



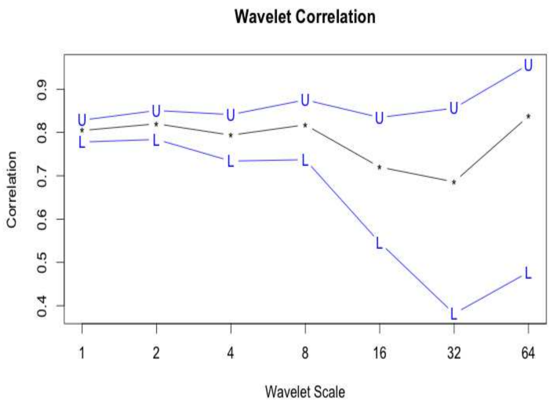
	wavecor	lower	upper
d1	0.463963	0.405935	0.518266
d2	0.581179	0.510021	0.644436
d3	0.50169	0.386316	0.601607
d4	0.465593	0.290368	0.610607
d5	0.404364	0.132594	0.619585
d6	0.447047	0.042716	0.725549
d7	0.656351	0.093153	0.901342
s7	0.92677	0.244066	0.995257

2.3 MODWT-Wavelet Correlation France-Germany



	wavecor	lower	upper
d1	0.849746	0.828591	0.868479
d2	0.865036	0.837138	0.888445
d3	0.82466	0.772748	0.865613
d4	0.795842	0.707274	0.859813
d5	0.810837	0.682614	0.890626
d6	0.834634	0.643987	0.927679
d7	0.881234	0.596897	0.968913
s7	0.936017	0.308589	0.995874

2.4 MODWT-Wavelet Correlation France-Italy



	wavecor	lower	upper
d1	0.804594	0.777845	0.828433
d2	0.820046	0.783925	0.850634
d3	0.79376	0.734033	0.841303
d4	0.817621	0.737118	0.875242
d5	0.720076	0.545764	0.834652
d6	0.685888	0.381563	0.856061
d7	0.837462	0.47744	0.956711
s7	0.8957	0.064043	0.993141