Slutsky equation and negative elasticity of labor supply: behavioral bias or optimal consumption-leisure choice?

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Slutsky Equation and Negative Elasticity of Labor Supply: behavioral bias or optimal consumption-leisure choice?
(For the centenary *of Sulla teoria del bilancio del consumatore*)

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Abstract
One of the applications of the prospect theory is the behavioral phenomenon of the negative elasticity of the individual labor supply. This paper argues that the negative elasticity of labor supply can be understood better with the help of the interpretation of the Slutsky equation with regard to the common consumption-leisure choice. The interpretation of the Slutsky equation corresponds to the empirical evidence that leisure is a net complement for an important part of consumption.

Key words: *Slutsky equation, prospect theory, labor supply, consumption-leisure choice*

JEL Classification: *D11.*

1.Introduction and Literature Review

In July 1915 the Italian *Giornale degli Economisti* published the article *Sulla teoria del bilancio del consumatore* written by Russian statistician and economist Eugen Slutsky. The further fate of the paper is well known. From the present point of view the destiny of this article looks like a real detective story even in the discreet and profound presentation of J.S. Chipman and J.-S. Lenfant: "As it now well known, Slutsky's article is one of the most famous examples of those neglected and ignored works whose originality and importance are recognized only after similar results have been obtained by others." (Chipman and Lenfant 2002, p.553)

Another discovery of Eugen Slutsky was accompanied by an even greater detective story. However, when R.G.D.Allen, J.R.Hicks, and H.Schultz recognized the Eugen Slutsky's priority in the discovery of the substitution and the income effects, R.Frisch appreciated much more moderately the importance of Eugen Slutsky's article on the summation of random causes as the source of cyclical processes (Slutsky 1927 [1937]), although "later historians have suggested that it was Slutsky's 1927 article that helped Frisch to construct a mathematical model of the trade cycle in which the oscillations were caused by exogenous shocks." (Barnett 2006, p.420). Hence the name *Economometrics*, given by Eugen Slutsky to the new economic science at birth, did not stick and gave way to the name of *Econometrics*.

However, the question of priority is of secondary importance with regard to the outcome of both Eugen Slutsky's discoveries that provided foundations for much of both neo-classical consumer theory and real business cycle theory. These findings have one common feature in its base – Eugen Slutsky's belief that if an economic phenomenon occurs, than no matter how random or irrational it looks, it should have a rigorous mathematical explanation. There is no doubt that M.Friedman who got a real historical chance to participate in the rediscovery of *Sulla teoria del bilancio del consumatore* in H.Schultz's team shared that belief when he described the positive approach to the economic theory. Unfortunately, the occasional reduction of the positive approach to the famous "as if" notation from the famous billiard metaphor of M.Friedman and L.J.Savage when they compared economic agents with billiard players, who made their shots *as if* they knew the complicated mathematical formulas (Friedman 1953), initiates rather excessive psychological generalizations of inconsistencies of economic behavior. Almost all of these generalizations try to challenge, more or less successfully, the formal economic modeling. Some of these generalizations are presented like applications of the prospect theory regarding the positive theory of consumer choice (Thaler 1980). However, the prospect theory, which is widely used in enlightenments of "anomalies and puzzles" of economic behavior, can well explain the choice of the billiards player between two risky shots but it cannot replace the natural laws underlying the trajectories of the balls. It has been already presented that some of applications of the prospect theory, i.e., "behavioral

inconsistencies", like the search for big-ticket items, the endowment effect, and the sunk costs sensitivity, could be explained by the marginal analysis of the consumer search behavior (Malakhov 2014a, 2014b).
There is another application of the prospect theory that challenges the traditional economic analysis. In 1997 Camerer et al. presented the results of the analysis of labor supply of New York City cab drivers (Camerer et al. 1997 [2000]). The authors of the paper discovered the negative elasticity of labor supply of inexperienced drivers that seemed to be inconsistent with the classical labor-leisure trade-off. And the revised version of that paper was presented in famous *Choices, Values, and Frames* of D.Kahneman and A.Tversky as one of the applications of the prospect theory.
The present paper argues that the negative elasticity of labor supply of inexperienced cab drivers can be understood better with the help of the interpretation of the Slutsky equation with regard to the common consumption-leisure choice.

2.Interpretation of the Slutsky equation

In 1972 the *American Economic Review* published the article of Ph.J.Cook, graduate student of the University of California, with the elegant interpretation of the Slutsky equation (Cook 1972) Later "Microeconomic Theory: basic principles and extensions" provided the illustrative adaptation of that "one-line" proof for students and instructors (Nicholson 1992, pp.148-150). If we slightly change the way of the "one-line" proof, we can get the illustrative interpretation of the Slutsky equation for the consumption-leisure choice.
When we analyze the negative labor supply elasticity it might be better to choose the indirect statement of the problem. We can replace the question "why the inequality $\partial L/\partial w<0$ occurs?" by the question "why the interrelated inequalities $\partial L/\partial P>0$ and hence $\partial H/\partial P<0$ take place for the given wage rate?" If we follow this indirect statement with regard to the consumption-leisure choice we come to the differential $dQ(P,H(P))$ where we can await both income and substitution effects. It is easy to show that this consumption differential is irrelevant to the labor-leisure choice for the given time horizon, or $dQ(P,H(P))= dQ(P,L(P))$. And we get:

$$dQ(P,H(P)) = dQ(P,L(P))$$

$$dQ(P,H(P)) = dP\left(\left.\frac{\partial Q}{\partial P}\right|_{Hconst} + \frac{\partial Q}{\partial H}\frac{\partial H}{\partial P}\bigg|_{U(Q,H)const}\right) = dP\left(\left.\frac{\partial Q}{\partial P}\right|_{Lconst} + \frac{\partial Q}{\partial L}\frac{\partial L}{\partial P}\bigg|_{U(Q,H)const}\right) \quad (1)$$

$$\partial Q/\partial H < 0; \partial H/\partial P > 0; \partial Q/\partial L > 0; \partial L/\partial P < 0$$

where the bottom line represents the set of common theoretic assumptions underlying consumer behavior.
We can compare graphically this interpretation with the Slutsky equation itself (Fig.1):

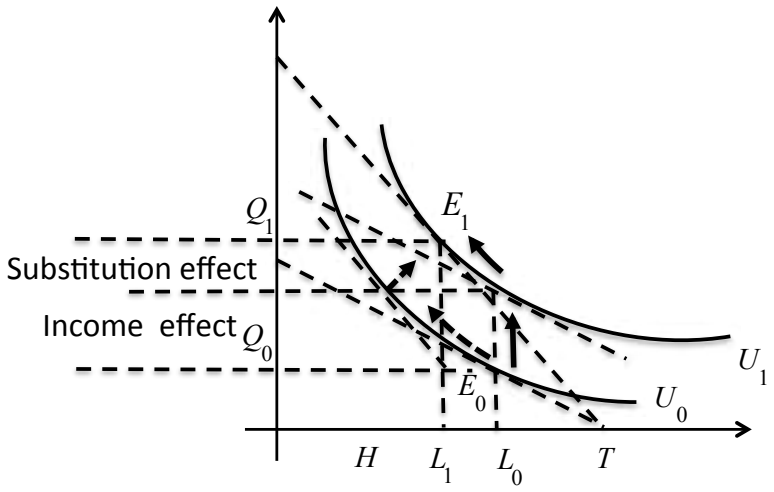

Figure 1.Graphical interpretation of the Slutsky equation

We can follow the prices' fall from E_0 to E_1 along the dotted arrows. However, it is also possible to get the same way along the bold arrows. First, we come to the new utility level for the given allocation of time (L_{const}; H_{const}). This shift gives us the net income effect for the given income wL. Second, we get the substitution differential dQ where we multiply the change in labor supply by the original marginal rate of

substation of leisure for consumption ($dQ=dL\times\partial Q_0/\partial L_0=dL\times w/P_0=-dL\times\partial Q_0/\partial H_0$). Then we can include the constant wage rate into the substitution differential and get the total derivative $dQ(P,L(P))/dP$:

$$\frac{dQ(P,L(P))}{dP}=\frac{\partial Q}{\partial P}\bigg|_{Lconst}+\frac{\partial Q}{\partial L}\frac{\partial L}{\partial P}\bigg|_{U(Q,H)const}=\frac{\partial Q}{\partial P}\bigg|_{wLconst}+\frac{\partial Q}{\partial wL}\frac{\partial wL}{\partial P}\bigg|_{U(Q,H)const} \quad (2)$$

This is the final result of the interpretation of the Slutsky equation, where the value $\partial Q/\partial P|_{wLconst}$ represents the income effect and the value $\partial Q/\partial P|_{U(Q,H)const}$ represents the substitution effect. However, it gives us only approximated results and it looks not yet illustrative. Nevertheless, its elasticity form can justify the interpretation itself as well as its approximated results:

$$\frac{P}{Q}\frac{dQ(P,L(P))}{dP}=\frac{P}{Q}\frac{\partial Q}{\partial P}\bigg|_{wLconst}+\frac{P}{Q}\frac{\partial Q}{\partial wL}\frac{\partial wL}{\partial P}\bigg|_{U(Q,H)const}\frac{wL}{wL};$$

$$e_{Q,P}=e_{Q,P}\big|_{wLconst}+e_{Q,wL}\,e_{wL,P}\big|_{U(Q,H)const}=-1+1\times e_{wL,P}\big|_{U(Q,H)const} \quad (3)$$

$$e_{Q,P}=-1+e_{wL,P}\big|_{U(Q,H)const}$$

However, even if the total price elasticity of consumption is held negative ($e_{Q,P}<0$), it gives us two different outcomes:

$$\partial H/\partial P\,|_{U(Q,H)const}>0\Rightarrow\partial wL/\partial P\,|_{U(Q,H)const}<0\Rightarrow e_{Q,P}<-1;$$

$$\partial H/\partial P\,|_{U(Q,H)const}<0\Rightarrow\partial wL/\partial P\,|_{U(Q,H)const}>0\Rightarrow e_{Q,P}>-1. \quad (4)$$

We see that for the inelastic demand ($-1<e_{Q,P}<0$) the ***leisure becomes the net complement for consumption***. While this conclusion doesn't correspond to the theoretic properties of the world of two goods, it finds the confirmation in the real world, where the «*empirical evidence indicates that leisure is a net complement for an important part of total consumption*." (Rousslang and Tokarick 1995,p.83). Moreover, the graphical presentation of the prices' fall with regard to stable preferences and the stable north-east-east consumption path (Q/H_{const}) tells us that the net leisure complementarity is really the common case (Fig.2):

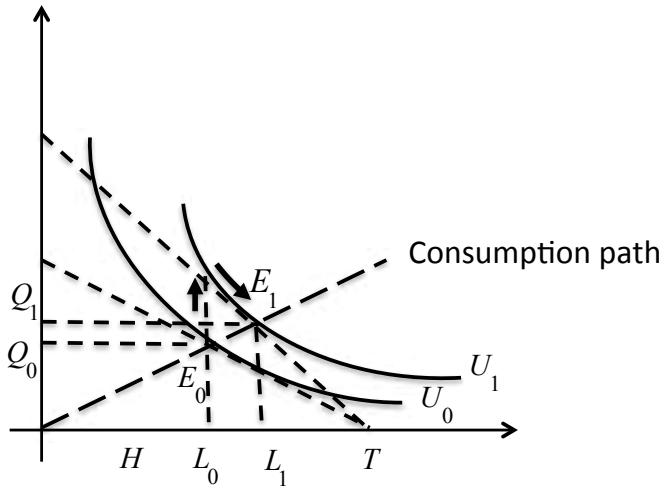

Figure 2. Graphical interpretation of the Slutsky equation for the stable inelastic demand

We can see that here the substitution effect decreases the income effect. This example tells us more about individual labor supply under inelastic demand that the behavioral bias as the application of the prospect theory, used by C.Camerer and his colleagues.

3.Conclusion

The authors of the paper '*Labor Supply of New York City Cab Drivers: one day at a time*' (Part V. *Applications* in *Choices, Values, and Frames*) ask readers in the conclusion to their paper: "*Critics who think our findings of negative elastiticities are an econometric fluke must explain why we did not find negative elasticities for experience drivers.*" The answer could be very simple – because the consumption of experience drivers was elastic while the consumption of inexperienced drivers, i.e., newcomers and beginners with basic needs, was inelastic and it resulted in the negative labor supply elasticity.

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