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Abstract. The current implementation of the Fan Chart displays equal tail probability bands and do not take into account that the variable of interest may be subject to data revision. In this note I propose the use of Highest Probability Density, HPD, bands and include flexibility to display the risks related to data revision. "Click here to obtain a Visual Basic for Excel routine." Please save the file as FanChartGdpGrowth.xls and enable Macros to run the program.

1. Introduction

The current implementation of the Fan Chart displays equal tail probability bands and do not take into account that the variable of interest may be subject to data revision. See Julio [4] and [5].

The *Fan Chart* was proposed for the first time by the *Bank of England*, who has included it as a part of its *Inflation Report* ever since 1997. See Britton, Fisher and Whitley [2] and Blix and Sellin [1].

For an inflation targeting Central Bank, the Fan Chart meets two objectives; First, it reveals to the public opinion the economic foresight of the Central Bank based on its *best knowledge* of the economy, a goal related to the transparency of the inflation targeting regime and the credibility of the policies to reach these targets. And second, the construction of the Fan Chart underlies the making of the forecasts, which has to do with the production and organization of the inflation report.

The current implementation of the Fan Chart used in the Colombian Central Bank follows the lines of the Britton, Fisher and Whitley [2] and Blix and Sellin [1], and presents equal tail probability bands. For highly skewed densities these bands may be misleading as the mode might lay outside the central band, which is centered at the median of the forecasting distribution. In this note we propose the use of HPD bands. HPD bands are not only narrower, by definition, but also its central band centers at the mode of the forecasting distribution. See Casella and Berger [3].

Moreover, many variables of interest may be subject to data revision at the end of the observed sample, a fact that might have a substantial effect on the present

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and near past foresight of the variable. Substantial data revision may be observed, for instance, in GDP figures in emerging economies.

A Visual Basic for Excel routine that includes these improvements, and hypothetical GDP growth data is used to illustrate its results. Apart from the introduction this note has only one section in which we summarize the computation of the HPD regions for a split normal density, the relevant information regarding the data revision and an example.

2. HPD Regions, Data Revision and Example

2.1. **HPD Regions for the Split Normal Density.** For a unimodal probability density function $f_X(x)$, a size α Highest Probability Density corresponds to the shortest interval $[x_l, x_u]$ such that $P[x_l \leq X \leq x_u] = \alpha$. If the interval $[x_l, x_u]$ satisfies

- i. $\int_{x_l}^{x_u} f_X(x)dx = \alpha$,
- ii. $f_X(x_l) = f_X(x_u) > 0$, and
- iii. $x_l \leq \mu_o \leq x_u$ where μ_o is the unique mode of $f_X(x)$

Then $[x_l, x_u]$ is the shortest interval satisfying (i). See Casella and Berger [3].

It is easily seen that the HPD region of size α for the split normal density is determined by

$$\begin{aligned} x_u &= \mu_0 + \sigma_2 \Phi^{-1}(1-\alpha/2) \\ x_l &= \mu_0 - \sigma_1 \frac{x_u - \mu_o}{\sigma_2} \end{aligned} \tag{2.1}$$

2.2. **Data Revision.** For a particular horizon, h, before the end of the sample, let the historical RMSE of revision be denoted as σ^2_{T-h}, the estimated mode of the variable be denoted as μ_{T-h}, and the balance of risks, $p_{T-h} = P[X_{T-h} < \mu_{T-h}]$ for $h = 0, 1, 2, ..., H$. The historical RMSE of revision is corrected according to an uncertainty multiplier that accounts for idiosyncratic uncertainty shifts with respect to the average historical uncertainty.

The triplet $(\mu_{T-h}, \sigma^2_{T-h}, p_{t-h})$ is transformed into the triplet $(\mu_{T-h}, \sigma^2_{1,T-h}, \sigma^2_{2,T-h})$, which determines the split normal density at time $T-h$, for $h = 0, 1, 2, ..., H$. This triplet is then used to compute the HPD regions 2.1 and the associated probability table. See Julio [5].

2.3. **Example.** Hypothetical data on four factors, RMSE of revision, impulse responses, etc., with data until 2008Q4 help us illustrate the results for hypothetical GDP growth, which are shown in figure 1 and table 1.

FIGURE 1. The Fan Chart

TABLE 1. Probability Distribution Of Future GDP Grpwth for Selected Ranges

Prob	2009 Q1	2009 Q2	2009 Q3	2009 Q4	2010 Q1	2010 Q2	2010 Q3	2010 Q4	2011 Q1
Pr.{ <-6%}	0.00%	0.00%	0.00%	0.11%	0.69%	1.05%	1.07%	0.75%	0.55%
Pr.{-6%–5.29%}	0.00%	0.00%	0.00%	0.19%	0.62%	0.75%	0.72%	0.53%	0.40%
Pr.{-5.29%–4.58%}	0.00%	0.00%	0.02%	0.45%	1.05%	1.18%	1.12%	0.84%	0.65%
Pr.{-4.58%–3.87%}	0.00%	0.00%	0.10%	0.97%	1.72%	1.78%	1.66%	1.28%	1.01%
Pr.{-3.87%–3.16%}	0.00%	0.00%	0.42%	1.92%	2.66%	2.59%	2.38%	1.88%	1.51%
Pr.{-3.16%–2.45%}	0.00%	0.05%	1.44%	3.46%	3.90%	3.61%	3.29%	2.65%	2.17%
Pr.{-2.45%–1.74%}	0.01%	0.55%	3.97%	5.72%	5.46%	4.85%	4.39%	3.62%	3.02%
Pr.{-1.74%–1.03%}	0.31%	3.30%	8.77%	8.63%	7.26%	6.25%	5.64%	4.76%	4.05%
Pr.{-1.03%–0.32%}	3.34%	11.71%	15.47%	11.87%	9.17%	7.75%	6.99%	6.04%	5.24%
Pr.{-0.32%-0.39%}	15.52%	24.55%	21.82%	14.92%	11.02%	9.23%	8.36%	7.38%	6.54%
Pr.{0.39%-1.1%}	31.23%	30.28%	24.38%	17.12%	12.59%	10.58%	9.63%	8.70%	7.87%
Pr.{1.1%-1.81%}	29.25%	20.64%	16.99%	17.69%	13.67%	11.65%	10.71%	9.89%	9.13%
Pr.{1.81%-2.52%}	15.12%	7.39%	5.67%	12.17%	14.04%	12.33%	11.48%	10.84%	10.22%
Pr.{2.52%-3.23%}	4.42%	1.38%	0.89%	4.09%	10.74%	12.35%	11.86%	11.45%	11.03%
Pr.{3.23%-3.94%}	0.73%	0.14%	0.06%	0.66%	4.38%	9.05%	10.80%	11.58%	11.48%
Pr.{3.94%-4.65%}	0.07%	0.01%	0.00%	0.05%	0.93%	3.89%	6.60%	9.55%	11.08%
Pr.{4.65%-5.36%}	0.00%	0.00%	0.00%	0.00%	0.10%	0.97%	2.57%	5.44%	8.05%
Pr.{5.36%-6.07%}	0.00%	0.00%	0.00%	0.00%	0.01%	0.14%	0.64%	2.13%	4.10%
Pr.{ >6.07%}	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.11%	0.69%	1.90%
Pr. {<Mode}	44.87%	52.67%	65.23%	74.45%	78.70%	79.09%	78.17%	76.52%	76.07%

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