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Leverage, return, volatility and contagion: Evidence from the portfolio framework

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Abstract

When regulating the financial system, the volatility phenomenon seems to emerge, practically, as a phenomenon which is intrinsic to the capital market behaviour. Theoretically, the leverage of the firms appears to be a major determinant of the volatility of prices and returns. At the same time, the leverage has also got a role at both levels: the capital structure of the firm and the investors' strategy. We examine the return and volatility in relation to leverage by considering different sized portfolios constructed based on the firm's level of debt and taken from a panel of 320 firms distributed over eight European countries and classified by their level of debt and their size. The optimal portfolio weights are computed for each quarter by maximizing the value of Sharpe ratio. We analyze the return, the volatility and the Value at Risk (VaR) based on different investors' strategies with a view to taking into account the capital structure and the level of the debt of the firms. Our findings tend to indicate that in the case of two separate equity funds (low debt and high debt), the optimal portfolio is obtained for a weight with high low debt fund. Overall, the leverage seems to have a big role for the portfolio return, volatility and value at risk (VaR). The high leverage is indicative of having a big role in making worse the portfolio return and volatility under shocks. Finally, we explore the value of systematic risk in the case of several portfolio strategies based on high and low debt in regard to the benchmark index (the MSCI Europe index). The presence of these effects is further explored through the response of the model's variables to market-wide return and volatility shocks.

Keywords: Volatility, leverage, contagion, Mean Variance Efficient Frontier, Wavelet Timefrequency analysis

1. Introduction

Investors have been seeking greater returns while minimizing risk. They may accept to bear more risk in exchange of higher returns that can be earned (Fischer, 1991). Since the investors are tempted to add any stock to their portfolio whose future returns are high, it is important to consider the leverage factor as the latter may affect the level of risk besides return. The level of debt (leverage) of a firm can play a significant role in decision-making for optimal allocation of resources within the portfolio management framework and market mechanism that can operate in a more efficient allocation of financial resources.

This study attempts to analyze the impact of the leverage on the portfolio behavior in terms of return and volatility in the European stock market.

To do so, we are using tools such Sharpe ratio, Capital Market line (CML) and Value at Risk (VaR) and Portfolio optimization based mean variance efficient frontier (MVEF) to elucidate the leverage effect on the portfolios return.

The rest of this paper is organized as follows. Section 2 presents the literature review. In section 3, we present the Optimal portfolio, the Capital Market Line (CML), the portfolio optimization based Sharpe ratio in general and in the case of two assets and and highlight how to compute its systematic risk. We also define the portfolio evaluation and the European portfolio construction used in this study. In section 4, we analyze the sensitivity in terms of returns and volatility of the proposed portfolio policies to changes in the leverage (Low debt versus high debt). In section 5, we compare the different policies related to the portfolio evaluation. Section 6 concludes.

2. Literature review

When regulating the financial system, the volatility phenomenon seems to emerge, practically, as a phenomenon which is intrinsic to the capital market behaviour. Theoretically, the leverage of the firms appears to be a major determinant of the volatility of prices and returns. Investors are interested in maximizing the return and minimizing the risk of their portfolios by finding the best optimal-weighted portfolio. Therefore, they have to hold a portfolio on the mean-variance efficient frontier, which was first defined by Markowitz (1952). In this paper we aim to analyze the impact of leverage on volatility of different equity portfolios taken from eight European countries. To do so, we consider, low and high debt firms and the combined portfolios.

Polasek and Pojarliev (2008) have compared the performance of different strategies with the MSCI (Europe index as benchmark) using VAR-GARCH model for European countries.

They also support their analysis by using cumulative return, the Sharpe ratio, the geometric mean, the Success rate, etc..

They conclude that the multivariate volatility timing strategies outperform the benchmark index and even a small country can be used to contribute to a better overall portfolio return.

3. Methodology and data collection

3.1 Optimal portfolio for the Investor

The optimal-weighted and equal-weighted portfolios are constructed on a daily basis, where the allowed VaR is set at a confidence level of 5% for each portfolio.

When the factors change (i.e - oil prices go up, or growth goes down), the sensitivities of stocks may be affected by it. This is called Active Factor Risk.

Active Specific Risk has to do with the particular stocks you have picked to be in your portfolio. Their subsequent performance and volatility directly affect your portfolio.

Two types of risk should be taken into account by a portfolio manager having a number of stocks in its equity fund which are exposed to macro and micro-economic factors:

1. How does the portfolio's sensitivity change when the active factors change?

2. How does the portfolio's return and volatility change when we add or remove the individual stocks in the portfolio? (active specific)

3.2 The Capital Market Line (CML) or Capital asset line (CAL)

The CAL with the highest Sharpe ratio is the CAL with respect to the tangency portfolio. In equilibrium, the market portfolio is the tangency portfolio.

The market portfolio's CAL is called the Capital Market Line (CML)

The CML gives the risk-return combinations achieved by forming portfolios from the risk-

free security and the market portfolio.

Risk-averse investors prefer lower to higher risk for a given level of expected return. Investors accept high risk investment only if expected return are greater:

1- Risk neutral: expected return is 16 what ever risk is ++> CAL Capital Allocation line utility curve is represented by a horizontal line

2- Risk-averse, utility curve -- vertical line for the same risk we are getting higher return

for one unit of risk, B is less risk-averse . the steeper the curve is the more risk averse ..

U = E(r) - 1/2 * A * Variance ==> Higher utility ==> happier investor with high U ==> E(r) = 1/2 A sigma 2 + U

The optimal portfolio for the Investor will be the curve with the higher Utility and intersection with the CML which is obtained with the maximum of Sharpe ratio.

3.3 Sharpe ratio optimization

The Sharpe ratio given as follows:

$$SR = \left(\frac{r_i - \sigma_f}{r_i}\right)$$
 (1)

The optimal portfolio weights are computed for each quarter by maximizing the value of Sharpe ratio using the expected return (minus the mean risk- free return) of a portfolio and its volatility. In this context the risk-free return refers to the mean of the short term interest rate of the eight European countries. Furthermore, transaction costs are supposed to be equal zero between trading quarters. The VaR has to be at its maximum and this is also implemented in the optimization model.

To be able to use the Shrape ratio, we consider the fact that a risk-free asset is available for investment based on the mean of the short term interest rate for the eight European countries.

So, we need to find the weights for a portfolio of minimum variance that has a fixed expected return. The minimum variance is reached at the point with lowest possible variance. Finding the portfolio with the lowest variance for a given expected return will provide the mean-variance frontier based on the marginal utility obtained at the First Order Condition (FOC) used in the Asset Pricing Theory (Back, 2010).

3.4 Portfolio optimization in the case of two assets

By using the Lagrangean multiplier, the First Order Condition (FOC) in the case of a portfolio of two assets and with minimum variance is given as follows:

$$w_1^* = (\sigma_2^2 - \sigma_{12})/(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) (2)$$

and the diversification principle applying the second derivative from the First Order Condition (FOC), then we get:

$$\frac{\partial \sigma_p^2}{\partial w_1} (w_1 = 0) = 2 \sigma_1 \sigma_2 \left(\rho_{12} - \sigma_2 / \sigma_1 \right) (3)$$

 w_1 is the weight of the first portfolio, then $(1 - w_1)$ will be the weight of the second portfolio in the combination of the two portfolios in one.

- If $\rho_{12} < 0$ or [if $\rho_{12} > 0$ but $\frac{\sigma_2}{\sigma_1} > \rho_{12}$], then $\frac{\partial \sigma_p^2}{\partial w_1}$ ($w_1 = 0$) < 0, in this case, we should increase w_1 (i.e. buying p1).

- If If $\rho_{12} > 0$ but $\frac{\sigma_2}{\sigma_1} < \rho_{12}$, then $\frac{\partial \sigma_p^2}{\partial w_1}$ ($w_1 = 0$) > 0, so we should decrease w_1 (i.e. short-sell p1).

3.5 Systematic risk for a portfolio with two assets

In general, the systematic risk is given as follows:

$$\beta = \left(\frac{\cos \left(r_{1}, r_{m}\right)}{\sigma_{m}^{2}}\right) \qquad (4)$$

In the case of two assets, the systematic risk can be expressed as:

$$\beta = \rho_{1m} \left(\frac{\sigma_1}{\sigma_m}\right) \quad (5)$$

3.6. European portfolio construction

In this section we briefly describe the different portfolio strategies we have used for the eight European countries (The list of the countries is given in the appendix.). For each firm the weights are determined by the following simple formula:

Portfolios have been classified into three categories (i) low debt (LD), (ii) high debt (HD) and (iii) the combined portfolio (LD + HD) based on the *debt ratio threshold*. This *threshold is determined* as the ratio of total debt to total assets of the portfolio. It is computed as follows: High Debt: HD (D2TASSETS > 0.33 and Low Debt - LD (D2TASSETS ≤ 0.33)

The total weights of each portfolio is equal to 1 and determined by the following simple formula in which $w_{i,t}$ is the weight of each firm within the portfolio:

 $1 = \Sigma w_{i,t}$

3.7. Portfolio evaluation

For our analysis, we are using the buy-and-hold portfolio strategy. This will allow us to compare the leverage effect between different portfolios.

To be able to compare the results of different portfolio strategies for different quarters, different returns and different VaR, we are using the cumulative normalized variables. The returns, volatility and VaR of the MSCI Europe index has been taken as benchmark of our comparison.

In our portfolio evaluation for the whole studied period we use the following criteria:

1. The cumulative normalized return based volatility is calculated as the integral function of the return related to the volatility. (See Appendix 1).

2. The cumulative normalized volatility (standard deviation) based return is calculated as the integral function of the volatility related to the return (See Appendix 1):

3. The cumulative normalized VaR (Value at Risk) based return is calculated as the integral function of the volatility related to the return (See Appendix 1).

4. The cumulative normalized VaR (Value at Risk) based volatility is calculated as the integral function of the VaR related to the volatility (See Appendix 1).

5. The Sharpe ratio for quarter Q is defined as the expected excess return of the portfolio divided by the standard deviation of the portfolio. Using the equation (1), we compute the Sharpe ratio as the ratio of the average return and the SD of the returns for the same quarter.

5. The Value At Risk (VaR) measures the maximum potential loss in the value of a portfolio over one period of time with a certain level of confidence. Here we take 95% as the level of confidence. The minimum VaR of a portfolio is located on the efficient frontier in the y axis and the volatility in x axis.

3.8 Data collection for the sample

The statistics for the return, volatility and the level of debt (D2TA) per country are given below:

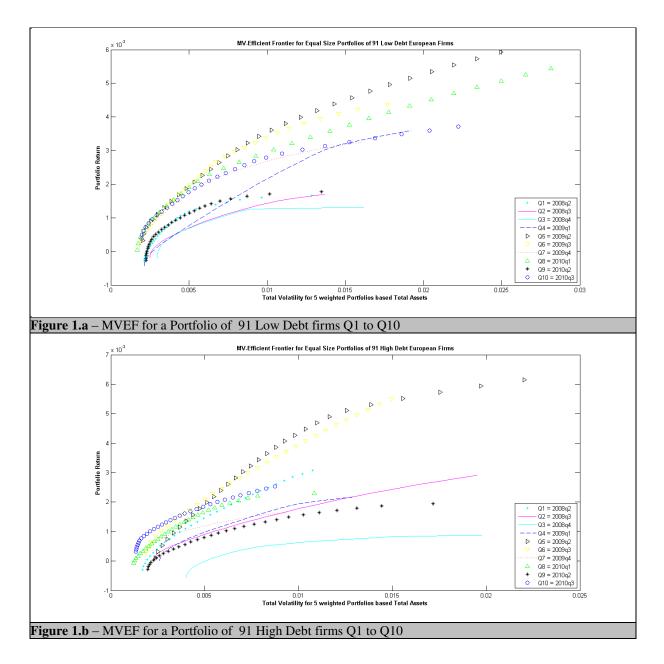
4. Results and discussion

In this section we have considered three portfolios: (i) 182 firms as a combined portfolio of low and high debt portfolios detailed in (ii) and (iii). (ii) A portfolio of 91 Low Debt firms, (iii) A portfolio of 91 high Debt firms.

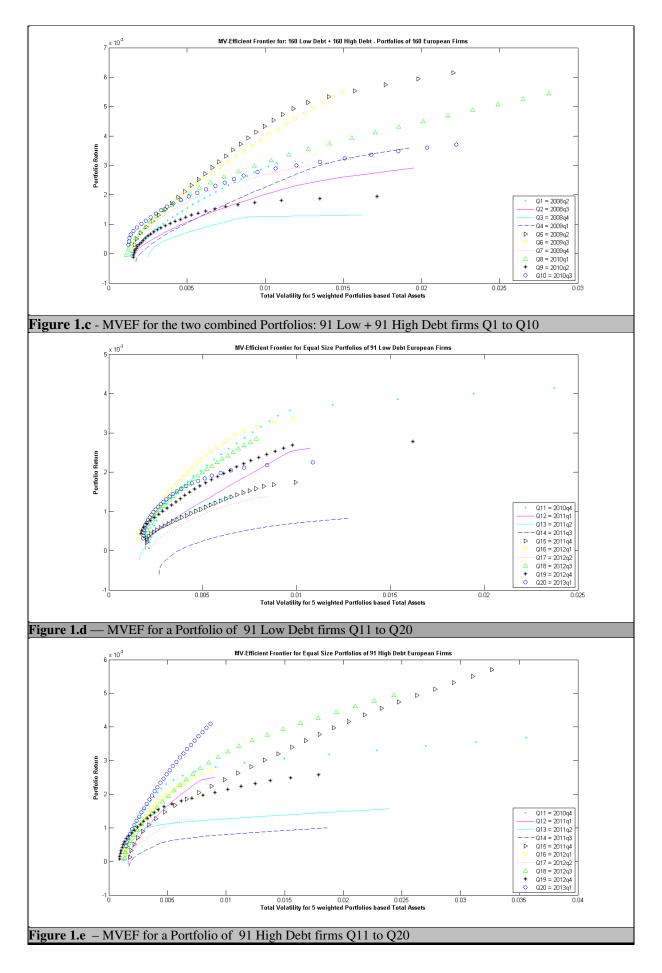
Figure 6.a show that during the GFC-2008, the combined 320 firms portfolio of high and low debt has very large variations volatility coupled with negative returns compared to the 160 firms portfolio of low debt for same period (see Figure 7.a). This shows that the diversification was not helping during the period of GFC-2008 and the low debt is offering more protection in terms of volatility.

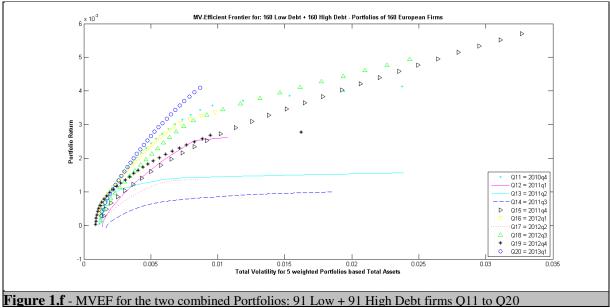
However, the quarter 3, 2008 (Q6) is showing more dispersion in the low debt portfolio (ii) in terms of volatility without offering any noticeable better return than the combined portfolio while the high debt portfolio (iii) is giving less volatility than the two previous one. This could be explained that outside the period of the GFC-2008, portfolio with high debt could offer less volatility than ones with low debt.

In most cases in terms of the 20 studied quarters, low debt portfolios are showing less dispersion of volatility and outperforming the high debt ones, while the combined portfolio is showing less volatility than the two other portfolios. We cannot conclude whether this result is due to the low debt effect or to the diversification effect. Additional analysis should be conducted in future studies in order to elucidate this issue.



4.1. Case of Combined portfolios of 182 European firms: 91 Low Debt and 91 High debt

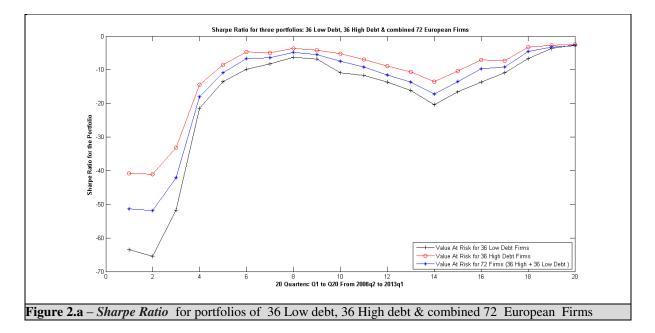


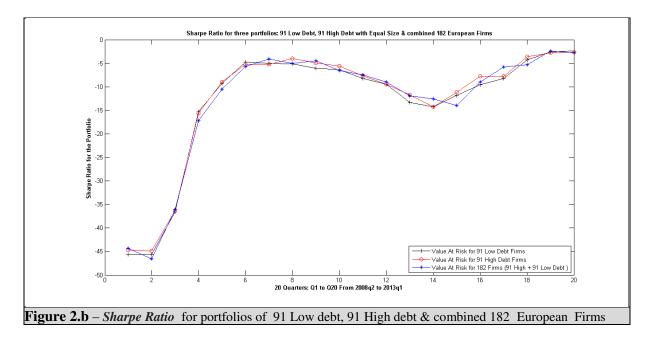


4.1. Sharpe Ratio for individual and combined portfolios of European Firms

Heretofore, we have optimized the weights related of the portfolios uing the MVEF without talking into account the CML (Capital Market line) based on the risk free rate. Which is involved when we are to maximizing the Sharpe ratio. In this section, we report the maximum Sharpe ratio for the 20 studied quarters of the three strategies with 36 and 91 firms: LD, HD and combined LD+HD equity portfolios.

In this section, we are computing the Sharpe ratio (Sharpe 1966) which measures the returnto-risk of a portfolio. Specifically, a portfolio the maximum of the Sharpe ratio is represented by the intersection between the tangency portfolio on the efficient frontier. To maximize the Sharpe ratio portfolio, first we use *portopt* in Matlab to get the weight, and the risk-return for the same portfolio for 30 different distributions of the weights. Then, this allows us to compute the Sharpe ratio. Then we determine the maximum among the 30 portfolios.





The figure 2.a shows that the LD portfolio present the best Sharpe ration compared to HD and the combined portfolios. It shows also a certain benefit to combine the LD and HD portfolios. However, in the case of figure 2.b, the values of Sharpe ratio are very close between the three portfolios showing no benefit to combine the LD and HD portfolios because of the existing over-diversification (91x2 = 182 compared to 36x2 = 72 firms).

The third point is the fact that there is a structural break, in the Sharpe ratio, happening before and after the GFC 2008. The latter seems to be a break point in the economy: a decrease in the absolute value of the Sharpe ration has become a permanent phenomenon 18 quarters after the crisis.

4.2. Sharpe Ratio maximized for the best combination between the low debt portfolio and high debt portfolio

In this section we consider the two portfolios as separate funds that could provide efficient investment service without any need to buy individual stocks separately. We have only to find the best combination between the two portfolios (Low debt and High debt) to the get the best return with the minimum volatility. This leads us to fin the maximum value of the Sharpe ratio.

However, two restrictive assumptions should be considered: (i) the Investors care only about mean and variance of returns, and (ii) there is a fixed investment horizon (buy and hold).

Table 10 in the Appendix 3 shows that the $\rho_{12} < 0$ which tends to indicate that the portfolio formed as a combination of the LD and HD portfolios is not optimized and we should increase the weight of the LD portfolio since the mean weight of the two portfolios in the third one are 0.1397 for the LD and 0.8603 for the HD. In this composition the formed portfolio of the two is not optimal. We will show in the next section that it is possible to get a higher μ for the less volatility by giving more weight to the LD portfolio.

It follows that the investors should choose put more weight on portfolio with low debt than the one with high debt to maintain higher pair of (μ, σ_p) .

From the table above, to obtain the market portfolio, the relative proportion of the low debt portfolio should be always higher than the weight of the high debt portfolio regardless of the level of the Sharpe ratio values.

Table 1: Maximizing the Sharpe Ratio for the combination of two separate funds in one Unified									
portfolio	<u>– (46LD</u>	+46HD) th	en - (91I	<u>_D+91HD</u>)				
Combinatio	on	Two separate	funds – 46l	LD + 46HD pc	ortfolios	Two separate	funds – 91L	D + 91HD port	folios
Quarter Number	20 Quarters 5 years	Sharpe Katio	Value At Risk	Weight of the Low Debt Portfolio	Weight of the Hight Debt Portfolio	Maxi. Of Sharpe Ratio	Value At Risk	Weight of the Low Debt Portfolio	Weight of the Hight Debt Portfolio
Q1	2008q2	47.035	0.00347	0.778	0.222	47.3473	0.00340	0.5251	0.4749
Q2	2008q3	46.968	0.00308	0.735	0.265	47.4222	0.00276	0.5021	0.4979
Q3	2008q4	37.670	0.00376	0.753	0.247	38.1649	0.00360	0.5175	0.4825
Q4	2009q1	15.559	0.00146	0.807	0.193	16.1895	0.00190	0.5604	0.4396
Q5	2009q2	9.637	0.00074	0.860	0.140	9.5697	0.00071	0.5972	0.4028
Q6	2009q3	5.096	0.00000	0.844	0.156	5.1675	0.00000	0.5900	0.4100
Q7	2009q4	5.394	0.00122	0.891	0.109	5.3804	0.00130	0.6369	0.3631
Q8	2010q1	5.298	0.00127	0.937	0.063	5.0010	0.00113	0.6930	0.3070
Q9	2010q2	6.158	0.00185	0.941	0.059	5.9604	0.00182	0.7010	0.2990
Q10	2010q3	6.500	0.00081	0.924	0.076	6.3890	0.00083	0.6973	0.3027
Q11	2010q4	8.586	0.00109	0.890	0.110	8.3656	0.00103	0.6603	0.3397
Q12	2011q1	9.771	0.00083	0.954	0.046	9.8091	0.00113	0.7329	0.2671
Q13	2011q2	13.774	0.00228	0.953	0.047	13.3371	0.00221	0.7344	0.2656
Q14	2011q3	14.612	0.00278	0.928	0.072	14.8864	0.00333	0.7082	0.2918
Q15	2011q4	12.093	0.00102	0.912	0.088	12.0674	0.00118	0.6895	0.3105
Q16	2012q1	9.906	0.00128	0.942	0.058	9.3679	0.00096	0.7299	0.2701
Q17	2012q2	8.436	0.00250	0.904	0.096	8.4051	0.00255	0.7025	0.2975
Q18	2012q3	4.145	0.00118	0.854	0.146	4.2206	0.00127	0.6554	0.3446
Q19	2012q4	2.962	0.00150	0.786	0.214	2.8097	0.00131	0.5942	0.4058
Q20	2013q1	2.372	0.00087	0.802	0.198	2.6202	0.00110	0.6126	0.3874

Case of two separate	Equity funds.	case of 46 firms and 91 firms
Cuse of two separate	Equity junus.	

It follows that the investors should choose put more weight on portfolio with low debt than the one with high debt to maintain higher pair of (μ, σ_p) for the two separate equity funds in the case of 46 firms and 91 firms.

4.3. Minimizing the Value At Risk: VaR

To be able to compare the VaR, we have chosen two equal sized portfolios based on their total assets and computed for the whole analyzed period from quarter 1 until quarter 20.

When we deal with risky assets, it is obvious that not only we have to maximize the Sharpe ratio but also we have to minimize the Var especially during bear periods.

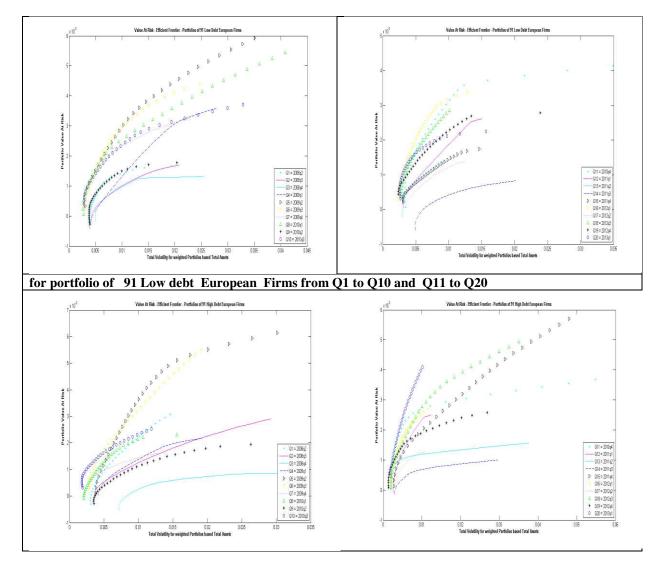
In this section, we have computed Value at Risk (VaR) for three kinds of portfolios: LD portfolio, HD portfolio and the combined portfolio between the two previous ones. We have taken 40 firms (20 LD and 20 HD) from each country. That makes 320 firms gathered from 8

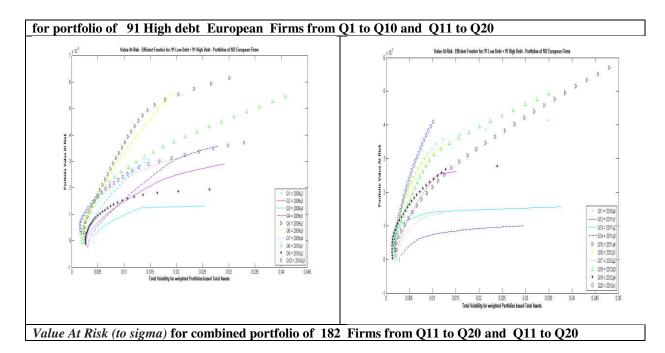
European studied countries. The VaR has been computed based on the volatility (sigma) then on the return as well for equal size portfolios of 91 low debt and 91 high debt firms, then for the combined portfolio from Q1 to Q20 (from quarter 2008 to quarter 1 2013)

Furthermore, we have added the capital structure of each portfolio and the level of debt (Debt over total assets) for each 20 quarters: from quarter 2, 2008 to quarter 1 2013 in order to encompass the GFC 2008 period.

4.3.1. Value At Risk in relation to volatility and return for portfolios with 91 Firms

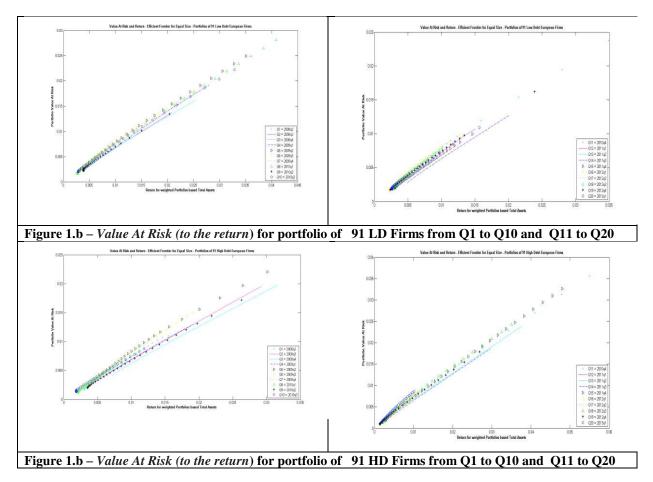
In this section, we report the graphs for Q1 to Q10 and Q11 to Q20 for quasi-equal size portfolio of 91 firms low and high deb; then combined portfolio. The quasi-equal size notion is based on the total assets of each portfolio. The difference of size of the two portfolios should not be beyond 5%.

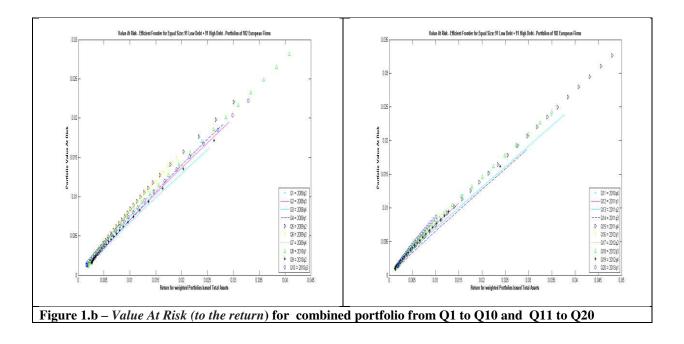




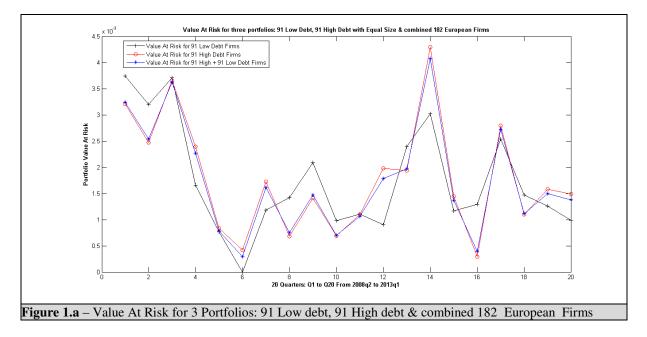
4.3.2. Value At Risk in relation to the return and return for portfolios with 91 Firms

- Equal size portfolio of 91 firms low and high debt for Q1 to Q10 and Q11 to Q20 and the combined portfolio of 182 firms.





4.4. Value at Risk for the three portfolios across the 20 analyzed quarters

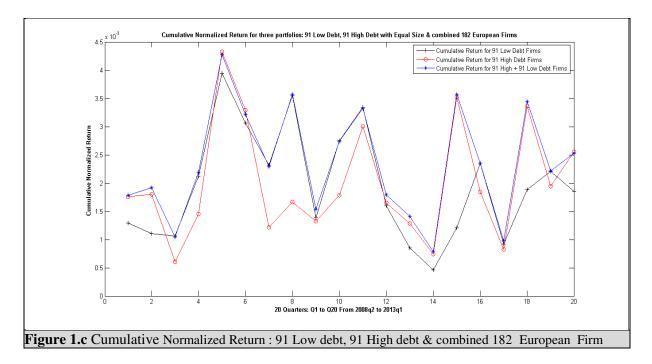


4.5. Cumulative Volatility, return and risk

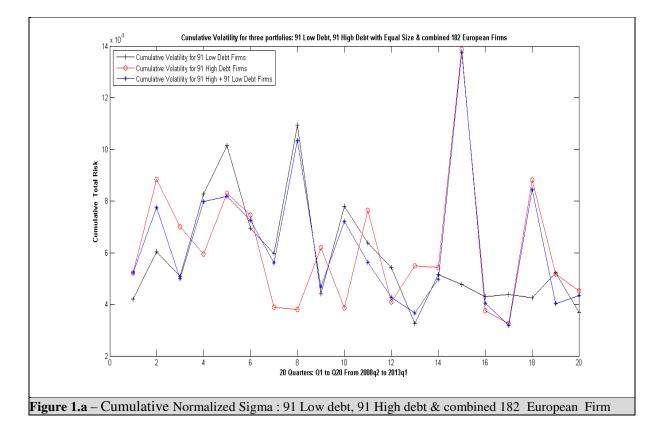
Figure XX and Table XX (Appendix 2) summarize the results according to the above criteria defined in section 5 (Portfolio Evaluation).

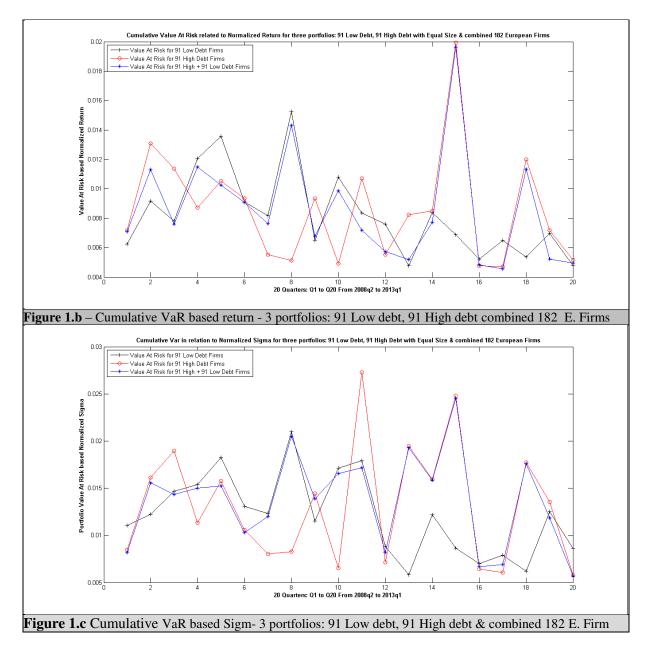
We compare three schemes of cumulative return: LD, HD and combined (LD+HD) portfolios over 20 quarters of evaluation period. The results show that LD and combined portfolio (LD+HD) with a large difference both are moving together in the same trend, yield about a

maximum of 55% (maximum at quarter 8) more returns than the HD portfolio in the evaluation period between Q3 and Q12. This happened just one quarter after the starting time of the GFC 2008. Those two portfolios seem to be as good strategies during this period of time.



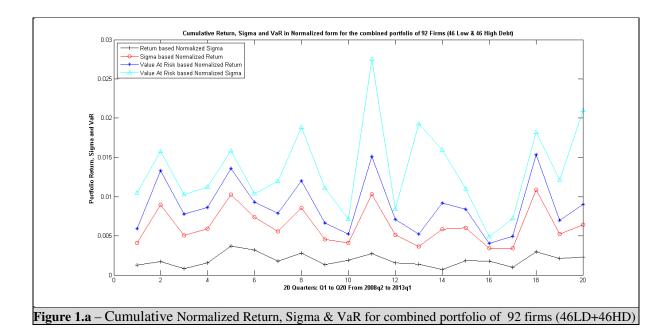
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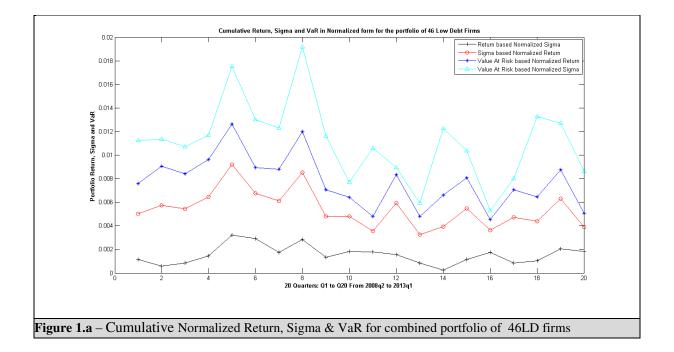


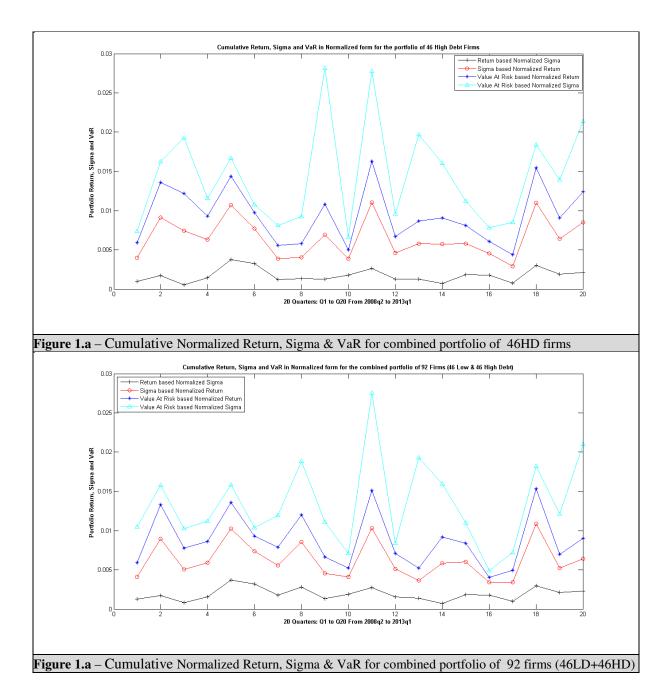


4.6. Case of Combined portfolios of 92 European firms: 46 Low Debt and 46 High debt

4.2.3.2 Cumulative Volatility, return and risk





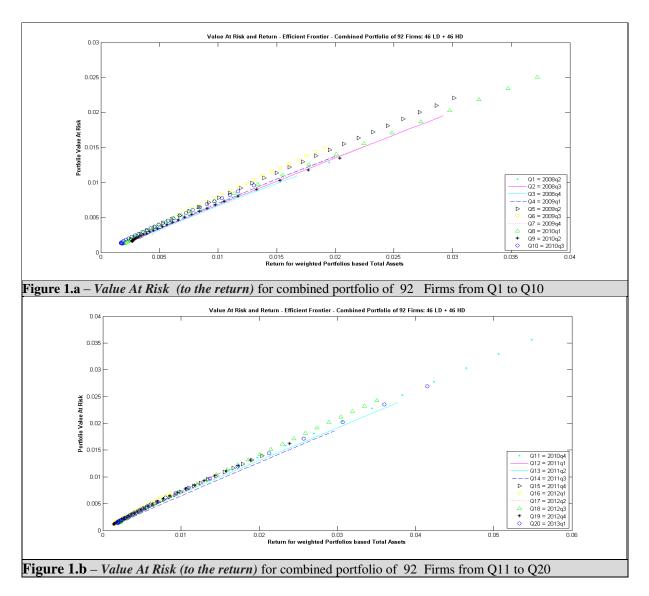


4.2.3.1 Value At Risk

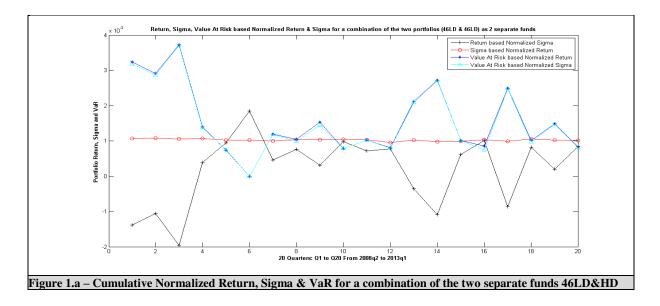
> Case of Value At Risk in relation to the return

- Equal size portfolio of 46 firms low and high debt for Q1 to Q10 and Q11 to Q20

- Combined portfolio 92 firms for Q1 to Q10 and Q11 to Q20



4.7. Return, Sigma and Value at Risk for a combination of the two portfolios (46LD & 46LD) as 2 separate funds across the 20 analyzed quarters



6. Conclusions

This paper obtains the optimal portfolio based on two separate equity funds: low debt and high debt. Overall, the leverage seems to have a big role for the portfolio return, volatility and value at risk (VaR). However, high leverage is indicative of having a big role in making worse the portfolio return and volatility under shocks.

Nonetheless, in most cases, the low debt portfolios management is quite successful and can give less volatility and higher returns with low debt portfolios compared to high debt portfolios.

We conclude returns of portfolios related to the high-level debt strategies for European countries can be improved considerably if those portfolios are combined with low-level debt strategies, while highlevel debt strategies alone could be detrimental for the performance and volatility.

Further research including more countries may show a better insight into the changing volatility structure of the European markets by, for example, extending the studied period of time. (e.g. using dynamic macro-economic models) and by creating sub-levels of debt (Very high level of debt and very low level of debt) in order to examine more accurately their effects on volatility and return.

Appendix 1	1
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Cumulative normalized						
return	Volatility					
CN _{Return}	CN _{VaR/σ}	CN _{VaR/r}	CN _{Return}			
$= \int_{0}^{1} r_i (\frac{d\sigma_i}{\sigma_{max} - \sigma_{min}})$	$= \int_{0}^{1} VaR_i \left(\frac{d\sigma_i}{\sigma_{max} - \sigma_{min}}\right)$	$= \int_{0}^{1} VaR_i \left(\frac{dr_i}{r_{max} - r_{min}}\right)$	$= \int_{0}^{1} \sigma_i \left(\frac{dr_i}{r_{max} - r_{min}} \right)$			
$(\sigma_{max} - \sigma_{min})$ stands for the range of the volatility $(r_{max} - r_{min})$ stands for the range of the return.						
r_i , VaR and	σ_i are the quarterly portfo	olio returns Value at Ris	k and volatility			

Table 10: FOC	Cable 10: FOC's Derivation for the Combination of the LD & HD portfolios (91 firms each)								
20 Quarters	σ1	σ2	<mark>و</mark> 12	σ2/σ1	FOC's Derivative*	σр			
2008q2	0.00105984	0.001069406	0.82129	1.009026	-4.2556E-07	0.0010602			
2008q3	0.001071347	0.001072146	0.82129	1.000745	-4.1226E-07	0.0010591			
2008q4	0.001081171	0.001087957	0.82129	1.006276	-4.3519E-07	0.0010602			
2009q1	0.001089282	0.001113069	0.82129	1.021837	-4.8631E-07	0.0010617			
2009q2	0.001062593	0.001100276	0.82129	1.035463	-5.0080E-07	0.0010610			
2009q3	0.001059948	0.00109469	0.82129	1.032777	-4.9078E-07	0.0010602			
2009q4	0.001033616	0.001085779	0.82129	1.050466	-5.1434E-07	0.0010599			
2010q1	0.001009898	0.001082996	0.82129	1.072381	-5.4925E-07	0.0010603			
2010q2	0.0010049	0.001080835	0.82129	1.075564	-5.5235E-07	0.0010600			
2010q3	0.001011221	0.001086149	0.82129	1.074096	-5.5533E-07	0.0010592			
2010q4	0.001010989	0.001071163	0.82129	1.059519	-5.1597E-07	0.0010581			
2011q1	0.00099214	0.001079967	0.82129	1.088520	-5.7267E-07	0.0010587			
2011q2	0.000987672	0.001075727	0.82129	1.089154	-5.6919E-07	0.0010589			
2011q3	0.001009808	0.001089022	0.82129	1.078444	-5.6558E-07	0.00105895			
2011q4	0.001018874	0.001091195	0.82129	1.070981	-5.5521E-07	0.0010595			
2012q1	0.001001617	0.001089065	0.82129	1.087306	-5.8036E-07	0.0010598			
2012q2	0.001012696	0.00108984	0.82129	1.076176	-5.6263E-07	0.0010604			
2012q3	0.001025698	0.001084774	0.82129	1.057595	-5.2585E-07	0.0010589			
2012q4	0.001042412	0.001078197	0.82129	1.034329	-4.7888E-07	0.0010584			
2013q1	0.001029745	0.0010722	0.82129	1.041228	-4.8567E-07	0.00105838			
	*	Derivative of	FOC: Firs	st Order C	ondition				

Appendix 2

Appendix 3

Table 11: Maximum Sharpe Ratio, VaR, and corresponding total weight of Low and High Debt firms for combined portfolio of 92 firms							
Quarter Number	20 Quarters. 5 years	Maxi. Of Sharpe Ratio (Abs Value)	Value At Risk	Total Weight of the Low Debt firms	Total Weight of the Hight Debt firms		
Q1	2008q2	33.267	0.002371	0.343	0.657		
Q2	2008q3	26.998	0.002922	0.345	0.655		
Q3	2008q4	14.719	0.004388	0.712	0.288		
Q4	2009q1	9.797	0.003179	0.615	0.385		
Q5	2009q2	5.908	0.002541	0.698	0.302		
Q6	2009q3	4.563	0.002294	0.622	0.378		
Q7	2009q4	4.110	0.002481	0.634	0.366		
Q8	2010q1	4.213	0.001990	0.320	0.680		
Q9	2010q2	3.578	0.002718	0.339	0.661		
Q10	2010q3	5.334	0.001876	0.187	0.813		
Q11	2010q4	7.382	0.001928	0.158	0.842		
Q12	2011q1	7.090	0.002331	0.325	0.675		
Q13	2011q2	9.140	0.002141	0.433	0.567		
Q14	2011q3	7.849	0.002922	0.075	0.925		
Q15	2011q4	7.760	0.002243	0.448	0.552		
Q16	2012q1	7.906	0.001583	0.376	0.624		
Q17	2012q2	5.268	0.002393	0.260	0.740		
Q18	2012q3	3.008	0.002188	0.207	0.793		
Q19	2012q4	2.410	0.001458	0.215	0.785		
Q20	2013q1	2.068	0.001949	0.334	0.666		

Table of	portfolio	of 92	firms
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able 12: Cumulative Return, Sigma, VaR (based Return & Sigma) for ombined portfolio of 92 firms (46 Low and 46 High Debt firms)						
Quarter Number	20 Quarters. 5 years		Cumulative Sigma		Cumulative VaR based Sigma	
Q1	2008q2	0.00126	0.00408	0.00592	0.01046	
Q2	2008q3	0.00171	0.00896	0.01330	0.01578	
Q3	2008q4	0.00084	0.00505	0.00776	0.01028	
Q4	2009q1	0.00156	0.00592	0.00861	0.01121	
Q5	2009q2	0.00373	0.01025	0.01361	0.01585	
Q6	2009q3	0.00318	0.00738	0.00929	0.01036	
Q7	2009q4	0.00176	0.00554	0.00789	0.01195	
Q8	2010q1	0.00280	0.00853	0.01200	0.01885	
Q9	2010q2	0.00132	0.00457	0.00666	0.01109	
Q10	2010q3	0.00188	0.00409	0.00524	0.00706	
Q11	2010q4	0.00271	0.01032	0.01512	0.02754	
Q12	2011q1	0.00158	0.00511	0.00710	0.00841	
Q13	2011q2	0.00141	0.00367	0.00522	0.01932	
Q14	2011q3	0.00072	0.00583	0.00916	0.01596	
Q15	2011q4	0.00181	0.00599	0.00837	0.01096	
Q16	2012q1	0.00180	0.00340	0.00406	0.00490	
Q17	2012q2	0.00097	0.00342	0.00496	0.00720	
Q18	2012q3	0.00299	0.01089	0.01534	0.01819	
Q19	2012q4	0.00214	0.00523	0.00700	0.01214	
Q20	2013q1	0.00228	0.00642	0.00902	0.02100	

Table of portfolio of 46 LD firms and 46 HD firms - Maxi Sharpe Ratio

able 13: Maximum Sharpe Ratio, VaR for 46 Low and 46 High Debt firms							
		Low debt portfoli	o of 46 firms	High debt portfolio of 46 firms			
Quarter Number	20 Quarters. 5 years	Maxi. Of Sharpe Ratio (Abs Value)	Value At Risk	Maxi. Of Sharpe Ratio (Abs Value)	Value At Risk		
Q1	2008q2	21.450	0.00388	24.949	0.00330		
Q2	2008q3	18.921	0.00435	19.960	0.00397		
Q3	2008q4	12.272	0.00523	8.844	0.00794		
Q4	2009q1	7.948	0.00405	6.025	0.00458		
Q5	2009q2	5.049	0.00322	3.720	0.00433		
Q6	2009q3	3.542	0.00293	3.265	0.00310		
Q7	2009q4	3.134	0.00294	3.033	0.00306		
Q8	2010q1	3.187	0.00279	3.860	0.00238		
Q9	2010q2	2.564	0.00391	2.879	0.00382		
Q10	2010q3	3.429	0.00295	4.993	0.00199		
Q11	2010q4	3.936	0.00360	6.431	0.00224		
Q12	2011q1	4.820	0.00308	6.032	0.00289		
Q13	2011q2	7.360	0.00311	7.445	0.00257		
Q14	2011q3	4.930	0.00516	7.706	0.00303		
Q15	2011q4	5.828	0.00320	6.727	0.00292		
Q16	2012q1	5.943	0.00239	6.102	0.00230		
Q17	2012q2	3.287	0.00369	4.130	0.00319		
Q18	2012q3	2.172	0.00295	2.613	0.00255		
Q19	2012q4	1.544	0.00242	2.060	0.00187		
Q20	2013q1	1.434	0.00264	1.671	0.00234		

E.

able 14: Cumulative Return, Sigma, VaR (based Return & Sigma) for aortfolio of 46 Low Debt firms							
Quarter Number	20 Quarters. 5 years	Cumulative Return	Cumulative Sigma	Cumulative VaR based Retrun	Cumulative VaR based Sigma		
Q1	2008q2	0.00114	0.00504	0.00761	0.01122		
Q2	2008q3	0.00059	0.00575	0.00906	0.01137		
Q3	2008q4	0.00084	0.00546	0.00842	0.01071		
Q4	2009q1	0.00143	0.00647	0.00961	0.01170		
Q5	2009q2	0.00321	0.00920	0.01266	0.01752		
Q6	2009q3	0.00292	0.00677	0.00896	0.01302		
Q7	2009q4	0.00176	0.00614	0.00878	0.01228		
Q8	2010q1	0.00283	0.00851	0.01200	0.01918		
Q9	2010q2	0.00134	0.00479	0.00707	0.01162		
Q10	2010q3	0.00182	0.00482	0.00641	0.00770		
Q11	2010q4	0.00178	0.00356	0.00480	0.01061		
Q12	2011q1	0.00156	0.00594	0.00833	0.00894		
Q13	2011q2	0.00085	0.00327	0.00482	0.00593		
Q14	2011q3	0.00023	0.00394	0.00661	0.01226		
Q15	2011q4	0.00115	0.00548	0.00808	0.01040		
Q16	2012q1	0.00176	0.00365	0.00453	0.00528		
Q17	2012q2	0.00085	0.00471	0.00705	0.00802		
Q18	2012q3	0.00103	0.00439	0.00647	0.01328		
Q19	2012q4	0.00205	0.00630	0.00875	0.01273		
Q20	2013q1	0.00184	0.00391	0.00508	0.00866		

	Table 15: Cumulative Return, Sigma, VaR (based Return & Sigma) for a combination of the two portfolios (46LD & 46LD) as 2 separate funds							
Quarter Number	20 Quarters. 5 years	Cumulative Return	Cumulative Sigma	Cumulative VaR based Retrun	Cumulative VaR based Sigma			
Q1	2008q2	-0.00139	0.00107	0.00324	0.00318			
Q2	2008q3	-0.00106	0.00108	0.00291	0.00286			
Q3	2008q4	-0.00196	0.00106	0.00372	0.00370			
Q4	2009q1	0.00039	0.00107	0.00139	0.00137			
Q5	2009q2	0.00095	0.00103	0.00075	0.00075			
Q6	2009q3	0.00185	0.00103	0.00000	0.00000			
Q7	2009q4	0.00046	0.00100	0.00119	0.00118			
Q8	2010q1	0.00077	0.00104	0.00105	0.00099			
Q9	2010q2	0.00031	0.00103	0.00153	0.00144			
Q10	2010q3	0.00097	0.00104	0.00078	0.00079			
Q11	2010q4	0.00072	0.00103	0.00103	0.00102			
Q12	2011q1	0.00077	0.00096	0.00081	0.00080			
Q13	2011q2	-0.00036	0.00102	0.00212	0.00209			
Q14	2011q3	-0.00110	0.00098	0.00272	0.00270			
Q15	2011q4	0.00061	0.00099	0.00101	0.00101			
Q16	2012q1	0.00102	0.00104	0.00086	0.00074			
Q17	2012q2	-0.00086	0.00099	0.00249	0.00248			
Q18	2012q3	0.00081	0.00106	0.00101	0.00097			
Q19	2012q4	0.00020	0.00102	0.00148	0.00148			
Q20	2013q1	0.00084	0.00101	0.00083	0.00081			

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