Non-linear effects of the U.S. Monetary Policy in the Long Run

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Abstract

We find non-linearities in the U.S. long-run relationships among trend inflation, growth rate and financial frictions. Moreover, our results show that mismeasurements of the natural rate of interest deviate the trend inflation from its target, which is especially clear when monetary policy reacts preventively against inflation deviations. The long-run growth rate, the trend inflation and the natural rate of interest, specified as time-varying, are jointly estimated over the period 1960:Q1-2013:Q2 by applying the Kalman filter, following mainly Laubach and Williams (2003).

JEL code: C32; D52; E31; E52

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1 Introduction

The existence of rigidities and frictions in the markets leads to non-neutral monetary policies in the long run and non-linear effects of some key variables according to neokeynesian dynamic models with endogenous growth (Amano *et al.*, 2009; Olmos and Sanso, 2014a,b). These two features are especially clear when monetary policy is conducted following some type of inflation targeting using the short-term interest rate as the instrument or, in other words, following a Taylor rule (Taylor, 1993). In this paper, we search for these nonlinearities for the case of the U.S. monetary policy. In particular, we are interested in three non-linear effects of the monetary policy in the long run: the non-linear relationships between the trend inflation and the growth rate, between the trend inflation and the external finance premium and between the growth rate and the error in the estimation of the natural rate of interest. But the possibility of finding this type of evidence is hindered by the problem of the non-observable character of the long-term variables.

In fact, the relevance of the long-term variables is crucial for the performance of the monetary policy carried out by central banks. Most specifications of Taylor rules include one or more long-term variables, whose unobservability is an intrinsic characteristic. The long-term variables which are usually incorporated into the monetary policy rules are the natural interest rate, the inflation target and the potential output[1]. As a result of the importance of these variables for the policy design, many

[1]We consider these variables as long-term variables instead of medium-term references.

contributions have been made on their estimation. Nevertheless, this task is not straightforward.

There are several approaches to estimating unobservable variables. The simplest techniques are the univariate filters such as that of Hodrick-Prescott, but these methods are only based on the statistical properties of the series and ignore the connections with other variables. Equilibrium models can also be built in order to estimate unobserved series as is done in, among others, Neiss and Nelson (2003), Smets and Wouters (2003), Giammaroli and Valla (2004) and Andrés, López-Salido and Nelson (2009), but the resulting estimates are based on subjective assumptions and are prone to be more volatile (Edge *et al.*, 2008). As an alternative to the foregoing methods, and admitting that this routine has been the object of some criticism[2], our approach is based on the Kalman filter applied to a semi-structural econometric model. This procedure has been implemented by many studies to estimate long-term values of several economic variables. However, most of the papers that follow this technique do not jointly estimate all the long-term variables involved in the monetary policy rules[3] nor emphasize the long-term perspective. By contrast, our approach simultaneously includes the natural rate of interest, the long-run growth rate and the steady-state inflation in the estimation process in order to capture all the long-run interactions we are interested in.

[2] As discussed in Weber, Lemke and Worms (2008).

[3] An exception is Bjørnland, Leitemo and Maih (2011), but they combine Bayesian and Kalman filter procedures. Benati and Vitale (2007) also obtain estimates of all the variables, but their purposes are far from ours as we focus on the long-term perspective.

The first long-run variable we estimate is the natural rate of interest defined as the long-run real rate of interest that ensures inflation stability and the reaching of the potential output. The estimation of this variable has attracted the interest of the literature since central banks conduct monetary policy through rules with this rate as the intercept. Moreover, the gap between the natural rate of interest and the actual real rate is very useful because it measures the monetary policy stance and has predictive power for future inflation. Many empirical studies have tried to assign a value to this rate, which initially was considered constant over time. Afterwards, in a seminal paper, Laubach and Williams (2003) drop the assumption of a fixed value[4] and estimate the time-varying natural rate of interest (TVNRI) for the U.S. by applying the Kalman filter. The papers that have followed this methodology are not few. Crespo-Cuaresma *et al.* (2004), Mésonnier and Renne (2007) and Garnier and Wilhelmsen (2009) estimate the TVNRI for the euro zone, Larsen and McKeown (2003) for the U.K., Manrique and Marques (2004) for the U.S. and Germany, Basdevant *et al.* (2004) for New Zealand, Brzoza-Brzezina (2006) for Poland and, recently, Bouis, *et al.* (2013) for Canada, the euro zone, Japan, Sweden, Switzerland, the U.K. and the U.S. Combining Bayesian methods with the Kalman filter, Edge, Kiley and Laforte (2008) and Bjørnland, Leitemo and Maih (2011) estimate the TVNRI for the U.S.[5].

[4]They argue that this rate changes in response to shifts in preferences and in the trend growth rate of output. Trehan and Wu (2007) compare the implications of considering the natural rate of interest to be fixed or variable.

[5]Other methods have been applied in Christensen (2002), Vitek (2005) and Horváth (2009).

We also focus on the implications generated by the potential error that central banks could commit in the estimation of the natural rate of interest. This issue has been theoretically studied by, among others, Orphanides and Van Norden (2002), Orphanides and Williams (2002), Tristani (2009) and Olmos and Sanso (2014b). We approximate the gap between the correct and the estimated TVNRI and compute its effects on long-term inflation dynamics.

Estimating the TVNRI requires the estimation of the long-run trend of the potential output because, in the theoretical dimension, both variables are closely related. In addition, this estimation is necessary because monetary policy rules are specified in terms of output deviations from the steady state level. Therefore, potential output and, consequently, its growth rate, is the second unobservable variable we estimate.

But the TVNRI and the potential output are not the only variables involved in the long run that are relevant for monetary policy. Trend inflation is another long-term variable that plays an important role in its design and in its outcomes[6]. Like the potential output, it serves as a reference in the deviation measure of the inflation rate as long as it coincides with the inflation target of the monetary policy rule. Moreover, as it is pointed out in Olmos and Sanso (2014b), the potential incorrect estimation of the TVNRI generates a gap between the inflation target and its steady-state value that sets off distortions in the long-run equilibrium. Therefore, by including this variable in the estimation process, an increase in the robustness of the analysis is expected as well as an expansion of the range of conclusions. In this

[6] In this regard, Ascari (2004) and Cogley and Sbordone (2008), among others, analyze the effects of non-zero trend inflation on short-term dynamics.

line, Leigh (2008) estimates the TVNRI for the U.S. through the Kalman filter, but all other unknown variables are overlooked. Moreover, a relevant topic we want to study is the relationship between the long-run growth rate and the trend inflation. Previous theoretical literature, such as Amano *et al.* (2009) and Olmos and Sanso (2014a), shows the existence of a non-linear relationship between these two long-term variables. And, despite the fact that the Kalman filter provides a linear estimation, we use the outcomes of the model to check the kind of connection between them through a quadratic and a quantile regression.

Another issue we want to discuss is the role of financial frictions in the determination of the long-term main variables. Olmos and Sanso (2014a) show a connection between financial frictions, the growth rate and the trend inflation in the long run with some non-linear relationships. Öğünç and Batmaz (2011) follow the Laubach and Williams (2003) procedure and include the risk premia for Turkey. They conclude that the long-run evolution of this spread determines the natural rate of interest. This link between financial frictions and the TVNRI is also studied empirically in Archibald and Hunter (2001) for the New Zealand case.

Our database comprises time series for the U.S. during the period 1960:Q1-2013:Q2. The evolution of the estimates of the TVNRI, the long-run growth rate of the economy and the steady-state inflation rate are in line with foregoing results. Our estimates prove the negative effect that financial frictions would cause on the long-run growth rate, that potential misunderstandings of the TVNRI would deviate the trend inflation from the inflation target and that the relationship between the long-run growth rate and the trend inflation is described as a nearly hump-shaped curve.

The remainder of the paper is organized as follows. The second section describes the methodology applied. In the third section, we present the estimation results, carry out a quadratic and a quantile analysis of the relationships among the long-run growth rate, the trend inflation and financial frictions, and study the effects of misunderstandings of the natural rate of interest. Finally, Section 4 summarizes the main conclusions. The Kalman filter procedure is detailed in the first appendix and the state-space form of the model is explained in the second.

2 Estimation methodology for the unobservable variables

To achieve the objectives stated in the previous section about the estimation of the long-run unobservable variables, we extend the Laubach and Williams (2003) semi-structural model by making some modifications. The core of the procedure, a state-space model devoted to implementing the Kalman filter, remains unchanged. However, we include a new state variable, the trend inflation. This extension complicates the model but adds robustness to the whole estimation since it jointly estimates all relevant variables in the long-term horizon. We also introduce some changes into the model specification in order to improve the consistency of the long-run implications and to check some theoretical outcomes.

This empirical model is a small-scale simplification of the New Keynesian macroeconomic model developed in Olmos and Sanso (2014a,b), where the main findings we want to evaluate are the relationships among the long-run economic growth rate, the trend inflation and financial frictions, as well as other relevant conclusions like the effects of potential errors in the estimation of the natural rate of interest. The connections established among these variables show the non-linear effects of the monetary policy in the long run.

The first equation of the model corresponds to the Phillips curve and describes the evolution of the inflation rate (π_t), an observable value. We define the inflation rate as the core consumer price index, which includes all items except food and energy, and take the data from the Bureau of Labor Statistics. The quarterly series is obtained as the monthly average value and then is seasonally adjusted with the Tramo/Seats methodology. Once we have computed the quarterly inflation rate, the data is annualized. We consider the inflation rate as a function of its own lags, the output gap (z_t), the trend inflation (Π_t) and a serially uncorrelated error term ε_t^π. In this way, we ensure the consistency of the model in the long term because inflation rate would equal its steady-state value, the trend inflation. The resulting equation is the following:

$$\pi_{t+1} = \rho^\pi(L)\pi_t + \beta^z z_t + (1 - \rho^\pi(L))\,\Pi_t + \varepsilon_{t+1}^\pi \tag{1}$$

where $\rho^\pi(L)$ is a lag-polynomial and β^z is interpreted as the slope of the Phillips curve.

The next relationship is a state equation equivalent to the reduced form of the IS curve that explains the output gap, the percentage deviation of the real output from its potential level. This variable depends on its own lags and on the measurement error of the natural rate of interest, defined as the difference between the ex-ante real interest rate (R_t) and the natural rate of interest (R_t^n). In turn, real interest rate is gauged by subtracting the inflation expectations ($E_t\pi_{t+1}$) from the short-term nominal interest rate R_t^{st}, which is obtained from the Federal Reserve System database. Inflation expectations are computed by an 8-quarters forward-moving average and nominal interest rate is equivalent to the federal funds effective rate. Again, a serially uncorrelated error term ε_t^z is included. This relationship is also consistent with the long run because, in the absence of shocks and mismeasurement problems, the output gap would be zero in the steady state:

$$z_{t+1} = \rho^z(L)z_t + \beta^r\left(R_t - R_t^n\right) + \varepsilon_{t+1}^z \tag{2}$$

where $\rho^z(L)$ is a lag-polynomial. We assume, following Mésonnier and Renne (2007), a definition of the natural interest rate based on standard optimal growth models. However, the specification is slightly different and follows Bouis *et al.* (2013), where the natural rate of interest is related to the long-run growth rate (g_t) corrected by a parameter[7] ∂ and augmented by ϵ, the inverse of the intertemporal elasticity of substitution in consumption, also interpreted as the relative risk aversion. An intercept ϱ is also included, which represents the time preference of consumers:

[7]In terms of the Ramsey model, parameter ∂ could be interpreted as a measure of the effects of the average productivity and population growth rates.

$$R_t^n = \varrho + \epsilon \left(g_t - \partial \right) \tag{3}$$

The long-run growth rate, equivalent to the growth of the potential output y_t^*, is explained as a function of its first lag, financial frictions (f_t) and the trend inflation. Financial frictions are proxied by the spread between the average majority prime rate charged by banks on short-term loans to business and the 3-month Treasury bill rate, both series collected from the Federal Reserve System database. This external finance premium is a standard simple measure of the frictions present in the financial markets. An intercept and a serially uncorrelated error term are also included. In the steady state, the growth rate would depend on a fixed value ∂ and also on the trend inflation, as is theoretically shown in Olmos and Sanso (2014a):

$$g_t = \partial \left(1 - \rho^g \right) + \rho^g g_{t-1} + \zeta f_t + \varkappa \Pi_{t-1} + \varepsilon_t^g \tag{4}$$

As can be seen, one main difference between our specification and those of Laubach and Williams (2003) and Mésonnier and Renne (2007) is that growth of the potential output is defined as a function of state and observed variables instead of a simple AR(1) process. Moreover, our hypothesis regarding the order of integration of both R_t^n and g_t follows the approach of Mésonnier and Renne (2007) assuming highly persistent but stationary variables driven by unobservable processes which capture common low-frequency variations in R_t^n and g_t as well as idiosincratic fluctuations of g_t.

Trend inflation is determined by its first lag and the inflation expectations. As noted in equation (1), trend inflation would equal the inflation rate in the long run:

$$\Pi_{t+1} = \rho^{\Pi}\Pi_t + \left(1-\rho^{\Pi}\right) E_t\pi_{t+1} + \varepsilon_{t+1}^{\Pi} \tag{5}$$

Finally, the last equation is the identity that defines the output gap as the difference between the output (y_t), built as the log of the real chain-weighted GDP in billions of chained 2009 dollars taken from the Bureau of Economic Analysis, and the log of its potential level:

$$z_t = y_t - y_t^{*} \tag{6}$$

Summing up, the unobservable variables that we jointly estimate are $(y_t^{*}, g_t, R_t^{n}, \Pi_t)$, whilst the observed variables are $(\pi_t, R_t^{st}, y_t, f_t)$. We should note that shocks $\left(\varepsilon_t^{\pi}, \varepsilon_t^{z}, \varepsilon_t^{g}, \varepsilon_t^{\Pi}\right)$ are independently and normally distributed and their variances are $\left(\sigma_{\pi}^2, \sigma_z^2, \sigma_g^2, \sigma_{\Pi}^2\right)$, respectively.

Having introduced the equations of the semi-structural model, we have to articulate the state-space model. Appendix A1 is devoted to presenting the state-space representation which consists of the measurement equation and the transition equation. Afterwards, we are able to implement the Kalman algorithm (Kalman, 1960). The basic intuition behind this procedure follows two steps. In the first, the system makes a prediction based on the information available at a specific point of time. In the next period, the filter corrects this prediction by uploading the new information. The maximum likelihood method is used to estimate the conditionally unbiased and efficient estimators of the state variables. In Appendix A2, we formally detail the

Kalman filter mechanism[8].

3 Results for the U.S. economy

We now estimate the model specified in the previous section. The quarterly data set we have used refers to the United States in 1960:Q1-2013:Q2. It should be noted that the Kalman filter is very sensitive to the initial conditions. The technique we have implemented to overcome this issue consists of several steps. Firstly, we carry out a univariate estimation of each unobserved variable. To that end, we have applied the Hodrick-Prescott filter to the inflation rate, the real GDP and its growth rate in order to obtain preliminary estimates of the trend inflation, the potential output and the long-run growth rate, respectively. Secondly, we have estimated each equation including the series provided by the HP filter with the purpose of assigning the initial values to the parameters. This first estimation of the Kalman filter generates variances biased towards zero[9] and, therefore, unsatisfactory results outside the acceptable values for the unobservable variables. Thus, we have to use the common method of restricting some coefficients by calibrating the following parameters:

- Following Bouis *et al.* (2013), one of the best candidates to gauge ϱ is the average of the actual real interest rate because it measures the trend value of the natural rate of interest approximately. This approach is also used to set the value of the intercept of the long-run growth rate equation ∂, equating it to the

[8] Good references to understand this procedure are Harvey (1989) and Hamilton (1994).

[9] Because of the pile-up problem described in Stock (1994).

sample average of the real output growth rates.

- Due to the lack of consensus about the value of the parameter ϵ, we choose the value 4.167, used in our theoretical model of reference, i.e. Olmos and Sanso (2014a).

- Finally, we calibrate σ_{Π}^2 so that trend inflation accounts for 50% of the inflation rate fluctuations.

Table 1: Coefficient estimates

ρ_1^{π}	0.814	(12.72)
β^z	0.158	(2.43)
ρ_1^z	0.901	(14.06)
β^r	-0.179	(-4.33)
ρ^g	0.813	(8.20)
ζ	-0.020	(-1.69)
$\varkappa$	0.009	(1.69)
ρ^{Π}	0.883	(13.55)
σ_{π}^2	1.746	(12.84)
σ_z^2	0.3456	(8.72)
σ_g^2	0.033	(7.70)
LF	-633.2	

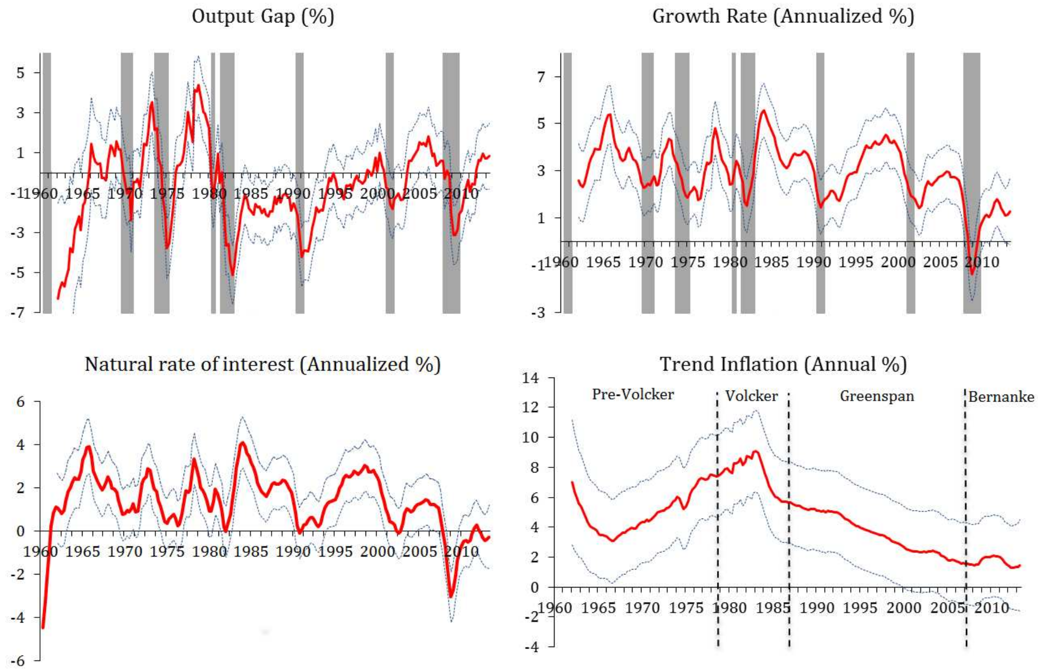
z-Statistic in parenthesis. LF: Likelihood function.

We now explore the results of the model by analyzing the estimated coefficients shown in Table 1. As can be seen, all the coefficients have the expected sign and are statistically significant. Regarding the lags included for the inflation rate in (1), we impose order 1 for the $\rho^{\pi}(L)$ lag-polynomial, whose coefficient is ρ_1^{π}. Otherwise, the coefficient associated with Π_t in (1) loses weight and, consequently, the state estimates become distorted. The significativity criterion reveals that the lag-polynomial of the output gap in (2) is of order 1. Both the slope of the Phillips curve β^z and the coefficient β^r, which drives the output gap in accordance with fluctuations in the difference between the actual interest rate and its natural level, are higher than those estimated by Laubach and Williams (2003) and Bouis *et al.* (2013)[10], but remain within reasonable values. Financial frictions negatively affect the long-run growth rate, which can be seen from the negative value of ζ. Trend inflation exerts the opposite effect because coefficient $\varkappa$ has a positive sign, though the size is very low. As the statistical significance of these two linear effects is at the limit of 10%, we go deeper into this issue in the last part of the paper when we pose the question of the non-linear effects.

The estimation of the model with the features described above yields the evolution of the unobserved variables displayed in Figure 1. We should clarify that these series are two-sided estimates or smoothed estimates, that is, to compute them, the Kalman algorithm has used the information of the full sample. In addition, we have discarded the first few quarters because the estimates are outside the admissible range.

[11] Other values of the literature are summarized in Mésonnier and Renne (2007).

Figure 1: Estimates of the unobserved variables



Grey bars refer to the official recession dates provided by the National Bureau of Economic Research.

Dashed lines represents the 90% confidence interval.

Table 2 displays the statistical properties of the unobserved variables. Output gap shows an expected path in the range (-6%,5%) with eight slowdowns corresponding to the official recession dates of the U.S. economy[11], which also can be appreciated in the long-run growth rate trajectory. This first inference seems to verify the accuracy of the estimates. The sharpest declines of the output gap are situated at the beginning of the sample and in the early 1980s recession, whilst the long-run growth rate reaches its minimum value in 2008 during the financial crisis. The trajectory of the natural

[11]The crisis periods are 1960:Q2-1961:Q1, 1969:Q4-1970:Q4, 1973:Q4-1975:Q1, 1980:Q3, 1981:Q3-1982-Q4, 1990:Q3-1991:Q1, 2001:Q1-2001:Q4 and 2007:Q4-2009:Q2.

rate of interest is obviously analogous to the long-run growth rate evolution because the former is defined as a linear combination of the latter. Values of the long-run growth rate and the natural rate of interest in 2008:Q4 seem to be atypical since the troughs of both series are anomalously low for long-term references. Finally, the trend inflation rises sharply from the late sixties to 1980, during the Pre-Volcker era. After reaching its peak in the middle of Volcker's presidency of the Federal Reserve System, the trend inflation has constantly decreased leading to the so-called Volcker disinflation. From the late-nineties, under the leadership of Greenspan, the trend inflation has stabilized around a value of approximately 2%, level at which it is assumed that FED locates its inflation rate target for the medium and long term.

Table 2: Statistical properties of the estimated series

	Mean	Standard deviation	Minimum	Maximum
Output Gap	-0.70	1.95	-6.32	4.40
			(1962:Q1)	(1978:Q4)
Growth Rate	2.91	1.21	-1.38	5.57
			(2008:Q4)	(1984:Q1)
Natural Rate of Interest	1.38	1.24	-3.06	4.07
			(2008:Q4)	(1984:Q1)
Trend Inflation	4.45	2.08	1.30	9.09
			(2012:Q2)	(1983:Q1)

3.1 Searching for nonlinearities

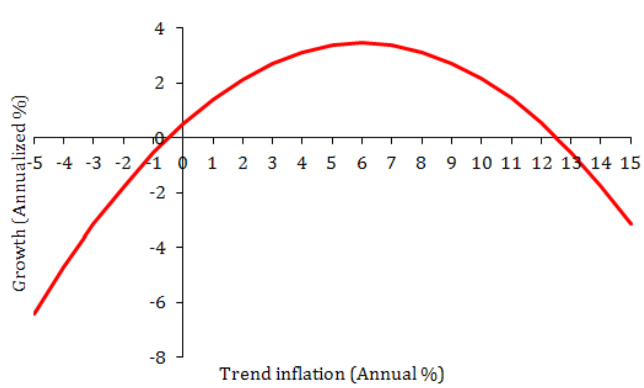
We have seen, in Table 1, that the estimation of the coefficient $\varkappa$ is near zero and that its corresponding p-value is slightly lower than 10%, so the linear relationship between the long-run growth rate and the trend inflation is very weak. The same remark can be made about the relationship between the external finance premium and the growth rate. These are not two counterintuitive results in the light of the findings of Olmos and Sanso (2014a), because these outcomes may not mean the absence of a relationship between the trend inflation and the long-run growth rate and between the latter and the external finance premium, but perhaps the model specification used is veiling relevant bivariate movements. The theoretical results we have proposed to test in this paper are the presence of nonlinearities between these two pairs of variables but, unfortunately, in the model to which the Kalman filter is applied, non-linear specifications can not be included. Although the Extended Kalman filter can integrate such specifications, its operation is very complex, so we have opted for a two-step analysis. The first, already done, is to obtain estimates of the state variables. In the second, we use these estimates of the unobservable long-run variables to test the hypothesis established in Olmos and Sanso (2014a), a hump-shaped relationship between the long-run growth rate and the trend inflation, on the one hand, and a U-shaped relationship between the trend inflation and the external finance premium, on the other.

A simple way to look for non-linear relationships is to define a quadratic equation. By doing so, the result obtained for the regression is the folowing:

$$\hat{g}_t = \underset{(1.42)}{0.52} + \underset{(5.88)}{0.98\hat{\Pi}_t} - \underset{(-4.84)}{0.08\hat{\Pi}_t^2} + \hat{u}_t^g \tag{7}$$

where $\hat{\Pi}_t$ and $\hat{g}_t$ are the trend inflation and the long-run growth rate series estimated by the Kalman filter, $\hat{u}_t^g$ refers to the residuals and the t-ratios are presented in parentheses. In line with the foregoing theoretical results, the coefficient values show a hump-shaped relationship between $\hat{g}_t$ and $\hat{\Pi}_t$ plotted in Figure 2. This very significant non-linear relationship indicates that, for low levels of trend inflation, the long-run growth rate increases until $\hat{\Pi}_t = 6\%$ and, after this value, the growth rate decreases with the trend inflation, reaching zero when it is 12.7% and -0.5%. The annualized maximum potential growth is near 4%. But this outcome varies depending on the sample considered. If we contemplate a period of inflation stability, such as the subsample beginning in 1994 during which the Federal Reserve has reacted preemptively against deviations of inflation from its target, the level of trend inflation for which estimated growth is maximized drops markedly to $\hat{\Pi}_t = 4\%$.

Figure 2: Quadratic relationship between $\hat{g}_t$ and $\hat{\Pi}_t$

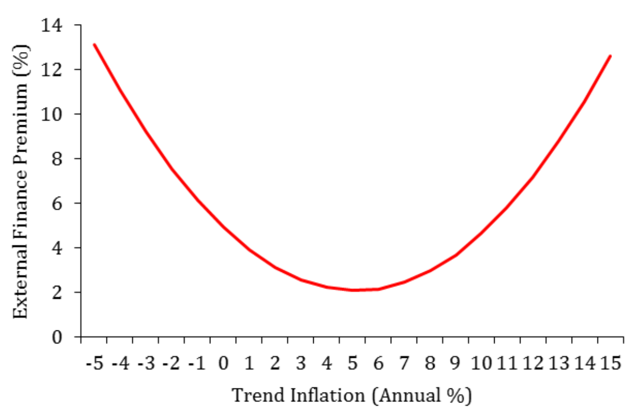


As in our previous exercise, we again perform this analysis to test for the link between financial frictions and the trend inflation in the long run. Olmos and Sanso (2014a) conclude that this connection has a U shape. Equation (8) presents the estimated coefficients, where it can be seen that the U-shaped relationship found in that paper, displayed in Figure 3, is corroborated. It should be noted that the level of trend inflation for which estimated growth is maximized nearly matches the minimum value of financial frictions.

$$
\begin{array}{ccccccccc}
f_t = & 4.90 & - & 1.11\hat{\Pi}_t & + & 0.11\hat{\Pi}_t^2 & + & \hat{u}_t^f \\
 & (16.04) & & (-7.96) & & (7.69) & &
\end{array} \tag{8}
$$

where $\hat{u}_t^f$ are the residuals.

Figure 3: Quadratic relationship between f_t and $\hat{\Pi}_t$



Another way of searching for the nonlinearties we are interested in is to take into account that the state estimates have been obtained using a specific linear model. Hence, maintaining this specification we analyze the relationship of both the trend inflation and financial frictions with the long-term growth rate, looking for possible nonlinearities. In doing so, we again try to verify if the theoretical results obtained in Olmos and Sanso (2014a) are validated. We take the sample 1962:Q1-2013:Q2 in order to avoid the initial distorted observations of the state estimates. The methodology we adopt is based on a quantile regression but, in contrast to the common practice of ordering observations according to the endogenous variable, we arrange the quantiles according to an exogenous variable, the trend inflation. Thus, the specification of our equation of interest, which relates the long-term growth rate, financial frictions and the trend inflation, is the same[12] as (4), although levels of the explanatory variable $\hat{\Pi}_t$ are distinguished. We opt for six quantiles because this is the largest number that provides an acceptable number of observations in each quantile. Table 3 shows the lower and the upper limit of each quantile, the estimated coefficients, the variance and the coefficient of determination.

We expect a positive sign of $\varkappa$ in the lower quantiles and a negative one in the higher, what would resemble the inverted parabola obtained previously. For all the quantiles except the last one and, consequently, for most of the sample, the relationship can be described as an inverted U curve. However, when trend inflation is above 6.67% (last quantile), which occurs between 1973 and 1984, the estimated coefficient is positive. Nevertheless, this scenario could be considered as an anomalous pattern since, during those years, the two economic recoveries after strong downturns were attached to a monetary policy that did not react severely to inflation deviations.

[12] We no longer include the trend inflation as a lag because that was a specific constraint of the Kalman filter.

Table 3: Quantile regression

	Quantiles					
	1st	2nd	3rd	4th	5th	6th
Minimum $\hat{\Pi}_t$	1.30%	2.09%	3.42%	4.35%	5.22%	6.67%
Maximum $\hat{\Pi}_t$	2.06%	3.40%	4.31%	5.21%	6.52%	9.09%
$\partial\left(1-\rho^g\right)$	0.324	-0.262	0.085	0.227	0.263	-0.454
	(1.68)	(-2.77)	(0.26)	(1.10)	(1.74)	(-2.22)
ρ^g	0.973	0.804	1.054	0.947	0.905	0.921
	(25.77)	(14.91)	(9.86)	(10.34)	(20.83)	(15.06)
ζ	-0.161	0.026	0.006	-0.024	-0.014	-0.046
	(-4.02)	(2.60)	(0.70)	(-1.11)	(-1.27)	(-3.31)
$\varkappa$	0.120	0.118	-0.035	-0.026	-0.031	0.085
	(2.23)	(2.82)	(-0.56)	(-0.52)	(-1.21)	(3.21)
σ^2	0.004	0.002	0.003	0.005	0.003	0.009
R^2	0.96	0.97	0.92	0.88	0.94	0.90

t-ratios are reported in parentheses.

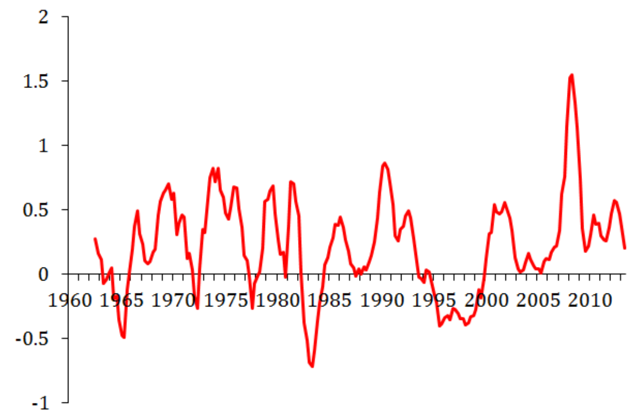
With respect to the relationship between financial frictions and the long-run growth rate, coefficient ζ is statistically significant at 5% only for the extreme lower and higher values of the trend inflation, so the influence is coherent with the U-shaped relationship between the trend inflation and the external finance premium. For medium inflation rate levels, when the external finance premium does not reach the upper levels and, therefore, credit markets operate flexibly, their fluctuations do not affect, or positively affect, the long-term growth rate. However, when the degree of financial frictions increases, long-run growth could be negatively affected by such rigidities.

3.2 Long-run effects of natural rate of interest mismeasurements

In order to capture other non-neutral and non-linear effects of the monetary policy in the long run, we approximate the real-time gap between the estimated and the correct value of the natural rate of interest as the difference between the one-sided ($\hat{R}_t^*$, filtered) and the two-sided ($\hat{R}_t^n$, smoothing) estimates following Mésonnier and Renne (2007). This is a proxy of the mismeasurement gap since the former estimation takes into account the information available at the time of the estimation, as central banks do, and the latter uses the full sample information of the signal variables, which approaches the true value. In Figure 4, the evolution of the mismeasurement gap is displayed.

This gap reaches a substantial size and, even if we do not consider the highest deviations, the gap moves around values of $(-0.5\%, 1\%)$. When the mismeasurement gap takes positive values, monetary policy tends to be more contractionary since the intercept of the rule is higher than the endogenous value. Analogously, when the gap is negative, monetary policy is more expansive. The average of the gap is near 0.25% meaning that, on average, Federal Reserve implements a restrictive monetary policy through the overestimation of the natural rate of interest.

Figure 4: Gap $(\hat{R}_t^* - \hat{R}_t^n)$

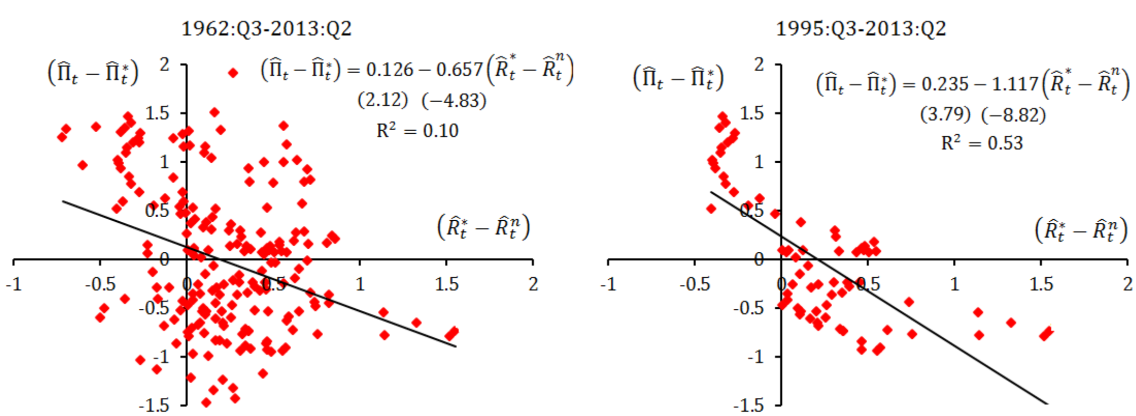


Plot of the 2-quarter central moving average of the gap between the filtered $(\hat{R}_t^*)$ and the smoothing $(\hat{R}_t^n)$ estimation of the TVNRI

We now explore the potential implications of the existence of this gap from the long-run perspective. This exercise is based on the theoretical work developed in Olmos and Sanso (2014b), where it is concluded that central banks' misunderstandings in the estimation of the natural rate of interest affect the long-run equilibrium by deviating the steady-state inflation rate from its target. When the natural rate of interest estimated by the central bank is higher than the correct value, trend inflation is below its target and vice versa. Accordingly, the relationship between the deviation of the natural rate and the gap of the trend inflation is negative.

Firstly, in order to carry out this analysis, we have to transform the trend inflation series to be comparable with the mismeasurement gap. Hence, we have to calculate the deviations of the trend inflation from the estimated target, the latter proxied as its statistical mean. However, the evolution of the estimated trend inflation exhibits a clear non-linear pattern of behavior. To verify this point, we have estimated the mean of this variable by way of the Bai-Perron methodology, which allows for the presence of structural changes (see Bai and Perron, 1998, 2003) throughout the sample 1962:Q1-2013:Q2. The application of this methodology leads us to observe the existence of 5 differentiated periods in the evolution of the trend inflation with the following break points: 1970:Q3, 1979:Q2, 1986:Q4 and 1995:Q2. The estimated annualized values for the mean of the trend inflation in the five subperiods are 4.1%, 6.1%, 7.5%, 4.9% and 2.3%, respectively. Secondly, with these averages, we can compute the deviations of the trend inflation from the estimated target and relate them to the mismeasurement gap. In order to smooth both series, characterized by strong fluctuations, we construct the 2-quarter central moving averages for the trend inflation and the mismeasurement gap.

Figure 5: Trend inflation deviations from its target and mismeasurement gap



Scatter plot between the 2-quarter central moving average of $(\hat{\Pi}_t - \hat{\Pi}_t^*)$ and $(\hat{R}_t^* - \hat{R}_t^n)$.

t-ratios in parentheses

Figure 5 shows the scatter plot of the trend inflation deviations $\left(\hat{\Pi}_t - \hat{\Pi}_t^*\right)$, where

$\hat{\Pi}_t$ is the estimated trend inflation and $\hat{\Pi}_t^*$ is the estimated target, and the mismeasurement gap $\left(\hat{R}_t^* - \hat{R}_t^n\right)$, where $\hat{R}_t^*$ is the estimated intercept of the Taylor rule and $\hat{R}_t^n$ is the estimated natural rate of interest.

This analysis reveals that the relationship is negative for the full sample as the theoretical analysis predicts. However, if we divide the total sample into the specified periods detailed above, the results are not homogeneous. In the final period, which is the largest one of the time intervals considered, the negative relationship is much clearer than in the whole sample. These results lead us to think that, when monetary policy was conducted through monetary aggregates and the implicit or explicit estimation of the natural rate of interest was not required, or monetary authorities do not react against inflation deviations tightly enough, the mismeasurement gap was not so relevant and was not so clearly transferred to the long-term inflation. So, we can conclude that, with policies derived from Taylor rules whose priority is the inflation stability, the natural rate of interest and its measurement become essential for the monetary policy because measurement errors could influence the long-term equilibrium. These preliminary results seem to support this intuition and, therefore, verify the negative relationship between the two gaps specially for the period in which the Federal Reserve conducts monetary policy through inflation targeting rules which reacts severely and preemptively against inflation deviations.

4 Conclusions

In this paper, we analyze the long-run interactions of the U.S. unobservable variables included in the Taylor rules. Firstly, we look for the existence of nonlinearities between the long-run growth rate, the trend inflation and financial frictions in the long run. Through quadratic equations, we confirm the existence of a hump-shaped connection between the long-run growth rate and the trend inflation and a U-shaped relationship between the latter and financial frictions. Then, by estimating a quantile regression which distinguishes among levels of trend inflation, we corroborate that hump-shaped connection if the trend inflation does not exceeds an upper threshold. Moreover, especially for low and high levels of trend inflation, financial frictions negatively affect the long-run growth rate.

Furthermore, we approximate the gap between the real-time estimate and the correct value of the natural rate of interest and study its effects on the trend inflation deviations from the target level. We prove that, for the whole sample, there is a negative relationship between the error in the estimation of the natural rate of interest and the gap of the actual trend inflation and its target, as is concluded in the theoretical models developed in Olmos and Sanso (2014a,b). This negative relationship is especially clear and significant when monetary policy reacts aggressively against inflation deviations. In turn, deviations of the trend inflation from its target could also affect financial frictions and the long-run growth rate, since have been demonstrated the interactions among these three key variables.

In order to obtain all these results, we have jointly estimated the natural rate of interest, the potential output and the trend inflation using U.S. data for the 1960:Q1-2013:Q2 period. Our procedure extends the methodology of Laubach and Williams (2003), who implement the Kalman filter to a semi-structural econometric model in order to obtain the unobservable series, by including the trend inflation as a state variable and financial frictions as an exogenous factor. The state estimates show that the long-run growth rate and the natural rate of interest have experienced an unprecedented decline during the financial crisis triggered in 2007, a pattern that is not observed for the output gap. Meanwhile, the trend inflation has stabilized since the mid-nineties around a value of 2%.

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A 1. State-space Form of the Model

To implement the Kalman filter procedure, equations (1-6) have to be expressed in the state-space form. This appendix describes the state-space model respecting the notation in the main text.

The measurement equation describes how the observations are derived from the internal state vectors:

$$\begin{bmatrix} \Delta y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 1 \\ 0 & \beta & 0 & 1-\rho_1^\pi & 0 \end{bmatrix} \begin{bmatrix} z_t \\ z_{t-1} \\ \Pi_t \\ \Pi_{t-1} \\ g_t \end{bmatrix} + \begin{bmatrix} 0 \\ \rho_1^\pi \end{bmatrix} \pi_{t-1} + \begin{bmatrix} 0 \\ \varepsilon_t^\pi \end{bmatrix} \tag{A1.1}$$

where ρ_1^π is the first element of the $\rho^\pi(L)$ lag-polynomial. The representation of the state equation indicates that the new state vector is modeled as a linear combination of the previous state and an error process:

$$\begin{bmatrix} z_t \\ z_{t-1} \\ \Pi_t \\ \Pi_{t-1} \\ g_t \end{bmatrix} = \begin{bmatrix} \rho_1^z & -\beta^r \epsilon & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho^\Pi \\ 0 & 0 & 1 \\ 0 & \rho^g & \varkappa \end{bmatrix} \begin{bmatrix} z_{t-1} \\ g_{t-1} \\ \Pi_{t-1} \end{bmatrix} + \begin{bmatrix} \beta^r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1-\rho^\Pi \\ 0 & 0 & 0 \\ 0 & \zeta & 0 \end{bmatrix} \begin{bmatrix} R_{t-1} \\ f_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \beta^r \left(\epsilon\partial - \varrho\right) \\ 0 \\ 0 \\ 0 \\ \partial\left(1-\rho^g\right) \end{bmatrix} + \begin{bmatrix} \varepsilon_t^z \\ 0 \\ \varepsilon_t^\Pi \\ 0 \\ \varepsilon_t^g \end{bmatrix} \tag{A1.2}$$

A 2. The Kalman Filter

The Kalman filter is a semi-structural method to estimate unobserved variables. It is a suitable procedure because it provides a Minimum Mean Squared Error estimator if the observed variables and the noises are jointly Gaussian. To show the operation of the filter, let us define o_t as an $n \times 1$ vector, where o_t is an observable variable. This time series is a function of an $m \times 1$ vector, u_t, whose value and variance are unobservable. In order to simulate the latent variable, we have to specify a model as follows:

$$o_t = \delta_{1,t} + \delta_{2,t} u_t + \varepsilon_t^w \tag{A2.1}$$

$$u_{t+1} = \delta_{3,t} + \delta_{4,t} u_t + \varepsilon_t^x \tag{A2.2}$$

where $\delta_{i,t}$ are vectors and $\varepsilon_t^o, \varepsilon_t^u$ are vectors of Gaussian noises. The first equation (A2.1) is the measurement or observation equation whilst equation (A2.2) is the state or transition equation. Disturbance errors $\varepsilon_t^o, \varepsilon_t^u$ are serially independent, with the following variance structure:

$$\Omega_t = \operatorname{var}\begin{bmatrix} \varepsilon_t^o \\ \varepsilon_t^u \end{bmatrix} = \begin{bmatrix} H_t & J_t \\ J_t' & B_t \end{bmatrix} \tag{9}$$

where H_t is an $n \times n$ symmetric variance matrix, B_t is an $m \times m$ symmetric variance matrix, and J_t is an $n \times m$ matrix of covariances.

The smoothing procedure generates the estimates of the state variables $\hat{u}_t \equiv E_T(u_t)$ with variance $V_t^u \equiv \mathrm{var}_T(u_t)$ and estimates of the signal variables $\hat{o}_t \equiv E(o_t \mid \hat{u}_t) = \delta_{1,t} + \delta_{2,t}\hat{u}_t$. The one-step ahead prediction error is $\breve{\varepsilon}_t^o = \varepsilon_{t|t-1}^o \equiv o_t - \breve{o}_{t|t-1}$ and the prediction error variance is $\breve{V}_t^o = V_{t|t-1}^o \equiv \mathrm{var}\left(\varepsilon_{t|t-1}^o\right) = \delta_{2,t} P_{t|t-1}^o \delta_{2,t}' + H_t$, where $P_{t|t-1}^o$ is the mean square error of the one-step ahead mean.

The Kalman filter updates the one-step ahead estimate of the state mean and variance with new information and computes the one-step ahead estimates of the state and the associated mean square error matrix, the contemporaneous or filtered state mean and variance and the one-step ahead prediction, prediction error and prediction error variance.