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ABSTRACT: As per the conventional wisdom there should be provision for public assistance for skills acquirement for improving relative wage inequality in the future. This paper attempts to explore the validity of this traditional perception with the help of a two-sector, specific factor general equilibrium model with endogenous skills formation and provision for public subsidy aimed to encourage acquisition of skills on the part of unskilled labour. The analysis questions the desirability of the policy on the ground that it may not only aggravate inequality in the current period but also does not necessarily improve the wage disparity at a future date.

Keywords: Skills Formation, Wage Inequality, Skilled Labour, Unskilled Labour, Education Subsidy, Intertemporal Analysis, General Equilibrium.

JEL Classification: J31, I24, I28, D58.

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1. Introduction and motivation

The aspect of skills formation is extremely crucial in all countries irrespective of whether developed or developing. It promotes human capital formation which is conducive to economic growth and prosperity of an economy as well as has an egalitarian effect because it is likely to lower the difference in wages between the two groups of worker differentiated with respect to their skills. The conventional wisdom is that skills acquisition, given the size of the workforce, lowers the endowment of unskilled labour in the short run and raises the supply of skilled labour in the future. In the current period, relative wages should move in favour of unskilled labour as the supply of this type of labour decreases although the endowment of skilled labour does not change. In future unskilled workers going for skill acquisition in period 1 become skilled and therefore, the endowment of skilled labour goes up while that of unskilled labour does not change. The wage inequality again improves because of increased supply of skilled labour. If these arguments are valid then logically it follows that there should be provision for public assistance for skill acquirement. In the post-reform regime, the need for this type of public assistance has become extremely critical in view of the empirical evidences of strong symmetrical wage movements against unskilled labour in different countries across the globe.

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1 Different facets of skills formation have been discussed in works like Autor (2014), Becker (1964), Brown et al. (2001), Crouch et al. (1999), Heckman and Krueger (2003), and Vanhuysse (2007).


3 In this connection, it is important to mention that Beyer et al. (1999), while studying the consequence of trade liberalization on wages in Chile, have found that an increase in the proportion of labour force with higher education exerted downward pressure to reduce the skilled-unskilled wage inequality. Therefore, as per their findings education had some equalizing
This paper purports to examine the validity of this conventional wisdom in terms of a two sector-three factor general equilibrium model with endogenous skill formation and provision for public subsidy designed at encouraging acquisition of expertise on the part of the workers. It is a two-period model where the number of people going for skills acquisition and hence the supply of unskilled labour in period 1 is endogenously determined from the intertemporal utility maximization exercise of the unskilled working families where the population size including skilled and unskilled workers remains the same over time. The question of skills acquisition on the part of skilled workers does not arise. Each unskilled working family in its maximization exercise takes the wage rates as datum. Although both the wages in period 2 change the family cannot foresee them as their decision for skills formation is taken in period 1. In the given circumstances, the consequences of the public subsidy provided in period 1 on the relative wage inequality in both the periods are examined. We find that the subsidy aggravates the inequality in the current period if the high-skill sector is capital-intensive and that the policy does not necessarily improve the inequality in the future period. These results can at least question the desirability of public assistance for skills formation especially when it may fail to ensure the desired effect on wage inequality in future even though it may aggravate income inequality among the working class at present.

effects on the wage gap in that country which fortifies the necessity of providing public incentives to skills acquisition.

4 We have basically incorporated endogenous skills formation in an otherwise Jones’ (1971) model.

5 That the difference in distributive shares of capital between high-skill and low-skill sectors play a crucial role in determining the final outcome on the relative wage inequality resulting from any exogenous shock has been pointed out in many theoretical works like Chaudhuri (2004), Marjit and Kar (2005), Chaudhuri and Yabuuchi (2007), Beladi et al. (2008) and Chaudhuri (2008). However, Chaudhuri (2008) has found a special case where these distributive shares do not matter. Although there are a couple of papers in the literature like Kar and Beladi (2004) and Yabuuchi and Chaudhuri (2009) that have introduced skills formation in terms of a separate education sector in a static general equilibrium framework and have studied its impact on the relative wage inequality we do not come across any theoretical work that has analyzed the consequence of the skills-promoting subsidy policy in a dynamic set-up with endogenous skills formation and has simultaneously demonstrated the possibility of its failure in improving wage inequality in both present and future periods.
2. The Model and Analysis

We consider a small open economy with two sectors: low-skill (sector 1) and high-skill (sector 2). Sector 1 produces the export commodity \((X_1)\) with the help of unskilled labour \((L)\) and capital \((K)\) while sector 2 produces the import good \((X_2)\) by means of skilled labour \((S)\) and capital. Markets are competitive and all the standard assumptions including CRS hold. While capital is perfectly mobile intersectorally unskilled labour and skilled labour are specific to sectors 1 and 2, respectively. The endowments of both skilled labour and capital are given exogenously while the aggregate supply of unskilled labour in the economy is endogenously determined. Commodity prices, \(P_i\) s are given by the small open economy assumption. Finally, commodity 1 is taken to be the numeraire.

The economy is endowed with \(L\) number of unskilled families each possessing 1 unit of labour. The supply of unskilled labour by each family is endogenously determined from its intertemporal utility maximizing behavior in period 1. Let \(W\) and \(S\) denote the existing unskilled and skilled wages, respectively. As \(W > W_s\) in period 1, a fraction of family labour, \(l_s\) goes for skills formation while the rest, \((1-l_s)\) works in the unskilled labour market at the wage, \(W\). The family receives a government assistance (subsidy) designed at encouraging skill formation at the rate, \(s\) for the part of family labour that goes for skill formation. There are no direct costs of acquiring skills.\(^6\) The family consumes its entire income in each period. Hence, the question of savings or borrowing at the time of their decision-making (period 1) does not arise. The indirect (opportunity) cost of skills formation is \((W - s)\). The endogenously determined aggregate supply function of unskilled labour in the economy in general form is given as follows.\(^7\)

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\(^6\) One can introduce direct costs of acquiring skills which would not affect the basic results of the paper. For further details see the concluding section.

\(^7\) It has been derived in Appendix 1.
\[ L = L(W, W_s, s) \text{ with } \frac{\partial L}{\partial W} > 0; \frac{\partial L}{\partial W_s} > 0; \frac{\partial L}{\partial s} < 0. \] (1)

Besides, \( E_W = \left( \frac{\partial L}{\partial W} \right) > 0; \ E_{W_s} = \left( \frac{\partial L}{\partial W_s} \right) < 0; \ \text{and, } \ E_s = \left( \frac{\partial L}{\partial s} \right) < 0 \) respectively, denote the elasticities of \( L(.) \) with respect to \( W, W_s \) and \( s \), respectively. We present the properties of the \( L(.) \) function in terms of the following proposition.\(^8\)

**Proposition 1:** The aggregate supply of unskilled labour in the economy is an increasing function of unskilled wage and decreasing in both skilled wage and rate of subsidy on skills formation provided in period 1.

The reasons for the signs of the partial derivatives are obvious. An increase (a decrease) in unskilled wage, \( W \) (rate of subsidy, \( s \)) raises the opportunity cost of acquiring skills, \( (W - s) \) and hence leads to higher supply of unskilled labour by each family and vice versa. On the other hand, an increase in skilled wage, \( W_s \) raises the present discounted return from skills formation which in turn lowers the supply unskilled labour.

Note that the number of unskilled working families, \( L \) is exogenously given and does not change over time.

The general equilibrium structure of the model consists of the following set of equations.

\[ W a_{1} L_{1} + ra_{1} K_{1} = 1 \] (2)

\[ W_s a_{2} S_{2} + ra_{2} K_{2} = P_{2} \] (3)

\[ a_{1} X_{1} = L(W, W_s, s) \] (4)

\[ a_{2} S_{2} X_{2} = S \] (5)

\[ a_{1} K_{1} X_{1} + a_{2} K_{2} X_{2} = K \] (6)

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\(^8\) This result holds in period 2 as well. See proposition 4.
where $a_{ji}$ denotes the per unit requirement of the $j$th input in the $i$th sector with $j = L, S, K$ and $i = 1, 2$ while $r$ is the return to capital. Besides, we define $\theta_{ji} = \left( \frac{w_j a_{ji}}{P_i} \right)$ as the distributive share of the $j$th input in the $i$th sector which would be used subsequently in stating the results.

Equations (2) and (3) are the two zero-profit conditions for the low-skill and high-skill sectors, respectively. Equations (4) – (6) are the full-employment conditions for unskilled labour, skilled labour and capital.

Using equations (7) and (8), equation (9) can be rewritten as follows.

$$\frac{aK1}{aL1} = \frac{aK2}{aS2}$$

This is an indecomposable production system. Factor prices, $W, W_S$ and $r$ are determined simultaneously from equations (2), (3) and (6.1) as functions of the system parameters including $s$. Then $X_1$ and $X_2$ are obtained from equations (4) and (5), respectively as $a_{ji}$s are now known.

As $W_S > W$, the absolute wage gap is $(W_S - W)$ and the relative wage inequality is given by $(\hat{W}_S - \hat{W})$ where $\hat{\cdot}$ implies proportional change e.g. $\hat{W}_S = \left( \frac{dW_S}{W_S} \right)$. The skilled–unskilled wage gap improves (worsens) in absolute terms if the gap between $W_S$ and $W$ falls (rises). On the other hand, the wage inequality improves (deteriorates) both in absolute and relative terms if $(\hat{W}_S - \hat{W}) < (>0)$. 
3. Results

Differentiating equations (1) – (3) and (6.1), using (1) and simplifying the following two propositions can be ascertained.

**Proposition 2:** An increase in the subsidy designed to encourage skills formation lowers the supply of unskilled labour in the economy if

\[
\frac{E_w}{E_{w_s}} \geq \left( \frac{\theta_{l1} \theta_{k2}}{\theta_{k1} \theta_{s2}} \right). \tag{9}
\]

**Proposition 3:** The relative wage inequality worsens (in period 1) due to the subsidy policy if

\[
\frac{E_w}{E_{w_s}} \geq \left( \frac{\theta_{l1} \theta_{k2}}{\theta_{k1} \theta_{s2}} \right) > 1. \quad \text{On the contrary, it improves if } \frac{E_w}{E_{w_s}} \geq \left( \frac{\theta_{l1} \theta_{k2}}{\theta_{k1} \theta_{s2}} \right) < 1.
\]

Propositions 2 and 3 can intuitively be explained in the following fashion. If the subsidy rate, \( s \) increases there would be a direct negative effect on \( L(.) \). As the supply of unskilled labour initially falls, the unskilled wage, \( W \) rises and the low-skill sector (sector 1) contracts releasing capital to the high-skill sector (sector 2). Consequently, sector 2 expands and demands more skilled labour that raises the skilled wage, \( W_s \). So, both \( W \) and \( W_s \) increase which produce two opposite induced (indirect) effects on \( L(.) \). The supply of unskilled labour rises as \( W \) rises while it falls as \( W_s \) increases. Therefore, there are three different effects on \( L(.) \). Our analysis shows that the sum of the two negative effects dominates over the positive effect of an increase in \( W \) and the net effect would be a fall in the aggregate supply of unskilled labour in the economy under the sufficient condition that

\[
\frac{E_w}{E_{w_s}} \geq \left( \frac{\theta_{l1} \theta_{k2}}{\theta_{k1} \theta_{s2}} \right).
\]

We have already noted that both \( W \) and \( W_s \) have increased and sector 1 (sector 2) has contracted (expanded). The contracting sector 1 releases capital to sector 2. However, as in our model capital is fully utilized, the released capital by sector 1 would be absorbed by sector 2 only if its rate of return, \( r \) falls. Thus, \( r \) plummets which is also clear from the two

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\( ^9 \) This result also holds in period 2. See proposition 4 in this context.
zero-profit conditions (equations 2 and 3). The fall in $r$ leads to saving on capital cost in both the sectors. So, the proportions of increase in $W$ and $W_s$ depend on the extent of saving on capital costs in the two sectors which in turn depends on their distributive shares of capital, $\theta_{ki}$. If $\theta_{k2} > (\leq)\theta_{k1}$ i.e. if $\theta_{l1}\theta_{k2} > (\leq)\theta_{k1}\theta_{s2}$, the saving on capital cost would be higher in sector 2 (sector 1) vis-à-vis that in sector 1 (sector 2) and the relative wage inequality worsens (improves).

**Period 2**

We consider the same broad production system in period 2 also. It has already been assumed that the aggregate workforce (skilled plus unskilled) i.e. $(\bar{L} + S)$ does not change over time although their composition changes due to skills formation. In period 2 unskilled labours that went for skills acquisition (and not working in the unskilled labour market) in period 1 becomes skilled labour. So, the endowment of skilled labour increases from $S$ (in period 1) to $(\bar{S} = S + \bar{L} - L(\cdot))$ (in period 2) while that of unskilled labour remains the same, $L(\cdot)$. As in our indecomposable production structure all factor prices depend on factor endowments apart from commodity prices, $W, W_s$ and $r$ would take different values in equilibrium and so would be the values of $a_\mu$ s, and $\theta_\mu$ s in period 2. In order to avoid complications in the use of notations we leave them unchanged.

Equations (5) and (6.1) would now have to be replaced by the following.

\[
as_{s2}X_2 = \bar{S}
\]

\[
\frac{a_{K1}}{a_{L1}} L(W, W_s, s) + \frac{a_{K2}}{a_{S2}} \bar{S} = K
\]

\[
\bar{S} = S + (\bar{L} - L(\cdot))
\]

where,

Differentiating equations (1) – (3) and (6.2), using (1) and simplifying the following two propositions follow.
Proposition 4: Propositions 1 and 2 continue to hold even in period 2.

Proposition 5: In period 2 the skilled-unskilled wage inequality improves under all situations except when $\theta_{L1}K_2 < \theta_{K1}S_2$ and the $\left(\frac{W_S}{W}\right)$ ratio is sufficiently high.

Although the intuitive explanations of proposition 4 remain the same as before those of the remaining one would be somewhat different. These are as follows. First, let us see what happens to the wage inequality in period 2 resulting from skills formation in period 1. Given the rate of skills subsidy, $s$, (or in the absence of any subsidy) the skilled wage, $W_S$, falls as the supply of skilled labour, $\bar{S}$, rises following the joining of skill-acquired unskilled labour in the army of skilled workers in period 2. This raises the return to capital, $r$ (equation (3)) which in turn lowers the unskilled wage, $W$ (equation (2)). Hence, in complete contrast to our findings in period 1, we here find that both the wages fall. The working class irrespective of whether skilled or unskilled becomes worse-off. In each of the two sectors the cost on capital has increased. What happens to the relative wage inequality now depends on the difference in intersectoral distributive shares of capital i.e. $\theta_{s}$. The higher the value of $\theta_{s}$, the higher would be the fall in the wage rates. Thus, given the rate of skills acquiring subsidy, $s$ ($\geq 0$) the wage inequality rises due to skills formation if $\theta_{K1} > \theta_{K2}$. Let us now consider the situation where $s > 0$ and $s$ rises. $L$ falls in both the periods while $\bar{S}$ rises even further. Sector 1 contracts while sector 2 expands as unskilled labour and skilled labour are specific to those sectors, respectively. Consequently, the expanding sector 2 demands more capital while the contracting sector 1 releases capital. If $\left(\frac{aK_2}{aS_2}\right) > \left(\frac{aK_1}{aL_1}\right)$, the net effect would be an increase in demand for capital that raises its return, $r$. Both $W$ and $W_S$ decrease.

When $\theta_{L1}K_2 < (\leq)\theta_{K1}S_2$ i.e. when $\theta_{K2} > (\leq)\theta_{K1}$ the proportionate fall in $W_S$ would be greater (less) than that in $W$. Hence the wage inequality improves (worsens). However, if

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10 See verbal explanations provided for propositions 1 and 2.
both the wage rates increase. In this case, the wage inequality improves due to the subsidy if \( \frac{\theta_{L1} K_2}{\theta_{K1}} < \frac{\theta_{K1} S_2}{\theta_{S1}} \).

3. Concluding remarks

This theoretical note questions the validity of the conventional wisdom which suggests that endogenous skills formation lowers the skilled-unskilled wage inequality in a small open economy in both present and future periods and therefore the endeavour on the part of unskilled workers for skills acquisition should be encouraged by public subsidy. For analytical purpose, we have used a two-sector, specific factor general equilibrium model with two types of labour, skilled and unskilled, where unskilled workers have the opportunity to go for skills acquisition, become skilled and earn a higher wage in future. The aggregate supply of unskilled labour in the economy is determined from the intertemporal utility maximizing-behaviour of the unskilled working families in the current period (period 1). The unskilled workers after acquiring skills join the army of skilled labour in future thereby increase the endowment of skilled labour. The aggregate size of the workforce is assumed to remain unchanged over time. A public subsidy designed to promote skills formation lowers the supply of unskilled labour in both the periods but raises that of skilled labour in future. Our analysis has found that an increase in the public subsidy raises the skilled-unskilled wage inequality in the present period if the high-skill sector is capital-intensive (in the Jones-Neary

\[ \frac{a_{K2}}{a_{S2}} < \frac{a_{K1}}{a_{L1}} \], the net effect would be a fall in the demand for capital that leads to a decrease in \( r \). Both the wage rates increase. In this case, the wage inequality improves due to the subsidy if \( \frac{\theta_{L1} K_2}{\theta_{K1}} < \frac{\theta_{K1} S_2}{\theta_{S1}} \).

\[ \frac{\theta_{L1} K_2}{\theta_{K1}} < \frac{\theta_{K1} S_2}{\theta_{S1}} \] if the ratio \( \frac{W_s}{W} > 1 \) is very low and close to unity. But, in the opposite case we always have

\[ \frac{a_{K2}}{a_{S2}} > \frac{a_{K1}}{a_{L1}} \] since \( W_s > W \). So, the demand for capital cannot fall in this situation.
sense) and may accentuate inequality even in period 2 under certain parametric restrictions.\footnote{See Jones and Neary (1984).} In period 1 both the wages increase while the return to capital falls; thereby leading to a redistribution of income in favour of the working class although the income inequality may become more uneven among the two groups of the working population differentiated according to skills. However, in period 2 both the wages fall while the return to capital increases except in a special case that leads to a redistribution of income against the working class although their inter-group inequality is most likely to diminish.

Finally, it should be pointed out that in this analysis some important issues like labour market imperfection especially those of unskilled labour, unemployment problem, efficiency wage consideration, and collective bargaining have not been considered. Furthermore, we do not take into account savings and direct costs associated with skills acquisition and its financing problem. Hence, the role of capital market imperfection has not been taken care of. The absence of any non-traded skill-intensive sector like services is also a limitation of the analysis. If some of these features are taken into consideration the results of the model may hold subject to one or two additional sufficient conditions. However, despite simplicity and abstraction the present analysis deserves some attention because it questions the desirability of providing public subsidy to assist skills formation from the perspective of improving the relative wage inequality in both present and future periods.

**References:**


