An Improvement on Sticky Price Assumptions

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An Improvement on Sticky Price Assumptions

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Abstract

This paper suggests an improvement to the assumptions underlying the New Keynesian Phillips Curve. The Curve rests on the assumption that price constrained producers commit to align their production along the demand curve, bypassing profit maximization. This assumption unnecessarily makes demand-driven supply a postulate instead of a result. This paper shows price constrained producers align their production along the demand curve, without commitment, if faced with constant marginal costs, and that this supposes additional agents, retailers. Furthermore, the paper restates how without this commitment, but with increasing marginal costs, the New Keynesian Phillips Curve is invalid and prices are acyclical, not procyclical.

Keywords: New Keynesian Phillips Curve, micro-foundations, price rigidity, marginal cost, Retailers.

JEL: E31, E12, D43.

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1 Introduction

Research on price rigidity culminated, with Roberts (1995) and Yun (1996), on the New Keynesian Phillips Curve (NKPC). Helped by the popularity of Dynamic Stochastic General Equilibrium (DSGE) models, the NKPC dominates as the standard model of inflation in modern macroeconomics. DSGE models came to the forefront of macroeconomics starting with Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003). Academia uses them to test theories while central banks and international institutions rely on them as forecast models. The NKPC emerged as a central part and permanent fixture of these popular models.

However, an assumption, made in the middle of constructing the curve, is unusual. The NKPC manipulations set the production of constrained producers (those that cannot reset price in the NKPC specification) along the product’s demand curve. According to marginal behavior, producers that cannot reset their price should choose production levels so marginal costs are coherent with the imposed prices. Therefore, a binding price constraint should lead to a surplus or penury in the product’s market, akin to a price ceiling or floor. These producers would reduce production when facing low prices instead of increasing them (while unconstrained producers react only to relative prices). Consequently, this type of price rigidity will not produce procyclical inflation.

The commitment assumption is akin to posing a Keynesian result instead of deducing it from assumptions that are more general. My contention is that the assumption was never a big deal because it was deemed necessary. The paper suggest otherwise.

To fix the issue, Section 2 investigates constant marginal costs. Constant marginal costs make producing along the product’s demand curve not an assumption, but a result. It comes at the price of additional, though more realistic, assumptions. Specifically, additional agents, retailers, buy from producers, who act in perfect competition with increasing marginal costs, then sell to consumer under monopolistic competition with a constant (the wholesale price) marginal cost.

The wholesale price makes the retailer’s supply curve horizontal. The markup from

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1 Sims (2012) and Christiano, Trabandt and Walentin (2010) discuss the historical development and current importance of DSGE modeling in academic work, while Dotsey (2013), Smets, Christoffel, Coenen et al. (2010) and Botman, Karam, Laxton et al. (2007) discuss its use by central banks and international institutions.

2 For example, in (Woodford, 2003, Part 1, Chapter 3, p. 155): “I assume that the supplier of good i is committed to supply whatever quantity buyers may wish to purchase at the predetermined price $p_t(i)$, and hence to purchase whatever quantity of inputs may turn out to be necessary to fill orders.”
monopolistic competition acts as a buffer keeping them profitable when relative prices drop. With constant prices, marginal changes in profits depend on the evolution of marginal costs, and constant marginal costs mean marginal profits do not decrease with each additional unit. The retailer wants to supply the maximum that demand allows without being forced to by a previous commitment.

As for producers, they cannot face constant marginal costs in a macroeconomic model where production involves diminishing returns to scale. Most macroeconomic models use constant returns to scale at the steady state. But in the short term (of interest to Keynesians), microeconomics expect diminishing returns. In fact, real rigidities on capital or investments create short-term diminishing returns. Putting a retailer between producer and consumer alleviate this difficulty by permitting diminishing return at an earlier stage of production.

In a way, the assumption of commitment is acceptable: people put what they want in contracts. The contract stipulates a one sided commitment that does appear in reality, though not frequently. By contrast, retailers exist, and they are the natural intermediary between producers and consumers. Consumers usually deal with retailers to buy their goods\(^3\). Furthermore, putting monopolistic retailers between producers and consumers in this context is common. For example, Bernanke, Gertler and Gilchrist (1999) incorporates retailers to the price rigidity model to avoid aggregation issues relating to producers engaging in financial transactions, and Walsh (2005), in labor market transactions.

Section 3 shows how much damage comes from avoiding both rival assumptions. Simple algebra shows standard microeconomic assumptions result in an intractable model with uninteresting results. Consequently, price rigidity relies on one of these assumptions.

The paper is structured as follows. Section 2 explains how a model with retailers leads back to the original NKPC, without the need of the commitment assumption. Section 3 details results from the model with neither the commitment, nor the constant-marginal-costs assumptions. Finally, Section 4 offers concluding remarks.

\(^3\)Although retailers are much less frequent in services, appointments are. Appointments means there is no commitment to supply whatever quantity buyers may wish to purchase.
2 Consumers, producers and retailers

This section is divided into three subsections. The first two subsections are conventional and meant to establish the problem and the equations used later. Subsection 2.1 details the choices of the consumer and Subsection 2.2 poses constant marginal costs and shows the results. Finally, Subsection 2.3 applies those results to a model with retailers.

2.1 Consumers

The NKPC combines monopolistic competition and price rigidity to create a structural model where marginal costs drive inflation. Dixit and Stiglitz (1977) aggregation implements monopolistic competition and Calvo (1983) contracts implements price rigidity. As a result, producers individually set their prices as a function of marginal costs and of the probability of being able to change them in the future; aggregation and log-linearization do the rest. The resulting equation is the NKPC,

\[ \pi_t = \gamma c_t + \beta E_t \pi_{t+1}, \]

where \( \pi \) represents inflation, \( c \), marginal cost, \( \beta \), the discount factor and \( \gamma = (1 - \omega)(1 - \beta \omega)/\omega \), where \( \omega \) represents a producer’s probability of not being able to reset price in a given period. A presents the other standard assumptions behind the model. In addition, Woodford (2003) provides further details and discussion.

In the familiar consumer side equations, a representative consumer optimizes using cost minimization subject to a given utility level, or

\[ \min_{y_{i,t}} \int_0^1 p_{i,t} y_{i,t} \, di \quad \text{subject to} \quad Y_t = \left( \int_0^1 y_{i,t} \, di \right)^{\frac{\varepsilon}{\varepsilon - 1}}, \]

where \( p_{i,t} \) and \( y_{i,t} \) respectively represent the price and production of producer \( i \) at time \( t \) and \( Y_t \) represents the aggregate production measure at time \( t \).\(^4\) The constraint represents a CES utility function for a continuum of products. The minimization results in the first order conditions that consist of the utility constraint and

\[ p_{i,t} - P_t Y_t^{\frac{1}{\varepsilon}} y_{i,t}^{\frac{\varepsilon - 1}{\varepsilon}} = 0, \]  

\(^4\)For readability, throughout the paper, subscript \( i \) refers to producers who may be constrained or not, \( j \), to unconstrained producers and \( k \), to constrained producer. Subscript \( r \) refers to retailers. In one (obvious) instance, the subscript will not include all the producers of its type.
where $P_t$ represents the Lagrange multiplier, but also represents the price index, or the marginal utility of an additional unit of aggregate production.

2.2 Producers

This section supposes a common constant marginal cost for producers. The specification’s analysis shows that, although some producers may refuse to produce, the NKPC fares well.

If marginal costs are increasing, constrained producers see their marginal profits drop when they produce more. This is textbook microeconomics; it leads to the increasing supply curve shown in Figure 1. But if marginal cost are constant, the supply curve flattens and constrained producers rationally produce as much as the consumer asks, making the commitment assumption unnecessary. The only problem arises when relative prices drop below real marginal costs.

With constant marginal costs, constrained producers refuse to produce with prices below a certain level. The problem admits only a border solution,

$$y_{k,t} = \begin{cases} 
0 & \text{if } p_{k,s} < P_t\bar{c}_t \\
\left(\frac{p_{k,t}}{P_t}\right)^{-\varepsilon} Y_t & \text{if } p_{k,s} \geq P_t\bar{c}_t
\end{cases},$$

where $\bar{c}_t$ represents the common marginal cost. Low enough relative prices mean decreased production, with some producers halting production.

Using the fact that by construction, $p_{j,t} \geq P_t\bar{c}_t$ for unconstrained producers and inserting the solution in the utility constraint equation, yields

$$Y_t = \left(\int_{[p_{k,t}<P_t\bar{c}_t]} 0 \, dk + \int_{[p_{i,t}\geq P_t\bar{c}_t]} \left(P_t^{\varepsilon}Y_t(p^{-\varepsilon}_{i,t})^{\frac{\varepsilon-1}{\varepsilon}} \, di\right)^{\frac{\varepsilon}{\varepsilon-1}} \right)^{\frac{\varepsilon-1}{\varepsilon}}. $$

Then, isolating $P_t$ generates the price index,

$$P_t = \left(\int_{[p_{i,t}\geq P_t\bar{c}_t]} p_{i,t}^{1-\varepsilon} \, di\right)^{\frac{1}{1-\varepsilon}}. $$

This index does not behave well. Specifically, the integral’s support depends on the price index as it more or less counts the number of $p_{i,t}$ above the cutoff point.

Because of the markup though, production continues. Here the markup actually serves a purpose unlike in Section 3, a section robust to whether producers price over
or at marginal cost. Producers take a hit, but continue production by *gnawing* at their markup since they still make a profit as long as they face a small enough price change. For large-enough markups, say larger than the largest inflation-prediction error, the price index, equation (2), coincides with the New Keynesian one (one without the constrained support) because the inequality restriction never binds.

### 2.3 Retailers

Assuming constant marginal cost in a macroeconomic model leads to difficulties because macroeconomic models often pose increasing marginal costs in the short term, as with investment adjustment costs or variable capital utilization. The solution is to separate production and distribution into two types of agents: producers and retailers. That way, production can involve increasing marginal costs, yet the consumer still deals with sellers that *produces* (distributes) under constant marginal costs.

In the first stage of a product’s life, a continuum of producers produces with increasing marginal costs. They act in perfect competition, selling at marginal cost to retailers. As a result, every producer sets marginal cost equal to the competitive wholesale price. A wholesale price (in real terms for simplicity) of \( q_{i,t} \) from producer \( i \), yields

\[
q_{i,t} = c(y_{i,t}) = q_t,
\]

where \( c \) is the real marginal cost function, and competition makes every \( q_{i,t} \) the same, \( q_t \).

In the second stage, retailers act in monopolistic competition, selling to the consumer at a markup over the price they paid, the wholesale price. For a retailer, the wholesale price represents its marginal cost, the rest of the costs consisting of fixed costs (rent on the store surface, clerk wages or other). For example, we could suppose that monopolistic competition arises from the consumer’s need to travel to stores, positioned at different parts of town generating finite elasticity of substitution between stores, shoe leather costs essentially.

Consequently, real total cost equals the real wholesale price times product, \( q_t y_{r,t} \) for retailer \( r \), and real marginal cost equals the real wholesale price, \( q_t \). The intertemporal profit maximization problem of retailer \( r \) becomes

\[
(3) \quad \max_{p_{r,t}} \left\{ E_t \sum_{s=0}^{\infty} \omega^s \beta^s \left( \frac{Y_{t+s}}{Y_t} \right)^{-\rho} (p_{r,t} y_{r,t+s} - P_{t+s} q_{t+s} y_{r,t}) \right\},
\]
where $\omega^s$ represents the probability, uniform across producers, for a producer of not being able to reset price until time $t+s$. The CRRA stochastic discount factor discounts profits, with $\beta$, the ordinary discount factor, and $\rho$, the relative-risk-aversion coefficient.

The first order conditions come after inserting $y_{r,t}$ from equation (1) into equation (3) before optimization, yielding

$$E_t \sum_{s=0}^{\infty} \omega^s \beta^s \left( \frac{Y_{t+s}}{Y_t} \right)^{1-\rho} \left( \frac{P_{t+s}}{P_t} \right)^\varepsilon \left( (1-\varepsilon)p_{r,t} + \varepsilon P_{t+s}q_{t+s} \right) = 0.$$  

Consequently, the usual manipulations behind the NKPC apply for a high enough markup, and lead to an inflation equation,

$$\pi_t = \gamma q_t + \beta E_t \pi_{t+1}.$$  

Furthermore, because $q_t = c_t(y_{i,t})$ for every producer $i$, that inflation equation reverts back to the NKPC.

In practice, most retailers’ markups go over even 1970s levels of inflation. For example, most small retailers choose a markup at about two times the wholesale price. Retail calls it the keystone markup. Barsky, Bergen, Dutta et al. (2002) describe other examples of markups. In practice, the inequality restriction rarely binds.

Thus, adding retailers to the list of the agents in the model fixes the NKPC at little conceptual costs.

### 3 Increasing marginal costs without commitment

This section makes the point that without one or the other assumption made earlier, price rigidity is not Keynesian. The arguments will remind older readers of disequilibrium economics. But they bare repeating in a succinct exposition, since, as shown in Backhouse and Boianovsky (2005), disequilibrium economics is fading into obscurity.

Subsection 3.1, on the choices of constrained producers, explains the difference between standard optimizing behavior and commitment to fulfill demand. It parallels the disequilibrium literature. Subsection 3.2 simply applies the results to aggregate production, and shows that standard optimizing behavior leads to an intractable price...

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5For a description and a history of disequilibrium economics, see Backhouse and Boianovsky (2013), or the working paper version, Backhouse and Boianovsky (2005).
index. Finally, Subsection 3.3 shows why equilibrium is impossible if price rigidity matters.

### 3.1 Constrained producers

The arguments of this exposition hinge on the constrained producer’s supply curve. Constrained producers face a static, one-period optimization problem. A price constrained producer \( k \) chooses \( y_{k,t} \) to maximize profits, \( p_{k,t}y_{k,t} - P_tC_t(y_{k,t}) \), where \( C \) represents the total cost function. Replacing \( p_{k,t} \) from equation (1) before profit maximization yields Lerner’s formula,

\[
p_{k,t} = \frac{\varepsilon}{\varepsilon - 1} P_t c_t(y_{k,t}),
\]

where \( c \) represents the marginal cost function. Lerner’s formula determines supply of a constrained producer whenever the producer can sell all its output.

Whenever the producer cannot sell all its output, he/she produces along the demand curve. Since constrained producers face price constraints, not production constraints, they choose a production level along the consumer demand curve when the relative price they face is too high. Producers produce only what they can sell when facing a relative price too high for a perishable product. If the relative price is too low, constrained producers choose a production level along the supply curve (marginal cost and markup). Hence, these producers set production to the minimal production level between the supply and demand curves.

Considering both supply and demand constraints yields the rationing rule,

\[
y_{k,t} = \min \left\{ c_t^{-1} \left( \frac{\varepsilon - 1}{\varepsilon} \frac{p_{k,t}}{P_t} \right), \left( \frac{p_{k,t}}{P_t} \right)^{-\varepsilon} Y_t \right\},
\]

where \( c^{-1} \) represents the inverse of the marginal cost function.

The lower part of the supply curve — the part with sufficient demand — simply represents marginal cost augmented by a markup. The markup does not vary; it does not act as a buffer permitting producers to act irrational. Only a constant marginal cost permits the markup to behave like a buffer. In fact, if constrained producers price without markup, at marginal cost \( p_{k,t} = P_t c_t(y_{k,t}) \), every point made in this section remains valid.

For its part, the NKPC poses that constrained producers commit to a production level along the consumer demand curve so the demand function, equation (1), always
Figure 1: Production determination for a constrained producer

Without commitment

With commitment

Note: A low relative price can mean a decrease or an increase in production for constrained producers depending on how producers set supply. Unconstrained producers are unaffected as they adjust to changes in relative prices.

applies. Hence, the point of contention of this paper: producers ignore profit maximization and their own supply incentives. In other terms, the NKPC supposes some kind of commitment possible to keep producers from behaving rationally in the future.

Furthermore, marginal costs end up the same for every producer as a result of NKPC assumptions. Imagine a producer forced to sell at lower price than other producers did. That producer can choose its production level, he or she should decrease production since this producer needs to decrease production to decrease marginal cost and regain or increase profitability. Otherwise, the producer is irrational.

Figure 1 illustrates both approaches. In the figure, $P^o$ represents the optimal price index from the standpoint of some constrained producer, while $P^l$ and $P^h$ represent respectively lower and higher price indexes than $P^o$. The commitment assumption, from standard NKPC manipulations, associates $P^h$ with increased production yielding a model where producers, when forced to sell at low relative prices, produce more. If using standard-microeconomic behavior instead, any deviation from $P^o$ lowers production. A surprise rise in the price index does not raise production; in fact, any surprise lowers production.
3.2 Aggregate production

The standard manipulations pose the intertemporal profit maximization problem of the unconstrained producer $j$ as

$$\max_{p_{j,t}} \left\{ E_t \sum_{s=0}^{\infty} \omega^s \beta^s \left( \frac{Y_{t+s}}{Y_t} \right)^{-\rho} \left( p_{j,t} y_{j,t+s} - P_{t+s} C_{t+s} (y_{j,t+s}) \right) \right\},$$

where $C$ represents real total cost as a function of production, which is common to all producers.

The first order conditions come after inserting $y_{j,t}$ from equation (1) into equation (6) before optimization, yielding

$$E_t \sum_{s=0}^{\infty} \omega^s \beta^s \left( \frac{Y_{t+s}}{Y_t} \right)^{1-\rho} \left( \frac{P_{t+s}}{P_t} \right)^{\varepsilon} ((1 - \varepsilon)p_{j,t} + \varepsilon P_{t+s} C_{t+s} (y_{j,t+s})) = 0.$$  

With the commitment assumption, the demand function applies to every product, and yields a simple price index,

$$P_t = \left( \int_0^1 p_{t,1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}},$$

from a straightforward insertion of the demand functions into the utility constraint. In turn, because of the symmetry of producers, the price index solves recursively to

$$P_t^{1-\varepsilon} = (1 - \omega)(p_t^*)^{1-\varepsilon} + \omega P_{t-1}^{1-\varepsilon},$$

where $\omega$ also acts as the share of constrained producers and $p_t^*$ represents the optimal price common to unconstrained producers. Lagged prices appear because every producer has a uniform probability, $\omega$, of not being able to reset its price, thus creating a mirror image of the price index.

The NKPC therefore relies on the equation (9) to yield contemporary inflation through $P_{t-1}$ by combining the equation with equation (7) after manipulations involving log-linearization. Note the unidentified level of the $P$’s in equation (7), a homogeneous function in only contemporary or future (not fixed in advance) prices. Past prices in equation (9) and marginal costs in equation (7) produce an inflation equation, the NKPC. I will spare readers the straightforward subsequent manipulations.

On the other hand, a supply function consistent with microeconomics does not
produce a well-behaved price index. The price index, generated from equation (5), is solved by isolating $P_t$ from

$$Y_t = \left( \int_{0}^{z_t} c_t^{-1} \left( \frac{\varepsilon - 1}{\varepsilon} p_{k,t} \right)^{\frac{\varepsilon - 1}{\varepsilon}} dk + \int_{z_t}^{1} \left( \frac{p_{k,t}}{P_t} \right)^{\frac{\varepsilon - 1}{\varepsilon}} Y_t \, dk \right)^{\frac{\varepsilon}{\varepsilon - 1}},$$

where $z_t$ represents the proportion of producers that set price equal to marginal cost augmented by a markup at time $t$. In addition, the proportion of producers that along with facing price constraints, also set production along the demand curve equals $(\omega - z_t)$, while the proportion of unconstrained producers equals $(1 - \omega)$. Moreover, $z_t$ depends on the other variables. This proportion varies through time according to the movement of that very price index, a price index that makes it impossible to generate the aggregation in equation (9).

Of course, generating the aggregation equation is not the point. The point is, the resulting inflation equation, if such an inflation equation possible, will not imply procyclical inflation. The rest of this paper will not determine what this inflation equation looks like, because the equation would be uninteresting. New Keynesians justify the assumptions behind the NKPC with a certain result, procyclical inflation. Finding an inflation equation free of that result offers little value.

### 3.3 The impossibility of equilibrium

This subsection supposes that somehow there exists a macroeconomic argument restricting the constrained producers’ product equilibrium always in the intersection between the demand curve and the supply curve. This specification’s analysis shows the model creates a proportionality between prices and nominal marginal costs that makes the NKPC vanish and a model of inflation impossible.

When demand equals supply for every constrained producer, the first order condition, equation (7), yields, by imposing $p_{j,t} = \frac{c}{c_{t+s}}(y_{j,t+s})$, $\forall s > 0$ (note the summation now starts at 1),

$$\omega^0 \beta^0 \left( \frac{Y_t}{Y_t} \right)^{1-\rho} \left( \frac{P_t}{P_t} \right)^{\varepsilon} \left( (1 - \varepsilon) P_{j,t} + \varepsilon c_t(y_{j,t}) \right) + E_t \sum_{s=1}^{\infty} \omega^s \beta^s \left( \frac{Y_{t+s}}{Y_t} \right)^{1-\rho} \left( \frac{P_{t+s}}{P_t} \right)^{\varepsilon} \left( (1 - \varepsilon) P_{j,t} + (\varepsilon - 1) P_{j,t+s} \right) = 0,$$
which simplifies to
\[ p_{j,t} = \frac{\varepsilon}{\varepsilon - 1} P_t c_t(y_{j,t}). \]

This equation for unconstrained producers is the same as equation (4) for constrained producers. Therefore, both constrained and unconstrained producers set marginal costs according to the same rule, Lerner formula.

As a result, the price index, equation (8), becomes
\[ P_t = \left( \int_0^1 \left( \frac{\varepsilon}{\varepsilon - 1} P_t c_t(y_{i,t}) \right)^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}}. \]

Meaning nominal marginal costs proportional to the price index, or simply put,
\[ \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{1-\varepsilon} \int_0^1 c_t(y_{i,t})^{1-\varepsilon} \, di = 1. \]

The price index disappears; real marginal cost does not influence it. Consequently, because of the symmetry between producers, the price level does not influence production. This result holds not only when some producers are constrained in the price they set, but also when those that can set their price have to consider its effects on future profits. Although still technically valid, equation (9) becomes irrelevant.

Ergo, equilibrium between supply and demand for every producer leads to the price index disappearing and the NKPC not existing. The disappearance of the price index, essentially a numéraire, appears a logical consequence of posing equilibrium, because the numéraire always disappear in ordinary microeconomic models.

This specification shows something already suspected: the NKPC fundamentally rests on disequilibrium, thus highlighting, as did Mankiw (2006), the kinship between disequilibrium economics and New Keynesian economics. New Keynesian markets clear though because the producers (with the commitment assumption) or the retailers (without it) assume the risks of this disequilibrium.

4 Conclusion

The paper showed the NKPC can rely on a more realistic assumption involving retailers. Furthermore, it restates how no commitment to fulfill future demand combined with increasing marginal costs lead to a complicated and acyclical inflation equation, but also how equilibrium between demand and an increasing supply means no inflation
equation is possible.

Overall, the paper aims to contribute to a better understanding of what the Phillips Curve is and means. This will have implications as research extending the NKPC could use, or have to contend with, the new assumptions.

Furthermore, the beauty of incorporating retailers to the price rigidity model comes from researchers not having to alter their models or codes, but just writing a short description of the new assumptions.

References


A Standard assumptions behind the NKPC

This appendix presents the other assumptions of the model:

**Assumption 1** There is one representative consumer who values products with a constant elasticity of scale (CES) function with an elasticity parameter, $\varepsilon$.

**Assumption 2** The consumer faces time preferences represented by a constant relative risk aversion (CRRA) function with a risk-aversion coefficient, $\rho$ and an ordinary discount factor, $\beta$.

**Assumption 3** The consumer ultimately owns every producer, so the consumer’s stochastic discount factor determines profit discounting.

**Assumption 4** Producers are symmetrical and each produces a differentiated and perishable product.

**Assumption 5** Every producer faces a constant probability $\omega$ of not being able to reset its price at the next period.

**Assumption 6** All production goes into consumption, none into capital.

For simplicity and exposition, the standard model makes these assumptions intentionally too restrictive. Researchers can replace Assumptions 2 and Assumptions 3 by posing another reasonable stochastic discount factor. They usually also generalize Assumption 6.