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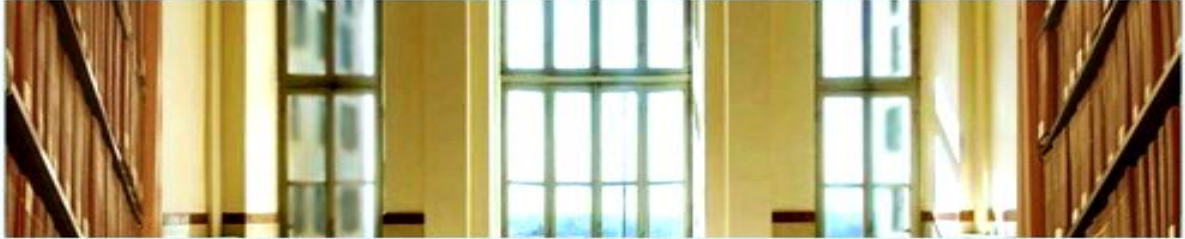
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AGGLOMERATION ECONOMIES AND OPTIMAL EFFICIENCY WAGE

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Abstract

In this paper we consider a model of optimal efficiency and agglomeration economies. Given the presence of urban and social externalities and in the absence of corrective policy, the efficiency wage chosen in decentralized market economy is too high. Indeed, in her optimal choice of wage and employment, the representative firm does not take into account of these agglomeration positive externalities which leads to a sub optimal city size. We have shown that an optimal wage employment allocation exists and can be implemented through a subvention taxation policy if it is available.

Key Words: Efficiency wage, Agglomeration Economies, City Size, Taxation Subvention

1.Introduction

During the 80s, after the two oil shocks, market economies (especially in Europe) experienced unusually high levels of persistent unemployment. This worrying macroeconomic scenario lead economists to develop new theories aimed at explaining the reasons underlying bad employment situation prevailing at that time. The labor market theories framed during this period can be grouped in two categories: those stating that unemployment is caused by too high wages and those who state exactly the opposite, i.e., that unemployment is the upshot of low wages. One group of theories collects the theory of efficiency wages (Akerlof and Yellen, 1986) and the theories of trade unions or insiders-outsiders (Lindbeck and Snower, 1988).

According to the theory of efficiency wages, there is an increasing causal relationship between the wage paid by firms and the level of effort provided by workers (Akerlof and Yellen, 1986, Katz, 1986). This kind of behavior might find its rationale on adverse selection or moral hazard factors affecting the workforce. Therefore ,in equilibrium, firms may find it profitable to pay wage in excess of market clearing. This theory is able to explain a great number of stylized labor markets facts, including real wage rigidity, dual labor markets, wage distributions for workers with identical productive characteristics and discrimination among observationally distinct groups. Because of the impact of the wage setting on the workers' effort function, profit-maximizing firms are expected to set an optimal wage such that the elasticity of effort function with

respect to wage is equal to one. This well-known result of the standard efficiency wage model is due to Solow (1979) and is known as the Solow condition. The optimal efficiency wage minimizes the employer's wage cost per effective units of service employed and each firm hires labor up to the point where the marginal product is equal to the efficiency wage.

However, it has been suggested that the Solow condition does not hold in general. In particular, Akerlof and Yellen (1986) point out that an effort-wage elasticity of unity is undoubtedly excessive.

This is an important issue, since it casts doubt on the possibility of equilibrium with unemployment in an efficiency wage model. A large number of papers have been proposed in the literature to illustrate an effort-wage elasticity lower than one. Among other papers, Akerlof and Yellen (1986) present a static model with external costs to account for the downside risk from shirking labor. In Schmidt-Sørensen (1990), fixed employment costs per worker are introduced in the profit function. Pisauro (1991) sets out a model with specific taxes on labor. Lin and Lai (1994) show that the Solow condition does not hold in an intertemporal maximizing framework with turnover costs. Marti (1997) and Faria (2000) examine models that combine the shirking and the turnover models of efficiency wage, with the possibility of managerial supervision. The role of the quality of job matching on efficiency wages is analyzed by Jellal and Zenou (1999). When job matching is unobservable, firms can either set wages such that the effort-wage elasticity is lower or greater than one. Finally, Jellal and Zenou (2000) consider a more general dynamic efficiency wage model with learning by doing, where workers accumulate a stock of knowledge that allows them to increase their effort.

Rather than relying on microeconomic foundations for the efficiency wage model, such as shirking or labor turnover costs, we propose different idea in this paper to show that the Solow condition does not hold in general. Indeed, we analyze the social optimal efficiency wage policy in an urban labor market with agglomeration economies. To our knowledge, the theoretical literature on the link between urban agglomeration externalities and optimal efficiency wages is not very developed.

Our model aims to fill this gap by extending a model proposed by Stark and Shulka (1985) where the urban wage is exogenous. These authors consider a model that proves that it is possible to utilize urban economics to identify socially optimal levels of urbanization and, by implication, optimal levels of rural-to-urban migration. After identifying the reason that decentralized allocation do not add up to the social optimum, they propose an analysis of instruments which could confer efficiency gains by closing the gap between the decentralized efficient and centralized optimal urban concentrations.

The remainder of the paper is organized as follows. In section 2, we present a labor markets model with decentralized efficiency wage. The centralized Solow condition and optimal public policy are discussed in section 3. Concluding comments are in section 4.

2. The Decentralized Efficiency Wage

The Literature of efficiency wage (Akerlof and Yellen ,1986) argue that the Solow condition depends on a production function that is labor augmenting in effort $F(ne(w))$ where $e(w)$ is the aggregated effort function for the urban workers, w is the level of wage in the urban firm and n is the number of workers in the representative primary firm, with the standard assumption of concavity $F'(\cdot) > 0$, $F''(\cdot) < 0$ and $e'(w) > 0$, $e''(w) < 0$. It is mentioned that other plausible production functions are expected to have lower wage-equilibrium wage elasticity. Indeed, in the recent efficiency wage literature, it is known that the Solow condition may be invalid if a general production function with separate arguments $F(n, e(w))$ is taken into account (Rasmaswamy and Rowthorn, 1991). However, this production function does not include productivity due to agglomeration externalities or the city size. Let us consider the impact of agglomeration economies on the wage employment policy as follows.

Consider an infinitely lived identical urban firms that have a general production function given by $A(N)F(n, e(w))$ where the function $A(N)$ captures the productivity related to agglomeration externalities or the city size. It depends on the aggregate urban employment or the city size given by $N = \int n(i)di$. It is assumed that $A'(N) > 0$ and $A''(N) < 0$. In a decentralized economy, urban firms do not take into account the economic effects of agglomeration externalities related to the size of the urban city given by the function $A(N)$. Hence, the optimization program for the representative firm is :

$$\max_{l,w} \Pi(n, w) = A(N)F(n, e(w)) - wn \quad (1)$$

With Ex post $n = N = \int n(i)di$

In this context, given the effort function and given the assumption on the production function, the maximization of the profit function with respect to \mathbf{n} and \mathbf{w} yields :

$$\frac{\partial \Pi(\mathbf{n}, \mathbf{w})}{\partial \mathbf{n}} = A(N)F_n(\mathbf{n}, e(\mathbf{w})) - \mathbf{w} = 0 \quad (2)$$

$$\frac{\partial \Pi(\mathbf{n}, \mathbf{w})}{\partial \mathbf{w}} = A(N)F_e(\mathbf{n}, e(\mathbf{w}))e'(\mathbf{w}) - \mathbf{n} = 0 \quad (3)$$

with $F_n = \frac{\partial F}{\partial n}$ and $F_e = \frac{\partial F}{\partial e}$.

We now find a more general solution for the effort-wage elasticity function.

Definition:

Let $\eta_n = \frac{n}{F} \frac{\partial F}{\partial n}$ and $\eta_e = \frac{e}{F} \frac{\partial F}{\partial e}$ be the elasticity of the production with respect to effort and employment in the urban sector, $\eta_a = \frac{N}{A} \frac{dA}{dN}$ the productivity-agglomeration elasticity and $\varepsilon(\mathbf{w}) = \frac{\mathbf{w}}{e} \frac{e'(\mathbf{w})}{e(\mathbf{w})}$ the effort-wage elasticity.

PROPOSITION 1:

Under decentralized economy, the optimal value for the efficiency wage is given by:

$$\varepsilon(\mathbf{w}) = \frac{\mathbf{w}}{e} \frac{e'(\mathbf{w})}{e(\mathbf{w})} = \frac{\eta_n}{\eta_e}$$

Proof :

From (2), we have $\mathbf{n}A(N)F_n(\mathbf{n}, e(\mathbf{w})) = \mathbf{w}n$. Using (3) and after some manipulations, we obtain the following equality:

$$F_e(\mathbf{n}, e(\mathbf{w}))e'(\mathbf{w})\mathbf{w} = \mathbf{n}F_n(\mathbf{n}, e(\mathbf{w}))$$

We finally deduce that the decentralized efficiency wage is such that:

$$e(w) \frac{F_e(n, e(w))}{F(n, e(w))} \frac{e'(w)}{e(w)} w = \frac{nF_n(n, e(w))}{F(n, e(w))}$$

Given the definitions of η_n and η_e , this proves the announced result :

$$\varepsilon(w) = \frac{w e'(w)}{e e(w)} = \frac{\eta_n}{\eta_e} \quad \text{Q.E.D}$$

COROLLARY 1:

The value of effort-wage elasticity is given by the following equivalence:

$$\varepsilon(w) \leq 1 \Leftrightarrow \eta_n \leq \eta_e$$

We observe that in a labor market model with a more general production function of the type $A(N)F(n, e(w))$, the optimal wage is independent of agglomeration economic externalities. The effort-wage elasticity function depends only on the internal technology of the urban firms .It can be lower, equal or greater than one, hence the Solow condition is likely to be invalid with a more general production function.

We can finally interpret this result. Let us consider the case where there exist high wage levels in the urban sector. When η_n is low in comparison with η_e , a low urban city size is expected in the urban sector and it is in the interest of the firm to set a high wage value. Conversely, when the value of η_n exceeds that of η_e , this means that the production function is not really sensitive to the workers' level of

effort which leads to a large size of urban city. Indeed, more labor is needed in the urban sector and the level of wage is set at a low value in that sector. In that case, it is useless to provide incentives for primary workers to work hard.

3. The Centralized Optimal Efficiency Wage

Since the urban firms do not internalize the externality of agglomeration induced by urban interactions, in decentralized market economy the level of labor force in the urban city is suboptimal. Indeed, given the assumptions and constraints facing urban firms ex ante, in centralized economy, a benevolent planner recognizes that firms are identical and that their wage employment allocation will be the same ex post, he is then naturally lead to internalize the agglomeration externality by assuming ex ante that employment levels $n = N$. Hence, in a centralized economy, the optimal labor force or city size is a solution of the planner's problem. This solution is given by the following optimization problem:

$$\max_{l,w} \Pi(n, w) = A(N)F(n, e(w)) - wn \quad (4)$$

With Ex ante $n = N = \int n(i)di$

The first order conditions are:

$$\frac{\partial \Pi(n,w)}{\partial n} = A'(N)F(N, e(w)) + A(N)F_n(N, e(w)) - w = 0 \quad (5)$$

$$\frac{\partial \Pi(n,w)}{\partial w} = A(N)F_e(N, e(w))e'(w) - N = 0 \quad (6)$$

From these conditions we obtain the following result.

PROPOSITION 2:

Under centralized economy, the optimal value for the efficiency wage is given a generalized Solow condition :

$$\varepsilon(w^*) = \frac{\eta_n}{\eta_e} + \frac{\eta_a}{\eta_e}$$

Proof :

From (5), we have $NA'(N)F(N, e(w)) + NA(N)F_n(n, e(w)) = wN$.

Using (6) and after some manipulations, we have the following equality:

$$NA'(N)F(N, e(w)) + NA(N)F_n(N, e(w)) = A(N)F_e(N, e(w))e'(w)w \quad (7)$$

Given that $\eta_n = \frac{n}{F} \frac{\partial F}{\partial n}$, $\eta_e = \frac{e}{F} \frac{\partial F}{\partial e}$ and $\eta_a = \frac{N}{A} \frac{dA}{dN}$ we have

$$NA(N)F_n(N, e(w)) \left[1 + \frac{\eta_a}{\eta_e} \right] = A(N)e(w)F_e(N, e(w)) \frac{e'(w)w}{e(w)}$$

From which we derive our announced result:

$$\varepsilon(w^*) = \frac{\eta_n}{\eta_e} + \frac{\eta_a}{\eta_e} \quad \text{Q.E.D.}$$

COROLLARY 2:

Under centralized market, the value of effort-wage elasticity is given by the following results:

$$\varepsilon(w^*) = \varepsilon(w) + \frac{\eta_a}{\eta_e}$$

$$\varepsilon(w^*) \leq 1 \Leftrightarrow \eta_n + \eta_a \leq \eta_e \quad .$$

This result tells us that in a centralized labor market model with a more general production function of the type $A(N)F(n, e(w))$, the optimal wage depends directly on the impact of agglomeration externalities which is given by the term η_a . Once again Solow condition is likely to be invalid in a more technological context. Therefore compared with decentralized allocation, the internalization of agglomeration externalities leads to lower centralized efficiency wage and higher aggregate employment or large city size.

We learn immediately that the efficiency wage choice in a centralized economy differs from that chosen in a decentralized economy by the fact that the planner takes into account the positive agglomeration externalities. Therefore, now, the optimal efficiency wage depends on the agglomeration elasticity. Hence, higher is this elasticity, higher is level of labor force and lower is amount of efficiency wage. Given this first best allocation, we now examine how the government can use the available taxation policy to make possible the implementation of centralized allocation market economy. Let us now examine the condition of Solow if the government is able to subsidize employment.

In this context the representative urban firm solves:

$$\max_{l,w} \Pi(n, w) = A(N)F(n, e(w)) - wn + sn - T \quad (8)$$

With Ex post $n = N = \int n(i)di$ and $T = \int sn(i)di$

Where s is the subsidy rate and T is a tax paid by the firms.

PROPOSITION 3:

Under decentralized economy with tax-subsvention scheme, the optimal value for the efficiency wage is given by:

$$\varepsilon(\mathbf{w}) = \frac{\eta_n}{\eta_e} \left(\frac{\mathbf{w}}{\mathbf{w}-s} \right)$$

Proof:

It is obtained by the same algebraic manipulations and is omitted.

We now observe that the wage depends on the size of subsidy and we ask what size subsidy rate could restore the optimality of first best.

PROPOSITION 4:

There exists an optimal subvention scheme which leads to an efficiency wage optimal in market economy and this subvention is given by the following rate:

$$s^* = \frac{\eta_a}{\eta_n + \eta_a} \mathbf{w}^*$$

Proof:

The implemented centralized optimal efficiency wage is given by:

$$\varepsilon(\mathbf{w}^*) = \frac{\eta_n}{\eta_e} + \frac{\eta_a}{\eta_e}$$

While the decentralized efficiency wage in a market economy is by:

$$\varepsilon(\mathbf{w}) = \frac{\eta_n}{\eta_e} \left(\frac{\mathbf{w}}{\mathbf{w}-s} \right)$$

Therefore we have $\mathbf{w}^* = \mathbf{w}$ if and only if:

$$\frac{\eta_n}{\eta_e} + \frac{\eta_a}{\eta_e} = \frac{\eta_n}{\eta_e} \left(\frac{w}{w-s} \right)$$

This equality holds if and only if we have:

$$s = s^* = \frac{\eta_a}{\eta_n + \eta_a} w^* \quad \text{Q.E.D}$$

We observe that optimal tax subvention is proportional to the optimal wage rate depends only on the parameters of the productivity. Further, the relative optimal subsidy depends positively on the agglomeration economies contribution rate and negatively on size labor force productivity. These results are quite intuitive. Then, we showed how we can implement an optimal urban city size in a market economy with efficiency wages if an appropriate taxation subvention policy is available and feasible.

4. Conclusion

In this paper we analyzed a model of optimal efficiency and agglomeration economies. Given the presence of urban externalities and in the absence of corrective policy, the efficiency wage chosen in decentralized market economy is too high. Indeed, in her optimal choice of wage and employment, the representative firm does not take into account of these agglomeration positive externalities which leads to a sub optimal city size. We have shown that an optimal wage employment allocation exists and can be implemented through a subvention taxation policy if it is available.

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