On Ratifiability of Efficient Cartel Mechanisms in First-Price Auctions with Participation Costs and Information Leakage

Cao, Xiaoyong and Hsueh, Shao-Chieh and Tian, Guoqiang

University of International Business and Economics, Southwestern University of Finance and Economics, Texas AM University

August 2009

On Ratifiability of Efficient Cartel Mechanisms in First-Price Auctions with Participation Costs and Information Leakage*

Xiaoyong Cao†
Department of Economics
University of International Business and Economics
Beijing, China, 100029

Shao-Chieh Hsueh‡
School of Economics
Southwestern University of Finance and Economics
Sichuan, China, 610074

Guoqiang Tian§
Department of Economics
Texas A&M University
College Station, TX 77843

Abstract

This paper investigates whether an efficient all-inclusive cartel mechanism studied by McAfee and McMillan (1992) can still preserve its efficiency when bidders can update their information through the cartel's collusive mechanism and there is a cost to participate in the seller's auction within the independent private values setting. It is shown that, when the seller uses the first-price auction, the usual efficient cartel mechanisms will no longer be ratifiable in the presence of both participation costs and potential information leakage. The bidder with the highest value in the cartel will have incentive to betray, sending a credible signal of his high value and thus discouraging other bidders from participating in the seller's auction. However, the cartel mechanisms would still be efficient if either participation cost or information leakage, but not both, is present.

Keywords: Ratifiability, efficient cartel mechanism, first-price auction, information leakage, participation cost.

JEL Classification Number: D42, D62, D82

*We wish to thank an anonymous referee for helpful comments and suggestions that significantly improved the exposition of the paper.

†Financial support from National Natural Science Foundation of China (NSFC-71201030) and "the Fundamental Research Funds for the Central Universities" in UIBE (14YQ02) is gratefully acknowledged. E-mail address: yongcx2000@uibe.edu.cn

‡E-mail address: schsuechieh@swufe.edu.cn

§Financial support from the National Natural Science Foundation of China (NSFC-71371117) is gratefully acknowledged. E-mail address: gtian@tamu.edu

1 Introduction

In this paper, we study the ratifiability of an efficient ring when the seller uses the first-price sealed-bid auction with participation costs and potential information leakage in the sense that bidders may update their information through pre-auction knockouts within the independent private values setting.

Auction is an effective way to extract private information by improving the competitiveness of potential buyers and thus can increase allocation efficiency from the perspectives of both sellers and the social planner. Efficiency, however, is diminished when buyers' collusion occurs. Nevertheless, asymmetric information between bidders makes it difficult to profitably collude and share the collusive surplus, which may destabilize a collusive ring. Some members in the ring may have incentive to veto a cartel mechanism. As Osborne (1976) indicates, when an efficient cartel mechanism is not freely implemented, a cartel may face both external and internal problems. A cartel has to anticipate and prevent outside production in order to avoid external threat. The internal problems include those in designing the rule, dividing the profit, detecting, and deterring cheating.

Much of the literature on collusion then tries to overcome cartel's problems mentioned above. For the external problems, vindictive strategies are a method which is usually used to enforce collusion (Cooper, 1977). For the internal problems, Roberts (1985) shows that the method of side payments is a dominant strategy for every member to join the cartel. Under different assumptions, the requirements of a successful collusive mechanism may be different: Graham and Marshall (1987) analyze a knockout mechanism organized by an outsider is efficient and sustainable. The knockout mechanism is also ex ante budget-balanced among bidders. Mailath and Zemsky (1991) discover that for any subset of bidders, there is an incentive compatible and individually rational collusive mechanism, which is ex post budget-balanced and efficient. Graham, Marshall, and Richard (1990) find when heterogeneous members exist, nested coalition structures are observed at each level of nesting. Cramton and Palfrey (1990) show when uncertainty exists, regardless of the ratification rule, a perfect collusion is possible in a large cartel.

As the synthesis of standard mechanism design literature, McAfee and McMillan (1992) explore the bidding strategies in first-price sealed-bid auctions under two cases: weak cartel and strong cartel. In a weak cartel, bidders cannot make transfer payments, and in a strong cartel, the members can exclude new entrants and make transfer payments among themselves. The model consists of one seller and n bidders. The seller's behavior is passive. She sets a reserve price for the object, and holds a legitimate auction to sell the object. She does not know whether she faces a cartel or not. Bidders can place identical bids in the seller's legitimate auction to achieve the optimal outcome without colluding. When side payments are prohibited, the incentive compatibility condition requires that the object should be allocated with equal probability to anyone whose value is greater than the seller's price. Bidders treat the seller as their randomizing device by placing equal bids.

In a strong cartel, the optimal cartel mechanism can reach the efficiency by implementing a prior auction before the seller's legitimate auction. The cartel will choose the bidder with the highest value to participate in the seller's auction. Since all bidders are in the cartel originally but only one participates in the seller's auction, he can submit the lowest bid that is equal to the seller's reserve price, and win the auction with certainty. The transfer payment is the difference between the winning bid in the prior auction and seller's reserve price. This transfer payment is distributed equally among all members in the cartel. With this extra payment, everyone in the cartel is better off than they would be in the non-collusive case, so no one has incentive to betray the cartel.

Despite the solutions mentioned above, many still argue that first-price auctions are effective in reducing the gain from the collusive behavior. The solutions are obtained from the comparison of bidders' interim payoffs. As Marshall and Marx (2007) state, when cartels cannot control members' bids, in first-price auctions, the cartel cannot eliminate all members' competition. Besides, shill bidding strategies reduce the profitability of collusive behavior at first-price auctions.

However, under (interim) individual rationality constraints, there may be a strategic interaction among bidders in the seller's auction, and its outcome would be affected by bidders' beliefs about others' values. Besides, there may be a participation cost involved in the seller's auction, which may make an efficient collusive mechanism inefficient, whatever the auction format is.

Thus, there are two constraints that a cartel mechanism must pay attention to. One is the information leakage problem in the sense that bidders may update their information from their participation decisions in the prior auction. Cramton and Palfrey (1995) show that when the information leakage problem exists, bidders' participation decisions in the prior auction would disintegrate the optimal cartel mechanism. Bidders can observe which bidder's value is highest from his choice of whether to veto the collusive mechanism, which in turn may affect the seller's revenue. Bidders may then set up a two-stage game: all players simultaneously vote for or against the collusive mechanism. This vote is held at the interim stage. If the collusive mechanism is unanimously accepted, it is implemented; otherwise they participate in the seller's auction with updated information.

The other constraint is that bidders may incur some costs, such as entry fees or sunk costs, when they participate in an auction. After the costs are incurred, a bidder may submit a bid. Mills (1993) points out that the bidding cost in a government procurement auction often runs in the range of millions of dollars. If a bidder's expected revenue from an auction is less than the participation costs, he will not attend the auction.

Tan and Yilankaya (2007) apply the ratifiability introduced by Cramton and Palfrey's (1995) to investigate whether efficient collusive bidding mechanisms are affected by potential information leakage and participation costs. They show that when the seller uses a second-price auction with participation costs, the standard efficient cartel mechanisms are not ratified by cartel members. However, as Tan and Yilankaya (2006) point out, there are several empirical studies that offer evidence of collusion in many auction markets, including highway construction contracts, federal offshore oil and gas lease auctions, etc. In these auctions, it is a first-price sealed-bid auction that is generally used. Since the first-price auction is often used in practice, one then is wondering whether an efficient all-inclusive cartel mechanism can still preserve its efficiency when seller uses a first-price auction with participation costs and bidders can update their information through a cartel's prior auction within the independent private values setting.

We will answer this question by considering a two-stage ratification game, following Cramton and Palfrey (1995) and Tan and Yilankaya (2007). In the first stage, the ring uses the knockout mechanism to coordinate bidding. Members send their bids to the ring in the (pre-auction) collusive mechanism which indicates the target that they are interested in, and how much they are willing to pay for it. The ring collects all members' bids, and determines the winner of the target and the side payments after the seller's auction concludes. Side payments are used by the cartel to compensate cartel members for not competing for the target in seller's auction. They simultaneously vote (interim) for or against the efficient cartel mechanism. In the second stage, if the cartel mechanism is unanimously accepted, it then is implemented; otherwise, bidders participate in the first-price auction, knowing who had vetoed the cartel mechanism and thus having updated beliefs about vetoers' values.

We will determine a veto set, such that if a bidder's value belongs to the set, he will choose to betray the cartel. With the veto set and the optimal inverse bidding function obtained in Cao and Tian (2010), we allow the presence of both participation costs and information leakage in the strong cartel studied by McAfee and McMillan (1992). However, we obtain a different result. We show that, when the seller uses a first-price auction, the usual efficient cartel mechanisms will no longer be ratifiable in the sense that a bidder vetoes a pre-auction in the presence of both participation costs and potential information leakage. The bidder with a value greater than a critical point in the cartel will have an incentive to veto. By vetoing the mechanism, a bidder sends a credible signal that he has a relatively high value, which discourages other bidders from joining the seller's auction when there are positive participation costs. However, even if there is information leakage so that bidders can update their information through a cartel's collusive mechanism, the usual efficient cartel mechanisms would still be ratifiable provided there is no participation cost. As such, there is a discontinuity about efficient collusive mechanisms at zero participation cost. A policy implication for this is that, in practice, the seller can charge some entry fee to decrease the possibility that bidders may form a cartel.

The remainder of the paper is structured as follows. Section 2 describes the economic environment. Section 3 considers the benchmark case where no information leakage is allowed. Section 4 allows the presence of information leakage and investigates the ratifiability of efficient cartel mechanisms. Section 5 concludes. All proofs are included in the appendix.

2 Economic Environment

Consider a standard independent private-valued economic environment with one seller and n ($n \geq 2$) potential bidders. The set of bidders is denoted by N. The seller values her object at $r \in [0, 1]$. She announces the reserve price r and sells an indivisible object to the bidder at the highest bid in a first-price sealed-bid auction, in which the bidder who submits the highest bid wins the object and pays what he bids. In order to simplify the calculation and notation, we assume that the reserve price $r = 0$. The analysis and results hold when there is a binding reserve price.

Bidder i's value is v_i, $i \in N$, which represents i's willingness to pay for the object in the auction, and $v = (v_1, ..., v_n)$ is the vector of n bidders' profile. v_i is private information which is a random draw from the same cumulative distribution function $F(\cdot)$ with continuous and strictly positive density function $f(\cdot)$ supported on $[0, 1]$.

In order to submit a bid, each bidder needs to pay non-refundable participation costs common to all bidders and denoted by $c \in [0, 1)$. Bidders do not know others' participation decisions when they make their own decisions. When a bidder is indifferent between participating and not participating in the seller's auction, we assume that he participates for illustration convenience. When a bidder submits a bid, he cannot observe the number of bidders who submit bids. Only the winner in the seller's auction must pay his bid, and all participants in seller's auction have to pay the participation costs.

Bidders may form a cartel. The seller is assumed to be passive; i.e., she does not know whether she faces a cartel.

3 Non-Collusive Auction and Efficient All-Collusive Cartel Mechanisms

In this section, we assume there is no information leakage problem in the sense that no bidders can update their information through the cartel's prior mechanism. We consider two extreme cases. One is that no bidder forms a cartel, i.e., non-collusive auction, and the other involves everyone in a ring, i.e., all-collusive mechanism. We compare bidders' choices between the non-collusive and collusive games without information leakage.

3.1 Non-Collusive First-Price Auction

The auction format for the seller is the first-price auction. Let v^* be the cutoff point, which is determined by $c = v^* F(v^*)^{n-1}$. It is obvious that $v^* > c$. Following Menezes and Monteiro (2000), there exists a unique (up to changes for a measure zero set of values) symmetric (Bayesian-Nash) equilibrium where each bidder's bidding function $\gamma(v_i)$ is monotonically increasing when bidder i with value v_i participates in the seller's auction, given by

$$\gamma(v_i) = \frac{\int_{v^*}^{v_i} y dF(y)^{n-1}}{F(v_i)^{n-1}},$$

when $v_i \geq v^*$. If $v_i < v^*$, bidder i does not participate in the auction. When $v_i \geq v^*$, the non-collusive profit $\pi_i^s(v_i)$ for bidder i is

$$\pi_i^s(v_i) = [v_i - \gamma(v_i)]F(v_i)^{n-1} - c.$$

Thus, with integration by parts and some simplifications, bidder i's expected revenue is given as follows.

$$\pi_i^s(v_i) = \begin{cases} 0 & v_i < v^* \\ \int_{v^*}^{v_i} F(y)^{n-1} dy & v_i \geq v^*. \end{cases}$$

3.2 Efficient All-Collusive Cartel Mechanism

An efficient cartel designs an incentive-compatible all-inclusive (symmetric) mechanism, which maximizes the sum of bidders' expected revenues with transfer payments.[1] This efficient cartel mechanism exists for $n > 2$.[2] Bidder i reports his value to the cartel mechanism before the seller's auction and $\pi_i^m(0)$ is the transfer payment received by each cartel member. As the assumption in McAfee and McMillan (1992), some punishment is available to the cartel, so no cartel member will break the ring when the ring dictates that he bids to lose.[3]

---

[1] This efficient cartel mechanism is what we called "strong cartel", which is shown to be ratifiable in McAfee and McMillan (1992).

[2] Che and Kim (2006) show that agents' collusion imposes no cost in a broad class of circumstances with more than two agents ($n \geqslant 3$) for correlated types and more than one agent ($n \geqslant 2$) for uncorrelated types. As for two-agent nonlinear pricing environments with correlated types, Meng and Tian (2013) show that collusive behavior cannot be prevented freely.

[3] This is because we focus on the constraints of the cartel that result from the privacy of the cartel members' information. The cartel should ensure obedience to the cartel's orders when the cartel is ratified.

The efficient cartel mechanism works as follows. While the bidders report their values to the cartel mechanism, the ith bidder is awarded the object with probability $p_i(w_i, v_{-i})$, where w_i is the value reported from bidder i and v_{-i} is a vector of other bidders' values. $Z_i(\cdot)$ is the total transfer payment which the winner has to pay in the cartel.[4] Then, bidder i's expected revenue in the cartel $\pi_i^m(w_i, v_{-i})$ in the prior auction with value v_i and reports w_i, can be written as:

$$\pi_i^m(w_i, v_{-i}) = E_{-i}[p_i(w_i, v_{-i})(v_i - Z_i(w_i, v_{-i}) - c)] + \frac{1}{n-1} E_{-i}[(1 - p_i(w_i, v_{-i}))Z_{-i}(w_i, v_{-i})],$$

where the first term on the right side is the expected revenue if he wins the prior auction, the second term represents the expected revenue if he loses, and E_{-i} is the expectation over v_{-i}. For any bidder, after dropping the bidder's indices to simplify the notation, $\pi^m(w, v)$ becomes:

$$\begin{aligned} \pi^m(w, v) &= [v - Z(w) - c]F(w)^{n-1} \\ &+ [1 - F(w)^{n-1}] \int_w^1 [\frac{Z(u)}{n-1}] \frac{(n-1)F(u)^{n-2}f(u)}{1 - F(w)^{n-1}} du \\ &= [v - Z(w) - c]F(w)^{n-1} \\ &+ \int_w^1 Z(u)F(u)^{n-2}f(u)du. \end{aligned}$$

Bidders choose w to maximize $\pi^m(w, v)$. In an incentive compatible mechanism, $w = v$. Note that $\frac{\partial \pi^m(w,v)}{\partial v} = F(w)^{n-1}$ and denote $F(w)^{n-1} = G(w)$, we can write

$$\pi^m(v) = \begin{cases} \pi^m(0) & if \quad v < c. \\ \pi^m(0) + \int_c^v G(u)du & if \quad v \geq c. \end{cases} \tag{1}$$

The cartel's total revenue is the expected difference between the winner's value and the participation costs. The density function of the winner's value is $nF(v)^{n-1}f(v)$. The total revenue for the cartel, $\pi^c(v)$, must equal total expected revenues:

$$\begin{aligned} \pi^c(v) &= \frac{1}{n} \int_c^1 (v - c)nF(v)^{n-1}f(v)dv \\ &= \int_0^1 \pi^m(v)f(v)dv \\ &= \pi^m(0) + \int_c^1 \int_c^v F(u)^{n-1}duf(v)dv \\ &= \pi^m(0) + \int_c^1 [1 - F(v)]F(v)^{n-1}dv. \end{aligned}$$

Thus, the transfer payment $\pi^m(0)$ is

$$\begin{aligned} \pi^m(0) &= \frac{1}{n} \int_c^1 (v - c)nF(v)^{n-1}f(v)dv \\ &- \int_c^1 [1 - F(v)]F(v)^{n-1}dv \\ &= \int_c^1 [y - \frac{1 - F(y)}{f(y)} - c]G(y)dF(y). \end{aligned}$$

[4] $\pi_i^m(0) = \frac{1}{n-1} Z_i(\cdot)$

Table 1: Side payments from an efficient cartel with \$0 reserve price

Knockout auction	Bid	Side payments
Bidder A	0.9	$\frac{0.9-0.1}{4} = 0.2$
Bidder B	0.7	$\frac{0.9-0.1}{4} = 0.2$
Bidder C	0.6	$\frac{0.9-0.1}{4} = 0.2$
Bidder D	0.3	$\frac{0.9-0.1}{4} = 0.2$
Reserve price	0	
Participation costs	0.1	

Since $v^* > c$ and $\pi^m(0) > 0$, we obtain

$$\pi^m(v) > \pi^s(v). \tag{1}$$

This means that the efficient cartel mechanism satisfies bidders' individual rationality conditions. Bidders in the cartel earn more profit, whether they win the object or not. For instance, suppose that target price is \$0 as our assumption in Section 2, and consider the participation cost \$0.1, every member in the efficient cartel could earn an extra \$0.2 in side payments as compared to the non-collusive case (as we show in Table 1).

As compared with non-collusive auction, all bidders prefer to form a cartel. This is true for all $0 \leq c < 1$; i.e., it does not matter whether there is a participation cost in the seller's auction or not. We then have the following proposition.

Proposition 1 *Suppose there is no information leakage. Then the strong cartel mechanism is efficient and interim individually rational with respect to symmetric equilibrium payoffs in the seller's auction no matter whether there is a participation cost.*

The assumption that there is no information leakage is certainly unrealistic. As long as bidders are rational, they would update their information, if any, through the cartel's prior auction before they participate in the seller's auction. Will this cartel mechanism still result in efficient outcome in the presence of participation costs? We address this question in the next section.

4 Ratifiability of Efficient Cartel Mechanisms

In this section, we investigate the ratifiability of efficient cartel mechanisms with both participation costs and the possibility that bidders may update their beliefs through the bidders' participation decisions. In this case, the bidding strategy of a bidder in the seller's auction usually depends on the bidder's beliefs about others' values, which in turn may be affected by the decision of participation in the cartel.

Again, we assume that the seller's auction is the first-price auction with participation costs. The cartel mechanism is as follows. The cartel holds a prior auction before the seller's auction. After bidders report their bids to the cartel, the cartel chooses the bidder with the highest bid to represent the cartel in the seller's auction.

With information leakage, we design a knockout cartel mechanism with $c \geq 0$. When $c > 0$, bidders have to pay when they participate in the seller's auction. Specifically, the timing of possible cartel formation between date 0 and date 1 works as follows:

- At date 0, an efficient cartel mechanism exists and all bidders belong to the cartel.
- At date $\frac{1}{3}$, nature draws a private valuation for each bidder.
- At date $\frac{2}{3}$, bidders vote for or against the efficient cartel mechanism simultaneously.
- At date 1, if all bidders accept the efficient cartel mechanism at date $\frac{2}{3}$, it is implemented and the winner in the prior auction represents the cartel to bid in the seller's auction, and he will compensate the losers with transfer payments. Otherwise, if at least one bidder rejects the collusive mechanism, no collusion occurs. Bidders bid in the seller's auction at date 1 noncooperatively.

In order to show that bidders may have incentive to exit the cartel, we define a veto set A_i for each $i \in N$. If the vetoer's value is in this set, he will veto the cartel; that is, vetoing the cartel brings more profit to the vetoer than he would gain in the collusive case. If a bidder is indifferent between staying in and vetoing the cartel, he will choose to stay in the cartel. $\pi_i^v(v_i, b)$ denotes the vetoer's payoff at equilibrium b, and $\pi_i^m(v_i)$ is the payoff when he stays in the cartel. Formally, we have the following definitions.

Definition 1 A set A_i with $\emptyset \neq A_i \subseteq [0,1]$ for bidder i is said to be a credible veto set if there exists an equilibrium b in the post-veto auction such that $\pi_i^v(v_i, b) > \pi_i^m(v_i) \Leftrightarrow v_i \in A_i$.

Definition 2 The cartel mechanism is ratifiable, if there is no credible veto set for all $i \in N$.

If bidder i vetoes the cartel,[5] other bidders will update their beliefs about the vetoer i's value and identify that the vetoer's value belongs to the credible veto set A_i after bidder i chooses to exit the cartel, and the cartel mechanism may not be supported. Suppose that when bidder i vetoes the cartel, others believe that his value is in $(v_N, 1]$, where "N" is for saying "no" to

[5]Since we are looking for the possibility that all bidders unanimously ratify the cartel mechanism, it is sufficient to be only concerned about the unilateral deviations.

the cartel mechanism (the vetoer), and v_N is an upper bound at which the bidder is indifferent between vetoing and staying in the cartel. $A_i = (v_N, 1]$ is then a credible veto set for bidder i. We will show that there is an asymmetric equilibrium of the auction with these updated beliefs, such that the vetoer's payoff at the equilibrium is larger than his payoff in the cartel if his value is larger than v_N. As such, the vetoer has an incentive to betray.

When bidder i vetoes the cartel, his value is updated to be distributed on $(v_N, 1]$ according to $F_N(v) \equiv \frac{F(v)-F(v_N)}{1-F(v_N)}$, which is derived from $F(.)$ using the Bayes' rule. For all other bidders, the expected revenue of participating in the seller's auction is a non-decreasing function of their true values. Thus with participation cost, a bidder uses cutoff strategy in which he submits a bid if and only if his value is greater than or equal to a cutoff.

Now, for simplicity, suppose there is only symmetric equilibrium,[6] i.e., the cutoff points used by all collusive bidders to decide whether or not to participate in the seller's auction are the same and denoted by v_Y, where Y is for saying yes (ratifiers). Thus when both the vetoer and the other bidders participate in the seller's auction, we have an asymmetric first-price auction in the sense that bidders' valuation distributions are on different supports. Now we consider this asymmetric first-price auction in which the ratifiers' values are distributed on $[0, 1]$ according to $F(.)$ and vetoer i's value is distributed on $[v_N, 1]$ according to $F_N(.)$ given earlier. Let $\lambda_i(v_i)$ and $\lambda_j(v_j)$ be the bidding functions of the vetoer and the ratifiers when they participate in the seller's auction. Then, by Maskin and Riley (2003), there is a unique optimal bidding strategy and bidders with the same cutoff use the same bidding function when participating. For the vetoer,

$$b_i^*(v_i) = \lambda_i(v_i) \quad \forall v_i \in (v_N, 1].$$

For a typical ratifier bidder j ($j \neq i$),

$$b_j^*(v_j) = b_{-i}^*(v_j) = \begin{cases} N_0 & v_j < v_Y \\ \lambda_j(v_j) = \lambda(v_j) & v_j \geq v_Y \end{cases} \quad \forall j \neq i.$$

Note that $v_Y \geq v_N$ since otherwise a collusive bidder with value v_Y has no chance to win in the seller's auction while still incurring the participation cost c, which makes a v_Y type collusive bidder's participation irrational. Thus if there are any ratifiers in the seller's auction, the rival's value is distributed on $[v_Y, 1]$, which makes a v_N type vetoer have no chance of winning and thus bid zero.

When both vetoer and ratifier bidders have a chance of winning in the seller's auction, the

[6] It should be noted that the collusive bidders may use different cutoffs as studied in Cao and Tian (2010), which may complicate our analysis considerably. However, if $F(\cdot)$ is inelastic, i.e., $F(v) \geq vf(v)$ for all $v \in [0, 1]$, then there is a unique equilibrium (cf. Cao and Tian (2010)).

vetoer bidder i's bid b can be determined by the following maximization problem:

$$\max_b F(v_j(b))^{n-1}(v_i - b).$$

Similarly, a ratifier bidder j's bid can be solved by the following problem:

$$\max_b \frac{F(v_i) - F(v_N)}{1 - F(v_N)} F(v_j(b))^{n-2}(v_j - b).$$

The optimal inverse bidding functions $v_i(b)$ and $v_j(b)$ when participating,[7] according to Cao and Tian (2010), are uniquely given by the first order conditions: For all $\underline{b} < b \leq \bar{b}$ with $\underline{b} = \max \arg \max_b \frac{F(b)-F(v_N)}{1-F(v_N)} F(v_Y)^{n-2}(v_Y - b)$,

$$v_i(b) = b + \frac{F(v_j(b))}{(n-1)f(v_j(b))v_j'(b)}, \tag{2}$$

$$v_j(b) = b + \frac{[F(v_i(b)) - F(v_N)]F(v_j(b))}{(n-2)f(v_j(b))v_j'(b)[F(v_i(b)) - F(v_N)] + F(v_j(b))f(v_i(b))v_i'(b)}, \tag{3}$$

with boundary conditions $v_j(\underline{b}) = v_Y$, $v_i(\underline{b}) = \underline{b}$ and $v_i(\bar{b}) = v_j(\bar{b}) = 1$.

Remark 1 When $v_N < v_i < \underline{b}$, the vetoer bidder has no chance of winning when there is any ratifier bidder participating in the seller's auction and thus he can do no better than bidding zero. On the other hand, when no ratifier bidder participates, his best choice is still to bid zero. As a vetoer bidder with $v_N < v_i < \underline{b}$, he can win the auction only when there are no other bidders participating in the seller's auction.

Remark 2 The information structure we adopted follows Menezes and Monteiro (2000). A bidder does not know who else is in the auction when he submits a bid, which is a different specification from Cao and Tian (2010). Menezes and Monteiro (2000) only focus on the symmetric equilibrium at which all bidders use the same cutoff point (which is equal to v^*) and submit bids via the same bidding function. We focus on the asymmetric equilibria where bidders use different cutoff points.

Rewriting equation (2), we have

$$v_j'(b) = \frac{F(v_j(b))}{(v_i(b) - b)(n-1)f(v_j(b))}.$$

Substituting in $v_j(b)$ gives:

$$\begin{aligned} v_j(b) &= b + \frac{[F(v_i(b)) - F(v_N)]F(v_j(b))}{(n-2)[F(v_i(b)) - F(v_N)]f(v_j(b))[\frac{F(v_j(b))}{(v_i(b)-b)(n-1)f(v_j(b))}] + F(v_j(b))f(v_i(b))v_i'(b)} \\ &= b + \frac{F(v_i(b)) - F(v_N)}{(\frac{n-2}{n-1})(\frac{F(v_i(b))-F(v_N)}{v_i(b)-b}) + f(v_i(b))v_i'(b)}. \end{aligned} \tag{4}$$

[7] $v_i(b)$ and $v_{-i}(b)$ are from $b_i = \lambda_i(v_i)$ and $b_{-i} = \lambda_j(v_j) = \lambda(v_j)$ with $b_i = b_j = b$.

Since the inverse bidding function is monotonically increasing in b, the relationship is uniquely determined. Define $Q(v_i(b)) \equiv v_j(v_i)$, where $Q(v_i(b))$ is the relationship between v_j and v_i when $b = b_j = b_i$, and $k(v_i)$ is the probability that given $v_i > v_N$, the vetoer i's bid is greater than bidder j's.

$$\begin{aligned}
k(v_i) &= P(b_i > b_j | v_i > v_N) \\
&= P(v_j < Q(v_i) | v_i > v_N) \\
&= \frac{P(v_j < Q(v_i), v_i > v_N)}{P(v_i > v_N)} \\
&= \frac{\int_{v_N}^{1} f(v_i) \int_0^{Q(v_i)} f(v_j) dv_j dv_i}{1 - F(v_N)} \\
&= \frac{[1 - F(v_N)] F(Q(v_i))}{1 - F(v_N)} \\
&= F(Q(v_i)).
\end{aligned}$$

Let $H(v_i) = k(v_i)^{n-1}$ be the probability that all other bidders' bids are less than vetoer i's. For any other bidder, the distribution of the maximum of others' bids is given by $\hat{H}(y) \equiv k(y)F(y)^{n-2}$, with $y \in (v_N, 1]$. Let $\tilde{v}_Y$ be the solution to

$$[\tilde{v}_Y - b^*_{-i}(\tilde{v}_Y)]\hat{H}(\tilde{v}_Y) = c.$$

The payoff of a $\tilde{v}_Y$ type bidder is equal to his participation costs, whenever $\tilde{v}_Y \leq 1$. We have $v_Y = min\{1, \tilde{v}_Y\}$. Notice that v_Y is the cutoff point where other bidders are indifferent between staying in and vetoing the cartel. An increase in v_N leads to a higher v_Y. Thus we have that v_Y is a strictly increasing function of v_N until it reaches 1 for some value of v_N and stays there for greater value of v_N. The payoff of vetoer i is

$$\pi_i^v(v_i, b^*) = max\{[v_i - \lambda_i(v_i)]H(v_i) - c, 0\}.$$

We first consider the simple case where $c = 0$ under the assumption that bidders can update their beliefs from the cartel's mechanism. With the information gained through the cartel's mechanism, if there is a vetoer of the cartel after the cartel's auction, other bidders can enter the seller's auction for free and bid as much as possible to make the vetoer earn a profit that is less than or equal to his cartel revenue. The game becomes the basic collusive model in McAfee and McMillan (1992). $\pi_i^m(v_i) \geq \pi_i^v(v_i, b^*) = \pi_i^s(v_i)$.

Proposition 2 *The efficient cartel mechanism is ratifiable when $c = 0$, even if the information leakage problem exists.*

Remark 3 This conclusion is different from that in Tan and Yilankaya (2007) who claimed without proof that the efficient cartel mechanism is not ratifiable when $c = 0$.[8] This is because the vetoer's betraying signal becomes an "incredible threat" without participation costs.

Now we consider the general case where participation costs are positive and the information leakage problem exists, and have the following proposition.

Proposition 3 *In a first-price sealed-bid auction, suppose $c > 0$ and the information leakage problem exists. Then the strong efficient cartel mechanism is no longer ratifiable.*

Remark 4 With the unilateral deviation in the very beginning efficient all-inclusive cartel, the remaining $n - 1$ bidders may still possibly form another non-all-inclusive cartel to participate in the seller's auction. Marshall and Marx (2007) investigate similar cases in the absence of participation costs. If this is the case in our model, then in the seller's auction, there will be only two potential participants including the vetoer (contrary to n potential participants in the seller's auction in our setting). The approach we employed in this paper can still be applied in this case and our main result remains true.

By combining the results of Propositions 2 and 3, one can see that there is a discontinuity at $c = 0$. When there is no participation cost, the strong cartel mechanism is still efficient even though information leakage is allowed. However, when participation cost is positive and information leakage is possible, the strong efficient cartel mechanism is no longer ratifiable. The intuition is that, having updated their beliefs that the vetoer's value belongs to A_i, other bidders with low values would not participate in the seller's auction because they would have to pay the non-refundable participation costs and earn negative profits, which in turn results in a positive effect on the expected revenue of the vetoer with a sufficiently high value. As a result, the cartel mechanism is not ratifiable.

Thus, we show that the efficient cartel mechanism cannot be ratifiable when both the information leakage problem and positive participation costs exist. When participation costs exist without information leakage problem, as Proposition 1 addresses in the previous section, the strong cartel mechanism is still efficient. This efficient cartel mechanism is designed to maximize bidders' ex post profit. Without the information leakage problem, bidders cannot update their beliefs through cartel's auction, so no one has incentive to exit the cartel. On the other hand, when the information leakage problem exists without participation costs, this cartel mechanism is still ratifiable. Since bidders can submit a bid in the seller's auction for free, the vetoer cannot earn extra profit from betraying the cartel. Thus, the efficient cartel mechanism is still possible even if there is participation cost or the information leakage problem, but not

[8]Actually, they are even not quite sure about their claim without further investigating why.

when both are present. However, when both exist simultaneously, the efficient cartel is no longer ratifiable.

Remark 5 When bidders have different participation costs, which means that bidders report the level at their values minus the participation costs, the highest-value bidder may not be the winner in a cartel because he may have higher participation costs. As in Proposition 3, we can get a similar result: when the information leakage problem exists, the bidder with the largest net value defined by the difference between his value and his participation cost would have an incentive to betray the cartel to maximize his payoff.

To illustrate the main conclusion of this section, we give the following example.

Example 1 Suppose $F(v) = v$, the uniform distribution on $[0, 1]$ and $n = 2$. v_N is determined by the indifference between staying in and vetoing the cartel and thus we have

$$\pi^m(0) + \int_c^{v_N} F(u)du = v_N F(v_Y) - c,$$

where the left side is the expected revenue for staying in the cartel while the right side is the expected revenue from participating in the seller's auction minus participation cost c (when $v_i = v_N$, he can win the auction only if he is the single bidder because if there is any other bidder, it must be $v_j \geq v_Y > v_N$.). With $F(v) = v$, the above equation is equivalent to

$$v_N v_Y - c = \frac{1}{2}v_N^2 - \frac{c}{2} - \frac{1}{6}c^3 + \frac{1}{6}.$$

v_Y is determined by the zero net payoff for a ratifier bidder who participates in the seller's auction. For the ratifier bidder j with $v_j = v_Y$, his expected payoff from participating in the auction is given by (assuming $v_Y \leq 1$ since otherwise the ratifier bidder never participates)

$$\max_b \frac{F(b) - F(v_N)}{1 - F(v_N)}(v_Y - b) - c = 0.$$

The above maximization problem gives $b = \frac{v_Y + v_N}{2}$. Zero net payoff gives

$$v_Y - v_N = \sqrt{4c(1 - v_N)}.$$

Now we have two equations on v_Y and v_N. Our numerical examples show that, when $c \geq 0.1755$, the ratifier bidder never participates and v_N is determined by

$$v_N - c = \frac{1}{2}v_N^2 - \frac{c}{2} - \frac{1}{6}c^3 + \frac{1}{6}.$$

For example, when $c = 0.1755$, $v_N = 0.2980$; when $c = 2/3$, $v_N = 0.686$.

Now consider an extreme case where c approaches zero. From the first condition, we can get $v_Y v_N = \frac{1}{2}v_N^2 + \frac{1}{6}$, and from the second condition, we can get $v_Y - v_N = 0$. The two conditions give $v_N = 0.577$, which implies that even when $c = 0$, the cartel mechanism is not ratifiable. However, the problem here is when $c = 0$, the vetoer bidder needs to worry about not only the ratifier bidder with value in $[v_Y, 1]$, but also the ratifier bidder with value in $[0, v_Y)$[9]. Considering this, bidders will not have any incentive to deviate. To see this, consider the first-price auction in which the vetoer bidder's value is distributed on $[v_N, 1]$ with $F_{v_N}(v) = \frac{v - v_N}{1 - v_N}$ and the ratifier bidder's value is distributed on [0,1] with $F(v) = v$. For the vetoer with value v_N, he maximizes $(v_N - b)F(b)$ with respect to b, which gives an expected payoff of $\frac{1}{4}v_N^2$. The payoff for staying in the cartel is $\frac{1}{2}v_N^2 + \frac{1}{6}$. Then in this case it is impossible to find a $v_N \in [0, 1]$ such that $\frac{1}{4}v_N^2 = \frac{1}{2}v_N^2 + \frac{1}{6}$. Thus, when $c = 0$, the vetoer does not have incentive to veto the cartel. There is a discontinuity between $c = 0$ and $c > 0$.

5 Conclusion

Buyers' collusion usually brings some losses to the seller's expected revenue and reduces the allocation efficiency. When participation costs exist, bidders cannot bid in the legitimate auction for free. When bidders' actions are strategically interactive and their outcome is affected by bidders' beliefs about others' values, the standard mechanism design approach may suffer from the information leakage problem. After the prior auction, if the winner chooses to betray the cartel, he will be the only bidder in the legitimate auction. He can bid the lowest value and win with certainty. With the above facts, it is clear that the winner in the cartel has incentive to betray the cartel; hence the efficient cartel mechanism is no longer sustainable in the presence of positive participation costs and the information leakage problem. This observation may provide important suggestions for the government to differentiate cartels or to prevent the formation of a cartel.

[9] A positive participation cost will eliminate the ratifier bidder of this type.

Appendix: Proofs

Proof of Equation (1):

To see equation 1, we only need to consider the following three cases:

Case 1: if $v > v^* > c$, then $\pi^m(v) = \pi^m(0) + \int_c^v F(u)^{n-1}du$ and thus $\pi^s(v) = \int_{v^*}^v F(y)^{n-1}dy < \int_c^v F(y)^{n-1}dy < \pi^m(v)$ with $\pi^m(0) > 0$.

Case 2: if $v^* \geq v > c$, then $\pi^m(v) = \pi^m(0) + \int_c^v F(u)^{n-1}du$ and $\pi^s(v) = 0$; it is obvious that $\pi^m(v) > \pi^s(v)$.

Case 3: if $v \leq c$, then $\pi^m(v) = \pi^m(0) > 0$ and $\pi^s(v) = 0$. Again we have $\pi^m(v) > \pi^s(v)$.

Proof of Proposition 2:

Consider the case when information leakage problem exists and $c = 0$. There are two groups of bidders, the vetoer with the cutoff point v_N and the cartel members with the cutoff point v_Y with $v_Y = min\{1, \tilde{v}_Y\}$. As $\tilde{v}_Y$ is the solution to

$$[\tilde{v}_Y - b^*_{-i}(\tilde{v}_Y)]\hat{H}(\tilde{v}_Y) = c,$$

when $c = 0$, we get

$$[\tilde{v}_Y - b^*_{-i}(\tilde{v}_Y)]\hat{H}(\tilde{v}_Y) = 0.$$

In a first-price auction, the ratifier bidder with value V_Y will bid below his true value, so $\hat{H}(\tilde{v}_Y) = 0$.

Noting that $\hat{H}(y) \equiv k(y)F(y)^{n-2}$ with $y \in (v_N, 1]$, when $y = v_N$, $\hat{H}(v_N) = 0$. Thus we get

$$[v_N - b^*_{-i}(v_N)]\hat{H}(v_N) = 0.$$

Compared with

$$[\tilde{v}_Y - b^*_{-i}(\tilde{v}_Y)]\hat{H}(\tilde{v}_Y) = 0,$$

we have $\tilde{v}_Y = v_N = v^* = 0$ because $c = v^*F(v^*)^{n-1}$, which means that the bidders in the cartel and the vetoer use the same cutoff point 0. The optimal bidding function becomes

$$\gamma(v_i) = \frac{\int_0^{v_i} ydF(y)^{n-1}}{F(v_i)^{n-1}}.$$

In this case, if the cartel does not exist, the game becomes the non-collusive game without participation costs. Thus, the winner's expected payoff is

$$\pi_i^v(v_i, b^*) = \pi_i^s(v_i) = \int_0^{v_i} F(y)^{n-1}dy,$$

which is less than

$$\pi_i^m(v_i) = \pi_i^m(0) + \int_0^{v_i} G(y)dy.$$

As we show in Section 3.2, when $c = 0$, the vetoer's revenue is always less than or equal to the revenue he would obtain from staying in the cartel. This is because the losers in cartel's auction can participate in seller's auction with no participation cost. With updated information, they can bid as much as possible to cause the vetoer to earn less profit than he would from the cartel. Thus, the bidder with value $v_i \in A_i$ does not veto the cartel in this case.

Q.E.D.

Proof of Proposition 3:

To prove this result, we only need to show there exists a $v_N \in (c, 1)$ such that $\pi_i^v(v_i, b^*) > \pi_i^m(v_i)$ if and only if $v_i > v_N$.

We examine three possible cases when vetoer i vetoes the cartel. The first case is that $v_i G(v_Y) < c$; i.e., his expected payoff is less than his participation costs. No matter whether other bidders participate in the seller's auction or not, the vetoer will not join the auction. The second case occurs when $c \le v_i G(v_Y) \le \underline{b} G(v_Y)$, with $\underline{b} = \max \arg \max_b \frac{F(b) - F(v_N)}{1 - F(v_N)} F(v_Y)^{n-2}(v_Y - b)$. In this case, the vetoer can win the auction only when no ratifier bidder participates. The vetoer chooses to participate in seller's auction because his expected payoff is larger than c. The third case occurs when $v_i > \underline{b}$. The expected revenues in the three cases are

$$\pi_i^v(v_i, b^*) = \begin{cases} 0 & v_i < \frac{c}{G(v_Y)} \\ v_i G(v_Y) - c & \frac{c}{G(v_Y)} \le v_i \le \underline{b} \\ v_i H(v_i) - \lambda_i(v_i) H(v_i) - c & v_i > \underline{b}. \end{cases}$$

The expected revenue in the cartel is:

$$\pi_i^m(v_i) = \begin{cases} \pi_i^m(0) & v_i < c. \\ \pi_i^m(0) + \int_c^{v_i} G(y)dy & v_i \ge c. \end{cases}$$

We want to show:

$$\pi_i^v(v_i, b^*) > \pi_i^m(v_i) \quad \forall v_i > v_N.$$

We will first find a v_N for which $\pi_i^v(v_N, b) = \pi_i^m(v_N)$, and then check the inequality.

Step 1: To show $\exists v_N \in (c, 1)$ such that $\pi_i^v(v_N, b) = \pi_i^m(v_N)$. Since $\frac{c}{G(v_Y)} \ge c$, if $v_N \le c$, we have $\pi_i^m(v_N) = \pi_i^v(v_N, b) = 0$, which is impossible. When $v_Y \ge v_N > c$, we have $\pi_i^v(v_N, b) =$

$v_N G(v_Y(v_N)) - c$, and $\pi_i^m(v_N) = \pi_i^m(0) + \int_c^{v_N} G(y)dy$, i.e., we need $v_N G(v_Y(v_N)) - c - \pi_i^m(0) - \int_c^{v_N} G(y)dy = 0$. Let

$$\phi(v_i) = v_i G(v_Y(v_i)) - \int_c^{v_i} G(y)dy - c - \pi_i^m(0),$$

and

$$\begin{aligned}\phi'(v_i) &= G(v_Y(v_i)) + v_i G'(v_Y(v_i))v_Y'(v_i) - G(v_i) \\ &= [G(v_Y(v_i)) - G(v_i)] + v_i G'(v_Y(v_i))v_Y'(v_i).\end{aligned}$$

Since $G(\cdot)$ is an increasing function and $v_Y(v_i) \geq v_i$, we have $G(v_Y(v_i)) - G(v_i) > 0$ and $v_Y'(v_i) > 0$. Therefore, $\phi'(v_i) > 0$ and

$$\phi(c) = cG(v_Y(c)) - c - \pi_i^m(0) = c[G(v_Y(c)) - 1] - \pi_i^m(0) < 0,$$

$$\begin{aligned}\phi(1) &= G(v_Y(1)) - \int_c^1 G(y)dy - c - \pi_i^m(0) \\ &= 1 - c - \int_c^1 G(y)dy - \pi_i^m(0).\end{aligned}$$

Now we prove $\int_c^1 [1 - G(y)]dy > \pi_i^m(0)$, where $\pi_i^m(0) = \int_c^1 [y - c - \frac{1-F(y)}{f(y)}]G(y)dF(y)$. We know

$$\begin{aligned}\phi(1) &= 1G(v_Y(1)) - \int_c^1 G(y)dy - c - \pi_i^m(0) \\ &= 1 - c - \int_c^1 G(y)dy - \int_c^1 [y - \frac{1-F(y)}{f(y)} - c]G(y)dF(y) \\ &= \int_c^1 (1 - F(y)^n)dy - \int_c^1 yG(y)dF(y) + \int_c^1 cG(y)dF(y) \\ &= \int_c^1 (1 - F(y)^n)dy - \frac{1}{n}[1 - \int_c^1 F(y)^n dy] + \frac{c}{n} \\ &= \frac{n-1}{n}[1 - c - \int_c^1 F(y)^n dy].\end{aligned}$$

Since $F(y)^n \leq 1$, $\int_c^1 F(y)^n dy \leq \int_c^1 1dy = 1 - c$. We have $\phi(1) \geq 0$.

For $v_i < 1$ and $v_Y(v_i) \geq v_i$, since $\phi(v_i)$ is continuous, $\phi(c) < 0$ and $\phi(1) \geq 0$, a unique solution to $\phi(v_i) = 0$ exists, and is our candidate for v_N.

Step 2: We want to show $\pi_i^v(v_i, b^*) > \pi_i^m(v_i)\ \ \forall v_i > v_N$. Fix c, hence v_N and v_Y are fixed, and we have $c \leq \frac{c}{G(v_Y)} \leq v_N \leq v_Y$. The payoff difference $\pi_i^v(v_i, b^*) - \pi_i^m(v_i)$ is continuous and given by:

$$\pi_i^v(v_i,b^*) - \pi_i^m(v_i) = \begin{cases} -\pi_i^m(0) & v_i < c & I \\ -\pi_i^m(0) - \int_c^{v_i} G(y)dy & c \leq v_i < \frac{c}{G(v_Y)} & II \\ v_i G(v_Y) - c - \pi_i^m(0) - \int_c^{v_i} G(y)dy & \frac{c}{G(v_Y)} \leq v_i \leq \underline{b} & III \\ v_i H(v_i) - \lambda_i(v_i)H(v_i) - c - \pi_i^m(0) - \int_c^{v_i} G(y)dy & v_i > \underline{b} & IV \end{cases}$$

I,II: $\pi_i^v(v_i, b^*) - \pi_i^m(v_i) < 0, \quad when \ \ v_i < \frac{c}{G(v_Y)}$.

III: Let $\varphi(v_i) = v_i G(v_Y) - c - \int_c^{v_i} G(y)dy - \pi_i^m(0)$, as $\varphi(v_N) = \phi(v_N) = 0$, we have $\varphi'(v_i) = G(v_Y) - G(v_i) > 0$ when $v_i < \underline{b}$, and $\varphi'(v_Y) = 0$. Therefore,

$$\varphi(v_i) \begin{cases} < 0 & v_i < v_N \\ = 0 & v_i = v_N \\ > 0 & v_i > v_N. \end{cases}$$

Thus, $\varphi(v_Y)$ is strictly positive.

IV: $H(v_i) = F(Q(v_i))^{n-1}$.

$$\begin{aligned} \frac{d}{dv_i}[\pi_i^v(v_i, b^*) - \pi_i^m(v_i)] &= \frac{d}{dv_i}[v_i H(v_i) - \lambda_i(v_i)H(v_i) - c - \int_c^{v_i} G(y)dy - \pi_i^m(0)] \\ &= H(v_i) + v_i H'(v_i) - \lambda_i(v_i)H'(v_i)) - \lambda_i'(v_i)H(v_i) - G(v_i) \\ &= (v_i - \lambda_i(v_i))H'(v_i) + (1 - \lambda_i'(v_i))H(v_i) - G(v_i). \end{aligned}$$

Denote $X = (v_i - \lambda_i(v_i))H'(v_i) + (1 - \lambda_i'(v_i))H(v_i) - G(v_i)$. $\lambda_i(v_i) = v_i - \frac{F(v_j)}{(n-1)f(v_j)\frac{\partial v_j}{\partial b}}$.

$$\begin{aligned} X &= [\frac{F(v_j)}{(n-1)f(v_j)\frac{\partial v_j}{\partial b}}](n-1)F(Q(v_i))^{n-2}f(Q(v_i))Q'(v_i) \\ &+ F(Q(v_i))^{n-1}[\frac{f(v_j)^2\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b} - F(v_j)f'(v_j)\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b} - F(v_j)f(v_j)\frac{\partial^2 v_j}{\partial b \partial v_i}}{(n-1)f(v_j)^2(\frac{\partial v_j}{\partial b})^2}] - F(v_i)^{n-1} \\ &= \frac{(n-1)F(v_j)^{n-1}f(v_j)^2\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b} + F(v_j)^{n-1}f(v_j)^2\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b}}{(n-1)f(v_j)^2(\frac{\partial v_j}{\partial b})^2} \\ &- \frac{F(v_j)^n f'(v_j)\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b} + F(v_j)^n f(v_j)\frac{\partial^2 v_j}{\partial b \partial v_i}}{(n-1)f(v_j)^2(\frac{\partial v_j}{\partial b})^2} - F(v_i)^{n-1} \\ &= \frac{nF(v_j)^{n-1}f(v_j)^2\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b} - F(v_j)^n[f'(v_j)\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b} + f(v_j)\frac{\partial^2 v_j}{\partial b \partial v_i}]}{(n-1)f(v_j)^2(\frac{\partial v_j}{\partial b})^2} - F(v_i)^{n-1} \\ &= \frac{F(v_j)^{n-1}\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b}[nf(v_j)^2 - F(v_j)f'(v_j)] - F(v_j)^n f(v_j)\frac{\partial^2 v_j}{\partial b \partial v_i}}{(n-1)f(v_j)^2(\frac{\partial v_j}{\partial b})^2} - F(v_i)^{n-1}. \end{aligned}$$

With

$$\frac{\partial v_j}{\partial b} = \frac{F(v_j)}{(v_i - b)(n-1)f(v_j)},$$

and

$$\begin{aligned} \frac{\partial^2 v_j}{\partial b \partial v_i} &= \frac{f^2(v_j)\frac{\partial v_j}{\partial v_i}(v_i - b)(n-1) - F(v_j)(n-1)f(v_j)}{[(n-1)f(v_j)(v_i - b)]^2} \\ &- \frac{F(v_j)(v_i - b)(n-1)f'(v_j)\frac{\partial v_j}{\partial v_i}}{[(n-1)f(v_j)(v_i - b)]^2}, \end{aligned}$$

we have

$$X = (n-1)F(v_j)^{n-2}(v_i - b)\frac{\partial v_j}{\partial v_i}f(v_j) + F(v_j)^{n-1} - F(v_i)^{n-1}.$$

Replace v_j with $Q(v_i)$ in $F(v_j)^{n-1}$,

$$X = (n-1)F(v_j)^{n-2}(v_i - b)\frac{\partial v_j}{\partial v_i} f(v_j) + [F(Q(v_i))^{n-1} - F(v_i)^{n-1}].$$

The first part of X, $(n-1)F(v_j)^{n-2}(v_i - b)\frac{\partial v_j}{\partial v_i} f(v_j)$, is positive because the bidding function is monotonically increasing with $Q'(v_i) = \frac{\partial v_j}{\partial v_i} > 0$.[10]

For the second part of X, $[F(Q(v_i))^{n-1} - F(v_i)^{n-1}]$. $Q(v_i)$ is the relationship between v_i and v_j when $b_i = b_j$. Since $v_i \in (v_N, 1]$, when $b_i = b_j$, at least we should have $v_j \geq v_i$. Otherwise, bidder i can bid $\underline{b} = v_N$ to win. Thus, we have $Q(v_i) \geq v_i$, which means that $X > 0$.

When $X > 0$, we have $[\pi_i^v(v_i, b^*) - \pi^m(v_i)]$ is increasing given $v_i > \underline{b}$. Thus, $\pi_i^v(v_i, b^*) > \pi^m(v_i)$ when $v_i > v_N$. **Q.E.D.**

[10] $v_j(v_i) = Q(v_i)$ is the relationship when $v_i(b_i) = v_j(b_j)$. When v_i increases, b_i increases, which means that b_j increases with a larger v_j.

References

Cao, X. and Tian, G., "Equilibria in First Price Auctions with Participation Costs", *Games and Economic Behavior*, 2, (2010), 258-273.

Che, Y. and J. Kim, "Robustly Collusion-Proof Implementation", *Econometrica*, 74, (2006), 1063-1107.

Cooper, Jeremy C., "Under the Hammer: the Auctions and Auctioneers of London", London: Constable, 1977.

Cramton, P.C. and Palfrey, T.R., "Cartel Enforcement with Uncertainty about Costs", *International Economic Review*, 31(1), (1990), 17-47.

Cramton, P.C. and Palfrey, T.R., "Ratifiable mechanisms: Learning from disagreement", *Games and Economic Behavior*, 10(2), (1995), 255-283.

Graham, D., Marshall, R., "Collusive bidder behavior at single-object second-price and English auctions", *Journal of Political Economy*, 95, (1987), 97-119.

Graham, D., Marshall, R., and Jean-Francois, Richard., "Differential Payments within a Bidder Coalition and the Shapley Value", *American Economic Review*, 80(3), (1990), 493-510.

Kaolan, R. T., Zamir, S., "Asymmetric First-Price Auctions with Uniform Distributions: Analytic Solutions to the General Case", *Economic Theory*, 50, (2012), 269-302.

Mailath, G., Zemsky, P., "Collusion in second price auctions with heterogeneous bidders", *Games and Economic Behavior*, 3, (1991), 467-503.

Marshall, R.C. and Marx, L.M., "Bidder collusion", *Journal of Economic Theory*, 133, (2007), 374-402.

Maskin, E.S. and Riley, J.G., "Uniqueness of Equilibrium in Sealed High-bid Auctions", *Games and Economic Behavior*, 45, (2003), 395-409.

McAfee, R.P. and McMillan, J., "Bidding Rings", *American Economic Review*, 67, (1992), 579-599.

Menezes, F.M. and Monterio, P.K., "Auctions with Endogeneous Participation", *Review of Economic Design*, 5, (2000), 71-89.

Meng, D., and Tian, G., "Two-Agent Collusion-Proof Mechanism Design with Both Correlations and Arbitrage", working paper, (2013).

Mills, M., “Giving Contractors a Break”, *Cong. Quart*, 51, (1993), 2947.

Osborne D. K., “Cartel Problems”, *American Economic Review*, 66, (1976), 835-844.

Roberts, Kevin, “Cartel Behavior and Adverse Selection”, *Journal of Industrial Economics*, 33, (1985), 401-413.

Tan, G. and Yilankaya, O., “Ratifiability of efficient collusive mechanisms in second-price auctions with participation costs”, *Games and Economic Behavior*, 59, (2007), 383-396.