Entry-Deterring Nonlinear Pricing with Bounded Rationality

Meng, Dawen and Tian, Guoqiang

Shanghai University of Finance and Economics, Texas AM University

August 2013

Entry-Deterring Nonlinear Pricing with Bounded Rationality

Dawen Meng *   Guoqiang Tian †

May 7, 2014

Abstract

This paper considers an entry-deterring nonlinear pricing problem faced by an incumbent firm of a network good. The analysis recognizes that the installed user base/network of incumbent monopolist has preemptive power in deterring entry if the entrant's good is incompatible with the incumbent's network. This power is, however, dramatically weakened by the bounded rationality of consumers in the sense that it is vulnerable to small pessimistic forecasting error when the marginal cost of entrants falls in some medium range. These findings provide a formal analysis that helps reconcile two seemingly contrasting phenomena: on one hand, it is very difficult for a new, incompatible technology to gain a footing when the product is subject to network externalities; on the other hand, new technologies may frequently escape from inefficient lock-in and supersede the old technologies even in the absence of backward incompatibility. Our results therefore shed light on how the market makes transition between incompatible technology regimes.

Keywords: Nonlinear pricing, Entry deterrence, Network Externalities, Bounded rationality.

JEL Codes: D42, D62, D82

1 Introduction

New entrants challenge the monopoly power and depress profits of incumbent firms. Incumbents are therefore strongly motivated to deter entry of newcomers. In fact, entry deterrence is among a firm's most important strategic decisions and has long been a central issue in industrial organization theory.

*Institute for Advanced Research, Shanghai University of Finance and Economics; Key Laboratory of Mathematical Economics (SHUFE), Ministry of Education, Shanghai 200433, China, devinmeng@hotmail.com

†Department of Economics, Texas A&M University, tian@econmail.tamu.edu

When facing threat of entry, incumbent firms must decide how to respond. Early theoretical research, notably by Bain (1956), Modigliani (1958), and Sylos-Labini (1962), emphasizes limit pricing: an incumbent firm setting its pre-entry price low enough to make entry appear unprofitable. Various researches have offered explanations for limit pricing. Milgrom and Roberts (1982a) show that when the entrant is uncertain about the incumbent's cost, the incumbent tries to signal its low cost and then discourage entry by setting a low price. Kreps and Wilson (1982) and Milgrom and Roberts (1982b) show that when the entrant is uncertain about the incumbent's payoffs, the incumbent may have an incentive to cut prices even after entry as a way to build a tough reputation for fighting future entry. Firms may use other types of signals instead of or in addition to price to deter entry. Bagwell and Ramey (1988), Linnemer (1998), and Bagwell (2007), among many others, show that advertising also has an entry-deterring effect. Espínola-Arredondo et al. (2014) investigate the signaling role of tax policy in promoting, or hindering, the ability of a monopolist to practice entry deterrence.

In complete information environment, however, the limit pricing theory is criticized on the grounds of credibility of commitment. Game-theoretic research has considered a variety of strategies and conditions that might provide credible deterrents to entry. Some literature (Spence (1977), Dixit (1980), Bulow et al. (1985), Maskin (1999), Allen et al. (2000), etc.) suggests that, firms might hold excess capacity in order to deter entrants. Other devices such as R&D expenditures, increased advertising, capital structure, exclusive contract, may also be used as commitments to deter entry. (See Fudenberg and Tirole (1984), Aghion and Bolton (1987), Tirole (1988), Segal and Whinston (2000) and Tarzijan (2007), among many others, for detailed discussion.) In multiproduct environment, it is shown that strategies such as product proliferation, bundling, and diversification, are all effective entry-deterrent measures. (See Schmalensee (1978), Omori and Yarrow (1982), Whinston (1990), Choi and Stefanadis (2001, 2006), Carlton and Waldman (2002), and Nalebuff (2004), etc., for detailed discussion.)

In many cases, network externalities may serve as barriers to market entry even when the newcomers have superior technologies and offer lower prices. An obvious example is a telephone network. In a world without interconnection, a user will not switch to a new telephone network featuring better technologies at lower prices as long as there are no subscribers on that network to communicate with. If all consumers postpone purchase of a product with network externalities until the critical mass is reached, then new entrants will not be able to establish themselves. A cheaper product or a product of better quality would not be sufficient to gain a user base in the face of strong network effects which guide users to previously established networks.

In industrial practice, incumbent firms often purposely create network externalities to impede entry. Examples are numerous, such as IBM's famous practice of requiring purchasers of its tabulating machines to also purchase tabulating cards from IBM; Microsoft's attempts to bundle Internet Explorer and Office with Windows operating system to keep out a rival product (e.g., Linux)[1]; China Mobile's fighting against China Unicom by releasing Fetion, an instant messenger software used only between China Mobile's subscribers.

It is of great importance to determine how the incumbent's strategies are affected by threat of entry; how the installed user base of a network good can serve as an incumbent's preemptive power to deter potential entry; and what factors influence an incumbent's entry-deterring ability. We answer these questions in the present paper by analyzing a nonlinear pricing problem faced by an entry-deterring monopolist under asymmetric information and network externalities. [2] Our model differs from the existing literature along several dimensions.

First, this paper investigates the influence of the consumers' bounded rationality on the incumbent firm's entry-deterring ability. Even though entry deterrence has long been a central issue in industrial organization theory, not much attention has been given to consumers. In most existing literature, firms (usually an incumbent monopolist and a potential entrant) compete with various forms of strategic weapons, while consumers do not play an active role. In our model, however, the market power of incumbent monopolist depends crucially on consumers' belief. We provide an entry-deterrence model where consumers choose whether or not to bypass the present network based on their expectations. Equilibria at which consumers' initial belief on network size is fulfilled in the sense that it is consistent with the actual outcome of the entry game are characterized. We also show that, under certain conditions, these equilibria are unstable in the sense that they are vulnerable to perturbation of initial expectation. Therefore, the incumbent firm's entry-deterring ability is weakened when facing boundedly rational consumers. This result rationalizes a substantial number of stylized facts that new technologies/brands which have not yet built their own networks could successfully supersede the old and mature technologies/brands with installed networks. It therefore throws light on how the market system escapes from inefficient lock-in due to network effects.

Second, this paper draws from and adds to the literature on mechanism design under bounded

[1] This practice stifles competition by reducing the desirability of entry of competing firms into the market of operating systems, and it is the main allegation in the antitrust case against Microsoft in 1998.

[2] In the context of network goods, Fudenberg and Tirole (2000) have shown that an incumbent monopolist may have an incentive to charge a low price to build a large installed user base in order to deter entry with an incompatible product. Their model, however, assumes away the possibility of performing price discrimination across different groups of consumers, which is the focus of our analysis.

rationality. The traditional mechanism design theory assumes that agents are able to forecast correctly the key parameters in the economy and then respond rationally to the principal's offer, so the correct design of mechanisms is decisive for achieving economic systems with good welfare properties. Yet as agents usually lack full rationality, the designer's desirable outcome may not be obtained any more. Intuitively, in a society populated by boundedly rational agents, the central planner's policy objective may often fail or even converge to an undesirable outcome. A great number of studies, both experimentally and theoretically, focus on the evolutionary properties of mechanism under bounded rationality (See Chen and Gazzale (2004), Healy (2006), Chen (2005, 2008), De Trenqualye (1988), Walker (1984), Cabrales (1999), Sandholm (2002, 2005), Cabrales and Serrano (2011), Healy and Mathevet (2012), etc., for detailed discussion.) In these existing studies, agents' bounded rationality is mainly modeled as their non-optimal reports to the mechanism designer due to behavioral biases or forecasting error. We focus instead on a setup where the agents are assumed to update gradually their participation decisions rather than reports. This makes a main difference between the previous works and ours: the equilibrium in our model is at least semi-stable from above since the agents' consumption is controlled by the principal as long as they accept the contract.

Third, this paper challenges the "no distortion at the top" convention by integrating two classes of literatures: network externalities and countervailing incentives. The canonical principal-agent model makes two simplifying assumptions. First, there is no externality among agents; second, an agent is assumed to obtain a minimal type-independent level of utility if he rejects the offer made by the principal. Under these assumptions, the optimal contract exhibits no distortion for the "best" type agent and downward distortions for all other types. (See for example Mussa and Rosen (1978), Maskin and Riley (1984), Baron and Myerson (1982), Myerson (1981), Hellwig (2010), Chade and Schlee (2012), among many others, for detailed discussion.) Many studies challenge this result by relaxing the above two assumptions. Lockwood (2000) shows that when the cost of any agent depends positively on his own effort and that of his co-workers, the optimal contract offered by the principal exhibits a two-way distortion in output. Hahn (2003) builds a model of telecommunication to examine the role of call and network externalities in nonlinear pricing model. It is shown that the presence of call externalities leads to downward distortions of the outgoing call quantity for all types of subscribers including the highest type. Also in a nonlinear pricing setup, Csorba (2008) shows that the joint presence of asymmetric information and positive network effects leads to a strict downward distortion for all types of consumers in the quantities provided. The failure of "no distortion at the top" result makes an appearance also in the countervailing incentives problem where an agent's reservation utility is type-dependent. Lewis and Sappington (1989) show in a regulation model that, when the fixed cost of a regulated firm is type-dependent, the output can be distorted above the efficient level for some types and below the efficient level for others. Maggi and Rodriguee-Clare (1995) give a complete analysis of the principal-agent problem with countervailing incentives. They show that the pattern of the two-way distortion depends crucially on whether the reservation utility of the agent is convex or concave in his private information. In our model, the monopolist must provide each customer with surplus at least equal to the maximum utility they could obtain from an outside market. As a result, the agent's participation constraint may be type-dependent and countervailing incentives could arise. We fully characterize the optimal pure monopoly and entry-deterring nonlinear pricing contracts for different kinds of network effects. It is shown that the nonlinear pricing contract exhibits either one-way or two-way distortion in quantities depending on the curvature properties of network. Therefore, our work incorporates as special cases many existing studies, such as Lockwood (2000) and Csorba (2008). It is also shown that the distortionary pattern of entry-deterring contract depends on both the properties of network and efficiency of outside competitors.

The remainder of this paper is organized as follows. Section 2 sets up the basic model. Section 3 gives the benchmark results without entry threats. Section 4 analyzes the entry-deterring model with, respectively, fully and boundedly rational agents. Finally, concluding remarks are offered in Section 5.

2 The model

Consider a monopolist who produces a good exhibiting network effects at a constant marginal cost c. There exists a continuum of consumers with heterogenous preferences for the good, and this heterogeneity is captured by the one-dimensional parameter θ with distribution function $F(\theta)$ and density $f(\theta)$ over $[\underline{\theta},\overline{\theta}]$. As usual in the standard adverse selection model, we assume the following monotone hazard rate properties.

ASSUMPTION 1 $\frac{d}{d\theta}\left[\frac{1-F(\theta)}{f(\theta)}\right] \leqslant 0 \leqslant \frac{d}{d\theta}\left[\frac{F(\theta)}{f(\theta)}\right]$.

The preference of a customer of type θ is represented by the linearly separable utility function

$$U(\theta,q,Q,t) = \theta v(q) + \Omega(Q) - t, \tag{1}$$

where q is the amount he consumes (individual consumption), Q is the total quantity of the product used by all customers in the market (often referred to as gross consumption or network size) and t is the tariff charged by the seller. $\theta v(q)$ and $\Omega(Q)$ are often referred to, respectively, as intrinsic value and network value. The above specification shows that the network effect is homogeneous, i.e., network value depends only on gross consumption rather than individual consumption. We assume $v' > 0, v'' < 0$, and the Inada conditions $v'(q) \to \infty$ for $q \to 0$ and $v'(q) \to 0$ for $q \to \infty$. We also assume $\sup_{Q \in [0,+\infty)} \Omega'(Q) < c$, the monopolist may otherwise wish to produce unbounded output levels.

In the presence of network externalities, each agent's utility depends not only on his own trade with the principal, but also on others' trades. The following properties of network externalities are defined according to the influences of an agent's trade on the other agents' total and marginal utilities.

DEFINITION 1 (The sign of network externalities) Externalities are positive (negative, absent) if for all θ, $U(\theta, q(\theta), Q, t(\theta))$ is increasing (decreasing, constant) in $q(\theta')$, for all $\theta' \neq \theta$.

DEFINITION 2 (The curvature properties of network externalities) Externalities are (strictly) increasing (decreasing, constant) if for all $\theta \in [\underline{\theta}, \overline{\theta}]$, $U_q(\theta, q(\theta), Q, t(\theta))$ is (strictly) increasing (decreasing, constant) in $q(\theta')$, for all $\theta' \neq \theta$.

In network having positive externalities, a buyer benefits from an increase in the consumption of other buyers. In the situation with negative externalities, however, an increase of one consumer's consumption hurts the others. This may arise due to congestion or internal competition. Various kinds of networks, such as transportation, communication, and computer networks exhibit congestion effects, whereby increased demand for certain network elements (e.g., roads, telecommunication lines, and servers) tends to downgrade their performance or increase the cost of using them. The curvature properties of externalities reflect the change of marginal utilities of an agent with the consumption of others. With increasing (decreasing) externalities, an agent is more (less) eager to trade more when other agents trade more because the externalities at higher trades are larger (smaller) than at lower trades.

If the utility function is twice differentiable, the property of positive (negative, absent) externalities involves the sign of first order derivative $\frac{\partial U}{\partial q(\theta')}$, while the property of increasing (decreasing, constant) externalities involves the sign of cross partial derivative $\frac{\partial^2 U}{\partial q(\theta) \partial q(\theta')}$. With the special utility structure in this paper, the externalities are positive (negative, absent) if and only if $\Omega'(Q) > 0, (< 0, = 0)$ for all Q; while the externalities are increasing (decreasing, constant) if and only if $\Omega''(Q) > 0, (< 0, = 0)$ for all Q.

Suppose there exists an outside market composed of many identical potential entrants. Let $U^0(\theta)$ represent the reservation utility available to an agent of type θ when he rejects the contract and switches to the outside product. The seller cannot explicitly distinguish between customer types prior to contracting. Thus, the entire menu of quantity-price pairs must be available to all customers. The revelation principle ensures that the firm can restrict its attention to a direct mechanism, i.e., a menu of quantity-price pairs $\{q(\hat{\theta}), t(\hat{\theta})\}$, where $\hat{\theta} \in [\underline{\theta}, \bar{\theta}]$. A contract $\{q(\hat{\theta}), t(\hat{\theta})\}$ is said to be incentive feasible for Q if, for each θ, it satisfies incentive compatibility and individual rationality constraints [3]:

$$\theta v(q(\theta)) + \Omega(Q) - t(\theta) \geqslant \theta v(q(\hat{\theta})) + \Omega(Q) - t(\hat{\theta}), \forall \hat{\theta} \in [\underline{\theta}, \overline{\theta}] \tag{2}$$

$$\theta v(q(\theta)) + \Omega(Q) - t(\theta) \geqslant U^0(\theta). \tag{3}$$

The incumbent's objective is to choose an optimal contract to maximize his expected payoff and successfully deter entry under constraints (2) and (3).

3 Nonlinear pricing without entry threats

In order to give a benchmark result against which entry-deterring contract is compared, we suppose in this section that the incumbent does not face the threat of entry. In this case, the agents' reservation utility would be zero, i.e., $U^0(\theta) = 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. We now proceed to analyze the incumbent's problem by considering in turn fully and boundedly rational agents.

3.1 Fully rational agents

In this subsection, the consumers are assumed to be fully rational in the sense that their common expectation on the network size will be actually fulfilled, i.e., $Q = \int_{\underline{\theta}}^{\bar{\theta}} q(\theta) f(\theta) \mathrm{d}\theta$. Let $U(\theta) = \theta v(q(\theta)) - t(\theta) + \Omega(Q)$ be the rent obtained by type θ, then it follows from the well-known nonlinear pricing result of Maskin and Riley (1984) that the optimal fully rational (fulfilled expectation) contract, denoted by $\{q^-(\theta), t^-(\theta)\}$, solves the following program.

$$[\mathbf{P}] : \begin{cases} \underset{\{q(\theta), U(\theta)\}}{max} \displaystyle\int_{\underline{\theta}}^{\overline{\theta}} [\theta v(q(\theta)) - cq(\theta) - U(\theta)] \, f(\theta) \, d\theta + \Omega(Q) \\ s.t. : U'(\theta) = v(q(\theta)), U(\theta) \geqslant 0, q'(\theta) \geqslant 0, Q = \displaystyle\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) f(\theta) \, d\theta \end{cases} \tag{4}$$

Neglecting momentarily the monotonicity condition $q'(\theta) \geqslant 0$ and applying integration by parts technique, the profit function can be rewritten as a functional of $q(\theta)$,

$$\Pi[q(\theta)] \equiv \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta - \frac{1 - F(\theta)}{f(\theta)} \right) [v(q(\theta)) - cq(\theta)] \, f(\theta) \, d\theta + \Omega \left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) f(\theta) \, d\theta \right). \tag{5}$$

[3] Since every agent is of measure zero, each agent takes Q as fixed when deciding on his own announcement $\hat{\theta}$.

Applying the calculus of variations, we can reformulate the principal's optimization problem as

$$[\mathbf{P}_\varepsilon] : \underset{\varepsilon\in\mathbb{R}}{max}\Pi(\varepsilon), \tag{6}$$

[4] where

$$\begin{aligned}\Pi(\varepsilon) \equiv & \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta - \frac{1-F(\theta)}{f(\theta)}\right) v(q^-(\theta)+\varepsilon q(\theta)) - c(q^-(\theta)+\varepsilon q(\theta))\right] f(\theta)\, d\theta \\ & + \Omega\left(\int_{\underline{\theta}}^{\overline{\theta}} (q^-(\theta)+\varepsilon q(\theta)) f(\theta) d\theta\right)\end{aligned}.$$

If $\varepsilon^* = 0$ maximizes $\mathbf{P}_\varepsilon$ (and thus $q^-(\theta)$ maximizes $\mathbf{P}$), then, for any fixed admissible function $q(\theta)$, we must have

$$\Pi'(0) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta - \frac{1-F(\theta)}{f(\theta)}\right) v'(q^-(\theta)) - c + \Omega'(Q^-)\right] q(\theta) f(\theta)\, d\theta = 0. \tag{7}$$

Since this is true for any variation $q(\theta)$, it follows that the continuous function which is the coefficient of $q(\theta)$ under the integral sign must vanish identically on $[\underline{\theta}, \overline{\theta}]$. We thus have the Euler's equation:

$$\left[\theta - \frac{1-F(\theta)}{f(\theta)}\right] v'(q^-(\theta)) + \Omega'(Q^-) = c. \tag{8}$$

Note that (8) equates the consumer's marginal revenue —taking into account informational rent and the externality— to the marginal cost to the principal of an increase in $q(\theta)$. The first term on the left-hand side of (8) is the marginal intrinsic value in terms of virtual valuation of raising $q(\theta)$ incrementally, and the second term is the marginal network value of raising $q(\theta)$. Letting $q^-(\theta, Q) \equiv (v')^{-1}\left[(c - \Omega'(Q))/\left(\theta - \frac{1-F(\theta)}{f(\theta)}\right)\right]$ and $\varphi^-(Q) \equiv \int_{\underline{\theta}}^{\overline{\theta}} q^-(\theta, Q) f(\theta)\, d\theta$, we have that the optimal network size Q^- is a fixed point of $\varphi^-(Q)$, i.e., $Q^- = \varphi(Q^-)$ and $q^-(\theta) = q(\theta, Q^-)$. The existence of Q^- is guaranteed by the boundedness of $\Omega'(Q)$ and the Inada conditions.[5]

If the externalities are nonincreasing, i.e., $\Omega''(Q) \leqslant 0$ for all Q, then $\varphi^-(Q)$ is nonincreasing, and Q^- is thus determined uniquely. However, the uniqueness of fixed points is not guaranteed in the case of increasing externalities, wherein an increasing curve $\varphi^-(Q)$ might cross the 45° line several times. The monopolist thus simply picks one from the set $\mathbf{Q} = \{Q \in \mathbb{R}^+ \,|\varphi^-(Q) = Q\}$ to maximize

$$\Pi^-(Q) \equiv \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta - \frac{1-F(\theta)}{f(\theta)}\right) v(q^-(\theta, Q)) - cq^-(\theta, Q)\right] f(\theta)\, d\theta + \Omega\left(\int_{\underline{\theta}}^{\overline{\theta}} q^-(\theta, Q) f(\theta)\, d\theta\right).$$

[4] With a slight abuse of notation, Π still represents the profit of the principal.

[5] It follows from condition $\sup_{x\in[0,+\infty)} \Omega'(x) < c$ and Inada conditions that $\varphi(0) = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta, 0) f(\theta) d\theta > 0$, $\varphi(+\infty) = \lim_{x\to+\infty} \int_{\underline{\theta}}^{\overline{\theta}} q(\theta, x) f(\theta) d\theta < +\infty$. Since $q^-(\theta, Q)$ is continuous, so is $\varphi^-(Q)$. Therefore, there exists a Q^- satisfying $\varphi(Q^-) = Q^-$.

Consider the following parametrized form

$$\Pi(Q,\epsilon) \equiv \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta - \epsilon \frac{1-F(\theta)}{f(\theta)} \right) v(q(\theta,Q,\epsilon)) - cq(\theta,Q,\epsilon) \right] f(\theta) d\theta + \Omega \left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta,Q,\epsilon) f(\theta) d\theta \right),$$

where $q(\theta,Q,\epsilon) = (v')^{-1} \left\{ [c - \Omega'(Q)] / \left[\theta - \epsilon \frac{1-F(\theta)}{f(\theta)} \right] \right\}$, $\epsilon = 0$ and $\epsilon = 1$ correspond respectively to the first-best and second-best cases. Note that

$$\frac{\partial \Pi}{\partial Q} = \int_{\underline{\theta}}^{\overline{\theta}} \left[\Omega'(\varphi(Q,\epsilon)) - \Omega'(Q) \right] \frac{\partial q}{\partial Q}(\theta,Q,\epsilon) f(\theta) d\theta = [\Omega'(\varphi(Q,\epsilon)) - \Omega'(Q)] \frac{\partial \varphi}{\partial Q},$$

where $\varphi(Q,\epsilon) = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta,Q,\epsilon) f(\theta) d\theta$, so in the case of strictly increasing externalities, any interior maximizer of $\Pi(Q,\epsilon)$ must be a fixed point of $\varphi(Q,\epsilon)$. This allows us to search the optimal network size $Q(\epsilon)$ over the whole real line, i.e., $arg \max_{Q \in \mathbf{Q}} \Pi(Q,\epsilon) = arg \max_{Q \in \mathbb{R}} \Pi(Q,\epsilon)$.

LEMMA 1 *If $\Omega''(Q)$ is of constant sign, given condition $\sup_{Q \in [0,+\infty)} \Omega'(Q) < c$, a fixed point Q is optimal only if $\frac{\partial \varphi}{\partial Q} \leqslant 1$.*

PROOF. See appendix. ■

It could be seen from the expression of $d\Pi/dQ$ that, at the optimum $\varphi(Q,\epsilon)$ must cross the $45°$ line from above. The points at which $\varphi(Q,\epsilon)$ is tangent to or crosses the $45°$ line from below are not optimal. It follows directly from the first order condition $\varphi(Q,\epsilon) - Q = 0$ that $dQ/d\epsilon = -\varphi_\epsilon(Q,\epsilon)/[\varphi_Q(Q,\epsilon) - 1] < 0$. We thus obtain the following result.

LEMMA 2 *If $\Omega''(Q)$ is of constant sign, the network size with incomplete information shrinks relative to the first-best case, i.e., $Q(1) < Q(0)$.*

We can now turn to analyze the distortions induced by the presence of both asymmetric information and externalities at the second-best consumption $q^-(\theta) \equiv q(\theta, Q(1), 1)$ relative to the first-best consumption $q^f(\theta) \equiv q(\theta, Q(0), 0)$. In the standard model without externalities, quantity is distorted downward for all values of θ except the highest. In the case with externalities, however, this outcome can no longer be sustained.

THEOREM 1 *The pattern of distortion in the second-best contract may depend on the curvature properties of externalities:*

- *If network externalities are increasing, i.e., $\Omega''(Q) > 0$ for all Q, there is one-way distortion: $q^-(\theta) < q^f(\theta)$, for all $\theta \in [\underline{\theta}, \overline{\theta}]$;*
- *If externalities are decreasing, i.e., $\Omega''(Q) < 0$ for all Q, there exists two-way distortion: $\exists \tilde{\theta} \in (\underline{\theta}, \overline{\theta})$, $q^-(\theta) > q^f(\theta)$ for $\theta \in (\tilde{\theta}, \overline{\theta}]$ and $q^-(\theta) < q^f(\theta)$ for $\theta \in [\underline{\theta}, \tilde{\theta})$;*

- *If externalities are constant, i.e.,* $\Omega''(Q) = 0$ *for all* Q*, then the standard result still holds:* $q^-(\theta) < q^f(\theta), \forall\theta \in [\underline{\theta}, \overline{\theta}), q^-(\overline{\theta}) = q^f(\overline{\theta})$.

PROOF. See appendix. ■

In the standard nonlinear pricing setup, the principal's rent extraction purpose motivates her to reduce the consumptions for all but the most efficient types. This effect may still exist in our model. Moreover, in our model, the principal also has an incentive to increase every type's consumption and thus make the network more attractive. If the network is increasing, the rent-extraction effect dominates the network-expansion effect for all types. As a result, every type's consumption is lower than that in the first-best case. If the network is decreasing, however, for the higher types the network-expansion effect dominates, while for the lower types the rent-extraction effect dominates, so two-way distortion is the outcome.

With decreasing externalities, an agent would be even more eager to trade more if he expects the other agents trade less with the principal. Therefore, whenever an outcome Q^- is fulfilled, any initial expectation smaller (larger) than Q^- will induce an actual aggregate consumption larger (smaller) than Q^-, thus the optimal contract must be uniquely implemented. With increasing externalities, however, an agent would be more eager to trade more if he expects the other agents trade more. In this case, multiple equilibria are more of a norm than an exception. This is due to the positive feedback associated with expectations: if all agents in the economy believe something will not succeed, it will usually fail; on the contrary, if they expect it to succeed, it usually will. Now we need some conditions ensuring the uniqueness of the fixed point of $\varphi^-(Q)$. Let ϕ be defined as $\phi(Q) \equiv \varphi^-(Q) - Q$. Then a fixed point of $\varphi^-(Q)$ is a solution of equation $\phi(Q) = 0$. If Q^- is unique, then $\phi(Q^-) = 0$ and $\phi(0) > 0$ implies that ϕ is nonnegative everywhere on the line from Q^- to 0. The following definition formalizes this property.

DEFINITION 3 The function $g : \mathbb{R} \to \mathbb{R}$ is radially quasiconcave ("$\mathbb{R}-$concave") if $g(x) = 0$ for some $x > 0$ implies $g(\lambda x) \geqslant 0$ for all $\lambda \in [0, 1]$. If the strict inequality holds for all $\lambda \in (0, 1)$, then g is strictly $\mathbb{R}-$concave.

Armed with this concept, we are now able to give a uniqueness result.

PROPOSITION 1 *(The Uniqueness Result) Suppose that* $\sup_{Q\in[0,\infty)} \Omega'(Q) < c$*, then* $\varphi^-(Q)$ *has a unique positive fixed point if and only if* $\phi(Q) = \varphi^-(Q) - Q$ *is strictly* $\mathbb{R}-$*concave.*

PROOF. See appendix. ■

Remarks. (i) Since quasiconcavity (quasiconvexity) means convexity of upper (lower) contour sets, a function $\phi(Q)$ is (strictly) $\mathbb{R}$-concave if it is either (strictly) quasiconcave or (strictly) quasiconvex provided that $\phi(0) > 0$ and $\lim_{Q\to+\infty}\phi(Q) = -\infty$, but not vice versa. Therefore, any one of the following assumptions on $\phi(Q)$ is sufficient (but not necessary) for equilibrium to be unique: (a) it is strictly concave; (b) it is strictly convex; (c) it is decreasing; (d) it increases slower than Q. Conditions (c) and (d) hold for decreasing or mildly increasing externalities. (ii) Note that quasiconcavity (quasiconvexity) of $\varphi^-(Q)$ does not necessarily imply the quasiconcavity (quasiconvexity) of $\phi(Q)$. (iii) It is clear that the strict version of $\mathbb{R}$-concavity cannot be replaced by the weak version. For example, function $\phi(Q) = -(Q-1)^2(Q-2), Q \in [0,\infty)$ satisfies $\phi(0) > 0, \lim_{Q\to\infty}\phi(Q) = -\infty$. Also, it is $\mathbb{R}$-concave but not strictly $\mathbb{R}$-concave. Of course the uniqueness result fails for this function. With increasing externalities, $\varphi^-(Q)$ is increasing, but $\phi(Q)$ is not necessarily $\mathbb{R}$-concave. Therefore, multiple fixed points may arise when function $\varphi^-(Q)$ crosses the 45° line more than once. Throughout the rest of the paper, we assume $\phi(Q)$ to be strictly $\mathbb{R}$-concave to avoid multiple equilibria.

3.2 Boundedly rational agents

In the preceding subsection, the second-best contract $\{q^-(\theta), t^-(\theta)\}$ is fulfilled if agents could form rational expectations without any systematic forecasting errors about the real network size. This is in line with the traditional neoclassical view propagated by Muth (1961) and Lucas (1972). This result is in fact attributing too much information and rationality on the part of agents. It is more reasonable to assume that agents are boundedly rational in their ability of forecasting and decision-making. They modify adaptively their expectations over time on the basis of observations of past performance. In this subsection, we will discuss whether or not the fully rational equilibria may emerge as outcomes from repeated adaptive learning process of boundedly rational agents.

Agents must form an expectation on network size one period ahead and make their participation decisions based on this expectation. Their adaptive learning process works as follows. At time t, all the agents form a common expectation Q_t^e of the network size, then consumers with nonnegative expected utilities, i.e., $\theta \in \left\{\theta \in [\underline{\theta}, \overline{\theta}] | \int_{\underline{\theta}}^{\theta} v(q^-(\theta))\,d\theta \geqslant \Omega(Q^-) - \Omega(Q_t^e)\right\}$, accept the contract $\{q^-(\theta), t^-(\theta)\}$.[6] The rest of the agents will reject it and quit the market. This forms

[6] In this paper, the agents' beliefs are assumed to be homogeneous, i.e., all the agents have identical expectations Q_t^e. Please see Hommes (2006), LeBaron (2006), Hommes and Wagener (2009) and Chiarella et al. (2009), among many others, for detailed discussion of models with heterogeneous expectations.

an actual network size $Q_t = \rho(Q_t^e)$, where

$$\rho(x) = \begin{cases} Q^- & \text{if} \quad \Omega(x) \in [\Omega(Q^-), +\infty) \\ \int_{\theta^*(x)}^{\overline{\theta}} q^-(\theta) f(\theta)\, d\theta & \text{if} \quad \Omega(x) \in \left[\max\{0, \Omega(Q^-) - \int_{\underline{\theta}}^{\overline{\theta}} v(q^-(\theta))\, d\theta\}, \Omega(Q^-)\right) \\ 0 & \text{if} \quad \Omega(x) \in \left[0, \max\{0, \Omega(Q^-) - \int_{\underline{\theta}}^{\overline{\theta}} v(q^-(\theta))\, d\theta\}\right) \end{cases},$$

$\theta^*(x)$ is given implicitly by $\int_{\underline{\theta}}^{\theta^*} v(q^-(\theta))\, d\theta + \Omega(x) = \Omega(Q^-)$ (see FIGURE 1). If $Q_t = Q_t^e$, the

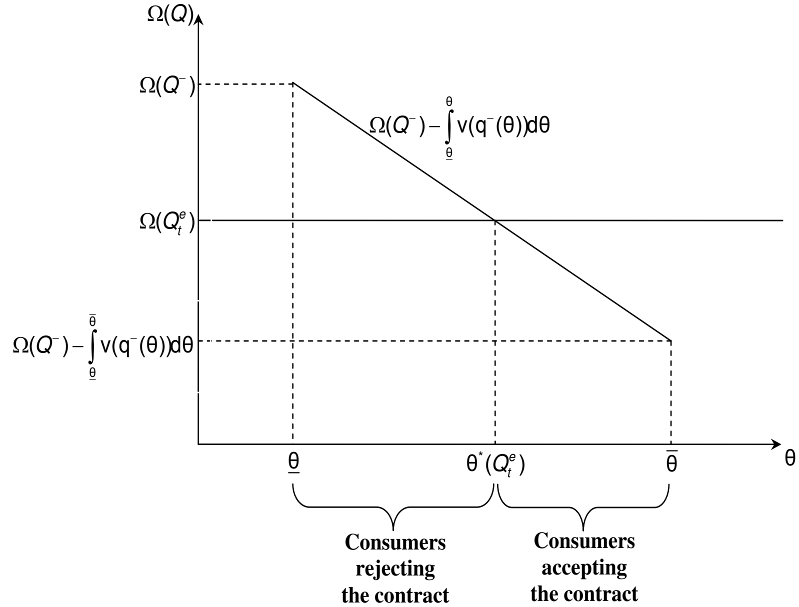


FIGURE 1.

rational expectation outcome is reached; otherwise, the expectation is updated according to an adaptive learning rule $Q_{t+1}^e = \alpha Q_t + (1-\alpha) Q_t^e$, where $\alpha \in (0, 1]$ is the expectations weight factor. The expected network size is a weighted average of yesterday's expected and realized values, or equivalently, the expected network size is adapted by a factor α in the direction of the most recent realization. This process is repeated until an expectation is self-fulfilled, i.e., $Q_t^e = Q_t$. Q^- is obviously a fixed point of the feedback function $\tilde{\rho}(Q) \equiv \alpha\rho(Q) + (1-\alpha)Q$. In what follows, we will discuss whether or not the adaptive learning procedures will lead the economy to converge to fully rational equilibrium (FRE).[7] We first introduce a few preliminary definitions and lemmas.

[7]Throughout this paper, it is assumed implicitly that there is a new independent draw of the agents' types every period, so there is no dynamic contracting issue arising from the gradual elimination of the informational asymmetry over time. Moreover, the principal is assumed to believe that the agents are or will eventually be rational. That is, the principal is irrational on the rationality of agents. Therefore, she will always provide the fully rational contract $\{q^-(\theta), t^-(\theta)\}$ in every period.

DEFINITION 4 A fixed point x^* of $f(x)$ is called attractive if there exists a neighborhood $U(x^*)$ such that the iterated function sequence $\{f^{(n)}(x)\}$ converges to x^* for all $x \in U(x^*)$.

DEFINITION 5 A fixed point x^* of $f(x)$ is called Lyapunov stable if, for each $\epsilon > 0$, there is a $\delta > 0$ such that for all x in the domain, if $|x - x^*| < \delta$, $|f^{(n)}(x) - f^{(n)}(x^*)| < \epsilon$ for all $n \in \mathbb{N}$.

DEFINITION 6 A fixed point x^* of $f(x)$ is called asymptotically stable (an attractor) if it is Lyapunov stable and attractive; it is called neutrally stable if it is Lyapunov stable but not attractive[8]; it is called unstable (a repeller) if it is not Lyapunov stable.

Lyapunov stability of an equilibrium means that solutions starting "close enough" to the equilibrium remain "close enough" forever. If x^* is an unstable fixed point, then there always exists a starting value x very near to it so that the system moves far away from x^* upon iteration: $\exists$ an open interval I containing x^*, $\forall x \in I\backslash\{x^*\}$, $\exists n > 0$ such that $f^{(n)}(x) \notin I$. Asymptotic stability means that solutions that start close enough not only remain close enough but also converge to the equilibrium eventually. Neutral stability means for all initial values x near x^*, the solution stays near but does not converge to x^*.[9]

LEMMA 3 *Let x^* be a fixed point of the discrete time dynamical system $x_{n+1} = f(x_n)$,*

- *if $0 \leqslant f'_+(x^*) < 1(0 \leqslant f'_-(x^*) < 1)$, then x^* is asymptotically stable from above (below);*
- *if $f'_+(x^*) > 1(f'_-(x^*) > 1)$, then x^* is unstable from above (below).*

PROOF. See appendix. ■

The above lemma leaves out the case with neutral fixed point, i.e., $f'(x^*) = 1$, wherein the stability of x^* could not be determined until further information regarding higher-order terms of Taylor expansion are available.[10] Armed with Lemma 3, we are now able to characterize the stability of Q^-.

PROPOSITION 2 *If $\Omega'(Q^-) > v(q^-(\underline{\theta}))/f(\underline{\theta})q^-(\underline{\theta})$, then Q^- is unstable from below and asymptotically stable from above; if $\Omega'(Q^-) < v(q^-(\underline{\theta}))/f(\underline{\theta})q^-(\underline{\theta})$, then Q^- is asymptotically stable from both sides.*

[8] It is also possible for a fixed point to be attractive but not Lyapunov stable. For example, trajectories starting at any point x_0 may always go to a circle of radius r before converging to x^*.

[9] The center of a linear homogeneous system with purely imaginary eigenvalues is an example of a neutrally stable fixed point.

[10] When x^* is neutral, "nothing definitive can be said about the behavior of points near x^*" (Holmgren (1991), p.53), and several situations are possible: x^* may be stable, unstable, semistable or neutrally stable.

PROOF. See appendix. ■

This theorem shows that when the externalities are negative or weakly positive at optimum, the equilibrium is robust to small initial forecasting errors. Adaptive learning processes of consumers lead the economy to converge toward the unique rational expectation equilibrium. When the externalities are strongly positive, however, the equilibrium is only semi-stable from above. The basic intuition behind this result is that the principal is capable of preventing the overheating but not necessarily the overcooling of economy. The overoptimistic expectations of consumers boost their participation in contracting but the good is not overconsumed since the principal controls their total consumptions. However, overpessimistic expectations will decrease the real network size since a set of consumers with positive measure will choose to quit the market. [11] In the case with negative externalities, if all consumers initially form a shared over-pessimistic prior on the network size, then a fraction of consumers will quit the market and the good is actually underconsumed. The adaptive learning process will lead the consumers gradually to an overoptimistic expectation. After that, all the consumers participate in contracting and the expectation converges increasingly to the fully rational equilibrium. Things are different for the case with positive externalities. If externalities are weakly positive and consumers form an overpessimistic expectation, the realized network size will outperform their expectation, then the consumers' over-pessimism disappears eventually. In contrast, with strongly positive externalities, the low initial expectation reduces the real aggregate consumptions, which in turn confirms the expectations. The effects that pessimism and depression reinforce each other lead the network size to a very low level. Proposition 2 encompasses several special cases of interest, which are depicted in the following FIGUREs 2 to 5 with $\alpha = 1$.

- *Case a.* $\Omega'(Q) > 0$ for all $Q \in [0, +\infty)$, $\Omega(Q^-) > \int_{\underline{\theta}}^{\overline{\theta}} v(q^-(\theta))d\theta$, $\Omega'(Q^-) < v(q^-(\underline{\theta}))/f(\underline{\theta})q^-(\underline{\theta})$ $\rho(Q)$ is strictly concave in $\left[\Omega^{-1}\left(\Omega(Q^-) - \int_{\underline{\theta}}^{\overline{\theta}} v(q^-(\theta))d\theta\right), Q^-\right]$. In this case, $\exists Q'$ such that $Q \geqslant \rho(Q), \forall Q \in [0, Q')$ and $Q < \rho(Q), \forall Q \in (Q', Q^-)$. When the curve $\rho(Q)$ lies below the 45° line, we have downward pressure on the consumption of the good: the realized network size outcome will underperform the consumers' expectation, and there will be a downward spiral in consumption. Correspondingly, when the curve $\rho(Q)$ lies above the 45° line, we have upward pressure on the consumption of the good. Q' is not just an unstable equilibrium, but is actually a critical point, or a tipping point, in the success of the good. If the firm producing the good can get the consumer's expectations for the total

[11] In the case with positive externalities, agents with expectations larger than the equilibrium value are called "optimistic", and those who have expectations smaller than the equilibrium value are called "pessimistic". It is opposite for negative externalities.

consumption above Q', then it can use the upward pressure of demand to get its market share to the stable equilibrium at Q^-. On the contrary, if the consumer's expectations are even slightly below Q', then the downward pressure will tend to drive the market to shut down. This result suggests that the success of a product depends crucially on consumers' initial confidence on it.

- *Case b.* $\Omega'(Q) > 0$ for all $Q \in [0, +\infty)$, $\Omega(Q^-) > \int_{\underline{\theta}}^{\overline{\theta}} v(q^-(\theta))d\theta$, and function $\sigma(Q) = Q - \rho(Q)$ is strictly $\mathbb{R}$-concave on $(0, +\infty)$. In this case, function $\rho(Q)$ has two fixed points 0 and Q^-. Since the curve $\rho(Q)$ lies below the 45° line, there is always downward pressure on the consumption of the good. Any initial overoptimistic expectation will disappear gradually. However, even a very small perturbation in the left will lead the market to shut down.

- *Case c.* $\Omega'(Q) > 0$ for all $Q \in [0, +\infty)$, $\Omega(Q^-) \leqslant \int_{\underline{\theta}}^{\overline{\theta}} v(q^-(\theta))d\theta$, $-\sigma(Q) = \rho(Q) - Q$ is strictly $\mathbb{R}$-concave on $(0, +\infty)$. In this case $\rho(Q)$ has a unique fixed point, where $\rho(Q)$ crosses the 45° line from above. Therefore, Q^- is globally stable. It is robust to perturbation in any amount and either direction.

- *Case d.* $\Omega'(Q) \leqslant 0$ for all $Q \in [0, +\infty)$. In this case Q^- is also globally stable due to the negative feedback effect.

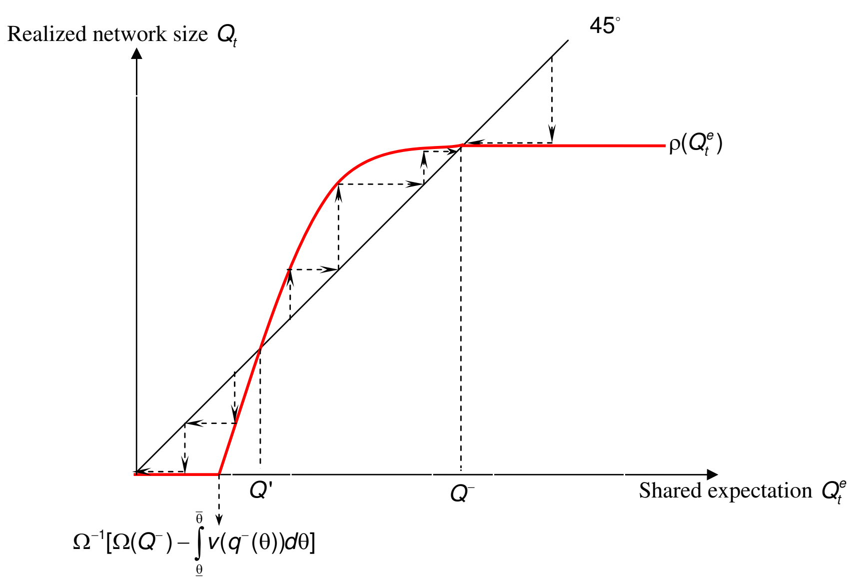


FIGURE 2. Case a

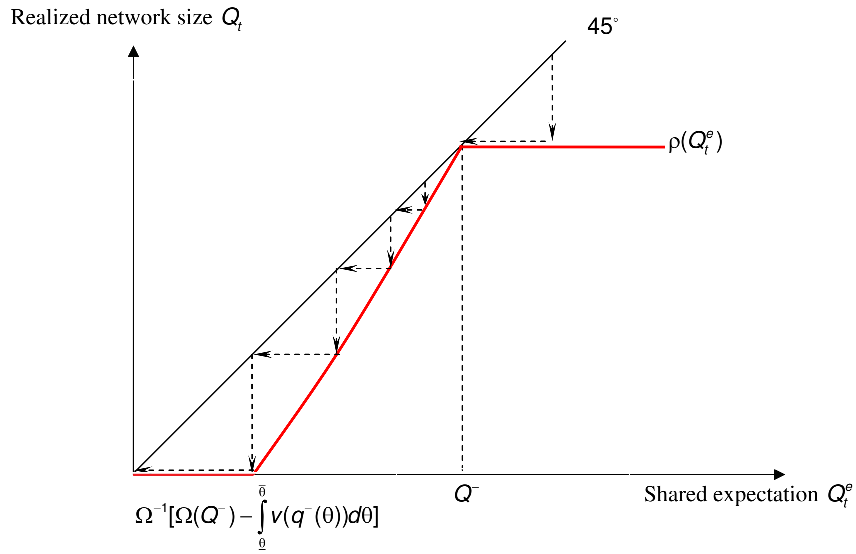


FIGURE 3. Case b

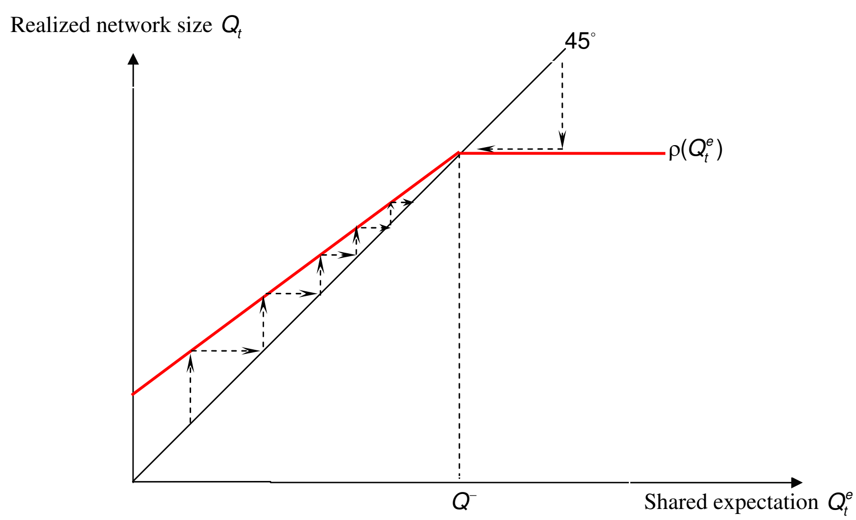


FIGURE 4. Case c

4 Nonlinear pricing with entry threats

The monopolist in the above model may face a threat of entry from rival firms, whose product is intrinsically a perfect substitute for the monopolist's product, but is incompatible with the existing network. By virtue of being the incumbent, the monopolist's product generates network value for all customers. The entrant's product, on the other hand, is assumed to provide only its intrinsic value to the customers. This section discusses the optimal entry-deterring nonlinear pricing contracts when consumers are, respectively, fully and boundedly rational.

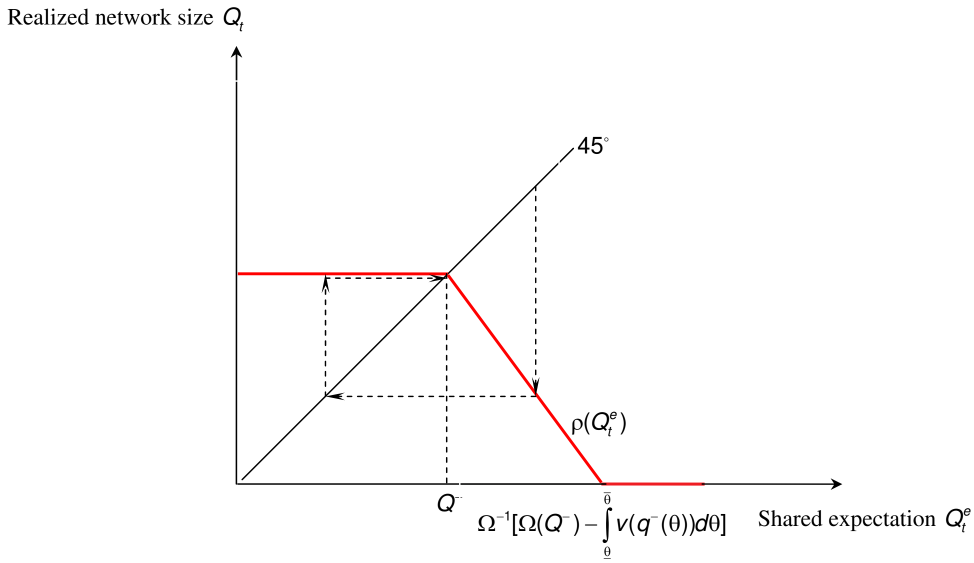


FIGURE 5. Case d

4.1 Fully rational agents

We assume that the competitive outside rivals set their price equal to their marginal cost ω. Therefore, in order to deter entry, the monopolist's pricing scheme must provide customers of type θ with a surplus of at least $U^0(\theta) = \max_{q\in[0,\infty)}[\theta v(q) - \omega q]$. Theoretically, this is a principal-agent model with type-dependent individual rationality constraints. Let $q^0(\theta) \equiv arg\max_{q\in[0,\infty)}[\theta v(q) - \omega q]$, $U(\theta) \equiv \theta v(q(\theta)) - t(\theta) + \Omega(Q) - U^0(\theta)$. Facing fully rational consumers, the monopolist's entry-deterring problem can be written as:

$$[\mathbf{P}_e] : \max_{q(\theta),U(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta v(q(\theta)) - cq(\theta) - U^0(\theta) - U(\theta)\right] f(\theta)d\theta + \Omega(Q)$$

$$subject\ to:$$

$$U'(\theta) = v(q(\theta)) - v(q^0(\theta)), q(\theta) is\ nondecreasing, U(\theta) \geqslant 0, Q = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)f(\theta)d\theta.$$

Note that the consumer's rents may either increase or decrease with θ depending on the comparison between $q(\theta)$ and $q^0(\theta)$. If the quantity a consumer purchases from the incumbent firm exceeds that from the outside rivals, rents rise with θ; otherwise they fall with θ. Neglecting momentarily the monotonicity condition of $q(\theta)$ and checking it ex post, we can write the principal's problem as a standard control problem with U as state variable and q as control variable. The Hamiltonian function for this control problem would take the form

$$\begin{aligned} H(U, q, \mu, \theta) = &\left[\theta v(q(\theta)) - cq(\theta) + U^0(\theta) - U(\theta)\right] f(\theta) \\ &+ \Omega\left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)f(\theta)d\theta\right) + \mu(\theta)[v(q(\theta)) - v(q^0(\theta))] \end{aligned}$$

where μ is the costate variable. The Lagrangian function is

$$L = H + \gamma(\theta)U(\theta),$$

where $\gamma(\theta)$ is the multiplier of constraint $U(\theta) \geqslant 0$. The first-order condition for the maximization of the Hamiltonian with respect to q is

$$\left[\theta + \frac{\mu(\theta)}{f(\theta)}\right] v'(q(\theta)) + \Omega'(Q) = c. \tag{9}$$

For a fixed network size Q, all variables (q, μ, U, γ) being represented as functions of θ and Q, the following conditions must be satisfied:

$$\begin{aligned}
\text{costate equation:} \quad & \mu_\theta = -\frac{\partial L}{\partial U} = f(\theta) - \gamma(\theta) & (10)\\
\text{state equation:} \quad & U_\theta = v(q(\theta, Q)) - v(q^0(\theta)) & (11)\\
\text{complementary slackness:} \quad & \gamma(\theta, Q)U(\theta, Q) = 0, \gamma(\theta, Q) \geqslant 0, U(\theta, Q) \geqslant 0 & (12)\\
\text{transversality conditions:} \quad & \mu(\underline{\theta}, Q)U(\underline{\theta}, Q) = \mu(\overline{\theta}, Q)U(\overline{\theta}, Q) = 0, & (13)\\
& \mu(\underline{\theta}, Q) \leqslant 0, \mu(\overline{\theta}, Q) \geqslant 0. &
\end{aligned}$$

Let $\hat{q}(\mu, \theta, Q)$ denote the value of q that maximizes the Hamiltonian given μ, θ and Q defined implicitly by (9), and let $\hat{\mu}(\theta, Q)$ be the solution in μ to the following equation $\hat{q}(\mu, \theta, Q) = q^0(\theta)$. It can be easily obtained $\hat{\mu}(\theta, Q) = [(c - \Omega'(Q))/\omega - 1]\,\theta f(\theta)$, $\hat{\mu}(\theta, Q)$ is the value of costate variable such that the agent's utility is constant ($U_\theta = 0$). From (10) and (12), we have $\gamma(\theta, Q) = f(\theta) - \mu_\theta(\theta, Q) \geqslant 0$, then there must be $\mu_\theta \leqslant f(\theta)$. If the IR constraint is binding on a nondegenerate interval, then $\mu(\theta, Q)$ must be equal to $\hat{\mu}(\theta, Q)$ and $\mu_\theta = \hat{\mu}_\theta < f(\theta)$ on that interval. To get an optimal costate function $\mu(\theta)$, we impose the following assumption.

ASSUMPTION 2 *Function $\frac{F(\theta)}{\theta f(\theta)}$ is nondecreasing.*

This assumption is obviously a bit stronger than the usual monotone hazard rate condition which only requires $F(\theta)/f(\theta)$ to be nondecreasing. In order to solve the problem, we conjecture a solution and then verify whether or not it satisfies conditions (10) to (13). The critical work is to construct the right solution for $\mu(\theta, Q)$. Consider the following function,

$$\mu^*(\theta, Q) = \begin{cases} F(\theta) & \text{if} \quad \hat{\mu}(\theta, Q) \geqslant F(\theta) \\ \hat{\mu}(\theta, Q) & \text{if} \quad F(\theta) - 1 < \hat{\mu}(\theta, Q) < F(\theta) \\ F(\theta) - 1 & \text{if} \quad \hat{\mu}(\theta, Q) \leqslant F(\theta) - 1. \end{cases} \tag{14}$$

For simplicity, we define Θ_1,Θ_2, and Θ_3 as the subintervals of Θ where $\mu^*(\theta, Q)$ is equal to $F(\theta)$, $\hat{\mu}(\theta, Q)$ and $F(\theta) - 1$, respectively. In order for $\mu^*(\theta, Q)$ to satisfy the conditions (10) to (13), we need to check the condition $\mu_\theta^* \leqslant f(\theta)$. It is obviously satisfied on Θ_1 and Θ_3. So we need only to check $\mu_\theta^* = \hat{\mu}_\theta(\theta, Q) \leqslant f(\theta)$ for $\theta \in \Theta_2$. Notice that $\hat{\mu}_\theta(\theta, Q) = [f(\theta) + \theta f'(\theta)]\hat{\mu}(\theta, Q)/\theta f(\theta)$. If $f + \theta f' \geqslant 0$, then we have

$$\hat{\mu}_\theta(\theta, Q) < \frac{F(\theta)[f(\theta) + \theta f'(\theta)]}{\theta f(\theta)} \leqslant f(\theta),$$

where the last inequality follows directly from **Assumption** 2. If $f + \theta f' < 0$, then we also have

$$\begin{aligned}
\hat{\mu}_\theta(\theta, Q) &< \frac{[F(\theta) - 1][\theta f'(\theta) + f(\theta)]}{\theta f(\theta)} \\
&\leqslant \frac{\theta f^2(\theta) - [1 - F(\theta)]\, f(\theta)}{\theta f(\theta)} \\
&= f(\theta) - \frac{1 - F(\theta)}{\theta} \\
&\leqslant f(\theta).
\end{aligned}$$

The second inequality follows from **Assumption** 1. Therefore, the optimal quantity

$$q^*(\theta, Q) \equiv \hat{q}(\mu^*(\theta, Q), Q, \theta) = \begin{cases} q^+(\theta, Q) & \text{if } \frac{c - \Omega'(Q)}{\omega} \geqslant 1 + \frac{F(\theta)}{\theta f(\theta)} \\ q^0(\theta) & \text{if } 1 - \frac{1 - F(\theta)}{\theta f(\theta)} < \frac{c - \Omega'(Q)}{\omega} < 1 + \frac{F(\theta)}{\theta f(\theta)} \\ q^-(\theta, Q) & \text{if } \frac{c - \Omega'(Q)}{\omega} \leqslant 1 - \frac{1 - F(\theta)}{\theta f(\theta)} \end{cases} \tag{15}$$

is obviously nondecreasing in θ, where $q^-(\theta, Q) = (v')^{-1}\left[(c - \Omega'(Q))/\left(\theta - \frac{1 - F(\theta)}{f(\theta)}\right)\right]$, $q^+(\theta, Q) = (v')^{-1}\left[(c - \Omega'(Q))/\left(\theta + \frac{F(\theta)}{f(\theta)}\right)\right]$. We then need to distinguish the following cases, depicted in FIGURE 6 to FIGURE 10, depending on the values of Q and ω:

- **CASE 1** $\frac{c - \Omega'(Q)}{\omega} < 1 - \frac{1}{\underline{\theta} f(\underline{\theta})}$. *In this case* $\hat{\mu}(\theta, Q) < F(\theta) - 1$ *and thus* $q^*(\theta, Q) = q^-(\theta, Q)$ *for all* $\theta \in [\underline{\theta}, \overline{\theta}]$;

- **CASE 2** $1 - \frac{1}{\underline{\theta} f(\underline{\theta})} \leqslant \frac{c - \Omega'(Q)}{\omega} < 1$. *In this case* $F(\theta) - 1 \leqslant \hat{\mu}(\theta, Q) < 0, \forall \theta \in [\underline{\theta}, \theta^-(Q))$ *and* $\hat{\mu}(\theta, Q) < F(\theta) - 1, \forall \theta \in (\theta^-(Q), \overline{\theta}]$. *Therefore, we have*

$$q^*(\theta, Q) = \begin{cases} q^0(\theta) & \text{if } \theta \in [\underline{\theta}, \theta^-(Q)) \\ q^-(\theta, Q) & \text{if } \theta \in [\theta^-(Q), \overline{\theta}] \end{cases},$$

where $\theta^-(Q)$ *is the critical type given implicitly by* $\frac{c - \Omega'(Q)}{\omega} = 1 - \frac{1 - F(\theta)}{\theta f(\theta)}$;

- **CASE 3** $\frac{c - \Omega'(Q)}{\omega} = 1$. *In this case* $\hat{\mu}(\theta, Q) = 0$, *and* $q^*(\theta, Q) = q^f(\theta, Q) \equiv (v')^{-1}[(c - \Omega'(Q))/\theta]$ *for all* $\theta \in [\underline{\theta}, \overline{\theta}]$;

- **CASE 4** $1 < \frac{c - \Omega'(Q)}{\omega} < 1 + \frac{1}{\overline{\theta} f(\overline{\theta})}$. *In this case* $\hat{\mu}(\theta, Q) > F(\theta)$ *for* $\theta \in [\underline{\theta}, \theta^+(Q))$; $0 < \hat{\mu}(\theta, Q) < F(\theta)$ *for* $\theta \in [\theta^+(Q), \overline{\theta})$. *Therefore, we have*

$$q^*(\theta, Q) = \begin{cases} q^+(\theta, Q) & \text{if } \theta \in [\underline{\theta}, \theta^+(Q)) \\ q^0(\theta) & \text{if } \theta \in [\theta^+(Q), \overline{\theta}) \end{cases}$$

where $\theta^+(Q)$ *is the critical type given implicitly by* $\frac{c-\Omega'(Q)}{\omega} = 1 + \frac{F(\theta)}{\theta f(\theta)}$*;*

- **CASE 5** $\frac{c-\Omega'(Q)}{\omega} \geqslant 1 + \frac{1}{\overline{\theta} f(\overline{\theta})}$. *In this case,* $\hat{\mu}(\theta, Q) \geqslant F(\theta)$ *and thus* $q^*(\theta, Q) = q^+(\theta, Q)$ *for all* $\theta \in [\underline{\theta}, \overline{\theta}]$.

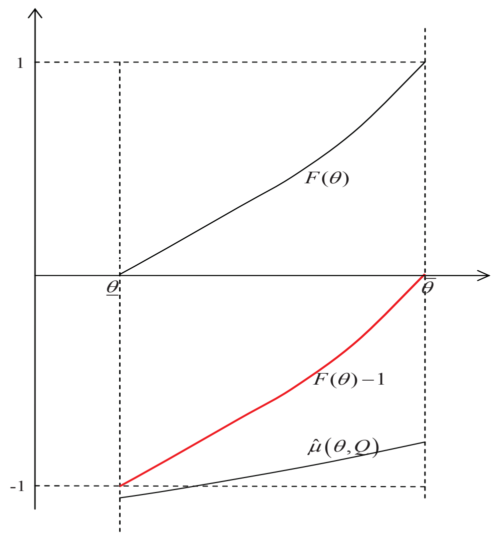


FIGURE 6. Case 1: $\frac{c-\Omega'(Q)}{\omega} < 1 - \frac{1}{\underline{\theta} f(\underline{\theta})}$

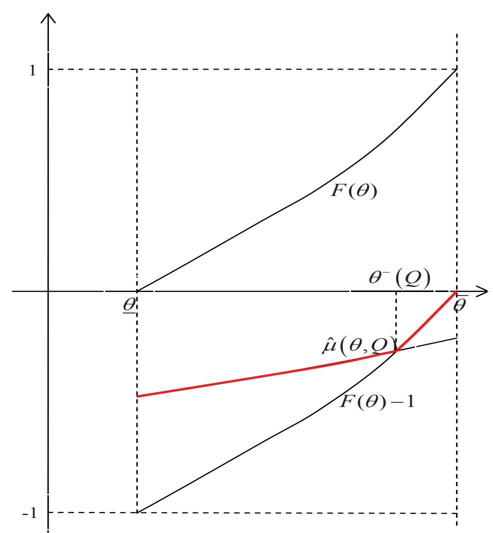


FIGURE 7. Case 2: $1 - \frac{1}{\underline{\theta} f(\underline{\theta})} \leqslant \frac{c-\Omega'(Q)}{\omega} < 1$

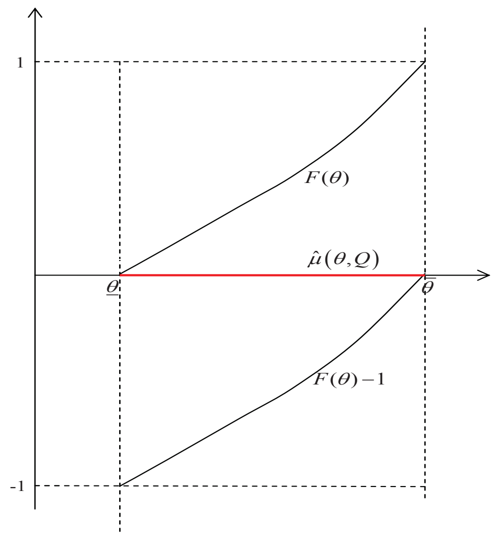


FIGURE 8. Case 3: $\frac{c-\Omega'(Q)}{\omega}=1$

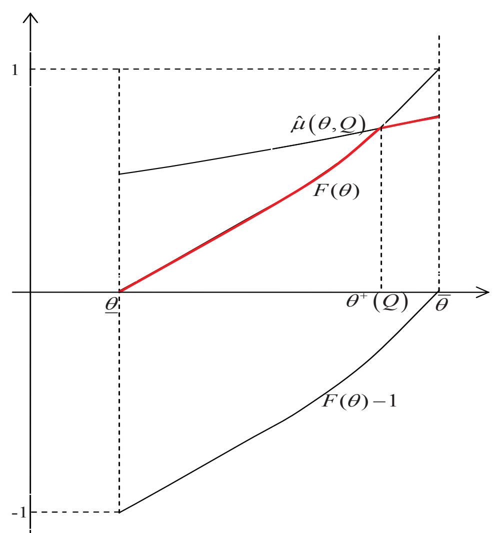


FIGURE 9. Case 4: $1<\frac{c-\Omega'(Q)}{\omega}<1+\frac{1}{\overline{\theta}f(\overline{\theta})}$

$\varphi^*(Q)\equiv\int_{\underline{\theta}}^{\overline{\theta}}q^*(\theta,Q)f(\theta)d\theta$ is consequently a piecewise function given by

$$\varphi^*(Q)=\begin{cases}\varphi^+(Q) & \text{if} \quad \frac{c-\Omega'(Q)}{\omega}\geqslant 1+\frac{1}{\overline{\theta}f(\overline{\theta})}\\ \int_{\underline{\theta}}^{\theta^+(Q)}q^+(\theta,Q)f(\theta)d\theta+\int_{\theta^+(Q)}^{\overline{\theta}}q^0(\theta)f(\theta)d\theta & \text{if} \quad 1<\frac{c-\Omega'(Q)}{\omega}<1+\frac{1}{\overline{\theta}f(\overline{\theta})}\\ \varphi^f(Q) & \text{if} \quad \frac{c-\Omega'(Q)}{\omega}=1\\ \int_{\underline{\theta}}^{\theta^-(Q)}q^0(\theta)f(\theta)d\theta+\int_{\theta^-(Q)}^{\overline{\theta}}q^-(\theta,Q)f(\theta)d\theta & \text{if} \quad 1>\frac{c-\Omega'(Q)}{\omega}\geqslant 1-\frac{1}{\underline{\theta}f(\underline{\theta})}\\ \varphi^-(Q) & \text{if} \quad \frac{c-\Omega'(Q)}{\omega}<1-\frac{1}{\underline{\theta}f(\underline{\theta})}\end{cases}, \qquad (16)$$

where $\varphi^i(Q)=\int_{\underline{\theta}}^{\overline{\theta}}q^i(\theta,Q)f(\theta)d\theta$, $i\in\{-,f,+\}$. Let Q^i be the fixed point of $\varphi^i(Q)$, and $q^i(\theta)\equiv q^i(\theta,Q^i),\forall i\in\{+,f,-\}$. We assume throughout this section that $\varphi^i(Q),i\in\{+,f,-\}$ are all

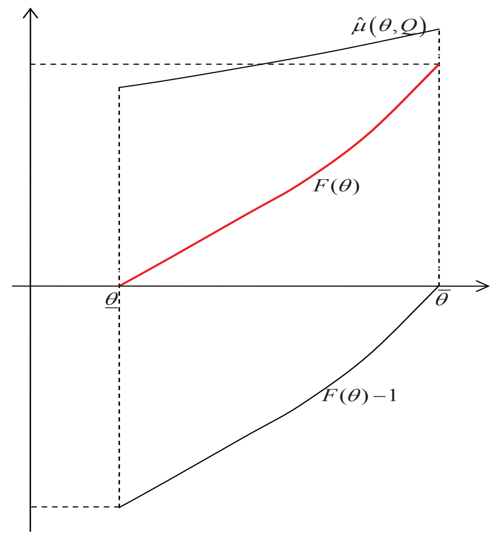


FIGURE 10. Case 5: $\frac{c-\Omega'(Q)}{\omega} \geqslant 1 + \frac{1}{\overline{\theta} f(\overline{\theta})}$

strictly $\mathbb{R}$-concave, then Q^i are uniquely determined. It is easy to find that $Q^- < Q^f < Q^+$ and $[c - \Omega'(Q^-)] / \left[1 - \frac{1}{\underline{\theta} f(\underline{\theta})}\right] = \underline{\theta} v'(q^-(\underline{\theta})) > \underline{\theta} v'(q^f(\underline{\theta})) = c - \Omega'(Q^f) = \overline{\theta} v'(q^f(\overline{\theta})) > \overline{\theta} v'(q^+(\overline{\theta})) = [c - \Omega'(Q^+)] / \left[1 + \frac{1}{\overline{\theta} f(\overline{\theta})}\right]$. [12] We now proceed to determine the fixed point Q^* of $\varphi^*(Q)$ and then characterize the optimal contract $\{U^*(\theta), q^*(\theta)\}$. In particular, it is interesting to compare the optimal allocation and network size with those in the full-information case.

THEOREM 2 *Given assumptions (1) and (2), if the sign of $\Omega''(Q)$ is constant on $[0, +\infty)$, $\sup_{Q \geqslant 0} \Omega'(Q) < c$, and $\varphi^i(Q), \forall i \in \{-, f, +\}$ are all strictly $\mathbb{R}$-concave, then the optimal nonlinear pricing contract of incumbent firm changes with the marginal cost of potential entrants:*

- *If $\omega \in \left[\frac{c-\Omega'(Q^-)}{1-1/\underline{\theta} f(\underline{\theta})}, +\infty\right)$, then $Q^* = Q^-$, $q^*(\theta) = q^-(\theta)$, $U^*(\theta) = \int_{\underline{\theta}}^{\theta} \left[v(q^-(\theta)) - v(q^0(\theta))\right] d\theta, \forall \theta \in [\underline{\theta}, \overline{\theta}]$;*
- *If $\omega \in \left(c - \Omega'(Q^f), \frac{c-\Omega'(Q^-)}{1-1/\underline{\theta} f(\underline{\theta})}\right)$, then $Q^* < Q^f$,*

$$q^*(\theta) = \begin{cases} q^0(\theta) & \text{if} \quad \theta \in [\underline{\theta}, \theta^-(Q^*)) \\ q^-(\theta, Q^*) & \text{if} \quad \theta \in [\theta^-(Q^*), \overline{\theta}] \end{cases},$$

$$U^*(\theta) = \begin{cases} 0 & \text{if} \quad \theta \in [\underline{\theta}, \theta^-(Q^*)) \\ \int_{\theta^-(Q^*)}^{\theta} \left[v(q^-(\theta, Q^*)) - v(q^0(\theta))\right] d\theta & \text{if} \quad \theta \in [\theta^-(Q^*), \overline{\theta}] \end{cases};$$

[12] Analogous to THEOREM 1, we have the following result: if $\Omega''(Q) > 0, \forall Q$, then $q^+(\theta) > q^f(\theta), \forall \theta \in [\underline{\theta}, \overline{\theta}]$; if $\Omega''(Q) < 0, \forall Q$, then $\exists \tilde{\theta} \in [\underline{\theta}, \overline{\theta}]$ such that $q^+(\theta) < q^f(\theta), \forall \theta \in [\underline{\theta}, \tilde{\theta})$, $q^+(\theta) > q^f(\theta), \forall \theta \in (\tilde{\theta}, \overline{\theta}]$; if $\Omega''(Q) = 0, \forall Q$, then $q^+(\underline{\theta}) = q^f(\underline{\theta})$, $q^+(\theta) > q^f(\theta), \forall \theta \in (\underline{\theta}, \overline{\theta}]$. Therefore, if $\Omega''(Q)$ has constant sign, we always have $q^f(\overline{\theta}) < q^+(\overline{\theta})$.

- *If* $\omega = c - \Omega'(Q^f)$, *then* $Q^* = Q^f$, $q^*(\theta) = q^0(\theta) = q^f(\theta), U^*(\theta) \equiv 0, \forall\theta \in [\underline{\theta}, \overline{\theta}]$;

- *If* $\omega \in \left(\frac{c-\Omega'(Q^+)}{1+1/\overline{\theta}f(\overline{\theta})}, c - \Omega'(Q^f)\right)$, *then* $Q^* > Q^f$,

$$q^*(\theta) = \begin{cases} q^0(\theta) & \text{if} \quad \theta \in [\theta^+(Q^*), \overline{\theta}] \\ q^+(\theta, Q^*) & \text{if} \quad \theta \in [\underline{\theta}, \theta^+(Q^*)) \end{cases},$$

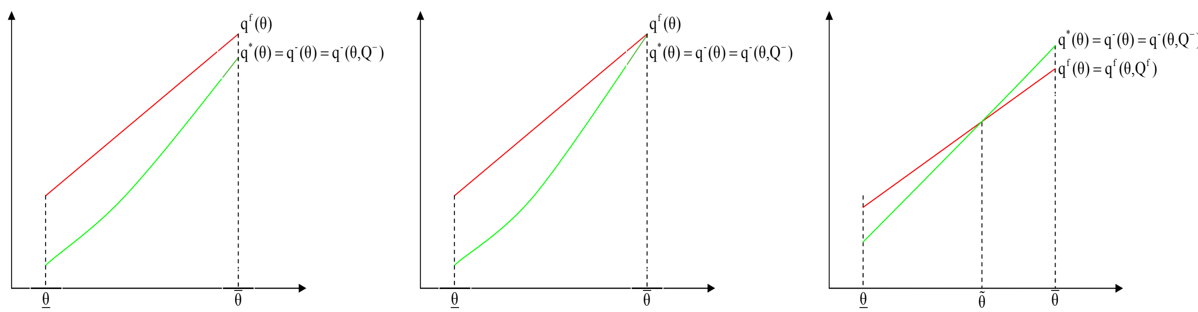
$$U^*(\theta) = \begin{cases} \int_{\theta^+(Q^*)}^{\theta} \left[v(q^+(\theta, Q^*)) - v(q^0(\theta))\right] d\theta & \text{if} \quad \theta \in [\underline{\theta}, \theta^+(Q^*)) \\ 0 & \text{if} \quad \theta \in [\theta^+(Q^*), \overline{\theta}] \end{cases};$$

- *If* $\omega \in \left[0, \frac{c-\Omega'(Q^+)}{1+1/\overline{\theta}f(\overline{\theta})}\right]$, *then* $Q^* = Q^+$, $q^*(\theta) = q^+(\theta)$, $U^*(\theta) = \int_{\overline{\theta}}^{\theta} \left[v(q^+(\theta)) - v(q^0(\theta))\right] d\theta, \forall\theta \in [\underline{\theta}, \overline{\theta}]$.

PROOF. See appendix. ■

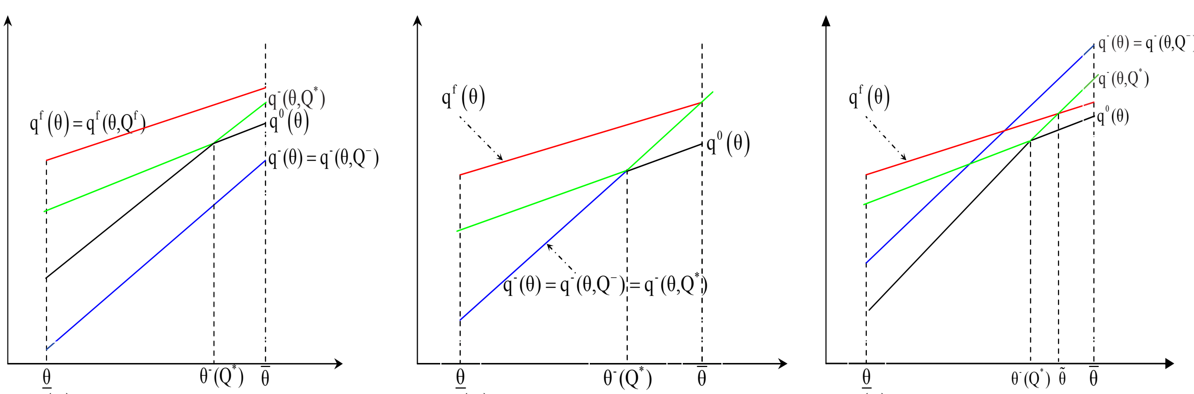
Facing potential entrants, consumers have an opportunity of using an alternative, incompatible but competitively supplied good and the incumbent firm's profit may be depressed, so it is in the interest of incumbents to deter entry if possible. Whether and to what extent would the incumbent firm's pricing strategy be affected by entry threat has constituted a major theme in the marketing and industrial organization literature. In the standard nonlinear pricing model, where the reservation utility of consumers is independent of their types and normalized to zero, a consumer has incentive to underreport his type to earn information rents. This will also be the case in our model if the potential entrants are very inefficient ($\omega \geqslant [c - \Omega'(Q^-)]/[1 - 1/\underline{\theta}f(\underline{\theta})]$). In this case, for all types of consumers, the incentive to understate their valuations to get information rents always outweighs the incentive to bypass the present network and get their reservation utilities. Therefore, the pure monopoly pricing contract remains optimal. As the potential entrants become more efficient ($\omega \in (c - \Omega'(Q^f), [c - \Omega'(Q^-)]/1 - \underline{\theta}f(\underline{\theta}))$), the low demand consumers prefer bypassing to staying in the network and understating their types. Therefore, the principal has to offer them less distorted quantities to prevent them from quitting the present network. If $\omega = c - \Omega'(Q^f)$, all types have incentive to quit the market rather than to misreport their types. So the principal finds that it is no longer optimal to distort quantities away from the first-best level. The remaining tool available to the principal to retain all consumers in the market is the transfer t. In this case, all consumption levels are the efficient ones. As ω continues to decrease, the outside market becomes attractive to the high demand consumers, so the principal has to offer them quantities higher than the first-best level to prevent them from quitting. The low demand types are now attracted by the allocation given to the high demand types. That is, they have incentive to overstate their types and thereby convince the seller that greater reward is required to prevent them from switching to the outside market. If the potential entrants are highly efficient with $\omega \leqslant [c-\Omega'(Q^{+})] \,/\, \left[1+1/\overline{\theta} f(\overline{\theta})\right]$, then for all types of consumers, the incentive to overstate always dominates their incentive to quit the market, and the agent's participation constraint binds for the highest realization of θ.

For each type, if $\omega > c-\Omega'(Q^{f})$ ($\omega < c-\Omega'(Q^{f})$), decreasing (increasing) his consumption may (1) reduce the information rents obtained by the higher (lower) types; (2) decrease (increase) the whole network value; (3) reduce (intensify) the congestion of network if the externalities are decreasing. The joint presence of these three effects leads to complex patterns of contract distortions, which are depicted in the following FIGUREs 11 to 15. In each figure, the red curve represents the benchmark case with complete information, the blue curve represents the pure monopoly case, while the green curve represents the case with entry threat. The distortionary way of quantities is described in TABLE 1.



(a) increasing externalities (b) constant externalities (c) decreasing externalities

FIGURE 11. $\omega \geqslant \frac{c-\Omega'(Q^{-})}{1-\frac{1}{\underline{\theta} f(\underline{\theta})}}$



(a) increasing externalities (b) constant externalities (c) decreasing externalities

FIGURE 12. $c-\Omega'(Q^{f}) < \omega < \frac{c-\Omega'(Q^{-})}{1-\frac{1}{\underline{\theta} f(\underline{\theta})}}$

The above results show that when network externalities are positive, subtracting a term $\Omega'(Q)$ from the incumbent firm's marginal cost c makes him more competitive in fighting against

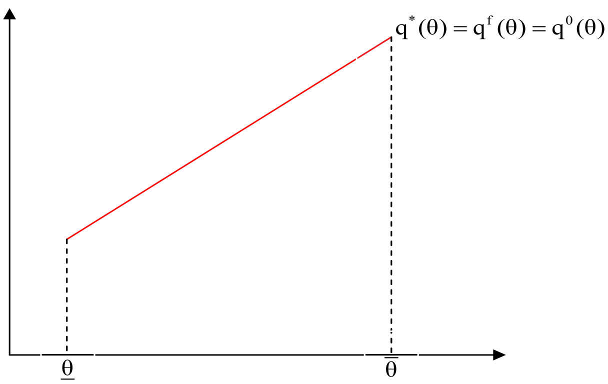
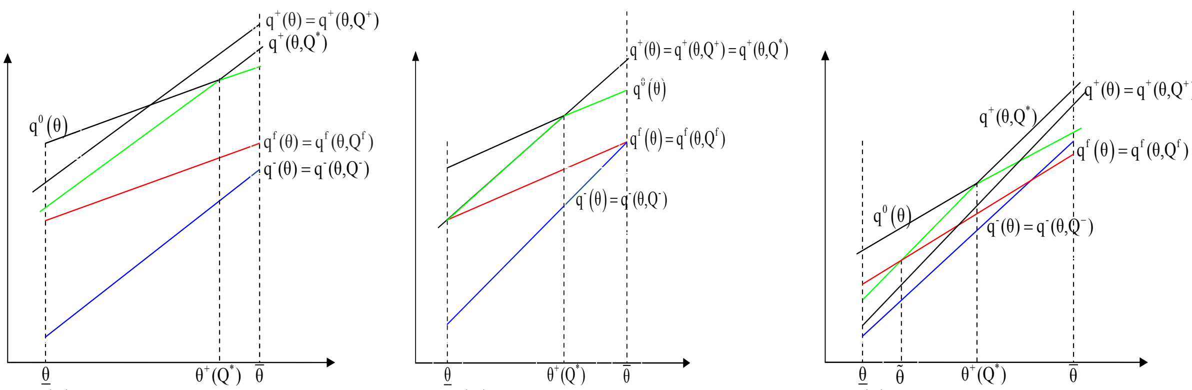


FIGURE 13. $\omega = c - \Omega'(Q^f)$

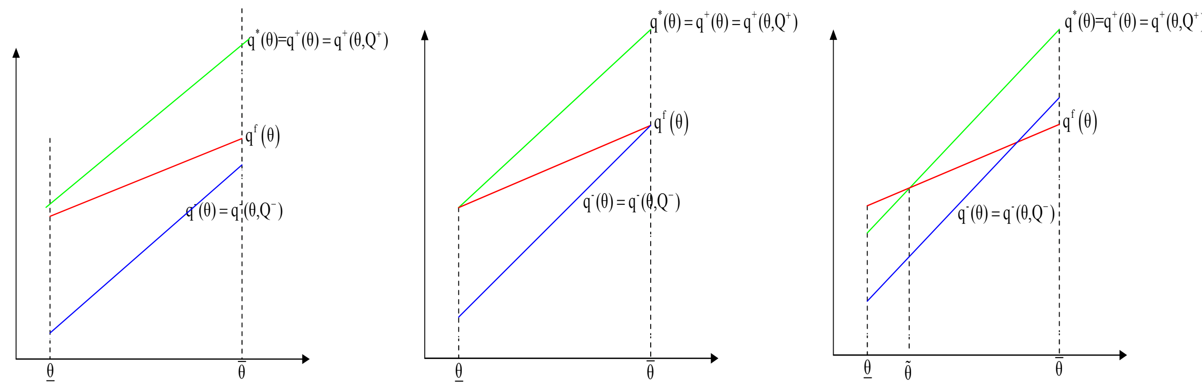


(a) increasing externalities (b) constant externalities (c) decreasing externalities

FIGURE 14. $\frac{c-\Omega'(Q^+)}{1+\frac{1}{\bar{\theta} f(\bar{\theta})}} < \omega < c - \Omega'(Q^f)$

his rivals. That implies a network with positive externalities may serve as a powerful anti-competitive weapon. It allows an inefficient incumbent to successfully keep out an efficient rival. It is very difficult for newcomers with more advanced but incompatible technology to establish a market position. As a result, the incumbent firm may be reluctant to explore a superior but incompatible technology in the presence of network externalities. In this sense, our findings are consistent with the famous lock-in argument [13]: the difficulty of gaining a footing by a new, incompatible technology when a product is subject to network externalities. Despite this argument's popularity, ample historical evidence suggests that many new, incompatible technologies have been successfully introduced. For example, Microsoft Word was introduced after WordPerfect dominated the market; MS-DOS was introduced and subsequently cornered the market after CP/M-80 became established as the industry standard operating system. Cases of lock-in to inferior technologies are rare in the long history of technological change (Liebowitz

[13] See Farrell and Saloner (1985, 1986), Katz and Shapiro (1985), Farrell and Klemperer (2004), among many others, for detailed discussion.



(a) increasing externalities (b) constant externalities (c) decreasing externalities

FIGURE 15. $0 \leqslant \omega \leqslant \frac{c-\Omega'(Q^+)}{1+\frac{1}{\bar{\theta} f(\bar{\theta})}}$

Marginal Cost ω \ Curvature Properties	IE	CE	DE
$\left[\frac{c-\Omega'(Q^-)}{1-1/\underline{\theta} f(\underline{\theta})}, +\infty\right)$	OWD	OWD+NDT	TWD
$\left(c-\Omega'(Q^f), \frac{c-\Omega'(Q^-)}{1-1/\underline{\theta} f(\underline{\theta})}\right)$	OWD	OWD+NDT	TWD
$c-\Omega'(Q^f)$	ND	ND	ND
$\left(\frac{c-\Omega'(Q^+)}{1+1/\bar{\theta} f(\bar{\theta})}, c-\Omega'(Q^f)\right)$	OWU	OWU+NDB	TWD
$\left[0, \frac{c-\Omega'(Q^+)}{1+1/\bar{\theta} f(\bar{\theta})}\right]$	OWU	OWU+NDB	TWD

TABLE 1. IE: increasing externalities; CE: constant externalities; DE: decreasing externalities; OWD: one-way downward distortion; OWU: one-way upward distortion; NDT: no distortion at the top; ND: no distortion; NDB: no distortion at the bottom; TWD: two-way distortion.

and Margolis (1995)). These observations raise a question: under what circumstances and to what extent will the entry-deterring power of network externalities be limited? In the sequel, we are trying to answer these questions from the perspective of bounded rationality.

4.2 Boundedly rational agents

Analogous to the pure monopoly case described in Section 3, the optimal contract $\{q^*(\theta), U^*(\theta)\}$ depends heavily on the rationality of agents. If agents are boundedly rational, this contract may not necessarily be fulfilled. Given a common prior expectation Q_t^e, consumers belonging to $\Theta(Q_t^e) \equiv \left\{\theta \in [\underline{\theta}, \overline{\theta}] | U^*(\theta) \geqslant \Omega(Q^*) - \Omega(Q_t^e)\right\}$ will accept the contract proposed by the principal, then a network size of $Q_t = \rho(Q_t^e) \equiv \int_{\Theta(Q_t^e)} q^*(\theta)\, d\theta$ is actually realized. The feedback function $\rho(\cdot)$ is given as follows:

- If $\omega > c - \Omega'(Q^f)$, then

$$\rho(x) = \begin{cases} Q^* & \text{if} \quad \Omega(x) \in [\Omega(Q^*), +\infty) \\ \int_{\theta^*(x)}^{\overline{\theta}} q^*(\theta) f(\theta)\, d\theta & \text{if} \quad \Omega(x) \in \left[\max\{0, \Omega(Q^*) - \int_{\underline{\theta}}^{\overline{\theta}} [v(q^*(\theta)) - v(q^0(\theta))]\, d\theta\}, \Omega(Q^*)\right) \\ 0 & \text{if} \quad \Omega(x) \in \left[0, \max\{0, \Omega(Q^*) - \int_{\underline{\theta}}^{\overline{\theta}} [v(q^*(\theta)) - v(q^0(\theta))]\, d\theta\}\right) \end{cases},$$

  where $\theta^*(x)$ is given implicitly by $\int_{\underline{\theta}}^{\theta^*} [v(q^*(\theta)) - v(q^0(\theta))]\, d\theta + \Omega(x) = \Omega(Q^*)$.

- If $\omega = c - \Omega'(Q^f)$, then

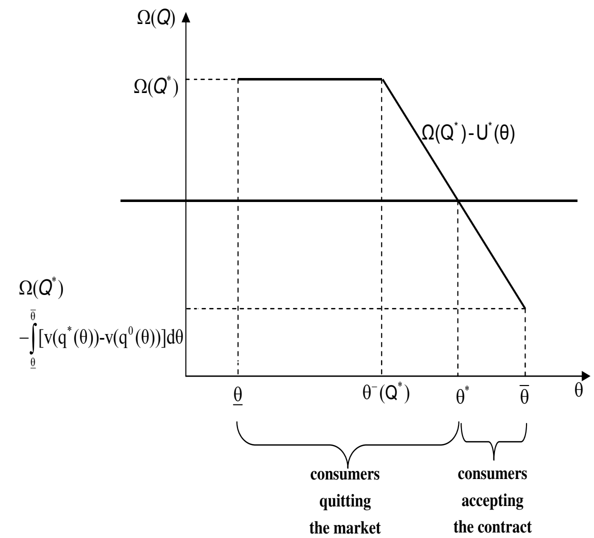
$$\rho(x) = \begin{cases} Q^* & \text{if} \quad \Omega(x) \in [\Omega(Q^*), +\infty) \\ 0 & \text{if} \quad \Omega(x) \in [0, \Omega(Q^*)) \end{cases},$$

- If $\omega < c - \Omega'(Q^f)$, then

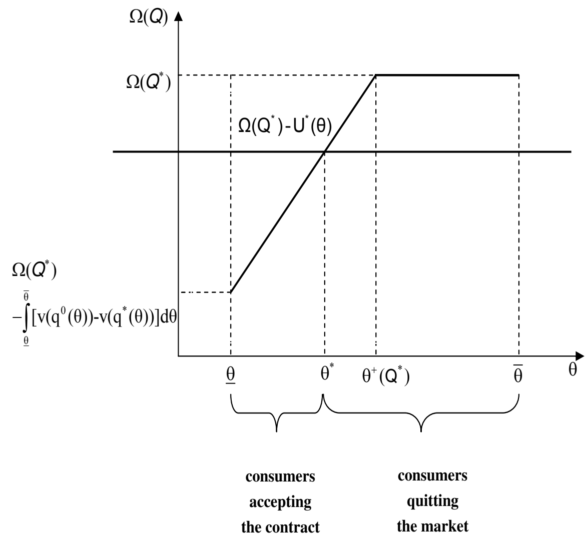
$$\rho(x) = \begin{cases} Q^* & \text{if} \quad \Omega(x) \in [\Omega(Q^*), +\infty) \\ \int_{\underline{\theta}}^{\theta^*(x)} q^*(\theta) f(\theta)\, d\theta & \text{if} \quad \Omega(x) \in \left[\max\{0, \Omega(Q^*) - \int_{\underline{\theta}}^{\overline{\theta}} [v(q^0(\theta)) - v(q^*(\theta))]\, d\theta\}, \Omega(Q^*)\right) \\ 0 & \text{if} \quad \Omega(x) \in \left[0, \max\{0, \Omega(Q^*) - \int_{\underline{\theta}}^{\overline{\theta}} [v(q^0(\theta)) - v(q^*(\theta))]\, d\theta\}\right) \end{cases},$$

  where $\theta^*(x)$ is given implicitly by $\int_{\theta^*}^{\overline{\theta}} [v(q^0(\theta)) - v(q^*(\theta))]\, d\theta + \Omega(x) = \Omega(Q^*)$.

As shown in the preceding section, the consumers then update their expectation according to $Q_{t+1}^e = (1 - \alpha) Q_t^e + \alpha Q_t$, and this adaptive learning process will not end until an equilibrium is reached or the market is totally shut down. We can easily obtain that if $\Omega'(Q^*) < 0$, then $\rho'_-(Q^*) = 0, \rho'_+(Q^*) = \Omega'(Q^*) q^*(\theta^*(Q^*)) f(\theta^*(Q^*)) / |v(q^*(\theta^*(Q^*))) - v(q^0(\theta^*(Q^*)))| < 0$; if $\Omega'(Q^*) = 0$, then $\rho'_+(Q^*) = \rho'_-(Q^*) = 0$; if $\Omega'(Q^*) > 0$, then $\rho'_+(Q^*) = 0, \rho'_-(Q^*) = \Omega'(Q^*) q^*(\theta^*(Q^*)) f(\theta^*(Q^*)) / |v(q^*(\theta^*(Q^*))) - v(q^0(\theta^*(Q^*)))| > 0$. Following the same logic as in the proof of Proposition 2, we find that Q^* is asymptotically stable from both sides when $\Omega'(Q^*) \leqslant 0$; when $\Omega'(Q^*) > 0$, however, it is more likely to be unstable from below. The following theorem summarizes the analysis above.



(a) $c - \Omega'(Q^f) \leqslant \omega \leqslant \frac{c-\Omega'(Q^-)}{1-1/\underline{\theta} f(\underline{\theta})}$



(b) $\frac{c-\Omega'(Q^+)}{1+1/\bar{\theta} f(\bar{\theta})} \leqslant \omega \leqslant c - \Omega'(Q^f)$

FIGURE 16.

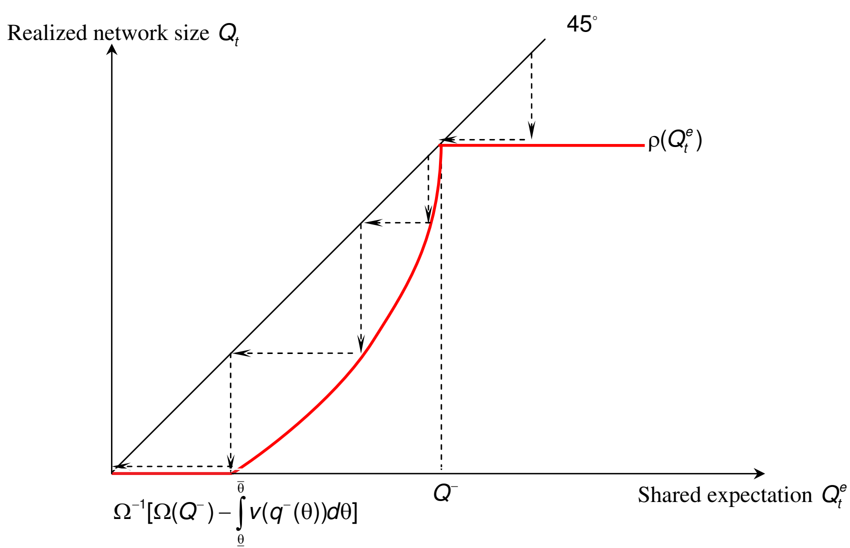


FIGURE 17.

THEOREM 3 *The stability of Q^* depends on $\Omega'(Q^*)$.*

- *If*

$$\Omega'(Q^*) > \frac{|v(q^*(\theta^*(Q^*))) - v(q^0(\theta^*(Q^*)))|}{f(\theta^*(Q^*))q^*(\theta^*(Q^*))}, \tag{17}$$

then Q^* is unstable from below and asymptotically stable from above;

- *if*

$$\Omega'(Q^*) < \frac{|v(q^*(\theta^*(Q^*))) - v(q^0(\theta^*(Q^*)))|}{f(\theta^*(Q^*))q^*(\theta^*(Q^*))}, \tag{18}$$

then Q^* is asymptotically stable from both sides.

If externalities are positive at the optimum, i.e., $\Omega'(Q^*) > 0$, and ω falls in the intermediate range, i.e., $[c-\Omega'(Q^+)]/[1+1/\overline{\theta}f(\overline{\theta})] \leqslant \omega \leqslant [c-\Omega'(Q^-)]/[1-1/\underline{\theta}f(\underline{\theta})]$, $q^0(\theta^*) = q^*(\theta^*)$[14], then it is obvious that (17) holds. The following corollary summarizes this result.

COROLLARY 1 *If $\Omega'(Q^*) > 0$ and $\omega \in \left[[c-\Omega'(Q^+)]/[1+1/\overline{\theta}f(\overline{\theta})], [c-\Omega'(Q^-)]/[1-1/\underline{\theta}f(\underline{\theta})]\right]$, then Q^* is asymptotically stable from above but unstable from below.*

With intermediate ω, a strictly positive measure of consumers earn zero rents. They are indifferent between accepting and rejecting the contract (see FIGURE 16). A slightly pessimistic initial expectation, i.e., $Q_0^e < Q^*$, will ignite positive network feedback, in which pessimism and quitting alternatively reinforce each other, so the market may eventually collapse. There is no way to reach point Q^* unless the initial expectation starts above or right at it (see FIGURE 17). This result suggests that the fulfilled expectation equilibrium is more vulnerable to pessimistic expectation than in the case without entry threat. In the presence of positive network externalities, the incumbent market is sure to be occupied gradually by rivals with medium cost. When a product is subject to network externalities, bounded rationality among users will tend to make entry easy and therefore will boost technological innovations. In this sense, our results also throw light on the transition between incompatible technology regimes and how the market system escapes from lock-in.

5 Conclusion

In this paper we show how an incumbent monopolist performs price discrimination among customers having different preferences and responses to potential entry threats. Network effects can distort consumption levels across customers away from the canonical principal-agent model. Curvature properties of network externalities are identified as factors responsible for the different distortionary patterns. We also investigate the stability of contract when consumers are boundedly rational. We find that the optimal equilibrium is always robust to optimistic expectation, but is vulnerable to pessimistic expectation for strong positive externalities. Facing entry

[14]If $c-\Omega'(Q^f) \leqslant \omega \leqslant [c-\Omega'(Q^-)]/[1-1/\underline{\theta}f(\underline{\theta})]$, then $\theta^* \in [\underline{\theta}, \theta^-(Q^*)]$; if $[c-\Omega'(Q^-)]/[1+1/\overline{\theta}f(\overline{\theta})] \leqslant \omega \leqslant c-\Omega'(Q^f)$, then $\theta^* \in [\theta^+(Q^*), \overline{\theta}]$. In both cases, $q^*(\theta^*) = q^0(\theta^*)$.

threat, the incumbent firm's nonlinear pricing contracts exhibit a complex pattern of distortions and are more likely to be unstable. Our results suggest that an incumbent may make use of his installed network/user base to impede potential entry, but bounded rationality of consumers imposes severe limitations on the entry-deterring ability of the incumbent firm.

APPENDIX

Proof of Lemma 1. For non-increasing externalities, i.e., $\Omega''(Q) \leqslant 0, \forall Q$, the result is obvious. We need only to prove it in the case with increasing externalities. Suppose that $\frac{\partial \varphi}{\partial Q} > 1$. The boundedness condition $\sup_{Q \in [0,+\infty)} \Omega'(Q) < c$ ensures that for any $\epsilon \in [0,1]$, $\varphi(0,\epsilon) > 0$ and $\varphi(+\infty,\epsilon) < +\infty$. So there exist at least another two fixed points $Q_1(\epsilon) \in (0, Q(\epsilon))$ and $Q_2(\epsilon) \in (Q(\epsilon), +\infty)$ such that $\varphi(Q,\epsilon) < Q$ for all $Q \in (Q_1(\epsilon), Q(\epsilon))$ and $\varphi(Q,\epsilon) > Q$ for all $Q \in (Q(\epsilon), Q_2(\epsilon))$, which implies that $d\Pi/dQ < 0$ for all $Q \in (Q_1(\epsilon), Q(\epsilon))$ and $d\Pi/dQ > 0$ for all $Q \in (Q(\epsilon), Q_2(\epsilon))$. This contradicts the fact that $Q(\epsilon)$ is optimal (see FIGURE 18).

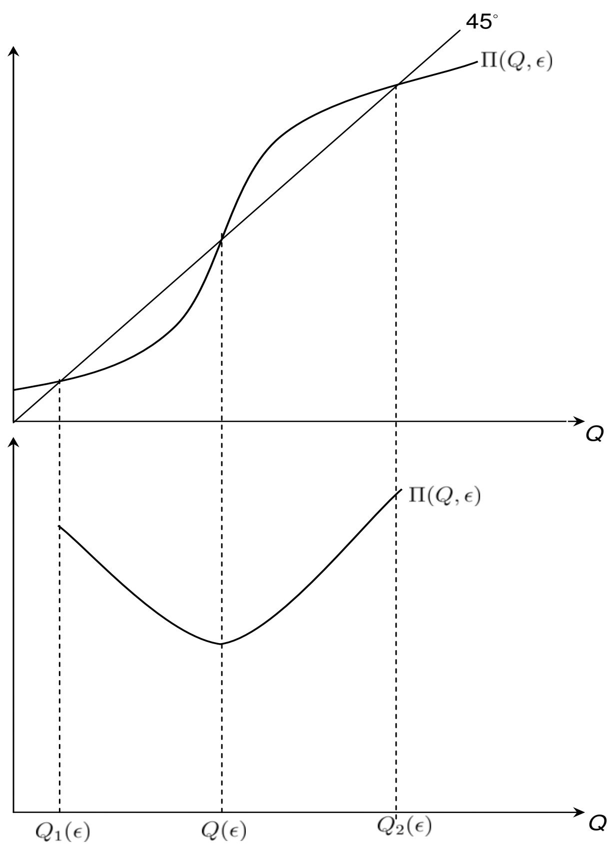


FIGURE 18.

Proof of Theorem 1. Lemma 2 shows that $Q^- < Q^f$.

- If $\Omega''(Q) > 0, \forall Q$, then function $q(\theta, Q, \epsilon)$ is increasing in Q and decreasing in ϵ. Therefore, $q^-(\theta) = q(\theta, Q^-, 1) < q(\theta, Q^f, 0) = q^f(\theta), \forall \theta \in [\underline{\theta}, \overline{\theta}]$.

- If $\Omega''(Q) < 0, \forall Q$, then $q(\overline{\theta}, Q, \epsilon)$ is decreasing in Q. We have $q^-(\overline{\theta}) = q(\overline{\theta}, Q^-, 1) = q(\overline{\theta}, Q^-, 0) > q(\overline{\theta}, Q^f, 0) = q^f(\overline{\theta})$. Again from $Q^- < Q^f$, we must have $q^-(\underline{\theta}) < q^f(\underline{\theta})$. Hence, there exists a type $\tilde{\theta} \in [\underline{\theta}, \overline{\theta}]$ such that $q^f(\tilde{\theta}) = q^-(\tilde{\theta})$. The final step is to show that such a $\tilde{\theta}$ is unique. Since $\tilde{\theta}$ is given by $\left[\theta - \frac{1-F(\theta)}{f(\theta)}\right] / \theta = \frac{c-\Omega'(Q^-)}{c-\Omega'(Q^f)}$ and the LHS is strictly decreasing in θ, the critical type $\tilde{\theta}$ is unique.

- For the case $\Omega''(Q) = 0, \forall Q$, the result is obvious.

Proof of Proposition 1.

- *Sufficiency*: Given $\sup_{Q \in [0,\infty)} \Omega'(Q) < c$, we have $\phi(0) > 0$ and $\phi(Q) < 0$ for sufficiently large Q, then $\phi(\cdot)$ has at least one zero point. Suppose that there exist two distinct points $Q_1 < Q_2$ such that $\phi(Q_1) = \phi(Q_2) = 0$. Then $\phi(\cdot)$ is obviously not strictly $\mathbb{R}$-concave. Therefore, $\varphi^-(Q)$ has a unique fixed point.

- *Necessity*: Suppose that $Q^* > 0$ is the unique zero point. If there exists a $\tilde{Q} \in (0, Q^*)$ such that $\phi(Q) < 0$, then, considering $\phi(0) > 0$, there must exist another point $Q^{**} \in (0, \tilde{Q})$ such that $\phi(Q^{**}) = 0$. It contradicts the assumption that Q^* is the unique zero point. Therefore, we have $\phi(\lambda Q^*) > 0$ for all $\lambda \in (0, 1)$.

Proof of Lemma 3. We only prove the cases from above, as the proof for the cases from below is analogous. Linearizing $f(x)$ at x around and larger than x^* with Taylor expansion:

$$f(x) = f(x^*) + f'_+(x^*)(x - x^*) + o(x - x^*).$$

Applying triangular inequality, we get

- *The case with* $0 \leqslant f'_+(x^*) < 1$. In this case, we can take $b \in [0, 1)$ to be any number larger than $f'_+(x^*)$, then for $x - x^*$ sufficiently small,

  $$f(x) - f(x^*) \leqslant b(x - x^*).$$

  So starting with x and iterating $x_{n+1} = f(x_n)$ gives a sequence of points with

  $$f^{(n)}(x) - x^* \leqslant b^n(x - x^*).$$

Hence, $\lim_{n\to\infty} f^n(x) = x^*$ whenever x is close enough to x^*; for every $\epsilon > 0$, there exists a $\delta(\epsilon) = \frac{\epsilon}{b^n}$ such that $f^{(n)} - x^* \leqslant \epsilon$ whenever $x - x^* \leqslant \delta(\epsilon)$ for all $n \in \mathbb{R}$. Subsequently, x^* is both Lyapunov stable and attractive, and it is therefore asymptotically stable.

- *The case with $f'_+(x^*) > 1$.* If $f'_+(x^*) > 1$, then for an arbitrary neighborhood $U(x^*)$, there exists $x \in U(x^*)$ such that

$$f(x) - x^* \geqslant b(x - x^*)$$

for some $b \in (1, f'_+(x^*))$. Running iteration of this inequality yields

$$f^{(n)}(x) - x^* \geqslant b^n(x - x^*).$$

Hence, $f^{(n)}(x) - x^*$ goes to infinity, and x^* is therefore unstable.

Proof of Proposition 2.

- If $\Omega'(Q^-) > v(q^-(\underline{\theta}))/f(\underline{\theta})q^-(\underline{\theta})$, then $\exists\delta > 0$ such that $\rho(Q) = Q^-$ for all $Q \in (Q^-, Q^-+\delta)$ and $\rho(Q) = \int_{\theta^*(Q)}^{\overline{\theta}} v(q^-(\theta))d\theta$ for all $Q \in (Q^- - \delta, Q^-)$. Therefore, $\tilde{\rho}'_+(Q^-) = \alpha\rho'_+(Q^-) + (1-\alpha) = 1-\alpha \in [0,1)$, $\tilde{\rho}'_-(Q^-) = \alpha\rho'_-(Q^-) + (1-\alpha) = \alpha\Omega'(Q^-)f(\underline{\theta})q^-(\underline{\theta})/v(q^-(\underline{\theta})) + (1-\alpha) \in (1, +\infty)$. It follows from Lemma 3 that Q^- is asymptotically stable from above and unstable from below.

- If $0 \leqslant \Omega'(Q^-) < v(q^-(\underline{\theta}))/f(\underline{\theta})q^-(\underline{\theta})$, we still have $\tilde{\rho}'_+(Q^-) = \alpha\rho'_+(Q^-) + (1-\alpha) \in [0,1)$, but now $\tilde{\rho}'_-(Q^-) = \alpha\rho'_-(Q^-) + (1-\alpha) = \alpha\Omega'(Q^-)f(\underline{\theta})q^-(\underline{\theta})/v(q^-(\underline{\theta})) + (1-\alpha) \in [0,1)$. It follows from Lemma 3 that Q^- is asymptotically stable from both sides.

- If $\Omega'(Q^-) < 0$, then $\exists\delta > 0$ such that $\rho(Q) = Q^-$ for all $Q \in (Q^- - \delta, Q^-)$ and $\rho(Q) = \int_{\theta^*(Q)}^{\overline{\theta}} v(q^-(\theta))d\theta$ for all $Q \in (Q^-, Q^- + \delta)$. We have $\tilde{\rho}'_-(Q^-) = 1-\alpha \in [0,1)$, $\tilde{\rho}'_+(Q^-) = \alpha\Omega'(Q^-)f(\underline{\theta})q^-(\underline{\theta})/v(q^-(\underline{\theta})) + (1-\alpha) < 1-\alpha$. If $\tilde{\rho}'_+(Q^-) \in [0, 1-\alpha)$, then the stability of Q^- follows directly from Lemma 3, and we only need to consider the case of $\tilde{\rho}'_+(Q^-) \in (-\infty, 0)$. Given $\tilde{\rho}'_-(Q^-) \in [0,1)$ and $\tilde{\rho}'_+(Q^-) < 0$, there exists a $\delta' > 0$ such that $Q^e_t \equiv \tilde{\rho}^{(t)}(Q^e_0)$ is increasing and converges to Q^- for all initial expectations $Q^e_0 \in (Q^- - \delta', Q^-)$. There also exists a $\delta'' > 0$ such that $\tilde{\rho}(Q^e_0) \in (Q^- - \delta', Q^-), \forall Q^e_0 \in (Q^-, Q^- + \delta'')$. Let $\delta = \min\{\delta', \delta''\}$, then we have all terms of $\tilde{\rho}^{(t)}(Q^e_0)$ fall in $(Q^- - \delta, Q^- + \delta)$ and converge to Q^- whenever $Q^e_0 \in (Q^- - \delta, Q^- + \delta)$. Therefore, Q^- is asymptotically stable.

Proof of Theorem 2. The uniqueness of fixed point Q^* is guaranteed by the boundedness of $\Omega'(Q)$ and strict $\mathbb{R}$-concavity of $\varphi^i(Q), \forall i \in \{-, f, +\}$. If $\Omega''(Q) = 0$ for all $Q \in [0, +\infty)$, then $[c - \Omega'(Q)]/\omega$ is constant, the optimal consumption $q^*(\theta)$ is obtained directly from 15, and from

(10) to (13) we obtain the optimal utilities $U^*(\theta)$. In the sequel, we will prove the results hold for the cases of strictly increasing and decreasing externalities. Let Q_1, Q_0, Q_2 be points defined as follows: $[c-\Omega'(Q_1)]/\omega = 1+1/\overline{\theta}f(\overline{\theta}), [c-\Omega'(Q_0)]/\omega = 1, [c-\Omega'(Q_2)]/\omega = 1-1/\underline{\theta}f(\underline{\theta})$. It is clear that $Q_1 < Q_0 < Q_2$ when $\Omega''(\cdot) > 0$ and $Q_1 > Q_0 > Q_2$ when $\Omega''(\cdot) < 0$.

- If $\omega \in \left[\frac{c-\Omega'(Q^-)}{1-1/\underline{\theta}f(\underline{\theta})}, +\infty\right)$, then the 45° line intersects $\varphi^*(Q)$ at point $(Q^-, \varphi^-(Q^-))$. We have $Q^* = Q^- \geqslant (\leqslant) Q_2$ whenever $\Omega'' > (<) 0$ (see FIGURE 19). Consequently, $[c-\Omega'(Q^*)]/\omega < 1-1/\underline{\theta}f(\underline{\theta})$ for either $\Omega''(Q) > 0$ or $\Omega''(Q) < 0$. From the analysis in main text (**CASE** 1), it is clear that $q^*(\theta) = q^-(\theta, Q^-) \equiv q^-(\theta)$ and $U^*(\theta) = \int_{\underline{\theta}}^{\theta} \left[v(q^-(\theta)) - v(q^0(\theta))\right] d\theta, \forall \theta \in [\underline{\theta}, \overline{\theta}]$.

- If $\omega \in \left(c-\Omega'(Q^f), \frac{c-\Omega'(Q^-)}{1-1/\underline{\theta}f(\underline{\theta})}\right)$, then the intersection of 45° line and $\varphi^*(Q)$ lies between $(Q_0, \varphi^f(Q_0))$ and $(Q_2, \varphi^-(Q_2))$, and $Q^* < Q^f$. If $\Omega''(Q) > 0, \forall Q \in [0, +\infty)$, we have $Q_0 < Q^* < Q_2$, therefore $1 = [c-\Omega'(Q_0)]/\omega > [c-\Omega'(Q^*)]/\omega > [c-\Omega'(Q_2)]/\omega = 1-1/\underline{\theta}f(\underline{\theta})$; if $\Omega''(Q) < 0, \forall Q \in [0, +\infty)$, then $Q_2 < Q^* < Q_0$, and we also have $1 = [c-\Omega'(Q_0)]/\omega > [c-\Omega'(Q^*)]/\omega > [c-\Omega'(Q_2)]/\omega = 1-1/\underline{\theta}f(\underline{\theta})$ (see FIGURE 20). It falls in **CASE** 2 of the main text analysis. Therefore, we have

$$q^*(\theta) = \begin{cases} q^0(\theta) & \text{if} \quad \theta \in [\underline{\theta}, \theta^-(Q^*)) \\ q^-(\theta, Q^*) & \text{if} \quad \theta \in [\theta^-(Q^*), \overline{\theta}] \end{cases},$$

$$U^*(\theta) = \begin{cases} 0 & \text{if} \quad \theta \in [\underline{\theta}, \theta^-(Q^*)) \\ \int_{\theta^-(Q^*)}^{\theta} \left[v(q^-(\theta, Q^*)) - v(q^0(\theta))\right] d\theta & \text{if} \quad \theta \in [\theta^-(Q^*), \overline{\theta}] \end{cases}.$$

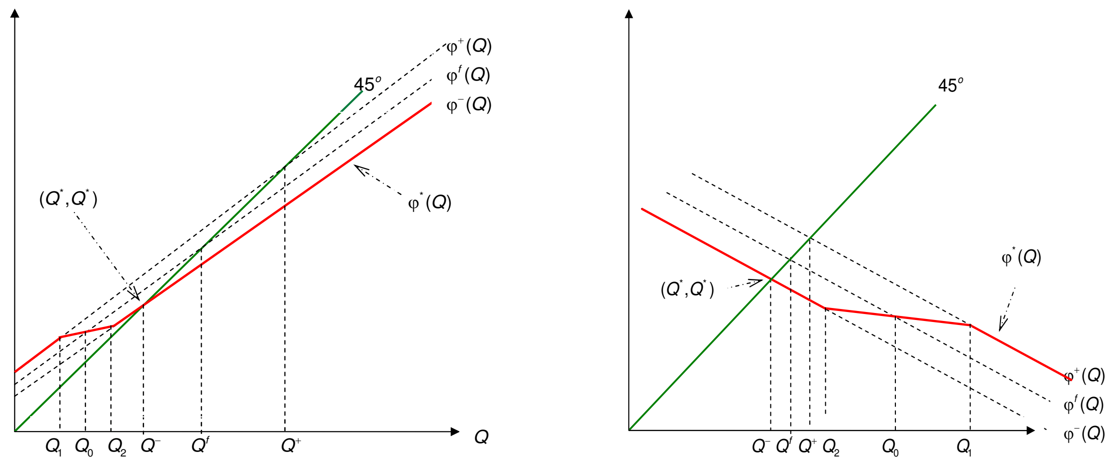
- If $\omega = c-\Omega'(Q^f)$, then the 45° line goes through the point $(Q_0, \varphi^f(Q_0))$ (see FIGURE 21). It is clear that $Q^* = Q_0 = Q^f$. As shown in **CASE** 3 of the preceding discussions, $q^*(\theta) = q^f(\theta, Q^f) \equiv q^f(\theta)$ and $U^*(\theta) \equiv 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

- If $\omega \in \left(\frac{c-\Omega'(Q^+)}{1+1/\overline{\theta}f(\overline{\theta})}, c-\Omega'(Q^f)\right)$, then $Q^* > Q^f$ and $Q^* \in (Q_1, Q_0)$ (resp. $Q^* \in (Q_0, Q_1)$) whenever $\Omega''(Q) > 0$ (resp. $\Omega''(Q) < 0$), $\forall Q \in [0, +\infty)$ (see FIGURE 22). We have $1 < [c-\Omega'(Q^*)]/\omega < 1+1/\overline{\theta}f(\overline{\theta})$ for either increasing or decreasing externalities. Our discussion of **CASE** 4 shows that

$$q^*(\theta) = \begin{cases} q^0(\theta) & \text{if} \quad \theta \in [\theta^+(Q^*), \overline{\theta}] \\ q^+(\theta, Q^*) & \text{if} \quad \theta \in [\underline{\theta}, \theta^+(Q^*)) \end{cases},$$

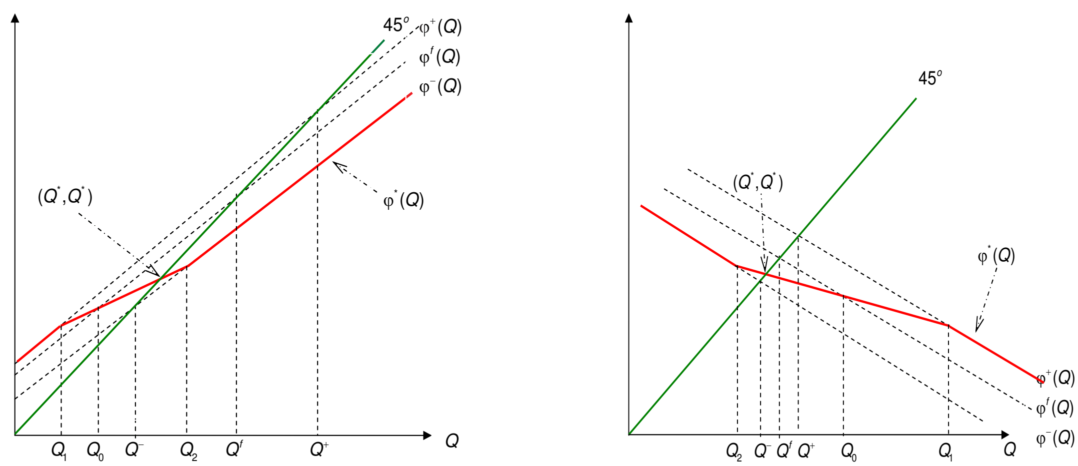
$$U^*(\theta) = \begin{cases} \int_{\theta^+(Q^*)}^{\theta} \left[v(q^+(\theta, Q^*)) - v(q^0(\theta))\right] d\theta & \text{if} \quad \theta \in [\underline{\theta}, \theta^+(Q^*)) \\ 0 & \text{if} \quad \theta \in [\theta^+(Q^*), \overline{\theta}] \end{cases}.$$

- If $\omega \in \left[0, \frac{c-\Omega'(Q^+)}{1+1/\overline{\theta}f(\overline{\theta})}\right]$, then the 45° line intersects $\varphi^*(Q)$ at point $(Q^+, \varphi^+(Q^+))$ (see FIGURE 23). It follows from the preceding discussion of **CASE** 5 that $q^*(\theta) = q^+(\theta)$, $U^*(\theta) = \int_{\underline{\theta}}^{\theta} \left[v(q^+(\theta)) - v(q^0(\theta))\right] d\theta, \forall \theta \in [\underline{\theta}, \overline{\theta}]$.



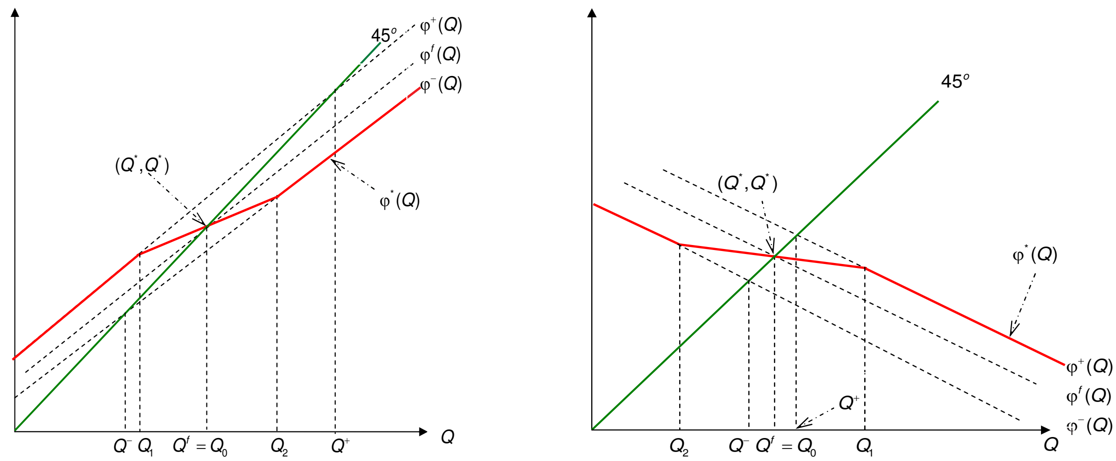
(a) increasing externalities (b) decreasing externalities

FIGURE 19. $\omega \geqslant \frac{c-\Omega'(Q^-)}{1-\frac{1}{\underline{\theta}f(\underline{\theta})}}$



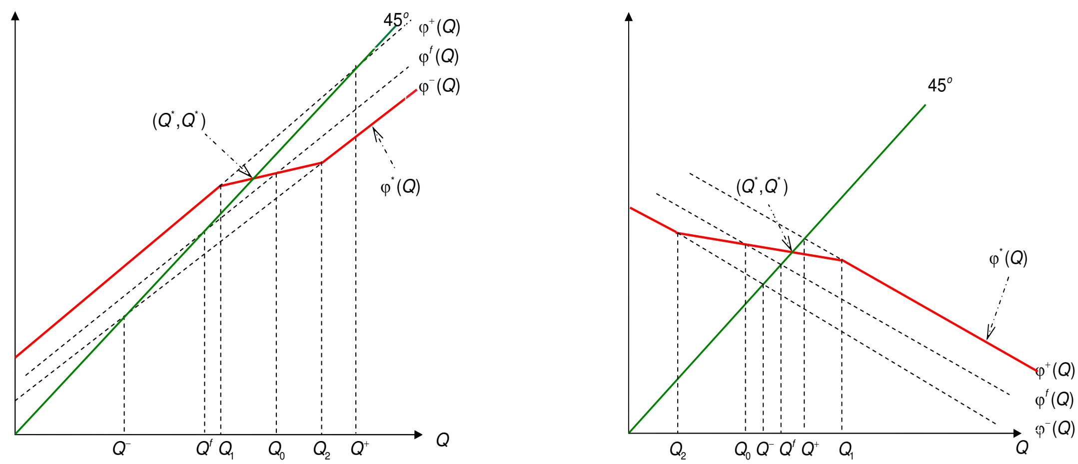
(a) increasing externalities (b) decreasing externalities

FIGURE 20. $c - \Omega'(Q^f) < \omega < \frac{c-\Omega'(Q^-)}{1-\frac{1}{\underline{\theta}f(\underline{\theta})}}$



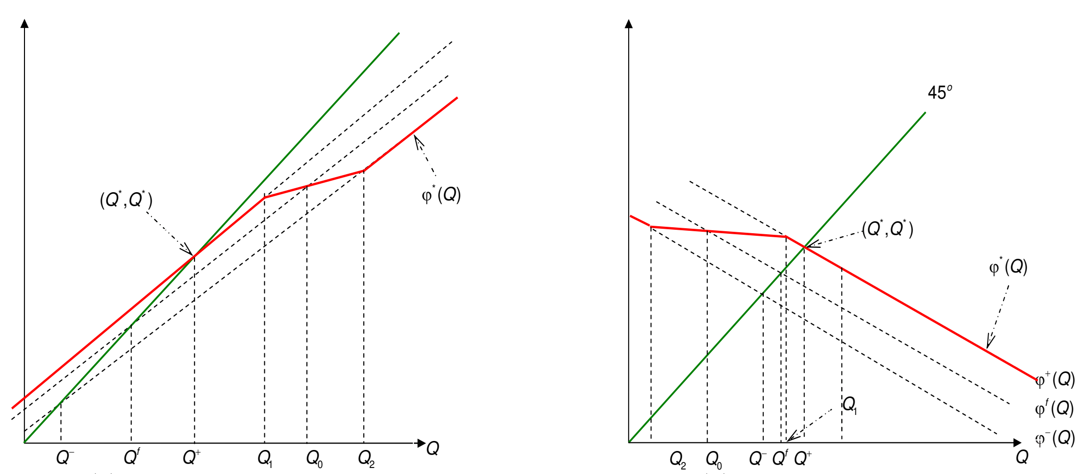
(a) increasing externalities (b) decreasing externalities

FIGURE 21. $\omega = c - \Omega'(Q^f)$



(a) increasing externalities (b) decreasing externalities

FIGURE 22. $\frac{c-\Omega'(Q^+)}{1+\frac{1}{\bar{\theta} f(\bar{\theta})}} < \omega < c - \Omega'(Q^f)$



(a) increasing externalities (b) decreasing externalities

FIGURE 23. $0 \leqslant \omega \leqslant \frac{c-\Omega'(Q^+)}{1+\frac{1}{\bar{\theta} f(\bar{\theta})}}$

References

[1] Aghion, P. and Bolton, P., (1987), "Contracts as a Barrier to Entry", *American Economic Review*, 77, 388-401

[2] Allen, B., Deneckere, R., Faith, T., and Kovenock, D., (2000), "Capacity precommitment as a barrier to entry: a Bertrand-Edgeworth approach", *Economic Theory*, 15, 501-530

[3] Bagwell, K. (2007), "Signalling and Entry Deterrence: A Multi-dimensional Analysis", *Rand Journal of Economics*, 38, 670-697

[4] Bagwell, K., and G. Ramey, (1988), "Advertising and limit pricing", *Rand Journal of Economics*, 19, 59-71

[5] Bagwell, K., and G. Ramey, (1990), "Advertising and pricing to deter or accommodate entry when demand is unknown", *International Journal of Industrial Organization*, 8, 93-113

[6] Bagwell, K., and G. Ramey, (1991), "Oligopoly limit pricing", *Rand Journal of Economics*, 22, 155-172

[7] Bain, Joe S., (1956), "Barriers to New Competition", Cambridge, Mass.: *Harvard Univ. Press*

[8] Baron, D., and R. Myerson, (1982), "Regulating a Monopolist with Unknown Costs", *Econometrica*, 50, 911-930.

[9] Bulow, J., Geanakoplos, J., and P. Klemperer, (1985), "Holding idle capacity to deter entry", *Economic Journal*, 95, 178-182.

[10] Cabrales, A., (1999), "Adaptive dynamics and the implementation problem with complete information", *Journal of Economic Theory*, 86, 159-184.

[11] Cabrales, A., and Serrano, R., (2011), "Implementation in adaptive better-response dynamics: Towards a general theory of bounded rationality in mechanisms", *Games and Economic Behavior*, 73, 360-374

[12] Carlton, D., and Waldman, M., (2002), "The Strategic Use of Tying to Preserve and Create Market Power in Industries", *The Rand Journal of Economics*, 33, 194-220

[13] Chade, H., and Schlee, E., (2012), "Optimal Insurance in Adverse Selection", *Theoretical Economics*, 7, 571-607.

[14] Chen, Y., (2005), "Dynamic stability of Nash-efficient public goods mechanisms: Reconciling theory and experiments". In: Zwick, R., Rapoport, A. (Eds.), *Experimental Business Research*. Vol. II. Kluwer Academic Publishers, Norwell, MA.

[15] Chen, Y., (2008), "Incentive compatible mechanisms for pure public goods: A survey of experimental research", In: Plott, C. R., Smith, V. (Eds.), *The Handbook of Experimental Economics Results*, Vol. 1. North Holland, Amsterdam.

[16] Chen, Y., and Gazzale, R., (2004), "When does learning in games generate convergence to Nash equilibria? The role of supermodularity in an experimental setting", *American Economic Review*, 95, 1505-1535.

[17] Choi, J., and Stefanadis, C., (2001), "Tying, Investment, and the Dynamic Leverage Theory", *Rand Journal of Economics*, 32, 52-71.

[18] Choi, J., and Stefanadis, C., (2006), "Bundling, Entry Deterrence, and Specialist Innovators", *The Journal of Business*, 79, 2575-2594

[19] Csorba, G. (2008), "Screening Contracts in The Presence of Positive Network Effects", *International Journal of Industrial Organization*, 26, 213-226

[20] De Trenqualye, P., (1989), "Stable implementation of Lindahl allocations", *Economics Letters*, 29, 291-294.

[21] Dixit, A., (1980), "The role of investment in entry-deterrence", *Economic Journal*, 90, 95-106.

[22] Espínola-Arredondo, A., Muñoz-Garcia, F., and Jude Bayham, (2014), "The Entry-Deterring Effects of Inflexible Regulation", *Canadian Journal of Economics*, 47, 298-324

[23] Farrell, J. and Saloner, G., (1985), "Standardization, Compatibility, and Innovation", *Rand Journal of Economics*, 16, 70-83.

[24] Farrell, J., Saloner, G., (1986), "Installed base and compatibility: innovation, product preannouncements, and predation", *American Economic Review*, 76, 940-955.

[25] Farrell, J., P. Klemperer. (2004), "Coordination and lock-in: Competition with switching costs and network effects", M. Armstrong, R. H. Porter, eds. *Handbook of Industrial Organization*, Vol. 3. North Holland, Amsterdam, The Netherlands.

[26] Fudenberg, D., J. Tirole, (1984), “The fat cat effect, the puppy dog ploy and the lean and hungry look”, *American Economic Review, Papers and Proceedings*, 74, 361-366

[27] Fudenberg, D. and Tirole, J., (2000), “Pricing a Network Good to Deter Entry”, *Journal of Industrial Economics*, 48, 373-390

[28] Hahn, J., (2003), “Nonlinear Pricing of Telecommunication with Call and Network Externalities”, *International Journal of Industrial Organization*, 21, 949-967

[29] Healy, P., (2006), “Learning dynamics for mechanism design: An experimental comparison of public goods mechanisms”, *Journal of Economic Theory*, 129, 114-149.

[30] Healy, P., and Mathevet, L., (2012), “Designing stable mechanisms for economic environments”, *Theoretical Economics*, 7, 609-661

[31] Hellwig, M., (2010), “Incentive problems with unidimensional hidden characteristics: a unified approach”, *Econometrica*, 78, 1201–1237

[32] Holmgren, R., (1991), “A First Course in Discrete Dynamical Systems”, *Springer-Verlag.*

[33] Hommes, C., (2006), “Heterogeneous agent models in economics and finance”, In: Tesfatsion, L. and Judd, K.L., (eds), *Handbook of Computational Economics*, Volume 2: Agent-Based Comp. Econ., chapter 23, North-Holland, 1109-1186.

[34] Hommes, C., and Wagener, F., (2009), “Complex Evolutionary Systems in Behavioral Finance”, In: T. Hens and K.R. Schenk-Hoppé (Eds.), *Handbook of Financial Markets: Dynamics and Evolution*, Elsevier, 217-276.

[35] Katz, M. L., and Shapiro, C., (1985), “Network externalities, competition, and compatibility”, *American Economic Review*, 75, 424-440.

[36] Kreps, D.M., Wilson, R., (1982), “Reputation and imperfect information”, *Journal of Economic Theory*, 27, 253-279.

[37] LeBaron, B. (2006), “Agent-based Computational Finance”, *Handbook of Computational Economics*, Volume 2: Agent-Based Computational Economics, chapter 24, L. Tesfatsion and K.L. Judd (Eds.), North-Holland, Amsterdam, 1187-1233.

[38] Liebowitz,S. J., Margolis, S., (1995), “Path dependence, lock-in, and history”, *Journal of Law, Economics and Organization*, 11, 205-226.

[39] Lewis, T., and Sappington, D., (1989), "Countervailing Incentives in Agency Problems", *Journal of Economic Theory*, 49, 294-313

[40] Linnemer, L., (1998), "Entry deterrence, product quality: price and advertising as signals", *Journal of Economics and Management Strategy*, 7, 615-646.

[41] Lockwood, B., (2000), "Production Externalities and Two-way Distortion in Principal-multiagent Problems", *Journal of Economic Theory*, 92, 144-166.

[42] Lucas, R., (1972), "Expectations and the Neutrality of Money", *Journal of Economic Theory*, 4, 103-124.

[43] Maggi, G., and Rodriguez-Clare., A., (1995), "On countervailing incentives", *Journal of Economic Theory*, 66, 238-263.

[44] Maskin, E. and Riley, J., (1984), "Monopoly with Incomplete Information", *Rand Journal of Economics*, 15, 171-196.

[45] Maskin, E., (1999), "Uncertainty and entry deterrence", *Economic Theory*, 14, 429-437

[46] Milgrom, P. and Roberts, J., (1982a), "Limit Pricing and Entry under Incomplete Information", *Econometrica*, 50, 443-460.

[47] Milgrom, P. and Roberts, J., (1982b), "Predation, Reputation, and Entry Deterrence", *Journal of Economic Theory*, 27, 280-312.

[48] Modigliani, F., (1958), "New Developments on the Oligopoly Front", *Journal of Political Economy*, 66, 215-232.

[49] Mussa, M. and Rosen, S. (1978), "Monopoly and Product Quality", *Journal of Economic Theory*, 18, 301-317.

[50] Muth, John F. (1961), "Rational Expectations and the Theory of Price Movements", *Econometrica*, 29, 315-335

[51] Myerson, R. (1981), "Optimal Auction Design", *Mathematics of Operations Research*, 6, 58-73.

[52] Nalebuff. B, (2004), "Bundling as an Entry Barrier", *The Quarterly Journal of Economics*, 119, 159-187

[53] Omori, T. and Yarrow, G. (1982), "Product Diversification, Entry Prevention and Limit Pricing", *Bell Journal of Economics*, 13, 242-248.

[54] Sandholm, W., (2002), "Evolutionary implementation and congestion pricing", *Review of Economic Studies*, 69, 667-689.

[55] Sandholm, W., (2005), "Negative externalities and evolutionary implementation", *Review of Economic Studies*, 72, 885-915.

[56] Schmalensee, R. (1978), "Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry", *Bell Journal of Economics*, 9, 305-327.

[57] Segal, I., (1999), "Contracting with Externalities", *Quarterly Journal of Economics*, 114, 337-388.

[58] Segal, I., and Whinston, M., (2000), "Exclusive contracts and protection of investments", *The Rand Journal of Economics*, 31, 603-633.

[59] Spence, A., (1977), "Entry, capacity, investment and oligopolistic pricing", *Bell Journal of Economics*, 8, 534-544.

[60] Sylos-Labini, P., (1962), "Oligopoly and Technical Papers", Cambridge: *Harvard University Press*

[61] Tarzijan, J., (2007), "Capital Structure and Entry Deterrence with Multiple Incumbents", *The B.E. Journal of Economic Analysis & Policy*, 7, 1-18

[62] Tirole, J., (1988), "The Theory of Industrial Organization", Cambridge, MA: *MIT Press*

[63] Topkis, D., (1998), "Supermodularity and Complementarity", *Princeton University Press*, Princeton, N.J.

[64] Walker, M., (1984), "A simple auctioneerless mechanism with walrasian properties", *Journal of Economic Theory*, 32, 111-127.

[65] Whinston, M., (1990), "Tying, foreclosure, and exclusion", *American Economic Review*, 80, 837-859