# War Size Distribution: Empirical Regularities Behind the Conflicts

Rafael, González-Val

Universidad de Zaragoza

14 August 2014

# War Size Distribution: Empirical Regularities Behind the Conflicts

Rafael González-Val

Universidad de Zaragoza & Instituto de Economía de Barcelona

***Abstract:*** This paper analyses the statistical distribution of war size. We find strong support for a Pareto-type distribution (power law) using data from different sources (COW and UCDP) and periods. A power law describes accurately the size distribution of all wars, but also the distribution of the sample of wars in any given period. The estimated Pareto exponent is always less than 1, indicating that the distribution is heavy-tailed; this means that the war average loss is controlled by the largest conflicts. Furthermore, the study of battle deaths' growth rates reveals a clear decreasing pattern; the growth of deaths declines faster the greater the number of initial deaths.

***Keywords:*** war size distribution, battle deaths, power law, Pareto distribution.

***JEL:*** D74, F51, N40,

## 1. Introduction

In one of the first analyses of the statistics of war, Richardson (1948) studied the variation of the frequency of fatal quarrels with magnitude. He collected a data set of violent incidents (wars and homicides) measured by the number of victims from 1820 to 1945 and his calculations revealed that the relationship between magnitude (size) and the frequency (number) of both wars and small crime incidents could be satisfactorily fitted by a straight decreasing line with negative slope, pointing to a power law function. This striking empirical regularity could have important implications, but it has remained almost unexplored from either a theoretical or empirical point of view for many years.

Only a few papers follow Richardson's approach (Roberts and Turcotte, 1998; Cederman, 2003; Clauset et al., 2007), finding evidence of power law behaviour too. Roberts and Turcotte (1998) find a power law dependence of number on intensity taking into consideration several alternative measures of the intensity of a war in terms of battle deaths, using Levy's (1983) data set of 119 wars from 1500 to 1974 and Small and Singer's (1982) data set of 118 wars during the period 1816–1980. Cederman (2003) finds strong support for a power law distribution using interstate war data from 1820 to 1997 from the Correlates of War project. Based on this empirical evidence, he also proposes an agent-based model of war and state formation that exhibits the same kind of power law regularities. Clauset et al. (2007) extend Richardson's analysis to study the frequency and severity of terrorist attacks worldwide since 1968, also finding a linear relationship between the frequency and severity of these deadly incidents.

The results of these studies are similar to the original result of Richardson. However, as Levy and Morgan (1984) point out, all these studies focus on the distribution of all wars rather than on those occurring in a given period, although the frequency of wars in a given period is also assumed to be inversely related to their seriousness. Levy and Morgan (1984) try to address this latter point by calculating Pearson correlation indexes between frequency and intensity, finding a negative correlation. They use Levy's (1983) data set of wars between 1500 and 1974, aggregating wars in 25-year periods.

Finally, there is also another strand of literature related. All the previous studies use between-conflict data, but there are other papers (Bohorquez et al., 2009; Johnson et al., 2011) that focus on within-conflict incidents (attacks). Surprisingly, these studies conclude that the size distribution or timing of within-conflict events is also power law distributed. Bohorquez et al. (2009) show that the sizes and timing of 54,679 violent events reported within nine diverse insurgent conflicts exhibit remarkable similarities. In all cases they cannot reject the hypothesis that the size distribution of the events follows a power law, but they can reject log-normality. They build on this empirical evidence to propose a unified theoretical model of human insurgency that reproduces these features, explaining conflict-specific variations quantitatively in terms of underlying rules of engagement. Johnson et al. (2011) uncover a similar dynamical pattern using data of fatal attacks by insurgent groups in both Afghanistan and Iraq, and by terrorist groups operating worldwide. They estimate the escalation rate and timing of fatal attacks, finding that the average number of fatalities per fatal attack is fairly constant in a conflict. Furthermore, when they calculate the progress curve they obtain a straight line, which is best fitted by a power law.

This paper contributes in several ways. First, in the spirit of Richardson (1948) we estimate the distribution of a pool of all wars. Second, using yearly data we estimate the war size distribution by year from 1989 to 2010 to study whether there are differences between the overall distribution of all wars and the year-by-year distribution. Third, we analyse the evolution of the distribution over time and its stability. Finally, we study the behaviour of the growth rates for the conflicts that last more than one period.

The paper is organised as follows. Section 2 introduces the databases we use. Section 3 contains the statistical analysis of war size distribution and its evolution over time and Section 4 concludes.

## 2. Data

We measure war size using the number of recorded battle deaths, i.e. the battle-related combatant fatalities. Data come from two international data sets: the Correlates of War (COW) (Version 4.0) (2010) project and the Uppsala Conflict Data Program (UCDP/PRIO) Armed Conflict Dataset (Version 5) (2011).

Like Cederman (2003), we only consider COW interstate wars. According to the COW war typology, an interstate war must involve sustained combat, involving organised armed forces, resulting in a minimum of 1,000 battle-related combatant fatalities within a 12-month period; for a state to be considered a war participant, the minimum requirement is that it has to either commit 1,000 troops to the war or suffer 100 battle-related deaths. This requisite condition was established by Small and Singer (1982). The COW data contain information of 95 different interstate wars from 1823 to 2003.[1] Thus, the COW data set covers all the conflicts within a long-term period and enables us to estimate the size distribution of a wide pool of modern wars.

The UCDP defines conflict as: "a contested incompatibility that concerns government and/or territory where the use of armed force between two parties, of which at least one is the government of a state, results in at least 25 battle-related deaths."[2] There are two important differences from the COW data. First, the UCDP data set includes four different types of conflict: extrasystemic, interstate, internal and internationalised internal. Second, the UCDP data set contains information about conflicts by year from 1989 to 2010. Thus, we can estimate the year-by-year size distribution.

Table 1 shows the sample sizes for each year and the descriptive statistics. There is a decrease in the number of wars over time, especially marked in the last few years (the average number of wars by year from 1989 to 2000 is 43.8, while in the 2001–2010 period it is 33.3). Moreover, the conflicts in the last few years are also less intense; the average number of battle deaths by war also decreases over time.

Roberts and Turcotte (1998) indicate that a pool of wars from different periods (like the COW data set) can be criticised because there is a substantial change in the global population over such a long time period. Therefore, the same number of battle deaths would not represent the same war intensity if there has been a huge change in the world population. Some authors try to correct for this using relative measures of size: Levy (1983) defines the intensity of a war as the quotient of battle deaths over the population of Europe in millions at the time of the war because estimates of the total world population may not be reliable for early periods. Here we also define a relative measure of size as the ratio of battle deaths to the European population (in thousands) the year prior to the start of the conflict. Population data are taken from Maddison (2003). Thus, this ratio represents the number of deaths per thousand European

[1] The list of interstate wars included in the database is in Sarkees and Wayman (2010).
[2] More information about the UCDP-PRIO Armed Conflict Dataset can be found in Gleditsch et al. (2002).

inhabitants. However, note that this normalisation is not necessary when all the conflicts are in the same period.

## 3. Results

### 3.1 War size distribution

Let us denote $S$ as the war size (measured by recorded battle deaths); if it is distributed following a power law, also known as Pareto distribution, the density function is $p(S) = \frac{a\underline{S}^a}{S^{a+1}}$ $\forall S \geq \underline{S}$ and the cumulative density function $P(S)$ is $P(S) = 1 - \left(\frac{\underline{S}}{S}\right)^a$ $\forall S \geq \underline{S}$, where $a > 0$ is the Pareto exponent and $\underline{S}$ is the battle deaths of the war at the truncation point. The relationship with the empirically observed rank $R$ (1 for the largest conflict, 2 for the second largest and so on) is $R = \underline{N} \cdot (1 - P(S)) = \underline{N} \cdot \left(\frac{\underline{S}}{S}\right)^a$, where $\underline{N}$ is the number of wars above the truncation point (minimum casualties threshold,[3] $\underline{S}$). Note that the rank includes the cumulative density function and, thus, can also represent the frequency.

Making $A = \underline{N} \cdot \underline{S}^a$ we obtain the simple expression $R = A \cdot S^{-a}$. This expression is applied to the study of very varied phenomena, such as the distribution of the number of times different words appear in a book (Zipf, 1949), the intensity of earthquakes (Kagan, 1997), the losses caused by floods (Pisarenko, 1998), forest fires (Roberts and Turcotte, 1998), city size distribution (Soo, 2005) or country size distribution (Rose, 2006).

Taking natural logarithms, we obtain the linear specification that is usually estimated

$$\ln R = \ln A - a \ln S + u\,, \tag{1}$$

where $u$ represents a standard random error ($E(u) = 0$ and $Var(u) = \sigma^2$) and $\ln A$ is a constant, $\ln A = \ln \underline{N} + a \ln \underline{S}$. The greater the coefficient $\hat{a}$, the more homogeneous are the war sizes. Similarly, a small coefficient (lower than 1) indicates a heavy-tailed distribution.

[3] The COW data set uses the relatively high threshold of 1,000 battle-deaths, while the UCDP dataset has a lower threshold, 25 annual battle-deaths (Gleditsch et al., 2002).

Gabaix and Ibragimov (2011) propose specifying Equation (1) by subtracting $1/2$ from the rank to obtain an unbiased estimation of $a$ in small samples:

$$\ln\left(R-\frac{1}{2}\right)=b-a\ln S+v\,. \qquad (2)$$

First, we replicate Cederman's (2003) results, considering only COW interstate wars from 1823 to 2003. Eq. (1) can be represented as a graph. Figure 1 shows the logarithmic plot for the COW data, covering interstate wars from 1823 to 2003. Data are fitted by a power law, and its exponent is estimated by using the Gabaix and Ibragimov (2011) estimator. For illustrative purposes a log-normal distribution is also fitted to the data by maximum likelihood. The power law provides a very good fit to the real behaviour of the distribution ($R^2=0.97$) with an estimated Pareto exponent of 0.464. In contrast, the fit by the log-normal distribution is very poor, especially at the upper tail. These results are robust to a change in the measure of size; by using the relative measure of size (battle deaths divided by European population) we obtain a similar good fit, with an estimated Pareto exponent of 0.459. This evidence confirms Cederman's (2003) results and the original result of Richardson (1948).

Remember that this is the distribution of a pool of all wars over a long period. Next, we use the yearly UCDP data set to estimate the war size distribution by year from 1989 to 2010. Eq. (2) was estimated by OLS for our yearly sample of wars; Figure 2 displays the results for two representative years (1992 and 2007).[4] Again, the power law provides a very good fit to the real behaviour of the distribution and the fit improves over time ($R^2$ increases from 0.87 in 1989 to 0.94 in 2010). However, the log-normal distribution provides a very poor fit, even worse than in the size distribution for all the 1823–2003 period.

Figure 3 shows the evolution over time of the estimated exponent. We estimated using all the observations available in each year. The results show that the distribution remained stable around the 0.5 value, although in the last ten years there has been a slight increase in the exponent. Thus, during this period the year-by-year distribution has been stable, with an estimated value similar to that of the size distribution of all wars. This exponent is always lower than 1, indicating that the distribution is heavy-tailed; this means that the war average loss is controlled by the largest conflicts.

[4] Results for all the years are available from the author upon request.

Therefore, a power law describes accurately the size distribution of all wars, but also the distribution of the sample of wars in any given period.

**3.2 Growth analysis**

The above results show what we consider to be a snapshot of the size distribution of wars from 1989 to 2010. For each year we obtained the estimated coefficients of the Pareto exponent, which enabled us to conclude that war size distribution is fairly stable over time. Literature studying financial assets (Gabaix et al., 2006), firm (Sutton, 1997) and city (Gabaix, 1999) size distributions usually concludes that this kind of Pareto-type distribution is generated by a random growth process. The hypothesis tested is that the growth of the variable is independent of its initial size.[5] To check whether this is true for war sizes we carry out a dynamic analysis of growth rates using two different non-parametric tools. The UCDP data set enables us to calculate the yearly growth rates of battle deaths for the conflicts that last more than one year. We define $g_i$ as the growth rate $\left(\ln S_{it} - \ln S_{it-1}\right)$ normalised (subtracting the contemporary mean and dividing by the standard deviation in the relevant year), with $S_{it}$ being the *i*th war's size (battle deaths).[6] We build a pool with all the growth rates between two consecutive years; there are 639 battle deaths-growth rate pairs in the period 1989–2010.

First, we study how the distribution of growth rates is related to the distribution of initial battle deaths (Ioannides and Overman, 2004). Figure 4 shows the stochastic kernel estimation of the distribution of normalised growth rates, conditional on the distribution of initial battle deaths at the same date. In order to make the interpretation easier, the contour plot is also shown. The plot reveals a slight negative relationship between both distributions, although there is a great deal of variance. However, most of the observations are concentrated into two peaks of density; the greater corresponds to conflicts with a small number of deaths (lower than 5 on the logarithmic scale, i.e. less than 150 casualties), and the second to the less numerous group of conflicts with a high number of battle deaths (7 on the logarithmic scale, which means around 1,100 casualties). Note that the conditional distribution of growth rates is equal to zero for

[5] In firm and city size literature this hypothesis is called "Gibrat's law".

[6] Growth rates need to be normalised because we are considering growth rates from different periods jointly in a pool.

both types of war, indicating that both distributions are independent for most of the observations.

To get a clearer view of the relationship between growth and initial battle deaths we also perform a non-parametric analysis using kernel regressions (Ioannides and Overman, 2003). It consists of taking the following specification:

$$g_i = m(s_i) + \varepsilon_i ,$$

where $g_i$ is the normalised growth rate and $s_i$ the logarithm of the *i*th war's number of initial battle deaths. Instead of making assumptions about the functional relationship $m$, $\hat{m}(s)$ is estimated as a local mean around the point $s$ and is smoothed using a kernel, which is a symmetrical, weighted and continuous function in $s$.

To estimate $\hat{m}(s)$, the Nadaraya-Watson method is used, as it appears in Härdle (1990, Chapter 3), based on the following expression:

$$\hat{m}(s) = \frac{n^{-1}\sum_{i=1}^{n} K_h(s - s_i) g_i}{n^{-1}\sum_{i=1}^{n} K_h(s - s_i)} ,$$

where $K_h$ denotes the dependence of the kernel $K$ (in this case an Epanechnikov) on the bandwidth $h$. We use the bandwidth $h = 0.5$.[7] As the growth rates are normalised, if growth was independent of the initial number of deaths the non-parametric estimate would be a straight line on the zero value, and values different from zero would involve deviations from the mean.

Results are shown in Figure 5. The graph also includes the bootstrapped 95% confidence bands (calculated from 500 random samples with replacement). The estimates confirm the negative relationship between size and growth observed in Figure 4, although we cannot reject the fact that growth is different from zero (random growth) for most of the distribution. Random growth would explain the observed war size distribution, because it implies a Pareto (power law) distribution if there is a lower bound to the distribution (which can be very low) (see Gabaix, 1999). Nevertheless, the decreasing pattern is clear: the greater the number of initial deaths, the lower the growth

[7] Results using Silverman's optimal kernel bandwidth were similar.

rate. We can interpret this result as evidence of the 'explosive' behaviour of conflicts, because the growth of deaths declines faster the greater the number of initial deaths.

## 4. Conclusions

Richardson's (1948) seminal study established a negative relationship between the frequency and severity of wars, introducing a new empirical regularity. The aim of this paper is to provide robust evidence on whether Richardson's claim holds.

First, we estimate the distribution of a pool of all wars using COW interstate war data from 1823 to 2003. Our estimates confirm Cederman's (2003) results and the original result of Richardson (1948); the power law provides a very good fit to the real behaviour of the distribution. Second, using UCDP yearly data we estimate the war size distribution by year from 1989 to 2010, finding that a power law describes accurately the size distribution of wars in any given period. Furthermore, during this period the year-by-year distribution has been stable, with an estimated value similar to that of the size distribution of all wars. If we add that some studies conclude that the size distribution or timing of within-conflict events is also power law distributed (Bohorquez et al., 2009; Johnson et al., 2011), all this evidence points to a universal pattern across and within war sizes.

Finally, the study of battle deaths' growth rates reveals that random growth cannot be rejected for most of the distribution, which could explain the resulting Pareto (power law) size distribution. Nevertheless, a clear decreasing pattern is also observed: the growth of deaths declines faster the greater the number of initial deaths.

## References

Bohorquez, J. C., S. Gourley, A. R. Dixon, M. Spagat, and N. F. Johnson. (2009). Common ecology quantifies human insurgency. Nature, 462: 911–914.

Cederman, L.-E. (2003). Modeling the size of wars: From billiard balls to sandpiles. The American Political Science Review, 97(1): 135–150.

Clauset, A., M. Young, and K. S. Gleditsch. (2007). On the frequency of severe terrorist events. The Journal of Conflict Resolution, 51(1): 58–87.

Correlates of War (Version 4.0), (2010). http://www.correlatesofwar.org/

Gabaix, X. (1999). Zipf's Law for cities: An explanation. Quarterly Journal of Economics, 114(3): 739–767.

Gabaix, X., P. Gopikrishnan, V. Plerou, and H. E. Stanley. (2006). Institutional investors and stock market volatility. The Quarterly Journal of Economics, 121(2): 461–504.

Gabaix, X., and R. Ibragimov. (2011). Rank-1/2: A simple way to improve the OLS estimation of tail exponents. Journal of Business & Economic Statistics, 29(1): 24–39.

Gabaix, X., and Y. M. Ioannides. (2004). The Evolution of City Size Distributions. Handbook of Urban and Regional Economics, Vol. 4, J. V. Henderson and J. F. Thisse, eds. Amsterdam: Elsevier Science, 2341–2378.

Gleditsch, N. P., P. Wallensteen, M. Eriksson, M. Sollenberg, and H. Strand. (2002). Armed conflict 1946–2002: A new dataset. Journal of Peace Research, 39(5): 615–637.

Härdle, W. (1990). Applied nonparametric regression. In Econometric Society Monographs. Cambridge, New York and Melbourne: Cambridge University Press.

Ioannides, Y. M., and H. G. Overman. (2003). Zipf's Law for cities: An empirical examination. Regional Science and Urban Economics, 33: 127–137.

Ioannides, Y. M., and H. G. Overman. (2004). Spatial evolution of the US urban system. Journal of Economic Geography, 4(2): 131–156.

Johnson, N., S. Carran, J. Botner, K. Fontaine, N. Laxague, P. Nuetzel, J. Turnley, and B. Tivnan. (2011). Pattern in escalations in insurgent and terrorist activity. Science, 333: 81–84.

Kagan, Y. Y. (1997). Earthquake size distribution and earthquake insurance. Communications in Statistics. Stochastic Models, 13(4): 775–797.

Levy, J. S. (1983). War in the Modern Great Power System, 1495–1975. Lexington: University Press of Kentucky.

Levy, J. S., and T. C. Morgan. (1984). The frequency and seriousness of war. Journal of Conflict Resolution, 28(4): 731–749.

Maddison, A. (2003). The World Economy: Historical Statistics. Paris: OECD.

Pisarenko, V. F. (1998). Non-linear growth of cumulative flood losses with time. Hydrological Processes, 12(3): 461–470.

Richardson, L. F. (1948). Variation of the frequency of fatal quarrels with magnitude. Journal of the American Statistical Association, 43(244): 523–546.

Roberts, D. C., and D. L. Turcotte. (1998). Fractality and self-organized criticality of wars. Fractals, 6(4): 351–357.

Rose, A. K. (2006). Cities and countries. Journal of Money, Credit and Banking, 38(8): 2225–2245.

Sarkees, M. R., and F. W. Wayman. (2010). Resort to War: A Data Guide to Inter-State, Extra-State, Intra-state, and Non-State Wars, 1816–2007. Washington, DC: CQ Press.

Small, M., and J. D. Singer. (1982). Resort to Arms. Beverly Hills: Saga Publications.

Soo, K. T. (2005). Zipf's Law for cities: A cross-country investigation. Regional Science and Urban Economics, 35: 239–263.

Sutton, J. (1997). Gibrat's legacy. Journal of Economic Literature, 35(1): 40–59.

Uppsala Conflict Data Program (UCDP/PRIO) Armed Conflict Dataset (Version 5), (2011). http://www.pcr.uu.se/research/ucdp/datasets/

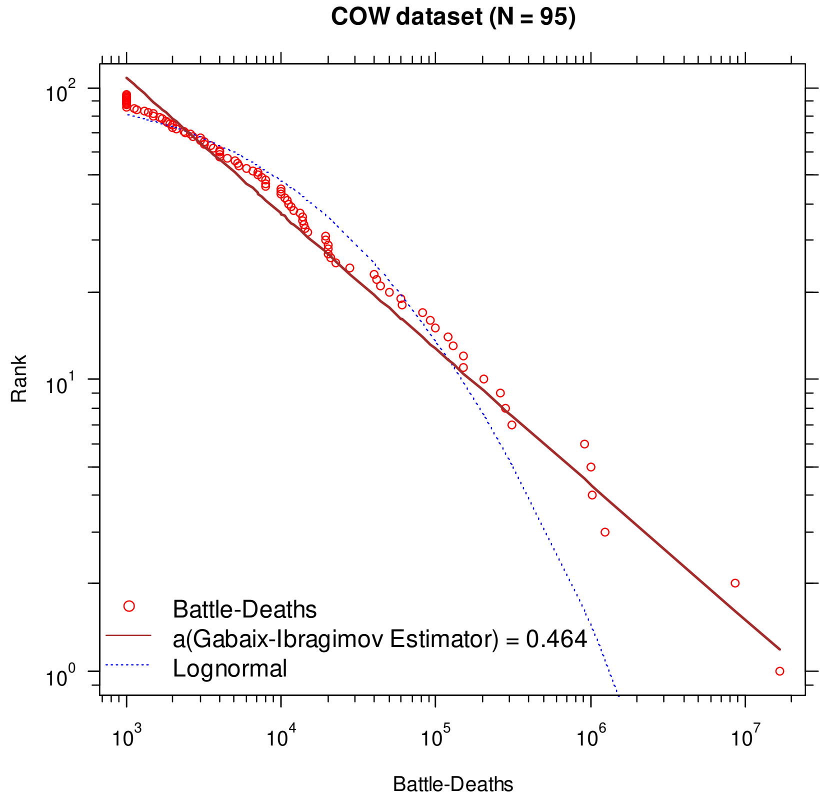
Zipf, G. (1949). Human Behaviour and the Principle of Least Effort. Cambridge, MA: Addison-Wesley.

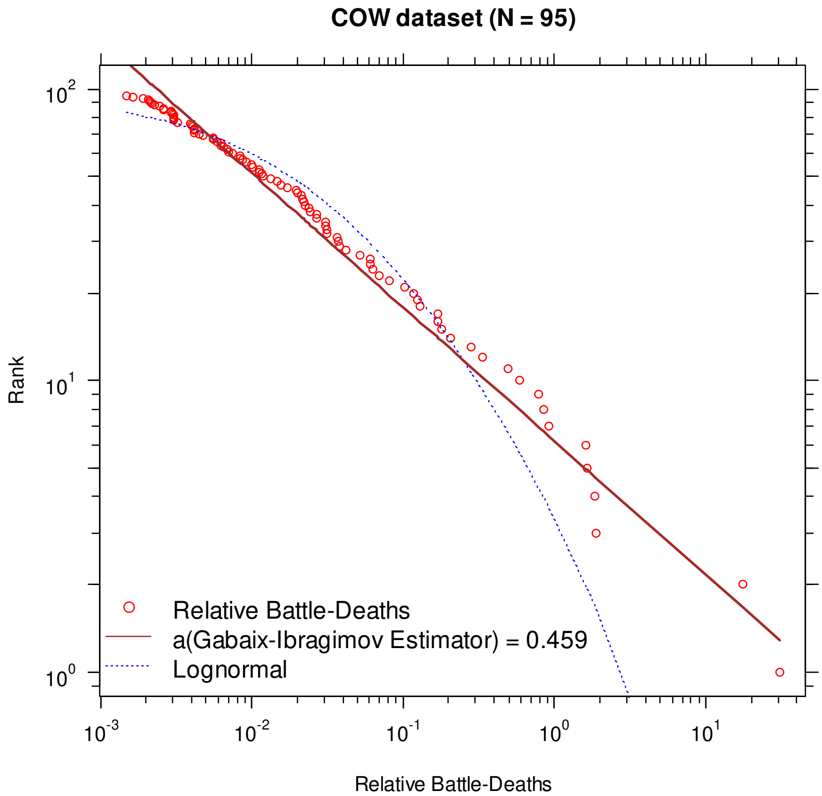
**Table 1. Armed conflict battle deaths: descriptive statistics by year**

| Year | Observations | Mean Size | Standard Deviation | Minimum | Maximum | Max. Location |
|---|---|---|---|---|---|---|
| 1989 | 43 | 1,256.651 | 3,023.588 | 25 | 18,403 | Ethiopia |
| 1990 | 50 | 1,631.6 | 5,057.416 | 25 | 30,633 | Ethiopia |
| 1991 | 51 | 1,372.471 | 3,436.919 | 25 | 21,790 | Iraq, Kuwait |
| 1992 | 53 | 676.2453 | 1,142.743 | 25 | 4,989 | Bosnia-Herzegovina |
| 1993 | 45 | 852.6889 | 1,955.79 | 25 | 12,054 | Angola |
| 1994 | 47 | 727.0213 | 1,505.68 | 25 | 8,829 | Afghanistan |
| 1995 | 41 | 698.7318 | 1,249.098 | 25 | 5,061 | Afghanistan |
| 1996 | 41 | 591.0732 | 955.7285 | 25 | 3,533 | Turkey |
| 1997 | 39 | 927.3075 | 1,948.249 | 25 | 10,033 | Congo |
| 1998 | 40 | 881.8 | 1,297.505 | 25 | 4,891 | Sudan |
| 1999 | 39 | 2,035.283 | 7,521.621 | 25 | 47,192 | Eritrea, Ethiopia |
| 2000 | 37 | 2,016.649 | 8,161.813 | 25 | 50,000 | Eritrea, Ethiopia |
| 2001 | 36 | 603.6111 | 800.9718 | 25 | 3,407 | Sudan |
| 2002 | 32 | 551.4063 | 787.1231 | 25 | 3,947 | Nepal |
| 2003 | 30 | 697.5001 | 1,512.132 | 25 | 8,202 | Australia, Iraq, United Kingdom, United States of America |
| 2004 | 32 | 566.6875 | 891.6652 | 25 | 3,499 | Iraq |
| 2005 | 32 | 358.0313 | 533.4645 | 25 | 2,364 | Iraq |
| 2006 | 33 | 527.2122 | 853.8419 | 25 | 3,656 | Iraq |
| 2007 | 35 | 487.7714 | 1,049.312 | 25 | 5,828 | Afghanistan |
| 2008 | 37 | 738.6217 | 1,586.588 | 25 | 8,413 | Sri Lanka |
| 2009 | 36 | 858.3056 | 1,805.982 | 25 | 8,162 | Sri Lanka |
| 2010 | 30 | 640.6 | 1,425.88 | 25 | 6,374 | Afghanistan |

Source: UCDP Battle-related deaths dataset v5 (2011), available at:
www.pcr.uu.se/research/ucdp/datasets/

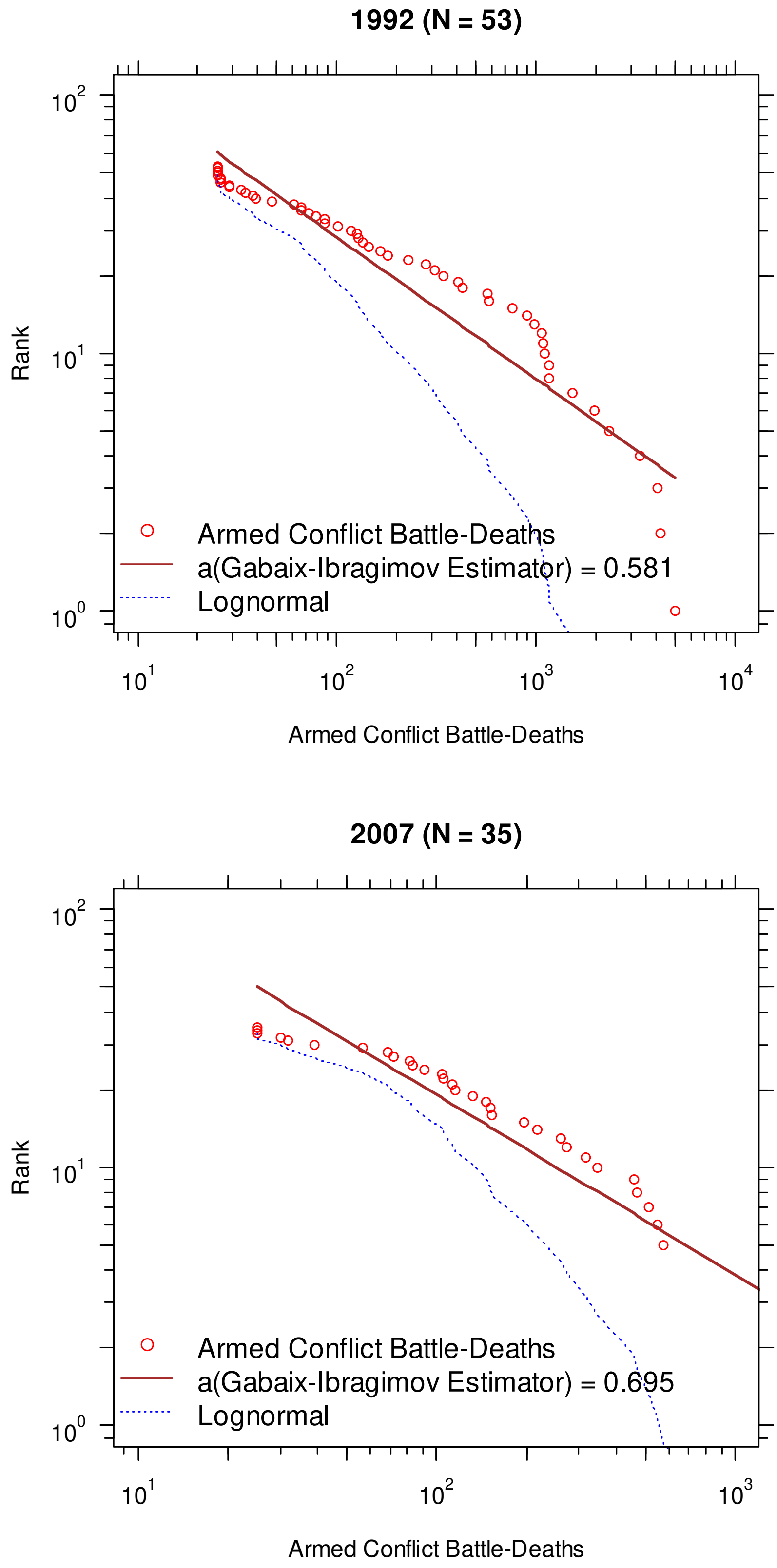
**Figure 1. Log-log plot, interstate wars from 1823 to 2003**




Note: COW InterState War Data (v4.0). The slope of the line is fitted by OLS using the Gabaix-Ibragimov (2011) specification. $R^2 = 0.97$ in both cases.

**Figure 2. Log-log plots in 1992 and 2007**

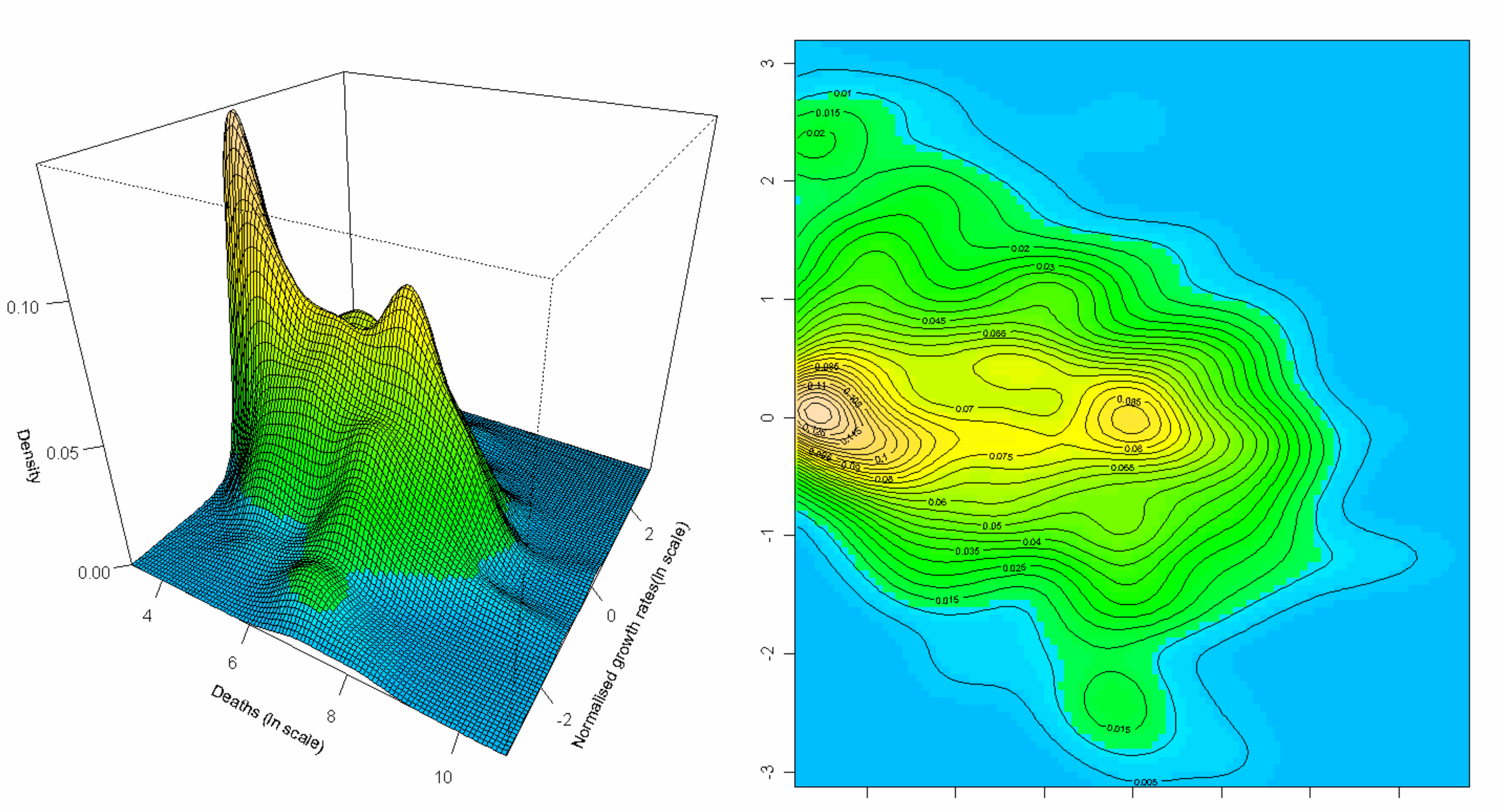



Note: UCDP Battle-related deaths dataset v5 (2011). The slopes of the lines are fitted by OLS using the Gabaix-Ibragimov (2011) specification. $R^2 = 0.85$ in 1992 and $R^2 = 0.92$ in 2007.

**Figure 3. Evolution of the estimated Pareto exponents**

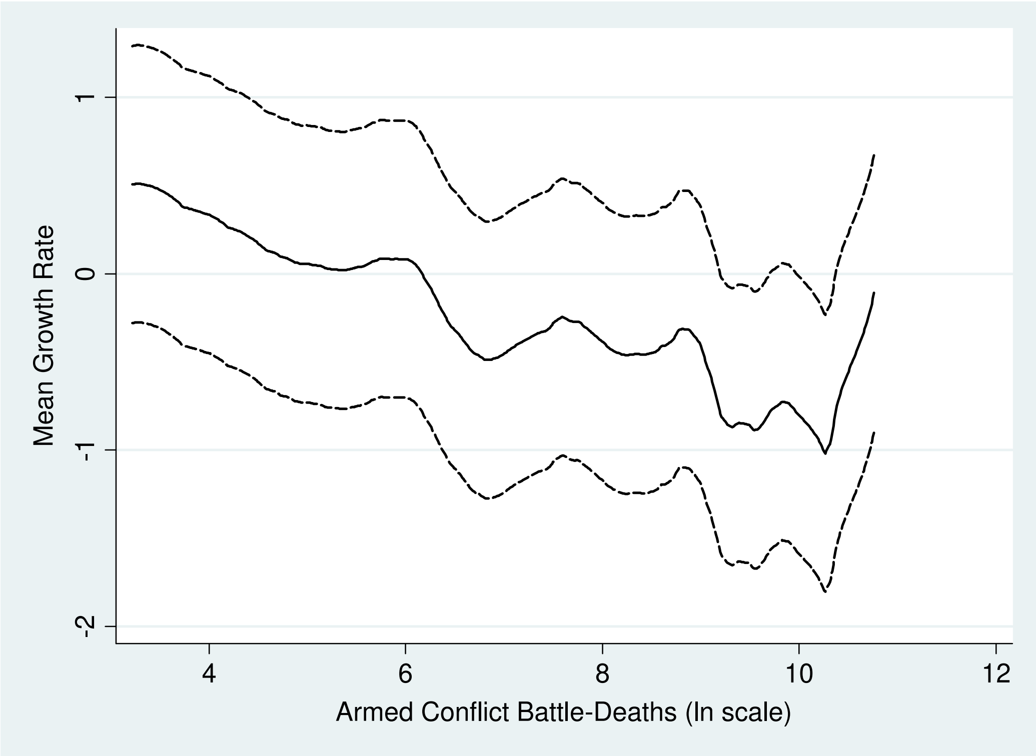

Notes: UCDP Battle-related deaths dataset v5 (2011). The Pareto exponent is estimated using Gabaix and Ibragimov's Rank-$1/2$ estimator. Dashed lines represent the standard errors calculated applying Gabaix and Ioannides's (2004) corrected standard errors: $\text{GI s.e.} = \hat{a} \cdot (2/N)^{1/2}$, where $N$ is the sample size.

**Figure 4. Stochastic kernel, battle deaths to growth rates**




Note: UCDP Battle-related deaths dataset v5 (2011), 639 observations.

**Figure 5. Kernel estimate of growth (bandwidth 0.5), 639 observations**




Note: UCDP Battle-related deaths data set v5 (2011).