Option Pricing in an Oligopolistic Setting

Villena, Marcelo and Villena, Mauricio

Adolfo Ibáñez University

1 March 2011

Option Pricing in an Oligopolistic Setting

Marcelo J. Villena* and Mauricio G. Villena†

August 16, 2014

Abstract

Option valuation models are usually based on frictionless markets. This paper extends and complements the literature by developing a model of option pricing in which the derivative and/or the underlying asset have an oligopolistic market structure, which produces an expected return on these assets that exceeds (or goes below) their fundamental value, and hence affects the option valuation. Our formulation begins modeling a capital asset pricing model that takes into account an oligopolistic setting, and hence the standard option pricing formula is derived, but this time considering the level of market power into the model. Our results show that higher levels of market power will lower the required expected return, in comparison to the perfectly competitive CAPM model. Similarly, simulations show that higher levels of market power in the derivative markets tend to increase the call option values in comparison to those values given by the standard Black and Scholes formulation, while the impact of market power in the underlying asset market tends to lower the option price.

JEL Classification: G13, D43

Keywords: Capital Asset Pricing, Option Pricing, Oligopolistic Markets.

*Faculty of Engineering and Sciences, Adolfo Ibáñez University, Av. Diagonal las Torres #2640 Peñalolén, Santiago, Chile. Corresponding author: marcelo.villena@uai.cl

†School of Business, Adolfo Ibáñez University, Chile.

1 Introduction

[4] and [14] in their seminal works showed that the construction of a risk-less hedge between the option and its underlying asset, allows the derivation of an option pricing formula regardless of investors risk preferences. In these derivations, it is assumed that taxes and transaction costs are zero, and that the market works perfectly, or in other words, that the capital asset pricing applies at each instant of time. The main advantages of the Black–Scholes model are that their formula is a function of "observable" variables and that the model can be extended to the pricing of any type of option. Thus, the Black and Scholes model has been extended since the seventies by several authors to account for other observable variables and to tackle several financial issues[1]. Nevertheless, all this literature has been constructed assuming frictionless markets and the absence of market power.

[9] was one of the first authors to point out that the impact of market imperfections on option pricing could be large and that could be even larger than many researchers had realized. He found that, in actual markets such as the stock index options, the standard arbitrage is exposed to such large risk and transactions costs that only very wide bounds on equilibrium options prices can be established. This evidence has important implications for price determination in options markets, as well as for the testing of valuation models. Consequently, it is important to take into account that option arbitrage could account for some kind of market imperfections.

Much more recently, [5] reported widespread violations of stochastic dominance by 1-month S&P 500 index call options over the period 1986–2006. They pointed out that a trader can improve expected utility by engaging in a zero net-cost trading net of transaction costs and bid-ask spread. They also found that although pre-crash option prices conform to the Black-Scholes model reasonably well, they are incorrectly priced if the distribution of the index return is estimated from time-series data.

Additionally to the academic works discussed above, even though today most financial markets are characterized by high levels of competition (with players relatively well-financed and well-informed), the behavior of some financial instruments suggests the existence of different kinds of market imperfections. Indeed, adverse selection, iliquidity, transaction costs, and the level of market competition are all problems that have been discussed in the academic literature, especially in light of the recent financial crises. Market power issues, such as oligopolistic competition in financial markets, are much less common in the academic literature. Despite this, in practice, some financial markets present important levels of concentration. For instance, derivatives activity in the U.S. banking system is dominated by a small group of large financial institutions. The OCC's quarterly report on trading revenues and bank derivatives activities [2], in his Fourth Quarter 2012 issue, points out that the top 4 banks in the US (i.e. JP Morgan Chase Bank, Bank of America, Citibank, and Goldman Sachs Bank) account for a very important part of the derivative risk in the financial system. Specifically, as shown in Table 1, of the $223 trillion in gross notional amount of derivative contracts, just 4 banks account for 93.2% of all derivative contracts.

[1] See [6], [10], [17] and [3] for some complete reviews of these extensions.

[2] This report is based on Call Report information provided by all insured U.S. commercial banks and trust companies, reports filed by U.S. financial holding companies, and other published data.

Table 1. Concentration of Derivative Contracts - 4th Quarter 2012 ($ in Billions)

Derivative Contracts	Top 4 Bks	Top Derivs	All Other Bks	Tot Derivs	All Bks	Tot Derivs
Futures & Fwds	39,006	17.5%	4,437	2.0%	43,443	19.5%
Swaps	126,773	56.8%	8,165	3.7%	134,938	60.5%
Options	29,669	13.3%	1,884	0.8%	31,553	14.1%
Credit Derivatives	12,605	5.6%	585	0.3%	13,190	5.9%
Total	**208,053**	**93.2%**	**15,071**	**6.8%**	**223,124**	**100%**

Source: OCC's quarterly report on trading revenues and bank derivatives activities

One of the first symptoms of market failures in financial markets is the miss-pricing regarding the risk-based valuation. Considering homogeneous information and perfect competition the capital assets pricing models will usually apply, however, in the presence of market imperfection capital assets pricing models will eventually produce an expected return on these assets that may exceed (go below) their fundamental value. This abnormal return can be econometrically estimated taking into account the deviations from the capital asset pricing model (CAPM) as presented by [16], [13] and [15] or any other multi-factor asset pricing model, e.g.[8].

For instance, [2] studied the relationship between risk and returns under imperfect competition. They argue that when the model of expected returns is based on imperfect competition, the level of market competition and the degree of adverse selection affects asset prices in conjunction with market risk. In their empirical model, they use Fama and French three-factors model on a database of firms sort into five quintiles, such that firms in the fifth quartile have the most shareholders and therefore the greatest degree of market competition. Each portfolio contained an average of 949 firm months over period 1976 to 2006. They found a modest effect on the level of market competition on expected returns, and also find that expected returns were increasing in the degree of adverse selection when there is a relatively low degree of market competition. Collectively, their results suggest that while it may be appropriate to characterize capital markets as a whole by standard asset-pricing models based on price-taking behavior, there exist sub-markets within the wider capital market that are better characterized by imperfect competition. In a similar paper, [12] analyze the role of information in pricing and cost of capital in securities markets characterized by imperfect competition among investors. They found that imperfect competition results in markets being less than perfectly liquid. Their analysis shows that the interaction between illiquid markets and asymmetric information gives rise to a role for information in estimating the cost of capital that is absent in perfect competition settings such as the CAPM. In terms of liquidity risk, [1] develop a simple liquidity adjusted CAPM, where a security's required return depends on its expected liquidity as well as on the covariances of its own return and liquidity with the market return and liquidity. The model provides a framework for understanding the various channels through which liquidity risk may affect asset prices. Their empirical results shed light on the total and relative economic significance of these channels and provide evidence of flight to liquidity. Finally, another related application associating CAPM with option pricing in a context of incomplete markets is provided by[11]. Specifically, they consider the case of an option on an arbitrary asset in an incomplete lognormal market and analyse the situation in which the planning horizon of the individual investor is shorter than the time-to-maturity of the option. Their derived pricing equations depend explicitly on the planning horizon of the individual investor.

This paper extends and complements the depicted stream of literature by developing a model of option pricing in which the derivative and/or the underlying asset have an oligopolistic market structure, which produces an expected return on these assets that exceeds (or goes below) their fundamental value, and hence affects the option valuation. Our formulation begins modeling a CAPM that takes into account an oligopolistic setting, and thus the standard option pricing formula is derived, but this time considering the level of market power into the model. This ensures that the results are easy to use in practice. In particular, financial analysts could find useful and benefit from an option pricing model that admits some type of disequilibrium, in terms for example of abnormal returns in the derivative and underlying assets markets, specially considering market power as a consequence of an oligopolistic market structure.

This paper is structured as follows. Firstly, the basic model is presented. Secondly, some numerical illustrations are developed in order to quantify the potential impact of market imperfections in the option valuation model. Finally, some conclusions and further research are outlined.

2 The Model Formulation

In this section, an oligopolistic option pricing model is developed, using a CAPM that captures this kind of market structure. First, an oligopolistic CAPM is put forward, and second the implications of such formulation for the derivative and/or the underlying asset markets are taken into account in the development of our oligopolistic option pricing model.

2.1 Capital Asset Pricing Model in an Oligopolistic Setting

Let us assume a market composed by m investors and n assets. Each investor will determine his/her portfolio. The portfolio return for the investor i is defined as:

$$r_p^i = \sum_j^n \left(x_j^i r_j\right) \tag{1}$$

where x_j^i is the fraction of the wealth of the investor i, invested in asset j; and $\bar{r_j}$ is the expected value of the asset return. The portfolio variance is defined as:

$$var(r_p^i) = \sum_j^n \sum_k^n \left(x_j^i x_k^i \sigma_{jk}\right) \tag{2}$$

being $\sigma_{jk} = \mathrm{cov}(r_j, r_k)$.

Traditionally, each investor wants to minimize the variance of the portfolio return subject to a specified expected return for its portfolio $\bar{R}^i$, and at the same time considering his/her budget constraint, in this case $\sum_j^n x_j^i = 1$. It is well known that one of the results of this problem is the standard form of the CAPM.

Now, let us assume an oligopolistic structure in the financial market of asset j, in which there are only a few participants that demand the asset, maintaining different levels of market power. It is assumed that investor i has the ability to individually affect the prevailing market price by modifying the demanded quantity of asset j in the market. Specifically, we will assume that there is a positive relationship between the volume of the demanded asset in the market and its price, in other words, an inverse demand function, of the form:

$$P(X_j) = \bar{P_j^1} + \theta_j X_j \tag{3}$$

where, $P(X_j)$ is the market price of asset j, $\bar{P_j^1}$ is the expected price of asset j in the next period assuming a perfectly competitive setting (reflecting exactly its true fundamental value), θ_j is a measure of the market power, or how the price is affected by the volume of assets demanded, and X_j represents the demanded quantity of asset j in the market. In this very simple demand equation, it is clear that a positive and greater θ_j implies e a more elastic market. Moreover, given the positive slope of the demand curve, more elastic here means that when the price of the asset goes up, the demand for the asset will be greater than in the case of a more inelastic asset market.

X_j is equivalent to the total amount of money invested in asset j by all investors, divided by the the current price of the asset: $\frac{\sum_i^m \left(x_j^i w^i\right)}{P_j^0}$, where w^i represent the initial wealth of investor i. Hence, the demand function becomes:

$$P(X_j) = \bar{P_j^1} + \theta_j \frac{\sum_i^m \left(x_j^i w^i\right)}{P_j^0} \tag{4}$$

In this context, the return of asset j is given by:

$$\bar{r_j} = \frac{\bar{P_j^1}}{P_j^0} + \frac{\theta_j}{\left(P_j^0\right)^2} \sum_i^m \left(x_j^i w^i\right) - 1 \tag{5}$$

where P_j^0 is the current price of asset j. Finally, in a more compact form, the expected return of an asset in an oligopolistic setting is:

$$\bar{r_j} = \theta_j^{'} \sum_i^m \left(x_j^i w^i\right) + r_j' \tag{6}$$

where $\theta_j^{'} = \frac{\theta_j}{\left(P_j^0\right)^2}$ and $r_j'$is equivalent to the return of the asset in a competitive market $r_j' = \frac{\bar{P_j^1}}{P_j^0} - 1$. From equation 6 it is clear that when there is not market power $\theta_j = 0$, the return of the asset equals the percentage change of prices as it is common in competitive markets.

Thus, the investor's problem is given by:

$$\begin{aligned} \min \sum_j^n \sum_k^n \left(x_j^i x_k^i \sigma_{jk}\right) \\ \text{s.t.} \sum_j^n x_j^i \left(\theta_j^{'} \sum_i^m \left(x_j^i w^i\right) + r_j'\right) = \bar{R}^i \\ \sum_j^n x_j^i = 1 \end{aligned} \tag{7}$$

The Lagrangian, of the i^{th} investor, associated with this problem is:

$$L^i = \sum_{j}^{n}\sum_{k}^{n}\left(x_j^i x_k^i \sigma_{jk}\right) - \lambda^i\left(\sum_{j}^{n} x_j^i\left(\theta_j^{'}\sum_{i}^{m}\left(x_j^i w^i\right) + r_j^{\prime}\right) - \bar{R}^i\right) - \mu^i\left(\sum_{j}^{n} x_j^i - 1\right) \tag{8}$$

where $\lambda^i$and μ^i are the Lagrange's multipliers.

Given that our decision variable is the percentage of wealth invested in each asset, the first order condition of the problem, for investor i and asset j, takes the form:

$$\begin{aligned}\frac{\partial L^i}{\partial x_j^i} &= 2\sum_{k}^{n}\left(x_k^i\sigma_{jk}\right) - \lambda^i\left(\sum_{j}^{n} x_j^i\right)\theta_j^{'} w^i - \lambda^i\theta_j^{'}\sum_{i}^{m}\left(x_j^i w^i\right) - \lambda^i r_j^{\prime} - \mu^i = 0\\ &= 2\sum_{k}^{n}\left(x_k^i\sigma_{jk}\right) - \lambda^i x_j^i\theta_j^{'} w^i - \lambda^i\left(\bar{r_j} - r_j^{\prime}\right) - \lambda^i r_j^{\prime} - \mu^i = 0\\ &= 2\sum_{k}^{n}\left(x_k^i\sigma_{jk}\right) - \lambda^i\left(x_j^i\theta_j^{'} w^i + \bar{r_j}\right) - \mu^i = 0 \end{aligned} \tag{9}$$

Equations 9 must be fulfilled by all of the n assets and represent the best response functions of each investor. Clearly, as it is common in oligopolistic markets, the optimum decision of player i depends upon the decision of other players, represented implicitly in $\bar{r_j}$.

In order to solve the system in terms of λ^i and μ^i, we assume that there are only two assets in which to invest: a risk free asset and a "mutual fund", that invest in all the assets in the market (this asset is known to be mean-variance efficient). It is worth noting that for this portfolio of two assets, the first-order condition becomes:

$$2\sigma_{jk} - \lambda^i\left(x_j^i\theta_j^{'} w^i + \bar{r_j}\right) - \mu^i = 0$$

- If the asset j is risk-free (f) we obtain:

$$-\lambda^i\left(x_f^i\theta_f^{'} w^i + \bar{r_f}\right) = \mu^i \tag{10}$$

- If the asset j is a market portfolio (M) we get:

$$2\sigma_M^2 - \lambda^i\left(x_M^i\theta_M^{'} w^i + \bar{r_M}\right) = \mu^i \tag{11}$$

Solving (10) and (11) for λ^i, we have:

$$-\lambda^i\left(x_f^i\theta_f^{'} w^i + \bar{r_f}\right) = 2\sigma_M^2 - \lambda^i\left(x_M^i\theta_M^{'} w^i + \bar{r_M}\right)$$

$$\lambda^i = \frac{2\sigma_M^2}{\left[\left(\bar{r_M} - \bar{r_f}\right) + \left(x_M^i\theta_M^{'} w^i - x_f^i\theta_f^{'} w^i\right)\right]} \tag{12}$$

replacing λ^i in (10)

$$\mu^i = -\frac{2\sigma_M^2\left(x_f^i\theta_f^{'}w^i+\bar{r_f}\right)}{\left[(\bar{r_M}-\bar{r_f})+(x_M^i\theta_M^{'}w^i-x_f^i\theta_f^{'}w^i)\right]} \tag{13}$$

Finally, using equations (9), (12) , (13)we get:

$$2\sigma_{jk}-\frac{2\sigma_M^2}{\left[(\bar{r_M}-\bar{r_f})+(x_M^i\theta_M^{'}w^i-x_f^i\theta_f^{'}w^i)\right]}\left(x_j^i\theta_j^{'}w^i+\bar{r_j}\right)+\frac{2\sigma_M^2\left(x_f^i\theta_f^{'}w^i+\bar{r_f}\right)}{\left[(\bar{r_M}-\bar{r_f})+(x_M^i\theta_M^{'}w^i-x_f^i\theta_f^{'}w^i)\right]}=0$$

$$2\sigma_{jk}\left[(\bar{r_M}-\bar{r_f})+(x_M^i\theta_M^{'}w^i-x_f^i\theta_f^{'}w^i)\right]-2\sigma_M^2\left(x_j^i\theta_j^{'}w^i+\bar{r_j}\right)+2\sigma_M^2\left(x_f^i\theta_f^{'}w^i+\bar{r_f}\right)=0$$

$$\bar{r_j}=\beta_j\left[(\bar{r_M}-\bar{r_f})+(x_M^i\theta_M^{'}w^i-x_f^i\theta_f^{'}w^i)\right]+\left[\left(-x_j^i\theta_j^{'}w^i+x_f^i\theta_f^{'}w^i\right)+\bar{r_f}\right]$$

Hence we can obtain the following proposition:

Proposition 1: *For any asset j, the basic formula for an oligopolistic capital asset pricing model is given by:*

$$\bar{r_j}=\beta_j\left(\bar{r_M}-\bar{r_f}+\alpha_j^1\right)+\bar{r_f}+\alpha_j^2$$

where the beta of asset j is: $\beta_j=\sigma_{jM}/\sigma_M^2$, $\alpha_j^1=\left(x_M^i\theta_M^{'}w^i-x_f^i\theta_f^{'}w^i\right)$ *and* $\alpha_j^2=\left[x_f^i\theta_f^{'}w^i-x_j^i\theta_j^{'}w^i\right]$.

Compared with the traditional CAPM model, oligopolistic behavior implies two distinct effects. First, there is a risk premium associated with oligopolistic market power, α_j^1, analogous to the risk premium of a perfectly competitive market, but now given by the difference between the percentage effect of the money invested in the market portfolio asset and the percentage effect of the money invested in the risk free asset, instead of the returns as in the perfectly competitive case. As pointed out above, the term θ is related to the price elasticity of demand, the lower this parameter the more elastic the market. In our oligopolistic model of the asset market, each additional unit of idiosyncratic risk is rewarded with a risk premium of market power, which depends positively upon the amount of money expended in the market portfolio and negatively in terms of investments in the risk free asset. Furthermore, there is an additional rate, α_j^2, that grows with investments in the risk free asset and that decreases with investments in the same evaluated asset.

From proposition 1, we can infer the following Corollary:

Corollary 1: *Assuming perfect competition in the risk free asset market and in the mutual fund market, in other words assuming no market power i.e.* $\theta_f^{'} = 0$ *and* $\theta_M^{'} = 0$*, we obtain the following asset pricing equation:*

$$\bar{r_j} = \beta_j \left(\bar{r_M} - \bar{r_f}\right) + \bar{r_f} - \theta_j^{'} x_j^i w^i \tag{14}$$

$$\bar{r_j} = \beta_j \left(\bar{r_M} - \bar{r_f}\right) + \bar{r_f} - \alpha_j \tag{15}$$

where $\alpha_j = \left(\theta_j^{'} x_j^i w^i\right)$.

It is clear that more investment in the asset with market power, and more market power, imply a lower return valuation for the asset. The alpha term represents the level of market power, that is, the ability of big firms to influence market prices. In other words, two assets with the same intrinsic risk, the market with more market power will have lower levels of returns. This is consistent with the three factor model of Fama and French, in which size matters at the time of evaluating returns, and that empirically small cap and value portfolios imply higher expected returns [7]. Finally, in the absence of market power, equation 14 will collapse into the traditional CAPM. Consequently, even in this simplified case, the oligopolistic model is different to that of the traditional CAPM valuation.

2.2 The Oligopolistic Option Pricing Model

Following [4], the problem is stated as follows: let t be time and St be the price of stock. Consider a derivative security whose price depends on S and t, so we call it $\pi(S,t)$ or just π. Then, the task becomes to find the equation which π satisfies. First, we assume that there is a risk-free bond B which earns a risk-free rate r.

$$dB = rBdt \tag{16}$$

In addition, we assume that the stock price S_t follows the geometric Brownian motion:

$$dS = \mu S dt + \sigma S dz \tag{17}$$

Regarding the derivative $\pi(S,t)$, by Ito's lemma, the following holds:

$$d\pi = \left(\frac{\partial \pi}{\partial t} + \mu S \frac{\partial \pi}{\partial S} + \frac{1}{2}\sigma^2 S^2 \pi_{ss}\right)dt + \sigma S \frac{\partial \pi}{\partial S} dz \tag{18}$$

We rearrange using 17 and obtain:

$$d\pi = \frac{\partial \pi}{\partial S} dS + \left(\frac{\partial \pi}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \pi}{\partial S^2}\right)dt \tag{19}$$

Hence:

$$\frac{d\pi}{\pi} = \frac{S}{\pi}\frac{\partial \pi}{\partial S}\frac{dS}{S} + \frac{1}{\pi}\left(\frac{\partial \pi}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \pi}{\partial S^2}\right) dt \tag{20}$$

If we assume that the value of α is different to zero, as a product of market power, we have:

$$\bar{r_S} = E\left(\frac{dS}{S}\right) = \beta_S p dt + \bar{r_f} dt - \alpha_S dt \tag{21}$$

$$\bar{r_\pi} = E\left(\frac{d\pi}{\pi}\right) = \beta_\pi p dt + \bar{r_f} dt - \alpha_\pi dt \tag{22}$$

where $p = \bar{r_M} - \bar{r_f}$ and $\alpha_j = \left(\theta_j^{'} x_j^i w^i\right)$. The alpha term can be interpreted as the level of market power, i.e. the ability of big firms to influence market prices. As pointed out by [4], the following relation between the derivative's betas β^π and the stock's betas β^S holds:

$$\beta_\pi = \frac{S}{\pi}\frac{\partial \pi}{\partial S}\beta_S \tag{23}$$

The coefficient $\frac{S}{\pi}\frac{\partial \pi}{\partial S}$ can be interpreted as the "elasticity" of the derivative price with respect to the stock price, in other words, it is the ratio of the percentage change in the derivative price to the percentage change in the stock price, for small percentage changes.

Using 23 and 22 and multiplying both sides by π we obtain:

$$E(d\pi) = S\frac{\partial \pi}{\partial S}\beta_S p dt + \bar{r_f}\pi dt - \alpha_\pi \pi dt \tag{24}$$

Taking the expected value of 19 and now using 21 we get:

$$E(d\pi) = S\bar{r_f}\frac{\partial \pi}{\partial S}dt - S\alpha_S\frac{\partial \pi}{\partial S}dt + S\frac{\partial \pi}{\partial S}\beta_S p dt + \frac{\partial \pi}{\partial t}dt + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 \pi}{\partial S^2}dt \tag{25}$$

Combining 24 and 25 we have:

$$S\frac{\partial \pi}{\partial S}\beta_S p + \bar{r_f}\pi - \alpha_\pi \pi = S\bar{r_f}\frac{\partial \pi}{\partial S} - S\alpha_S\frac{\partial \pi}{\partial S} + S\frac{\partial \pi}{\partial S}\beta_S p + \frac{\partial \pi}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 \pi}{\partial S^2} \tag{26}$$

After rearranging, we finally obtain the following result:

Proposition 2: *An extended version of the Black & Scholes equation that takes into account market imperfections, through the potential abnormal returns in the stock and derivative markets is given by:*

$$\frac{\partial \pi}{\partial t} + S\frac{\partial \pi}{\partial S}\left[\bar{r_f} - \alpha_S\right] + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \pi}{\partial S^2} \quad = \quad \pi\left[\bar{r_f} - \alpha_\pi\right]. \tag{27}$$

It transpires that when α_S and α_π are zero 27 becomes the traditional Black & Scholes equation. It is interesting to note that there is not any factor in the equation that allows the quantification of the trade-off between risk and expected return, being still a formulation in which the risk of the investor is not relevant. On the one hand, when deviations of the state of equilibrium are considered, as a product of some market imperfections, such as the ones discussed in the introduction; the classical non-arbitrage assumption of the Black-Scholes model is violated, implying a non risk-free portfolio. On the other hand, the impact of market imperfections implies marks-up in the derivative and stock markets that are asymmetric. Indeed, while α_π tends to increase the value of the call, α_S goes in opposite direction, decreasing the call's price.

The following call option pricing result can be established from our extended Black & Scholes formulation.

Proposition 3: *For a call option, with T as maturity date and K as the strike price, we use the following contract function $\Phi(S)$:*

$$\Phi(S) = \begin{cases} 0 & 0 < S < K \\ S - K & K < S \end{cases}$$

The Extended Black-Scholes solution $\pi(S, t, \alpha_\pi, \alpha_S)$ is given by

$$\pi(S,t) = Se^{(\alpha_\pi - \alpha_S)(T-t)}N(d_1) - Ke^{-(\bar{r_f} - \alpha_\pi)(T-t)}N(d_2)$$

where

$$d_1(S,t) = \frac{ln\, \frac{S}{K} + (\bar{r_f} + \frac{\sigma^2}{2} - \alpha_S)(T-t)}{\sigma\sqrt{(T-t)}}$$

$$d_2(S,t) = d_1 - \sigma\sqrt{T-t}$$

and $N(x)$ is the normal distribution function $N(0,1)$.

Finally, the comparative statics of our extended Black-Scholes model is presented in the following proposition:

Proposition 4: *The sensitivity of the price of derivatives, π, to a change in underlying parameters, S and t, on which is dependent, in the presence of oligopolistic competition in the underlying and derivative assets, is given by:*

$$\triangle = \frac{\partial \pi}{\partial S} = e^{(\alpha_\pi - \alpha_S)(T-t)} N(d_1)$$

$$\Theta = \frac{\partial \pi}{\partial t} = -(\bar{r_f} - \alpha_\pi) K e^{-(\bar{r_f} - \alpha_\pi)(T-t)} N(d_2)$$

$$\nu = \frac{\partial \pi}{\partial \sigma} = \sqrt{T-t} S e^{(\alpha_\pi - \alpha_S)(T-t)} N(d_1)$$

$$\Gamma = \frac{\partial^2 \pi}{\partial S^2} = \frac{\partial \triangle}{\partial S} = \frac{e^{(\alpha_\pi - \alpha_S)(T-t)}}{S\sigma\sqrt{T-t}} N'(d_1)$$

.

It can be noted that results vary from the traditional greek analysis when oligopolistic competition is considered. From Proposition 4 it is clear that all values are consistent with a call paying dividends equal to $\alpha_s - \alpha_\pi$ at a rate $r_f - \alpha_\pi$. As we established above, more investment in the asset with market power, and more market power, imply a lower return valuation for the asset in the underlying market α_s and derivative market α_π, which in turn modifies the risk free rate in each market respectively, as a dividend would do it. In the next section these results are numerically shown.

3 Some numerical illustrations

In order to explore the importance of the stock and derivative potential market imperfections in the option valuation some numerical simulations are carried out, considering the following parameters: $K = 100$, $S = K$, $t = 0.0$, $T = 0.5$, $\sigma = 0.3$ and $r = 0.05$. In table 2, it is possible to see how the option price varies, given different values of α_π and α_S. Clearly for the case $\alpha_\pi = \alpha_S = 0$ the Black and Scholes valuations are obtained, and the value reached is 9.63.

Our simulations show that higher levels of market power in the derivative markets (α_π) tend to increase the call option values compared with those values given by the standard Black and Scholes formulation. In contrast, the impact of market power in the underlying asset market (α_S) tends to lower the option price, see Table 2.

Table 2. Option pricing considering α_S and α_π variations

		α_π										
		0.0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
	0.0	9.63	10.12	10.64	11.19	11.76	12.37	13.00	13.67	14.37	15.11	15.88
	0.1	6.98	7.34	7.72	8.12	8.53	8.97	9.43	9.91	10.42	10.96	11.52
	0.2	4.90	5.15	5.42	5.70	5.99	6.30	6.62	6.96	7.31	7.69	8.08
	0.3	3.32	3.49	3.67	3.86	4.06	4.27	4.49	4.72	4.96	5.21	5.48
	0.4	2.17	2.28	2.40	2.52	2.65	2.79	2.93	3.08	3.24	3.40	3.58
α_S	**0.5**	1.36	1.43	1.50	1.58	1.66	1.75	1.84	1.93	2.03	2.14	2.25
	0.6	0.82	0.86	0.91	0.95	1.00	1.05	1.11	1.16	1.22	1.29	1.35
	0.7	0.47	0.50	0.525	0.55	0.58	0.61	0.64	0.67	0.71	0.74	0.78
	0.8	0.26	0.27	0.29	0.30	0.32	0.33	0.35	0.37	0.39	0.41	0.43
	0.9	0.14	0.14	0.15	0.16	0.17	0.18	0.18	0.19	0.20	0.22	0.23
	1.0	0.07	0.07	0.07	0.08	0.08	0.09	0.09	0.10	0.10	0.11	0.11

In Table 3, it is shown the oligopolistic option pricing model now considering variations in $\bar{r_f}$, besides different values of α_S. In general, higher values of $\bar{r_f}$ produce higher option prices.

Table 3. Option pricing considering α_S and $\bar{r_f}$ variations

		$\bar{r_f}$									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
	0.1	9.63	9.88	10.13	10.39	10.65	10.91	11.17	11.44	11.71	11.98
	0.2	6.99	7.19	7.40	7.61	7.82	8.04	8.25	8.48	8.70	8.93
	0.3	4.91	5.07	5.23	5.39	5.56	5.74	5.91	6.09	6.27	6.46
	0.4	3.33	3.45	3.57	3.70	3.83	3.96	4.09	4.23	4.37	4.52
α_S	**0.5**	2.17	2.26	2.35	2.44	2.54	2.63	2.73	2.84	2.94	3.05
	0.6	1.37	1.43	1.49	1.55	1.62	1.69	1.76	1.83	1.91	1.99
	0.7	0.82	0.86	0.91	0.95	0.99	1.04	1.09	1.14	1.19	1.24
	0.8	0.48	0.50	0.53	0.56	0.58	0.61	0.65	0.68	0.71	0.75
	0.9	0.26	0.28	0.30	0.31	0.33	0.35	0.37	0.39	0.41	0.43
	1.0	0.14	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.24

Figure 1 shows the difference between the modified Black and Scholes, which considers different values of α_π and α_S, and the standard Black and Scholes results, without market power. Investors should keep these conclusions and intuition in mind if they suspect deviations from the equilibrium returns, caused by market power, as those experienced during the last financial crisis. It is important to point out that this model can be calibrated empirically using the CAPM, through the alpha term. In Figure 2 and Figure 3, the differences between the modified Black and Scholes and the standard Black and Scholes results are presented, now considering $\bar{r_f}$ variations. As we already saw, the impacts of imperfections in the derivative market tend to increase the option valuation, while the market power of the underlying asset market tend to lower the variation, regardless the interest rate.

Figure 1. Difference between the Oligopolistic Black and Scholes and the standard Black and Scholes results: α_S and α_π variations

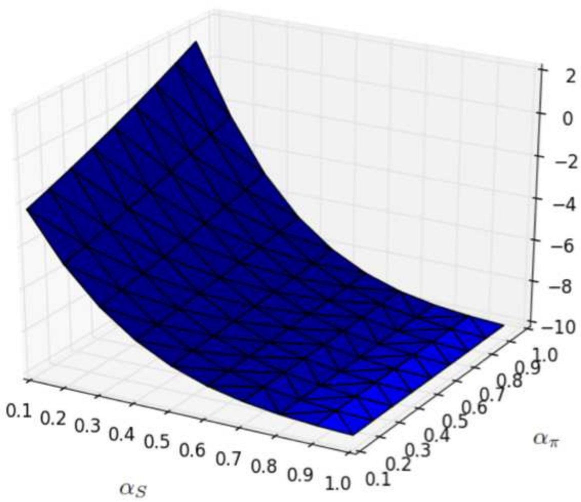


Figure 2. Difference between the Oligopolistic Black and Scholes and the standard Black and Scholes results: α_S and r variations

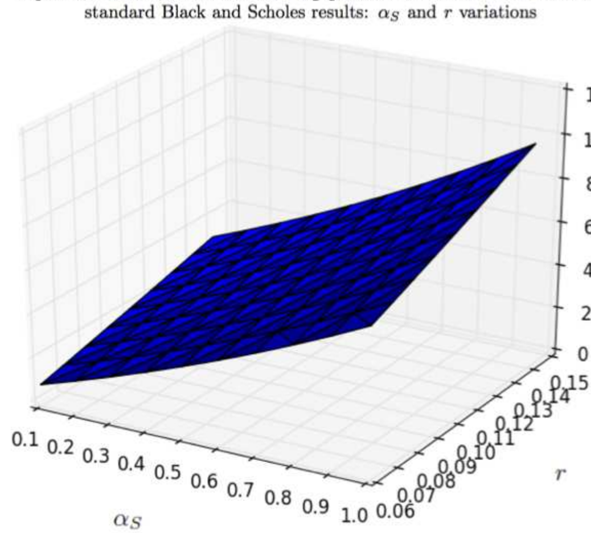
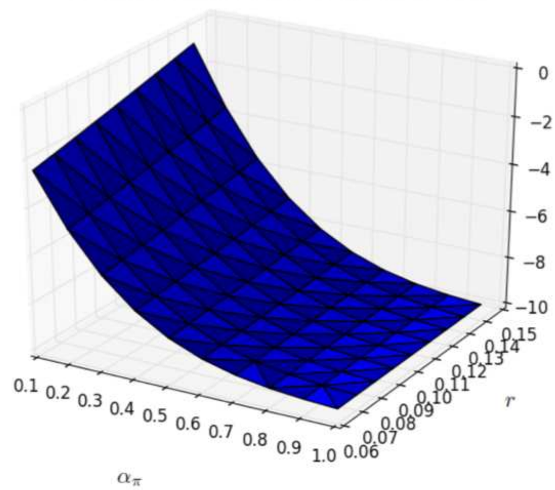


Figure 3. Difference between the Oligopolistic Black and Scholes and the standard Black and Scholes results: α_π and r variations



4 Conclusions and Further Research

We have extended the classical Black and Scholes approach in order to take into account imperfect market settings. Option valuation models are usually based on a frictionless market, however, as it has been established in the literature review, the behavior of some financial instruments suggests the existence of different kinds of market imperfections, specially when regarding the events ocurred during the recent financial crises. In this work an oligopolistic market power setting has been explored.

In particular in this paper a simple model of option pricing has been developed in which the derivative and the underlying assets present some level of market power, that produces a lower level of expected returns on these assets that goes below their fundamental value, and hence affects the option valuation. Our formulation extends the standard Black-Scholes model to account for market power in the derivative and the underlying asset markets.

One of the advantages of this formulation, besides its simplicity and theoretical consistency with the standard Black-Scholes model, is that since abnormal returns can be econometrically estimated through CAPM, it allows us to analyze the quantitative consequences of market frictions on the option pricing, which ensures that the results of this work are easy to use in practice.

Our simulations show that higher levels of market power in the option markets tend to increase the call option values compared with those values given by the standard Black and Scholes formulation. In addition, the impact of market power in the underlying asset market (α_S) tends to lower the option price. Let us remember that in our formulation the alpha term represents the level of market power, in other words, the ability of big firms to influence market prices.

These results can be useful to investors if they suspect deviations from the equilibrium returns, as those experienced in the last financial crisis. Indeed, financial analysts could benefit from an option pricing model that admits for some type of disequilibrium, in terms for example of abnormal returns in the derivative and underlying assets markets, specially considering the current levels of concentrations experienced in financial markets as well as in some of the underlying asset markets.

Further research in this area could point towards incorporating more elements of strategic interaction among big firms in financial markets, as well as more sophisticated game theoretic approaches. Finally, these models could be easily tested empirically, using current financial databases.

References

[1] Acharya, V., and Pedersen, L. (2005) “Asset pricing with liquidity risk”, *Journal of Financial Economics*, 77, 375-410.

[2] Armstrong, C., J. Core, D. Taylor, and R. Verrecchia, (2011) “When Does Information Asymmetry Affect the Cost of Capital?”,*Journal of Accounting Research*, 49, 1, doi: 10.1111/j.1475-679X.2010.00391.x

[3] Björk, T. (1998) *Arbitrage Theory in Continuous Time*, Oxford University Press.

[4] Black, F. and Scholes, M. (1973). “The Pricing of Options and Corporate Liabilities”,*Journal of Political Economy*, 81 (3), 637–654

[5] Constantinides, G. M., Jackwerth, J.C., Perrakis, S. (2009) "Mispricing of S&P 500 Index Options", *Review of Financial Studies*, 22, 1247-1277.

[6] Duffie, D. (1996) *Dynamic Asset Pricing Theory. 2nd Edition*, Princeton University Press: New Jersey.

[7] Fama, E., and French, K. (2012) "Size, value, and momentum in international stock returns",*Journal of Financial Economics*, 105.3: 457-472.

[8] Fama, E. and French, K. (1993) "Common risk factors in the returns on stocks and bonds", *Journal of Financial Economics*, 33, 3-56.

[9] Figlewski, S. (1989) "Options Arbitrage in Imperfect Markets", *Journal of Finance*, 44, 5

[10] Hull, J. C. (1997) *Options, futures, and other derivatives*, Englewood Cliffs, NJ, Prentice-Hall.

[11] Husmann, Sven and Neda Todorova (2011). "CAPM option pricing", *Finance Research Letters*, 8 (4): 213–219.

[12] Lambert, R. A., Leuz, C. and Verrecchia, R.E. (2012) "Information asymmetry, information precision, and the cost of capital",*Review of Finance*, vol 16.1, pp. 1-29.

[13] Lintner, J. (1965) "Security prices, risk and maximal gains from diversification", *Journal of Finance*, 587–516.

[14] Merton, R (1973). "Theory of rational option pricing", *Bell Journal of Economics and Management Science*, 4, 141–183.

[15] Mossin, J. (1966) "Equilibrium in a Capital Asset Market", *Econometrica* , 34: 768-783.

[16] Sharpe, W.F. (1964) "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk", *Journal of Finance*, 19(3): 425-442.

[17] Wilmott, P. (1998) *Derivatives: The Theory and Practice of Financial Engineering.* J. Wiley.