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Abstract: Parametric production frontier function has been commonly employed in stochastic frontier model but there was no proper test statistic for its plausibility. To fill into this gap, this paper develops two test statistics to test for a hypothesized parametric production frontier function based on local smoothing and global smoothing, respectively. We then propose the residual-based wild bootstrap approach to compute the p-values of our proposed test statistics. Our proposed test statistics are robust to heteroscedasticity. Simulation studies are carried out to examine the finite sample performance of the sizes and powers of the test statistics.

Keywords: Stochastic frontier; Specification testing; Wild bootstrap.

JEL Classification : C13, C14.

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1 Introduction

Getting inspiration by the pioneer work of Farrell (1957) on the estimation of the production function, Aigner *et al.* (1977) and Meeusen and van den Broeck (1977) introduce the stochastic frontier (SF) model to analyze production or cost frontiers. This is one of the most important approaches in studying productivity and efficiency analysis. Readers may refer to Fried *et al.* (2008) for updated review on the issue and literature.

Consider the following SF model:

$$Y = m(X) - U + V, \tag{1.1}$$

where Y is the logarithm of output, X is a p-dimensional set of the logarithm of inputs, $m(\cdot)$ is an unknown smooth production frontier function, U is the positive inefficiency term, and V is a symmetric random noise. In addition, we assume that $E(V|X) = 0$ and U and V are conditional independent given the inputs X.

Parametric SF model (Aigner *et al.*, 1977) specification requires both the functional form of the production frontier function $m(\cdot)$ and the distributions of the stochastic components U and V to be known. Academics, for example, Simar and Wilson (2014), thus, point out that the major drawback of the fully parametric SF model is its inflexiblity.

To circumvent the limitation, some academics suggest to reduce the parametric restriction on the production frontier function. For example, Fan *et al.* (1996) introduce the quasi-likelihood method so that it is not necessary to specify the production frontier. However, the model set-up requires the distribution of the stochastic components to be known. Kumbhakar *et al.* (2007) propose not to impose a parametric assumption on production frontier function but to impose the semi-parametric assumptions on U and V to obtain local maximum likelihood estimation. Recently, Simar *et al.* (2014) develop a nonparametric least-squares method to avoid the high computational complexity involved in the local maximum likelihood method introduced by Kumbhakar *et al.* (2007). Another merit of Simar *et al.* (2014)'s approach is that only local distribution assumption on U is required but it is still necessary to assume symmetry on V.

Nonetheless, the parametric specification is attractive for the production frontier function because it is easy to compute and the economic interpretation of the production process can fit into the production theory well. Hence, the parametric SF model is still dominating the area of productivity and efficiency analysis. We note that all these procedures are based on the assumed parametric form of the production frontier function. If the parametric assumption on $m(\cdot)$ is not adequate, the conclusions may not be valid and the inference drawn may be misleading. Thus, testing the form of the production frontier function is essential in the SF analysis. There are many regression theories (see, for example, González-Manteiga and Crujeiras (2013) for a review) could be used to test the form of functions being assumed in economic theories. However as far as we know, there is no paper explore the testing theory for the SF model. In this connection, we aim to fill into the gap in this paper to test whether the production frontier function can be described by some known parametric functions.

In this paper, we are interested in testing the following null hypothesis:

$$H_0 : m(X) = g(X, \beta_0), \tag{1.2}$$

for some β_0 against an alternative hypothesis

$$H_1 : m(X) \neq g(X, \beta), \tag{1.3}$$

for any β. Here, $g(X, \beta)$ is a known smooth function with unknown d-dimensional vector, β, of parameters. We will propose two test statistics based on local smoothing and global smoothing. To conduct these two test statistics in practice, we further suggest to employ the residual-based wild bootstrap approach to calculate the p-values for the test statistics. One advantage of using our proposed approach is that our proposed test statistics can efficiently detect the alternative hypothesis even under heteroscedasticity. To our best knowledge, our paper is the first work in the literature to test whether the hypothesized production frontier function is appropriate for the parametric SF models.

The rest of this paper is organized as follows: In Section 2, we construct the test statistics and use the residual-based wild bootstrap approach calculate the p-values for the test statistics. In Section 3, simulation results are reported to examine the performance of the tests. Section 4 concludes the paper.

2 The Theory

Before proposing the test statistics to the SF analysis, we first discuss the estimation procedure for the parametric SF model without making specific distributional assumptions on U and V in the following subsection.

2.1 Preliminary Estimation

We suppose that $\mu_U(X) = E(U|X)$, $\epsilon = V - U + \mu_U(X)$, and $m_1(X) = Y - \epsilon$. In this model set-up, we note that the condition $E(\epsilon|X) = 0$ always holds and under the null hypothesis stated in (1.2), the model in (1.1) can be expressed as:

$$Y^1 = Y + \mu_U(X) = g(X, \beta) + \epsilon. \tag{2.1}$$

We note that if we can get the value of $\mu_U(X)$, we can obtain the "generated" observation Y^1 by (2.1). For the "generated" data set (Y^1, X), the model in (2.1) turns out to be the traditional parametric regression model, and thereafter, we can estimate the parameter β by using nonlinear least squares based on (Y^1, X). Thus, to estimate β, we need to first estimate the term $\mu_U(X)$. Simar *et al.* (2014) have developed an approach to estimate $\mu_U(X)$. We apply their approach to estimate $\mu_U(X)$. If the null hypothesis in (1.2) is true, by comparing equation in (2.1), we can rewrite the model in (1.1) to be

$$Y = m_1(X) + \epsilon \ , \tag{2.2}$$

where $m_1(X) = g(X, \beta) - \mu_U(X)$. In the revised model stated in (2.2), we can still have $E(\epsilon|X) = 0$. This enables the model become a standard nonparametric regression model. One could then employ different nonparametric methods, such as kernel, local polynomial, and spline (Härdle, 1990) to obtain the estimate, $\hat{m}_1(X)$, for $m_1(X)$. In this paper, we recommend to use the kernel-type of estimator given by

$$\hat{m}_1(x) = \sum_{i=1}^{n} W_{ni}(x) Y_i \quad \text{with} \tag{2.3}$$

$$W_{ni}(x) = \frac{K_h(x - X_i)}{\sum_{j=1}^{n} K_h(x - X_j)} \quad \text{and} \quad K_h(\cdot) = \frac{K(\cdot/h)}{h^p} \ ,$$

in which $K(\cdot)$ is a kernel function and h is the bandwidth.

We are now ready to bring back the assumptions of symmetry on V and the conditional independence of U and V given X. Imposing these assumptions to the model in (2.2), we can have the following results:

$$\begin{aligned} E\big(\epsilon^2|X\big) &= var_U(X) + var_V(X) \ , \\ E\big(\epsilon^3|X\big) &= -E\big[(U - \mu_U(X))^3|X\big]. \end{aligned}$$

Here, $var_U(X)$ and $var_V(X)$ are the conditional variances of U and V given X, respectively. When we get the estimation of $m_1(X)$, we can obtain the residual $\hat{\epsilon} = Y - \hat{m}_1(X)$. Let $m_j(X) = E(\epsilon^j|X)$ for $j = 2$ and 3. Then, by adopting an appropriate nonparametric technique, we obtain the following consistent estimators, $\hat{m}_j(x)$, for $m_j(X)$ $(j = 2, 3)$:

$$\hat{m}_j(x) = \sum_{i=1}^{n} W_{ni}(x)(Y - \hat{m}_1(X_i))^j \quad \text{for} \ \ j = 2, 3.$$

To estimate $\mu_U(X)$, setting some local parametric assumptions on the distributions of $U|X=x$ is necessary. To do so, we assume that $U|X=x \sim |N(0,\sigma_U^2(x))|$ and assume that, conditional on X, U and V are independent. These are the same assumptions Kumbhakar *et al.* (2007) and others are using. With the assistance of these assumptions, we get

$$\begin{aligned}
\mu_U(X) &= E(U|X) = \sqrt{\frac{2}{\pi}}\sigma_U(X) , \\
E(\epsilon^2|X) &= \frac{\pi-2}{\pi}\sigma_U^2(X) + var_V(X) , \\
E(\epsilon^3|X) &= \sqrt{\frac{2}{\pi}}\Big(1-\frac{4}{\pi}\Big)\sigma_U^3(X) .
\end{aligned}$$

Using the above equations, we obtain the following results:

$$\begin{aligned}
\hat{\sigma}_U(X) &= \max\left\{0, \left[\sqrt{\frac{\pi}{2}}(\frac{\pi}{\pi-4})\hat{E}(\epsilon^3|X)\right]^{1/3}\right\} , \\
\hat{\mu}_U(X) &= \sqrt{\frac{2}{\pi}}\hat{\sigma}_U(X).
\end{aligned}$$

After obtaining $\hat{\mu}_U(X)$, we use (2.1) to obtain the generated data $\hat{Y}_i^1$ such that $\hat{Y}_i^1 = Y_i + \hat{\mu}_U(X_i)$. We then use (2.1) again to estimate β by using the nonlinear least square method based on the generated data set $\{(\hat{Y}_i^1, X_i)|i=1,\cdots,n\}$. That is, $\hat{\beta} = \arg\min_\beta \sum_{i=1}^n \big[\hat{Y}_i^1 - g(X_i,\beta)\big]^2$. Let $\epsilon_0 = Y^1 - g(X,\beta)$. Thereafter, the residuals $\hat{\epsilon}_{0i} = \hat{Y}_i^1 - g(X_i,\hat{\beta})$ under the null hypothesis can be obtained. We note that $\epsilon_0 = Y^1 - g(X,\beta) = Y + \mu_U(X) - g(X,\beta)$ and $\epsilon = V - U + \mu_U(X)$. Under the null hypothesis, they are the same. However, under the alternatives, they could be different.

2.2 Construction of Test Statistics

Under the null hypothesis, H_0, set in (1.2), we get

$$E(\epsilon_0|X) = E\big[Y+\mu_U(X)-g(X,\beta)|X\big] = E\big[g(X,\beta)+V-U+\mu_U(X)-g(X,\beta)|X\big] = 0.$$

On the other hand, under the alternative hypothesis, H_1, set in (1.3), we obtain

$$\begin{aligned} E(\epsilon_0|X) &= E\big[Y + \mu_U(X) - g(X,\beta)|X\big] = E\big[m(X) + V - U + \mu_U(X) - g(X,\beta)|X\big] \\ &= m(X) - g(X,\beta) \neq 0. \end{aligned}$$

These observations can be used as a base in the construction of our test statistics. We first develop the local smoothing based test statistic. Since

$$\begin{aligned} E\big[\epsilon_0 E(\epsilon_0|X) f(X)\big|H_0\big] &= E\big[E^2(\epsilon_0|X) f(X)\big|H_0\big] = 0 \quad \text{and} \\ E\big[\epsilon_0 E(\epsilon_0|X) f(X)\big|H_1\big] &> 0 , \end{aligned}$$

where $f(X)$ is the density function of X. We note that the empirical analogue of $E\big[\epsilon_0 E(\epsilon_0|X) f(X)\big]$ can be used to construct our proposed test statistic.

By using the leave-one-kernel method (Härdle, 1990) to estimate $E(\epsilon_0|X)$ and $f(X)$ and applying the specification testing independently introduced by Zheng (1996) and Fan and Li (1996), we obtain the following test statistic:

$$T_{1n} = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j\neq i}^{n} K_h(X_i - X_j)\hat{\epsilon}_{0i}\hat{\epsilon}_{0j}. \tag{2.4}$$

We note that due to its technical tractability and easy computation, the local smoothing method introduced by Zheng (1996) and Fan and Li (1996) has been used intensively. For instance, Guo *et al.* (2014), Lin *et al.* (2014), and Lahaye and Shaw (2014) have extended the theory of local smoothing to handel regression models with missing response, fixed effects panel data models and heterogenous autoregressive model.

One limitation of local-smoothing-based type of test statistics like T_{1n} in (2.4) is that its convergent rate during detecting the alternatives is relatively slow. For example, González-Manteiga and Crujeiras (2013) show that the local-smoothing-based type of test statistic when detecting the alternatives converging to the null hypothesis is at the rate of $n^{-1/2}h^{-p/4}$.

To improve the convergent rate, we now introduce the global-smoothing-based type of test statistic for the SF model. To do so, we first note that under the null hypothesis, the following equation holds:

$$E\Big[\epsilon_0 I(X \leq x)\Big] = 0, \quad \forall x \in \mathbb{R}^p.$$

This motivates us to construct the following residual-based empirical process:

$$R_n(x) \;=\; \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \hat{\epsilon}_{0i} I(X_i \leq x).$$

Thereafter, the Cramér-von Mises type of test statistic can be constructed by:

$$T_{2n} = \int \Big[R_n(x)\Big]^2 dF_n(x), \tag{2.5}$$

where $F_n(x)$ is the empirical distribution based on $X_1, X_2, \cdots, X_n$.

Readers may refer to Phillips and Jin (2014) and Hsu and Kuan (2014) for some extensions of the global smoothing based tests in several areas of economic theory. We note that the global smoothing based test statistics like T_{2n} in (2.5) can detect the alternatives converging to the null hypothesis at the rate of $n^{-1/2}$. However, we also note that the higher detecting rate of the global-smoothing-based type of tests do not mean that, in general, they have larger power compared with the local smoothing based tests. It only means that they can have some powers against closer alternatives. While these powers could also be very low. In fact, Fan and Li (2000) claim that global smoothing based tests generally yard low powers against high frequency alternatives. Our simulation study in next section supports their claim.

In classical regression models, it can be shown that the distributions of T_{in} ($i = 1, 2$) converge to a centered normal; that is, a normal distribution with zero mean as $n \to \infty$. However, in the context of the SF model, the asymptotic properties of T_{in} can be very complicated due to the fact that the term $\mu_U(X)$ is unknown. If we can estimate $\mu_U(x)$ exactly; that is, $P(\hat{\mu}_U(X) = \mu_U(X))$=1, then the conventional results hold. However, this is absolutely only an idealistic situation but it will rarely happen in reality. In practice, to formally study the asymptotic properties of T_{in}, we need to investigate the impact of the nonparametric estimation $\hat{\mu}_U(X)$ on the estimation of β. Furthermore, the generated data $\hat{Y}^1$ depends on $\hat{\mu}_U(X)$, and thus, investigating the asymptotic properties of the test statistics for the SF models could be so complicated that there may not be any solution. In this connection, in this paper we do not investigate the asymptotic properties of the test statistics for the SF models. Instead, we provide an approach to calculate the p-values of our proposed test statistics so that practitioners could construct a proper testing procedure by using our proposed test statistics. We discuss this issue in next subsection.

2.3 P-Value Construction

In this section we propose an approach to calculate the p-values of our proposed test statistics. To do so, we apply the residual-based wild bootstrap method (Stute *et al.* 1998) to determine whether to reject the null hypothesis by using the following steps:

Step 1. Obtain $\hat{\mu}_U(X)$, $\hat{\beta}$, and $\hat{\epsilon}_0$ by using the approach proposed in Section 2.1, and thereafter, construct T_{in} $(i = 1, 2)$ as discussed in Section 2.2.

Step 2. Generate bootstrap observations $Y_i^* = g(X_i, \hat{\beta}) - \hat{\mu}_U(X_i) + \hat{\epsilon}_{0i} \times V_i$. Here, $\{V_i\}_{i=1}^n$ is a sequence of i.i.d. random variables with zero mean, unit variance, and independent of the sequence $\{Y_i, X_i\}_{i=1}^n$. We note that $\{V_i\}_{i=1}^n$ can be chosen from i.i.d. Bernoulli variates with

$$P(V_i = \frac{1-\sqrt{5}}{2}) = \frac{1+\sqrt{5}}{2\sqrt{5}}, \quad \text{and} \quad P(V_i = \frac{1+\sqrt{5}}{2}) = 1 - \frac{1+\sqrt{5}}{2\sqrt{5}}.$$

Step 3. Let T_{in}^* be defined similarly as T_{in} $(i = 1, 2)$, basing on the bootstrap sample $\{Y_i^*, X_i\}_{i=1}^n$.

Step 4. Repeat Steps 2 and 3 B times and calculate the p-value as $p_i^B = \#\{T_{in}^* > T_{in}\}/B$.

Under the null hypothesis, the p-values become the sizes of our proposed test statistics. On the other hand, under the alternative hypotheses, the p-values are the empirical powers of our proposed test statistics. Thus, we expect that under the null hypothesis, the p-values will be close to the nominal level while under the alternative hypotheses, the p-values should be much larger than the nominal level, especially for large sample sizes. We demonstrate our conjecture in next section.

3 Simulation Studies

We now conduct the Monte Carlo simulation to examine the performance of our proposed test statistics T_{in} $(i = 1, 2)$ defined in (2.4) and (2.5) to the following SF models:

$$\begin{aligned} \text{Model 1} \quad &: \quad Y = 5 + 5X + a\exp\{X^2\} - U + V, \qquad (3.1) \\ \text{Model 2} \quad &: \quad Y = 5 + 5X + a\sin\{4\pi X\} - U + V, \end{aligned}$$

with the following null hypotheses H_{0i} versus the alternatives H_{1i}:

$$H_{0i} \;:\; a = 0 \quad \text{and} \quad H_{1i} \;:\; a \neq 0\ , \qquad (3.2)$$

where $i = 1$ for Model 1 and $i = 2$ for Model 2. We first consider the following case:

Study 1 *$X \sim U(0, 1)$, $U \sim |N(0, 1)|$, and $V \sim N(0, \sigma_v^2)$ where $\sigma_v = 0.75 \times \sqrt{(\pi - 2)/\pi}$.*

We note that in the above model settings, under the null hypotheses, H_{0i} $(i = 1, 2)$; that is, $a = 0$, it becomes Example 1 in Kumbhakar *et al.* (2007).

To conduct simulation to examine the size and power performances of our proposed test statistics T_{in} $(i = 1, 2)$ defined in (2.4) and (2.5) for Model 1 for the null hypotheses H_{01} versus the alternatives H_{11}, we take the sample size n to be 100 and 200 and the values of a to be $0.0, 0.3, \cdots, 1.5$. On the other hand, to conduct simulation to examine the performance of our proposed test statistics T_{in} $(i = 1, 2)$ for

Model 2 for the null hypotheses H_{02} versus the alternatives H_{12}, we take the sample size n to be 50 and 100 and the values of a to be $0.0, 0.2, \cdots, 1.0$. In each case (each n and a), the replication is $2,000$ and for each replication, $B = 500$ bootstrapped samples are generated. In the nonparametric regression estimation, the kernel function is taken to be $K(u) = 15/16(1-u^2)^2$, if $|u| \leq 1$; and 0 otherwise. The bandwidth is taken to be $h = \hat{\sigma}(X) \times n^{-1/5}$ for simplicity. Here, $\hat{\sigma}(X)$ is the empirical estimator of the standard deviation of variable X and the nominal level α is set to be 0.05.

From out simulation, we have the following observations: First, under all situations considered, the empirical sizes of these two test statistics are all close to the nominal level. This implies that our proposed test statistics can maintain the size very well. Second, when we turn to the empirical power, we can see clearly that our proposed tests are very sensitive to the alternatives. That is, when the value of a increases, the power increases quickly. Third, we find that for Model 1, the second test statistic, T_{2n}, has larger power than the first test statistic, T_{1n}. However, when we look into Model 2, T_{1n} becomes the winner. Fourth, the power of both tests improve significantly for larger n and when n is large enough (see, for example, $n = 200$ for Model 1 and $n = 100$ for Model 2), the powers of both tests are close to one.

We turn to study the SF model with heteroscedasticity in the distribution of the inefficiencies by using the following study:

Study 2 *Consider the model setting in Study 1, replace U by $U|X = x \sim N\big[0, (1+2x)^2\big]$.*

We note that in the above model settings, under the null hypotheses, H_{0i} $(i = 1, 2)$; that is, $a = 0$, the model setting in Study 2 becomes Example 2 in Kumbhakar *et al.* (2007). This study aims to investigate the impact of heteroscedasticity in the performance of our proposed two test statistics.

We also conduct the simulation results for Study 2. From our simulation, we have the following observations: First, the sizes of our proposed test statistics are all close to the nominal level. This implies that the heteroscedasticity has little impacts on the empirical sizes. Second, the empirical powers of our test statistics are still reasonable and acceptable. These two observations demonstrate that our proposed test statistics can efficiently detect the alternative hypothesis even under heteroscedasticity. Third, comparing with the results in Study 1 , the powers of the two test statistics reduce significantly. This implies that the heteroscedasticity in the distribution of the inefficiencies has a negative impact on the power performance but has no negative impact on the size performance of the test statistics. Fourth, we find that for Model 1, T_{2n} outperforms T_{1n}, while for Model 2, T_{1n} performs better. This observation suggests that these two test statistics should be viewed as complements to each other.

4 Concluding Remarks

Though SF models has been widely used in economics, finance, and other areas for very long period, a formal specification testing procedure for the production frontier function has not been developed. Thus, we propose two test statistics in this paper by adopting local smoothing and global smoothing methods to fill in the gap in the literature. However, as we discussed in Section 2.2, since the inefficiency term U is unknown, the asymptotic properties of the test statistics could be so complicated that there may not be any solution to it. Thus, we leave the development of the asymptotic properties of the test statistics in the future study. In this paper we propose to use a resampling technique to calibrate the critical values. To this end, residual-based wild bootstrap is suggested so that the p-values of our proposed test statistics can be estimated. This enables academics and practitioners to apply our proposed test statistics to test for any hypothesized SF model and make the testing of the empirical SF models become possible.

In addition, we find that our proposed test statistics can be used in the specification testing of production frontier function even under heteroscedasticity. Simulation studies show that the sizes of these two test statistics are close to the nominal level and the powers are satisfactory even when the sample size is small, say $n = 50$.

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