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Kakarot-Handtke, Egmont

University of Stuttgart, Institute of Economics and Law

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The Law of Supply and Demand: Here It Is Finally

Egmont Kakarot-Handtke*

Abstract

There is no such thing as a law of human or social behavior. The conceptual consequence of the present paper is therefore to discard the subjective-behavioral axioms and to take objective-structural axioms as new formal foundations. The central piece of economic theory is the interaction of demand and supply which determines prices and quantities. Demand and supply in turn are determined by subjective factors. In the structural axiomatic paradigm the Law of Supply and Demand follows from objective factors alone. The Law consists of measurable variables and is testable in principle. The results prove the superiority of the new paradigm.

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Keywords new framework of concepts; structure-centric; axiom set; harmonic structure

*Affiliation: University of Stuttgart, Institute of Economics and Law, Keplerstrasse 17, 70174 Stuttgart, Germany. Correspondence address: AXEC Project, Egmont Kakarot-Handtke, Hohenzollernstraße 11, 80801 München, Germany, e-mail: handtke@axec.de. Research reported in this paper is not the result of a for-pay consulting relationship; there is no conflict of interest of any sort.

1 From subjectivity to objectivity

There is no such thing as a law of human or social behavior. Therefore it is a vain hope that the functioning of the actual economy can be explained by applying behavioral principles. This is not a question of realism/unrealism but of theory design and methodology.

Standard economics rests on behavioral assumptions that are formally expressed as axioms (Debreu, 1959; Arrow and Hahn, 1991; McKenzie, 2008). Axioms are indispensable to build up a theory that epitomizes formal and material consistency. The fatal flaw of the standard approach is that there is no such thing as a behavioral axiom (for details see 2014). Moreover, the proper subject matter of theoretical economics is not human behavior but the behavior of the economic system.

The conceptual consequence of the present paper is therefore to discard the subjective-behavioral axioms and to take objective-structural axioms as new formal foundations.

The central piece of economic theory is the interaction of demand and supply which determines prices and quantities. Demand and supply in turn is determined by utility and profit maximization respectively. This subjective-behavioral approach is fully replaced by the objective-structural approach.

Section 2 first provides the new formal foundations with the set of three structural axioms. These minimalistic premises represent the elementary consumption economy. In Section 3 the Profit Law is derived and the conditions of zero profit in the individual firm are established. Then, the powerful formal tool is applied to the derivation of the structural axiomatic Law of Supply and Demand in Section 4. Section 5 concludes.

2 The structure of economic reality

From the axiomatic point of view, mathematics appears thus as a storehouse of abstract forms – the mathematical structures; and it so happens – without our knowing why – that certain aspects of empirical reality fit themselves into these forms, as if through a kind of preadaptation. (Bourbaki, 2005, p. 1276)

We now advance from behavioral axioms as formal incarnation of homo oeconomicus to structural axioms as formal incarnation of the economic system. Human beings are thereby moved to the analytical periphery. This amounts to a *decoupling* of behavioral assumptions and the axiomatic method. The formal foundations of theoretical economics define the interdependence of the real and nominal variables that constitutes the elementary monetary economy.

2.1 Axioms

The first three structural axioms relate to income, production, and expenditure in a period of arbitrary length. The period length is conveniently assumed to be the calendar year. Simplicity demands that we have for the beginning one world economy, one firm, and one product. Axiomatization is about ascertaining the *minimum* number of premises.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L , and distributed profit, i.e. the product of dividend D and the number of shares N . Nothing is implied at this stage about who owns the shares.

$$Y = WL + DN \quad (1)$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad (2)$$

The productivity R depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X .

$$C = PX \quad (3)$$

The axioms represent the pure consumption economy, that is, no investment, no foreign trade, and no government.

The economic content of the four axioms is transparent. One point to mention is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit. Conventional approaches come to grief at this first axiomatic step.

2.2 Definitions

Income categories

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. With (4) wage income Y_W and distributed profit Y_D is defined:

$$Y_W \equiv WL \quad Y_D \equiv DN. \quad (4)$$

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

Key ratios

We define the sales ratio as:

$$\rho_X \equiv \frac{X}{O}. \quad (5)$$

A sales ratio $\rho_X = 1$ indicates that the quantity bought/sold X and the quantity produced O are equal or, in other words, that the product market is cleared.

We define the expenditure ratio as:

$$\rho_E \equiv \frac{C}{Y}. \quad (6)$$

An expenditure ratio $\rho_E = 1$ indicates that consumption expenditures C are equal to total income Y , in other words, that the household sector's budget is balanced.

We define the factor cost ratio as:

$$\rho_F \equiv \frac{W}{PR}. \quad (7)$$

A factor cost ratio $\rho_F = 1$ indicates that the nominal value of one hour's labor input W is equal to the value of output PR which implies that profit per hour, respectively per unit of output, is zero.

We define the distributed profit ratio as:

$$\rho_D \equiv \frac{DN}{WL}. \quad (8)$$

The distributed profit ratio may, for instance, assume a value between zero and 10 percent.

The axioms and definitions constitute the most elementary economic configuration. Clearly, real economies are a bit more complex. In order to cover the greater part of real world phenomena the structural axiomatic framework therefore has to be differentiated and extended. Because of its pivotal role for the understanding of how the economy works, we first turn to the phenomenon of profit.

3 Monetary profit

Total profit consists of monetary and nonmonetary profit. Here we are at first concerned with monetary profit. Nonmonetary profit is treated at length in (2012).

The business sector's monetary profit/loss in period t is defined with (9) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditure C – and costs – here identical with wage income Y_W :

$$Q_m \equiv C - Y_W. \quad (9)$$

Because of (3) and (4) this is identical with:

$$Q_m \equiv PX - WL. \quad (10)$$

This form is well-known from the theory of the firm.

3.1 The Profit Law

From (9) and (1) follows:

$$Q_m \equiv C - Y + Y_D \quad (11)$$

or, using the definitions (6) and (8),

$$Q_m \equiv \left(\rho_E - \frac{1}{1 + \rho_D} \right) Y. \quad (12)$$

The four equations (9) to (12) are formally equivalent and show profit under different perspectives. The Profit Law (12) tells us that total monetary profit is zero if $\rho_E = 1$ and $\rho_D = 0$. Profit or loss for the business sector as a whole depends on the expenditure and distributed profit ratio and nothing else (for details see 2013). Total income Y is the scale factor.

It is a unique fact of the history of economic thought that neither Classical, nor Walrasian, nor Keynesian, nor Marxian, nor Institutionalist, nor Monetary Economist, nor Austrian, nor Sraffian, nor Evolutionist, nor game theorist, nor Econophysicist ever came to grips with profit.

3.2 Total zero profit

From (9) in combination with (4) and (6) follows for the differentiated monetary profits of two firms, respectively:

$$\begin{aligned}
Q_{m1} &\equiv \rho_{E1}Y - W_1L_1 \\
Q_{m2} &\equiv \rho_{E2}Y - W_2L_2 \\
Q_m &\equiv (\rho_{E1} + \rho_{E2})Y - Y_W \\
\text{with } Y_W &\equiv W_1L_1 + W_2L_2.
\end{aligned} \tag{13}$$

Monetary profit of the business sector as a whole is given as difference of total consumption expenditures and total wage income. This simplifies to:

$$\begin{aligned}
Q_m &\equiv (\rho_{E1} + \rho_{E2} - 1)Y \\
\text{if } Y_D = 0 &\rightarrow Y = Y_W.
\end{aligned} \tag{14}$$

The zero profit condition for the business sector as a whole then reads:

$$\begin{aligned}
\rho_{E1} + \rho_{E2} &= 1 \\
\text{if } Y_D &= 0.
\end{aligned} \tag{15}$$

This is the balanced budget condition for an economy with two firms.

Overall zero profit is compatible with any distribution of individual profit and loss among firms. In the case of two firms the profit of one firm is then equal to the loss of the other.

3.3 Individual zero profit

From (10) in combination with (5) follows for the differentiated monetary profit of firm 1:

$$Q_{m1} \equiv P_1\rho_{X1}R_1L_1 - W_1L_1. \tag{16}$$

Applying (7) this reduces to:

$$Q_{m1} \equiv \rho_{X1}P_1R_1L_1 \left(1 - \frac{\rho_{F1}}{\rho_{X1}}\right). \tag{17}$$

The zero profit condition for firm 1 then reads:

$$\rho_{X1} = 1 \text{ and } \rho_{F1} = 1 \quad \text{resp.} \quad P_1 = \frac{W_1}{R_1} \text{ or } \frac{W_1}{P_1} = R_1 \quad (18)$$

and analogous for all other firms. The condition implies: if the market is cleared and the factor cost ratio ρ_F is unity then the profit of the respective firm is zero. A factor cost ratio of unity means that the market clearing price is equal to unit wage costs, or in other words, that the real wage is equal to the productivity.

A zero profit for the economy as a whole according to the Profit Law (12) is compatible with profit and loss for individual firms.

4 The objective Law of Supply and Demand

The basic intuition of conventional economics is that prices and quantities are *uno actu* determined by the interaction of supply and demand and that the unhindered working of the price mechanism guarantees an optimal outcome. This intuition is formally translated into supply functions and demand functions which imply a host of behavioral assumptions. We now move homo oeconomicus out of focus and derive the structural axiomatic Law of Supply and Demand.

4.1 The initial state

The axiom set is – as it has to be – free of any assumptions about causality or directionality. Directionality is an add-on to the axiom set. When the price is now taken as the dependent variable, then from the 3rd axiom and the condition of market clearing $\rho_X = 1$ follows for firm 1:

$$P_1 = \frac{C_1}{X_1} = \frac{\rho_{E1}Y}{R_1L_1} \quad (19)$$

$$\text{if } \rho_{X1} = 1.$$

Distributed profit is set to zero and together with the 1st axiom this gives the price determinants in greater detail:

$$P_1 = \rho_{E1} \left(\frac{W_1}{R_1} + \frac{W_2 L_2}{R_1 L_1} \right) \quad (20)$$

$$\text{if } \rho_{X1} = 1, Y_D = 0.$$

The market clearing price for output 1 is equal to the product of demand, represented by the partial expenditure ratio ρ_{E1} , and supply, represented by a combination of

the unit wage costs W/R and the allocation of labor input between the firms. The markets are entangled and one market cannot be cut out of the picture and treated in isolation. That is, partial analysis in the style of Marshall's textbook models is ruled out *ab initio*. The minimum number of markets to be considered is in each and any case two. The methodological veto against partial analysis has always been that it is prone to the fallacy of composition, that is, to false generalizations.

Under the condition of equal wage rates in both firms the price equation becomes rather straightforward:

$$P_1 = \frac{\rho_{E1}}{R_1} \underbrace{\left(1 + \frac{L_2}{L_1}\right)}_c W \quad (21)$$

if $\rho_{X1} = 1, Y_D = 0, W_1 = W_2 = W$.

With wage rates equal and the allocation of labor input fixed, the price of product 1 rises with demand and falls with supply as shown in Figure 1. The familiar supply-demand cross is replaced by the structural axiomatic price surface. With demand, i.e. ρ_{E1} , and supply, i.e. R_1 , given the market clearing price is unambiguously determined. Because all variables in (21) are measurable the result is testable in principle.

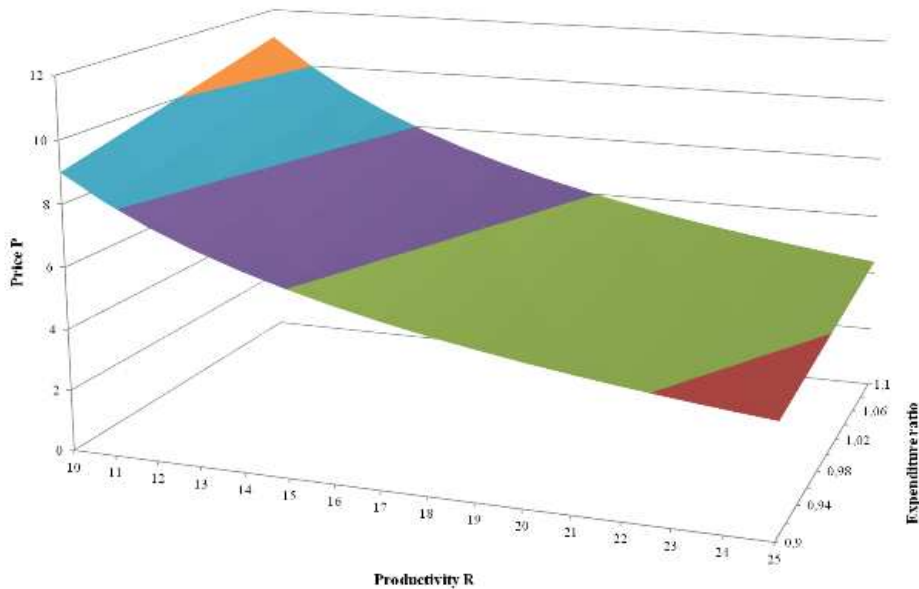


Figure 1: The price rises with demand and falls with supply under the condition of a fixed allocation of labor input and a fixed wage rate; demand is here represented by the expenditure ratio and supply by productivity

A proportional growth of labor input in both firms, which increases output O_1 , has no effect on the market clearing price. The price equation (21) states that a proportionally growing labor supply creates its own demand and this keeps the price perfectly constant. The point to emphasize is: an increase of L_1 affects the market clearing price if it is partial but not at all if it is due to a general and proportional increase of labor supply. The effect of an employment variation on the price depends on what happens in the other firm. This interrelation tends to be overlooked in partial analysis.

Variations of output O_1 , i.e. supply, that are due to a varying productivity R_1 at constant labor input move the market clearing price in the opposite direction. Thus (21) gives a precise formal underpinning to the general statement that the market clearing price rises with increasing demand and falls with increasing supply.

In analogy to (19) one gets for firm 2:

$$P_2 = \rho_{E2} \left(\frac{W_2}{R_2} + \frac{W_1 L_1}{R_2 L_2} \right) \quad (22)$$

if $\rho_{X1} = 1, Y_D = 0$.

From (19) follows for relative prices:

$$\frac{P_1}{P_2} = \frac{\frac{\rho_{E1} Y}{R_1 L_1}}{\frac{\rho_{E2} Y}{R_2 L_2}} \rightarrow \frac{P_1}{P_2} = \frac{\rho_{E1} R_2 L_2}{\rho_{E2} R_1 L_1} = \underbrace{\frac{\rho_{E1}}{\rho_{E2}}}_{\text{demand}} \underbrace{\frac{O_2}{O_1}}_{\text{supply}} \quad (23)$$

$$\text{if } \rho_{X1} = 1, \rho_{X2} = 1, Y_D = 0.$$

The price ratio is determined by the ratios of the expenditure ratios, the productivities and the allocation of labor input, or in loose words, by relative supply and demand.

On the other hand, from the individual zero profit condition (18) follows for relative prices:

$$\frac{P_1}{P_2} = \frac{\frac{W_1}{R_1}}{\frac{W_2}{R_2}} \rightarrow \frac{P_1}{P_2} = \frac{W_1 R_2}{W_2 R_1} \quad (24)$$

$$\rho_{X1} = 1, \rho_{F1} = 1, \rho_{X2} = 1, \rho_{F2} = 1.$$

Taken together (23) and (24) yield the proportionality condition:

$$\frac{W_1 L_1}{W_2 L_2} = \frac{\rho_{E1}}{\rho_{E2}} = \frac{1}{\frac{1}{\rho_{E1}} + 1} \quad (25)$$

$$\rho_{X1} = 1, \rho_{F1} = 1, \rho_{X2} = 1, \rho_{F2} = 1, Y_D = 0.$$

Under the condition of zero profit in each firm the relation of wage incomes of both firms must be equal to the relation of expenditure ratios. Because of the condition of budget balancing (15) the ratios are here not independent. The partitioning of consumption expenditures as given by ρ_{E1} expresses the household sector's preference, which in turn depends on the individual household's expenditure ratio, which in turn depends on individual preferences. It goes without saying that the households always realize the optimal partitioning of their consumption expenditures. This is a behavioral tautology without particular practical consequence. Utility maximization cannot determine the concrete value of ρ_{E1} .

It is assumed now that the labor supply for each firm is individually given. That is to say that the quality of the labor input is different for both firms. The distribution of the quantities and the specific qualities of the factor endowment are given. It is assumed further that the inhomogeneous labor supply is fully employed. The full employment supplies are given with L_1^θ, L_2^θ . From (25) then follows finally for the relation of the wage rates in both firms:

$$\frac{W_1}{W_2} = \frac{\frac{\rho_{E1}}{L_1^\theta}}{\frac{\rho_{E2}}{L_2^\theta}} \quad (26)$$

$$\rho_{X1} = 1, \rho_{F1} = 1, \rho_{X2} = 1, \rho_{F2} = 1, Y_D = 0.$$

The relation of the wage rates of firm 1 and 2 increases if final demand ρ_{E1} increases and decreases if full employment labor supply L_1^θ increases.

To get the absolute values, (26) is rewritten as:

$$W_1 = \frac{\frac{\rho_{E1}}{L_1^\theta}}{\frac{\rho_{E2}}{L_2^\theta}} W_2 \quad (27)$$

$$\rho_{X1} = 1, \rho_{F1} = 1, \rho_{X2} = 1, \rho_{F2} = 1, Y_D = 0.$$

If the wage rate of the second firm W_2 is taken as numéraire and exogenously fixed then the absolute values of all other nominal variables are determined.

We define the harmony ratio as a stacked ratio:

$$\rho_{\Phi} \equiv \frac{\frac{\rho_{E1}}{L_1}}{\frac{\rho_{E2}}{L_2}}. \quad (28)$$

And we define the labor input ratio of firm 1, and analogous for firm 2, as a fraction of particular labor input to total labor input:

$$\rho_{L1} \equiv \frac{L_1}{L} \quad (29)$$

$$\text{with } L \equiv L_1 + L_2 \quad \text{here } L^{\theta} \equiv L_1^{\theta} + L_2^{\theta}.$$

Eq. (27) is rewritten in its final form as:

$$W_1 = \frac{\frac{\rho_{L1}}{\rho_{E2}}}{\rho_{L2}} W_2 \quad \rightarrow \quad W_1 = \rho_{\Phi} W_2 \quad (30)$$

$$\text{with } \rho_{E1} + \rho_{E2} = 1, \rho_{L1} + \rho_{L2} = 1, Y_D = 0.$$

In the case of a harmonic structure with $\rho_{\Phi} = 1$ the (average) wage rates are equal in both firms. The harmonic structure is, in the initial period, simply a coincidence of demand structure and supply structure. Nothing is said at the moment about any adaptation processes. We are not concerned with human behavior only with structural necessities. If full harmony of final demand and factor endowment obtains then (average) wage rates must be equal in the zero profit consumption economy.

Doubling of labor inputs or any other proportional change leaves the harmonic ratio unaltered.

If labor input is not allocated in proportion to the partitioning of consumption expenditures then the wage rates between firms must be different. This is a structural property that follows from the zero profit condition. If, for example, the full employment input L_1^{θ} increases while the partitioning of consumption expenditures remains unchanged, then the wage rate W_1 must be lower than W_2 . Wage differentials between firms are due to a mismatch of demand, expressed by the expenditure ratios, and different qualities of labor supply. Quality means in this context: can be employed in firm 1 but not in firm 2 and vice versa. If workers could adapt this specific quality quickly to the demand structure then the wage rates of both firms would be equal according to (30). These wages rates are *averages* for each firm, that is, there can be any wage differentiation *within* the firm. The distribution of working time per quality category and wage rate per quality category in each firm gives the average wage rates W_1 and W_2 . While these wages rates must be equal in

the harmonic structure there can be at the same time wage differentiation within the firms.

If the wage rates $W_1 = W_2 = W$ are constant and total employment L is constant, total income Y is not affected by a changing allocation of labor input between the firms.

From (23) in combination with (28) now follows:

$$\frac{P_1}{P_2} = \rho_\Phi \frac{R_2}{R_1} \quad (31)$$

$$\rho_{X1} = 1, \rho_{F1} = 1, \rho_{X2} = 1, \rho_{F2} = 1.$$

In a more compact form:

$$\rho_P = \frac{\rho_\Phi}{\rho_R} \quad (32)$$

$$\text{with } \rho_P \equiv \frac{P_1}{P_2}, \rho_R \equiv \frac{R_1}{R_2}.$$

Relative prices depend on the value of the harmony ratio and the productivities. The harmony ratio captures the supply and demand relations. In the harmonic structure relative prices are inverse to relative productivities:

$$\rho_P = \frac{1}{\rho_R} \quad (33)$$

$$\text{if } \rho_{X1} = 1, \rho_{F1} = 1, \rho_{X2} = 1, \rho_{F2} = 1, \rho_\Phi = 1, Y_D = 0.$$

This elementary case is characterized by the objective conditions of market clearing, zero profit, a harmonic structure of supply and demand, full employment, and equal average wage rates $W_1 = W_2$ with W_2 as numéraire. From this follow the absolute values of the product prices under the given production conditions and the given factor endowment. The household sector's preferences play no role for the relative prices under the conditions of the harmonic structure, which says – roughly – that the partitioning of nominal demand is in proportion with the specific labor supplies.

The structural axiomatic Law of Supply and Demand (33) states that relative prices are, in the most elementary case, determined by supply which is represented by relative productivities. This follows from objective conditions. Subjective elements like preferences play no role.

It deserves explicit mention that market clearing, budget balancing, and zero profit are Walrasian conditions. Here we are on common ground. These objective conditions in conjunction with the axioms formally determine the system and there

is no room left for additional behavioral assumptions like profit maximization. This is a huge analytical advantage because the usual behavioral assumptions are unacceptable.

4.2 Demand variations

We take (27) as point of reference and now increase the demand for product 1, such that $\rho_{E1.1} > \rho_{E1.0}$:

$$W_1 = \frac{\frac{\rho_{E1}}{L_1^\theta}}{\frac{\rho_{E2}}{L_2^\theta}} W_2 \quad (34)$$

$$\rho_{X1} = 1, \rho_{F1} = 1, \rho_{X2} = 1, \rho_{F2} = 1, Y_D = 0.$$

This means $\rho_{E2.1} < \rho_{E2.0}$ because of (15). The demand shift effects an increase of the wage rate W_1 . Demand is no longer compatible with the initially given factor allocation, that is $L_{1.1} > L_{1.0}^\theta$ and $L_{2.1} < L_{2.0}^\theta$. In order to restore structural harmony it is necessary to change the specific quality of a part of the workforce of firm 2 and to initiate a move of this part to firm 1. The demand shift must be followed by an adaptation of the allocation of the workforce.

This, of course, takes time and a lot of practical measures. First of all, labor input of firm 1 is not stepped up immediately with an increase of demand. Thus output remains for a while unchanged. What happens in firm 1 under the condition of market clearing is that the price increases according to (21). However, the wage rate must increase according to (34). The immediate wage increase is necessitated by the zero profit condition. If the wage rate remains unchanged firm 1 makes a profit. By consequence, firm 2 makes a loss because its market clearing price falls. It is essential for the continuation of the adaptation that firm 2 does not immediately drop out of the market. As soon as part of the workforce moves from firm 2 to firm 1 prices and wage rates return to their former level. To follow this process through all details requires a simulation. This in turn requires additional assumptions about the direction and magnitude of the changes of each variable over time. This is an exercise for another occasion.

Here we take an analytical shortcut and assume that the reallocation of labor input happens *uno actu* with the demand shift. Hence the harmony ratio is unity in period 1 just as it was in the initial period. The wage rates remain equal and constant. The migration of labor takes place immediately and at equal wage rates. This scenario is like jumping from one equilibrium to the next; it is analytically convenient but descriptively false. However, this is the only scenario that conforms to the zero profit condition.

Under the condition of immediate adaptation prices remain unaffected by demand variations. The real effect of a demand shift is a reallocation of labor between firms. There is no effect on prices and wages. Hence, price and wage flexibility is not required. Under the condition of immediate adaptation there is no signaling of prices and no profit or wage incentive to get the agents going. The demand shift logically requires an immediate shift of labor input without any guidance by price signals. This implies that the agents have to be guided by other signals.

4.3 Supply variations

From (20), (27), and (28) follows alternatively for the market clearing price of product 1 in the zero profit consumption economy:

$$P_1 = \rho_\Phi \frac{W_2}{R_1} \quad \rightarrow \quad P_1 = \frac{W}{R_1} \quad (35)$$

if $\rho_\Phi = 1$.

An increase of supply, which is formally represented by an increase of the productivity R_1 , effects a fall of the market clearing price P_1 . If nothing else changes, the household sector has a greater quantity of product 1 at its disposal. That is, the composition of consumption goods changes. If the initial relation of quantities bought X_1/X_2 had been in some unspecified sense ‘optimal’ then the household sector has now ‘too much’ of product 1. In order to restore the former relation, the supply of and demand for product 1 has to be curtailed, and correspondingly expanded for product 2.

It is intuitively clear that a productivity boost which increases the supply of product 1 under the condition of market clearing, i.e. $O_1 = X_1$, effects a reshuffling of labor input between the firms, such that L_1 is reduced and L_2 is expanded in order to restore the relation that has been realized before the productivity change. This is to say that ‘taste’ or ‘preferences’ are fixed and do not change while the economy adapts to a productivity change. The inverted commas indicate that these concepts are poorly defined. That does not matter much because these concepts are not essential in the structural axiomatic context.

We simply add the objective condition that the relation of quantities bought by the household sector remains constant, or more precisely:

$$\frac{X_1}{X_2} = c \quad \rightarrow \quad \frac{X_1 P_1}{X_2 P_2} = \frac{c P_1}{P_2} = \frac{C_1}{C_2} = \frac{\rho_{E1}}{\rho_{E2}}. \quad (36)$$

In the harmonic structure holds according to (31):

$$\frac{P_1}{P_2} = \frac{R_2}{R_1}. \quad (37)$$

From (36), (37), and (15) follows:

$$\rho_{E1} = \frac{1}{\frac{R_1}{cR_2} + 1}. \quad (38)$$

In order to keep the relation of products constant, the household sector reduces the expenditure ratio ρ_{E1} if the productivity R_1 increases. The complementary expenditure ratio ρ_{E2} increases correspondingly.

From the constancy of the real consumption pattern (36) follows on the other hand:

$$\frac{X_1}{X_2} = c \quad \rightarrow \quad \frac{R_1 L_1}{R_2 L_2} = c \quad \rightarrow \quad \frac{R_1}{R_2} = \frac{c L_2}{L_1} \quad (39)$$

if $\rho_X = 1$.

From this follows in turn for the labor input:

$$L_1 = \frac{L}{\frac{R_1}{cR_2} + 1} \quad (40)$$

if $\rho_X = 1$.

An increase of the productivity R_1 reduces the labor input L_1 . In relation to the initial endowment now less than the former full employment supply is needed, i.e. $L_{1.1} < L_{1.0}^\theta$, and by consequence $L_{2.1} > L_{2.0}^\theta$. Therefore, a change of specific quality and a move of part of the workforce from firm 1 to 2 is required.

Taken together (38) and (40) yield:

$$\frac{\rho_{E1}}{L_1} = \frac{1}{L} \quad (41)$$

The ratio of demand, represented by the expenditure ratio, and supply, represented by labor input, remains unchanged because total labor input L is fixed. Both variables are proportionally reduced. In firm 2 both variables are proportionally increased. As a result, the harmony ratio ρ_Φ remains unchanged and equal to unity. This implies equal wage rates according to (30).

When we blank out the time consuming steps of the adaptation process then the reallocation has to take place at constant wage rates and zero profits. In sum we have for firm

1: price down, expenditure ratio down, labor input down, real supply $O_1 = X_1$ up because the positive productivity effect overcompensates the negative employment effect,

2: price constant, expenditure ratio up, labor input up, real supply $O_2 = X_2$ up.

The productivity increase in one firm ultimately results in more consumption of *both* products. The structure remains harmonic and the relation of quantities bought remains constant. The price ratio declines. Because the wage rates are equal and constant, i.e. $W_1 = W_2 = W = c$, and total employment L is constant too, total income Y remains unaffected by the reallocation of labor.

The structural axiomatic analysis tells what must happen under certain preset conditions, it does not claim that agents behave spontaneously in the appropriate way. Note in particular that under the zero profit condition, which is a Walrasian equilibrium condition, firms have no motive to move from unemployment to full employment because profit is zero in any case. Likewise, employees have no pecuniary motive to move from one firm to the next in the harmonic structure because average wage rates are equal. The point is: because price and profit signals are absent they cannot initiate the appropriate reactions. The behavior of the system and the behavior of agents are entirely different things and they almost certainly do not coincide. The claim that price signals and optimizing behavior are sufficient for the functioning of the market system is unsubstantiated. The textbook story of the price mechanism has to be rewritten.

5 Conclusion

The standard approach is based on indefensible subjective-behavioral axioms which are in the present paper replaced by objective-structural axioms. The main results of the structural axiomatic analysis of the price mechanism are:

- The familiar supply-demand-equilibrium approach implies a logically defective profit theory and is therefore of little or no value.
- There is no place for supply functions and demand functions in a consistent formalism that is composed of structural axioms and objective conditions.
- The structural axiomatic Law of Supply and Demand states that relative prices are, in the most elementary case, determined by supply which is represented by relative productivities. This follows from objective conditions. Subjective elements on the demand side like preferences play no role. With subjectivism, marginalism too is out.

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