Explaining GDP Quarterly Growth from its Components in the Context of the Annual Overlap Method: A Comparison of Approaches

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18 August 2014

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Abstract

The use of chain-linked methods reduces significantly the problem of price structure obsolescence present in fixed base environments and it has been, therefore, adopted by many countries to measure GDP. The price updating it involves introduces a dimension new to those accustomed to the fixed based methodology that may produce confusion if not accounted for. Probably the most notorious difficulty generated by the introduction of chain-linked indices to the measurement of GDP has been that the aggregate is not the direct sum of its components, thus making it harder to explain its behaviour in terms of the specific sectors. To alleviate this problem most countries publish sector contributions in conjunction with aggregate GDP growth, however, there is no consensus on a single way of calculating these contributions when the annual overlap method is applied. In this context, this document compares a number of different ways of calculating contributions that have been suggested in the relevant literature and highlights their strengths and weaknesses. The results show that the outcomes of using different measures may vary considerably under certain circumstances, such as high price volatility, and that some of the measures do not fulfil certain desirable properties. In an application to Chilean GDP we find that the differences are negligible between the measures that do account for the chain-linking but significant when compared to the traditional fixed base measure.

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* The ideas expressed in this paper were developed while working at the Central Bank of Chile. The contents, however, by no means represent views of the Bank. Any opinions and mistakes are the authors responsibility. Email: mcobb@uc.cl

1. Introduction

Macroeconomic analysis devotes a fair amount of effort to the economy's real variables, thus generating a need for aggregate measurements of volumes and quantities. When evaluating the economy's performance, Gross Domestic Product (GDP) is the most often used indicator. Traditionally, GDP and other real variables have been measured using the fixed-base-year method; however, in the last decades its shortcomings have become obvious and difficult to ignore. As explained by Steindel (1995) when presenting the U.S. Bureau of Economic Analysis (BEA) change in method of measuring the growth of the U.S. economy, the dramatic and recurring reduction in prices exhibited by computers in the mid 80's and 90's introduced a considerable upward bias in the fixed-base aggregate creating a problem that was significant enough to justify changing the way of measuring GDP.

In line with the recommendations of the System of National Accounts (SNA, 1993) many countries have moved to an annual update of the relative price structure for their real output series through the generation of chain-linked series. This procedure greatly reduces the problem of price structure obsolescence but introduces a new dimension in the analysis that makes understanding the evolution of the series less straightforward. Probably the most notorious difficulty has been that an aggregate is not the direct sum of its components. This induces some practical issues and on the other hand may blur the analysis. It is unadvisable, however, to ignore the fact and continue working as if the accounting properties of the fixed base methodology are still valid because it could lead to error if the environment has suffered significant changes in its price structure (OECD, 2006a).

To alleviate this specific problem, the aggregate measure is generally accompanied by the contributions of each component to its growth, where these contributions do sum up to the total. The way of calculating these contributions, however, is not unique and depends on which of the three annual chain-linking methods is implemented.[1] It also depends on the type of index that is used. Canada and the U.S., for example, use chain-linked Fisher indices while most of the other countries use Laspeyres (OECD, 2006a). According to Eurostat/ECB (2008), at least six different formulas have been proposed to calculate contributions to quarterly growth.

The annual overlap chain-linking method is a practical and, therefore, popular method for Laspeyres volume measures (IMF, 2001), however, the calculation of quarterly contributions is not straightforward.[2] The difficulty consists in finding the right weights that permit them to sum up to the total (OECD, 2006a). Some countries find the contributions from using pair of years valued at the first year's prices. Others, like France and Germany have proposed their own way (Eurostat/ECB, 2008). A different approach becomes evident from a methodological note published by the OECD that utilizes an approximation to calculate contributions acknowledging that it is not right in a strict sense (OECD, 2013).

This document compares different approaches for calculating the component's contributions to the growth of a composite index that is built using the annual overlap method. It examines how they measure up to a number of properties that could be described as desirable and it looks at how they perform under general and specific circumstances. Although this document focuses its attention primarily on quarterly GDP growth, as most of the relevant literature does, the results extend to any composite measure that is built using this method. Section 2 briefly explains the annual overlap methodology, the calculation of annual contributions and desirable properties an expression for contributions to quarterly growth should have. Section 3 presents the different ways that have been proposed in the literature to explain quarterly aggregate growth from its components and how they measure up to the desirable properties. Section 4 examines how the different measures perform under stylized circumstances. Section 5 examines how the different measures performance using a set of real data and Section 6 summarizes the main findings.

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[1] See IMF (2001) for information on the different methods.

[2] According to the last available survey of OECD (2009) updated to May 2014, out of the 37 OECD countries plus Brazil and Russian Federation, 27 use the annual overlap method, 5 use the quarterly overlap method, 4 use an unspecified indirect method and one still uses a fixed-base method. It's worth mentioning that the five countries that use the quarterly overlap method are Australia, Canada, Japan, United Kingdom and United States.

2. Annual overlap chain-linked indices, contribution to annual growth and desirable properties of an expression for contributions to quarterly growth

The development of real economic measures typically involves comparing two periods keeping prices unaltered. The simplest approach has been to value the whole series using the prices of a specific (or base) period. This approach, however, imposes the price structure of that period to the whole series meaning it is subject to the obsolescence of this structure as one moves further away from the base year.[3] Due to this known problem, following the guidelines of the System of National Accounts (SNA, 1993), many countries, have moved to a system where the price structure is updated annually.

2.1. The annual overlap method and component's contribution to annual growth

There are various methodologies to implement the annual update being the annual overlap method of Laspeyres indices one of the more popular ones. This technique involves creating a set of overlapping links with a length of two years, where in each link both the quantities of the relevant year (*y*) and the previous one (*y-1*) are valued using average prices of the previous year (*y-1*). Then, using the growth rates of these links an annual time series is built starting from the first link.[4]

A practical difficulty that arises from using this and other methods for constructing chain-linked indices is that aggregates are not the direct sum of their components. This loss of additivity generates some practical issues but, more importantly, it reflects the fact that the well-known accounting identities of GDP hold only approximately for the levels of the series.[5] To alleviate the inconveniences produced by the lack of additivity, the publishing agencies accompany the aggregate with the corresponding component's contributions to aggregate growth.

Due to the lack of additivity, the traditional way of calculating contributions ceases to be accurate, but given that aggregate growth in any year is equal to a two-year Laspeyres index, that is:

$$\frac{\Delta Q^{y}}{Q^{y-1}} = \frac{\sum_{j=1}^{J}\left(p_j^{y-1} \cdot q_j^{y}\right)}{\sum_{j=1}^{J}\left(p_j^{y-1} \cdot q_j^{y-1}\right)} - 1 \qquad (1)$$

where,

Q^{y} : chain-linked aggregate in year y

q_j^{y} : component j in year y

p_j^{y} : implicit price deflator of component j in year y (calculated as nominal value over chain-linked value)

the contributions may be calculated by introducing the updating of weights into the formula. Formally, the contribution to the growth rate of the annual totals for year *y* of component *j* is the growth of the component, q_j, times its relative nominal importance in the previous year:[6]

$$C_j^{y} = \frac{\left(p_j^{y-1} \cdot q_j^{y-1}\right)}{\sum_{j=1}^{J}\left(p_j^{y-1} \cdot q_j^{y-1}\right)} \cdot \left(\frac{q_j^{y}}{q_j^{y-1}} - 1\right) \qquad (2)$$

Given that the expression for annual contribution is derived directly from the chain-linking process, the sum of them adds up to total aggregate growth even when the sum of the components in levels do not.

As useful as this expression may be, often agents are interested in examining the dynamics of a given variable at a higher frequency (i.e. quarterly). As one would expect, quarterly series that are chain-linked using the annual overlap method suffer from the same problem of lack of additivity, but additionally, the introduction of weights that change on an annual basis, as opposed to every quarter, make the process of finding the appropriate weights that permit the component's quarterly contributions to sum up to the total less straightforward.

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[3] What matters in this context are the relative prices and all the discussion in this paper refers to them unless stated otherwise.

[4] The reference year is the year to which the prices of the first link belong.

[5] The accounting identities hold perfectly for the links only.

[6] Derivation presented in Annex 1. An example of the chain linking method and annual contributions is provided in Annex 2. The implementation of this particular method (and others) is documented extensively in IMF (2001).

From an economic perspective, the most obvious choice would seem to be to use the contributions that are derived from the two period Laspeyres links. These contributions would reflect, by definition, the true share of growth that is attributable to each component. The group of links, however, are not an actual time series. For descriptive purposes these contributions could be sufficient, but economic analysis is greatly concerned with the evolution of different variables and, therefore, requires time series for much of its work. To this avail, chain-linked time series are created and in this context it is necessary to fit the contributions of the links into the time series framework.

2.2. Desirable properties of an expression for contributions to quarterly growth

As it was mentioned before, given the difficulty of assigning weights for the contributions to be calculated, a number of options coexist. To compare these alternatives, however, first it is necessary to set some requirements one would expect an ideal measure to fulfil and then compare how the competing candidates perform. Before that, it is worth mentioning that any proposed measure to be considered should be economically meaningful and unique. This rules out, for example, methods that rely in some way on the distribution of unwanted discrepancies according to some *ad-hoc* rule or numerical procedure.

For the purpose of this comparison we consider the following requirements not as mandatory but as desirable:

1- Additivity of the contributions to annual growth of the aggregate quarterly series,

2- Consistency of the component's contribution to the annual growth of the aggregate quarterly series with the contributions to growth of the aggregate annual series,

3- Additivity of the contributions to quarterly growth of the aggregate quarterly series, and

4- Consistency of the component's contribution to the quarterly growth of the aggregate quarterly series with the contributions to growth of the aggregate annual series.

The additivity requirements are of first order and fairly straightforward. They simply state that the sum of contributions for all components in a specific period should add up to the aggregate growth in that period, that is:

$$\frac{\Delta Q_t}{Q_{t-1}} = \sum_{j=1}^{J} \hat{c}_{j,t}^{QoQ} \qquad \text{and} \qquad \frac{\Delta Q_t}{Q_{t-4}} = \sum_{j=1}^{J} \hat{c}_{j,t}^{YoY}$$

where, $\hat{c}_{j,t}^{QoQ}$ and $\hat{c}_{j,t}^{YoY}$ are a proposition for the contributions of component *j* to quarterly and annual aggregate growth.

To develop these restrictions, an expression for the aggregate growth of the quarterly series is required. To do this we resort to one of the properties of the annual overlap method. This is that, by definition, series valued at previous year average prices are cross-section consistent—meaning that the sum of the valued components equals the valued total (IMF, 2001). This means that at any time *t* the total may be valued at previous year average prices and that is equal to the sum of the appropriate components also valued at previous year average prices, that is:[7]

$$P_t^{y-1} \cdot Q_t = \sum_{j=1}^{J} p_{j,t}^{y-1} \cdot q_{j,t}$$

where,

Q_t : Aggregate chain-linked series in quarter *t*

$q_{j,t}$: Component *j* in quarter *t*

P_t^{y-1} : Annual aggregate price deflator of the year before the year to which quarter *t* belongs.

$p_{j,t}^{y-1}$: Component's *j* annual price deflator of the year before the year to which quarter *t* belongs.

Aggregate growth may then be expressed in the following way:

$$\frac{\Delta Q_t}{Q_{t-s}} = \frac{1}{Q_{t-s}} \sum_{j=1}^{J} \left(w_{j,t} \cdot q_{j,t} - w_{j,t-s} \cdot q_{j,t-s} \right) \qquad \text{where} \quad w_{j,t} = \frac{p_{j,t}^{y-1}}{P_{Q,t}^{y-1}} \tag{3}$$

where $s = 4$ when calculating the contribution to annual growth and $s = 1$ for quarterly growth.

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[7] Average prices refer to the annual implicit price deflators, which are those that result from dividing the annual nominal value by the annual chain-linked value (as opposed to averaging the quarterly prices).

It is worth highlighting the fact that although the weights have a quarterly subscript *t*, they are in fact always annual and do not change within a year. The "quarterly" weights are used, instead of directly using the annual weights, to allow for a unique formula. This becomes clear form examining expression (3), where if the respective annual weights were used directly, in the context of quarterly contributions, an expression for the first quarter would be different from the one appropriate for the remaining three quarters making it cumbersome to work with.[8]

In the context of formalizing the desired properties we find that the following equation must hold in order for requirement 1 to be fulfilled:

$$\sum_{j=1}^{J} \hat{c}_{j,t}^{YoY} = \frac{1}{Q_{t-4}} \cdot \sum_{j=1}^{J} \left(w_{j,t} \cdot q_{j,t} - w_{j,t-4} \cdot q_{j,t-4} \right) \tag{R.1}$$

As mentioned before, the additivity requirements are of primary importance given that one of the main reasons for providing contributions is for them to sum up to the total. The consistency requirements, however, refer to a desired consistency in a sense of that calculating equivalent contributions from two different frequencies should result in the same outcome.

Expressing the consistency requirements in a compact way is less straightforward than for the others, because quarterly growth rates are not expected to sum up to the yearly growth rate. However, an easy way of circumventing this is by concentrating on the changes in levels as these should be equivalent. With this in mind, the aggregate changes are easily attainable.

For the annual series we have that aggregate change is easily obtainable from multiplying its growth rate by Q^{y-1}. Then, we may define the contribution to the aggregate yearly change by multiplying expression (2) by Q^{y-1} as:[9]

$$K_j^y = \frac{Q^{y-1}}{q_j^{y-1}} \cdot \frac{\left(p_j^{y-1} \cdot q_j^{y-1} \right)}{\sum_{j=1}^{J} \left(p_j^{y-1} \cdot q_j^{y-1} \right)} \cdot \left(q_j^y - q_j^{y-1} \right) \quad = \quad \omega_j^y \cdot \left(q_j^y - q_j^{y-1} \right) \qquad \text{where} \quad \omega_j^y = \frac{p_j^{y-1}}{P_Q^{y-1}} \tag{4}$$

In the same way we define the corresponding propositions for the contribution to the respective annual and quarterly changes of the quarterly series as:

$$\hat{k}_{j,t}^{YoY} = Q_{t-4} \cdot \hat{c}_{j,t}^{YoY} \qquad \text{and} \qquad \hat{k}_{j,t}^{QoQ} = Q_{t-1} \cdot \hat{c}_{j,t}^{QoQ}$$

Regarding the contributions to annual change of the quarterly series, given that one of the properties of the annual overlap method is that annual figures are equal to the sum of the corresponding quarterly figures, both for the aggregate and the components, one would expect the same to be true for the respective contributions to change.

Relying on the desirability of (R.1), we would have that:

$$\sum_{j=1}^{J} \hat{k}_{j,t}^{YoY} = Q_t - Q_{t-4}$$

If the temporal additivity of contributions holds the following proposition is true when *l* is the first quarter of year *y*:

$$\sum_{j=1}^{J} \sum_{t=l}^{l+3} \left(\hat{k}_{j,t}^{YoY} \right) = \sum_{t=l}^{l+3} \left(Q_t - Q_{t-4} \right) = Q^y - Q^{y-1} = \sum_{j=1}^{J} K_j^y$$

Therefore, requirement 2 may be written in the following way:

$$K_j^y = \sum_{t=l}^{l+3} \hat{k}_{j,t}^{YoY} \tag{R.2}$$

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[8] In the first quarter $w_{j,t}$ is equal to w_j^y and $w_{j,t-1}$ to w_j^{y-1}, but for the remaining three quarters both $w_{j,t}$ and $w_{j,t-1}$ are equal to w_j^y.

[9] Given the mixing of frequencies within formulas it might be helpful to state explicitly that any variable that has the superscript *y* or *y-1* is an annual figure. Quarterly figures have a subscript *t*. The only exceptions are those that are the annual figures in reference to a certain quarter that have both a subscript *t* and the superscript *y* (or *y-1*).

As for requirement 1, requirement 3 is directly attainable from expression (3):

$$\sum_{j=1}^{J}\hat{c}^{QoQ}_{j,t} = \frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left(w_{j,t}\cdot q_{j,t} - w_{j,t-1}\cdot q_{j,t-1}\right) \tag{R.3}$$

The derivation of requirement 4 is less straightforward and is provided in Annex 3. It states that the following should hold:

$$\begin{aligned} K_j^y &= \sum_{t=l-3}^{l+3}\left(4-abs(t-l)\right)\cdot\hat{k}^{QoQ}_{j,t} \\ &= \hat{k}^{QoQ}_{j,l-3} + 2\cdot\hat{k}^{QoQ}_{j,l-2} + 3\cdot\hat{k}^{QoQ}_{j,l-1} + 4\cdot\hat{k}^{QoQ}_{j,l} + 3\cdot\hat{k}^{QoQ}_{j,l+1} + 2\cdot\hat{k}^{QoQ}_{j,l+2} + \hat{k}^{QoQ}_{j,l+3} \end{aligned} \tag{R.4}$$

Having established the desired requirements, we can proceed to evaluate how different approaches comply with them.

3. Different ways of explaining quarterly aggregate growth from its components and how they measure up to the desirable properties

Eurostat/ECB (2008) states the fact that the traditional way of calculating contributions, the formula for additive measures, does not yield satisfying results when applied on chain-linked volumes and numbers six different formulas that have been proposed. The mentioned formulas represent a compromise between user friendliness and performance.[10] For the purpose of this document we concentrate on four measures: the traditional formula of the additive case, a formula based on the previous year average prices, the formula proposed by INSEE (2007) for the French national accounts and a formula proposed by Cobb (2013) for the Chilean national accounts.

Taking into account the difficulties encountered in finding a unique expression for quarterly contributions, it is reasonable to ask the question of whether the traditional way of calculating contributions, understood as the change of the component in *t* over the aggregate in *t-1*, is a sufficiently good approximation. One would sense that this is not necessarily the case given that the shortcomings of the fixed base methodology justified radically changing the way GDP was measured in the U.S. (Steindel, 1995). Some researchers, however, could feel inclined to use it in favour of simplicity and therefore we use the fixed base or fixed-weights method (*FW*) for calculating contributions as the first approach to serve as an example of the possible errors one could incur in if one ignored the chain-linked nature of the aggregate:

$$c^{FW}_{j,t} = \omega_j^{ref.year}\cdot\frac{\left(q_{j,t}-q_{j,t-s}\right)}{Q_{t-s}} \quad \text{where } \omega_j^{ref.year} = \frac{p_j^{ref.year}}{P_Q^{ref.year}} \tag{E.1}$$

From an economic perspective, the most obvious choice would seem to be to use the contributions that are derived from the Laspeyres links. The group of links, however, are not an actual time series and therefore it is not possible to present a unique general expression for them. The nearest option is to build contributions using (annual) previous year prices to generate the relevant weights:

$$c^{PYP}_{j,t} = w_{j,t}\cdot\frac{\left(q_{j,t}-q_{j,t-s}\right)}{Q_{t-s}} \qquad \text{where, as before, } w_{j,t} = \frac{p_{j,t}^{y-1}}{P_{Q,t}^{y-1}} \tag{E.2}$$

Due to some shortcomings that will become clear further on, a proposition that builds on approach 2 is the expression provided by INSEE (2007).[11] This proposition builds on the contributions that are derived from the Laspeyres links but corrects them by an additive factor.

$$c^{\text{INSEE}}_{j,t} = w_{j,t}\cdot\frac{q_{j,t}-q_{j,t-s}}{Q_{t-s}} + \left(w_{j,t}-w_{j,t-s}\right)\cdot\left(\frac{q_{j,t-s}}{Q_{t-s}} - \frac{q_{j,t}^{y-1}}{Q_t^{y-1}}\right) \tag{E.3}$$

where,

$q_{j,t}^{y-1}$: Annual value for component *j* the year before the year to which quarter *t* belongs.
Q_t^{y-1} : Annual chain-linked aggregate the year before the year to which quarter *t* belongs.

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[10] The whole presentation is found in Annex 2 of Eurostat/ECB (2008).

[11] A summarized exposition in English may be found in Banque Nationale de Belgique (2010).

Finally, Cobb (2013) presents an expression similar to that of INSEE (2007) but suggests an alternative correction factor:[12]

$$c_{j,t}^{Cobb} \quad = \quad w_{j,t} \cdot \frac{q_{j,t} - q_{j,t-s}}{Q_{t-s}} \quad + \quad \left(w_{j,t} - w_{j,t-s}\right) \cdot \left(\frac{q_{j,t-s}}{Q_{t-s}} - \frac{1}{Q_{t-s}} \cdot \frac{q_{j,t}^{y-1}}{f} \right) \quad \text{, where } f = 4 \text{ for quarterly data} \qquad \text{(E.4)}$$

The compliancy of the different expressions for contributions is presented in Annex 4 and the results are summarized in Table 3.1. The additivity requirements are presented for each quarter.

Table 3.1: Compliancy of measures for calculating contributions with the specified requirements

Contribution to growth of the quarterly aggregate series:	**Annual growth**					**Quarterly growth**				
	R.1 Additivity				R.2 Consistency	R.3 Additivity				R.4 Consistency
	q1	q2	q3	q4		q1	q2	q3	q4	
Fixed weights (E.1)	-	-	-	-	-	-	-	-	-	-
Previous year prices (E.2)	-	-	-	-	Yes	-	Y	Y	Y	-
INSEE 2007 (E.3)	Y	Y	Y	Y	Yes	Y	Y	Y	Y	-
Cobb 2013 (E.4)	Y	Y	Y	Y	Yes	Y	Y	Y	Y	Yes

As one might expect the expression using fixed weights (E.1) does not fulfil any of the requirements we suggested as desirable for an expression for contributions. Using such an expression is equivalent to ignoring the implementation of chain-linked indices completely. This is not necessarily problematic depending on the setting, but a number of conditions have to be met. From the results presented in Annex 4, it is straightforward to see how the inaccuracy of using E.1 depends fundamentally on how different the prevailing price structure is from that of the reference year.

Passing on to compare the more relevant expressions, less intuitive is that, although it is the closest to the actual link with which the chain-linked aggregate is built, the expression that uses previous year prices, E.2, only fully satisfies requirement 2. This is not so counterintuitive given that within a two-year link all periods are valued at the prices of the first year, while in the chain-linked series the current year is valued at previous year prices but the previous year is valued at prices of the year before that. Then, both E.3 and E.4 propose using this measure, that is economically meaningful, but correcting it due to the shifting of weights that occurs when constructing the time series to ensure additivity. Expression E.3 only fails in fulfilling the quarterly consistency, while E.4 fulfils all of them.

At this point, a respectable question to ask is whether it makes much of a difference what expression you use. Using E.1 seems unadvisable given that it ignores the chain-linking completely and therefore significant mistakes could be made (Cobb, 2014), but expressions E.2 to E.4 are only marginally different from each other and simply ignoring the correction could simplify a process without a considerable negative impact on the results. Abad et al. (2007), for example, in the context of a quarterly exercise acknowledge the correction factor proposed by INSEE (2007) but choose to ignore it under the assumption that it is generally small. Under their framework, ignoring the correction could be harmless, but it obviously depends on their specific dataset. As a generalization it is probably unadvisable, especially when dealing with more detailed breakdowns.

4. Performance of the different ways of explaining quarterly aggregate growth from its components under stylized circumstances

To have a notion on how the different measures perform and avoid getting surprised by an odd number when some unusual episode occurs, in this section we observe how the different measures perform under a set of stylized circumstances. It is worth mentioning that the probability of unusual events occurring will obviously

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[12] The expression from Cobb (2013) is reordered to show it as the contributions from the Laspeyres links plus a correction.

depend on the specific characteristics of the data. In particular one would expect macro aggregates to be more stable overall than a detailed breakdown of specific sectors. However, a good measure should perform well in all circumstances.

4.1. Contributions under smooth growth

To have an idea of what the different measures involve in practice and, in particular, get a feeling for the magnitude of the corrections to E.2 under a realistic setting, we perform a simple comparative exercise with an aggregate built from 2 components where quantities A and B grow smoothly at a 2 and 5% annual rate respectively, while their prices vary 10 and -5%.[13] It is worth noting that the parameters are chosen in such a way that chain-linking has a relevant effect. The parameters reflect heterogeneous real growth of the components and significant changes in price structure. Table 4.1 presents the overall annual information.

Table 4.1: Component's chain-link weights for the example

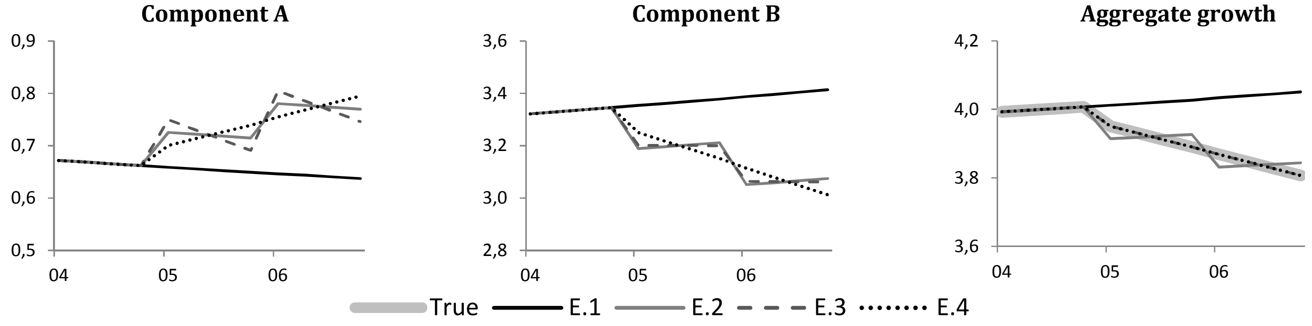
Year	*Quantities*		*Prices*		*Chain-linked (reference year=2003)*		*Chain-link weights*	
	A	*B*	*A*	*B*	*level*	*yoy*	*A*	*B*
2003	100	100	2.00	4.00	600		*1.00*	*1.00*
2004	102	105	2.20	3.80	624	***4.00%***	*1.00*	*1.00*
2005	104	110	2.42	3.61	648	***3.92%***	*1.10*	*0.95*
2006	106	116	2.66	3.43	673	***3.84%***	*1.21*	*0.90*

As it can be seen the price of A is relatively lower than B's in 2003 and therefore makes it weigh relatively less in the aggregate in that year. However, with the shifting price structure, A becomes progressively more important, as reflected by the chain-link weights, although it grows slower than B in real terms.

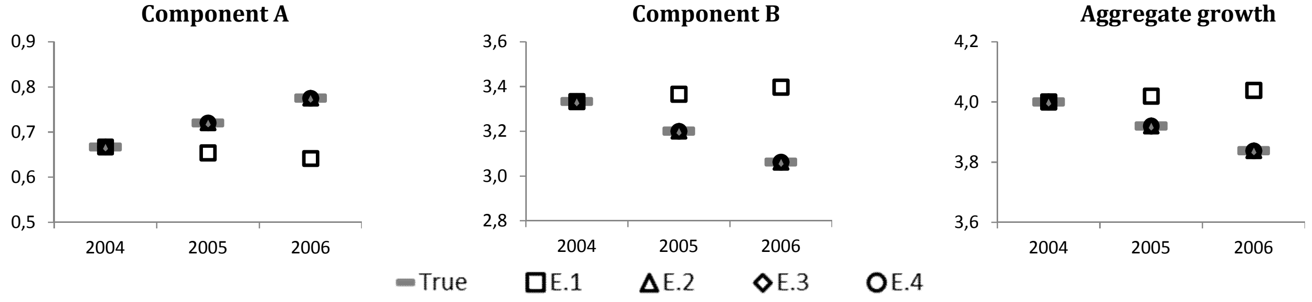
Figure 4.1: Contribution to annual growth of an aggregate quarterly series using different measures

(contributions in percentage points, growth as a percentage)

Quarterly contributions:



Implied annual contributions:



Based on the simple dataset, both contributions to annual growth of the quarterly series and contributions to quarterly growth are calculated using the four expressions. The results are presented in Figure 4.1 and 4.2 respectively. The first two charts present the contributions calculated for component A and B using each measure. The third chart presents the aggregate growth that is obtained from adding the component's contributions. Also,

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[13] This is the quarterly version of the chain-linking example in Annex 2. The numerical exercises are presented in Annex 5.

this chart includes the true chain-linked aggregate growth to highlight any differences with the sum of contributions.

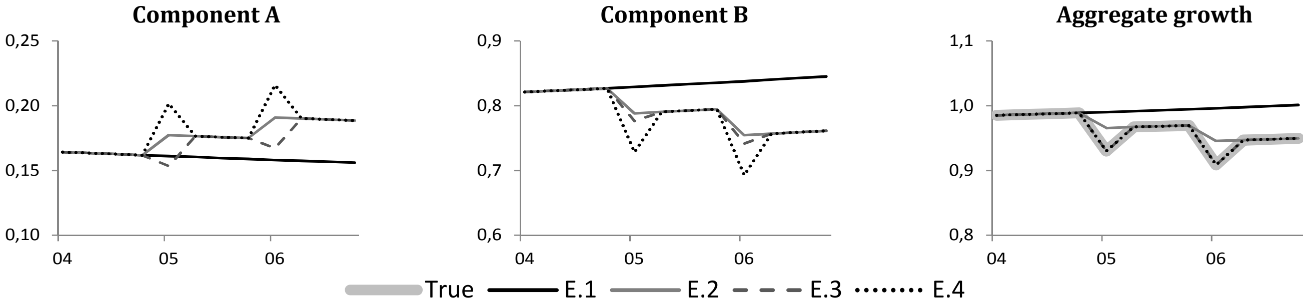
The implied contributions to growth of the annual series are presented in the bottom panels of the respective figures. These graphs highlight the compliancy of respective measures with the consistency requirements. It is worth remembering that the first two years of a chain-linked series are by definition a fixed base index and, therefore, measures E.2 to E.4 coincide with E.1 and fulfil all the requirements. The analysis therefore concentrates on the years that follow the initial link.

From Figure 4.1 it becomes clear that, regarding quarterly contributions to annual growth, E.1 shows results that differ greatly from those of the rest. In particular, for this example, it considerably underestimates the contribution of component A and overestimates that of component B. Then, as a result it generates an aggregate growth that is significantly higher than the actual growth. These significant discrepancies result in implied annual contributions that are considerably different from the true contributions calculated using expression 2.[14] All of the other measures, however, sum up to the true annual contributions, but both E.2 and E.3 exhibit "steps" in the component's contributions the firsts quarters while E.4 shows a relative smoothness similar to that of the aggregate. Comparing, the corrections applied to E.2 by E.3 and E.4, these corrections go in opposite directions for component A while for component B they go in the same direction. Regarding additivity, E.2 shows discrepancies with the true aggregate within years, but these cancel out in the implied annual contributions.

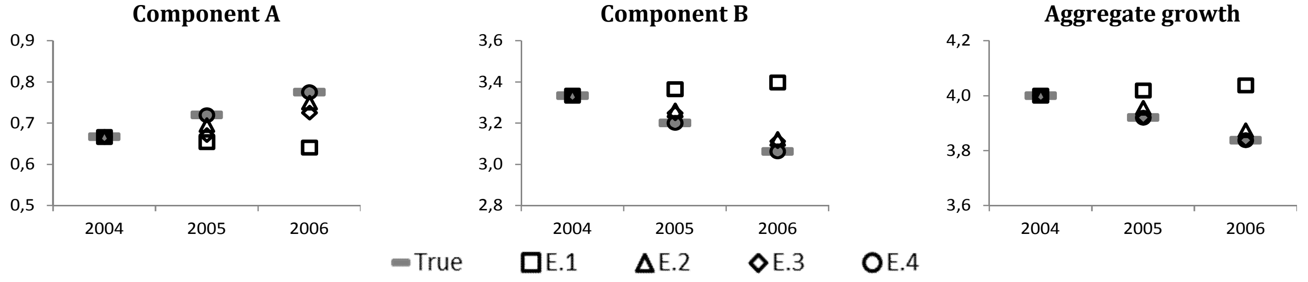
Figure 4.2: Contribution to quarterly growth of an aggregate series using different measures

(contributions in percentage points, growth as a percentage)

Quarterly contributions:



Implied annual contributions:



On a quarterly basis, as it can be seen in Figure 4.2, the shifting of weights at the beginning of each year becomes obvious due to the downward spikes that occur in aggregate growth every first quarter. Regarding the contributions to quarterly growth, E.1 shows similar deficiencies as before, but in this case E.2 fails to capture the aggregate shifts fully. As a result, the contributions from E.2 do not sum up to the total in the first quarters. For the remaining quarters, the three measures coincide and show no discrepancy. Using E.2 as the reference to be corrected, we observe, as before, that the corrections for A go in the opposite directions while the correction for B go the same way. Also, that the corrections of E.3 seem to be smaller in magnitude than those of E.4. This, however, is only due to the particular setting. Interestingly, as it can be seen from the implied annual contributions, the individual contributions of E.2 and E.3 are biased and suggest a contribution of A that is lower

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[14] Expression 2 refers to the component's contributions for the annual frequency, presented in section 2, not to be confused with expression E.2.

than its true contribution while suggesting a contribution of B that is higher. For E.3 this bias cancels out perfectly in the aggregate. The contributions calculated using E.4 are consistent with the annual figures both for the components and the aggregate.

4.2. Contributions under smooth annual growth with differing quarterly volatility

The previous example helps to shed some light on some points, but leaves out an important feature of high frequency data. The exercise assumed a constant growth rate for both components although high frequency data often exhibit significant volatility and/or seasonal patterns. To examine the impact of a lack of general smoothness on the previous exercise, we use the same basic information but introduce a significant seasonal pattern into one of the components, in this case B, as a way of increasing volatility in the series.[15]

Before looking at how the previous exercise is affected by the introduction of a seasonal component, it is worth spending a few lines on analysing what is going on with the aggregate. The three graphs to the left side of Figure 4.3 show the two components and the aggregate. At a first glance, it seems that the seasonal properties of component B are transferred directly into the aggregate. Due to the short length of the series it is not obvious, however, that the seasonal pattern of the aggregate shrinks over time due to the relative decrease in the weights of B, presented in Table 4.1. To appreciate the moderation of the pattern, the chart to the left of Figure 4.3 extends the time span dramatically. Here the shrinking becomes obvious. The attenuation in the dynamics of the aggregate would be faster if the relative increase of the price of A were larger, for example if the price of B fell by 50% yearly instead of 5%.

Figure 4.3: Chain-linked aggregate with a seasonal component

(Levels, reference year 2003)

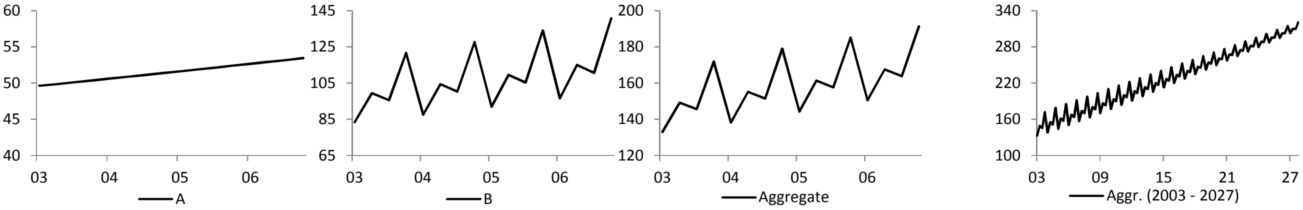
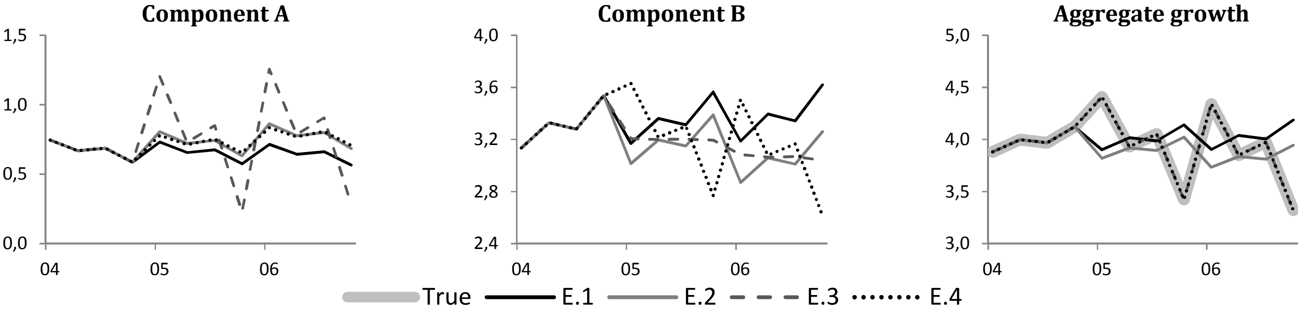


Figure 4.4: Contribution to annual growth of an aggregate with a seasonal component using different measures

(contributions in percentage points, growth as a percentage)



The effects of introducing a seasonal pattern into component B on the different measures of contributions to annual growth are appreciated in Figure 4.4. As expected, E.1 shows the overall weaknesses highlighted previously. E.2, however, shows an important deficiency when "assigning" the increase of volatility of the aggregate between the components. In fact, the dynamics of the contributions calculated with E.2 are quite similar to those of E.1 only that they differ to allow consistency with the annual contributions. The sum of the quarterly contributions, however, tracks the aggregate growth dynamics poorly.

Regarding the dynamics of the aggregate, it seems a bit odd that the true aggregate growth rate acquires a certain systematic pattern of its own. This pattern appears due to the way the exercise was set up because not only the seasonal pattern is systematic, but also the reduction in relative importance. For this reason, the volatility contributed by component B to the aggregate level decreases at the same rate every year and this fact is picked up in the annual growth rate. As it was mentioned before, the change in aggregate dynamics are not reflected by the

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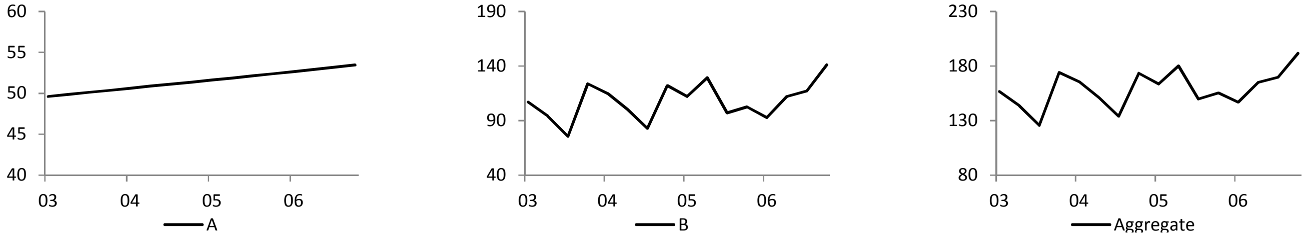
[15] The seasonal pattern; 0.85, 1.0, 0.95 and 1.20, multiplies the respective quarter of the constant growth component B.

sum of contributions of measures E.1 and E.2, both of which show relatively smooth contributions for the components. The only way of accounting for the shrinkage of the aggregate seasonal pattern is by introducing the volatility produced by the price shifts into the contributions. Although it may seem counterintuitive at first, given that the components grow at constant rates, this is what measures E.3 and E.4 attempt to do in their own way.

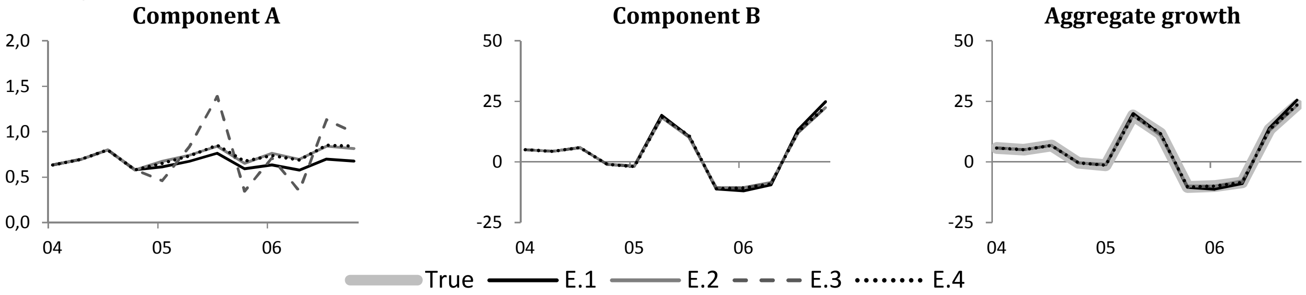
Figure 4.5: Contribution to annual growth of a quarterly aggregate with a volatile component

(Levels reference year 2003, contributions in percentage points, growth as a percentage)

Levels:



Quarterly contributions:



From observing Figure 4.4, a rather counterintuitive transfer of seasonality from component B to A becomes obvious in the contributions calculated using E.3. This transfer occurs because the distribution of the correction term in E.3 depends on the relative real importance of the component in the respective quarter compared to the annual real importance meaning that the aggregate change is distributed between components according to their respective weight in the total and not based on how much of the aggregate volatility comes from each component. This means, for example, that in the exercise a larger share of the positive overall correction for the first quarter (true aggregate growth is higher than the sum of contributions calculated using E.2) is assigned to component A given that component B is at a low seasonal level (seasonal factor 0.85), thus transferring B's volatility to A.[16] With E.4, component A's contribution remains practically unaltered compared to that of E.2 and almost all of the correction goes directly to component B. This is visible when comparing the dynamics of the contribution of component B with the dynamics of the true aggregate growth. All the discrepancies between E2, E.3 and E.4 cancel out within a year meaning their implied annual contributions are equal to the true annual contributions.

Regarding the odd transfer of seasonality that occurs when using E.3, one could argue that many countries use seasonally adjusted data meaning that this feature could be regarded as unimportant. The problem, however, arises due to the general volatility of the components and not only due to seasonality. Figure 4.5 shows the resulting graphs of the previous exercise but where the seasonal pattern of B has been replaced by a random multiplicative factor. In this case, the volatility would not be removed by a process of seasonal adjustment, and the transfer of volatility would happen anyway. This feature is probably undesirable under most circumstances.

By concentrating on the contributions to quarterly growth another odd feature arises. From looking at Figure 4.6, something that becomes obvious is the relatively large correction applied to component A by E.3 and the fact that the contributions become negative. This highlights another odd property of the contributions with an additive correction factor, which is that they may have the opposite sign than the components growth rate. This may seem

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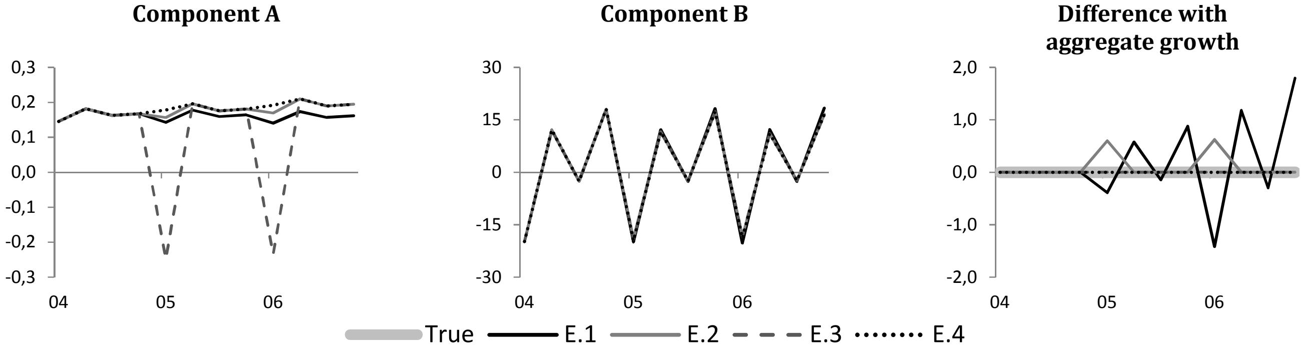
[16] Neither INSEE (2007) nor Banque Nationale de Belgique (2010) explicitly warn the readers about using the correction method only with seasonally adjusted data but their implementation and examples only use adjusted data. This is understandable given that both countries deal mainly with seasonally adjusted data. The author acknowledges that due to his basic knowledge of French he could have overlooked a less than obvious warning in INSEE (2007).

very counterintuitive and hard to explain, but it is something that may happen because of the mixing of frequencies involved in the annual overlap method. This problem, however, will occur only when the correction factor is large and the component's growth is not very different from zero. As some sort of proof for that, the correction factors applied to component B are indistinguishable given the magnitude of the quarterly rates. As with the exercise without a seasonal component, all measures except E.4 show a bias in the implied annual contributions of the individual components, only that in this case it is significantly larger.

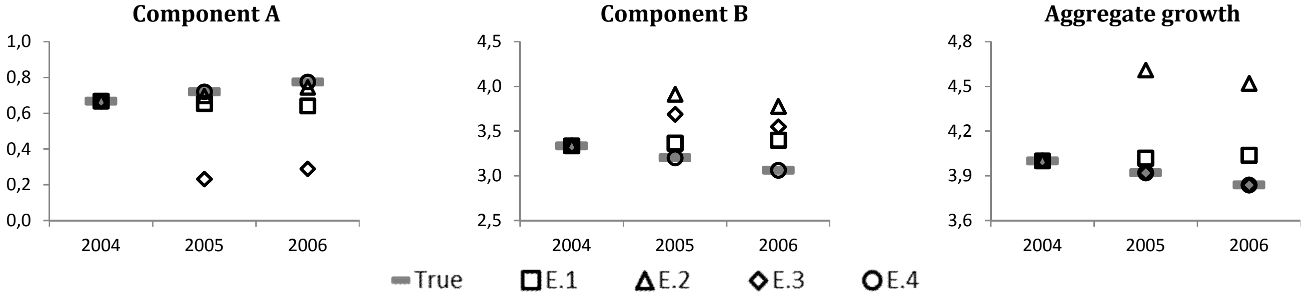
Figure 4.6: Contribution to quarterly growth of an aggregate with a seasonal component using different measures

(contributions in percentage points, growth as a percentage)

Contributions to quarterly growth:



Implied annual contributions:



4.3. Contributions under low annual growth and quarterly volatility

To explore in more detail the apparently counterintuitive aspects that may arise under low growth, volatility and large price changes, we repeat the previous exercise with different parameters. Quantities A and B grow both at 0.5% annually, ensuring an aggregate annual growth of 0.5%, while their prices change by -5 and 20%. In annual terms, as it can be seen from Table 4.2, everything looks as one would expect. The price of A is relatively lower than B's in 2003 and therefore makes it weigh relatively less in the aggregate in that year. More so, with the shifting price structure, B becomes increasingly more important, as reflected by the chain-link weights, although both components grow at the same rate in real terms. These circumstances, however, generate a number of events in the quarterly series that may seem odd at first.

The contributions to annual growth are presented in Figure 4.7. The first thing that looks odd is that, although the components grow consistently at a 0.5% annual rate, the aggregate annual growth rates of the four quarters differ and in particular, in the first quarters they are negative. Explaining these negative growth rates in isolation may become a challenging task. Looking at the components only contributes to the confusion, because both grow at a 0.5% rate. Using E.1 or E.2 is not of much help because they fail to add up to the total and, therefore, do not explain aggregate dynamics. Using the more appropriate E.3 or E.4 result invariably in that the contribution of at least one of the components is negative that is also counterintuitive. These seemingly counterintuitive events have their roots in the fact that the source of the aggregate quarterly series is primarily the annual series and not the other way around. The aggregate quarterly series is basically an annual series that is given quarterly dynamics based on the quarterly component's volatility and the corresponding weights meaning that each year's dynamics has a degree of discontinuity produced by the discrete shifting of prices that only becomes evident in extreme cases like this one.

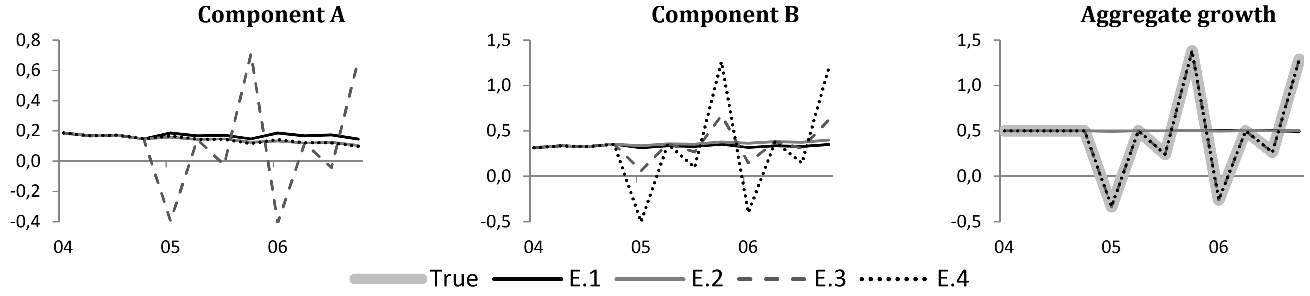
Table 4.2: Component's chain-link weights for the low growth example

Year	*Quantities*		*Prices*		*Chain-linked* *(reference year=2003)*		*Chain-link weights*	
	A	*B*	*A*	*B*	*level*	*yoy*	*A*	*B*
2003	100.0	100.0	2.00	4.00	600		*1.00*	*1.00*
2004	100.5	100.5	1.90	4.80	603	***0.50%***	*1.00*	*1.00*
2005	101.0	101.0	1.81	5.76	606	***0.50%***	*0.85*	*1.07*
2006	101.5	101.5	1.71	6.91	609	***0.50%***	*0.72*	*1.14*

It might be easier to understand this by looking at the levels of the aggregate in more detail. The levels for the previous exercise are presented in Figure 4.8. To highlight the point to be made, however, we amplify the relative price changes by letting the price of component A decrease annually by 50%. As it can be seen, both components grow at a low but constant rate. The aggregate, however, grows at the same rate on average, as shown by the dotted line (yearly avg.), but the first quarters actually decrease relative to the previous year while the fourth quarter grows at a significantly higher rate. For 2005, the first quarter falls 2.2% year over year while the last quarter grows by 3.3%.

Figure 4.7: Contribution to annual growth of a quarterly aggregate with a seasonal component and low growth

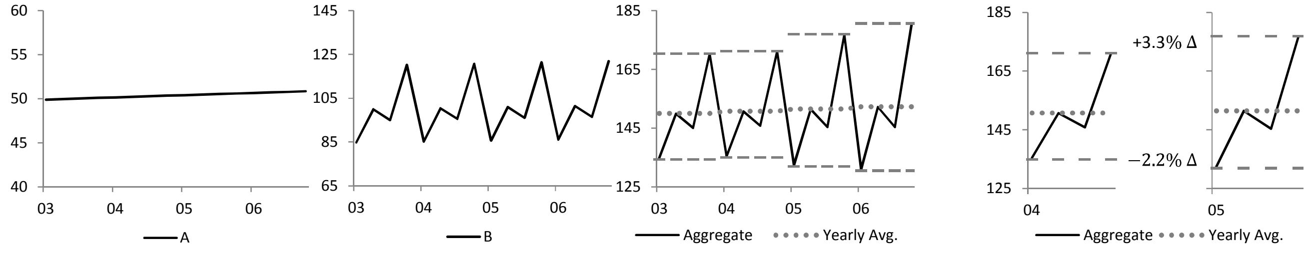
(contributions in percentage points, growth as a percentage)



What is happening is that the annual average grows steadily but so does the dispersion of the aggregate intra annual volatility (as highlighted by the dashed lines). This is due to the fact that with every change of year, in this case, the component with all the intra-annual volatility becomes progressively more important, resulting in that at the beginning of each year the aggregate dynamics show a leap to accommodate the increase in volatility. In a way, the quarterly dynamics of each year are determined independently from that of the other years and then stuck together to create the quarterly time-series.

Figure 4.8: Chain-linked aggregate with a seasonal component and low growth

(Levels, reference year 2003)



To be fair, the process is not independent in a full sense. On the one hand, the dynamics of different years are based on the same time series meaning that any systematic movements and long lasting shocks will be transmitted, at least to some extent, into a number of consecutive periods of the aggregate. On the other hand, the shifts in the price structure are probably not purely random. Taking all this into account, one gets the notion that the quarterly series that belong to an annual overlap framework are not "fully quarterly" in the way one would expect from building the series relying only on the basic quarterly information, but they are more like annual series that are transformed into quarterly series by introducing the quarterly dynamics in a rather unique way.

4.4. Comments on the findings under stylized circumstances

In this section, many things were presented. The primary objective was to show a broader view of the performance of the different measures for contributions that goes beyond how each measure complies or not with the specific requirements presented in section 3. In particular the idea was to highlight strengths and weaknesses of the different methods that might not become immediately obvious when examining examples of real data. In the process, some apparently odd features were exposed. The results suggested that, even in the case of very gradual changes in prices, the way in which the contributions are calculated are probably not irrelevant.

Based on their performance, by all means, method E.1 should be avoided given that the discrepancies that arise are cumulative and do not cancel out necessarily. The rest of the measures are very similar unless price changes are significant. In such a case, E.2 does not add up to the aggregate growth even approximately meaning that when additivity is important, although it is the measure that makes the most economic sense, a correction is mandatory and E.4 should be preferred to perform it. This is due to the fact that it is the only measure that fulfils all the presented requirements and also because E.3 could generate some odd distribution among components if the components where to exhibit very different volatilities.

An interesting point that stems from the previous analysis is that for the contributions to sum up to aggregate growth they may have the opposite sign to the growth of the component. This, far from being a mistake, reflects the shifting weights of the components and is sourced in the annual nature of the aggregate quarterly series. As mentioned in the previous section, the quarterly series is an annual series that is transformed by introducing the quarterly dynamics in a rather unique way. The apparently odd features of the contributions come from the way the annual links are fitted into a time-series framework.

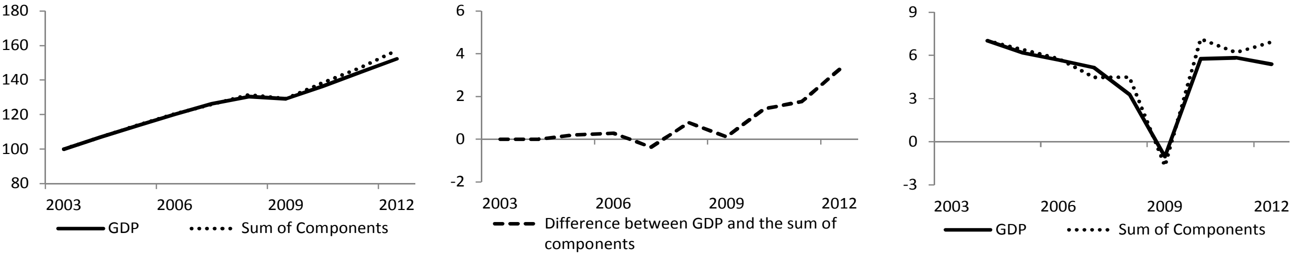
It is worth mentioning that, although the circumstances were chosen to highlight certain aspects, these are probably not altogether rare in real life. For example, the combination of low growth and volatility, both systematic and non-systematic, is probably not so far-fetched even at quite an aggregate level. Large price shifts are probably less common at a very aggregate level, but may be important when examining more detailed data within sectors.

5. Comparing the different ways of explaining GDP quarterly growth from the common macro aggregates

The previous section examined the performance of the different measures under particular circumstances that were suggested purposefully to highlight the differences between them. Nonetheless, as highlighted in Eurostat/ECB (2008), the problem of non-additivity at the level of macro aggregates is probably less important and might be negligible. In this section we proceed to compare the measures for a set of Chilean data. As it was highlighted, the difference between measures will depend primarily on how the relative prices have changed over time and then on the volatility of the actual components, therefore it is worth having a looking at these features in the data set.

Chart 5.1: Chilean GDP, the difference with the sum of components and aggregate annual growth rates

(level reference year 2003=100, percentage, percentage)



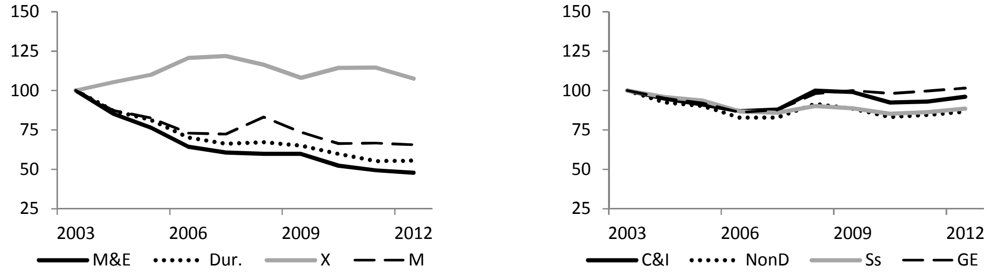
Source: Own calculation with data from the Central Bank of Chile

Chile adopted chain-linking for its accounts in 2012 and implemented the annual overlap method for Laspeyres indices (Guerrero et. al., 2012). For the purpose of this exercise we used quarterly expenditure data spanning from

2003, our reference year, to 2012.[17] As a first thing, Chart 5.1 shows the difference that arises between GDP and the sum of its components. As it can be seen, at the end of the sample the sum of components is considerably higher than the level of GDP as are the respective growth rates. This is in line with the findings of Steindel (1995) for the U.S. and suggests an overrepresentation of at least one of the components in the direct sum.

Chart 5.2: GDP expenditure component's relative prices

(Component Deflator/GDP Deflator, ratio, 2003=100)

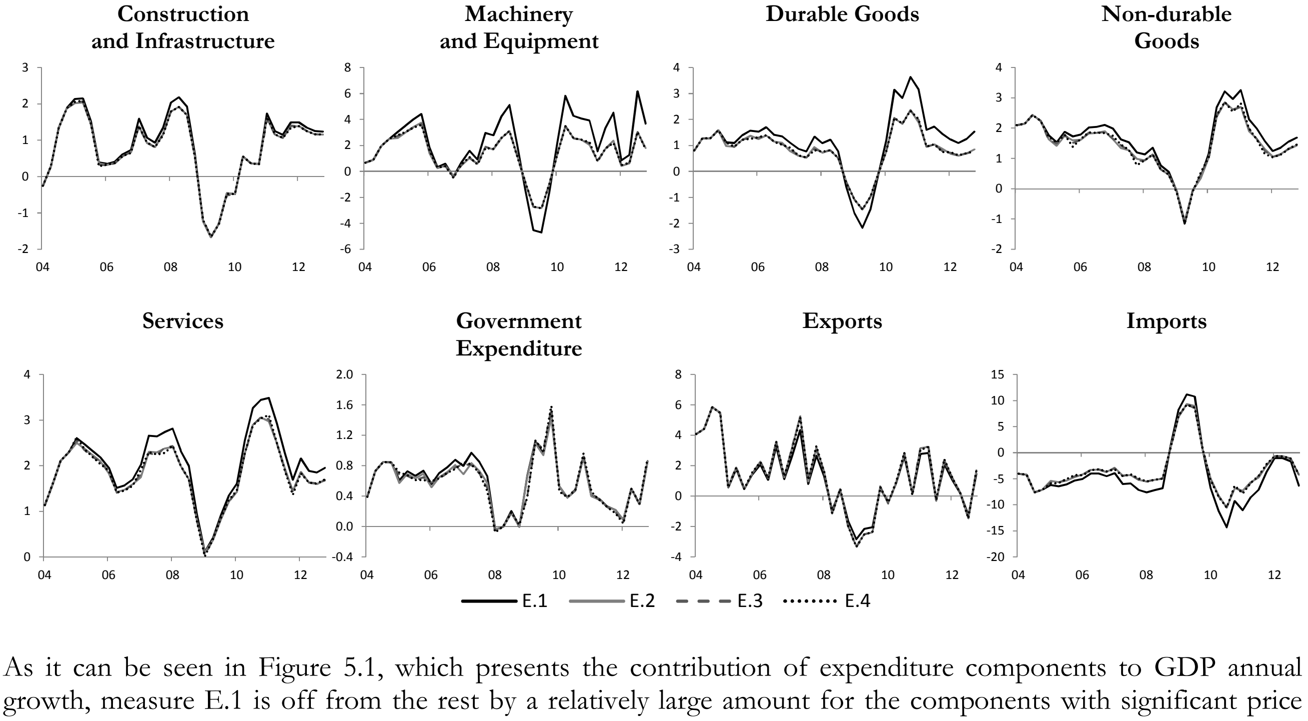


Source: Own calculation with data from the Central Bank of Chile

This immediately should make us want to look at the evolution of the relative prices that are presented in Chart 5.2.[18] On the left side are those that show larger changes; Machinery and Equipment, Durable Goods, Exports and Imports. Here it becomes clear how the prices of these components have evolved more or less consistently at a different pace highlighting the permanent changes in the relevant price structure. It is especially striking how the weights of both Durable Good and Machinery and Equipment are halved only in a matter of a decade.

Figure 5.1: Contribution of expenditure components to GDP annual growth using different measures

(percentage points)



As it can be seen in Figure 5.1, which presents the contribution of expenditure components to GDP annual growth, measure E.1 is off from the rest by a relatively large amount for the components with significant price changes, therefore, making the cumulative effect of ignoring the price shifts evident. From 2007 onwards the

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[17] The official series use the most recent compilation year, 2008, as reference year. We use 2003 as the reference year to avoid having to spline two series. This does affect the results due to allowing for larger changes in relative prices and therefore the magnitudes should only be taken within the context of this exercise. However, for the purpose of the exercise the reference year is not important. The level of disaggregation is the following; Construction and Infrastructure, Machinery and Equipment, Durable Goods, Non-durable Goods, Services, Government Expenditure, Exports, Imports and Change in Inventories.

[18] It is worth noting that one year's relative prices are the chain-linked weights of the components for the following year.

contribution of Machinery and Equipment and Durable Goods are grossly overestimated, due to its failing to account for changes in relative prices, but also Services and Non-Durable Goods, that do not exhibit dramatic price changes, show a significant overestimation. From this it becomes clear how misleading ignoring the chain-linking could be. On the contrary, the measures that account for the chain-linking are remarkably similar to the point of being nearly visually indistinguishable in the graphs.

To view how similar the measures actually are in this example Table 5.1 presents the mean average discrepancy for them. Due to the obvious drawbacks of E.1, we use E.2 as the benchmark given that it is the closest to the contributions that are derived from the Laspeyres links. In doing so, one quantifies the bias produced because of ignoring the chain-linking, by using E.1, and also it is possible to have a notion of the size of the corrections that needs to be made to achieve additivity. To give perspective to the absolute measures, the first column in Table 5.1 shows the mean absolute contribution for each expenditure component over the 2005-2012 period. Then, the following columns present the mean and maximum absolute discrepancies for the three measures.

Table 5.1: Mean average discrepancy of the measures for contribution of components to GDP annual growth

(percentage points)

	Mean Absolute Contribution of E.2	**Mean Absolute Discrepancy**			**Maximum Absolute Discrepancy**		
		E.1	**E.3**	**E.4**	**E.1**	**E.3**	**E.4**
Construction and Infrastructure	**1,1**	0,1	0,0	0,0	0,3	0,1	0,1
Machinery and Equipment	**1,8**	1,1	0,0	0,1	3,1	0,2	0,2
Durable Goods	**1,0**	0,5	0,0	0,0	1,3	0,1	0,1
Non-durable Goods	**1,4**	0,2	0,0	0,1	0,5	0,1	0,2
Services	**1,8**	0,2	0,0	0,0	0,5	0,1	0,2
Government Expenditure	**0,6**	0,0	0,0	0,1	0,1	0,2	0,3
Exports	**1,8**	0,2	0,0	0,0	0,9	0,2	0,1
Imports	**4,8**	1,5	0,1	0,2	3,8	0,4	0,5

In line with what Figure 5.1 shows, measure E.1 presents large discrepancies in the sectors with significant price changes, that is Machinery and Equipment, Durable Goods and Imports. For the first two, the mean discrepancy is about half the size of the mean contribution. This reinforces the idea that using this measure could lead to significant errors. For the other two measures, the mean discrepancies are significantly lower. This suggests that the correction factors are on average relatively small. However, the maximum absolute discrepancies do show that in a given period the correction may be significant.

Table 5.2: Contributions of expenditure components to quarterly growth in the first quarter of 2011

(percentage points)

Aggr.Growth: -4,4 %

	E.1	E.2	E.3	E.4
Construction and Infrastructure	0,5	0,5	0,5	0,4
Machinery and Equipment	-1,4	-0,7	-0,9	-0,9
Durable Goods	-1,7	-1,0	-1,1	-1,1
Non-durable Goods	-1,9	-1,6	-1,7	-1,8
Services	-2,3	-2,0	-2,0	-2,1
Government Expenditure	-4,5	-4,4	-4,4	-4,4
Exports	0,1	0,1	0,0	0,1
Imports	0,6	0,4	0,4	0,7
Changes in Inventories*	8,0	5,2	5,6	6,0
Sum of contributions	**-3,9**	**-4,3**	**-4,4**	**-4,4**

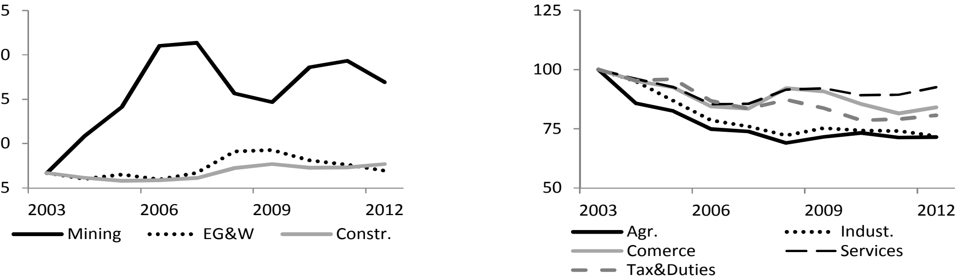
* For E.1 and E.2 the contribution of Changes in Inventories is calculated as the joint contribution of Imports plus Changes in Inventories minus the contribution of Imports.

The evaluation does not change much when examining the contributions to quarterly growth. For the calculation of contributions to quarterly growth of the aggregate all formulas but that of E.1 coincide for all quarters but the first and, therefore, given that E.1 has been proven to be fairly unreliable, to appreciate the discrepancies between the three "chain-linked" measures it is only necessary to check the outcomes for the first quarters. Table 5.2

presents the contributions to quarterly growth of the different measures for the first quarter of 2011.[19] In line with what can be seen in Figure 5.1, the contributions for Machinery and Equipment and Durable Goods using E.1 are far greater, in absolute terms, than those of the other three measures. It is interesting to see though, that this overstatement is compensated by the other components in the sense that the implied aggregate growth of E.1 is not so different from the actual aggregate growth. The corrections made to E.2 by E.3 and E.4 are relatively small being the largest in the vicinity of 0.3 percentage points.

Chart 5.3: GDP production component's relative prices

(Component Deflator/GDP Deflator, ratio, 2003=100)

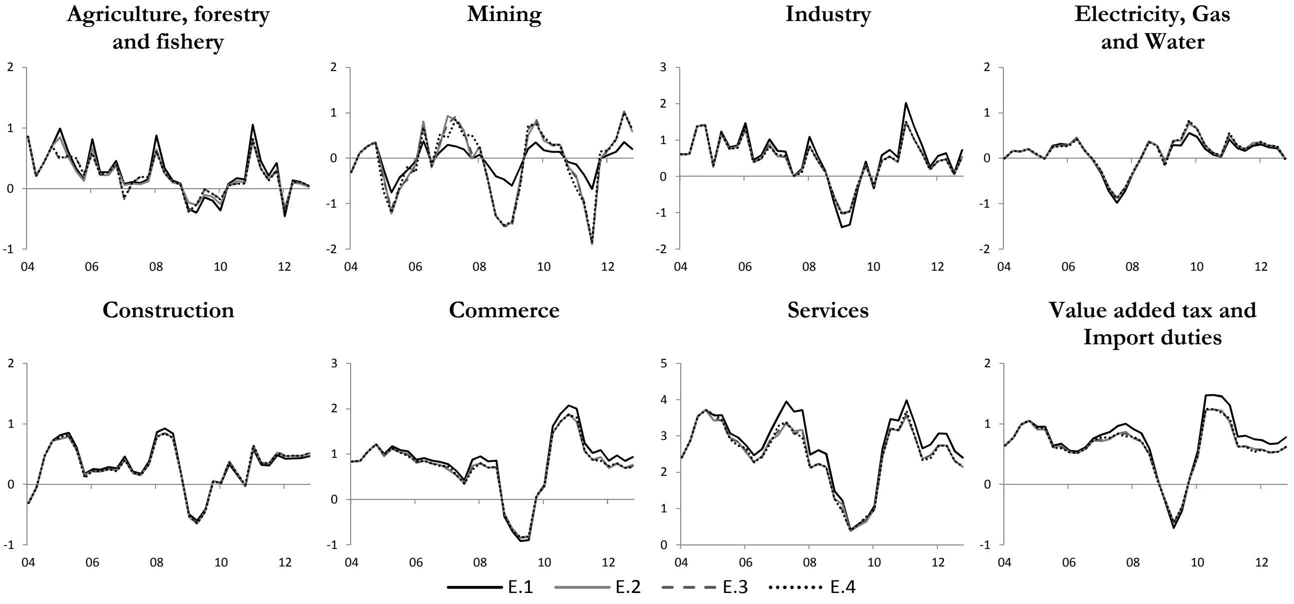


Source: Own calculation with data from the Central Bank of Chile

All this reinforces the idea that, at least at this level of aggregation, the contributions calculated using E.2, E.3 and E.4 are all very similar. For certain applications this could be good enough, however, to someone wanting to fully explain the source of the aggregate growth, E.2 would fall short by 0.1 percentage point.[20] The difference in this case is quite small, but nonetheless it is still 0.1 percentage points of growth from an unidentified source and this sort of explanation would be embarrassing for a publishing body. In such a case the only reasonable decision respecting the choice of measures is to go for one that fulfils the criteria of additivity.

Figure 5.2: Contribution of production components to GDP annual growth using different measures

(percentage points)



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[19] This year was chosen as the example because it is the one that presents the largest discrepancies between E.2, E.3 and E.4 excluding Changes in Inventories. The contributions for the other years may be found in Annex 6.

[20] To account for GDP fully it is necessary to incorporate Changes in Inventories, that is a series that may not be chain-linked. However, its contributions may be calculated residually for E.3 and E.4. For E.1 and E.2 we present a sort of implicit contribution by subtracting the contribution of Imports from the joint contribution of Changes in Inventories plus Imports.

For completeness, it is also interesting to look at the production side to see if the evaluation is more or less the same. Chart 5.3 presents the evolution of the relative prices.[21] From looking at the left graph, the explosive increase of the Mining relative price becomes obvious. This increase exerts downward pressure on all the other relative prices. With such an important shift in the relevant price structure, one would expect to see large discrepancies between the contributions calculated with the different measures. This is especially the case for Mining and Services (Figure 5.2).

6. Final remarks

The use of chain-linked methods reduces significantly the problem of price structure obsolescence present in fixed base environments, but introduces a new dimension that may produce confusion if not accounted for. The updating of the economy's price structure results in lack of additivity for the levels of the series. This means that the traditional accounting identities are not directly applicable and, therefore, explaining aggregate performance from the disaggregate data is not straightforward. To lessen the problem, contributions to growth are provided by statistical agencies. For the annual overlap method, that is a practical and popular method for chain-linking Laspeyres volume measures, there is no consensus on a single way of calculating the contributions to aggregate growth. In this paper, four potential candidates are examined both in the sense of how they comply with certain requirements and how they perform empirically.

The first thing that becomes obvious is that ignoring the chain-linking method by continuing to use the traditional way of calculating contributions may lead to significant errors. This is due to the fact that, even when the price structures change slowly, the bias in these measures is cumulative over time. This way for calculating contributions should be avoided by all means.

The second thing is that, although from an economic perspective using the contributions derived from the chain-linking Laspeyres links, that is the previous year prices measure, makes the most economic sense, it does not fulfil many of the desirable properties one would hope for in a measure of contributions. This occurs because the set of links do not represent a time series and, therefore, the aforementioned measure lacks the composition effects that take place when chain-linking the aggregate. It must be said, however, that at least at the macro level of the performed exercise the discrepancies were negligible. If additivity and consistency requirements are required, however, the only measure mentioned in this document that fulfils all the proposed properties is the one suggested in Cobb (2013).

A third thing is that the three measures that account for chain-linking provide very similar results when changes in price structures are modest. When shifts in relative prices are large, however, the correction factors that ensure additivity may be relatively large leading to some apparently odd results. Both the measures proposed by INSEE (2007) and Cobb (2013) may result in having the opposite sign than that of the component's growth if the correction factor due to the composition effects is large compared to the rate. The measure proposed by INSEE (2007) may also exhibit some transfer of volatility between components if the relative volatility is very different between them. The measure proposed by Cobb (2013) confines volatility to the contribution of the components that present it.

Relying on the criteria for choosing between formulae suggested by Eurostat/ECB (2008) and that is relevant for the annual overlap method, that is; user friendliness, additivity and interpretability, the contributions that use previous year prices and the correction suggested by Cobb(2013) seem to be the best choice. The uncorrected contribution that uses previous year prices is the one that has the most economically meaningful interpretation as it comes directly from the construction of the composite index but remains very straightforward to implement and communicate. If additivity is paramount, that would be the case for the publishing bodies or for practitioners that need to calculate contributions by difference or perform a detailed breakdown, based on the issue of volatility transfer and the compliancy with the additivity and consistency requirements, the correction proposed in Cobb (2013) should be preferred. Performing the mentioned correction will generally not affect the economic interpretation of the uncorrected previous year price contribution, but it will mean that aggregate growth is fully explained.

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[21] The level of disaggregation is the following; Agriculture, forestry and fishery, Mining, Manufacturing industry, Electricity, gas and water, Construction, Commerce, Services and Value added tax and Import duties.

This document presents the strengths and weaknesses of different measures for calculating contributions by highlighting how they perform for descriptive and presentation purposes only. However, beyond the scope of presenting data, a topic for further research could be to compare how the different measures affect the estimation and results of econometric models. Related to this, would be to analyse how these measures may be utilized to adapt models developed under the fixed base framework and how using a different measure may influence their outcome.

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Annex 1: Deriving the contributions to annual growth in an annual overlap framework

By definition, the annual overlap method links a series of consecutive overlapping two-period Laspeyres indices. Then GDP growth any given period is given by:

$$\frac{\Delta Q^{y}}{Q^{y-1}}=\frac{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y}\right)}{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y-1}\right)}-1 \qquad \text{(A1.1)}$$

where,

Q^{y} : chain-linked aggregate in year y

q_j^{y} : component j in year y

p_j^{y} : implicit price deflator of component j in year y (calculated as nominal value over chain-linked value)

We can reorder this expression in the following way:

$$\begin{aligned}
\frac{\Delta Q^{y}}{Q^{y-1}} &= \frac{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y}\right)-\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y-1}\right)}{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y-1}\right)}\\
&= \frac{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y}-p_j^{y-1}\cdot q_j^{y-1}\right)}{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y-1}\right)}\\
&= \sum_{j=1}^{J}\left[\frac{1}{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y-1}\right)}\cdot\left(p_j^{y-1}\cdot q_j^{y}-p_j^{y-1}\cdot q_j^{y-1}\right)\right]\\
&= \sum_{j=1}^{J}\left[\frac{\left(p_j^{y-1}\cdot q_j^{y-1}\right)}{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y-1}\right)}\cdot\frac{\left(p_j^{y-1}\cdot q_j^{y}-p_j^{y-1}\cdot q_j^{y-1}\right)}{\left(p_j^{y-1}\cdot q_j^{y-1}\right)}\right]\\
&= \sum_{j=1}^{J}\left[\frac{\left(p_j^{y-1}\cdot q_j^{y-1}\right)}{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y-1}\right)}\cdot\left(\frac{\left(p_j^{y-1}\cdot q_j^{y}\right)}{\left(p_j^{y-1}\cdot q_j^{y-1}\right)}-1\right)\right]\\
&= \sum_{j=1}^{J}\left[\frac{\left(p_j^{y-1}\cdot q_j^{y-1}\right)}{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y-1}\right)}\cdot\left(\frac{q_j^{y}}{q_j^{y-1}}-1\right)\right]
\end{aligned}$$

Then, we define the contribution of component *j* to annual aggregate growth in year *y* as:

$$C_j^{y}=\frac{\left(p_j^{y-1}\cdot q_j^{y-1}\right)}{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y-1}\right)}\cdot\left(\frac{q_j^{y}}{q_j^{y-1}}-1\right) \qquad \text{(A1.2)}$$

That is, the annual growth of *j* times its nominal share in the previous year (*y-1*).

Annex 2: Annual overlap chain-linked indices and contribution to annual growth

This technique of annual overlap involves calculating the variation between the current year (*y*) and the previous year (*y-1*) both valued using prices of the previous year (*y-1*) and building a time series from the variation between them. The following table shows a simple example based on two components of an annual overlap chain-linked index and the equivalent fixed base index. Quantities A and B grow at a 2 and 5% annual rate respectively, while their prices vary 10 and -5%.

Table A1.1: Annual overlap chain-linked index

Year	*Quantities*		*Prices*		*Total at current prices*	*Index 2003 base year*	*yoy*
	A	*B*	*A*	*B*			
2003	100	100	2.00	4.00	600	*600.0*	
2004	102	105	2.20	3.80	623	*624.0*	***4.00%***
2005	104	110	2.42	3.61	650	*649.1*	***4.02%***
2006	106	116	2.66	3.43	680	*675.3*	***4.04%***

	Constant prices from:										
		2003			***2004***			***2005***		***Chain-linked***	
Year	***Level***	***Index***	***yoy***	***Level***	***Index***	***yoy***	***Level***	***Index***	***yoy***	***index***	***yoy***
2003	600	100.0								*600.0*	
2004	624	104.0	***4.0%***	623	100.0					*624.0*	***4.00%***
2005				648	103.9	***3.9%***	650	100.0		*648.5*	***3.92%***
2006							675	103.8	***3.8%***	*673.3*	***3.84%***

Then, the contribution to the growth rate of the annual totals for year *y* of component *j*, c_j^y, is the growth of the component, q_j, times its relative nominal importance in the previous year, v_j^{y-1}, meaning $c_j^y = v_j^{y-1} \cdot (q_j^y / q_j^{y-1} - 1)$. For the previous example that is:

Table A1.2: Annual contribution to annual overlap chain-linked index

	Quantities		***Prices***		***Total***	***Chain-linked***	***yoy***
	A	***B***	***A***	***B***	***current prices***	***index***	***%***
2003	100	100	2.00	4.00	600	600	-
2004	102	105	2.20	3.80	623	624	**4.00**
2005	104	110	2.42	3.61	650	648	**3.92**
2006	106	116	2.66	3.43	680	673	**3.84**

	yoy quant. (%)		***nominal weights (v)***		***Contribution***	
	A	***B***	***A***	***B***	***A***	***B***
2003	-	-	0.33	0.67	-	-
2004	2.0	5.0	0.36	0.64	**0.67**	**3.33**
2005	2.0	5.0	0.39	0.61	**0.72**	**3.20**
2006	2.0	5.0	0.42	0.58	**0.77**	**3.06**

Annex 3: Contribution to annual change derived from the contributions to quarterly growth in an annual overlap framework

As mentioned before, expressing the consistency of the component's contribution to the quarterly growth of the aggregate quarterly series with the contributions to growth of the aggregate annual series in a compact way is not straightforward due to the fact that quarterly growth rates are not expected to sum up to the yearly growth rate.

In the case of the contribution to annual growth of the quarterly series the difficulties are easily resolved because the four quarterly contributions refer to the current and previous year and therefore together encompass the complete current and previous years. The contributions to quarterly growth, however, are linked to the previous period only. By concentrating on the changes in levels it is possible to derive a tractable expression. First, we find the share of the aggregate that comes from the real growth of the components in each quarter of a given year and then, by subtracting the share of the previous year, we find the contribution to yearly change in terms of the quarterly contributions.

In the reference year -the base year of the first link used to generate the chain-linked series- the aggregate level is the sum of the components meaning the share of the aggregate corresponding to component *j* in any of the quarters of the reference year is simply the component and, therefore, the share of the aggregate corresponding to component *j* in the reference year (*y*=0) is:

$$e_j^{y=0} = q_{j,1} + q_{j,2} + q_{j,3} + q_{j,4}$$

Alternatively the share corresponding to the real growth of component *j* in any given quarter, *t*, may be formulated as the share in the previous quarter, *t-1*, plus the contribution to aggregate change in *t*:

$$e_j^{y=0} = q_{j,1} + \left(q_{j,1} + \hat{k}_{j,2}^{QoQ}\right) + \left(q_{j,1} + \hat{k}_{j,2}^{QoQ} + \hat{k}_{j,3}^{QoQ}\right) + \left(q_{j,1} + \hat{k}_{j,2}^{QoQ} + \hat{k}_{j,3}^{QoQ} + \hat{k}_{j,4}^{QoQ}\right)$$

where the last parenthesis in the expression is the share of the aggregate that corresponds to the real growth of component *j* for the last quarter of that year (*t*=4).

In the same way, the shares corresponding to component *j* in the quarters of *y*=1 are the shares in the previous quarters plus the respective contributions to aggregate change. Then, the share of the aggregate corresponding to component *j* in the year following the reference year (*y*=1) is:

$$\begin{aligned} e_j^{y=1} &= \left(q_{j,4} + \hat{k}_{j,5}^{QoQ}\right) + \left(q_{j,4} + \hat{k}_{j,5}^{QoQ} + \hat{k}_{j,6}^{QoQ}\right) + \left(q_{j,4} + \hat{k}_{j,5}^{QoQ} + \hat{k}_{j,6}^{QoQ} + \hat{k}_{j,7}^{QoQ}\right) + \left(q_{j,4} + \hat{k}_{j,5}^{QoQ} + \hat{k}_{j,6}^{QoQ} + \hat{k}_{j,7}^{QoQ} + \hat{k}_{j,8}^{QoQ}\right) \\ &= \left[q_{j,1} + \sum_{t=2}^{5}\left(\hat{k}_{j,t}^{QoQ}\right)\right] + \left[q_{j,1} + \sum_{t=2}^{6}\left(\hat{k}_{j,t}^{QoQ}\right)\right] + \left[q_{j,1} + \sum_{t=2}^{7}\left(\hat{k}_{j,t}^{QoQ}\right)\right] + \left[q_{j,1} + \sum_{t=2}^{8}\left(\hat{k}_{j,t}^{QoQ}\right)\right] \end{aligned}$$

From here it becomes obvious that the share corresponding to the real growth of component *j* in any given quarter corresponds to the level in the first quarter of the reference year plus the sum of the contributions to change up to that quarter. Then, for any year *y*, for *y*>0 with *y*=0 being the reference year and *l* being the first quarter of year *y*, the corresponding share in year *y* is:

$$e_j^{y} = \left(4 \cdot q_{j,1}\right) + \sum_{t=2}^{l}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=2}^{l+1}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=2}^{l+2}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=2}^{l+3}\left(\hat{k}_{j,t}^{QoQ}\right)$$

Then, for any year *y* the contribution to annual change derived from the quarterly contributions is:

$$\begin{aligned} e_j^{y} - e_j^{y-1} &= \left[\left(4 \cdot q_{j,1}\right) + \sum_{t=2}^{l}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=2}^{l+1}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=2}^{l+2}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=2}^{l+3}\left(\hat{k}_{j,t}^{QoQ}\right)\right] \\ &\quad -\left[\left(4 \cdot q_{j,1}\right) + \sum_{t=2}^{l-4}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=2}^{l-3}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=2}^{l-2}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=2}^{l-1}\left(\hat{k}_{j,t}^{QoQ}\right)\right] \\ &= \sum_{t=l-3}^{l}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=l-2}^{l+1}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=l-1}^{l+2}\left(\hat{k}_{j,t}^{QoQ}\right) + \sum_{t=l}^{l+3}\left(\hat{k}_{j,t}^{QoQ}\right) \\ &= \hat{k}_{j,l-3}^{QoQ} + 2 \cdot \hat{k}_{j,l-2}^{QoQ} + 3 \cdot \hat{k}_{j,l-1}^{QoQ} + 4 \cdot \hat{k}_{j,l}^{QoQ} + 3 \cdot \hat{k}_{j,l+1}^{QoQ} + 2 \cdot \hat{k}_{j,l+2}^{QoQ} + \hat{k}_{j,l+3}^{QoQ} \end{aligned}$$

Annex 4: Compliance of the different suggestions for quarterly contributions with the proposed requirements

A.3.1. Additivity of the contributions to annual growth of the aggregate quarterly series

For (R.1) to be met always the following must be true:

$$\sum_{j=1}^{J} c_{j,t} - \frac{1}{Q_{t-4}} \cdot \sum_{j=1}^{J} \left(w_{j,t} \cdot q_{j,t} - w_{j,t-4} \cdot q_{j,t-4} \right) = 0$$

A.3.1.1: Fixed weights (E.1)

$$\begin{aligned}
\sum_{j=1}^{J} c_{j,t}^{FW} - \sum_{j=1}^{J} \hat{c}_{j,t}^{YoY} &= \sum_{j=1}^{J} \left[\frac{\omega_j^{ref.year}}{Q_{t-4}} \cdot \left(q_{j,t} - q_{j,t-4} \right) \right] - \frac{1}{Q_{t-4}} \cdot \sum_{j=1}^{J} \left(w_{j,t} \cdot q_{j,t} - w_{j,t-4} \cdot q_{j,t-4} \right) \\
&= \frac{1}{Q_{t-4}} \cdot \sum_{j=1}^{J} \left[\left(\omega_j^{ref.year} - w_{j,t} \right) \cdot q_{j,t} - \left(\omega_j^{ref.year} - w_{j,t-4} \right) \cdot q_{j,t-4} \right]
\end{aligned}$$

The result is not necessarily zero.

A.3.1.2: Previous year prices (E.2)

$$\begin{aligned}
\sum_{j=1}^{J} c_{j,t}^{PYP} - \sum_{j=1}^{J} \hat{c}_{j,t}^{YoY} &= \sum_{j=1}^{J} \left[\frac{w_{j,t}}{Q_{t-4}} \cdot \left(q_{j,t} - q_{j,t-4} \right) \right] - \frac{1}{Q_{t-4}} \cdot \sum_{j=1}^{J} \left(w_{j,t} \cdot q_{j,t} - w_{j,t-4} \cdot q_{j,t-4} \right) \\
&= \frac{1}{Q_{t-4}} \cdot \sum_{j=1}^{J} \left[\left(w_{j,t} - w_{j,t} \right) \cdot q_{j,t} - \left(w_{j,t} - w_{j,t-4} \right) \cdot q_{j,t-4} \right] \\
&= \frac{1}{Q_{t-4}} \cdot \sum_{j=1}^{J} \left[\left(w_{j,t} - w_{j,t-4} \right) \cdot q_{j,t-4} \right]
\end{aligned}$$

The result is not necessarily zero.

A.3.1.3: INSEE 2007 (E.3)

$$\begin{aligned}
\sum_{j=1}^{J} c_{j,t}^{INSEE} - \sum_{j=1}^{J} \hat{c}_{j,t}^{YoY} &= \sum_{j=1}^{J} \left[\frac{w_{j,t}}{Q_{t-4}} \cdot \left(q_{j,t} - q_{j,t-4} \right) + \left(w_{j,t} - w_{j,t-4} \right) \cdot \left(\frac{q_{j,t-4}}{Q_{t-4}} - \frac{q_{j,t}^{y-1}}{Q_t^{y-1}} \right) \right] - \frac{1}{Q_{t-4}} \cdot \sum_{j=1}^{J} \left(w_{j,t} \cdot q_{j,t} - w_{j,t-4} \cdot q_{j,t-4} \right) \\
&= \frac{1}{Q_{t-4}} \cdot \sum_{j=1}^{J} \left[w_{j,t} \cdot q_{j,t} - w_{j,t} \cdot q_{j,t-4} + w_{j,t} \cdot q_{j,t-4} - w_{j,t-4} \cdot q_{j,t-4} - \left(w_{j,t} - w_{j,t-4} \right) \cdot \left(\frac{q_{j,t}^{y-1}}{Q_t^{y-1}} \cdot Q_{t-4} \right) - w_{j,t} \cdot q_{j,t} + w_{j,t-4} \cdot q_{j,t-4} \right] \\
&= \frac{1}{Q_{t-4}} \cdot \sum_{j=1}^{J} \left[- \left(w_{j,t} - w_{j,t-4} \right) \cdot \left(\frac{q_{j,t}^{y-1}}{Q_t^{y-1}} \cdot Q_{t-4} \right) \right] \\
&= \left(\sum_{j=1}^{J} \left[\frac{Q_{t-4}^{y-1}}{\sum_{j=1}^{J} \left(p_{j,t-4}^{y-1} \cdot q_{j,t-4}^{y-1} \right)} \cdot p_{j,t-4}^{y-1} \cdot \left(\frac{q_{j,t}^{y-1}}{Q_t^{y-1}} \right) \right] - \sum_{j=1}^{J} \left[\frac{Q_t^{y-1}}{\sum_{j=1}^{J} \left(p_{j,t}^{y-1} \cdot q_{j,t}^{y-1} \right)} \cdot p_{j,t}^{y-1} \cdot \left(\frac{q_{j,t}^{y-1}}{Q_t^{y-1}} \right) \right] \right) \\
&= \left(\left(\frac{Q_{t-4}^{y-1}}{Q_t^{y-1}} \right) \cdot \sum_{j=1}^{J} \left[\frac{p_{j,t-4}^{y-1} \cdot q_{j,t}^{y-1}}{\sum_{j=1}^{J} \left(p_{j,t-4}^{y-1} \cdot q_{j,t-4}^{y-1} \right)} \right] - \sum_{j=1}^{J} \left[\frac{p_{j,t}^{y-1} \cdot q_{j,t}^{y-1}}{\sum_{j=1}^{J} \left(p_{j,t}^{y-1} \cdot q_{j,t}^{y-1} \right)} \right] \right) \\
&= \left(\left(\frac{Q_{t-4}^{y-1}}{Q_t^{y-1}} \right) \cdot \frac{\sum_{j=1}^{J} \left(p_{j,t-4}^{y-1} \cdot q_{j,t}^{y-1} \right)}{\sum_{j=1}^{J} \left(p_{j,t-4}^{y-1} \cdot q_{j,t-4}^{y-1} \right)} - \frac{\sum_{j=1}^{J} \left(p_{j,t}^{y-1} \cdot q_{j,t}^{y-1} \right)}{\sum_{j=1}^{J} \left(p_{j,t}^{y-1} \cdot q_{j,t}^{y-1} \right)} \right) \\
&= \left(\left(\frac{Q_{t-4}^{y-1}}{Q_t^{y-1}} \right) \cdot \left(\frac{Q_t^{y-1}}{Q_{t-4}^{y-1}} \right) - 1 \right) \\
&= 0
\end{aligned}$$

A.3.1.4: Cobb 2013 (E.4)

$$\begin{aligned}
\sum\nolimits_{j=1}^{J} c_{j,t}^{Cobb} - \sum\nolimits_{j=1}^{J} \hat{c}_{j,t}^{YoY} &= \sum\nolimits_{j=1}^{J}\left[\frac{w_{j,t}}{Q_{t-4}}\cdot\left(q_{j,t}-q_{j,t-4}\right)+\left(w_{j,t}-w_{j,t-4}\right)\cdot\left(\frac{q_{j,t-4}}{Q_{t-4}}-\frac{1}{Q_{t-4}}\cdot\frac{q_{j,t}^{y-1}}{4}\right)\right]-\frac{1}{Q_{t-4}}\cdot\sum\nolimits_{j=1}^{J}\left(w_{j,t}\cdot q_{j,t}-w_{j,t-4}\cdot q_{j,t-4}\right)\\
&= \frac{1}{Q_{t-4}}\cdot\sum\nolimits_{j=1}^{J}\left[w_{j,t}\cdot\left(q_{j,t}-q_{j,t-4}\right)+\left(w_{j,t}-w_{j,t-4}\right)\cdot q_{j,t-4}-\left(w_{j,t}-w_{j,t-4}\right)\cdot\frac{q_{j,t}^{y-1}}{4}-w_{j,t}\cdot q_{j,t}+w_{j,t-4}\cdot q_{j,t-4}\right]\\
&= \frac{1}{Q_{t-4}}\cdot\sum\nolimits_{j=1}^{J}\left[-\left(w_{j,t}-w_{j,t-4}\right)\cdot\frac{q_{j,t}^{y-1}}{4}\right]\\
&= \frac{1}{Q_{t-4}}\cdot\frac{1}{4}\cdot\left(\sum\nolimits_{j=1}^{J}\left[\frac{Q_{t-4}^{y-1}}{\sum\nolimits_{j=1}^{J}\left(p_{j,t-4}^{y-1}\cdot q_{j,t-4}^{y-1}\right)}\cdot p_{j,t-4}^{y-1}\cdot q_{j,t}^{y-1}\right]-\sum\nolimits_{j=1}^{J}\left[\frac{Q_{t}^{y-1}}{\sum\nolimits_{j=1}^{J}\left(p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right)}\cdot p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right]\right)\\
&= \frac{1}{Q_{t-4}}\cdot\frac{1}{4}\cdot\left(Q_{t-4}^{y-1}\cdot\frac{\sum\nolimits_{j=1}^{J}\left(p_{j,t-4}^{y-1}\cdot q_{j,t}^{y-1}\right)}{\sum\nolimits_{j=1}^{J}\left(p_{j,t-4}^{y-1}\cdot q_{j,t-4}^{y-1}\right)} - Q_{t}^{y-1}\cdot\frac{\sum\nolimits_{j=1}^{J}\left(p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right)}{\sum\nolimits_{j=1}^{J}\left(p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right)}\right)\\
&= \frac{1}{Q_{t-4}}\cdot\frac{1}{4}\cdot\left(Q_{t-4}^{y-1}\cdot\frac{Q_{t}^{y-1}}{Q_{t-4}^{y-1}} - Q_{t}^{y-1}\right)\\
&= 0
\end{aligned}$$

A.3.2. Consistency of the component's contribution to the annual growth of the aggregate quarterly series with the contributions to growth of the aggregate annual series

For (R.2) to be met always the following must be true:

$\sum\nolimits_{t=l}^{l+3} k_{j,t} - K_{j}^{y} = 0$ when l is the first quarter of year y

For the following derivations it is necessary to understand that quarterly weights within a year are the same and equal to the annual weight. Specifically for this exercise, for $t=l$ to $l+3$, $w_{j,t}=w_{j}^{y}$ and $w_{j,t-4}=w_{j}^{y-1}$

A.3.2.1: Fixed weights (E.1)

$$\begin{aligned}
\sum\nolimits_{t=l}^{l+3} k_{j,t}^{FW} - K_{j}^{y} &= \sum\nolimits_{t=l}^{l+3}\left[\omega_{j}^{ref.year}\cdot\left(q_{j,t}-q_{j,t-4}\right)\right]-w_{j}^{y}\cdot\left(q_{j}^{y}-q_{j}^{y-1}\right)\\
&= \omega_{j}^{ref.year}\cdot\left(\sum\nolimits_{t=l}^{l+3} q_{j,t}-\sum\nolimits_{t=l}^{l+3} q_{j,t-4}\right)-w_{j}^{y}\cdot\left(q_{j}^{y}-q_{j}^{y-1}\right)\\
&= \omega_{j}^{ref.year}\cdot\left(q_{j}^{y}-q_{j}^{y-1}\right)-w_{j}^{y}\cdot\left(q_{j}^{y}-q_{j}^{y-1}\right)\\
&= \left(\omega_{j}^{ref.year}-w_{j}^{y}\right)\cdot\left(q_{j}^{y}-q_{j}^{y-1}\right)
\end{aligned}$$

The result is not necessarily zero.

A.3.2.2: Previous year prices (E.2)

$$\begin{aligned}
\sum\nolimits_{t=l}^{l+3} k_{j,t}^{PYP} - K_{j}^{y} &= \sum\nolimits_{t=l}^{l+3}\left[w_{j,t}\cdot\left(q_{j,t}-q_{j,t-4}\right)\right]-w_{j}^{y}\cdot\left(q_{j}^{y}-q_{j}^{y-1}\right)\\
&= w_{j}^{y}\cdot\left(\sum\nolimits_{t=l}^{l+3} q_{j,t}-\sum\nolimits_{t=l}^{l+3} q_{j,t-4}\right)-w_{j}^{y}\cdot\left(q_{j}^{y}-q_{j}^{y-1}\right)\\
&= w_{j}^{y}\cdot\left(q_{j}^{y}-q_{j}^{y-1}\right)-w_{j}^{y}\cdot\left(q_{j}^{y}-q_{j}^{y-1}\right)\\
&= 0
\end{aligned}$$

A.3.2.3: INSEE 2007 (E.3)

$$\begin{aligned}
\sum_{t=l}^{l+3} k_{j,t}^{INSEE} - K_j^y &= \sum_{t=l}^{l+3}\left[w_{j,t}\cdot\left(q_{j,t}-q_{j,t-4}\right)+\left(w_{j,t}-w_{j,t-4}\right)\cdot\left(q_{j,t-4}-\frac{q_{j,t}^{y-1}}{Q_t^{y-1}}\cdot Q_{t-4}\right)\right]-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)\\
&= \sum_{t=l}^{l+3}\left[\left(w_{j,t}-w_{j,t-4}\right)\cdot\left(q_{j,t-4}-\frac{q_{j,t}^{y-1}}{Q_t^{y-1}}\cdot Q_{t-4}\right)\right]\\
&= \left(w_j^y-w_j^{y-1}\right)\cdot\left[\sum_{t=l}^{l+3}\left(q_{j,t-4}\right)-\frac{q_{j,t}^{y-1}}{Q_t^{y-1}}\cdot\sum_{t=l}^{l+3}\left(Q_{t-4}\right)\right]\\
&= \left(w_j^y-w_j^{y-1}\right)\cdot\left[q_{j,t}^{y-1}-\frac{q_{j,t}^{y-1}}{Q_t^{y-1}}\cdot Q_t^{y-1}\right]\\
&= 0
\end{aligned}$$

A.3.2.4: Cobb 2013 (E.4)

$$\begin{aligned}
\sum_{t=l}^{l+3} k_{j,t}^{Cobb} - K_j^y &= \sum_{t=l}^{l+3}\left[w_{j,t}\cdot\left(q_{j,t}-q_{j,t-4}\right)+\left(w_{j,t}-w_{j,t-4}\right)\cdot\left(q_{j,t-4}-\frac{q_j^{y-1}}{4}\right)\right]-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)\\
&= w_j^y\cdot\sum_{t=l}^{l+3}\left(q_{j,t}-q_{j,t-4}\right)+\left(w_{j,t}-w_{j,t-4}\right)\cdot\sum_{t=l}^{l+3}\left(q_{j,t-4}-\frac{q_j^{y-1}}{4}\right)-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)\\
&= \left(w_{j,t}-w_{j,t-4}\right)\cdot\left[q_j^{y-1} \quad - \quad q_j^{y-1}\cdot\sum_{t=l}^{l+3}\left(1/4\right)\right]\\
&= 0
\end{aligned}$$

A.3.3. Additivity of the contributions to quarterly growth of the aggregate quarterly series

For (R.3) to be met always the following must be true:

$$\sum_{j=1}^{J} c_{j,t}-\frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left(w_{j,t}\cdot q_{j,t}-w_{j,t-1}\cdot q_{j,t-1}\right)=0$$

A.3.3.1: Fixed weights (E.1)

$$\begin{aligned}
\sum_{j=1}^{J} c_{j,t}^{FW}-\sum_{j=1}^{J}\hat{c}_{j,t}^{YoY} &= \sum_{j=1}^{J}\left[\frac{\omega_j^{ref.year}}{Q_{t-1}}\cdot\left(q_{j,t}-q_{j,t-1}\right)\right]-\frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left(w_{j,t}\cdot q_{j,t}-w_{j,t-1}\cdot q_{j,t-1}\right)\\
&= \frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left[\left(\omega_j^{ref.year}-w_{j,t}\right)\cdot q_{j,t}-\left(\omega_j^{ref.year}-w_{j,t-1}\right)\cdot q_{j,t-1}\right]
\end{aligned}$$

The result is not necessarily zero.

A.3.3.2: Previous year prices (E.2)

$$\begin{aligned}
\sum_{j=1}^{J} c_{j,t}^{PYP}-\sum_{j=1}^{J}\hat{c}_{j,t}^{YoY} &= \sum_{j=1}^{J}\left[\frac{w_{j,t}}{Q_{t-1}}\cdot\left(q_{j,t}-q_{j,t-4}\right)\right]-\frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left(w_{j,t}\cdot q_{j,t}-w_{j,t-1}\cdot q_{j,t-1}\right)\\
&= \frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left[\left(w_{j,t}-w_{j,t}\right)\cdot q_{j,t}-\left(w_{j,t}-w_{j,t-1}\right)\cdot q_{j,t-1}\right]\\
&= \frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left[\left(w_{j,t}-w_{j,t-1}\right)\cdot q_{j,t-1}\right]
\end{aligned}$$

The result is not necessarily zero when *t* is the first quarter of any given year.

A.3.3.3: INSEE 2007 (E.3)

$$
\begin{aligned}
\sum_{j=1}^{J} c_{j,t}^{INSEE} - \sum_{j=1}^{J} \hat{c}_{j,t}^{YoY}
&= \sum_{j=1}^{J}\left[\frac{w_{j,t}}{Q_{t-1}}\cdot\left(q_{j,t}-q_{j,t-1}\right)+\left(w_{j,t}-w_{j,t-1}\right)\cdot\left(\frac{q_{j,t-1}}{Q_{t-1}}-\frac{q_{j,t}^{y-1}}{Q_{t}^{y-1}}\right)\right]-\frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left(w_{j,t}\cdot q_{j,t}-w_{j,t-1}\cdot q_{j,t-1}\right)\\
&= \frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left[w_{j,t}\cdot q_{j,t}-w_{j,t}\cdot q_{j,t-1}+w_{j,t}\cdot q_{j,t-1}-w_{j,t-1}\cdot q_{j,t-1}-\left(w_{j,t}-w_{j,t-1}\right)\cdot\left(\frac{q_{j,t}^{y-1}}{Q_{t}^{y-1}}\cdot Q_{t-1}\right)-w_{j,t}\cdot q_{j,t}+w_{j,t-1}\cdot q_{j,t-1}\right]\\
&= \frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left[-\left(w_{j,t}-w_{j,t-1}\right)\cdot\left(\frac{q_{j,t}^{y-1}}{Q_{t}^{y-1}}\cdot Q_{t-1}\right)\right]\\
&= \left(\sum_{j=1}^{J}\left[\frac{Q_{t-1}^{y-1}}{\sum_{j=1}^{J}\left(p_{j,t-1}^{y-1}\cdot q_{j,t-1}^{y-1}\right)}\cdot p_{j,t-1}^{y-1}\cdot\left(\frac{q_{j,t}^{y-1}}{Q_{t}^{y-1}}\right)\right]-\sum_{j=1}^{J}\left[\frac{Q_{t}^{y-1}}{\sum_{j=1}^{J}\left(p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right)}\cdot p_{j,t}^{y-1}\cdot\left(\frac{q_{j,t}^{y-1}}{Q_{t}^{y-1}}\right)\right]\right)\\
&= \left(\left(\frac{Q_{t-1}^{y-1}}{Q_{t}^{y-1}}\right)\cdot\sum_{j=1}^{J}\left[\frac{p_{j,t-1}^{y-1}\cdot q_{j,t}^{y-1}}{\sum_{j=1}^{J}\left(p_{j,t-1}^{y-1}\cdot q_{j,t-1}^{y-1}\right)}\right]-\sum_{j=1}^{J}\left[\frac{p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}}{\sum_{j=1}^{J}\left(p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right)}\right]\right)\\
&= \left(\left(\frac{Q_{t-1}^{y-1}}{Q_{t}^{y-1}}\right)\cdot\frac{\sum_{j=1}^{J}\left(p_{j,t-1}^{y-1}\cdot q_{j,t}^{y-1}\right)}{\sum_{j=1}^{J}\left(p_{j,t-1}^{y-1}\cdot q_{j,t-1}^{y-1}\right)}\quad-\quad\frac{\sum_{j=1}^{J}\left(p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right)}{\sum_{j=1}^{J}\left(p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right)}\right)\\
&= \left(\left(\frac{Q_{t-1}^{y-1}}{Q_{t}^{y-1}}\right)\cdot\left(\frac{Q_{t}^{y-1}}{Q_{t-1}^{y-1}}\right)\quad-\quad 1\right)\\
&= 0
\end{aligned}
$$

A.3.3.4: Cobb 2013 (E.4)

$$
\begin{aligned}
\sum_{j=1}^{J} c_{j,t}^{Cobb} - \sum_{j=1}^{J} \hat{c}_{j,t}^{YoY}
&= \sum_{j=1}^{J}\left[\frac{w_{j,t}}{Q_{t-1}}\cdot\left(q_{j,t}-q_{j,t-1}\right)+\left(w_{j,t}-w_{j,t-1}\right)\cdot\left(\frac{q_{j,t-1}}{Q_{t-1}}-\frac{1}{Q_{t-1}}\cdot\frac{q_{j,t}^{y-1}}{4}\right)\right]-\frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left(w_{j,t}\cdot q_{j,t}-w_{j,t-1}\cdot q_{j,t-1}\right)\\
&= \frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left[w_{j,t}\cdot\left(q_{j,t}-q_{j,t-1}\right)+\left(w_{j,t}-w_{j,t-1}\right)\cdot q_{j,t-1}-\left(w_{j,t}-w_{j,t-1}\right)\cdot\frac{q_{j,t}^{y-1}}{4}-w_{j,t}\cdot q_{j,t}+w_{j,t-1}\cdot q_{j,t-1}\right]\\
&= \frac{1}{Q_{t-1}}\cdot\sum_{j=1}^{J}\left[-\left(w_{j,t}-w_{j,t-1}\right)\cdot\frac{q_{j,t}^{y-1}}{4}\right]\\
&= \frac{1}{Q_{t-1}}\cdot\frac{1}{4}\cdot\left(\sum_{j=1}^{J}\left[\frac{Q_{t-1}^{y-1}}{\sum_{j=1}^{J}\left(p_{j,t-1}^{y-1}\cdot q_{j,t-1}^{y-1}\right)}\cdot p_{j,t-1}^{y-1}\cdot q_{j,t}^{y-1}\right]-\sum_{j=1}^{J}\left[\frac{Q_{t}^{y-1}}{\sum_{j=1}^{J}\left(p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right)}\cdot p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right]\right)\\
&= \frac{1}{Q_{t-1}}\cdot\frac{1}{4}\cdot\left(Q_{t-1}^{y-1}\cdot\frac{\sum_{j=1}^{J}\left(p_{j,t-1}^{y-1}\cdot q_{j,t}^{y-1}\right)}{\sum_{j=1}^{J}\left(p_{j,t-1}^{y-1}\cdot q_{j,t-1}^{y-1}\right)}\quad-\quad Q_{t}^{y-1}\cdot\frac{\sum_{j=1}^{J}\left(p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right)}{\sum_{j=1}^{J}\left(p_{j,t}^{y-1}\cdot q_{j,t}^{y-1}\right)}\right)\\
&= \frac{1}{Q_{t-1}}\cdot\frac{1}{4}\cdot\left(Q_{t-1}^{y-1}\cdot\frac{Q_{t}^{y-1}}{Q_{t-1}^{y-1}}\quad-\quad Q_{t}^{y-1}\right)\\
&= 0
\end{aligned}
$$

A.3.4. Consistency of the component's contribution to the quarterly growth of the aggregate quarterly series with the contributions to growth of the aggregate annual series

For (R.4) to be met always the following must be true:

$$\sum_{t=l-3}^{l+3}\left[\left(4-abs(t-l)\right)\cdot k_{j,t}^{QoQ}\right]-K_j^y \quad = \quad 0 \qquad \text{when } l \text{ is the first quarter of year } y$$

As before, the following derivations requires understanding that quarterly weights within a year are the same and equal to the annual weight. Specifically for this exercise, for *t=l* to *l+3*, $w_{j,t}=w_j^y$ and for *t=l-3* to *l*, $w_{j,t-1}=w_j^{y-1}$

For simplicity we will define, $f(k_{j,t}^{QoQ})=\sum_{t=l-3}^{l+3}\left[\left(4-abs(t-l)\right)\cdot k_{j,t}^{QoQ}\right]$

A.3.4.1: Fixed weights (E.1)

$$\begin{aligned}
f(k_{j,t}^{FW})-K_j^y \quad = \quad & \omega_j^{ref.year}\cdot\left(q_{j,l-3}-q_{j,l-4}\right)+2\cdot\omega_j^{ref.year}\cdot\left(q_{j,l-2}-q_{j,l-3}\right)+3\cdot\omega_j^{ref.year}\cdot\left(q_{j,l-1}-q_{j,l-2}\right)+4\cdot\omega_j^{ref.year}\cdot\left(q_{j,l}-q_{j,l-1}\right)\\
& +3\cdot\omega_j^{ref.year}\cdot\left(q_{j,l+1}-q_{j,l}\right)+2\cdot\omega_j^{ref.year}\cdot\left(q_{j,l+2}-q_{j,l+1}\right)+\omega_j^{ref.year}\cdot\left(q_{j,l+3}-q_{j,l+2}\right)-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)\\
\\
= \quad & \omega_j^{ref.year}\cdot\left[\left(q_{j,l-3}-q_{j,l-4}\right)+2\cdot\left(q_{j,l-2}-q_{j,l-3}\right)+3\cdot\left(q_{j,l-1}-q_{j,l-2}\right)+4\cdot\left(q_{j,l}-q_{j,l-1}\right)+3\cdot\left(q_{j,l+1}-q_{j,l}\right)\right]\\
& +\omega_j^{ref.year}\cdot\left[2\cdot\left(q_{j,l+2}-q_{j,l+1}\right)+\left(q_{j,l+3}-q_{j,l+2}\right)\right]-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)\\
\\
= \quad & \omega_j^{ref.year}\cdot\left[-q_{j,l-4}-q_{j,l-3}-q_{j,l-2}-q_{j,l-1}+q_{j,l}+q_{j,l+1}+q_{j,l+2}+q_{j,l+3}\right]-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)\\
= \quad & \omega_j^{ref.year}\cdot\left(q_j^y-q_j^{y-1}\right)-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)\\
= \quad & \left(\omega_j^{ref.year}-w_j^y\right)\cdot\left(q_j^y-q_j^{y-1}\right)
\end{aligned}$$

The result is not necessarily zero.

A.3.4.2: Previous year prices (E.2)

$$\begin{aligned}
f(k_{j,t}^{PYP})-K_j^y \quad = \quad & w_{j,l-3}\cdot\left(q_{j,l-3}-q_{j,l-4}\right)+2\cdot w_{j,l-2}\cdot\left(q_{j,l-2}-q_{j,l-3}\right)+3\cdot w_{j,l-1}\cdot\left(q_{j,l-1}-q_{j,l-2}\right)+4\cdot w_{j,l}\cdot\left(q_{j,l}-q_{j,l-1}\right)\\
& +3\cdot w_{j,l+1}\cdot\left(q_{j,l+1}-q_{j,l}\right)+2\cdot w_{j,l+2}\cdot\left(q_{j,l+2}-q_{j,l+1}\right)+w_{j,l+3}\cdot\left(q_{j,l+3}-q_{j,l+2}\right)-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)\\
\\
= \quad & w_j^{y-1}\cdot\left[\left(q_{j,l-3}-q_{j,l-4}\right)+2\cdot\left(q_{j,l-2}-q_{j,l-3}\right)+3\cdot\left(q_{j,l-1}-q_{j,l-2}\right)\right]\\
& +w_j^y\cdot\left[4\cdot\left(q_{j,t}-q_{j,t-1}\right)+3\cdot\left(q_{j,t+1}-q_{j,t}\right)+2\cdot\left(q_{j,t+2}-q_{j,t+1}\right)+\left(q_{j,t+3}-q_{j,t+2}\right)\right]-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)\\
\\
= \quad & w_j^{y-1}\cdot\left[-q_{j,l-4}-q_{j,l-3}-q_{j,l-2}-q_{j,l-1}+4\cdot q_{j,l-1}\right]+w_j^y\cdot\left[-4\cdot q_{j,l-1}+q_{j,t}+q_{j,t+1}+q_{j,t+2}+q_{j,t+3}\right]-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)\\
= \quad & w_j^y\cdot q_j^y-w_j^{y-1}\cdot q_j^{y-1}-4\cdot q_{j,l-1}\cdot\left(w_j^y-w_j^{y-1}\right)-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)\\
= \quad & \left(w_j^y-w_j^{y-1}\right)\left(q_j^{y-1}-4\cdot q_{j,l-1}\right)
\end{aligned}$$

The result is not necessarily zero.

A.3.4.3: INSEE 2007 (E.3)

$$
\begin{aligned}
f(k_{j,t}^{INSEE})-K_j^y \quad = \quad & w_{j,l-3}\cdot\left(q_{j,l-3}-q_{j,l-4}\right)+\left(w_{j,l-3}-w_{j,l-4}\right)\cdot\left(q_{j,l-4}-\frac{q_{j,l-3}^{y-1}}{Q_{l-3}^{y-1}}\cdot Q_{l-4}\right)+2\cdot\left[w_{j,l-2}\cdot\left(q_{j,l-2}-q_{j,l-3}\right)+\left(w_{j,l-2}-w_{j,l-3}\right)\cdot\left(q_{j,l-3}-\frac{q_{j,l-2}^{y-1}}{Q_{l-2}^{y-1}}\cdot Q_{l-3}\right)\right] \\
& +3\cdot\left[w_{j,l-1}\cdot\left(q_{j,l-1}-q_{j,l-2}\right)+\left(w_{j,l-1}-w_{j,l-2}\right)\cdot\left(q_{j,l-2}-\frac{q_{j,l-1}^{y-1}}{Q_{l-1}^{y-1}}\cdot Q_{l-2}\right)\right]+4\cdot\left[w_{j,l}\cdot\left(q_{j,l}-q_{j,l-1}\right)+\left(w_{j,l}-w_{j,l-1}\right)\cdot\left(q_{j,l-1}-\frac{q_{j,l}^{y-1}}{Q_{l}^{y-1}}\cdot Q_{l-1}\right)\right] \\
& +3\cdot\left[w_{j,l+1}\cdot\left(q_{j,l+1}-q_{j,l}\right)+\left(w_{j,l+1}-w_{j,l}\right)\cdot\left(q_{j,l}-\frac{q_{j,l+1}^{y-1}}{Q_{l+1}^{y-1}}\cdot Q_{l}\right)\right]+2\cdot\left[w_{j,l+2}\cdot\left(q_{j,l+2}-q_{j,l+1}\right)+\left(w_{j,l+2}-w_{j,l+1}\right)\cdot\left(q_{j,l+1}-\frac{q_{j,l+2}^{y-1}}{Q_{l+2}^{y-1}}\cdot Q_{l+1}\right)\right] \\
& +w_{j,l+3}\cdot\left(q_{j,l+3}-q_{j,l+2}\right)+\left(w_{j,l+3}-w_{j,l+2}\right)\cdot\left(q_{j,l+2}-\frac{q_{j,l+3}^{y-1}}{Q_{l+3}^{y-1}}\cdot Q_{l+2}\right)-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & w_j^{y-1}\cdot\left[\left(q_{j,l-3}-q_{j,l-4}\right)+2\cdot\left(q_{j,l-2}-q_{j,l-3}\right)+3\cdot\left(q_{j,l-1}-q_{j,l-2}\right)\right] \\
& +w_j^y\cdot\left[4\cdot\left(q_{j,t}-q_{j,t-1}\right)+3\cdot\left(q_{j,t+1}-q_{j,t}\right)+2\cdot\left(q_{j,t+2}-q_{j,t+1}\right)+\left(q_{j,t+3}-q_{j,t+2}\right)\right] \\
& +4\cdot\left(w_j^y-w_j^{y-1}\right)\cdot\left(q_{j,l-1}-\frac{q_j^{y-1}}{Q^{y-1}}\cdot Q_{l-1}\right)-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & w_j^y\cdot q_j^y-w_j^{y-1}\cdot q_j^{y-1}-4\cdot q_{j,l-1}\cdot\left(w_j^y-w_j^{y-1}\right)-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)+4\cdot\left(w_j^y-w_j^{y-1}\right)\cdot\left(q_{j,l-1}-\frac{q_j^{y-1}}{Q^{y-1}}\cdot Q_{l-1}\right) \\
= \quad & \left(w_j^y-w_j^{y-1}\right)\cdot q_j^{y-1}+\left(w_j^y-w_j^{y-1}\right)\cdot\left(-\frac{q_j^{y-1}}{Q^{y-1}}\cdot 4\cdot Q_{l-1}\right) \\
= \quad & \left(w_j^y-w_j^{y-1}\right)\cdot q_j^{y-1}\cdot\left(1 \quad - \quad \frac{4\cdot Q_{l-1}}{Q^{y-1}}\right)
\end{aligned}
$$

The result is not necessarily zero.

A.3.4.4: Cobb 2013 (E.4)

$$
\begin{aligned}
f(k_{j,t}^{Cobb})-K_j^y \quad = \quad & w_{j,l-3}\cdot\left(q_{j,l-3}-q_{j,l-4}\right)+\left(w_{j,l-3}-w_{j,l-4}\right)\cdot\left(q_{j,l-4}-\frac{q_{j,l-3}^{y-1}}{4}\right)+2\cdot\left[w_{j,l-2}\cdot\left(q_{j,l-2}-q_{j,l-3}\right)+\left(w_{j,l-2}-w_{j,l-3}\right)\cdot\left(q_{j,l-3}-\frac{q_{j,l-2}^{y-1}}{4}\right)\right] \\
& +3\cdot\left[w_{j,l-1}\cdot\left(q_{j,l-1}-q_{j,l-2}\right)+\left(w_{j,l-1}-w_{j,l-2}\right)\cdot\left(q_{j,l-2}-\frac{q_{j,l-1}^{y-1}}{4}\right)\right]+4\cdot\left[w_{j,l}\cdot\left(q_{j,l}-q_{j,l-1}\right)+\left(w_{j,l}-w_{j,l-1}\right)\cdot\left(q_{j,l-1}-\frac{q_{j,l}^{y-1}}{4}\right)\right] \\
& +3\cdot\left[w_{j,l+1}\cdot\left(q_{j,l+1}-q_{j,l}\right)+\left(w_{j,l+1}-w_{j,l}\right)\cdot\left(q_{j,l}-\frac{q_{j,l+1}^{y-1}}{4}\right)\right]+2\cdot\left[w_{j,l+2}\cdot\left(q_{j,l+2}-q_{j,l+1}\right)+\left(w_{j,l+2}-w_{j,l+1}\right)\cdot\left(q_{j,l+1}-\frac{q_{j,l+2}^{y-1}}{4}\right)\right] \\
& +w_{j,l+3}\cdot\left(q_{j,l+3}-q_{j,l+2}\right)+\left(w_{j,l+3}-w_{j,l+2}\right)\cdot\left(q_{j,l+2}-\frac{q_{j,l+3}^{y-1}}{4}\right)-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & w_j^{y-1}\cdot\left[\left(q_{j,l-3}-q_{j,l-4}\right)+2\cdot\left(q_{j,l-2}-q_{j,l-3}\right)+3\cdot\left(q_{j,l-1}-q_{j,l-2}\right)\right] \\
& +w_j^y\cdot\left[4\cdot\left(q_{j,t}-q_{j,t-1}\right)+3\cdot\left(q_{j,t+1}-q_{j,t}\right)+2\cdot\left(q_{j,t+2}-q_{j,t+1}\right)+\left(q_{j,t+3}-q_{j,t+2}\right)\right] \\
& +4\cdot\left(w_j^y-w_j^{y-1}\right)\cdot\left(q_{j,l-1}-\frac{q_j^{y-1}}{4}\right)-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & w_j^y\cdot q_j^y-w_j^{y-1}\cdot q_j^{y-1}-4\cdot q_{j,l-1}\cdot\left(w_j^y-w_j^{y-1}\right)-w_j^y\cdot\left(q_j^y-q_j^{y-1}\right)+4\cdot\left(w_j^y-w_j^{y-1}\right)\cdot\left(q_{j,l-1}-\frac{q_j^{y-1}}{4}\right) \\
= \quad & \left(w_j^y-w_j^{y-1}\right)\cdot q_j^{y-1}-\left(w_j^y-w_j^{y-1}\right)\cdot\left(4\cdot\frac{q_j^{y-1}}{4}\right) \\
= \quad & 0
\end{aligned}
$$

Annex 5: Examples of the discrepancies between different suggestions for quarterly contributions

Table A5.1: Contributions to annual growth of a quarterly aggregate series and the implied contributions to the growth of the corresponding annual series

Growth	A	B	Pa	Pb
	2%	5%	10%	-5%

	Chain-linked series				Annual contribution (pp.)	
Annual	A	B	**Total**	*y/y* %	A	B
2003	200	400	**600**			
2004	204	420	**624**	*4.00*	***0.67***	***3.33***
2005	208	441	**648**	*3.92*	***0.72***	***3.20***
2006	212	463	**673**	*3.84*	***0.77***	***3.06***

Quarterly	A	B	**Total**	*y/y* %		
Mar-03	50	98	**148**			
Jun-03	50	99	**149**			
Sep-03	50	101	**151**			
Dec-03	50	102	**152**			
Mar-04	51	103	**154**	*3.99*	-	-
Jun-04	51	104	**155**	*4.00*	-	-
Sep-04	51	106	**157**	*4.00*	-	-
Dec-04	51	107	**158**	*4.01*	-	-
Mar-05	52	108	**160**	*3.95*	-	-
Jun-05	52	110	**161**	*3.93*	-	-
Sep-05	52	111	**163**	*3.91*	-	-
Dec-05	52	112	**164**	*3.89*	-	-
Mar-06	53	114	**166**	*3.87*	-	-
Jun-06	53	115	**168**	*3.85*	-	-
Sep-06	53	116	**169**	*3.83*	-	-
Dec-06	53	118	**171**	*3.81*	-	-

E.1:

Fixed Weights

	Annual contribution implied by quarterly contributions (pp.)				
	Total (%)	A	B	*Diff. With* A	*Diff. With* B
2004	4.00	***0.67***	***3.33***		
2005	4.02	***0.65***	***3.37***	0.07	-0.17
2006	4.04	***0.64***	***3.40***	0.13	-0.33

Levels implied by quarterly contributions	
A	B
200	400
204	420
208	441
212	463

	Quarterly contribution to aggregate growth					
	Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	A	B
Mar-03					50	98
Jun-03					50	99
Sep-03					50	101
Dec-03					50	102
Mar-04	*3.99*	0.67	3.32	-	51	103
Jun-04	*4.00*	0.67	3.33	-	51	104
Sep-04	*4.00*	0.67	3.34	-	51	106
Dec-04	*4.01*	0.66	3.35	-	51	107
Mar-05	*4.01*	0.66	3.35	-0.06	52	108
Jun-05	*4.02*	0.66	3.36	-0.09	52	110
Sep-05	*4.02*	0.65	3.37	-0.11	52	111
Dec-05	*4.03*	0.65	3.38	-0.14	52	112
Mar-06	*4.03*	0.65	3.39	-0.16	53	114
Jun-06	*4.04*	0.64	3.40	-0.19	53	115
Sep-06	*4.04*	0.64	3.40	-0.22	53	116
Dec-06	*4.05*	0.64	3.41	-0.24	53	118

E.2:

Previous Year Prices

	Annual contribution implied by quarterly contributions (pp.)				
	Total (%)	A	B	*Diff. With* A	*Diff. With* B
2004	4.00	***0.67***	***3.33***		
2005	3.92	***0.72***	***3.20***	0.00	0.00
2006	3.84	***0.77***	***3.06***	0.00	0.00

Levels implied by quarterly contributions	
A	B
200	400
204	420
208	440
214	460

	Quarterly contribution to aggregate growth					
	Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	A	B
Mar-03					50	98
Jun-03					50	99
Sep-03					50	101
Dec-03					50	102
Mar-04	*3.99*	0.67	3.32	-	51	103
Jun-04	*4.00*	0.67	3.33	-	51	104
Sep-04	*4.00*	0.67	3.34	-	51	106
Dec-04	*4.01*	0.66	3.35	-	51	107
Mar-05	*3.91*	0.73	3.19	0.04	52	108
Jun-05	*3.92*	0.72	3.20	0.01	52	109
Sep-05	*3.92*	0.72	3.20	-0.01	52	111
Dec-05	*3.93*	0.71	3.21	-0.04	53	112
Mar-06	*3.83*	0.78	3.05	0.04	53	113
Jun-06	*3.84*	0.78	3.06	0.01	53	114
Sep-06	*3.84*	0.77	3.07	-0.01	54	116
Dec-06	*3.84*	0.77	3.07	-0.04	54	117

E.3:

INSEE 2007

	Annual contribution implied by quarterly contributions (pp.)				
	Total (%)	A	B	*Diff. With* A	*Diff. With* B
2004	4.00	***0.67***	***3.33***		
2005	3.92	***0.72***	***3.20***	0.00	0.00
2006	3.84	***0.77***	***3.06***	0.00	0.00

Levels implied by quarterly contributions	
A	B
200	400
204	420
208	440
214	460

	Quarterly contribution to aggregate growth					
	Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	A	B
Mar-03					50	98
Jun-03					50	99
Sep-03					50	101
Dec-03					50	102
Mar-04	*3.99*	0.67	3.32	-	51	103
Jun-04	*4.00*	0.67	3.33	-	51	104
Sep-04	*4.00*	0.67	3.34	-	51	106
Dec-04	*4.01*	0.66	3.35	-	51	107
Mar-05	*3.95*	0.75	3.20	-	52	108
Jun-05	*3.93*	0.73	3.20	-	52	109
Sep-05	*3.91*	0.71	3.20	-	52	111
Dec-05	*3.89*	0.69	3.20	-	52	112
Mar-06	*3.87*	0.80	3.06	-	53	113
Jun-06	*3.85*	0.78	3.06	-	53	114
Sep-06	*3.83*	0.77	3.06	-	53	116
Dec-06	*3.81*	0.75	3.06	-	54	117

E.4:

Cobb 2013

	Annual contribution implied by quarterly contributions (pp.)				
	Total (%)	A	B	*Diff. With* A	*Diff. With* B
2004	4.00	***0.67***	***3.33***		
2005	3.92	***0.72***	***3.20***	0.00	0.00
2006	3.84	***0.77***	***3.06***	0.00	0.00

Levels implied by quarterly contributions	
A	B
200	400
204	420
208	440
214	460

	Quarterly contribution to aggregate growth					
	Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	A	B
Mar-03					50	98
Jun-03					50	99
Sep-03					50	101
Dec-03					50	102
Mar-04	*3.99*	0.67	3.32	-	51	103
Jun-04	*4.00*	0.67	3.33	-	51	104
Sep-04	*4.00*	0.67	3.34	-	51	106
Dec-04	*4.01*	0.66	3.35	-	51	107
Mar-05	*3.95*	0.70	3.25	-	52	108
Jun-05	*3.93*	0.71	3.22	-	52	109
Sep-05	*3.91*	0.73	3.18	-	52	111
Dec-05	*3.89*	0.74	3.15	-	53	112
Mar-06	*3.87*	0.75	3.11	-	53	113
Jun-06	*3.85*	0.77	3.08	-	53	114
Sep-06	*3.83*	0.78	3.05	-	54	116
Dec-06	*3.81*	0.79	3.01	-	54	117

Chart A5.1: Levels of the quarterly components and aggregate series

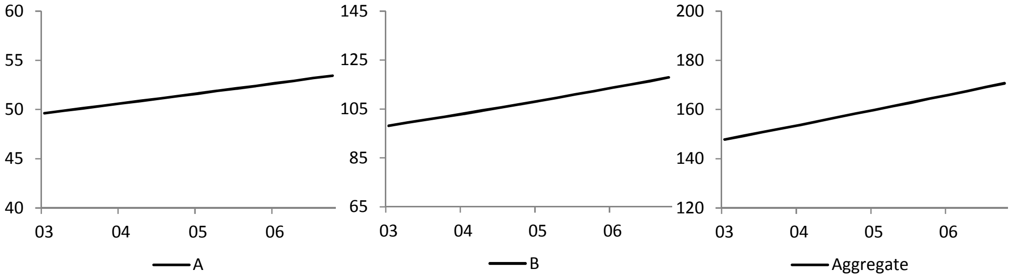

Table A5.2: Contributions to quarterly growth of an aggregate series and the implied contributions to the growth of the corresponding annual series

Growth	A	B	Pa	Pb
	2%	5%	10%	-5%

Annual	Chain-linked series A	B	**Total**	*y/y* %	Annual contribution (pp.) A	B
2003	200	400	**600**			
2004	204	420	**624**	*4.00*	***0.67***	***3.33***
2005	208	441	**648**	*3.92*	***0.72***	***3.20***
2006	212	463	**673**	*3.84*	***0.77***	***3.06***

Quarterly	A	B	**Total**	*q/q* %		
Mar-03	50	98	**148**			
Jun-03	50	99	**149**	*0.98*	-	-
Sep-03	50	101	**151**	*0.98*	-	-
Dec-03	50	102	**152**	*0.98*	-	-
Mar-04	51	103	**154**	*0.99*	-	-
Jun-04	51	104	**155**	*0.99*	-	-
Sep-04	51	106	**157**	*0.99*	-	-
Dec-04	51	107	**158**	*0.99*	-	-
Mar-05	52	108	**160**	*0.93*	-	-
Jun-05	52	110	**161**	*0.97*	-	-
Sep-05	52	111	**163**	*0.97*	-	-
Dec-05	52	112	**164**	*0.97*	-	-
Mar-06	53	114	**166**	*0.91*	-	-
Jun-06	53	115	**168**	*0.95*	-	-
Sep-06	53	116	**169**	*0.95*	-	-
Dec-06	53	118	**171**	*0.95*	-	-

E.1:

Fixed Weights

Annual contribution implied by quarterly contributions (pp.)

Annual	Total (%)	A	B	*Diff. With* A	B
2004	4.00	***0.67***	***3.33***		
2005	4.02	***0.65***	***3.37***	0.07	-0.17
2006	4.04	***0.64***	***3.40***	0.13	-0.33

Levels implied by quarterly contributions

A	B
200	400
204	420
208	441
212	463

Quarterly contribution to aggregate growth

Quarterly	Total (%)	A (pp.)	B (pp.)	*Diff. w/ q/q* (pp.)	Levels implied A	B
Mar-03					50	98
Jun-03	*0.98*	0.17	0.82	-	50	99
Sep-03	*0.98*	0.17	0.82	-	50	101
Dec-03	*0.98*	0.17	0.82	-	50	102
Mar-04	*0.99*	0.16	0.82	-	51	103
Jun-04	*0.99*	0.16	0.82	-	51	104
Sep-04	*0.99*	0.16	0.83	-	51	106
Dec-04	*0.99*	0.16	0.83	-	51	107
Mar-05	*0.99*	0.16	0.83	-0.06	52	108
Jun-05	*0.99*	0.16	0.83	-0.02	52	110
Sep-05	*0.99*	0.16	0.83	-0.02	52	111
Dec-05	*0.99*	0.16	0.84	-0.02	52	112
Mar-06	*1.00*	0.16	0.84	-0.09	53	114
Jun-06	*1.00*	0.16	0.84	-0.05	53	115
Sep-06	*1.00*	0.16	0.84	-0.05	53	116
Dec-06	*1.00*	0.16	0.85	-0.05	53	118

E.2:

Previous Year Prices

Annual contribution implied by quarterly contributions (pp.)

Annual	Total (%)	A	B	*Diff. With* A	B
2004	4.00	***0.67***	***3.33***		
2005	3.96	***0.70***	***3.26***	0.02	-0.06
2006	3.87	***0.75***	***3.12***	0.03	-0.06

Levels implied by quarterly contributions

A	B
200	400
204	420
208	440
213	461

Quarterly contribution to aggregate growth

Quarterly	Total (%)	A (pp.)	B (pp.)	*Diff. w/ q/q* (pp.)	Levels implied A	B
Mar-03					50	98
Jun-03	*0.98*	0.17	0.82	-	50	99
Sep-03	*0.98*	0.17	0.82	-	50	101
Dec-03	*0.98*	0.17	0.82	-	50	102
Mar-04	*0.99*	0.16	0.82	-	51	103
Jun-04	*0.99*	0.16	0.82	-	51	104
Sep-04	*0.99*	0.16	0.83	-	51	106
Dec-04	*0.99*	0.16	0.83	-	51	107
Mar-05	*0.97*	0.18	0.79	-0.04	52	108
Jun-05	*0.97*	0.18	0.79	-	52	109
Sep-05	*0.97*	0.18	0.79	-	52	111
Dec-05	*0.97*	0.17	0.79	-	53	112
Mar-06	*0.95*	0.19	0.75	-0.04	53	113
Jun-06	*0.95*	0.19	0.76	-	53	115
Sep-06	*0.95*	0.19	0.76	-	53	116
Dec-06	*0.95*	0.19	0.76	-	54	117

E.3:

INSEE 2007

Annual contribution implied by quarterly contributions (pp.)

Annual	Total (%)	A	B	*Diff. With* A	B
2004	4.00	***0.67***	***3.33***		
2005	3.92	***0.67***	***3.25***	0.05	-0.05
2006	3.84	***0.73***	***3.11***	0.05	-0.05

Levels implied by quarterly contributions

A	B
200	400
204	420
208	440
213	460

Quarterly contribution to aggregate growth

Quarterly	Total (%)	A (pp.)	B (pp.)	*Diff. w/ q/q* (pp.)	Levels implied A	B
Mar-03					50	98
Jun-03	*0.98*	0.17	0.82	-	50	99
Sep-03	*0.98*	0.17	0.82	-	50	101
Dec-03	*0.98*	0.17	0.82	-	50	102
Mar-04	*0.99*	0.16	0.82	-	51	103
Jun-04	*0.99*	0.16	0.82	-	51	104
Sep-04	*0.99*	0.16	0.83	-	51	106
Dec-04	*0.99*	0.16	0.83	-	51	107
Mar-05	*0.93*	0.15	0.78	-	52	108
Jun-05	*0.97*	0.18	0.79	-	52	109
Sep-05	*0.97*	0.18	0.79	-	52	111
Dec-05	*0.97*	0.17	0.79	-	52	112
Mar-06	*0.91*	0.17	0.74	-	53	113
Jun-06	*0.95*	0.19	0.76	-	53	114
Sep-06	*0.95*	0.19	0.76	-	53	116
Dec-06	*0.95*	0.19	0.76	-	54	117

E.4:

Cobb 2013

Annual contribution implied by quarterly contributions (pp.)

Annual	Total (%)	A	B	*Diff. With* A	B
2004	4.00	***0.67***	***3.33***		
2005	3.92	***0.72***	***3.20***	0.00	0.00
2006	3.84	***0.77***	***3.06***	0.00	0.00

Levels implied by quarterly contributions

A	B
200	400
204	420
208	440
214	460

Quarterly contribution to aggregate growth

Quarterly	Total (%)	A (pp.)	B (pp.)	*Diff. w/ q/q* (pp.)	Levels implied A	B
Mar-03					50	98
Jun-03	*0.98*	0.17	0.82	-	50	99
Sep-03	*0.98*	0.17	0.82	-	50	101
Dec-03	*0.98*	0.17	0.82	-	50	102
Mar-04	*0.99*	0.16	0.82	-	51	103
Jun-04	*0.99*	0.16	0.82	-	51	104
Sep-04	*0.99*	0.16	0.83	-	51	106
Dec-04	*0.99*	0.16	0.83	-	51	107
Mar-05	*0.93*	0.20	0.73	-	52	108
Jun-05	*0.97*	0.18	0.79	-	52	109
Sep-05	*0.97*	0.18	0.79	-	52	111
Dec-05	*0.97*	0.17	0.79	-	53	112
Mar-06	*0.91*	0.22	0.69	-	53	113
Jun-06	*0.95*	0.19	0.76	-	53	114
Sep-06	*0.95*	0.19	0.76	-	54	116
Dec-06	*0.95*	0.19	0.76	-	54	117

Chart A5.2: Levels of the quarterly components and aggregate series

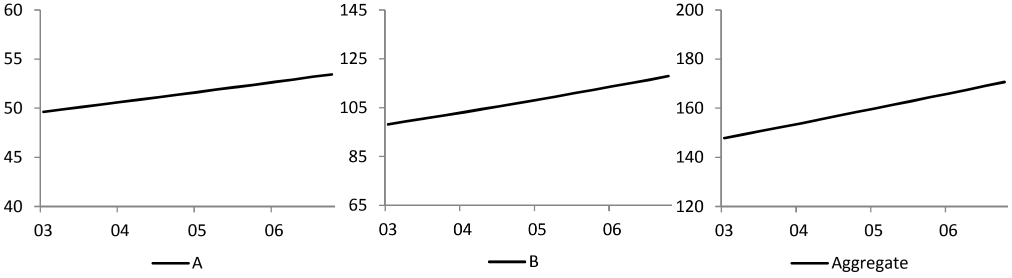

Table A5.3: Contributions to annual growth of a quarterly aggregate series with a seasonal component and the implied contributions to growth of the annual series

Growth	A	B	Pa	Pb
	2%	5%	10%	-5%

	Chain-linked series				Annual contribution (pp.)	
Annual	A	B	**Total**	*y/y* %	A	B
2003	200	400	**600**			
2004	204	420	**624**	*4,00*	***0,67***	***3,33***
2005	208	441	**648**	*3,92*	***0,72***	***3,20***
2006	212	463	**673**	*3,84*	***0,77***	***3,06***

Quarterly	A	B	**Total**	*y/y* %		
Mar-03	50	83	**133**			
Jun-03	50	99	**149**			
Sep-03	50	96	**146**			
Dec-03	50	122	**172**			
Mar-04	51	88	**138**	*3,88*	-	-
Jun-04	51	104	**155**	*4,00*	-	-
Sep-04	51	100	**151**	*3,97*	-	-
Dec-04	51	128	**179**	*4,12*	-	-
Mar-05	52	92	**144**	*4,41*	-	-
Jun-05	52	110	**161**	*3,93*	-	-
Sep-05	52	105	**158**	*4,05*	-	-
Dec-05	52	134	**185**	*3,42*	-	-
Mar-06	53	97	**151**	*4,34*	-	-
Jun-06	53	115	**168**	*3,85*	-	-
Sep-06	53	111	**164**	*3,97*	-	-
Dec-06	53	141	**191**	*3,32*	-	-

E.1:

Fixed Weights

	Annual contribution implied by quarterly contributions (pp.)					Levels implied by quarterly contributions	
Annual	Total (%)	A	B	*Diff. With* A	*Diff. With* B	A	B
2003						200	400
2004	4,00	***0,67***	***3,33***			204	420
2005	4,02	***0,65***	***3,37***	0,07	-0,17	208	441
2006	4,04	***0,64***	***3,40***	0,13	-0,33	212	463

	Quarterly contribution to aggregate growth					
Quarterly	Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	A	B
Mar-03					50	83
Jun-03					50	99
Sep-03					50	96
Dec-03					50	122
Mar-04	*3,88*	0,75	3,14	-	51	88
Jun-04	*4,00*	0,67	3,33	-	51	104
Sep-04	*3,97*	0,69	3,28	-	51	100
Dec-04	*4,12*	0,59	3,54	-	51	128
Mar-05	*3,90*	0,73	3,17	0,51	52	92
Jun-05	*4,02*	0,66	3,36	-0,09	52	110
Sep-05	*3,99*	0,68	3,31	0,06	52	105
Dec-05	*4,14*	0,57	3,57	-0,72	52	134
Mar-06	*3,90*	0,72	3,19	0,44	53	97
Jun-06	*4,04*	0,64	3,40	-0,19	53	115
Sep-06	*4,00*	0,66	3,34	-0,03	53	111
Dec-06	*4,19*	0,57	3,62	-0,87	53	141

E.2:

Previous Year Prices

	Annual contribution implied by quarterly contributions (pp.)					Levels implied by quarterly contributions	
Annual	Total (%)	A	B	*Diff. With* A	*Diff. With* B	A	B
2003						200	400
2004	4,00	***0,67***	***3,33***			204	420
2005	3,92	***0,72***	***3,20***	0,00	0,00	208	440
2006	3,84	***0,77***	***3,06***	0,00	0,00	214	460

	Quarterly contribution to aggregate growth					
Quarterly	Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	A	B
Mar-03					50	83
Jun-03					50	99
Sep-03					50	96
Dec-03					50	122
Mar-04	*3,88*	0,75	3,14	-	51	88
Jun-04	*4,00*	0,67	3,33	-	51	104
Sep-04	*3,97*	0,69	3,28	-	51	100
Dec-04	*4,12*	0,59	3,54	-	51	128
Mar-05	*3,82*	0,81	3,01	0,59	52	92
Jun-05	*3,92*	0,72	3,20	0,01	52	109
Sep-05	*3,89*	0,74	3,15	0,16	52	105
Dec-05	*4,02*	0,63	3,39	-0,60	53	134
Mar-06	*3,73*	0,86	2,87	0,61	53	96
Jun-06	*3,84*	0,78	3,06	0,01	53	114
Sep-06	*3,81*	0,80	3,01	0,16	54	110
Dec-06	*3,94*	0,68	3,26	-0,62	54	140

E.3:

INSEE 2007

	Annual contribution implied by quarterly contributions (pp.)					Levels implied by quarterly contributions	
Annual	Total (%)	A	B	*Diff. With* A	*Diff. With* B	A	B
2003						200	400
2004	4,00	***0,67***	***3,33***			204	420
2005	3,92	***0,72***	***3,20***	0,00	0,00	208	440
2006	3,84	***0,77***	***3,06***	0,00	0,00	214	460

	Quarterly contribution to aggregate growth					
Quarterly	Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	A	B
Mar-03					50	83
Jun-03					50	99
Sep-03					50	96
Dec-03					50	122
Mar-04	*3,88*	0,75	3,14	-	51	88
Jun-04	*4,00*	0,67	3,33	-	51	104
Sep-04	*3,97*	0,69	3,28	-	51	100
Dec-04	*4,12*	0,59	3,54	-	51	128
Mar-05	*4,41*	1,20	3,21	-	52	92
Jun-05	*3,93*	0,73	3,20	-	52	109
Sep-05	*4,05*	0,85	3,20	-	52	105
Dec-05	*3,42*	0,23	3,19	-	52	133
Mar-06	*4,34*	1,26	3,08	-	54	97
Jun-06	*3,85*	0,78	3,06	-	53	114
Sep-06	*3,97*	0,91	3,07	-	54	110
Dec-06	*3,32*	0,28	3,04	-	52	139

E.4:

Cobb 2013

	Annual contribution implied by quarterly contributions (pp.)					Levels implied by quarterly contributions	
Annual	Total (%)	A	B	*Diff. With* A	*Diff. With* B	A	B
2003						200	400
2004	4,00	***0,67***	***3,33***			204	420
2005	3,92	***0,72***	***3,20***	0,00	0,00	208	440
2006	3,84	***0,77***	***3,06***	0,00	0,00	214	460

	Quarterly contribution to aggregate growth					
Quarterly	Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	A	B
Mar-03					50	83
Jun-03					50	99
Sep-03					50	96
Dec-03					50	122
Mar-04	*3,88*	0,75	3,14	-	51	88
Jun-04	*4,00*	0,67	3,33	-	51	104
Sep-04	*3,97*	0,69	3,28	-	51	100
Dec-04	*4,12*	0,59	3,54	-	51	128
Mar-05	*4,41*	0,78	3,63	-	52	93
Jun-05	*3,93*	0,71	3,22	-	52	109
Sep-05	*4,05*	0,75	3,30	-	52	105
Dec-05	*3,42*	0,65	2,77	-	53	133
Mar-06	*4,34*	0,84	3,51	-	53	98
Jun-06	*3,85*	0,77	3,08	-	53	114
Sep-06	*3,97*	0,81	3,17	-	54	110
Dec-06	*3,32*	0,71	2,61	-	54	137

Chart A5.3: Levels of the quarterly components and aggregate series

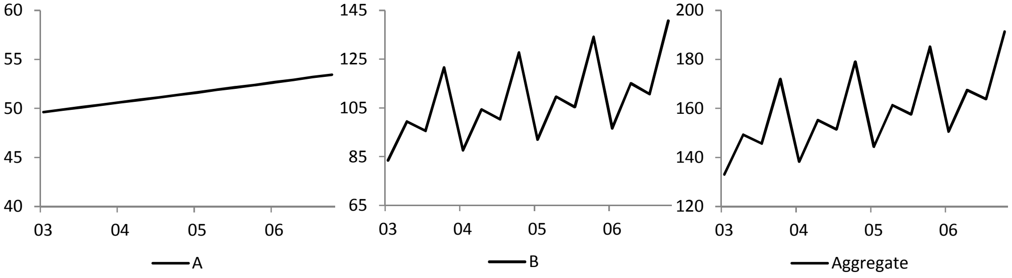

Table A5.4: Contributions to quarterly growth of an aggregate series with a seasonal component and the implied contributions to growth of the annual series

Growth	A	B	Pa	Pb
	2%	5%	10%	-5%

Annual	Chain-linked series A	B	Total	y/y %	Annual contribution (pp.) A	B
2003	200	400	**600**			
2004	204	420	**624**	*4,00*	***0,67***	***3,33***
2005	208	441	**648**	*3,92*	***0,72***	***3,20***
2006	212	463	**673**	*3,84*	***0,77***	***3,06***

Quarterly	A	B	Total	q/q %	Annual contribution (pp.) A	B
Mar-03	50	83	**133**			
Jun-03	50	99	**149**	*12,16*	-	-
Sep-03	50	96	**146**	*-2,39*	-	-
Dec-03	50	122	**172**	*18,03*	-	-
Mar-04	51	88	**138**	*-19,61*	-	-
Jun-04	51	104	**155**	*12,28*	-	-
Sep-04	51	100	**151**	*-2,41*	-	-
Dec-04	51	128	**179**	*18,21*	-	-
Mar-05	52	92	**144**	*-19,39*	-	-
Jun-05	52	110	**161**	*11,77*	-	-
Sep-05	52	105	**158**	*-2,30*	-	-
Dec-05	52	134	**185**	*17,49*	-	-
Mar-06	53	97	**151**	*-18,67*	-	-
Jun-06	53	115	**168**	*11,24*	-	-
Sep-06	53	111	**164**	*-2,18*	-	-
Dec-06	53	141	**191**	*16,75*	-	-

E.1:

Fixed Weights

Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	B
2004	4,00	***0,67***	***3,33***		
2005	4,02	***0,65***	***3,37***	0,07	-0,17
2006	4,04	***0,64***	***3,40***	0,13	-0,33

Levels implied by quarterly contributions A	B
200	400
204	420
208	441
212	463

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ q/q* (pp.)	Levels implied A	B
Mar-03					50	83
Jun-03	*12,16*	0,19	11,97	-	50	99
Sep-03	*-2,39*	0,17	-2,55	-	50	96
Dec-03	*18,03*	0,17	17,86	-	50	122
Mar-04	*-19,61*	0,15	-19,75	-	51	88
Jun-04	*12,28*	0,18	12,10	-	51	104
Sep-04	*-2,41*	0,16	-2,58	-	51	100
Dec-04	*18,21*	0,17	18,04	-	51	128
Mar-05	*-19,78*	0,14	-19,92	0,39	52	92
Jun-05	*12,35*	0,18	12,17	-0,58	52	110
Sep-05	*-2,44*	0,16	-2,60	0,14	52	105
Dec-05	*18,37*	0,16	18,20	-0,88	52	134
Mar-06	*-20,08*	0,14	-20,22	1,41	53	97
Jun-06	*12,42*	0,17	12,25	-1,18	53	115
Sep-06	*-2,48*	0,16	-2,63	0,29	53	111
Dec-06	*18,54*	0,16	18,38	-1,79	53	141

E.2:

Previous Year Prices

Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	B
2004	4,00	***0,67***	***3,33***		
2005	4,61	***0,70***	***3,91***	0,02	-0,71
2006	4,52	***0,74***	***3,78***	0,03	-0,71

Levels implied by quarterly contributions A	B
200	400
204	420
208	444
213	469

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ q/q* (pp.)	Levels implied A	B
Mar-03					50	83
Jun-03	*12,16*	0,19	11,97	-	50	99
Sep-03	*-2,39*	0,17	-2,55	-	50	96
Dec-03	*18,03*	0,17	17,86	-	50	122
Mar-04	*-19,61*	0,15	-19,75	-	51	88
Jun-04	*12,28*	0,18	12,10	-	51	104
Sep-04	*-2,41*	0,16	-2,58	-	51	100
Dec-04	*18,21*	0,17	18,04	-	51	128
Mar-05	*-18,79*	0,16	-18,94	-0,60	52	94
Jun-05	*11,77*	0,20	11,57	-	52	110
Sep-05	*-2,30*	0,18	-2,48	-	52	106
Dec-05	*17,49*	0,18	17,31	-	53	134
Mar-06	*-18,05*	0,17	-18,22	-0,62	53	100
Jun-06	*11,24*	0,21	11,03	-	53	117
Sep-06	*-2,18*	0,19	-2,37	-	53	113
Dec-06	*16,75*	0,19	16,56	-	54	140

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INSEE 2007

Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	B
2004	4,00	***0,67***	***3,33***		
2005	3,92	***0,23***	***3,69***	0,49	-0,49
2006	3,84	***0,29***	***3,55***	0,49	-0,49

Levels implied by quarterly contributions A	B
200	400
204	420
205	443
207	466

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ q/q* (pp.)	Levels implied A	B
Mar-03					50	83
Jun-03	*12,16*	0,19	11,97	-	50	99
Sep-03	*-2,39*	0,17	-2,55	-	50	96
Dec-03	*18,03*	0,17	17,86	-	50	122
Mar-04	*-19,61*	0,15	-19,75	-	51	88
Jun-04	*12,28*	0,18	12,10	-	51	104
Sep-04	*-2,41*	0,16	-2,58	-	51	100
Dec-04	*18,21*	0,17	18,04	-	51	128
Mar-05	*-19,39*	-0,25	-19,14	-	51	93
Jun-05	*11,77*	0,20	11,57	-	51	110
Sep-05	*-2,30*	0,18	-2,48	-	52	106
Dec-05	*17,49*	0,18	17,31	-	52	133
Mar-06	*-18,67*	-0,23	-18,44	-	51	99
Jun-06	*11,24*	0,21	11,03	-	52	116
Sep-06	*-2,18*	0,19	-2,37	-	52	112
Dec-06	*16,75*	0,19	16,56	-	52	139

E.4:

Cobb 2013

Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	B
2004	4,00	***0,67***	***3,33***		
2005	3,92	***0,72***	***3,20***	0,00	0,00
2006	3,84	***0,77***	***3,06***	0,00	0,00

Levels implied by quarterly contributions A	B
200	400
204	420
208	440
214	460

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ q/q* (pp.)	Levels implied A	B
Mar-03					50	83
Jun-03	*12,16*	0,19	11,97	-	50	99
Sep-03	*-2,39*	0,17	-2,55	-	50	96
Dec-03	*18,03*	0,17	17,86	-	50	122
Mar-04	*-19,61*	0,15	-19,75	-	51	88
Jun-04	*12,28*	0,18	12,10	-	51	104
Sep-04	*-2,41*	0,16	-2,58	-	51	100
Dec-04	*18,21*	0,17	18,04	-	51	128
Mar-05	*-19,39*	0,18	-19,56	-	52	93
Jun-05	*11,77*	0,20	11,57	-	52	109
Sep-05	*-2,30*	0,18	-2,48	-	52	105
Dec-05	*17,49*	0,18	17,31	-	53	133
Mar-06	*-18,67*	0,19	-18,86	-	53	98
Jun-06	*11,24*	0,21	11,03	-	53	114
Sep-06	*-2,18*	0,19	-2,37	-	54	110
Dec-06	*16,75*	0,19	16,56	-	54	137

Chart A5.4: Levels of the quarterly components and aggregate series

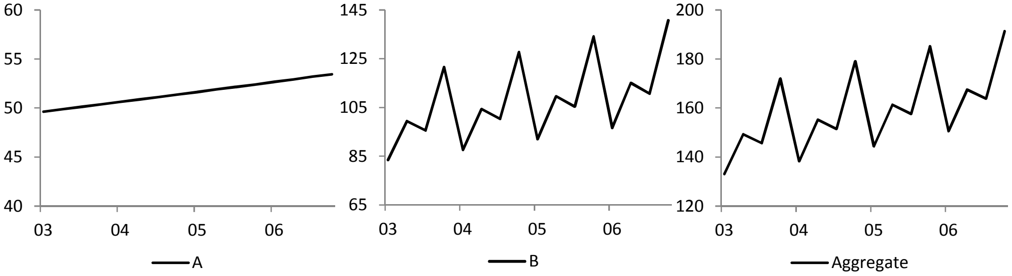

Table A5.5: Contributions to annual growth of a quarterly aggregate series with a volatile component and the implied contributions to growth of the annual series

Growth	A	B	Pa	Pb
	2%	5%	10%	-5%

Annual	Chain-linked series A	Chain-linked series B	Chain-linked series **Total**	Chain-linked series *y/y* %	Annual contribution (pp.) A	Annual contribution (pp.) B
2003	200	400	**600**			
2004	204	420	**624**	*4,00*	***0,67***	***3,33***
2005	208	441	**649**	*3,92*	***0,72***	***3,20***
2006	212	463	**673**	*3,84*	***0,77***	***3,06***

Quarterly	A	B	**Total**	*y/y* %	Annual contribution (pp.) A	Annual contribution (pp.) B
Mar-03	50	107	**157**			
Jun-03	50	94	**144**			
Sep-03	50	75	**126**			
Dec-03	50	124	**174**			
Mar-04	51	115	**166**	*5,70*	-	-
Jun-04	51	100	**151**	*5,05*	-	-
Sep-04	51	83	**134**	*6,75*	-	-
Dec-04	51	122	**173**	*-0,37*	-	-
Mar-05	52	112	**163**	*-1,32*	-	-
Jun-05	52	129	**180**	*19,22*	-	-
Sep-05	52	97	**150**	*11,68*	-	-
Dec-05	52	103	**155**	*-10,41*	-	-
Mar-06	53	93	**147**	*-10,02*	-	-
Jun-06	53	112	**165**	*-8,41*	-	-
Sep-06	53	117	**170**	*13,33*	-	-
Dec-06	53	141	**192**	*23,48*	-	-

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Fixed Weights

Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	*Diff. With* B
2003					
2004	4,00	***0,67***	***3,33***		
2005	4,02	***0,65***	***3,37***	0,07	-0,16
2006	4,04	***0,64***	***3,40***	0,13	-0,33

Levels implied by quarterly contributions A	B
200	400
204	420
208	441
212	463

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	Levels implied by quarterly contributions A	B
Mar-03					50	107
Jun-03					50	94
Sep-03					50	75
Dec-03					50	124
Mar-04	*5,70*	0,63	5,06	-	51	115
Jun-04	*5,05*	0,69	4,35	-	51	100
Sep-04	*6,75*	0,80	5,95	-	51	83
Dec-04	*-0,37*	0,58	-0,95	-	51	122
Mar-05	*-1,15*	0,61	-1,76	-0,17	52	112
Jun-05	*19,95*	0,67	19,28	-0,73	52	129
Sep-05	*11,30*	0,76	10,54	0,38	52	97
Dec-05	*-10,56*	0,59	-11,15	0,15	52	103
Mar-06	*-11,24*	0,63	-11,88	1,22	53	93
Jun-06	*-8,93*	0,58	-9,51	0,52	53	112
Sep-06	*14,05*	0,70	13,36	-0,73	53	117
Dec-06	*25,52*	0,67	24,85	-2,04	53	141

E.2:

Previous Year Prices

Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	*Diff. With* B
2003					
2004	4,00	***0,67***	***3,33***		
2005	3,92	***0,72***	***3,20***	0,00	0,00
2006	3,84	***0,77***	***3,06***	0,00	0,00

Levels implied by quarterly contributions A	B
200	400
204	420
208	440
214	460

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	Levels implied by quarterly contributions A	B
Mar-03					50	107
Jun-03					50	94
Sep-03					50	75
Dec-03					50	124
Mar-04	*5,70*	0,63	5,06	-	51	115
Jun-04	*5,05*	0,69	4,35	-	51	100
Sep-04	*6,75*	0,80	5,95	-	51	83
Dec-04	*-0,37*	0,58	-0,95	-	51	122
Mar-05	*-1,00*	0,67	-1,67	-0,32	52	112
Jun-05	*19,07*	0,74	18,33	0,15	52	128
Sep-05	*10,86*	0,84	10,03	0,82	52	96
Dec-05	*-9,95*	0,65	-10,61	-0,46	53	104
Mar-06	*-9,94*	0,76	-10,70	-0,08	53	95
Jun-06	*-7,87*	0,70	-8,57	-0,54	53	112
Sep-06	*12,88*	0,84	12,04	0,45	54	114
Dec-06	*23,20*	0,81	22,39	0,27	54	138

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INSEE 2007

Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	*Diff. With* B
2003					
2004	4,00	***0,67***	***3,33***		
2005	3,92	***0,72***	***3,20***	0,00	0,00
2006	3,84	***0,77***	***3,06***	0,00	0,00

Levels implied by quarterly contributions A	B
200	400
204	420
208	440
214	460

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	Levels implied by quarterly contributions A	B
Mar-03					50	107
Jun-03					50	94
Sep-03					50	75
Dec-03					50	124
Mar-04	*5,70*	0,63	5,06	-	51	115
Jun-04	*5,05*	0,69	4,35	-	51	100
Sep-04	*6,75*	0,80	5,95	-	51	83
Dec-04	*-0,37*	0,58	-0,95	-	51	122
Mar-05	*-1,32*	0,46	-1,78	-	51	112
Jun-05	*19,22*	0,84	18,38	-	52	128
Sep-05	*11,68*	1,39	10,29	-	53	97
Dec-05	*-10,41*	0,35	-10,76	-	52	103
Mar-06	*-10,02*	0,71	-10,73	-	53	94
Jun-06	*-8,41*	0,35	-8,76	-	53	112
Sep-06	*13,33*	1,13	12,19	-	55	115
Dec-06	*23,48*	0,99	22,49	-	54	138

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Cobb 2013

Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	*Diff. With* B
2003					
2004	4,00	***0,67***	***3,33***		
2005	3,92	***0,72***	***3,20***	0,00	0,00
2006	3,84	***0,77***	***3,06***	0,00	0,00

Levels implied by quarterly contributions A	B
200	400
204	420
208	440
214	460

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	Levels implied by quarterly contributions A	B
Mar-03					50	107
Jun-03					50	94
Sep-03					50	75
Dec-03					50	124
Mar-04	*5,70*	0,63	5,06	-	51	115
Jun-04	*5,05*	0,69	4,35	-	51	100
Sep-04	*6,75*	0,80	5,95	-	51	83
Dec-04	*-0,37*	0,58	-0,95	-	51	122
Mar-05	*-1,32*	0,65	-1,97	-	52	112
Jun-05	*19,22*	0,73	18,49	-	52	128
Sep-05	*11,68*	0,85	10,83	-	52	97
Dec-05	*-10,41*	0,67	-11,09	-	53	103
Mar-06	*-10,02*	0,74	-10,76	-	53	94
Jun-06	*-8,41*	0,69	-9,10	-	53	112
Sep-06	*13,33*	0,85	12,48	-	54	116
Dec-06	*23,48*	0,84	22,64	-	54	138

Chart A5.5: Levels of the quarterly components and aggregate series

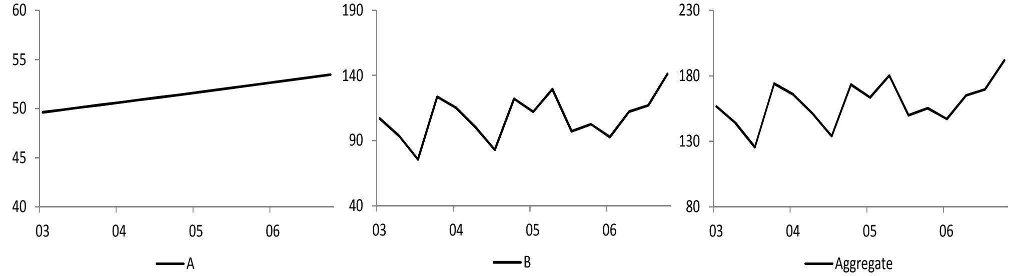

Table A5.6: Contributions to annual growth of a quarterly aggregate series with low overall growth and a seasonal component and the implied annual contributions

Growth	A	B	Pa	Pb
	0,5%	0,5%	-5%	20%

Annual	Chain-linked series A	B	**Total**	*y/y* %	Annual contribution (pp.) A	B
2003	200	400	**600**			
2004	201	402	**603**	*0,50*	***0,17***	***0,33***
2005	202	404	**606**	*0,50*	***0,14***	***0,36***
2006	203	406	**609**	*0,50*	***0,12***	***0,38***

Quarterly	A	B	**Total**	*y/y* %	Annual contribution A	Annual contribution B
Mar-03	50	85	**135**			
Jun-03	50	100	**150**			
Sep-03	50	95	**145**			
Dec-03	50	120	**170**			
Mar-04	50	85	**135**	*0,50*	-	-
Jun-04	50	100	**151**	*0,50*	-	-
Sep-04	50	96	**146**	*0,50*	-	-
Dec-04	50	121	**171**	*0,50*	-	-
Mar-05	50	86	**135**	*-0,33*	-	-
Jun-05	50	101	**151**	*0,50*	-	-
Sep-05	51	96	**146**	*0,24*	-	-
Dec-05	51	121	**173**	*1,38*	-	-
Mar-06	51	86	**135**	*-0,26*	-	-
Jun-06	51	101	**152**	*0,50*	-	-
Sep-06	51	96	**147**	*0,27*	-	-
Dec-06	51	122	**176**	*1,29*	-	-

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Fixed Weights

Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	B
2004	0,50	***0,17***	***0,33***		
2005	0,50	***0,17***	***0,33***	-0,02	0,02
2006	0,50	***0,17***	***0,33***	-0,05	0,05

Levels implied by quarterly contributions

A	B
200	400
201	402
202	404
203	406

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	Levels implied A	Levels implied B
Mar-03					50	85
Jun-03					50	100
Sep-03					50	95
Dec-03					50	120
Mar-04	*0,50*	0,19	0,31	-	50	85
Jun-04	*0,50*	0,17	0,33	-	50	100
Sep-04	*0,50*	0,17	0,33	-	50	96
Dec-04	*0,50*	0,15	0,35	-	50	121
Mar-05	*0,50*	0,19	0,31	-0,83	50	86
Jun-05	*0,50*	0,17	0,33	0,00	50	101
Sep-05	*0,50*	0,17	0,33	-0,26	51	96
Dec-05	*0,50*	0,15	0,35	0,88	51	121
Mar-06	*0,50*	0,19	0,32	-0,76	51	86
Jun-06	*0,50*	0,17	0,33	0,00	51	101
Sep-06	*0,50*	0,17	0,33	-0,24	51	96
Dec-06	*0,50*	0,15	0,35	0,79	51	122

E.2:

Previous Year Prices

Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	B
2004	0,50	***0,17***	***0,33***		
2005	0,50	***0,14***	***0,36***	0,00	0,00
2006	0,50	***0,12***	***0,38***	0,00	0,00

Levels implied by quarterly contributions

A	B
200	400
201	402
202	404
203	406

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	Levels implied A	Levels implied B
Mar-03					50	85
Jun-03					50	100
Sep-03					50	95
Dec-03					50	120
Mar-04	*0,50*	0,19	0,31	-	50	85
Jun-04	*0,50*	0,17	0,33	-	50	100
Sep-04	*0,50*	0,17	0,33	-	50	96
Dec-04	*0,50*	0,15	0,35	-	50	121
Mar-05	*0,50*	0,16	0,34	-0,83	50	86
Jun-05	*0,50*	0,14	0,36	0,00	50	101
Sep-05	*0,50*	0,15	0,35	-0,26	50	96
Dec-05	*0,50*	0,13	0,38	0,88	51	121
Mar-06	*0,50*	0,13	0,36	-0,76	51	86
Jun-06	*0,50*	0,12	0,38	0,00	51	102
Sep-06	*0,50*	0,12	0,38	-0,23	51	97
Dec-06	*0,50*	0,10	0,40	0,78	51	122

E.3:

INSEE 2007

Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	B
2004	0,50	***0,17***	***0,33***		
2005	0,50	***0,14***	***0,36***	0,00	0,00
2006	0,50	***0,12***	***0,38***	0,00	0,00

Levels implied by quarterly contributions

A	B
200	400
201	402
202	404
203	406

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	Levels implied A	Levels implied B
Mar-03					50	85
Jun-03					50	100
Sep-03					50	95
Dec-03					50	120
Mar-04	*0,50*	0,19	0,31	-	50	85
Jun-04	*0,50*	0,17	0,33	-	50	100
Sep-04	*0,50*	0,17	0,33	-	50	96
Dec-04	*0,50*	0,15	0,35	-	50	121
Mar-05	*-0,33*	-0,40	0,06	-	50	85
Jun-05	*0,50*	0,14	0,36	-	50	101
Sep-05	*0,24*	-0,02	0,27	-	50	96
Dec-05	*1,38*	0,71	0,67	-	52	122
Mar-06	*-0,26*	-0,41	0,15	-	49	86
Jun-06	*0,50*	0,12	0,38	-	51	102
Sep-06	*0,27*	-0,04	0,31	-	50	96
Dec-06	*1,29*	0,67	0,62	-	53	123

E.4:

Cobb 2013

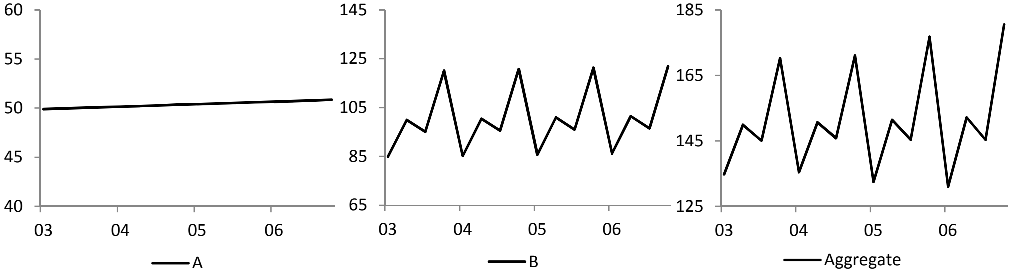
Annual	Annual contribution implied by quarterly contributions (pp.) Total (%)	A	B	*Diff. With* A	B
2004	0,50	***0,17***	***0,33***		
2005	0,50	***0,14***	***0,36***	0,00	0,00
2006	0,50	***0,12***	***0,38***	0,00	0,00

Levels implied by quarterly contributions

A	B
200	400
201	402
202	404
203	406

Quarterly	Quarterly contribution to aggregate growth Total (%)	A (pp.)	B (pp.)	*Diff. w/ y/y* (pp.)	Levels implied A	Levels implied B
Mar-03					50	85
Jun-03					50	100
Sep-03					50	95
Dec-03					50	120
Mar-04	*0,50*	0,19	0,31	-	50	85
Jun-04	*0,50*	0,17	0,33	-	50	100
Sep-04	*0,50*	0,17	0,33	-	50	96
Dec-04	*0,50*	0,15	0,35	-	50	121
Mar-05	*-0,33*	0,17	-0,50	-	50	85
Jun-05	*0,50*	0,14	0,36	-	50	101
Sep-05	*0,24*	0,14	0,10	-	50	96
Dec-05	*1,38*	0,12	1,26	-	51	123
Mar-06	*-0,26*	0,14	-0,40	-	51	84
Jun-06	*0,50*	0,12	0,38	-	51	102
Sep-06	*0,27*	0,12	0,14	-	51	96
Dec-06	*1,29*	0,10	1,19	-	51	125

Chart A5.6: Levels of the quarterly components and aggregate series



Annex 6: Discrepancies between different suggestions for quarterly contributions with Chilean GDP data

For the calculation of contributions to quarterly growth of the aggregate (chain-linked using the annual overlap method) all formulas but that of the fixed weight method (E.1) coincide for all quarters but the first. Given that E.1, has been proven to be unreliable to appreciate the discrepancies between the other three it is only necessary to check the outcomes for the first quarters. Table A6.1 presents the contributions to quarterly growth of the different measures for the first quarters.

It is worth noting, that the tables present the contribution of Changes in Inventories, that is a series that may not be chain-linked. This shortcoming is well known to practitioners and a reasonable amount of literature has been devoted to explore how to present the series in an economically meaningful way. However, its contributions may be calculated residually if the formula for contributions fulfils the additivity requirements. This is the case for E.3 and E.4. For E.1 and E.2 we present a sort of implicit contribution. To do this, we bundle Changes in Inventories with Imports, calculate the joint contribution and subtract the contribution of Imports.

Table A6.1: Contributions to quarterly growth in the first quarters of every year in the sample

	2005-I				2006-I				2007-I				2008-I			
	Aggr.Growth: -4,9 %				Aggr.Growth: -4,2 %				Aggr.Growth: -4,6 %				Aggr.Growth: -4,2 %			
	E.1	E.2	E.3	E.4	E.1	E.2	E.3	E.4	E.1	E.2	E.3	E.4	E.1	E.2	E.3	E.4
Construction and Infrastructure	-0,2	-0,2	-0,3	-0,3	-0,3	-0,3	-0,2	-0,3	0,5	0,4	0,4	0,4	1,0	0,9	0,9	0,9
Machinery and Equipment	0,2	0,2	0,1	0,0	-2,3	-1,7	-1,9	-1,9	-1,1	-0,7	-0,8	-0,8	-1,4	-0,8	-0,9	-0,9
Durable Goods	-0,9	-0,8	-0,8	-0,9	-0,9	-0,8	-0,8	-0,8	-1,2	-0,8	-0,9	-0,9	-1,4	-0,9	-1,0	-1,0
Non-durable Goods	-2,3	-2,1	-2,2	-2,3	-2,2	-1,9	-2,0	-2,0	-2,2	-1,8	-1,9	-2,0	-2,2	-1,8	-1,8	-1,8
Services	-1,8	-1,7	-1,7	-1,8	-2,0	-1,9	-1,9	-1,9	-1,7	-1,5	-1,4	-1,6	-1,7	-1,4	-1,4	-1,4
Government Expenditure	-3,6	-3,4	-3,5	-3,5	-3,6	-3,3	-3,3	-3,4	-3,5	-3,0	-3,1	-3,1	-4,0	-3,5	-3,5	-3,5
Exports	-1,0	-1,0	-1,1	-1,0	-0,4	-0,5	-0,5	-0,4	0,9	1,1	1,0	1,2	-0,5	-0,6	-0,6	-0,5
Imports	1,0	0,9	1,0	1,3	1,5	1,3	1,3	1,4	2,1	1,6	1,6	1,8	1,9	1,4	1,4	1,4
Changes in Inventories*	6,0	5,2	5,6	6,2	9,1	7,5	7,7	7,8	4,9	3,6	3,7	4,1	7,4	5,4	5,5	5,6
Sum of contributions	**-4,6**	**-4,8**	**-4,9**	**-4,9**	**-4,1**	**-4,1**	**-4,2**	**-4,2**	**-5,5**	**-4,3**	**-4,6**	**-4,6**	**-4,6**	**-4,2**	**-4,2**	**-4,2**

	2009-I				2010-I				2011-I				2012-I			
	Aggr.Growth: -6,9 %				Aggr.Growth: -7,0 %				Aggr.Growth: -4,4 %				Aggr.Growth: -4,5 %			
	E.1	E.2	E.3	E.4	E.1	E.2	E.3	E.4	E.1	E.2	E.3	E.4	E.1	E.2	E.3	E.4
Construction and Infrastructure	-0,7	-0,7	-0,8	-0,8	-0,7	-0,7	-0,7	-0,7	0,5	0,5	0,5	0,4	0,4	0,4	0,4	0,4
Machinery and Equipment	-4,8	-2,9	-2,9	-2,9	-1,1	-0,6	-0,6	-0,6	-1,4	-0,7	-0,9	-0,9	-4,9	-2,4	-2,5	-2,5
Durable Goods	-2,4	-1,6	-1,6	-1,6	-1,1	-0,7	-0,7	-0,7	-1,7	-1,0	-1,1	-1,1	-1,8	-1,0	-1,1	-1,1
Non-durable Goods	-2,9	-2,6	-2,5	-2,4	-2,1	-1,9	-1,9	-2,0	-1,9	-1,6	-1,7	-1,8	-2,2	-1,9	-1,8	-1,8
Services	-2,5	-2,3	-2,3	-2,2	-2,3	-2,0	-2,0	-2,1	-2,3	-2,0	-2,0	-2,1	-1,9	-1,6	-1,6	-1,6
Government Expenditure	-3,4	-3,3	-3,2	-3,1	-4,2	-4,2	-4,2	-4,2	-4,5	-4,4	-4,4	-4,4	-4,4	-4,4	-4,4	-4,3
Exports	-1,5	-1,8	-1,7	-1,8	-2,3	-2,5	-2,5	-2,7	0,1	0,1	0,0	0,1	-1,0	-1,1	-1,1	-1,1
Imports	8,6	7,1	7,4	7,3	1,6	1,2	1,4	1,6	0,6	0,4	0,4	0,7	3,1	2,1	2,1	2,1
Changes in Inventories*	18,5	15,4	15,4	15,2	9,0	6,6	7,1	7,6	8,0	5,2	5,6	6,0	14,6	9,7	9,7	9,6
Sum of contributions	**-8,2**	**-6,9**	**-6,9**	**-6,9**	**-6,4**	**-7,3**	**-7,0**	**-7,0**	**-3,9**	**-4,3**	**-4,4**	**-4,4**	**-4,2**	**-4,4**	**-4,5**	**-4,5**

* For E.1 and E.2 the contribution of Changes in Inventories is calculated as the joint contribution of Imports plus Changes in Inventories minus the contribution of Imports.