# Identifying the Sources of Seasonal Effects in an indirectly adjusted Chain-Linked Aggregate: A Framework for the Annual Overlap Method

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# Identifying the Sources of Seasonal Effects in an indirectly adjusted Chain-Linked Aggregate: A Framework for the Annual Overlap Method

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**Abstract**

The use of chain-linked methods reduces significantly the problem of price structure obsolescence present in fixed base environments. However, price updating introduces a new dimension that may produce confusion if not accounted for. Probably the most notorious difficulty generated by the introduction of chain-linked indices to the measurement of GDP has been that the aggregate is not the direct sum of its components, thus not only making it harder to explain its behaviour but also making it more cumbersome to work with the series in a consistent manner. Because of the non-additivity of the components, one of the processes that have been affected is that of the indirect seasonal adjustment. This document presents a consistent framework to identify and track down the sources of seasonal effects to its components in an aggregate measure chain-linked using the annual overlap method. This is done based on the decomposition of component's contributions and the indirect seasonal adjustment. The framework allows separating the effects on growth rates into non-systematic seasonal effects, systematic seasonality and changes in systematic seasonality.

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## 1. Introduction

Macroeconomic analysis devotes a fair amount of effort to the economy's real variables, thus generating a need for aggregate measurements of volumes and quantities. When evaluating the economy's performance, Gross Domestic Product (GDP) is the most often used indicator. Traditionally, GDP and other real variables have been measured using the fixed-base-year method; however, in the last decades its shortcomings have become obvious and difficult to ignore (Steindel, 1995).

In line with the recommendations of the System of National Accounts (SNA, 1993) many countries have moved to an annual update of the relative price structure for their real output series through the generation of chain-linked series. This procedure greatly reduces the problem of price structure obsolescence but introduces a new dimension into the analysis that makes understanding the evolution of the series less straightforward. Probably the most notorious difficulty has been that an aggregate is not the direct sum of its components, known as non-additivity, and therefore the fixed base accounting identities do not hold for the chain-linked levels. This induces some practical issues, makes working with the series less straightforward and may blur the analysis. It is unadvisable, however, to ignore the fact and continue working as if the accounting properties of the fixed base methodology are still valid because it could lead to significant error if the environment has suffered relevant changes in its price structure (OECD, 2006a).

In this context all processes that rely on the fixed-base accounting identities, like the case of the national accounting identities of GDP, may be affected to a varying degree. Cobb (2014a) illustrates this in a forecasting context showing that the discrepancy between an aggregate forecast generated by proper chain-linking and the one resulting of using the fixed-base identities directly may be significant and that these differences due to inaccurate aggregation are not necessarily small compared to the forecasting error. Another process that could be affected is indirect seasonal adjustment.

The process of seasonal adjustment is performed to aid economic analysis in uncovering the underlying movements of a series by removing systematic intra-annual movements and other effects that contribute to hide them. There are a number of approaches and methods used to perform this task. One of them, the indirect seasonal adjustment, involves generating the aggregate from the seasonal adjustment of its components. By doing so, this method generates an aggregate that is consistent with the adjusted components and, therefore, the aggregate performance may be broken down precisely to be accounted for by the components. This means that specific developments may be tracked down to their source and given a meaningful explanation. The loss of additivity due to the adoption of the chain-linked method, however, makes this process more cumbersome.[1]

To alleviate the problem of explaining a chain-linked aggregate's performance in terms of its components, publishing bodies generally accompany the aggregate measure with the contributions of each component to its growth, where these contributions do sum up to the total. These contributions have become the standard way of breaking down the performance of an aggregate and in this document we rely on them to break down the seasonal effects of an aggregate measure so they can be explained based on the properties of the individual components. Section 2 briefly explains the process of seasonal adjustment and the identification the seasonal effects for a series that is adjusted directly. Section 3 presents a framework for obtaining consistent indirectly adjusted aggregates in the context of the annual overlap methodology. Section 4 presents the derivation necessary to identify the sources of seasonal effects in a annual overlap chain-linked aggregate. Section 5 presents an example for the identification of seasonal effects using Chilean GDP data and Section 6 summarizes the main findings.

## 2. Direct seasonal adjustment and identifying the seasonal effects involved

Time series are at the heart of economic analysis and policy decision. However, often relevant series exhibit systematic patterns and other effects that mask their underlying movement. When these movements are the focus of interest, it is common to try to identify and remove these patterns and effects in order to allow for a more straightforward analysis.

[1] It is worth noting that other approaches to seasonal adjustment methods may be preferred when disaggregate consistency is not required.

## 2.1. Seasonal adjustment of a time-series

The seasonal adjustment process relies on the assumption that a time-series may be expressed as a combination of three distinctive unobservable components, where one of them reflects all the seasonal movements. The other two components are the trend and the irregular component, where the first reflects the underlying movement of the series and its general direction while the second contains all the erratic behaviour, both the regular unexplained non-systematic movement and specific, normally significant, non-periodic shocks that may result from natural disasters or civil events.

The seasonal component reflects the movement that is attributable to the features of a given period and may be separated into systematic and non-systematic. The systematic portion contains the intra-annual fluctuations that occur regularly every year. They are generally associated with weather conditions or certain aspects of the specific season. An example is an increase in the consumption of heating oil in winter. The non-systematic portion refers to the effects of differing daily composition of the periods being compared, like number of holydays, working days, etc. This component also contains events that occur systematically but with a frequency different form a year, like leap years that occur every four. An example of the effects or daily composition could be an increase of attendance to theatres and cinemas in months with more weekends and holidays.

Although the components could interact in a number of ways, the standard methods assume that $y_t$, either in its original form or as a transformation,[2] may be written as:

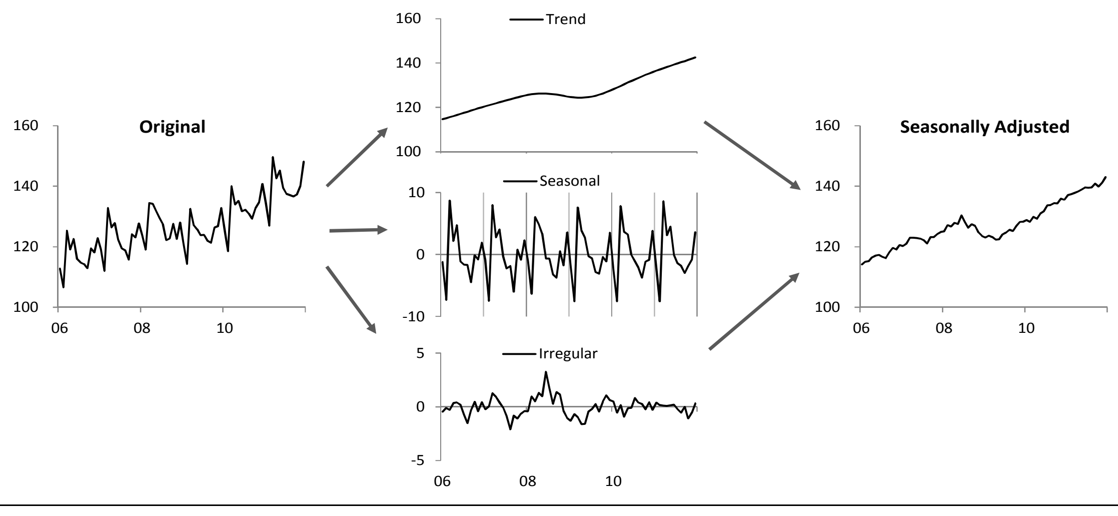
$$y_t = T_t + S_t + K_t + I_t \tag{2.1}$$

where $T_t$ is the trend, $S_t$ is the systematic seasonality, $K_t$ is the non-systematic seasonality and $I_t$ is the irregular component. The seasonally adjusted series is then:

$$y_t^{SA} \quad = \quad y_t - (S_t + K_t) \quad = \quad T_t + I_t \tag{2.2}$$

The outcome of the seasonal adjustment of an example series is presented in Chart 2.1. As it can be appreciated, the trend retains the unadjusted overall growth and level and the seasonal and irregular components are expressed as deviations from it. The seasonally adjusted series is generated as the sum of the trend and the irregular.

**Chart 2.1: Example of the identification of unobserved components**




Regarding the actual method used to identify the unobserved components, there are a number of approaches; however, two relatively similar time series methods stand out as the most used.[3] One of them is the non-parametric X12-ARIMA method (Findley, et. al., 1998) that relies on moving averages of different lengths to identify the components and the other is the parametric TRAMO-SEATS (Gómez and Maravall, 1997) that tries

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[2] Economic series in levels are typically modelled in logarithms.

[3] Foldesi et al. (2007) provide a complete survey of the different methods. OECD (2009) presents a survey of the methods that are used in each of the OECD countries.

to identify a unique model for each component. Although, both methods are based on different theoretical assumptions, in practice they are very similar. In fact, in the more recent implementations both methods have relied on the same procedure to identify models and perform prior adjustments and only differ in the actual unobserved component identification. In the most recent release of the X12-ARIMA family, that is the X13-ARIMA-SEATS (U.S. Census Bureau, 2013); both methods are contained in the same program and may be chosen depending on the analyst's preference. For the purpose of this document both methods provide the necessary information.[4]

### 2.2. Identifying seasonal effects in a time-series

In the process of analysing time-series one would typically wish to be able to identify the different effects that are at play. When examining a series, one might wonder how much of the level in a certain period is solely due to a seasonal phenomenon and what is responsible for it. When analysing the economy's performance, on the other hand, one might be more interested in knowing how much of the registered growth is only due to seasonal factors or how much is due to the specific calendar composition of the examined period. At least in regards to these factors, the seasonal adjustment process allows to separate a number of effects.

Recalling expression (2.1), we have that a series may be written as the sum of its trend, its systematic seasonality, its non-systematic seasonality and its irregular component, and that the seasonally adjusted series is simply the sum of the trend and the irregular component. By comparing the behaviour of both series, that is $y_t$ and $y_t^{SA}$, it is possible to determine how important the seasonal effects are. Regarding the level of a series at any point in time $t$, it is only necessary to look at the ratio between the both to obtain the percentage of the original that is estimated to be seasonal and by comparing the growth rates of both series over an interval it is possible to obtain the percentage points of growth that are attributable to seasonal factors, that is:

$$L_t^{SA} = \frac{y_t}{y_t^{SA}} - 1 \tag{2.3}$$

and

$$C_t^{SA} = \frac{y_t}{y_{t-s}} - \frac{y_t^{SA}}{y_{t-s}^{SA}} \tag{2.4}$$

where $L_t^{SA}$ is the estimated percentage seasonal deviation in $t$ and $C_t^{SA}$ is the estimated growth from $t$-$s$ to $t$ that is attributable solely to seasonal factors. The effects may be broken down further by building intermediate-steps in the adjustment process to infer the effect of each one of them. For the non-systematic seasonality we may build a series that only removes this effect. Let us define the original series excluding non-systematic seasonality as:

$$y_t^{exNSS} = y_t - K_t \tag{2.5}$$

Then it is easy to see how from (2.3) the estimated percentage seasonal deviation in $t$ may be separated into non-systematic and systematic by using (2.5):

$$L_t^{SA} = \frac{y_t}{y_t^{SA}} - 1 = \underbrace{\left(\frac{y_t - y_t^{exNSS}}{y_t^{SA}}\right)}_{L_t^{NSS}} + \underbrace{\left(\frac{y_t^{exNSS} - y_t^{SA}}{y_t^{SA}}\right)}_{L_t^{SS}} \tag{2.6}$$

where $L_t^{NSS}$ and $L_t^{SS}$ are the percentage points of $L_t^{SA}$ that are attributable to non-systematic and systematic seasonal effects.

An analogous separation may be performed for growth rates, where the difference between both of them is the percentage points of growth that are attributable to each seasonal effect, that is:

$$C_t^{SA} = \underbrace{\left(\frac{y_t}{y_{t-s}} - \frac{y_t^{exNSS}}{y_{t-s}^{exNSS}}\right)}_{C_t^{NSS}} + \underbrace{\left(\frac{y_t^{exNSS}}{y_{t-s}^{exNSS}} - \frac{y_t^{SA}}{y_{t-s}^{SA}}\right)}_{C_t^{SS}} \tag{2.7}$$

[4] For a detailed and comprehensive explanation of the seasonal adjustment process refer to ONS UK (2007).

where $C_t^{NSS}$ and $C_t^{SS}$ are the percentage points of $C_t^{SA}$ that are attributable to non-systematic and systematic seasonal effects. However, because seasonal patterns may change over time due to a number of reasons, like technological advancements and changes in the composition of the series, solely for the purpose of comparing growth rates it may be informative to isolate the changes in seasonal patterns. The desirability of this separation becomes apparent when looking at annual change, because one would expect yearly growth rates to be unaffected by the systematic seasonal effects. If the systematic seasonality evolves, however, it is not entirely neutral.

Accounting for the change in seasonal patterns is not that straightforward due to the fact that all of the components of expression (2.1) are unobservable and need to be estimated from the data. This means that they may change with the addition of new information. For the purpose of this document we will confine the change in the seasonal pattern to the difference between the patterns that are estimated concurrently. That is, the difference between the deviation estimated for *t-s* in *t-s* and the one estimated for *t-s* using the data available up to and including *t*. This approach focuses on singling out the update of the seasonal deviation.

This subdivision is not straightforward due to the fact that with every run of the seasonal adjustment program the systematic seasonal component is updated for the whole series. To perform this division then, it is necessary to have access to the systematic components of the adjustment performed with data up to *t-s*. Having that, we define:

$$y_t^{OSS} \quad = \quad y_t^{exNSS} - S_t^{SA=t} \tag{2.8}$$

That is, $y_{t-s}^{OSS}$ is the value of $y$ in period *t-s* that excludes non-systematic seasonality as estimated in the most recent seasonal adjustment, say in *t*, and is adjusted according to the systematic seasonality that was estimated with the series ending in *t-s*. Then the systematic portion of expression (2.7) may be written as:

$$C_t^{SS} \quad = \quad \left( \frac{y_t^{exNSS}}{y_{t-s}^{exNSS}} - \frac{y_t^{SA}}{y_{t-s}^{OSS}} \right) \quad + \quad \left( \frac{y_t^{SA}}{y_{t-s}^{OSS}} - \frac{y_t^{SA}}{y_{t-s}^{SA}} \right) \tag{2.9}$$

The need for a number of different estimations of the systematic components may prove to be rather cumbersome and, therefore, an alternative way of approaching the problem is by focusing on the update in the seasonal component in *t* due to the informational content of the information in *t*, instead of focusing on the update in *t-s*. This means comparing the systematic seasonality that was expected for *t* in *t-s* with the one effectively estimated in *t*.

This, in itself, would not reduce the effort implied because it would require keeping a log of forecasted systematic seasonality instead of the previously estimated systematic seasonality. However, as the series used to estimate the systematic seasonality is already free from non-systematic seasonality, that is $y^{exNSS}$, and that a forecast in *t-s* would only contain information up to that period, the systematic seasonality for the same period of the previous year estimated in *t* should be almost the same as the forecast for *t* made in *t-s*.[5]

Under this simplifying assumption we define $y_t$ corrected by the systematic seasonality of the previous year as:

$$y_t^{SSPY} \quad = \quad y_t^{exNSS} - S_{t-4} \tag{2.10}$$

and write the systematic portion of expression (2.7) as:

$$C_t^{SS} \quad = \quad \underbrace{\left( \frac{y_t^{exNSS}}{y_{t-s}^{exNSS}} - \frac{y_t^{SSPY}}{y_{t-s}^{SA}} \right)}_{C_t^{SSPY}} \quad + \quad \underbrace{\left( \frac{y_t^{SSPY}}{y_{t-s}^{SA}} - \frac{y_t^{SA}}{y_{t-s}^{SA}} \right)}_{C_t^{\Delta SS}} \tag{2.11}$$

From expression (2.11) it becomes clear that, at least for yearly growth, all the effects of updating the seasonality are picked up in $C_t^{\Delta SS}$. That is because in this case the exact same systematic seasonality is removed from $y_t^{SSPY}$ and $y_{t-4}^{SA}$.

[5] It is worth noting that all the unobserved components, except for the trend, are estimate as a deviation from the trend and centred on the mean. Then, if the last available estimated systematic deviation of any given period is $x$, in absence of any new relevant information, $x$ is probably the best forecast for the unknown given period in the following year.

Using expression (2.11) instead of (2.9) simplifies things greatly in the sense of only having to work with the outcome of one seasonal adjustment; therefore, we choose to express the contributions of the seasonal effects as:

$$C_t^{SA} = \underbrace{\left(\frac{y_t}{y_{t-s}} - \frac{y_t^{exNSS}}{y_{t-s}^{exNSS}}\right)}_{C_t^{NSS}} + \underbrace{\left(\frac{y_t^{exNSS}}{y_{t-s}^{exNSS}} - \frac{y_t^{SSPY}}{y_{t-s}^{SA}}\right)}_{C_t^{SSPY}} + \underbrace{\left(\frac{y_t^{SSPY}}{y_{t-s}^{SA}} - \frac{y_t^{SA}}{y_{t-s}^{SA}}\right)}_{C_t^{\Delta SS}} \qquad (2.12)$$

At first, it might seem that expression (2.6) and (2.12) are just different ways of presenting the unobserved components that are directly identified in the seasonal adjustment process. That is true for a series that is adjusted directly, but these expressions are valid for any series. For the aggregate series that are adjusted indirectly, however, it is necessary to construct the distinct unobservable components.

## 3. Indirect seasonal adjustment of an annual overlap chain-linked aggregate

The process of seasonal adjustment is performed to aid economic analysis in uncovering the underlying movements of a series by removing systematic intra-annual movements and other effects that contribute to hide them. The approaches and methods used will depend on which is the most appropriate for the final objective. When it comes to explaining economic performance, the indirect seasonal adjustment, which involves generating the adjusted aggregate from the seasonal adjustment of its components, generates an aggregate that is consistent with its components. This means the aggregate performance may be broken down to be accounted for by the components and, therefore, that developments may be given a meaningful explanation. The loss of additivity due to the adoption of the chain-linked method, however, makes this process more cumbersome. By following a procedure analogous to that of the non-adjusted series, it is possible to generate a chain-linked seasonally adjusted aggregate.[6]

### 3.1. The annual overlap method for quarterly series

There are various methodologies to implement the annual update of prices being the annual overlap method of Laspeyres indices one of the more popular ones. This technique involves creating a set of fixed-base overlapping links with a length of two years, where in each link both the quantities of the relevant year (*y*) and the previous one (*y-1*) are valued using average prices of the previous year (*y-1*). Then, using the growth rates of these links an annual time series is built starting from the first link. The quarterly series are then built based on the annual structures in a consistent way.[7] That means that the quarterly aggregate may be expressed in the following way:

$$Q_t^y = \frac{1}{P_Q^{y-1}} \cdot \sum_{j=1}^{J} \left( p_j^{y-1} \cdot q_{j,t}^y \right) \qquad (3.1)$$

where,

$Q_t^y$ : chain-linked aggregate in quarter $t$ that belongs to year $y$

$P_Q^{y-1}$ : implicit price deflator of the aggregate in year $y-1$

$q_{j,t}^y$ : component $j$ in quarter $t$ that belongs to year $y$

$p_j^{y-1}$ : implicit price deflator of component $j$ in year $y-1$

Expression (3.1) shows that to obtain the chain-linked aggregate in period *t*, it is necessary to know the value of the *J* components in quarter *t* and both the component's and aggregate price deflators for the year before the one quarter *t* belongs to. An annual deflator is calculated as the annual nominal value divided by the annual real or chain-linked value.[8] However, the annual value of the components and the chain-linked aggregate are equal to the sum of the respective quarters and, also, the value of the nominal aggregate is equal to the sum of the nominal components. This means that to build a chain-linked aggregate it is only necessary to have the quarterly real series and the annual nominal series of the *J* components.

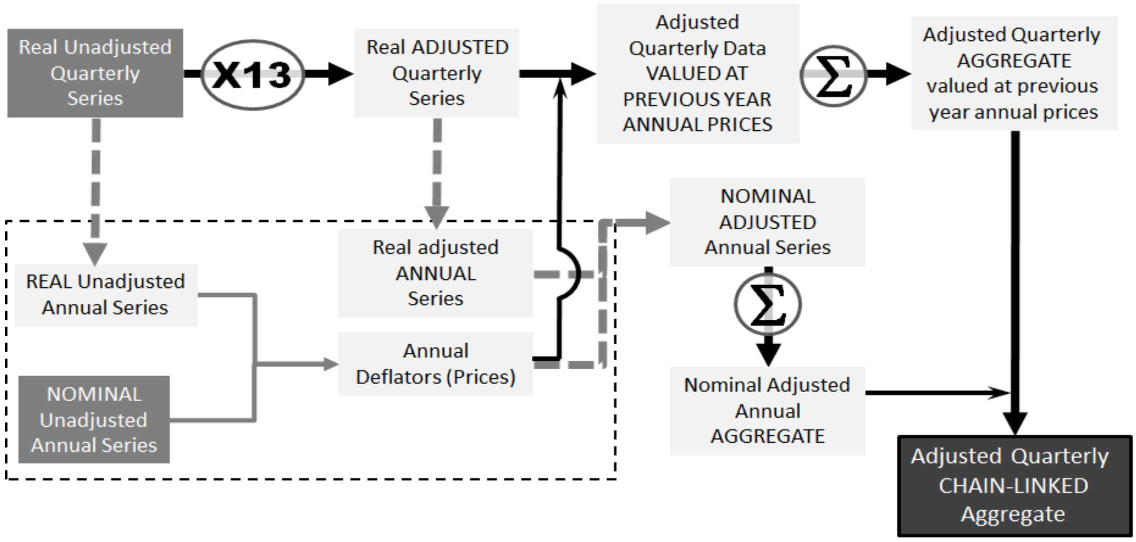
[6] It is worth noting that direct seasonal adjustment methods may be preferred when disaggregate consistency is not required.
[7] Annex 1 provides an example.
[8] The procedure is the same irrespective of whether the components are measured in physical quantities, as fixed base indices or as chain-linked sub-indices. These will be referred to simply as components, as opposed to variables valued at current prices, referred to as nominal, and those valued at previous year annual prices.

### 3.2. An indirect method for seasonally adjusting an annual overlap chain-linked aggregate

There are two alternatives to seasonally adjust an aggregate; to obtain it by adjusting it directly or to generate it from its seasonally adjusted components, the indirect approach. The discussion on which method is better is ongoing and depends crucially on the final objective.[9] Anyhow, the indirect method has a very desirable property in that it guarantees consistency between the aggregate and the components and allows, therefore, explaining its performance in terms of the latter.

The process of generating the seasonally adjusted aggregate is very similar to the chain-linking of the unadjusted one. However, it is necessary to provide seasonally adjusted nominal annual figures that are consistent with the adjusted component series. These adjusted annual figures may differ slightly from the unadjusted. This may initially seem counterintuitive due to the fact that one would expect the systematic portion of the seasonal component to cancel out within a year, however, the non-systematic portion does not necessarily. Fortunately, the process of adjusting the components provides the necessary information to be able to construct these seasonally adjusted nominal annual series. It is only necessary to make the reasonable assumption that the annual implicit price deflators are not affected by the non-systematic seasonal component. Based on this assumption and that the seasonal adjustment program provides the systematic and non-systematic seasonal component separately, it is fairly straightforward to aggregate the seasonally adjusted components. Chart 3.1 shows the process.

**Chart 3.1: Example of the identification of unobserved components**




Source: Cobb and Jara (2013)

The process is as follows:

1- The unadjusted quarterly components are seasonally adjusted.
2- The component's unadjusted annual price deflators are obtained by dividing the unadjusted annual nominal components by the annual unadjusted components.
3- The adjusted annual nominal components are obtained by multiplying the annual adjusted components by the respective unadjusted annual price deflators.
4- The adjusted annual nominal aggregate is obtained by adding up the adjusted annual nominal components.
5- The adjusted quarterly components are valued at previous year prices using the component's unadjusted annual price deflators.
6- The adjusted quarterly aggregate valued at previous year prices is generated by adding the adjusted quarterly components valued at previous year prices.
7- The adjusted quarterly chain-linked aggregate is obtained by dividing the adjusted quarterly aggregate valued at previous year prices by the previous year adjusted annual aggregate price deflator. This deflator is obtained from dividing the adjusted annual nominal aggregate by the adjusted annual chain-linked aggregate.

[9] Astolfi (2001) provides a brief exposition on the subject.

This procedure allows to generate the aggregate seasonally adjusted series but it may also be used to build the intermediate aggregate series that are needed to identify the seasonal effects, which are the original series excluding non-systematic seasonality and the series adjusted using the previous year seasonal pattern.

## 4. Explaining aggregate unobserved components in terms of the dissagregate series

When it comes to analysing an aggregate time series, the same questions regarding seasonal influences on the different measures may arise. However, the composite nature of the aggregate means that the aggregate seasonal effects are not the result of the economy's response to changing circumstances but are a combination of effects sourced in the underlying component's movements and, therefore, an explanation for aggregate performance should ideally be rooted in the specifics of the components. This makes being able to track down the sources of the aggregate effects desirable.

As with any chain-linked aggregate, however, the seasonally adjusted aggregate will not be equal to the sum of the seasonally adjusted components making the breakdown less straightforward. This difficulty is fairly easy to tackle when analysing the levels and for growth rates, as with the unadjusted series, the contributions may be used to explain the aggregate performance in terms of the components.

### 4.1. Identifying the sources of seasonal effects in aggregate levels

The analysis of the aggregate level is not very different from that of a component's only that, to be able to break it down to the component level, the annual updating of relative prices must be accounted for. Starting from expression (3.1) we may express the aggregate as the weighted sum of the components:

$$y_t = \sum_{j=1}^{J} \left( w_{j,t} \cdot q_{j,t} \right) \quad \text{with } w_{j,t} = \frac{p_{j,t}^{y-1}}{P_{Q,t}^{y-1}} \tag{4.1}$$

where,

$q_{j,t}$ : Component $j$ in quarter $t$
$w_{j,t}$ : Chain-linking weight of component $j$ in quarter $t$
$p_{j,t}^{y-1}$ : Component's $j$ annual price deflator of the year before the year to which quarter $t$ belongs.
$P_{Q,t}^{y-1}$ : Annual aggregate price deflator of the year before the year to which quarter $t$ belongs.

Using (4.1), expression (2.6) may be rewritten in terms of the components:

$$L_t^{SA} = \underbrace{\left( \frac{1}{y_t^{SA}} \cdot \sum_{j=1}^{J} \left( w_{j,t} \cdot q_{j,t} - w_{j,t}^{exNSS} \cdot q_{j,t}^{exNSS} \right) \right)}_{L_t^{NSS}} + \underbrace{\left( \frac{1}{y_t^{SA}} \cdot \sum_{j=1}^{J} \left( w_{j,t}^{exNSS} \cdot q_{j,t}^{exNSS} - w_{j,t}^{SA} \cdot q_{j,t}^{SA} \right) \right)}_{L_t^{SS}} \tag{4.2}$$

Then, the percentage points of the aggregate seasonal deviation in $t$ that are attributable to non-systematic and systematic seasonality of component $j$ may be calculated as:

$$L_{j,t}^{NSS} = \frac{1}{y_t^{SA}} \cdot \left( w_{j,t} \cdot q_{j,t} - w_{j,t}^{exNSS} \cdot q_{j,t}^{exNSS} \right) \quad \text{and} \quad L_{j,t}^{SS} = \frac{1}{y_t^{SA}} \cdot \left( w_{j,t}^{exNSS} \cdot q_{j,t}^{exNSS} - w_{j,t}^{SA} \cdot q_{j,t}^{SA} \right) \tag{4.3}$$

As it can be seen, both expressions simply reflect the difference between the component's levels in each measure weighted by the corresponding relative price. In practice, $w_{j,t}$, $w_{j,t}^{exNSS}$ and $w_{j,t}^{SA}$ are very similar but not necessarily identical due to the fact that the component's prices used to construct each of them are the same but the aggregate price deflator may not be given the differing aggregate compositions.

### 4.2. Identifying the sources of seasonal effects on aggregate growth rates

With the adoption of chain-linking, explaining aggregate growth in terms of the components has become less straightforward due to the annual change in the structure of relative prices. In this context, contributions have become the standard way of breaking down the performance of a chain-linked aggregate; however, the way of calculating these contributions is not unique and depends on which of the annual chain-linking methods is implemented. In the case of contributions to quarterly growth, IMF (2001) provides a unique formula for the quarterly overlap technique but none for the annual overlap technique. This is probably due to the fact that finding the right weights that permit the quarterly contributions to sum up to the total is far from direct.

This is quite inconvenient given that the annual overlap method is the most popular method in the OECD.[10] In this context, according to Eurostat (2008), at least six different formulas have been proposed to fill this gap. Some countries find the contributions from using pair of years valued at the first year's prices. Others, like France and Germany have proposed their own way. The OECD simply relies on an approximation to calculate its global contributions acknowledging that it is not right in a strict sense (OECD, 2013). The measures presented in Eurostat (2008) are basically a compromise between user friendliness, interpretability and core properties. The problem with all but one of them is that they do not necessarily produce additive contributions. This core property is fundamental when contributions are to be calculated as a difference and this, as it will become obvious later on, is a necessary feature for the identifications of effects.

The only measure contained in Eurostat (2008) that produces additive contributions is the one proposed for French Quarterly National Accounts (INSEE, 2007).[11] More recently the Chilean Quarterly National Accounts have adopted a similar measure suggested in Cobb (2013), which is also additive. Either measure fulfil the necessary requirements to be used for the analysis of the following sections, however, the measure proposed in INSEE (2007) does show some undesirable properties under seasonality or strong volatility and, therefore, we proceed by using the measure suggested in Cobb (2013).[12]

The contribution of component *j* to aggregate growth between *t-s* and *t* is defined as:[13]

$$c_{j,t} \;=\; w_{j,t}\cdot\frac{q_{j,t}-q_{j,t-s}}{Q_{t-s}} \;+\; \left(w_{j,t}-w_{j,t-s}\right)\cdot\left(\frac{q_{j,t-s}}{Q_{t-s}}-\frac{1}{Q_{t-s}}\cdot\frac{q_{j,t}^{y-1}}{4}\right)\quad \text{with } w_{j,t}=\frac{p_{j,t}^{y-1}}{P_{Q,t}^{y-1}} \tag{4.4}$$

where,

- $q_{j,t}$ : Component *j* in quarter *t*
- $Q_t$ : Aggregate chain-linked series in quarter *t*
- $q_{j,t}^{y-1}$ : Annual value for component *j* the year before the year to which quarter *t* belongs.
- $Q_t^{y-1}$ : Annual chain-linked aggregate the year before the year to which quarter *t* belongs.
- $w_{j,t}$ : Chain-linking weight of component *j* in quarter *t*
- $p_{j,t}^{y-1}$ : Component's *j* annual price deflator of the year before the year to which quarter *t* belongs.
- $P_{Q,t}^{y-1}$ : Annual aggregate price deflator of the year before the year to which quarter *t* belongs.

It is worth noting that, due to the nature of the linking method, expression (4.4) is only valid for comparisons within the same year or between consecutive years. This, however, should not be a major problem given that it covers the most common comparisons, which are comparing with the period immediately before and respect to the same period the previous year.

Taking that caveat into consideration, expression (2.7) and its individual parts may be broken down to component level by means of expression (4.4). We have that:

$$\frac{y_t}{y_{t-s}}-1 \;=\; \sum\nolimits_{j=1}^{J} c_{j,t}\;,\quad \frac{y_t^{exNSS}}{y_{t-s}^{exNSS}}-1 \;=\; \sum\nolimits_{j=1}^{J} c_{j,t}^{exNSS}\quad\text{and}\quad \frac{y_t^{SA}}{y_{t-s}^{SA}}-1 \;=\; \sum\nolimits_{j=1}^{J} c_{j,t}^{SA} \tag{4.5}$$

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[10] According to the last available survey of OECD (2009) updated to May 2014, out of the 37 OECD countries plus Brazil and Russian Federation, 27 use the annual overlap method, 5 use the quarterly overlap method, 4 use an unspecified indirect method and one still uses a fixed-base method. It's worth mentioning that the five countries that use the quarterly overlap method are Australia, Canada, Japan, United Kingdom and United States. The popularity is probably due to its desirable temporal aggregation properties. Annual figures are the simple sum of the quarterly figures. With the Quarterly Overlap method, if consistency is required, a benchmarking procedure is required.

[11] The same measure is used by Belgium (Banque Nationale de Belgique, 2010).

[12] Annex 2 presents an example of the issue. For a full comparison of methods, refer to Cobb (2014b).

[13] The expression from Cobb (2013) is reordered to show it as the contributions from the Laspeyres links plus a correction. The additivity and consistency properties of this measure are shown in Cobb (2014b). The measure not only produces additive contributions but these are consistent with those calculated with the formula for annual frequency.

where,

$c_{j,t}$ : contribution of component $j$ to the growth of unadjusted aggregate in quarter $t$

$c_{j,t}^{exNSS}$ : contribution of component $j$ to the growth of the unadjusted aggregate corrected for non-systematic seasonality in quarter $t$

$c_{j,t}^{SA}$ : contribution of component $j$ to the growth of the seasonally adjusted aggregate in quarter $t$

Then, the aggregate non-systematic seasonal effect may be written as:

$$C_t^{NSS} = \sum_{j=1}^{J}\left(c_{j,t} - c_{j,t}^{exNSS}\right)$$

and the contribution of component $j$ to the aggregate non-systematic seasonal effect as:

$$c_{j,t}^{NSS} = c_{j,t} - c_{j,t}^{exNSS} \tag{4.6}$$

In the same way, the contribution of component $j$ to the aggregate systematic seasonal effect may be written as:

$$c_{j,t}^{SS} = c_{j,t}^{exNSS} - c_{j,t}^{SA} \tag{4.7}$$

As it was the case of the single series, an aggregate series is also affected by changes in its seasonal pattern. Unfortunately, the division proposed in expression (2.11) is not as direct and, therefore, the derivation is provided in Annex 3. The formulas for the contributions are the following:

$$c_{j,t}^{SSPY} = c_{j,t}^{exNSS} - \frac{1}{y_{t-s}^{SA}}\cdot\left(\sum_{g=t^0+s}^{\left(t-t^0\right)/s}\left(c_{j,g}^{SSPY}\cdot y_{g-s}^{SSPY} - c_{j,g-s}^{SA}\cdot y_{g-2s}^{SA}\right)\right) \tag{4.8}$$

and:

$$c_{j,t}^{\Delta SS} = \frac{1}{y_{t-s}^{SA}}\cdot\left(\sum_{g=t^0+s}^{\left(t-t^0\right)/s}\left(c_{j,g}^{SSPY}\cdot y_{g-s}^{SSPY} - c_{j,g}^{SA}\cdot y_{g-s}^{SA}\right)\right) \tag{4.9}$$

where $t^0$ is the same quarter as $t$ but in the reference year.

This means that the overall contribution of seasonal effects to aggregate growth may be traced down to the components:

$$C_t^{SA} = \underbrace{\sum_j^J\left(c_{j,t}^{NSS}\right)}_{C_t^{NSS}} + \underbrace{\sum_j^J\left(c_{j,t}^{SSPY}\right)}_{C_t^{SSPY}} + \underbrace{\sum_j^J\left(c_{j,t}^{\Delta SS}\right)}_{C_t^{\Delta SS}} \tag{4.10}$$

Then, based on this expression it is possible to track down the sources of seasonal effects on growth rates. This should be especially useful for institutions that publish both original and seasonally adjusted series of the same variable.

## 5. An example using Chilean GDP

The previous section presents an expression to breakdown the contribution of seasonal effects to aggregate growth in order to identify their sources. In this section we perform the exercise with Chilean data. Chile adopted chain-linking for its accounts in 2012 and implemented the annual overlap method for Laspeyres indices (Guerrero et. al., 2012). Since then, it has adopted the indirect method for seasonal adjustment (Cobb and Jara, 2013). For the purpose of this exercise we used quarterly production data.[14]

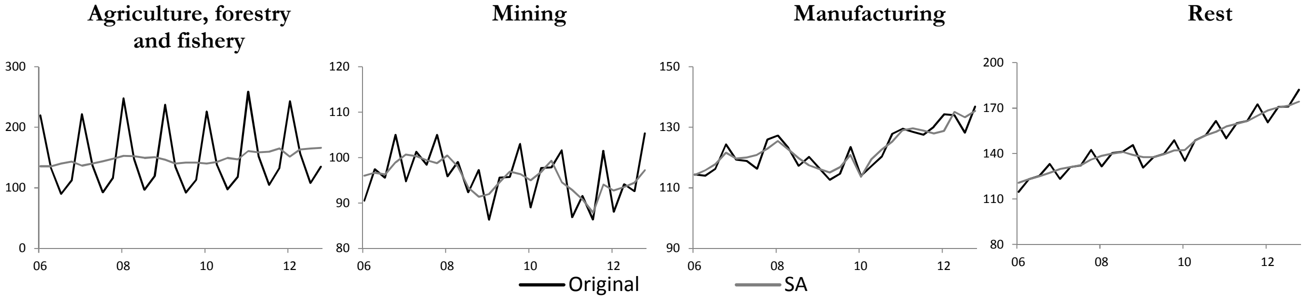
Chart 5.1 shows the original and seasonally adjusted components of GDP. As it can be seen, the different components show very different seasonal patterns. In particular, peaks and troughs do not necessarily coincide in

[14] The level of aggregation is the following; Agriculture, forestry and fishery, Mining, Manufacturing and the Rest. The selection of components is done specifically to allow for a diversity of seasonal patterns and effects among components but maintaining only a reduced number of them. It is worth mentioning that we perform a default seasonal adjustment in order to have the separate unobserved components and, therefore, it will not coincide with the official publication.

timings. Agriculture, forestry and fishery, for example, peaks in the first quarter while for the Rest that quarter is a seasonal low.

**Chart 5.1: Original and seasonally adjusted components of Chilean GDP**

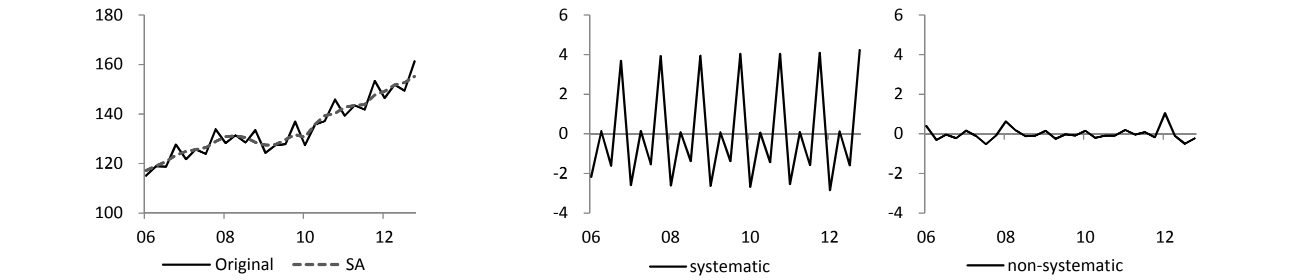
(Levels, reference year 2003=100)




As one would expect these individual seasonal patterns are then transferred to the aggregate as reported in the first graph of Chart 5.2. Also in this chart the estimated systematic and non-systematic seasonal components are presented. As one would expect, from the chart one can appreciate the significant systematic seasonality and, when observed on the same scale, the relatively less important non-systematic portion.[15] By examining the systematic seasonality it becomes obvious that the relative importance of the Rest overshadows in the aggregate the strong seasonal component of Agriculture. However, it is worth noting that, all the same, Agriculture's performance has a larger effect on the aggregate in the first quarter than in the other periods.

**Chart 5.2: Original and seasonally adjusted Chilean GDP and its seasonal unobserved components**

(Levels reference year 2003=100; deviation from trend in percentage)




To quantify this it is necessary to breakdown the aggregate growth down to the different sources of contributions. Chart 5.3 shows the exact breakdown of both the systematic and non-systematic seasonal effects for each quarter for the final four years of the series. Thanks to the breakdown it is possible to appreciate that of the -2.4% average systematic deviation of the first quarter, -3.5 pp. are attributable to the Rest, -0.8 pp. to Mining, -0.2 to Manufacturing and +2.2 to Agriculture. Also, most of the negative deviation of the third quarter is due to Agriculture (-1.3 pp. out of a total of -1.5%). Regarding the non-systematic effects one can appreciate that the 1.0% positive effect in the first quarter 2012 is mainly due to Manufacturing (+0.5 pp.) while the negative 0.5% of the third quarter is equally shared by Manufacturing and the Rest. This positive effect is probably partly due to the leap year effect that also becomes apparent in the first quarter of 2008. This overall breakdown is not in itself an explanation for the seasonal pattern but it provides the link between the component's specific features and the aggregate.

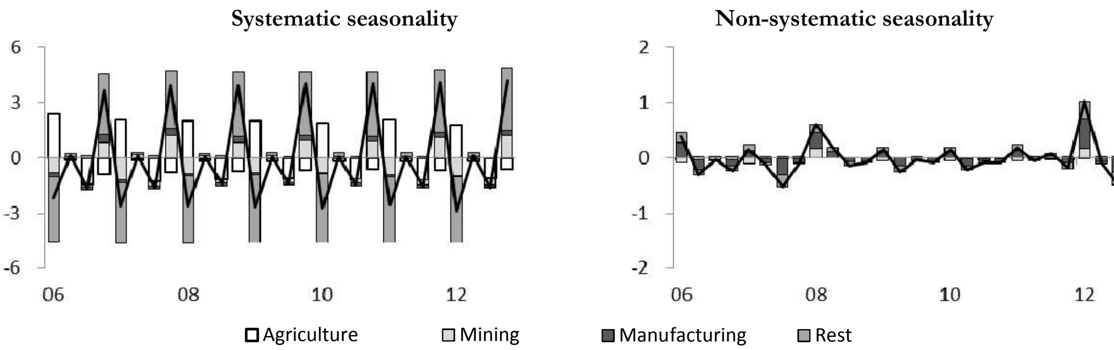
It is interesting to note that, although not immediately obvious, the aggregate systematic seasonality mutates slightly over the seven year span of the graph. It is not a smooth process but there is a noticeable difference

[15] It is worth mentioning that the implementation of the seasonal adjustment process does not permit to separate the aggregate trend from the irregular. This is not a problem for this exercise, but might be one for other applications. Also, given that the purpose of this section is to provide an example of the application of contributions to explain seasonal effects, the actual seasonal adjustment has been left on default and that no further effort has been made to obtain the "best" seasonal adjustment. In a real setting, the explanation of the aggregates effects will only be meaningful as long as the components effects are properly identified.

between the estimated pattern in 2006 and the one of 2012. Just to look at the peaks, over the seven year period, the deviation of the first quarter increases in absolute terms from -2.2 to -2.8% while the deviation in the fourth quarter increases from 3.7 to 4.2%. From a careful inspection of Chart 5.3 it can be seen that the contributions of Agriculture shrink marginally while those of Mining and the Rest expand very slightly. This change in the aggregate seasonal pattern, although slight, will undoubtedly show up as differences between the annual growth rates of the original and seasonal series.

**Chart 5.3: Breakdown of the systematic and non-systematic seasonality in the aggregate level**

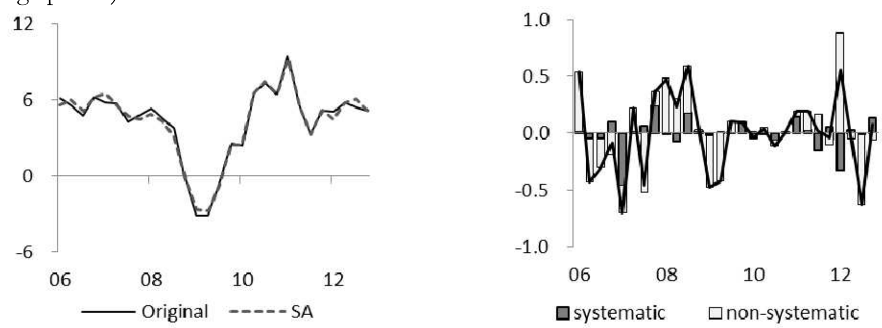
(percentage points)




Regarding the economic performance, Chart 5.4 shows the annual growth rates of the original and seasonally adjusted and the difference between them separated in what is attributable to systematic and non-systematic effects.[16] By examining the growth rates it is clear that the differences between them do not influence the overall assessment. However, in many periods the difference is in the vicinity of 0.5 pp. As one might expect, most of the difference is accounted for by the non-systematic seasonality, however, in some periods the influence of systematic seasonality is non negligible. As it was mentioned before, the only justification for this to happen is for the aggregate seasonal pattern to be changing. This change may be sourced in the component's seasonal patterns evolving, in a change in aggregate composition or a combination of both.

**Chart 5.4: Annual growth of the original and seasonally adjusted Chilean GDP and the difference between them**

(percentage; percentage points)



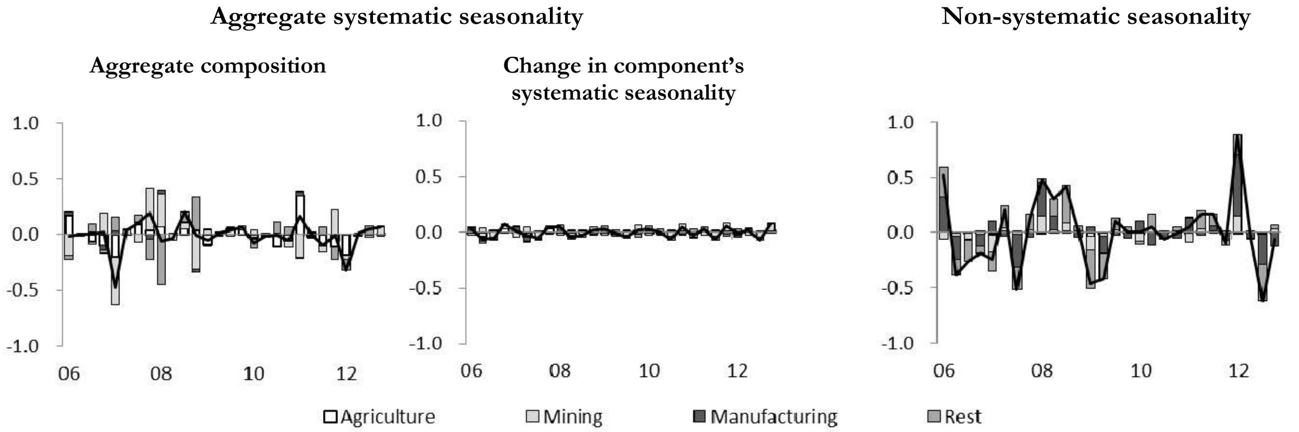

As with the levels, each one of the effects may be broken down to be accounted for by each component. Chart 5.5 shows the component's contributions to the difference in annual growth rates separated in what is attributable to systematic and non-systematic effects. By using expression (4.8) and (4.9), the difference due to the systematic effects is separated in what is attributable to the differences in the aggregate composition and what is due to the changes in the component's seasonal patterns.

The first thing that becomes apparent is that, by separating the difference due to systematic seasonality, it becomes clear that most of the effect is sourced in the differences between the compositions of the seasonally adjusted aggregate and the original. By looking at the three effects on the same scale, one can appreciate that the

[16] The analysis is performed with annual growth rates but the method is directly applicable to quarterly rates. However, quarterly growth rates of original series or seldom looked at precisely due to the seasonal effects that affect them and in fact, examining the levels is a more straightforward way of measuring the seasonal effects that are present in a quarterly comparison.

differences due to the changes in the component's systematic seasonality are almost negligible. This should not come as a surprise due to the way in which the component's seasonal patterns are estimated. If these were directly observable it would be possible to observe a higher incidence. The second thing is that the non-systematic seasonality drives the overall differences. This is because the effects of the components on the aggregate composition tend to compensate each other and only in a couple of periods the total difference stands out.

**Chart 5.5: Breakdown of the systematic and non-systematic seasonality in the aggregate growth rate**
(percentage points)




Up until now, all of the analysis has focused on building a general picture of how the component's features determine the aggregate series. However, the possibility of identifying all the effects that are involved in the aggregate performance will be most useful when examining a specific phenomenon. This could be the case for individuals or institutions analysing both original and seasonally adjusted series of the same variable where the assessment based on each one of them differs slightly. In the example, such a period could be the first quarter of 2012.I that based Graph 5.4 shows a relatively big difference between adjusted and original growth rates but also shows a significant systematic effect.

The decomposition of effects for 2012.I is shown in Table 5.1. The first column shows the seasonal deviation of that quarter relative to the yearly average, the following set of columns show the annual growth rate of the four industries for the original and seasonally adjusted series as well as for the intermediate series; the original series excluding non-systematic seasonality and the series adjusted by the systematic seasonality of the previous year. The second set of columns show the contributions to aggregate growth of the four industries for the four measures calculated using expression (4.4). The third set of columns show the three seasonal effects as calculate from the contributions.

**Table 5.1: Annual growth, industry contributions and contributions to seasonal effects to Chilean GDP in 2012.I**

| | | Annual Growth | | | | Contributions | | | | Seasonal Effects | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Seas. Level (%) | Orig. | ex NSS | PYSS | SA | Orig. | ex NSS | PYSS | SA | non-system. | Agg. Comp. | Change comp SS | Total |
| Agriculture, forestry and fishery | 51.3 | -6.02 | -6.02 | -6.02 | -5.67 | -0.38 | -0.38 | -0.20 | -0.19 | 0.00 | -0.18 | -0.01 | **-0.19** |
| Mining | -7.4 | 1.38 | -0.20 | -0.20 | -0.10 | 0.17 | -0.06 | -0.02 | -0.01 | 0.23 | -0.04 | -0.01 | **0.18** |
| Manufacturing | 0.7 | 3.66 | -0.23 | -0.23 | -0.12 | 0.42 | -0.03 | -0.03 | -0.01 | 0.44 | 0.00 | -0.01 | **0.43** |
| Rest | -6.2 | 7.07 | 6.76 | 6.76 | 6.71 | 4.87 | 4.66 | 4.76 | 4.73 | 0.21 | -0.10 | 0.03 | **0.14** |
| **TOTAL** | **-3.8** | **5.08** | **4.20** | **4.52** | **4.52** | **5.08** | **4.20** | **4.52** | **4.52** | **0.88** | **-0.32** | **-0.01** | **0.55** |

The difference between the original and adjusted growth rates is +0.55 pp. From looking at the calculated seasonal effects, the first thing that becomes obvious is the relatively large non-systematic effects in Mining, Manufacturing and the Rest that account for more than the total difference (+0.88 pp.). To have solid explanation for this effect would require digging deeper into each sector. It is probable that, to some extent, the effect is due to 2012 being a leap year and that Easter fell in April meaning three extra working days relative to an average quarter. The compositional effects work in the opposite direction reducing the gap by -0.32 pp. From examining the seasonal level of the first quarter relative to the yearly average, we observe that Agriculture is at its seasonal high while Mining and the Rest are at a seasonal trough. This means that, in principle, Agriculture's performance affects the original relatively more than the adjusted series while for the other two sectors it is the other way around.[17] This can be appreciated for Agriculture that falls 6% after removing the non-systematic effect and contributes -0.38 pp. to original growth and -0.20 pp. to adjusted aggregate growth. This is observable to a lesser degree for the Rest that grows 6.8% contributing +4.66 pp. to original growth and +4.76 pp. to adjusted growth. Finally, small changes in the component's seasonal patterns do occur, but nearly cancel each other out making the total effect negligible.

It is worth noting, that the findings regarding the relative importance of each effect are specific to this example and therefore may not be directly generalized to other datasets. The method however is.

## 6. Final remarks

The use of chain-linked methods reduces significantly the problem of price structure obsolescence present in fixed base environments, but introduces a new dimension that may produce confusion if not accounted for. The updating of the economy's price structure results in lack of additivity for the levels of the series. This means that the traditional accounting identities are not directly applicable and, therefore, explaining aggregate performance from the disaggregate data is not straightforward.

This document presents a consistent framework to identify the sources of seasonal effects in an aggregate measure chain-linked using the annual overlap method and adjusted through the indirect method. Based on the decomposition of component's contributions, the framework allows separating by industry the contribution of the individual effects; that is non-systematic seasonal effects, systematic seasonality and the change in systematic seasonality.

It is worth pointing out that the breakdown of the aggregate seasonal effects is just a procedure based on the outcome of the seasonal adjustment of the components. This means that the explanation for the aggregates effects will only be meaningful as long as the component's effects are properly identified and the overall seasonal adjustment is acceptable. In cases where the adjustment of components is unreliable the use of the indirect seasonal adjustment method may not be advisable and, therefore, this method of identifying the sources of aggregate seasonal effects will also suffer. Being able to break down the aggregate effects, however, may permit tracking down the sources of the effects and validate whether the outcome of the seasonal adjustment process is sound.

[17] For chain-linked series the causality is not direct. If the changes in aggregate composition are very small it holds relatively well, but chain-linked series are affected by discrete compositional changes due to the updating of relative prices and quarterly aggregates built using the annual overlap method also exhibit compositional effects due to the distribution of the updates. Annex 4 breaks down the effects even further to account for the chain-linking procedure.

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## Annex 1: Annual overlap chain-linked indices

This technique of annual overlap involves calculating the variation between the current year (*y*) and the previous year (*y-1*) both valued using prices of the previous year (*y-1*) and building a time series from the variation between them. By definition, the annual overlap method links a series of consecutive overlapping two-period Laspeyres indices. Then aggregate growth in any given period is given by:

$$\frac{\Delta Q^y}{Q^{y-1}} = \frac{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^y\right)}{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y-1}\right)} - 1 \qquad \text{(A1.1)}$$

where,

$Q^y$ : chain-linked aggregate in year $y$

$q_j^y$ : component $j$ in year $y$

$p_j^{y-1}$ : implicit price deflator of component $j$ in year $y-1$ (calculated as nominal value over chain-linked value)

The following table shows a simple example based on two components of an annual overlap chain-linked index and the equivalent fixed base index. Quantities A and B grow at a 2 and 5% annual rate respectively, while their prices vary 10 and -5%.

**Table A1.1: Annual series chain-linked using the Annual Overlap Method**

| Year | Quantities A | Quantities B | Prices A | Prices B | Total at current prices |
|---|---|---|---|---|---|
| 2003 | 100 | 100 | 2.00 | 4.00 | 600 |
| 2004 | 102 | 105 | 2.20 | 3.80 | 623 |
| 2005 | 104 | 110 | 2.42 | 3.61 | 650 |
| 2006 | 106 | 116 | 2.66 | 3.43 | 680 |

| | Constant prices from: 2003 | | | 2004 | | | 2005 | | | Chain-linked | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Year | Level | Index | yoy | Level | Index | yoy | Level | Index | yoy | index | yoy |
| 2003 | 600 | 100.0 | | | | | | | | *600.0* | |
| 2004 | 624 | 104.0 | ***4.00%*** | 623 | 100.0 | | | | | *624.0* | ***4.00%*** |
| 2005 | | | | 648 | 103.9 | ***3.92%*** | 650 | 100.0 | | *648.5* | ***3.92%*** |
| 2006 | | | | | | | 675 | 103.8 | ***3.84%*** | *673.3* | ***3.84%*** |

When it comes to quarterly series it becomes slightly different, given that it is the annual prices that are used for the chain-linking as opposed to the average of the quarterly prices.

The annual overlap method expresses the quarterly aggregate in the following way:

$$Q_t^y = \frac{1}{P_Q^{y-1}}\cdot\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_{j,t}^y\right) \qquad \text{(A1.2)}$$

where,

$Q_t^y$ : chain-linked aggregate in quarter $t$ that belongs to year $y$

$q_{j,t}^y$ : component $j$ in quarter $t$ that belongs to year $y$

$P_Q^{y-1}$ : implicit price deflator of the aggregate in year $y-1$

It is relatively easy to show that expression (A1.2) is consistent with (A1.1)

.

Given that under the annual overlap method annual totals are the sum of the quarterly series we can decompose (A1.1) in the following way proving the point:

$$Q^y = \frac{Q^{y-1}}{\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_j^{y-1}\right)}\cdot\sum_{j=1}^{J}\left(p_j^{y-1}\cdot\sum_{t=1}^{4}q_{j,t}^y\right) = \sum_{t=1}^{4}\left[\frac{1}{P_Q^{y-1}}\cdot\sum_{j=1}^{J}\left(p_j^{y-1}\cdot q_{j,t}^y\right)\right] = \sum_{t=1}^{4}Q_t^y \qquad \text{(A1.3)}$$

The equivalent exercise with quarterly figures is the following. Here, it becomes clear that the yearly growth rates of the chain-linked series differ from those of the constant price links. This is due to the fact that the aggregate composition of the aggregate and, therefore, the intra-annual dynamics, always depend on previous year prices while in each two-year link the aggregate composition depends on current year average prices for the first year of the link.

**Table A1.2: Quarterly series chain-linked using the Annual Overlap Method**

| Year | | Quantities | | Prices | | Total at current prices | Constant prices from: 2003 | | 2004 | | 2005 | | Chain-linked | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | A | B | A | B | | Level | yoy | Level | yoy | Level | yoy | index | yoy |
| 2003 | q1 | 24.8 | 24.5 | 1.9 | 4.1 | **148.0** | 147.8 | | | | | | *147.8* | |
| | q2 | 24.9 | 24.8 | 2.0 | 4.0 | **149.3** | 149.3 | | | | | | *149.3* | |
| | q3 | 25.1 | 25.2 | 2.0 | 4.0 | **150.7** | 150.7 | | | | | | *150.7* | |
| | q4 | 25.2 | 25.5 | 2.1 | 3.9 | **152.1** | 152.2 | | | | | | *152.2* | |
| 2004 | q1 | 25.3 | 25.8 | 2.1 | 3.9 | **153.5** | 153.7 | ***3.99%*** | 153.6 | | | | *153.7* | ***3.99%*** |
| | q2 | 25.4 | 26.1 | 2.2 | 3.8 | **155.1** | 155.2 | ***4.00%*** | 155.1 | | | | *155.2* | ***4.00%*** |
| | q3 | 25.6 | 26.4 | 2.2 | 3.8 | **156.6** | 156.8 | ***4.00%*** | 156.6 | | | | *156.8* | ***4.00%*** |
| | q4 | 25.7 | 26.7 | 2.3 | 3.7 | **158.2** | 158.3 | ***4.01%*** | 158.1 | | | | *158.3* | ***4.01%*** |
| 2005 | q1 | 25.8 | 27.1 | 2.3 | 3.7 | **159.8** | | | 159.6 | ***3.91%*** | 160.2 | | *159.8* | ***3.95%*** |
| | q2 | 25.9 | 27.4 | 2.4 | 3.6 | **161.5** | | | 161.2 | ***3.92%*** | 161.7 | | *161.3* | ***3.93%*** |
| | q3 | 26.1 | 27.7 | 2.4 | 3.6 | **163.3** | | | 162.7 | ***3.92%*** | 163.2 | | *162.9* | ***3.91%*** |
| | q4 | 26.2 | 28.1 | 2.5 | 3.5 | **165.1** | | | 164.3 | ***3.93%*** | 164.7 | | *164.5* | ***3.89%*** |
| 2006 | q1 | 26.3 | 28.4 | 2.6 | 3.5 | **166.9** | | | | | 166.3 | ***3.83%*** | *166.0* | ***3.87%*** |
| | q2 | 26.5 | 28.8 | 2.6 | 3.5 | **168.9** | | | | | 167.9 | ***3.83%*** | *167.5* | ***3.85%*** |
| | q3 | 26.6 | 29.1 | 2.7 | 3.4 | **170.8** | | | | | 169.5 | ***3.84%*** | *169.1* | ***3.83%*** |
| | q4 | 26.7 | 29.5 | 2.8 | 3.4 | **172.9** | | | | | 171.1 | ***3.85%*** | *170.7* | ***3.81%*** |

## Annex 2: Seasonality leakage in the distribution of contributions

Cobb (2014b) performs a comparison between four measures for calculating contributions in an annual overlap chain-linking framework: traditional fixed-weight, previous year prices, INSEE (2007) and Cobb (2013). The last two produce additive results. Both are based on the previous year price measure but suggest a correction to guarantee additivity.

The measure suggested by INSEE (2007) is the following:[18]

$$c_{j,t}^{\text{INSEE}} = w_{j,t} \cdot \frac{q_{j,t} - q_{j,t-s}}{Q_{t-s}} + \left(w_{j,t} - w_{j,t-s}\right) \cdot \left(\frac{q_{j,t-s}}{Q_{t-s}} - \frac{q_{j,t}^{y-1}}{Q_t^{y-1}}\right) \quad \text{with } w_{j,t} = \frac{p_{j,t}^{y-1}}{P_{Q,t}^{y-1}} \tag{A2.1}$$

where,

$q_{j,t}$ : Component *j* in quarter *t*
$Q_t$ : Aggregate chain-linked series in quarter *t*
$q_{j,t}^{y-1}$ : Annual value for component *j* the year before the year to which quarter *t* belongs.
$Q_t^{y-1}$ : Annual chain-linked aggregate the year before the year to which quarter *t* belongs.
$w_{j,t}$ : Chain-linking weight of component *j* in quarter *t*
$p_{j,t}^{y-1}$ : Component's *j* annual price deflator of the year before the year to which quarter *t* belongs.
$P_{Q,t}^{y-1}$ : Annual aggregate price deflator of the year before the year to which quarter *t* belongs.

while the measure suggested in Cobb (2013) is:[19]

$$c_{j,t}^{Cobb} = w_{j,t} \cdot \frac{q_{j,t} - q_{j,t-s}}{Q_{t-s}} + \left(w_{j,t} - w_{j,t-s}\right) \cdot \left(\frac{q_{j,t-s}}{Q_{t-s}} - \frac{1}{Q_{t-s}} \cdot \frac{q_{j,t}^{y-1}}{f}\right) \text{, where } f = 4 \text{ for quarterly data} \tag{A2.2}$$

Although the difference between correction terms is quite small, the correction proposed by INSEE (2007) shows an undesirable transfer of volatility under seasonality or strong volatility. The transfer occurs because the distribution of the correction term in (A2.1) depends on the relative real importance of the component within the respective quarter as opposed to the annual real importance. This means that the aggregate correction is distributed between components according to their relative weights and not based on how much of the aggregate volatility comes from each. The measure proposed by Cobb (2013) confines volatility to the contribution of the components that presents it. This annex highlights this point through a simple comparative example.

Consider an aggregate built from 2 components where quantities A and B grow at a 2 and 5% annual rate respectively, while their prices vary 10 and -5%.[20] The parameters reflect heterogeneous real growth of the components and significant changes in price structure. Table A2.1 presents the overall annual information. Regarding quarterly dynamics, however, A's quantity grows smoothly while B's shows a significant seasonal pattern.[21] These selected circumstances have an important effect on the quarterly aggregate dynamics.

**Table A2.1: Component's chain-link weights for the example**

| *Year* | *Quantities* | | *Prices* | | *Chain-linked (reference year=2003)* | | *Chain-link weights* | |
|---|---|---|---|---|---|---|---|---|
| | *A* | *B* | *A* | *B* | *level* | *yoy* | *A* | *B* |
| **2003** | 100 | 100 | 2.00 | 4.00 | 600 | | *1.00* | *1.00* |
| **2004** | 102 | 105 | 2.20 | 3.80 | 624 | ***4.00%*** | *1.00* | *1.00* |
| **2005** | 104 | 110 | 2.42 | 3.61 | 648 | ***3.92%*** | *1.10* | *0.95* |
| **2006** | 106 | 116 | 2.66 | 3.43 | 673 | ***3.84%*** | *1.21* | *0.90* |

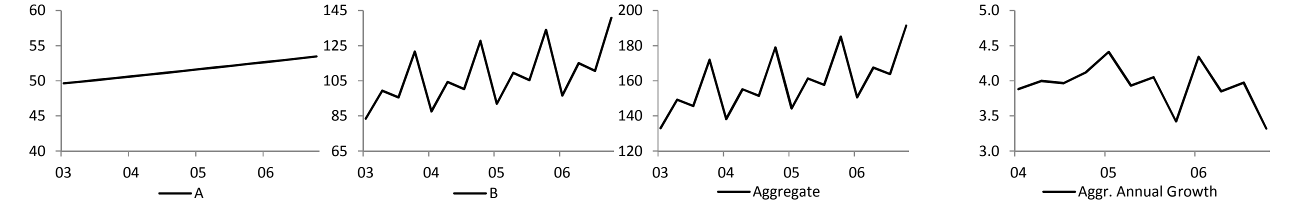
The quarterly series of the components and aggregate are presented as the three graphs to the left side of Figure A2.1. At a first glance, it seems that the seasonal properties of component B are transferred directly into the

[18] A summarized exposition in English may be found in Banque Nationale de Belgique (2010).
[19] The expression from Cobb (2013) is reordered to show it as the contributions from the Laspeyres links plus a correction.
[20] It is worth noting that the parameters are chosen in such a way that chain-linking has a relevant effect.
[21] The seasonal pattern; 0.85, 1.0, 0.95 and 1.20, multiplies the respective quarter of the constant growth component B. The series are provided in Table A2.2 at the end of this annex.

aggregate and one would expect, therefore, a relatively smooth growth rate. However, the plotted aggregate annual growth rate is relatively volatile and appears to develop a seasonal pattern of its own.

**Figure A2.1: Chain-linked aggregate with a seasonal component**

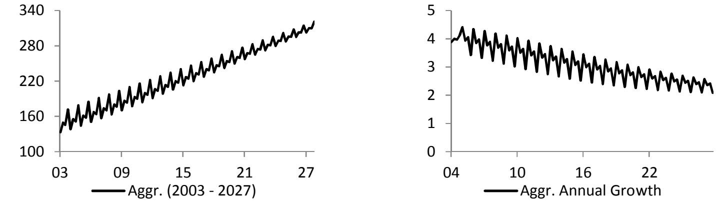
(Levels, reference year 2003; growth rate, percentage)




As it turns out, the aggregate growth rate looks like this because the aggregate seasonal pattern is actually shrinking over time due to the relative decrease in the weights of B, as presented in Table A2.1, but it is not obvious due to the length of the series. By extending the series considerably the shrinking of the pattern becomes obvious. This is shown in Figure A2.2. The seasonal pattern that appears in the annual growth rate reflects the gradual shrinkage of the aggregate seasonal pattern and it looks seasonal because the reduction in relative importance is also systematic.

**Figure A2.2: Extended chain-linked aggregate with a seasonal component**

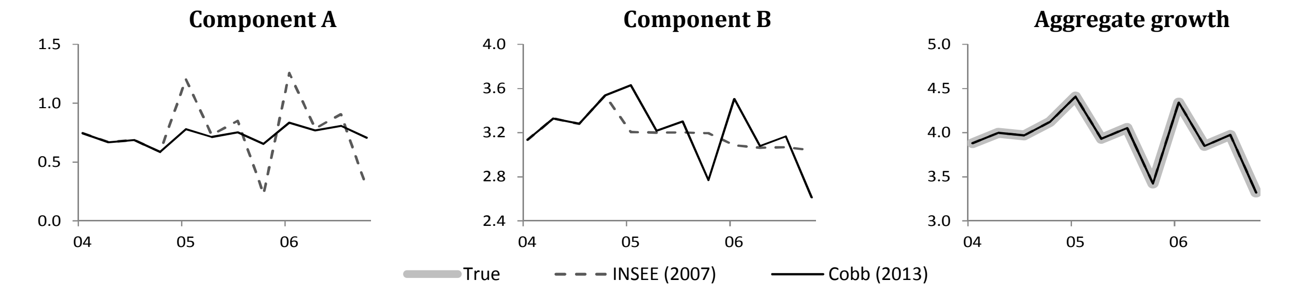
(Levels, reference year 2003; growth rate, percentage)




The ongoing decrease in relative importance of component B means that the volatility it contributes to the aggregate level decreases at the same rate every year and this fact is picked up in the annual growth rate. Although both components show smooth annual growth rates, their contributions should reflect this decrease in aggregate volatility. Figure A2.3 presents the example's contributions calculated using both measures. As it can be seen, the measure by INSEE(2007) transfers seasonality from component B to A while the measure by Cobb(2013) assigns all the seasonality to component B. As mentioned before, this transfer occurs because of the way the correction terms distribute the compositional effects.

**Figure A2.3: Contribution to annual growth of an aggregate with a seasonal component**

(contributions in percentage points, growth as a percentage)

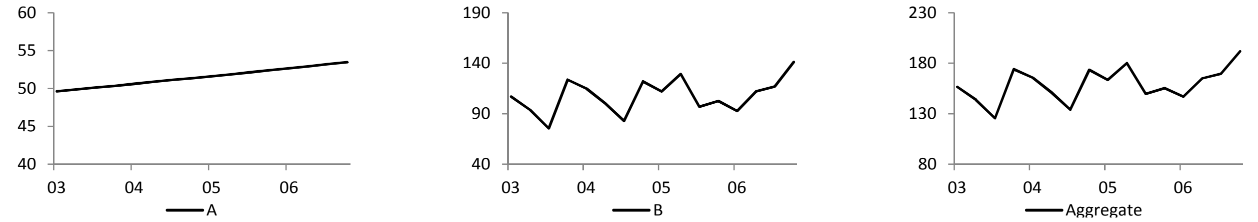



In the case of INSEE (2007) a larger share of the overall correction for the first quarter is assigned to component A given that component B is at a low seasonal level (seasonal factor 0.85), thus transferring B's volatility to A. One could argue that many countries use seasonally adjusted data meaning that this feature could be regarded as unimportant. The problem, however, arises due to the general volatility of the components and not only due to seasonality. Figure A2.4 shows the resulting graphs of the previous exercise but where the seasonal pattern of B has been replaced by a random multiplicative factor. In this case, the volatility would not be removed by a process of seasonal adjustment, and the transfer of volatility would happen anyway. This feature is probably undesirable under most circumstances. The series for both examples are provided in Table A2.2.[22]

**Figure A2.4: Contribution to annual growth of a quarterly aggregate with a volatile component**

(Levels reference year 2003, contributions in percentage points, growth as a percentage)

*Levels:*




*Quarterly contributions:*

**Component A** **Component B** **Aggregate growth**


**Table A2.2: Contribution to annual growth of a quarterly aggregate with a seasonal and a volatile component**

| Annual Growth | A | B | Pa | Pb |
|---|---|---|---|---|
| | 2% | 5% | 10% | -5% |

| | Contributions with a seasonal component | | | | | | | | Contributions with a volatile component | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | *INSEE 2007* | | *Cobb 2013* | | | | | | *INSEE 2007* | | *Cobb 2013* | |
| | | | | | A | B | A | B | | | | | A | B | A | B |
| **Quarterly** | A | B | **Total** | *y/y* % | (pp.) | (pp.) | (pp.) | (pp.) | A | B | **Total** | *y/y* % | (pp.) | (pp.) | (pp.) | (pp.) |
| Mar-03 | 50 | 83 | **133** | | | | | | 50 | 107 | **157** | | | | | |
| Jun-03 | 50 | 99 | **149** | | | | | | 50 | 94 | **144** | | | | | |
| Sep-03 | 50 | 96 | **146** | | | | | | 50 | 75 | **126** | | | | | |
| Dec-03 | 50 | 122 | **172** | | | | | | 50 | 124 | **174** | | | | | |
| Mar-04 | 51 | 88 | **138** | *3.88* | 0.75 | 3.14 | 0.75 | 3.14 | 51 | 115 | **166** | *5.70* | 0.63 | 5.06 | 0.63 | 5.06 |
| Jun-04 | 51 | 104 | **155** | *4.00* | 0.67 | 3.33 | 0.67 | 3.33 | 51 | 100 | **151** | *5.05* | 0.69 | 4.35 | 0.69 | 4.35 |
| Sep-04 | 51 | 100 | **151** | *3.97* | 0.69 | 3.28 | 0.69 | 3.28 | 51 | 83 | **134** | *6.75* | 0.80 | 5.95 | 0.80 | 5.95 |
| Dec-04 | 51 | 128 | **179** | *4.12* | 0.59 | 3.54 | 0.59 | 3.54 | 51 | 122 | **173** | *-0.37* | 0.58 | -0.95 | 0.58 | -0.95 |
| Mar-05 | 52 | 92 | **144** | *4.41* | 1.20 | 3.21 | 0.78 | 3.63 | 52 | 112 | **163** | *-1.32* | 0.46 | -1.78 | 0.65 | -1.97 |
| Jun-05 | 52 | 110 | **161** | *3.93* | 0.73 | 3.20 | 0.71 | 3.22 | 52 | 129 | **180** | *19.22* | 0.84 | 18.38 | 0.73 | 18.49 |
| Sep-05 | 52 | 105 | **158** | *4.05* | 0.85 | 3.20 | 0.75 | 3.30 | 52 | 97 | **150** | *11.68* | 1.39 | 10.29 | 0.85 | 10.83 |
| Dec-05 | 52 | 134 | **185** | *3.42* | 0.23 | 3.19 | 0.65 | 2.77 | 52 | 103 | **155** | *-10.41* | 0.35 | -10.76 | 0.67 | -11.09 |
| Mar-06 | 53 | 97 | **151** | *4.34* | 1.26 | 3.08 | 0.84 | 3.51 | 53 | 93 | **147** | *-10.02* | 0.71 | -10.73 | 0.74 | -10.76 |
| Jun-06 | 53 | 115 | **168** | *3.85* | 0.78 | 3.06 | 0.77 | 3.08 | 53 | 112 | **165** | *-8.41* | 0.35 | -8.76 | 0.69 | -9.10 |
| Sep-06 | 53 | 111 | **164** | *3.97* | 0.91 | 3.07 | 0.81 | 3.17 | 53 | 117 | **170** | *13.33* | 1.13 | 12.19 | 0.85 | 12.48 |
| Dec-06 | 53 | 141 | **191** | *3.32* | 0.28 | 3.04 | 0.71 | 2.61 | 53 | 141 | **192** | *23.48* | 0.99 | 22.49 | 0.84 | 22.64 |

[22] Neither INSEE (2007) nor Banque Nationale de Belgique (2010) explicitly warn the readers about using the correction method only with seasonally adjusted data but their implementation and examples only use adjusted data. This is understandable given that both countries deal mainly with seasonally adjusted data. The author acknowledges that due to his basic knowledge of French he could have overlooked a less than obvious warning in INSEE (2007).

## Annex 3: Explaining the contribution to changes in the systematic seasonality based on the components contribution to aggregate level

As it was mentioned in the body of this document, seasonal patterns may change over time due to a number of reasons, like technological advancements and changes in the composition of the series, and therefore, when comparing two periods, it might be desirable to divide the contribution of systematic seasonality in that which is attributable to a fixed identified seasonality and that which is attributable to the evolution of the systematic seasonality over time. In that context, we arrive at the following expression:

$$C_t^{SS} = \underbrace{\left(\frac{y_t^{exNSS}}{y_{t-s}^{exNSS}} - \frac{y_t^{SSPY}}{y_{t-s}^{SA}}\right)}_{C_t^{SSPY}} + \underbrace{\left(\frac{y_t^{SSPY}}{y_{t-s}^{SA}} - \frac{y_t^{SA}}{y_{t-s}^{SA}}\right)}_{C_t^{\Delta SS}} \tag{A3.1}$$

The two terms of this expression, however, are not so easily broken down to the individual component's contributions. That is because $C_t^{FSS}$ and $C_t^{\Delta SS}$ contain not only the growth rates of aggregate series but also the percentage difference between two series. This means that the formula for contributions is not directly applicable. However, there is a way around it that relies on the individual component's contributions to the aggregate level. The derivation is presented here.[23]

The expression for contribution of component *j* to aggregate growth, that is $c_{j,t}$,[24] may easily be transformed to represent the contribution to aggregate change in level by multiplying it by $Q_{t-s}$. Then, it is possible to formulate the contribution of component *j* to the aggregate level in period *t* as the contribution to the aggregate level in the previous period plus the contribution to aggregate change. That is:

$$k_{j,t} = k_{j,t-s} + c_{j,t} \cdot Q_{t-s} \tag{A3.2}$$

Then expression (A3.2) may be substituted recursively one interval, of length *s*, at a time until the reference year. In the reference year, *y=0*, the contribution to the aggregate level in any quarter is simply the level of component *j* in that quarter. Then component *j* in quarter $t^0$ of the reference year may be used as the initial condition to determine the contribution to the aggregate level in *t*:

$$\begin{aligned} k_{j,t} &= k_{j,t-s} + c_{j,t} \cdot Q_{t-s} \\ &= k_{j,t-2s} + c_{j,t-s} \cdot Q_{t-2s} + c_{j,t} \cdot Q_{t-s} \\ &\vdots \\ &= q_{j,t^0} + \sum_{g=t^0+s}^{(t-t^0)/s} \left[ c_{j,g} \cdot Q_{g-1} \right] \end{aligned} \tag{A3.3}$$

Then (A3.3) may be used to describe percentage difference between $y^{SSPY}$ and $y^{SA}$. That is:

$$\begin{aligned} \frac{y_t^{SSPY} - y_{t-s}^{SA}}{y_{t-s}^{SA}} &= \frac{1}{y_{t-s}^{SA}} \cdot \left[ \sum_{j=1}^{J} \left( q_{j,t^0}^{SSPY} + \sum_{g=t^0+s}^{(t-t^0)/s} \left[ c_{j,g}^{SSPY} \cdot y_{g-s}^{SSPY} \right] \right) - \sum_{j=1}^{J} \left( q_{j,t^0}^{SA} + \sum_{g=t^0+s}^{(t-t^0)/s} \left[ c_{j,g-s}^{SA} \cdot y_{g-2s}^{SA} \right] \right) \right] \\ &= \frac{1}{y_{t-s}^{SA}} \cdot \sum_{j=1}^{J} \left[ \left( q_{j,t^0}^{SSPY} - q_{j,t^0}^{SA} \right) + \sum_{g=t^0+s}^{(t-t^0)/s} \left( c_{j,g}^{SSPY} \cdot y_{g-s}^{SSPY} - c_{j,g-s}^{SA} \cdot y_{g-2s}^{SA} \right) \right] \end{aligned} \tag{A3.4}$$

It can be observed that expression (A3.4) contains the component *j* excluding non-systematic seasonality in $t^0$ corrected by the systematic seasonality of the previous year, that is $q_{j,t^0}^{SSPY}$. However, as $t^0$ belongs to the first year of the series, that is the reference year, there is no systematic seasonality of the previous year to correct with and therefore a backcast would be required. Following the same logic that was used to derive expression (2.11), that is

[23] The derivation relies on the temporal consistency of the measure for contributions that is shown in Cobb (2014b).

[24] It is worth remembering that, due to the nature of the linking method, the expression for contribution of component *j* to aggregate growth is valid for comparisons within the same year or between consecutive years. *s* is assumed to fulfil this requirement. Also, all contributions refer to the growth rate between *t-s* and *t*.

that the series of interest is already free from non-systematic seasonality and that a backcast in $t^0$ would not contain any information prior to that period, the systematic seasonality estimated for $t^0$ should be almost the same as the backcast of $t^0$-$s$.[25] This means that only for periods of the reference year, the initial value, $y_{t^0}$, corrected by the backcasted systematic seasonality of the previous year may be approximated by the estimated systematic seasonality of that period:

$$y_{t^0}^{SSPY} \quad = \quad y_{t^0}^{exNSS} - S_{t^0} \quad = \quad y_{t^0}^{SA} \tag{A3.5}$$

This fact can be substituted in (A3.4) resulting in:

$$\frac{y_t^{SSPY} - y_{t-s}^{SA}}{y_{t-s}^{SA}} \quad = \quad \frac{1}{y_{t-s}^{SA}} \cdot \sum_{j=1}^{J} \left[ \sum_{g=t^0+s}^{\left(t-t^0\right)/s} \left( c_{j,g}^{SSPY} \cdot y_{g-s}^{SSPY} - c_{j,g-s}^{SA} \cdot y_{g-2s}^{SA} \right) \right] \tag{A3.6}$$

Then the contribution of a previous year systematic seasonality may be expressed as:

$$\begin{aligned} C_t^{SSPY} \quad &= \quad \left( \frac{y_t^{exNSS}}{y_{t-s}^{exNSS}} - 1 \right) \quad - \quad \left( \frac{y_t^{SSPY} - y_{t-s}^{SA}}{y_{t-s}^{SA}} \right) \\ &= \quad \sum\nolimits_{j=1}^{J} \left( c_{j,t}^{exNSS} \right) \quad - \quad \frac{1}{y_{t-s}^{SA}} \cdot \sum_{j=1}^{J} \left[ \sum_{g=t^0+s}^{\left(t-t^0\right)/s} \left( c_{j,g}^{SSPY} \cdot y_{g-s}^{SSPY} - c_{j,g-s}^{SA} \cdot y_{g-2s}^{SA} \right) \right] \\ &= \quad \sum_{j=1}^{J} \left[ c_{j,t}^{exNSS} \quad - \quad \frac{1}{y_{t-s}^{SA}} \cdot \left( \sum_{g=t^0+s}^{\left(t-t^0\right)/s} \left( c_{j,g}^{SSPY} \cdot y_{g-s}^{SSPY} - c_{j,g-s}^{SA} \cdot y_{g-2s}^{SA} \right) \right) \right] \end{aligned} \tag{A3.7}$$

and the contribution of component $j$ to it may be written as:

$$c_{j,t}^{SSPY} \quad = \quad c_{j,t}^{exNSS} \quad - \quad \frac{1}{y_{t-s}^{SA}} \cdot \left( \sum_{g=t^0+s}^{\left(t-t^0\right)/s} \left( c_{j,g}^{SSPY} \cdot y_{g-s}^{SSPY} - c_{j,g-s}^{SA} \cdot y_{g-2s}^{SA} \right) \right) \tag{A3.8}$$

Proceeding in the same way with the contribution of the change in aggregate systematic seasonality the contribution of component $j$ to it may be written as:

$$\begin{aligned} c_{j,t}^{\Delta SS} \quad &= \quad \frac{1}{y_{t-s}^{SA}} \cdot \left( \sum_{g=t^0+s}^{\left(t-t^0\right)/s} \left( c_{j,g}^{SSPY} \cdot y_{g-s}^{SSPY} - c_{j,g-s}^{SA} \cdot y_{g-2s}^{SA} \right) \right) \quad - \quad c_{j,t}^{SA} \\ &= \quad \frac{1}{y_{t-s}^{SA}} \cdot \left( \sum_{g=t^0+s}^{\left(t-t^0\right)/s} \left( c_{j,g}^{SSPY} \cdot y_{g-s}^{SSPY} - c_{j,g}^{SA} \cdot y_{g-s}^{SA} \right) \right) \end{aligned} \tag{A3.9}$$

[25] This is particularly the case given that all the unobserved components, except for the trend, are estimate as a deviation from the trend.

## Annex 4: Seasonality and chain-linking effects

In the chain-linking framework the aggregate may be affected by a number of different effects. In first place, in the same way that fixed base indicators are affected, the aggregate may suffer compositional effects due to components growing at different rates. Additionally, the annual update of the structure of relative prices alters its composition in a discrete fashion. These sudden changes may not only dampen or reinforce the compositional effects due to differing component's growth rates but actually exceed them in magnitude making the overall effect go in the opposite direction.

In the context of explaining the seasonal features of chain-linked series the problem arises from comparing periods with different structures of relative prices. Given that many different things may be occurring at the same time, it can be useful to isolate the less intuitive chain-linking effects from the more familiar fixed weight effects.

To this we take expression (4.4) and separate three distinctive effects. The contribution may be written as follows:

$$c_{j,t} = \underbrace{\frac{q_{j,t} - q_{j,t-s}}{Q_{t-s}}}_{\substack{\text{Contributions using} \\ \text{reference year weights} \\ \text{(RYW)}}} + \underbrace{\left(w_{j,t} - 1\right) \cdot \frac{q_{j,t} - q_{j,t-s}}{Q_{t-s}}}_{\substack{\text{Effect of changing weights} \\ \text{(updating the economy's prices- PU)}}} + \underbrace{\left(w_{j,t} - w_{j,t-s}\right) \cdot \left(\frac{q_{j,t-s}}{Q_{t-s}} - \frac{1}{Q_{t-s}} \cdot \frac{q_{j,t}^{y-1}}{4}\right)}_{\substack{\text{Effect of linking periods with different weights} \\ \text{(CL)}}} \qquad \text{(A4.1)}$$

The first term reflects the impact of each component if the price structure continued to be exactly that of the reference year, making it equivalent to a fixed-base measure and in a sense more intuitive. The second term contains the effect of updating the economy's relative price structure. The sum of both these terms is equal to the contribution of the Laspeyres link that is used to build the chain-linked series (presented in Annex 1). The third term contains the compositional effects that occur in the process of transforming the separate links into a time-series. This third effect might at times be unintuitive due to it being directly the result of an aggregate correction. It is sourced in forcing a quarterly series to be consistent with an annual one and reflects, therefore, the effects of the movement of all the other components of the economy on the specific one.

From section 4 we have that the contribution of component $j$ to the aggregate seasonal effects as:

$$c_{j,t}^{SA} = \underbrace{\left(c_{j,t} - c_{j,t}^{exNSS}\right)}_{\text{non-systematic seas. } (NSS)} + \underbrace{\left(c_{j,t}^{exNSS} - \frac{1}{y_{t-s}^{SA}} \cdot \left[\sum_{g=t^0+s}^{\left(t-t^0\right)/s} \left(c_{j,g}^{SSPY} \cdot y_{g-s}^{SSPY} - c_{j,g-s}^{SA} \cdot y_{g-2s}^{SA}\right)\right]\right)}_{\text{fixed systematic seas. } (SSPY)} + \underbrace{\left(\frac{1}{y_{t-s}^{SA}} \cdot \left[\sum_{g=t^0+s}^{\left(t-t^0\right)/s} \left(c_{j,g}^{SSPY} \cdot y_{g-s}^{SSPY} - c_{j,g}^{SA} \cdot y_{g-s}^{SA}\right)\right]\right)}_{\text{change in systematic seas. } (\Delta SS)} \qquad \text{(A4.2)}$$

then, thanks to the additivity of the contributions each of the component's contribution to the specific seasonal effect, presented in expression (4.10), may be further broken down to account for how all of the nine effects interact. That means that the contribution of seasonal effects may be written as:[26]

$$C_t^{SA} = \sum_j^J \left[\left(c_{j,t}^{NSS,RYW} + c_{j,t}^{NSS,PU} + c_{j,t}^{NSS,CL}\right) + \left(c_{j,t}^{SSPY,RYW} + c_{j,t}^{SSPY,PU} + c_{j,t}^{SSPY,CL}\right) + \left(c_{j,t}^{\Delta SS,RYW} + c_{j,t}^{\Delta SS,PU} + c_{j,t}^{\Delta SS,CL}\right)\right] \qquad \text{(A4.3)}$$

This expression should provide enough information to pinpoint the sources of most of the forces involved in the contribution of seasonal effects to aggregate growth. It is quite probable that in most occasions that aggregate phenomena will be explained by a few aspects. To see this, we revisit the analysis of annual growth in the first quarter of 2012 performed in section 5 of the main text.

The decomposition of effects for 2012.I, where the difference between the original and adjusted growth rates is +0.55 pp., is shown in Table A4.1. The further breakdown shows that the non-systematic effects are contained mostly in the reference-year-weights component. These effects are compensated by the fact that Agriculture is at a seasonal high (51% above the annual average) but shows a negative growth rate, therefore reducing the original aggregate growth rate relatively more than the adjusted rate. Contributing in the same direction, the Rest is at a seasonal low (6% below the annual average) but shows a positive growth rate. The effect of updating prices reveals that, due to the compositional effects, both Agriculture and Mining contribute to reduce the gap between

[26] The actual expressions are presented at the end of this annex.

original and adjusted while the Rest contributes to increase it. The effects of chain-linking in this case are relatively small.

**Table A4.1: Annual growth, industry contributions and contributions to seasonal effects to Chilean GDP in 2012.I**

| | Seas. Level (%) | Annual Growth | | | | Contributions | | | | Seasonal Effects | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Orig. | ex NSS | PYSS | SA | Orig. | ex NSS | PYSS | SA | non-system. | Agg. Comp. | Change comp SS | Total |
| Agriculture, forestry and fishery | 51.3 | -6.02 | -6.02 | -6.02 | -5.67 | -0.38 | -0.38 | -0.20 | -0.19 | 0.00 | -0.18 | -0.01 | **-0.19** |
| Mining | -7.4 | 1.38 | -0.20 | -0.20 | -0.10 | 0.17 | -0.06 | -0.02 | -0.01 | 0.23 | -0.04 | -0.01 | **0.18** |
| Manufacturing | 0.7 | 3.66 | -0.23 | -0.23 | -0.12 | 0.42 | -0.03 | -0.03 | -0.01 | 0.44 | 0.00 | -0.01 | **0.43** |
| Rest | -6.2 | 7.07 | 6.76 | 6.76 | 6.71 | 4.87 | 4.66 | 4.76 | 4.73 | 0.21 | -0.10 | 0.03 | **0.14** |
| **TOTAL** | **-3.8** | **5.08** | **4.20** | **4.52** | **4.52** | **5.08** | **4.20** | **4.52** | **4.52** | **0.88** | **-0.32** | **-0.01** | **0.55** |

| | weight in total | | Seas level | Annual relative prices | | Reference year weights | | | | Price updating | | | | Chain-linking effect | | | | Total diff. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Orig. | SA | % | 2010 | 2011 | NSS | SSPY | ΔSS | Total | NSS | SSPY | ΔSS | Total | NSS | SSPY | ΔSS | Total | |
| Agriculture, f & f. | 5.3 | 3.2 | 51.3 | 1.06 | 1.03 | 0.00 | -0.12 | -0.01 | **-0.13** | 0.00 | -0.08 | 0.00 | **-0.08** | 0.00 | 0.02 | 0.00 | **0.02** | **-0.19** |
| Mining | 11.9 | 12.4 | -7.4 | 1.19 | 1.23 | 0.19 | 0.00 | -0.01 | **0.18** | 0.04 | -0.07 | -0.02 | **-0.04** | 0.00 | 0.03 | 0.02 | **0.05** | **0.18** |
| Manufacturing | 11.1 | 10.8 | 0.7 | 1.03 | 1.03 | 0.43 | 0.00 | -0.01 | **0.42** | 0.01 | 0.00 | 0.00 | **0.01** | 0.00 | 0.00 | 0.00 | **0.00** | **0.43** |
| Rest | 71.8 | 73.8 | -6.2 | 0.96 | 0.95 | 0.22 | -0.13 | 0.03 | **0.12** | -0.01 | 0.05 | 0.00 | **0.04** | 0.00 | -0.02 | 0.00 | **-0.02** | **0.14** |
| **TOTAL** | **100.0** | **100.0** | **-3.8** | **1.00** | **1.00** | **0.84** | **-0.25** | **0.00** | **0.59** | **0.04** | **-0.09** | **-0.02** | **-0.08** | **0.00** | **0.03** | **0.02** | **0.04** | **0.55** |

**Figure A4.1: Formulas for the nine individual effects:**

$$c_{j,t}^{NSS,RYW} = \frac{q_{j,t}-q_{j,t-s}}{y_{t-s}} - \frac{q_{j,t}^{exNSS}-q_{j,t-s}^{exNSS}}{y_{t-s}^{exNSS}}$$

$$c_{j,t}^{NSS,PU} = \left(w_{j,t}-1\right)\cdot\frac{q_{j,t}-q_{j,t-s}}{y_{t-s}} - \left(w_{j,t}^{exNSS}-1\right)\cdot\frac{q_{j,t}^{exNSS}-q_{j,t-s}^{exNSS}}{y_{t-s}^{exNSS}}$$

$$c_{j,t}^{NSS,CL} = \left(w_{j,t}-w_{j,t-s}\right)\cdot\left(\frac{q_{j,t-s}}{y_{t-s}}-\frac{1}{y_{t-s}}\cdot\frac{q_{j,t}^{y-1}}{4}\right) - \left(w_{j,t}^{exNSS}-w_{j,t-s}^{exNSS}\right)\cdot\left(\frac{q_{j,t-s}^{exNSS}}{y_{t-s}^{exNSS}}-\frac{1}{y_{t-s}^{exNSS}}\cdot\frac{q_{j,t}^{exNSS,y-1}}{4}\right)$$

$$c_{j,t}^{SSPY,RYW} = \frac{q_{j,t}^{exNSS}-q_{j,t-s}^{exNSS}}{y_{t-s}^{exNSS}} - \frac{1}{y_{t-s}^{SA}}\cdot\left[\sum_{g=t^0+s}^{(t-t^0)/s}\left(q_{j,g}^{SSPY}-q_{j,g-s}^{SSPY}-q_{j,g-s}^{SA}+q_{j,g-2s}^{SA}\right)\right]$$

$$c_{j,t}^{SSPY,PU} = \left(w_{j,t}^{exNSS}-1\right)\cdot\frac{q_{j,t}^{exNSS}-q_{j,t-s}^{exNSS}}{y_{t-s}^{exNSS}} - \frac{1}{y_{t-s}^{SA}}\cdot\left[\sum_{g=t^0+s}^{(t-t^0)/s}\left(\left(w_{j,g}^{SSPY}-1\right)\cdot\left(q_{j,g}^{SSPY}-q_{j,g-s}^{SSPY}\right)-\left(w_{j,g-s}^{SA}-1\right)\cdot\left(q_{j,g-s}^{SA}-q_{j,g-2s}^{SA}\right)\right)\right]$$

$$c_{j,t}^{SSPY,CL} = \left(w_{j,t}^{exNSS}-w_{j,t-s}^{exNSS}\right)\cdot\left(\frac{q_{j,t-s}^{exNSS}}{y_{t-s}^{exNSS}}-\frac{1}{y_{t-s}^{exNSS}}\cdot\frac{q_{j,t}^{exNSS,y-1}}{4}\right) - \frac{1}{y_{t-s}^{SA}}\cdot\left[\sum_{g=t^0+s}^{(t-t^0)/s}\left(\left(w_{j,g}^{SSPY}-w_{j,g-s}^{SSPY}\right)\cdot\left(q_{j,g-s}^{SSPY}-\frac{q_{j,g}^{SSPY,y-1}}{4}\right)-\left(w_{j,g-s}^{SA}-w_{j,g-2s}^{SA}\right)\cdot\left(q_{j,g-2s}^{SA}-\frac{q_{j,g-s}^{SA,y-1}}{4}\right)\right)\right]$$

$$c_{j,t}^{\Delta SS,RYW} = \frac{1}{y_{t-s}^{SA}}\cdot\left[\sum_{g=t^0+s}^{(t-t^0)/s}\left(q_{j,g}^{SSPY}-q_{j,g-s}^{SSPY}-q_{j,g}^{SA}+q_{j,g-s}^{SA}\right)\right]$$

$$c_{j,t}^{\Delta SS,PU} = \frac{1}{y_{t-s}^{SA}}\cdot\left[\sum_{g=t^0+s}^{(t-t^0)/s}\left(\left(w_{j,g}^{SSPY}-1\right)\cdot\left(q_{j,g}^{SSPY}-q_{j,g-s}^{SSPY}\right)-\left(w_{j,t}^{SA}-1\right)\cdot\left(q_{j,g}^{SA}-q_{j,g-s}^{SA}\right)\right)\right]$$

$$c_{j,t}^{\Delta SS,CL} = \frac{1}{y_{t-s}^{SA}}\cdot\left[\sum_{g=t^0+s}^{(t-t^0)/s}\left(\left(w_{j,g}^{SSPY}-w_{j,g-s}^{SSPY}\right)\cdot\left(q_{j,g-s}^{SSPY}-\frac{q_{j,g}^{SSPY,y-1}}{4}\right)-\left(w_{j,g}^{SA}-w_{j,g-s}^{SA}\right)\cdot\left(q_{j,g-s}^{SA}-\frac{q_{j,g}^{SA,y-1}}{4}\right)\right)\right]$$

* It is worth noting that in a spreadsheet environment it is easier to calculate each effect form the contribution to the aggregate levels, built using the ideas explained in Annex 3.