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Abstract

We construct a research and development (R&D) based endogenous growth model with exhaustible resources and investigate whether protection of intellectual property rights (IPR) can sustain perpetual growth. We show that relatively weak IPR protection is sufficient to sustain perpetual growth when goods production is more resource-intensive, whereas relatively strong IPR protection is needed for perpetual growth if production is less resource-intensive. If the resource intensity in goods production is medium, even the strictest IPR protection cannot sustain perpetual growth when the quality improvements brought about by innovations are small enough. In this case, we find that R&D subsidies can complement IPR protection in sustaining perpetual growth. We derive the socially optimal level of IPR protection, which is increasing in the resource intensity of goods production. Furthermore, we also consider a case where resource is essential for R&D activities and show a knife-edge condition for perpetual growth.

Keywords: Endogenous growth; Exhaustible resource; Innovation; Intellectual property rights protection; Patent breadth

JEL classification: L50, O30, P28

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1 Introduction

Technological change has been considered as one of the most important factors for economic growth, whereas scarcity of exhaustible resources has been considered as one of the barriers to economic growth. Because these opposing factors have a strong relation via energy price changes, a growing number of studies have focused on economic growth in the presence of exhaustible resources.[1] Theoretically, some authors, such as Stiglitz (1974), use models where the technological growth rate is exogenously given to examine the technological growth rate that overcomes the scarcity of exhaustible resources. Studies such as Barbier (1999) employ endogenous growth models to address the same issue. These studies usually assume that the intellectual property rights (IPR) of innovators of new technologies are perfectly protected; therefore, research firms have sufficient incentives for research and development (R&D) activities. However, in the real world, newly developed technologies are often imitated and IPR protection is imperfect. Then, there are global movements to enforce stronger IPR protection. Since the agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPs), all World Trade Organization (WTO) member countries adopted a set of minimum standard on IPRs There are some countries that have much stricter related laws on IPR protection. If IPR protection were too weak, no one would engage in R&D activities, resulting in technical progress becoming too slow to overcome the increasing resource scarcity. The main purpose of the present study is to examine IPR protection policies that overcome the scarcity of exhaustible resources and realize perpetual output growth. On the other hand, perpetual growth does not necessarily imply socially optimal allocation. Therefore, we are also interested in how the presence of exhaustible resources influences the welfare effects of IPR protection policies.

Assuming that an exhaustible resource is an essential production factor as in Stiglitz (1974), Barbier (1999), and others, we construct a simple quality-ladder Schumpeterian growth model (cf. Grossman and Helpman, 1991, Chapter 4), where the technological growth rate is endogenously determined. In our model, a single final good is produced by using intermediate goods. To produce the intermediate goods, firms in the intermediate good sector must employ labor and resources. Research firms conduct R&D activities. If a firm succeeds in its R&D activities, the quality of its intermediate good is improved. Assuming that the IPRs of successful innovators are protected legally by a patent, we examine IPR protection policies that overcome resource scarcity. The present paper considers the following two models; (i) labor is the only input in the R&D sector, and (ii) the exhaustible resource is essential for R&D activities. We show that these two models lead to quite different conclusions.[2]

In the model where labor is the only input in the R&D sector, we obtain the following results. First, strengthening IPR protection has two opposing effects on perpetual output growth. On the one hand, it stimulates R&D activities, which has a positive effect on growth. On the other, stronger IPR protection leads to exhaustible resources getting scarcer at a higher rate, which negatively affects the growth rate. Because the former effect dominates the latter, stronger IPR protection increases the growth rate. Second,

[1] According to the well-known Hotelling rule, the price of an exhaustible resource grows at the rate that equals the interest rate. Furthermore, empirical studies such as Popp (2002) show that an increase in energy prices stimulates technological change in industries that use these resources.

[2] Although capital accumulation has the potential to overcome resource scarcity, we abstract it because empirical studies such as Easterly and Levine (2001) point out that technical progress, rather than capital accumulation, is the main driving force of economic growth.

there exists a minimum level of IPR protection that is required to attain perpetual output growth. The minimum level of IPR protection depends on the resource intensity in goods production. Stiglitz (1974) shows that as the resource intensity gets higher, more rapid productivity growth is required to overcome increasing resource scarcity. Then, one may conjecture that IPR protection must be strong to sustain perpetual output growth if goods production is relatively resource-intensive. Contrary to this conjecture, we show that when production is sufficiently resource-intensive, relatively weak IPR protection is sufficient for perpetual growth, whereas if the resource intensity in goods production is sufficiently low, perpetual growth is possible only in the presence of relatively stronger IPR protection. This result arises because relatively high resource intensity in goods production implies relatively low demand for labor. Then, as the equilibrium wage rate becomes low, R&D activities increase because of decreased labor costs.

Third, depending on the resource intensity in goods production, IPR protection may not sustain perpetual output growth. Specifically, when the resource intensity in goods production is medium in level, even the strongest IPR protection cannot sustain perpetual growth if each innovation brings about a small quality improvement. In this case, in addition to sufficiently strong IPR protection, an R&D subsidy is needed. The present model shows that the subsidy must be introduced in the presence of IPR protection because the R&D subsidy alone does not stimulate R&D activities. Therefore, by setting sufficiently high subsidy rates and sufficiently strong IPR protection, perpetual growth can be sustained. The final result in the model where labor is the only input in the R&D sector is related to welfare analysis. The resource intensity in goods production affects the optimal IPR protection that maximizes social welfare. More precisely, as goods production becomes more resource intensive, the socially optimal level of IPR protection tends to be stronger.

If the exhaustible resource is essential for R&D activities, quite different results are obtained. Only when both the IPR protection policy and the subsidy rate satisfy a rather knife-edge condition, the perpetual output growth is possible. Then, for each set of economic parameter values, the policy maker has to accurately control the strength of IPR protection and the subsidy rate. Furthermore, if the initial stock of the exhaustible resource is small enough, the knife-edge condition is never satisfied regardless of the strength of IPR protection and the subsidy rate. Therefore, in this case, the IPR protection policy and the R&D subsidy cannot sustain perpetual growth.

Literature also has pointed out that parameter values associated with resource-productivity growth and resource intensity are important for perpetual growth. In exogenous growth models with infinitely-lived agents, studies like Stiglitz (1974) show that when the resource intensity in goods production is high, sufficiently high productivity growth is required to sustain perpetual growth. In an overlapping-generations model with exogenous productivity growth, Agnani *et. al.* (2005) obtain the similar results. However, because productivity growth rates are exogenously given, these studies do not examine how to realize such high productivity growth.

Many authors have considered the role of exhaustible resources in endogenous growth models. For example, Barbier (1999), Scholz and Ziemes (1999), and Grimaud and Rougé (2003, 2005) construct R&D-based endogenous growth models with exhaustible resources similar to our models. These studies derive the condition for perpetual growth and then show that the condition depends on preference and technology parameters including the resource intensity, as in exogenous growth models. Barbier (1999) and Scholz and Ziemes (1999) do not conduct any policy analyses. These studies then cannot answer what policies to be conducted to sustain perpetual growth when the model parameters do not satisfy the condition for perpetual growth. Grimaud and Rougé (2003, 2005) conduct policy analyses, focusing on the R&D subsidy and the tax on the resource use but paying no attention to IPR protection. Their main purpose is to characterize the policies that implement optimal allocations. In contrast, by explicitly modeling IPR protection, we find that the R&D subsidy alone cannot sustain perpetual growth because it has no impact on R&D activities in the absence of IPR protection. In addition, our model shows that the socially optimal IPR protection is increasing in the resource intensity.

More recently, some authors suppose multiple sectors to consider the direction of technical change and transformation of industrial structure. Extending the model of directed technical change developed by Acemoglu (2002), Di Maria and Valente (2008) construct a model where there are two final good sectors with different resource intensities.[3] They show that under some condition, the long-run growth rate of the economy is determined by the growth rate of the resource intensive sector alone. Constructing a model similar to Di Maria and Valente (2008), Pittel and Bretschger (2010) show that different resource intensities of the two sectors have exactly the same negative impact on the long-run growth rate of the economy (see equations (33) and (34) in their paper). Bretschger and Smulders (2012) also obtain the similar results in a model of structural change. Thus, even in the models of directed technical change or structural change, the long-run growth rate of economy is determined in manners similar to those of models that do not incorporate directed technical change or structural change. Therefore, our paper models a single final good but derives an analogous condition for perpetual growth.

Most of existing studies on growth and exhaustible resources assume that the exhaustible resource is not needed in R&D activities. In contrast, we consider the situation where the exhaustible resource is essential for R&D activities. In general, R&D activities often require inputs that are produced using exhaustible resources, such as electricity.[4] Therefore, it is significant to study a model where the exhaustible resource is needed in R&D activities. Besides, the existing literature on IPR protection does not investigate its role in an economy that faces resource scarcity.[5] In contrast, our study sheds light on IPR protection and whether it can overcome the resource scarcity and drive perpetual growth.

The present paper assumes that the IPR of innovators is protected legally by a patent. Broadly speaking, there are two policy instruments regarding patents: patent length and patent breadth.[6] Depending on their purposes, some authors focus on these instruments separately, whereas others consider both in one model.[7] The present paper analyzes the effects of patent breadth, assuming infinite patent length. There are two

[3] Acemoglu *et al.* (2012) also construct a directed technical change model with exhaustible resource. However, their focus is mainly on the environmental quality, rather than the role of exhaustible resource.

[4] According to U.S. Energy Information Administration, 79% of world total generation of electricity in kWh was produced with exhaustible resources including fossil fuels and nuclear in 2011.

[5] For example, see Judd (1985), Goh and Olivier (2002), Iwaisako and Futagami (2003), O'Donoghue and Zweimüller (2004), and Dinopoulos and Kottaridi (2008).

[6] Patent length is the length of time for which a patent is valid. Patent breadth prohibits imitation of patentees' products by firms that do not have the patent.

[7] Authors such as Judd (1985), Dinopoulos and Kottaridi (2008), and Iwaisako and Futagami (2003) consider patent length in dynamic general equilibrium models without exhaustible resources. Goh and Olivier (2002) and O'Donoghue and Zweimüller (2004) introduce patent breadth in endogenous growth models without exhaustible resources.

reasons for focusing on patent breadth. First, empirical evidence shows that most patents are replaced by new products and become obsolete before the end of their legal patent life (cf. Mansfield, 1984). This evidence suggests that patent breadth matters more than patent length in practice. Second, our assumption of infinite patent length makes it simpler to examine how the presence of exhaustible resources affects the socially optimal patent policy.

The remainder of the paper is structured as follows. Section 2 constructs an R&D-based endogenous growth model with exhaustible resources, where labor is the only input in R&D activities. Section 3 derives the equilibrium. Section 4 examines whether IPR protection policies can sustain perpetual growth. Section 5 conducts welfare analysis. Section 6 introduces R&D subsidies into the benchmark model and reexamines a condition for perpetual growth. Section 7 considers a model where the exhaustible resource is essential for R&D activities. Section 8 presents our concluding remarks.

2 The Model

As in Stiglitz (1974) and other authors, we assume that an exhaustible natural resource is an essential production factor. In order to examine the IPR protection policies that sustain positive long-run growth, we endogenize technical change, following the endogenous growth model proposed by Grossman and Helpman (1991), where the qualities of intermediate goods are improved through R&D activities.

2.1 Households and Exhaustible Resources

Time is continuous and denoted as $t \geq 0$. The economy is populated by identical households. The population size is normalized to be one. The lifetime utility of the representative household at time t is given by $U_t = \int_t^{\infty} \frac{{c_\tau}^{1-\sigma}}{1-\sigma} e^{-\rho(\tau - t)} d\tau$, where c_t is consumption at time t, $\rho > 0$ is the subjective discount rate, and $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution.[8] Because most of the empirical evidence suggests that the intertemporal elasticity of substitution is relatively small, we restrict our attention to the case where $\sigma > 1$. At each moment of time, the representative household inelastically supplies $L(> 0)$ unit of labor and sell $R_t(\geq 0)$ units of the exhaustible resource. The budget constraint of the representative household is given by $\dot{a}_t = r_t a_t + w_t L - c_t + p_{R,t} R_t$, where a_t is the asset holdings, r_t is the interest rate, w_t is the wage rate, and $p_{R,t}$ is the price of the resource at time t. The price of the consumption good is normalized to one. The stock of the exhaustible resource at time t is $S_t > 0$. Because the resource is not renewable, the following two constraints must be satisfied: $\dot{S}_t = -R_t$ and $\lim_{T\to\infty} S_T \geq 0$. The representative household maximizes the lifetime utility subject to the budget constraint, these two constraints on the exhaustible resource, and the No-Ponzi game condition, $\lim_{T\to\infty} a_T e^{-\int_t^T r_u du} \geq 0$. This yields $\dot{c}_t = (r_t - \rho)c_t/\sigma$ and the familiar Hotelling rule:

$$\dot{p}_{R,t} = r_t p_{R,t}. \tag{1}$$

In addition, the following two transversality conditions must be satisfied: $\lim_{T\to\infty} a_T {c_T}^{-\sigma} e^{-\rho T} = 0$ and $\lim_{T\to\infty} S_T = 0$. The second transversality condition, together with $\dot{S}_t = -R_t$,

[8] If $\sigma = 1$, the instantaneous utility function takes a logarithmic form; hence, the lifetime utility of the representative household becomes $U_t = \int_t^{\infty} (\ln c_\tau) e^{-\rho(\tau - t)} d\tau$.

implies

$$S_t = \int_t^\infty R_\tau d\tau. \tag{2}$$

Following Stiglitz (1974) and others, we define the resource extraction rate as R_t/S_t to focus on the ratio of resource extraction to resource stock.

2.2 Production

There is a unit continuum of intermediate goods industries. Each intermediate good industry is indexed by $j \in [0,1]$. If a firm succeeds in an innovation in industry j, the firm can produce a new generation of product j. Then, each intermediate good is classified by a number of generations, $m = 0, 1, 2, \cdots$. We assume that products of different generations have different qualities (or productivities). The quality (or productivity) of product j of generation m is denoted as $q_{j,m} = \lambda^m$, where $\lambda > 1$.

In the final good sector, identical firms produce a single consumption good competitively, by using intermediate goods as inputs. The number of firms is normalized to one. As in Grossman and Helpman (1991), the technology of the representative firm in the final good sector is

$$\ln Y_t = \int_0^1 \ln \left[\sum_m q_{j,m} x_{j,m,t} \right] dj, \tag{3}$$

where Y_t is the output and $x_{j,m,t}$ is the input of product j of generation m at time t. Because each generation of product j is a perfect substitute for other generations of the product, the final good firm purchases the single generation $\tilde{m}_{j,t}$ that carries the lowest quality-adjusted price, $p_{j,m,t}/q_{j,m}$, where $p_{j,m,t}$ is the price of product j of generation m at time t. Then, from the profit maximization of the final good firm, we obtain $x_{j,m,t} = Y_t/p_{j,m,t}$ for $m = \tilde{m}_{j,t}$ and 0 otherwise. The good market equilibrium condition is given by $c_t = Y_t$, which implies

$$\dot{Y}_t = \frac{1}{\sigma}(r_t - \rho) Y_t. \tag{4}$$

Each intermediate good of any generation is produced by using labor and the resource as inputs. The production technology of product j of generation m is

$$x_{j,m,t} = \bar{A} l_{j,m,t}^{1-\alpha} R_{j,m,t}^{\alpha}, \tag{5}$$

where $\alpha \in [0,1]$ is a parameter, $\bar{A} > 0$ is the total factor productivity, and $l_{j,m,t}$ and $R_{j,m,t}$ are labor and resource inputs, respectively. The resource intensity is given by α. A large (small) α indicates strong (weak) dependence on exhaustible resources in production. Letting $A = [\alpha^\alpha (1-\alpha)^{1-\alpha} \bar{A}]^{-1}$, we obtain the unit cost function from (5):

$$\omega(w_t, p_{R,t}) = A w_t^{1-\alpha} p_{R,t}^{\alpha} \equiv \omega_t. \tag{6}$$

We model IPR protection next. If a firm succeeds in inventing a state-of-the-art version of product j, it can take out a patent and supply the good monopolistically. Following Goh and Olivier (2002), we assume that firms other than the innovator of the latest-generation product may imitate the product at a constant per unit cost of imitation. However, the existence of the patent legally defends the inventor against imitations. If inventors' IPR is perfectly protected and no firms can imitate the latest-generation, the patentee of the state-of-the-art product charge a price, $p_{j,t} = \lambda \omega_t = (1+m)\omega_t$, where $m = \lambda - 1$ denotes the monopoly markup. If inventors' IPR is not fully protected, the patentee of the state-of-the-art product is forced to charge a price lower than $(1+m)\omega_t$. We measure the strength of the patent breadth by $\phi \in [0,1]$. As in Goh and Olivier (2002), we assume that the maximum price that the inventor of product j can charge is equal to $p_{j,t} = \beta\omega_t$, where $\beta \equiv (1+\phi m) \in [1, \lambda]$. The following discussion uses $\beta \in [1, \lambda]$ to measure the strength of IPR protection, instead of ϕ. The patent breadth of $\beta = \lambda$ implies perfect IPR protection. Most studies on R&D-based endogenous growth models focus on this case. When $\beta = 1$, there is no protection. Patent breadth within the interval of $(1, \lambda)$ indicates partial IPR protection.

Under the patent policy described above, the patentee the state-of-the-art product charges a price $p_{j,t} = \beta\omega_t \equiv p_t$. The output and profits of product j of the latest generation are, respectively,

$$x_{j,t} = \frac{Y_t}{\beta\omega_t} \equiv x_t, \tag{7}$$

$$\pi_{j,t} = \left(1 - \frac{1}{\beta}\right) Y_t \equiv \pi_t. \tag{8}$$

From $p_t = \beta\omega_t$, (7), and (8), all firms that produce the latest generation behave symmetrically. Then, (3) can be written as $Y_t = e^{Q_t}\bar{A} l_t^{1-\alpha} R_t^{\alpha}$, where l_t and R_t are the labor and resource inputs of the representative state-of-the-art firm, respectively. The term $e^{Q_t} \equiv e^{\int_0^1 \ln q_{j,t} dj}$ represents the productivity level whose growth rate is determined endogenously, where $q_{j,t}$ is the state-of-the-art quality of product j at time t. Then, our model is an endogenous growth version of Stiglitz's (1974) model without capital accumulation.

2.3 R&D

Following Grossman and Helpman (1991) and most R&D-based endogenous growth models with exhaustible resources, we assume that labor is the only input in R&D activities. If an R&D firm devotes $b\iota_t dt$ units of labor for a time interval of dt to R&D activities on product j, it succeeds in inventing a new-generation product j with probability $\iota_t dt$, where b is a positive constant. Then, ι_t represents the instantaneous intensity of R&D activities at time t. If a firm succeeds in inventing a new generation of product j, it can take out a patent for that generation. Let v_t denote the market value of a new invention. The expected profits of an R&D firm are $v_t \iota dt - w_t b \iota_t dt$. Because R&D intensity must be finite in equilibrium, we have

$$v_t \leq b w_t, \quad \text{with equality if } \iota_t > 0. \tag{9}$$

In addition, v_t must satisfy the following no-arbitrage condition:

$$r_t v_t = \dot{v}_t - \iota_t v_t + \pi_t. \tag{10}$$

2.4 Labor and Resource Markets

Each firm that produces a state-of-the-art product demands $(\partial\omega_t/\partial w_t)x_t$ units of labor and $(\partial\omega_t/\partial p_{R,t})x_t$ units of exhaustible resources. The labor demand of R&D firms equals $b\iota_t$. Using (6) and (7), the labor and resource market equilibrium conditions, respectively, are given by

$$L = \frac{\partial \omega_t}{\partial w_t} x_t + b\iota_t = \frac{(1-\alpha)\omega_t}{w_t} x_t + b\iota_t = \frac{(1-\alpha)Y_t}{\beta w_t} + b\iota_t, \tag{11}$$

$$R_t = \frac{\partial \omega_t}{\partial p_{R,t}} x_t = \frac{\alpha \omega_t}{p_{R,t}} x_t = \frac{\alpha Y_t}{\beta p_{R,t}}. \tag{12}$$

3 Equilibrium

We first derive the dynamic system of the model economy. We define $V_t \equiv Y_t/v_t$. V_t is a jump variable. From (4), (8), (10), and the definition of V_t, we have

$$\frac{\dot{V}_t}{V_t} = \frac{1}{\sigma}[(1-\sigma)r_t - \rho] - \iota_t + \frac{\beta-1}{\beta} V_t. \tag{13}$$

It is shown that ι_t is a function of V_t.[9]

$$\iota_t = \begin{cases} \frac{L}{b} - \frac{1-\alpha}{\beta} V_t, & \text{if} \quad V_t < \frac{\beta}{(1-\alpha)b} L, \\ 0, & \text{if} \quad V_t \geq \frac{\beta}{(1-\alpha)b} L. \end{cases} \tag{14}$$

Furthermore, we can show that r_t satisfies the following relation:[10]

$$r_t = \begin{cases} \frac{(1-\alpha)(\beta-1)}{\beta} V_t - (1-\alpha-\ln\lambda)\iota_t, & \text{if} \quad V_t < \frac{\beta}{(1-\alpha)b} L, \\ \frac{\sigma \ln \lambda}{1-\alpha+\alpha\sigma}\iota_t + \frac{\rho(1-\alpha)}{1-\alpha+\alpha\sigma}, & \text{if} \quad V_t \geq \frac{\beta}{(1-\alpha)b} L, \end{cases} \tag{15}$$

where ι_t is given by (14). Substituting (15) into (13) yields

$$\frac{\dot{V}_t}{V_t} = \begin{cases} \frac{1-\alpha+\alpha\sigma}{\sigma}\left[\frac{\beta-1}{\beta}V_t - \frac{(\sigma-1)\ln\lambda + (1-\alpha+\alpha\sigma)}{1-\alpha+\alpha\sigma}\iota_t - \frac{\rho}{1-\alpha+\alpha\sigma}\right] & \text{if} \quad V_t < \frac{\beta}{(1-\alpha)b} L, \\ \frac{\beta-1}{\beta}V_t - \frac{(\sigma-1)\ln\lambda + (1-\alpha+\alpha\sigma)}{1-\alpha+\alpha\sigma}\iota_t - \frac{\rho}{1-\alpha+\alpha\sigma}, & \text{if} \quad V_t \geq \frac{\beta}{(1-\alpha)b} L, \end{cases} \tag{16}$$

where ι_t is given by (14). The dynamic system is composed of (14) and (16).

Using phase diagrams, we next derive the steady state equilibrium where V_t is constant over time. Substituting $\dot{V}_t = 0$ into (16) yields

$$V_t = \frac{\beta}{\beta-1} \frac{\rho + \{(\sigma-1)\ln\lambda + (1-\alpha+\alpha\sigma)\}\iota_t}{1-\alpha+\alpha\sigma}. \tag{17}$$

[9] We show that if $V_t \geq \beta L/\{(1-\alpha)b\}$, $\iota_t = 0$ holds as follows: Suppose $V_t \geq \beta L/\{(1-\alpha)b\}$. If $\iota_t > 0$, (9) indicates $\beta L/\{(1-\alpha)b\} \leq Y_t/v_t = Y_t/(bw_t)$, which implies $L \leq (1-\alpha)Y_t/(\beta w_t)$. The last inequality contradicts (11) and $\iota_t > 0$. Then, $\iota_t = 0$ must hold. When $V_t < \beta L/\{(1-\alpha)b\}$, we can show $\iota_t > 0$ as follows: If $\iota_t = 0$ holds, (9) and $V_t < \beta L/\{(1-\alpha)b\}$ indicate $\beta L/\{(1-\alpha)b\} > Y_t/v_t > Y_t/(bw_t)$, which implies $L < (1-\alpha)Y_t/(\beta w_t)$ and contradicts (11). Then, we have $\iota_t > 0$. Solving (11) for ι_t yields $\iota_t = L/b - (1-\alpha)V_t/\beta$.

[10] Using (7), we rewrite (3) as $\ln Y_t = \int_0^1 [\ln q_{j,t} + \ln Y_t - \ln \beta\omega_t] dj$, which implies $\ln \beta\omega_t = \int_0^1 (\ln q_{j,t}) dj$. Because $\iota_t dt$ units of intermediate goods improve their quality for a time interval of length dt, we have $\ln \beta\omega_{t+dt} = \int_0^{\iota_t dt} (\ln \lambda q_{j,t}) dj + \int_{\iota_t dt}^1 (\ln q_{j,t}) dj = \iota_t (\ln\lambda) dt + \int_0^1 (\ln q_{j,t}) dj = \iota_t(\ln\lambda)dt + \ln\beta\omega_t$. Then, as $dt \to 0$, we have $\dot{\omega}_t/\omega_t = \iota_t \ln\lambda$. The last equation, together with (1) and (6), implies $(1-\alpha)\dot{w}_t/w_t + \alpha r_t = \iota_t \ln\lambda$. When $V_t \geq \frac{\beta}{(1-\alpha)b} L$, we have $\iota_t = 0$ from (14) and $L = (1-\alpha)Y_t/(\beta w_t)$ from (11). The second equation and (4) imply $\dot{w}_t/w_t = \dot{Y}_t/Y_t = (r_t - \rho)/\sigma$. If $V_t < \frac{\beta}{(1-\alpha)b} L$ holds, we have $\iota_t > 0$ from (14) and $v_t = bw_t$ from (9). The last equation and (10) imply $\dot{w}_t/w_t = \dot{v}_t/v_t = r_t + \iota_t - \pi/v_t = r_t + \iota_t - (\beta-1)V_t/\beta$. The discussion so far, together with $(1-\alpha)\dot{w}_t/w_t + \alpha r_t = \iota_t \ln\lambda$, implies (15).

In Figure 1, we draw the graphs of (14) and (17). An intersection of the two graphs corresponds to a steady state. The graph of (17) intersects with the vertical axis at $\underline{V} \equiv \frac{\beta}{\beta-1}\frac{\rho/\{(1-\alpha)\sigma\}}{1/\sigma+\alpha/(1-\alpha)} > 0$. If $\underline{V} < \beta L/\{(1-\alpha)b\}$, which is equivalent to $\beta > 1 + \frac{\rho/\sigma}{1/\sigma+\alpha/(1-\alpha)}\frac{b}{L} \equiv \underline{\beta}(\alpha) \geq 1$, the two graphs intersect at a point where ι^* is strictly positive (see Figure 1 (a)). An asterisk is used for a variable in the steady state. In contrast, if $\beta \leq \underline{\beta}(\alpha)$, we have $\iota^* = 0$ as shown in Figure 1 (b). Because the steady state is unstable, the economy is always in the steady state. We now can prove the next proposition.

[Figure 1]

Proposition 1
The economy is always on the unique steady state, where the following hold:

$$\iota^* = \begin{cases} 0, & \text{if} \quad \beta \leq \underline{\beta}(\alpha), \\ \frac{\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)\frac{\beta-1}{b}L-\frac{\rho}{\sigma}}{\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)(\beta-1)+\alpha+\frac{1-\alpha}{\sigma}-\frac{1-\sigma}{\sigma}\ln\lambda} > 0, & \text{if} \quad \beta > \underline{\beta}(\alpha), \end{cases} \tag{18a}$$

$$\frac{\dot{R}_t}{R_t} = -\frac{(\sigma-1)\iota^*\ln\lambda+\rho}{1-\alpha+\sigma\alpha} \equiv -\gamma_R^* < 0, \tag{18b}$$

$$R_t = \gamma_R^* S_t, \tag{18c}$$

$$r^* = \frac{\sigma\gamma_R^*-\rho}{\sigma-1} > 0, \tag{18d}$$

$$\frac{\dot{Y}_t}{Y_t} = \iota^*\ln\lambda - \alpha\gamma_R^* \equiv \gamma_Y^*. \tag{18e}$$

(Proof) Solving (14) and (17) yields (18a). In the steady state, labor allocated in the intermediate goods sector is constant because $\dot{V}_t = 0$. Thus, $Y_t = e^Q\bar{A}{l_t}^{1-\alpha}{R_t}^{\alpha}$ implies $\dot{Y}_t/Y_t = \iota^*\ln\lambda + \alpha\dot{R}_t/R_t$. From (12), we have $\dot{R}_t/R_t = \dot{Y}_t/Y_t - \dot{p}_{R,t}/p_{R,t}$. Solving these two equations, along with (4), yields (18b), (18d), and (18e). The inequality in (18d) holds because $\sigma\gamma_R^* \geq \frac{\sigma\rho}{1-\alpha+\sigma\alpha} > \rho$. We obtain (18c) directly from (2). In equilibrium, we have $a_t = v_t$ and hence $\dot{a}_t/a_t = \dot{c}_t/c_t = (r_t-\rho)/\sigma$. Then, it is easily shown that the transversality condition, $\lim_{T\to\infty} a_T {c_T}^{-\sigma}e^{-\rho T} = 0$, is satisfied. □

From (18a), we know that firms conduct R&D activities only when the patent breadth is sufficient, such that $\beta > \underline{\beta}(\alpha)$ holds. Then, $\underline{\beta}(\alpha)$ represents the lower bound of the patent breadth that ensures positive levels of R&D activities. If $\underline{\beta}(\alpha) \geq \lambda$, we have $\iota^* = 0$ for any $\beta \in [1, \lambda]$. To avoid this, we assume $\ln\lambda > \rho b/L$, which ensures $\underline{\beta}(\alpha) < \lambda$ because $\underline{\beta}(\alpha)$ is a decreasing function that satisfies $\underline{\beta}(\alpha) \leq \underline{\beta}(0) = 1 + \rho b/L < 1 + \ln\lambda < \lambda$. The last inequality holds because $\lambda > 1$ and $\ln\lambda$ is a concave function of λ with slopes less than one for $\lambda > 1$. In addition, the assumption $\ln\lambda > \rho b/L$ ensures that the socially optimal output growth rate, $\gamma_{Y,opt}$, is strictly positive, as we will see in Section 5.

Before closing this section, we examine the effects of β on R&D activities, allocation of exhaustible resources, and the output growth rate. Because a marginal change in β does not affect ι^* if $\iota^* = 0$, we assume $\beta > \underline{\beta}(\alpha)$. Differentiating ι^* with respect to β yields

$$\text{sign}\left\{\frac{\partial\iota^*}{\partial\beta}\right\} = \text{sign}\left\{\left(\alpha + \frac{1-\alpha}{\sigma} - \frac{1-\sigma}{\sigma}\ln\lambda\right)\frac{L}{b} + \frac{\rho}{\sigma}\right\} > 0.$$

Because a large β implies a large π_t, firms have a strong incentive to engage in R&D activities. Thus, stronger IPR protection stimulates R&D activities. Because $\sigma > 1$, when IPR protection is strengthened, the resource input decreases at a higher rate and the extraction rate of the resource, R_t/S_t, increases (see (18b) and (18c)). The intuition behind this result is simple. Stronger IPR protection increases the market value of a new invention, v_t. From (9), because labor demand of R&D firms increases, the wage rate increases relative to the resource price. Then, the demand for the resource relative to labor in the intermediate goods sector rises. Consequently, the extraction rate, R_t/S_t, increases with IPR protection and the resource input decreases at a higher rate. Remember that the price of the resource increases at the rate of interest (see (1)). Because the resource gets scarcer at a higher rate when IPR protection is stronger, the price of the resource also rises at a higher rate.

From (18e), we know that stronger IPR protection has two opposing effects on output growth. An increase in β accelerates technical progress, thereby affecting output growth positively. At the same time, stronger IPR protection increases the extraction of exhaustible resources, which depresses growth. In the present model, the former effect always dominates the latter as shown in the next equation:

$$\frac{\partial \gamma_Y^*}{\partial \beta} = \left\{1 - \frac{\alpha(\sigma - 1)}{1 - \alpha + \sigma\alpha}\right\}(\ln \lambda)\frac{\partial \iota^*}{\partial \beta} = \frac{\ln \lambda}{1 - \alpha + \sigma\alpha}\frac{\partial \iota^*}{\partial \beta} > 0.$$

The next proposition has been proved.

Proposition 2
Suppose $\beta > \underline{\beta}(\alpha)$. Strengthening IPR protection stimulates R&D activities, raises the extraction rate of exhaustible resources and the growth rate of resource price, and promotes output growth.

4 Patent Breadth and Growth

This section examines whether patent policies can sustain perpetual growth. Stiglitz (1974) and others show that as the resource intensity of goods production becomes high, the growth rate of technology must be high in order to sustain positive growth. Because R&D intensities increase with β in our model, one may conjecture that as the resource intensity of goods production becomes higher, stronger IPR protection must be required to sustain positive growth. This section examines the validity of this conjecture.

From (18e), we know that positive output growth is possible if and only if $\iota^* \ln \lambda > \alpha\gamma_R^*$ holds. If we use (18b), this inequality condition is rewritten as $\iota^* \ln \lambda > \alpha\rho$. Stiglitz (1974) showed that positive output growth is achieved if the exogenous growth rate of production technology is sufficiently large relative to the resource intensity in production. Then, the condition, $\iota^* \ln \lambda > \alpha\rho$, is analogous to that of Stiglitz (1974). Unlike Stiglitz (1974), the growth rate of technology, $\iota^* \ln \lambda$, is endogenous in our model.

If exhaustible resources are not needed in production ($\alpha = 0$), as long as R&D intensity is strictly positive ($\iota^* > 0$), positive output growth can be attained. In contrast, if exhaustible resources are essential in production ($\alpha > 0$), a strictly positive level of R&D activities does not necessarily sustain positive output growth. This is because the increasing scarcity of exhaustible resources negatively affects output growth, which is represented by the second term in (18e). To overcome rising resource scarcity, more rapid technological improvement is needed to sustain positive growth.

Solving $\iota^* \ln\lambda > \alpha\rho$ by using (18a), we know that perpetual output growth is possible if and only if IPR protection is sufficiently strong, such that β is large enough to satisfy

$$\beta > 1 + (1-\alpha)\rho \frac{\ln\lambda + \alpha}{\frac{L}{b}\ln\lambda - \alpha\rho} \equiv \hat{\beta}(\alpha) \left(\geq \underline{\beta}(\alpha)\right).$$

The inequality $\hat{\beta}(\alpha) \geq \underline{\beta}(\alpha)$ holds because $\beta > \hat{\beta}(\alpha)$ implies that $\iota^* > 0$. Because $\hat{\beta}(\alpha)$ depends on α, the patent policy that sustains perpetual output growth is affected by the resource intensity in goods production. When $\hat{\beta}(\alpha) < \lambda$, perpetual growth can be sustained if the government chooses β such that $\beta \in (\hat{\beta}(\alpha), \lambda]$. However, if $\hat{\beta}(\alpha) \geq \lambda$, even the strictest IPR protection, $\beta = \lambda$, cannot sustain perpetual growth. The magnitude relation between $\hat{\beta}(\alpha)$ and λ is crucial. Before examining this relation, we examine the properties of $\hat{\beta}(\alpha)$.

We can see that $\hat{\beta}(0) = \underline{\beta}(0) = 1 + \rho b/L (< 1 + \ln\lambda < \lambda)$ and $\hat{\beta}(1) = \underline{\beta}(1) = 1$, which implies $\hat{\beta}(0) > \hat{\beta}(1)$. The inequality $\hat{\beta}(0) > \hat{\beta}(1)$, together with the continuity of $\hat{\beta}(\alpha)$, implies that if the resource intensity in goods production is sufficiently low, relatively strong IPR protection is needed for perpetual growth; however, relatively weak IPR protection is sufficient for perpetual growth if goods production is sufficiently resource-intensive.

To obtain further results, we differentiate $\hat{\beta}(\alpha)$ with respect to α, which yields sign $\hat{\beta}'(\alpha) =$ sign $\Phi(\alpha)$, where $\Phi(\alpha) \equiv -\left[\frac{L}{b}\ln\lambda - \alpha\rho\right][\ln\lambda - 1 + 2\alpha] + (1-\alpha)\rho\ln\lambda + \alpha(1-\alpha)\rho$ has the following properties:

$$\Phi'(\alpha) = -2\rho\frac{L}{b}\left[\ln\lambda - (1-\alpha)\frac{\rho b}{L}\right] < 0, \tag{19a}$$

$$\Phi(0) = -\frac{L}{b}\ln\lambda[\ln\lambda - 1] + \rho\ln\lambda = -\frac{L}{b}\ln\lambda\left(\ln\lambda - 1 - \frac{\rho b}{L}\right), \tag{19b}$$

$$\Phi(1) = -\frac{L}{b}\left[\ln\lambda - \frac{\rho b}{L}\right][\ln\lambda + 1] < 0. \tag{19c}$$

The inequalities in (19a) and (19c) hold because $\ln\lambda > \rho b/L$. If $\ln\lambda < 1 + \rho b/L$ holds, $\Phi(0)$ is positive; therefore, there exists a unique $\hat{\alpha} \in (0,1)$ such that $\Phi(\hat{\alpha}) > (=)(<) 0$ holds if $\alpha < (=)(>) \hat{\alpha}$. On the other hand, when $\ln\lambda > 1 + \rho b/L$ is satisfied, $\Phi(0)$ takes a negative value. Hence, $\hat{\beta}(\alpha)$ is a decreasing function of α. The next proposition is thus derived.

Proposition 3
Suppose $\ln\lambda > \rho b/L$. *Then, if the resource intensity in goods production is sufficiently low, relatively strong IPR protection is needed for perpetual growth. However, if production is sufficiently resource-intensive, relatively weak IPR protection is sufficient for perpetual growth. In addition, the following results are obtained:*
(i) Suppose $\rho b/L < \ln\lambda < 1 + \rho b/L$. *If* $\alpha < (>) \hat{\alpha}$ *holds, as production becomes more resource-intensive, the degree of IPR protection that is required to sustain perpetual output growth increases (decreases).*
(ii) Suppose $\ln\lambda > 1 + \rho b/L$. *As production becomes more resource-intensive, the degree of IPR protection that is required to sustain perpetual output growth decreases.*

To understand the intuition behind Proposition 3, we note that changes in α have two opposing effects on output growth. First, increases in α stimulate R&D activities, which affects output growth positively. If the resource intensity of goods production is relatively high, labor demand in the intermediate goods sector is relatively low, leading to the wage rate, w_t, becoming relatively low. Because the cost of R&D activities, w_t, is relatively low, firms tend to conduct R&D activities intensively even when IPR protection is relatively weak. Second, increases in α affect output growth adversely, because the negative growth effects due to the increasing scarcity of exhaustible resources become stronger. When goods production is more resource-intensive, the first effect tends to dominate the second, as compared to the case where production is less resource-intensive. Thus, when goods production is sufficiently resource-intensive, even relatively weak IPR protection can sustain perpetual growth. Because a large λ implies rapid quality improvement given ι^*, the positive growth effect tends to increase with λ. When λ is large enough to satisfy $\ln \lambda > 1 + \rho b/L$, the positive effect always dominates the negative one. Thus, the patent strength that sustains perpetual output growth, $\hat{\beta}(\alpha)$, decreases as the resource intensity in goods production, α, increases. However, if λ is not as large, such that $\ln \lambda < 1 + \rho b/L$ is satisfied, the negative effect dominates the positive one for a small α, whereas the positive effect is dominant when α is sufficiently large. Therefore, $\hat{\beta}(\alpha)$ exhibits a hump-shaped pattern with respect to α.

We now know the properties of $\hat{\beta}(\alpha)$. Then, examining the magnitude relation between $\hat{\beta}(\alpha)$ and λ, we know whether a patent policy can sustain perpetual growth, as proved in the next proposition.

Proposition 4
(i) Suppose $\ln \lambda > 1+\rho b/L$, *or* $\rho b/L < \ln \lambda < 1+\rho b/L$ *and* $2 < e^{\rho b/L} - \rho b/L$. *Sufficiently strong IPR protection can then sustain perpetual growth for any* $\alpha \in [0, 1]$.
(ii) Suppose $\rho b/L < \ln \lambda < 1 + \rho b/L$ *and* $2 > e^{\rho b/L} - \rho b/L$. *(a) If* λ *is large enough, sufficiently strong IPR protection can sustain perpetual growth for any* $\alpha \in [0, 1]$. *(b) If* λ *is small enough, for* α *sufficiently close to* $\hat{\alpha} \in (0, 1)$, *even the strictest IPR protection cannot sustain perpetual growth. If* α *is sufficiently close to either zero or one, sufficiently strong IPR protection can sustain perpetual growth.*

(Proof) When λ is large enough to satisfy $\ln \lambda > 1 + \rho b/L$, we have $\hat{\beta}(\alpha) \leq \hat{\beta}(0) < \lambda$. Then, perpetual growth can be sustained by sufficiently strong IPR protection. Next, consider $\rho b/L < \ln \lambda < 1 + \rho b/L$. In this case, it is shown that $\hat{\alpha}$ decreases with λ.[11] In addition, using $\Phi(\hat{\alpha}) = 0$, we obtain $\lim_{\ln \lambda \to \rho b/L} \hat{\alpha} = 1$ and $\lim_{\ln \lambda \to 1+\rho b/L} \hat{\alpha} = 0$. Then, as $\ln \lambda$ increases from $\rho b/L$ to $1 + \rho b/L$, $\hat{\alpha}$ monotonically decreases from one to zero. If we use $\Phi(\hat{\alpha}) = 0$, $\hat{\beta}(\hat{\alpha})$ can be written as $\hat{\beta}(\hat{\alpha}) = 2\hat{\alpha} + \ln \lambda$. Note that $\hat{\beta}(\alpha)$ is maximized at $\alpha = \hat{\alpha}$. Then, if $2\hat{\alpha} < \lambda - \ln \lambda$, we have $\hat{\beta}(\alpha) < \lambda$ for all $\alpha \in [0, 1]$. This implies that perpetual growth can be sustained by IPR protection for all $\alpha \in [0, 1]$. However, if $2\hat{\alpha} > \lambda - \ln \lambda$, we have $\hat{\beta}(\hat{\alpha}) > \lambda$. Then, for α sufficiently close to $\hat{\alpha}$, even the strictest IPR protection cannot sustain perpetual growth. As $\ln \lambda$ increases from $\rho b/L$ to $1 + \rho b/L$, $2\hat{\alpha}$ monotonically decreases from 2 to 0, whereas $\lambda - \ln \lambda$ increases from $e^{\rho b/L} - \rho b/L$ to $e^{1+\rho b/L} - (1+\rho b/L)$. Then, if $2 < e^{\rho b/L} - \rho b/L$, we have $\hat{\beta}(\alpha) < \lambda$ for all $\alpha \in [0, 1]$. Otherwise, there exists a unique $\tilde{\lambda}$ such that $\hat{\beta}(\hat{\alpha}) > (<)\lambda$ holds for $\lambda < (>)\tilde{\lambda}$. Then, if $\lambda > \tilde{\lambda}$, perpetual growth can be sustained by sufficiently strong IPR protection. In contrast, if $\lambda < \tilde{\lambda}$, as long as α is close to $\hat{\alpha}$, even the strictest IPR protection cannot sustain perpetual growth. However, because we have $\hat{\beta}(0) = 1 + \rho b/L < 1 + \ln \lambda < \lambda$

[11] Rearranging $\Phi(\hat{\alpha}) = 0$, we know that $\hat{\alpha}$ satisfies $\hat{\beta}(\hat{\alpha}) = 2\hat{\alpha} + \ln \lambda$. Totally differentiating this equation yields $\hat{\beta}'(\hat{\alpha})d\hat{\alpha} - \frac{(1-\alpha)\alpha\rho\left(\rho+\frac{L}{b}\right)}{[\frac{L}{b}\ln \lambda - \alpha\rho]^2} d\ln \lambda = d\ln \lambda + 2d\hat{\alpha}$. Because of $\hat{\beta}'(\hat{\alpha}) = 0$, the above equation implies $d\hat{\alpha}/d\ln \lambda < 0$.

and $\hat{\beta}(1) = 1 < \lambda$, sufficiently strong IPR protection can then sustain perpetual growth when α is sufficiently close to either zero or one. $\square$

Proposition 4 suggests that when the resource intensity in goods production is medium in level, even the strictest IPR protection cannot sustain perpetual growth if λ is relatively small. However, if the resource intensity is sufficiently high or sufficiently low, perpetual growth can be sustained by IPR protection, even when λ is relatively small. Figure 2 presents a numerical example of the range of α for which even the strictest IPR protection cannot sustain perpetual growth.[12] As we analytically show in Proposition 4, if α is of medium magnitude, perpetual growth cannot be realized even under the strictest IPR protection when λ is small.

[Figure 2]

5 Welfare

This section derives the patent policy that maximizes social welfare and examine how this optimal patent policy is affected by the presence of exhaustible resources. It is rather easy to examine the welfare effects of patent policy in the present model, because there are no transitional dynamics and the economy always stays in the steady state equilibrium. For simplicity, we normalize the quality of the initial generation of any product to one, that is, $q_{j,0} = 1$. Then, we have $Y_t = Y_0 e^{\gamma_Y^* t} = (\bar{A} {l^*}^{1-\alpha} R_0^{\alpha}) e^{\gamma_Y^* t}$, where $l^* = L - b\iota^*$ represents labor allocated to the production of intermediate goods. Because $c_t = Y_t$ holds in equilibrium, social welfare is expressed as

$$U_0 = \frac{(\bar{A} {l^*}^{1-\alpha} R_0^{\alpha})^{1-\sigma}}{1-\sigma} \frac{1}{\rho + (\sigma - 1)\gamma_Y^*} = \frac{\bar{A}^{1-\sigma} S_0}{1-\sigma} \frac{l^{*(1-\alpha)(1-\sigma)}}{R_0^{1-\alpha+\sigma\alpha}}, \tag{20}$$

where R_0 is obtained by substituting $t = 0$ into (18c).[13] In the second equality of (20), we use (18d), (18e), and (18c). Because $\sigma > 1$ is assumed, we have $U_0 < 0$. The above equation shows that the patent policy, β, affects social welfare through its effects on the allocation of labor and exhaustible resources. Because $1 - \sigma < 0$ and $1 - \alpha + \sigma\alpha > 0$, increases in l^* and R_0 have positive effects on social welfare.

To derive the socially optimal level of R&D activities, we first differentiate U with respect to ι^* by using (18c) and $l^* = L - b\iota^*$. We then obtain $\partial U_\tau / \partial \iota^* = Z(\iota^*)(1-\sigma)U_0$, where

$$Z(\iota^*) = \frac{1 - \alpha + \sigma\alpha}{(\sigma - 1)\iota^* \ln\lambda + \rho} \ln\lambda - \frac{(1-\alpha)b}{L - b\iota^*}.$$

Because $(1 - \sigma)U_0$ is positive, $\partial U_0 / \partial \iota^*$ has the same sign as $Z(\iota^*)$. It is clear that we have $Z'(\iota^*) < 0$. The second derivative of U_0 has a negative sign: $\partial^2 U_0 / \partial {\iota^*}^2 = Z(\iota^*)^2 (1-\sigma)^2 U_0 + Z'(\iota^*)(1-\sigma)U_0 < 0$. The last inequality holds because $U_0 < 0$ and $Z'(\iota^*) < 0$. Then, social welfare is maximized if $Z(\iota^*) = 0$. We denote the socially optimal level of R&D activities as ι_{opt}. Solving $Z(\iota^*) = 0$ yields

$$\iota_{opt} = \left[1 + \frac{(1-\sigma)(1-\alpha)}{\sigma}\right] \frac{L}{b} - \frac{(1-\alpha)\rho}{\sigma \ln\lambda}.$$

[12] In Figure 2, we assume that $\rho = 0.05$, $b = 8$, and $L = 1$. Under these parameter values, we have $e^{\rho b/L} - \rho b/L = 1.0918 < 2$.

[13] Note that U_0 is bounded because $\rho + (\sigma - 1)\gamma_Y^* = \{(\sigma - 1)\iota^* \ln\lambda + \rho\}/(1 - \alpha + \alpha\sigma) > 0$.

It is clear that $\partial\iota_{opt}/\partial\alpha > 0$. The assumption $\ln\lambda > \rho b/L$ ensures $\iota_{opt}|_{\alpha=0} = (L\ln\lambda - \rho b)/(\sigma b\ln\lambda) > 0$, which implies $\iota_{opt} > 0$ for all $\alpha \in [0,1]$. Substituting ι_{opt} into (18e) yields the optimal output growth rate as follows: $\gamma_{Y,opt} = \frac{\alpha+\frac{1-\alpha}{\sigma}}{1-\alpha+\alpha\sigma}\left(\frac{L\ln\lambda}{b} - \rho\right) > 0$. The assumption, $\ln\lambda > \rho b/L$, ensures that the optimal output growth rate is strictly positive.

We are now in a position to derive the socially optimal level of IPR protection, β_{opt}. Because the assumption $\ln\lambda > \rho b/L$ ensures $\gamma_{Y,opt} > 0$ for all $\alpha \in [0,1]$, β_{opt} must be larger than $\hat{\beta}(\alpha)$. If there exists a $\beta \in [\hat{\beta}(\alpha), \lambda]$ such that $\iota^* = \iota_{opt}(> 0)$, we can find β_{opt} by solving $\iota^* = \iota_{opt}$ for β. However, if we have $\iota_{opt} > \iota^*$ for all $\beta \in [\hat{\beta}(\alpha), \lambda]$, we have $\beta_{opt} = \lambda$. The discussion so far yields $\beta_{opt} = \min\left\{\lambda,\ \tilde{\beta}_{opt}(\alpha)\right\}$, where

$$\tilde{\beta}_{opt}(\alpha) \equiv \alpha + \ln\lambda + \frac{\sigma L\ln\lambda}{(\sigma-1)L\ln\lambda + \rho b}.$$

Thus, $\tilde{\beta}_{opt}(\alpha)$ increases in α. The socially optimal IPR protection depends on the resource intensity and we have $\partial\beta_{opt}/\partial\alpha \geq 0$. In particular, if $\lambda > 1$ is small enough to satisfy $\lambda > \tilde{\beta}_{opt}(0) \equiv \ln\lambda + \frac{\sigma L\ln\lambda}{(\sigma-1)L\ln\lambda+\rho b}$, there exists a unique $\overline{\alpha} \in (0,1]$, such that we have $\beta_{opt} = \tilde{\beta}_{opt}(\alpha)$ and $\partial\beta_{opt}/\partial\alpha > 0$ for $\alpha < \overline{\alpha}$. The next proposition is derived.

Proposition 5
Suppose $\ln\lambda > \rho b/L$. *The socially optimal level of IPR protection tends to increase as production becomes more resource-intensive.*

The intuition behind Proposition 5 is simple. High resource intensity implies that utility depends largely on the resource input via the production processes of consumption goods. Then, social optimality requires a high extraction rate of the resource, R_t/S_t. Proposition 2 shows that the extraction rate of the resource increases with β. Therefore, the socially optimal level of IPR protection tends to be higher as production becomes more resource-intensive.

6 R&D Subsidies

We have shown that when the resource intensity in goods production is medium in level, even the strictest IPR protection cannot sustain perpetual growth if the quality increment brought about by innovations, λ, is relatively small. This section considers whether other policies complement or replace IPR protection. Can these policies sustain perpetual output growth for all parameter values?

To examine these questions, we now shed light on an R&D subsidy, which corresponds to a subsidy on labor input for R&D activities in our model.[14] Thus, the expected profits of an R&D firm are as follows: $v_t\iota_t dt - w_t b(1-s)b\iota_t dt$, where $s \in [0,1)$ is the subsidy rate. We suppose that a lump-sum tax on households finances this subsidy. As in the benchmark model, the economy is always on the unique steady state. In the presence of R&D subsidies, the R&D intensity in the steady state is given by

$$\iota_s^* = \begin{cases} 0, & \text{if} \quad \beta \leq \underline{\beta}_s(\alpha,s), \\ \frac{\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)\frac{\beta-1}{b}L-(1-s)\frac{\rho}{\sigma}}{\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)(\beta-1)+(1-s)(\alpha+\frac{1-\alpha}{\sigma}-\frac{1-\sigma}{\sigma}\ln\lambda)} > 0, & \text{if} \quad \beta > \underline{\beta}_s(\alpha,s), \end{cases} \tag{21}$$

[14]We retain the notations and assumptions made in the benchmark model.

where $\underline{\beta}_s(\alpha, s) \equiv 1 + \frac{\rho/\sigma}{1/\sigma+\alpha/(1-\alpha)} \frac{(1-s)b}{L}$ is the lower bound of the patent breadth for positive R&D activities. Here, we use the subscript s to indicate the presence of R&D subsidies. Replacing ι^* by ι_s^* in (18b)–(18e), we can obtain the steady state variables, r_s^*, γ_{Rs}^*, and γ_{Ys}^*, and the extraction rate R_t/S_t. By differentiating ι_s^* and $\underline{\beta}_s(\alpha, s)$ with respect to s, we obtain $\partial \iota_s^*/\partial s > 0$ and $\partial \underline{\beta}_s/\partial s < 0$. Then, the R&D subsidy complements IPR protection by stimulating R&D activities. However, (21) shows that in the absence of IPR protection ($\beta = 1$), R&D activities do not occur for any $s \in (0, 1)$. Therefore, the subsidy must be used along with the IPR protection policy to stimulate R&D activities.

We can easily show that in the presence of the R&D subsidy, the long-run growth rate, γ_{Ys}^*, is strictly positive if and only if

$$\beta > 1 + (1-\alpha)\rho(1-s)\frac{\ln\lambda + \alpha}{\frac{L}{b}\ln\lambda - \alpha\rho} \equiv \hat{\beta}_s(\alpha, s).$$

Apparently, we have $\hat{\beta}_s(\alpha, 0) = \hat{\beta}(\alpha)$. The assumption $\ln\lambda > \rho b/L$ ensures $\partial\hat{\beta}_s(\alpha, s)/\partial s < 0$. In addition, we have $\lim_{s\to 1}\hat{\beta}_s(\alpha, s) = 1 < \lambda$. Then, for any λ that satisfies $\ln\lambda > \rho b/L$, there exists a unique $\hat{s} \in [0, 1)$ such that we have $\hat{\beta}_s(\alpha, s) < \lambda$ for $s \in (\hat{s}, 1)$. We can conclude that if s and β satisfy $s \in (\hat{s}, 1)$ and $\beta \in (\hat{\beta}_s(\alpha, s), \lambda]$, perpetual growth can be sustained even if λ is relatively small and there is a medium level of resource intensity in goods production.

7 R&D Activities that Need Exhaustible Resource

Our results such that "relatively strong IPR protection is needed for perpetual growth when the resource intensity of goods production is sufficiently low" may depend on our assumption that the only input in R&D activities is labor. In this section, we modify our model so that the exhaustible resource is needed for R&D activities. For our purposes, we assume that the final good is used as an input in the R&D sector. Then, the labor market equilibrium conditions, (11), is replaced by $L = (1-\alpha)Y_t/(\beta w_t)$. The output of the final good sector is written as $Y_t = e^{Q_t}\bar{A}L^{1-\alpha}R_t^{\alpha}$ where $e^{Q_t} \equiv e^{\int_0^1 \ln q_{j,t} dj}$. Because the exhaustible resource is used in goods production, the R&D activities requires the exhaustible resource. Here, we retain the notations used in the benchmark model, wherever possible.

To conduct R&D activities with the intensity ι_t, an R&D firm must devote $bX_t\iota_t dt$ units of the final for a time interval of dt, where $X_t > 0$ is the R&D difficulty. Then, the final good market equilibrium condition and the free entry condition of the R&D sector, (9), are, respectively, given by

$$Y_t = c_t + bX_t\iota_t, \tag{22}$$

$$v_t \leq (1-s)bX_t, \quad \text{with equality if } \iota_t > 0, \tag{23}$$

where $s \in [0, 1)$ is the R&D subsidy rate. As in Segerstrom (1998), X_t evolves according to

$$\dot{X}_t = \mu X_t, \quad \mu > 0. \tag{24}$$

We first consider the economy where the exhaustible resource is not essential for goods production and R&D activities, $\alpha = 0$. Suppose $ln\lambda = \mu$ and the subsidy rate, s, is sufficiently close to zero. The condition $ln\lambda = \mu$ ensures the existence of the steady state where ι_t is constant over time. In Appendix, we show that the positive output growth is possible, if the next inequality is satisfied:

$$\left(1 - \frac{1}{\beta}\right) \frac{e^{Q_0} L / X_0}{(1-s)b} > \rho. \tag{25}$$

The economy is always on the steady state equilibrium, where the R&D intensity is given by

$$\iota^{**} = \frac{\left(1 - \frac{1}{\beta}\right) \frac{e^{Q_0} L / X_0}{(1-s)b} - \rho}{1 + (\sigma - 1)\mu} > 0. \tag{26}$$

The output growth rate is equal to $\mu\iota^{**} > 0$.

We finally consider the economy where the exhaustible resource is not essential for goods production and R&D activities, $\alpha > 0$. Appendix shows that only when all of the following three conditions are satisfied, there exists an equilibrium where perpetual growth is possible:

$$1 > 1/\beta > s(\geq 0), \tag{27}$$

$$\ln\lambda - \mu > (\sigma - 1)\alpha\mu (> 0), \tag{28}$$

$$S_0 = \frac{\ln\lambda - [1 + (\sigma - 1)\alpha]\mu}{\alpha\rho(\ln\lambda - \mu)} \left\{ \frac{(1-s)\rho b}{1 - 1/\beta} \frac{\ln\lambda - \mu + \alpha}{\ln\lambda - [1 + (\sigma - 1)\alpha]\mu} \frac{X_0}{e^{Q_0} L^{1-\alpha}} \right\}^{\frac{1}{\alpha}}. \tag{29}$$

In the equilibrium where perpetual growth is possible, the R&D intensity is given by

$$\iota_t = \frac{\alpha\rho}{\ln\lambda - [1 + (\sigma - 1)\alpha]\mu} \equiv \breve{\iota} > 0, \tag{30}$$

and the output growth rate is $\mu\breve{\iota} > 0$. The growth rate is independent of labor supply L. Hence, the scale effects are absent. More importantly, β and s have no effects on $\mu\breve{\iota}$.

If any of the above conditions are not satisfied, no R&D activities are conducted in equilibrium. Because the resource scarcity cannot be overcome, the output growth rate becomes negative. Among the three conditions, the third condition, (29), is especially knife-edge. If the government sets β and s that do not satisfy (29), the economy ends up to contract. The policy maker has no room to control β and s freely. Hence, in the model where the exhaustible resource is essential for R&D activities, we cannot examine the relationship between the resource intensity and the IPR protection policy. What is more, the first condition, (27), implies that the right-hand side of (29) is strictly positive. Hence, if the initial stock of the exhaustible resource S_0 is sufficiently small, (29) can never be satisfied and perpetual growth cannot be realized in the long-run whatever β and s the government chooses.

8 Conclusion

We constructed an endogenous growth model with vertical innovations and investigated whether IPR protection policies could sustain perpetual growth in the presence of exhaustible resources. We first consider the model where labor is the only input in the R&D activities. We find that R&D activities would not take place if IPR protection were sufficiently weak. The IPR protection required to sustain perpetual output growth is relatively weak (strong) for a more (less) resource-intensive economy because of smaller (larger) costs of R&D activities. When the quality improvements brought about by innovations are sufficiently small and there is a medium level of resource intensity in goods production, even the strictest IPR protections cannot sustain perpetual growth. We derive the socially optimal level of output growth and find that IPR protection should be stronger as an economy becomes more resource-intensive. If the exhaustible resource is essential for the R&D activities, perpetual growth cannot be realized in the long-run when the initial stock of the exhaustible resource is sufficiently small.

Appendix for Section 7

From (12) and $Y_t = e^{Q_t}\bar{A}L^{1-\alpha}R_t^{\alpha}$, we have $\dot{R}_t/R_t = \dot{Y}_t/Y_t - r_t$ and $\dot{Y}_t/Y_t = \iota_t \ln\lambda - \alpha\dot{R}_t/R_t$. Using these two equations, we obtain

$$\frac{\dot{Y}_t}{Y_t} = \frac{\iota_t \ln\lambda - \alpha r_t}{1-\alpha}, \text{ and } \frac{\dot{R}_t}{R_t} = \frac{\iota_t \ln\lambda - r_t}{1-\alpha}. \tag{31}$$

Define $\hat{c}_t \equiv c_t/Y_t$, $y_t \equiv Y_t/X_t$, and $z_t \equiv v_t/X_t \leq (1-s)b$. Note that we have $\iota_t = (>)0$ if $z_t < (=)(1-s)b$ from (23). We can derive the following equations.

$$\iota_t = (y_t - \hat{c}_t)/b, \tag{32}$$

$$\dot{\hat{c}}_t = (r_t - \rho - \sigma\mu\iota_t)\hat{c}_t/\sigma, \tag{33}$$

$$\dot{y}_t = [\ln\lambda - (1-\alpha)\mu]\iota_t - \alpha\iota_t)y_t/(1-\alpha), \tag{34}$$

$$\dot{z}_t = [r_t + (1-\mu)\iota_t]z_t - (1-1/\beta)\, y_t. \tag{35}$$

Further, it is shown that r_t satisfies

$$r_t = \begin{cases} \frac{(1-\alpha)\rho}{1+\alpha(\sigma-1)}, & \text{if } \ z_t < (1-s)b, \\ (\mu-1)\iota_t + (1-1/\beta)\frac{y_t}{(1-s)b}, & \text{if } \ z_t = (1-s)b, \end{cases} \tag{36}$$

When $z_t < (1-s)b$, we have $\iota_t = 0$ and $y_t = \hat{c}_t$. Then, we have $\dot{Y}_t/Y_t = -\alpha r_t/(1-\alpha) = (r_t - \rho)/\sigma = \dot{c}_t/c_t$, which yields the first line of (36). When $z_t = (1-s)b$, we have $\dot{z}_t = 0$. From (35), we obtain the second line of (36).

We now consider the economy where exhaustible resource is not needed for production and R&D activities and hence $\alpha = 0$ holds. We know from (32) that ι_t becomes constant when $\dot{Y}_t/Y_t = \dot{c}_t/c_t = \dot{X}_t/X_t$. To ensure $\dot{Y}_t/Y_t = \dot{X}_t/X_t$, we assume $\ln\lambda = \mu$. Then, y_t remains constant at $y_0 (= Y_t/X_t = e^{Q_0}L/X_0)$. If $z_t < (1-s)b$, we have $\dot{z}_t = \rho z_t - (1-1/\beta)\, y_0$ from (35), (36), and $\alpha = \iota_t = 0$. If (25) is satisfied, we have $\dot{z}_t = \rho z_t - (1-1/\beta)\, y_0 < 0$ for $z_t < (1-s)b$. Then, there exists no steady state in this case. If $z_t = (1-s)b$, using (32), (33), and (36), we obtain

$$\begin{aligned} \frac{\dot{\hat{c}}_t}{\hat{c}_t} &= \frac{1}{\sigma}\left\{[(1-\sigma)\mu - 1]\iota_t + \left(1 - \frac{1}{\beta}\right)\frac{y_0}{(1-s)b} - \rho\right\} \\ &= \frac{1}{\sigma}\left\{\frac{(\sigma-1)\mu + 1}{b}(\hat{c}_t - y_0) + \left(1 - \frac{1}{\beta}\right)\frac{y_0}{(1-s)b} - \rho\right\}. \end{aligned} \tag{37}$$

The second line of (37) monotonically increases with $\hat{c}_t$. If (25) is satisfied, the second line of (37) is positive at $\hat{c}_t = y_0$. After setting $s = 0$, we evaluate the second line of (37) at $\hat{c}_t = 0$.

$$\frac{1}{\sigma}\left\{-\frac{(\sigma-1)\mu + 1}{b}y_0 + \left(1 - \frac{1}{\beta}\right)\frac{y_0}{b} - \rho\right\} = -\frac{y_0}{b}\left[(\sigma-1)\mu + 1/\beta\right] - \rho < 0.$$

Then, if $s \in [0, 1)$ is sufficiently close to zero, $\dot{\hat{c}}_t = 0$ has a unique solution, $\hat{c}^{**}$, that satisfies $0 < \hat{c}^{**} < y_0$. When $\hat{c}_t = \hat{c}^{**}$, we have $\iota_t = \iota^{**} > 0$ from the first line of (37), where ι^{**} is defined by (26). In equilibrium, we have $a_t = v_t$ and $\dot{a}_t/a_t = \dot{c}_t/c_t = (r_t - \rho)/\sigma$. Then, it is easily shown that the transversality condition, $\lim_{T\to\infty} a_T {c_T}^{-\sigma} e^{-\rho T} = 0$, is satisfied in the steady state. There exists a unique steady state equilibrium. Because (37) has an upward slope, the steady state is unstable. Hence, the economy is always on the steady state equilibrium.

We next consider the case where exhaustible resource is essential for goods production and R&D activities. In this case, $0 < \alpha \leq 1$ holds. We need not assume $\ln \lambda = \mu$. If $z_t < (1 - s)b$, we have $\iota_t = 0$ and $\hat{c}_t = y_t$. From (33)–(36), the following differential equations are derived:

$$\dot{y}_t = -\frac{\alpha\rho}{1 + \alpha(\sigma - 1)} y_t, \tag{38}$$

$$\dot{z}_t = \frac{(1 - \alpha)\rho}{1 + \alpha(\sigma - 1)} z_t - \left(1 - \frac{1}{\beta}\right) y_t. \tag{39}$$

Here, we omit the differential equation for $\hat{c}_t$ because $\hat{c}_t = y_t$. z_t are jump variables. As we will see later, we have to treat y_t as a predetermined variable. Panel (a) of Figure 3 shows the dynamics of y_t and z_t. There exists a unique and stable steady state where $\hat{c}_t = y_t = z_t = 0$. We have to examine whether the transversality condition, $\lim_{T\to\infty} a_T {c_T}^{-\sigma} e^{-\rho T} = 0$, is satisfied along the transition to the steady state. Note that $\dot{a}_t/a_t - \sigma\dot{c}_t/c_t - \rho < 0$ ensures the transversality condition. Remember $\dot{c}_t/c_t = (r_t - \rho)/\sigma$. We define $Z_t \equiv y_t/z_t$. Note that a_t is equal to v_t in equilibrium and X_t is constant because $\iota_t = 0$. Then, we obtain $\dot{a}_t/a_t = \dot{v}_t/v_t = \dot{z}_t/z_t = r - \left(1 - \frac{1}{\beta}\right) Z_t$ using (36) and (39). Therefore, we have $\dot{a}_t/a_t - \sigma\dot{c}_t/c_t - \rho = -(1 - 1/\beta)Z_t$. From (38) and (39), we have $\dot{Z}_t = \left[(1 - 1/\beta) Z_t - \frac{\rho}{1+\alpha(\sigma-1)}\right] Z_t$. This equation tells us that along the transition to the steady state, Z_t remains constant at $\frac{\rho(1-1/\beta)^{-1}}{1+\alpha(\sigma-1)} > 0$. Then, we can conclude $\dot{a}_t/a_t - \sigma\dot{c}_t/c_t - \rho = -(1 - 1/\beta)Z_t < 0$; the transversality condition is satisfied. The path that converges to $(\hat{c}_t, y_t, z_t) = (0, 0, 0)$ is equilibrium. In this equilibrium, no R&D activities are conducted. The growth rate of per capita consumption is negative because of the increasing resource scarcity.[15] From the second equation of (31) and the first line of (36), we know that the resource input decreases at the rate of $-\rho/[1 + \alpha(\sigma - 1)](\equiv -\tilde{g}_R)$. Then, from (2), R_0 is uniquely determined: $R_0 = \tilde{g}_R S_0$. Therefore, the initial value, $y_0 (= e^{Q_0} L^{1-\alpha} {R_0}^{\alpha}/X_0)$, is also uniquely determined. We have to treat y_t as a predetermined variable.

[Figure 3]

If $z_t = (1 - s)b$, we can derive the following differential equations using (32), (33), (34), and (36):

$$\dot{\hat{c}}_t = \frac{\hat{c}_t}{\sigma} \left\{ [(1 - \sigma)\mu - 1]\iota_t + \left(1 - \frac{1}{\beta}\right) \frac{y_t}{(1 - s)b} - \rho \right\},$$

$$\dot{y}_t = \frac{y_t}{1 - \alpha} \left\{ (\ln \lambda - \mu + \alpha)\iota_t - \alpha \left(1 - \frac{1}{\beta}\right) \frac{y_t}{(1 - s)b} \right\},$$

[15] In this equilibrium, $\hat{c}_t = y_t$ holds. Then, from (38), the growth rate of per capita consumption is given by $-\alpha\rho/[1 + \alpha(\sigma - 1)] < 0$.

where ι_t is given by (32). The $\dot{\hat{c}}_t = 0$ and $\dot{y}_t = 0$ loci are, respectively, given by

$$\hat{c} = \frac{(\sigma - 1)\mu + \frac{1/\beta - s}{1-s}}{(\sigma - 1)\mu + 1} y + \frac{\rho b}{(\sigma - 1)\mu + 1}, \tag{40}$$

$$\hat{c} = \frac{\frac{\ln\lambda - \mu}{\alpha} + \frac{1/\beta - s}{1-s}}{\frac{\ln\lambda - \mu}{\alpha} + 1} y. \tag{41}$$

The intersection of these two loci is a candidate of the steady state equilibrium. In the steady state, $\hat{c}$ must be smaller than y for ι to be positive (see (32)). Then, the slope of (41) must be smaller than one, which is ensured by (27) and (28). If (41) has a steeper slope than (40), these two loci have a unique intersection, where both $\hat{c}$ and y are strictly positive (see point A in panel (b) of Figure 3) The inequalities, (27) and (28), ensure that (41) is steeper than (40). At point A, the R&D intensity and the output growth rate are given by (30) and $\dot{c}_t/c_t = \dot{Y}_t/Y_t = \mu\breve{\iota} \equiv \breve{g} > 0$, respectively. In addition, we have

$$y_t = \left(1 - \frac{1}{\beta}\right)^{-1} \frac{\rho(1-s)b[\ln\lambda - \mu + \alpha]}{\ln\lambda - [1 + (\sigma - 1)\alpha]\mu} \equiv \breve{y} > 0$$

$$\hat{c}_t = \frac{\frac{\ln\lambda - \mu}{\alpha} + \frac{1/\beta - s}{1-s}}{\frac{\ln\lambda - \mu}{\alpha} + 1} \breve{y} \equiv \breve{c} > 0,$$

$$r_t = \frac{\ln\lambda - (1-\alpha)\mu}{\alpha} \breve{\iota} \equiv \breve{r} > 0,$$

$$\frac{\dot{R}_t}{R_t} = -(\ln\lambda - \mu)\breve{\iota} \equiv -\breve{g}_R < 0.$$

The inequality, $\ln\lambda > [1 + (\sigma - 1)\alpha]\mu$, ensures $\hat{\iota} > 0$, $\hat{r} > 0$, and $\hat{g}_R > 0$.

To examine whether point A in panel (b) of Figure 3 is equilibrium, we have to consider the following two cases; (i) the economy is always on point A and (ii) the economy gradually converges to point A. We first consider the second case. As shown in panel (b) of Figure 3, there is no transitional path that converges to point A if $z_t = (1-s)b$. Then, if there is a transitional path that converges to point A, it must pass through the region where $z_t < (1-s)b$ holds. As discussed above, we have $\hat{c}_t = y_t$ if $z_t < (1-s)b$. In contrast, $\breve{c} < \breve{y}$ holds at point A because $\breve{\iota} > 0$. Then, any path that passes through the region where $z_t < (1-s)b$ cannot converges to point A. Therefore, the second case cannot happen in equilibrium.

We next consider the first case. Because the economy is always on point A, y_t is equal to $\breve{y}$ for all $t \geq 0$. Solving $\breve{y} = y_t (= e^{Q_t}\bar{A}L^{1-\alpha}R_t^{\alpha}/X_t)$ for R_t and substituting $t = 0$, we obtain

$$R_0 = \left\{ \frac{(1-s)\rho b}{1 - 1/\beta} \frac{\ln\lambda - \mu + \alpha}{\ln\lambda - [1 + (\sigma - 1)\alpha]\mu} \frac{X_0}{e^{Q_0}L^{1-\alpha}} \right\}^{\frac{1}{\alpha}}. \tag{42}$$

Then, we have $\int_0^\infty R_\tau d\tau = R_0/\breve{g}_R$. The condition, (29), is equivalent to $S_0 = R_0/\breve{g}_R$. Suppose that R_0 is large enough to satisfy $R_0/\breve{g}_R > S_0$. Then, if the economy is always on point A, the stock of resource becomes zero for a finite time period. This cannot happen in equilibrium. Conversely, suppose that R_0 is small enough to satisfy $R_0/\breve{g}_R < S_0$. Then, if the economy is always on point A, the equilibrium condition (2) is not satisfied. In other words, the transiversality condition, $\lim_{T\to\infty} S_T = 0$, is violated.

Then, point A cannot be equilibrium. In sum, if (29) is not satisfied, the path that converges to $(\hat{c}_t, y_t, z_t) = (0, 0, 0)$ is the only equilibrium. If (29) is satisfied, both point A and the path that converges to $(\hat{c}_t, y_t, z_t) = (0, 0, 0)$ are equilibria. Only when all of (27)–(29) are satisfied, perpetual growth may be possible.

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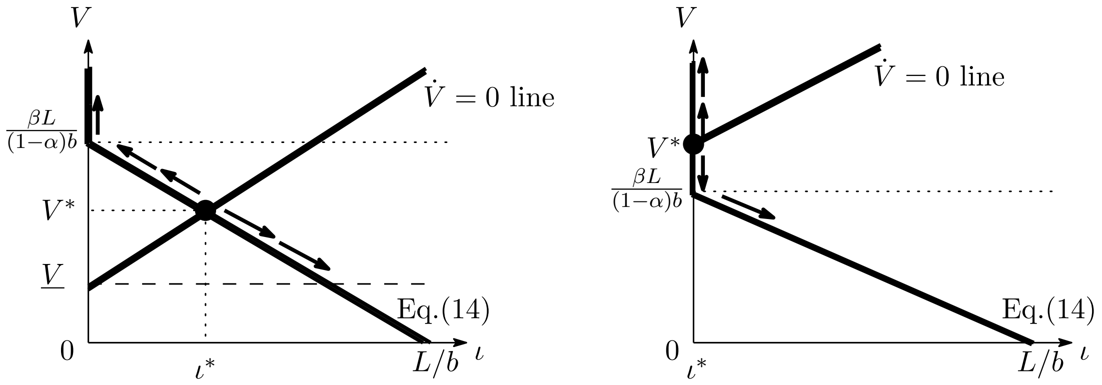


(a) $\beta > \underline{\beta}(\alpha)$ (b) $\beta \leq \underline{\beta}(\alpha)$

Figure 1: Phase Diagrams

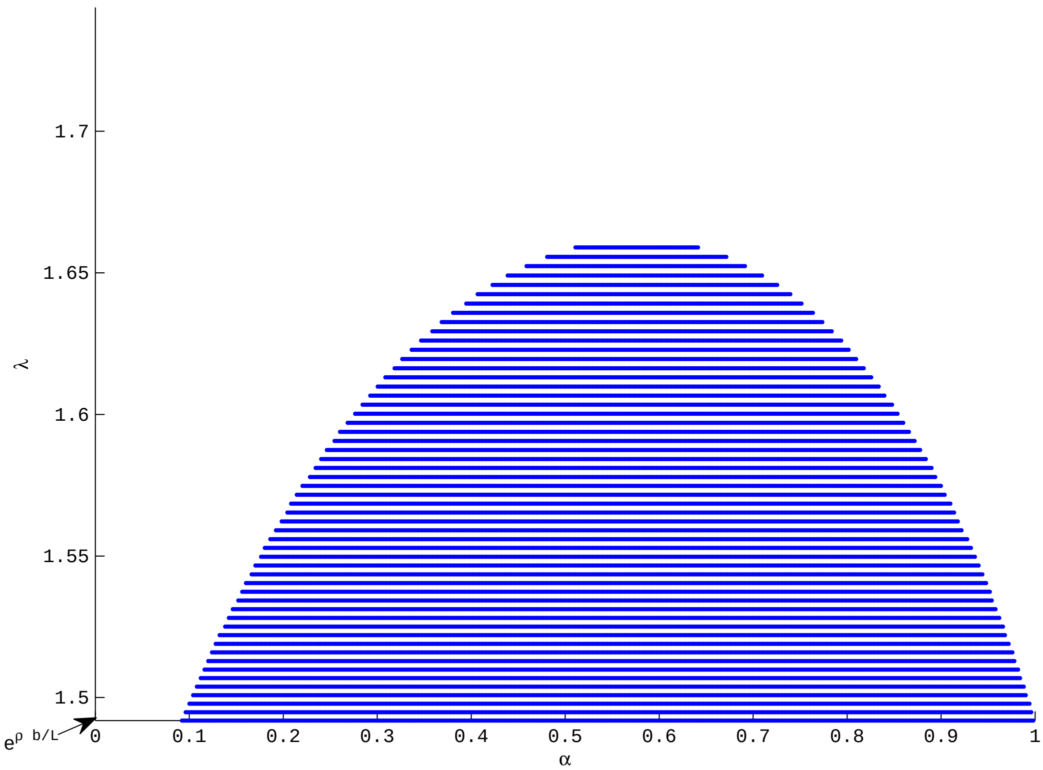
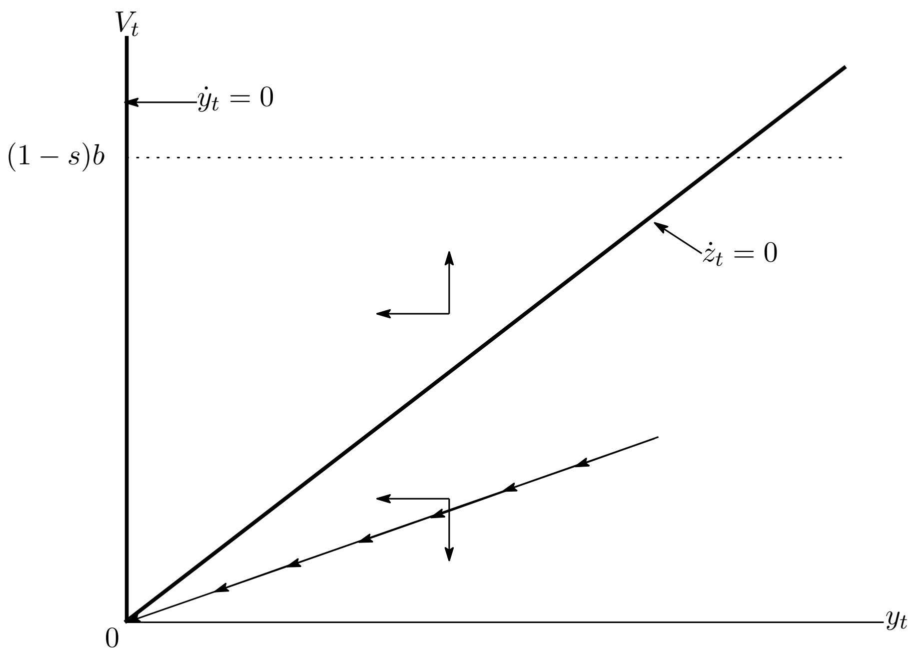


Figure 2: Perpetual Growth

(a) $z_t < (1-s)b$ ($\iota_t = 0$)



(b) $z_t = (1-s)b$ ($\iota_t > 0$)

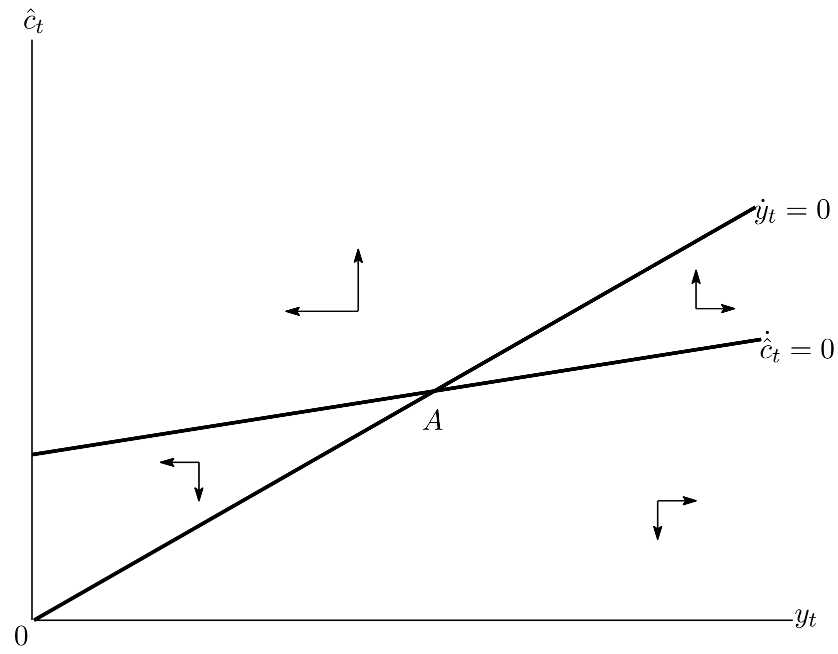


Figure 3: Dynamics when exhaustible resource is needed for R&D activities