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Abstract

This study explores the long-run effects of inflation in a two-country Schumpeterian growth model with cash-in-advance constraints on consumption and R&D investment. We find that increasing domestic inflation reduces domestic R&D investment and the growth rate of domestic technology. Given that economic growth in a country depends on both domestic and foreign technologies, increasing foreign inflation also affects the domestic economy. When each government conducts its monetary policy unilaterally to maximize the welfare of domestic households, the Nash-equilibrium inflation rates are generally higher than the optimal inflation rates chosen by cooperative governments who maximize the welfare of both domestic and foreign households. Under the CIA constraint on R&D (consumption), a larger market power of firms amplifies (mitigates) this inflationary bias. We use cross-country panel data to estimate the effects of inflation on R&D and also calibrate the two-country model to data in the Euro Area and the US to quantify the welfare effects of decreasing the inflation rates from the Nash equilibrium to the optimal level.

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1 Introduction

In this study, we explore the long-run effects of inflation on economic growth and social welfare in an open economy. Specifically, we develop a two-country version of the Schumpeterian growth model and introduce money demand into the model via a cash-in-advance (CIA) constraint on R&D investment in each country.[1] Empirical evidence supports the view that R&D investment is severely affected by liquidity requirements. For example, Hall (1992), Himmelberg and Petersen (1994), Opler *et al.* (1999) and Brown and Petersen (2009) find a positive and significant relationship between R&D and cash flows in US firms. According to Bates *et al.* (2009), the average cash-to-assets ratio in US firms increased substantially from 1980 to 2006, and this change is partly due to their increased R&D expenditures. Brown *et al.* (2009) provide empirical evidence that the increase in corporate cash flow in the 1990's drives the increase in R&D in that period. Recent studies by Brown and Petersen (2011) and Brown *et al.* (2012) explain this phenomenon by providing evidence that firms smooth R&D expenditures by maintaining a buffer stock of liquidity in the form of cash reserves. Furthermore, Brown and Petersen (2014) show that firms use cash reserves to finance R&D but not capital investment. Berentsen *et al.* (2012) argue that information frictions and limited collateral value of intangible R&D capital prevent firms from financing R&D investment through debt or equity forcing them to fund R&D projects with cash reserves. We capture these cash requirements on R&D using a CIA constraint. Given this CIA constraint on R&D, inflation that determines the opportunity cost of cash holdings affects R&D investment, economic growth and social welfare. In an open economy, monetary policy may also have spillover effects across countries through international trade.

The results from our growth-theoretic analysis confirm the above intuition and can be summarized as follows. An increase in the domestic inflation rate decreases domestic R&D investment and the growth rate of domestic technology. Given that economic growth in a country depends on both domestic and foreign technologies, an increase in the foreign inflation rate also affects the domestic economy. When each government conducts its monetary policy unilaterally to maximize the welfare of only domestic households, the Nash-equilibrium inflation rates are generally different from the optimal inflation rates chosen by cooperative governments who maximize the aggregate welfare of domestic and foreign households. Specifically, we find that under the special case of inelastic labor supply, the Nash-equilibrium inflation rates coincide with the optimal inflation rates. However, under the more general case of elastic labor supply, the Nash-equilibrium inflation rates become higher than the optimal inflation rates due to a cross-country spillover effect of monetary policy. The intuition of this result can be explained as follows. When the government in a country reduces the inflation rate, the welfare gain from increased R&D investment is shared by the other country through technology spillovers, whereas the welfare cost of increasing labor supply falls entirely on domestic households. As a result, the governments do not reduce inflation sufficiently in the Nash equilibrium.

The wedge between the Nash-equilibrium and optimal inflation rates depends on the market power of firms. Under the CIA constraint on consumption, a larger markup reduces this

[1] See also Chu and Cozzi (2014), who introduce a CIA constraint on R&D investment into a closed-economy version of the Schumpeterian growth model and analyze the effects of monetary policy.

wedge. This finding is consistent with the interesting insight of Arseneau (2007), who shows that the market power of firms has a dampening effect on the inflationary bias from monetary policy competition analyzed in an influential study by Cooley and Quadrini (2003). However, under the CIA constraint on R&D investment, we have the opposite result that a larger markup amplifies the inflationary bias from monetary policy competition. These different implications highlight the importance of the differences between the two CIA constraints. The main difference between the CIA constraint on consumption and the CIA constraint on R&D is that under the latter, an increase in the inflation rate leads to a reallocation of labor from R&D to production. As a result, higher inflation rates would be chosen by governments in the Nash equilibrium to depress R&D when the negative R&D externality in the form of a business-stealing effect determined by the markup becomes stronger. In contrast, under the CIA constraint on consumption, this reallocation effect is absent because an increase in the inflation rate reduces both R&D and production by decreasing labor supply. Given that increasing the markup worsens a monopolistic distortionary effect on the production of goods, governments would reduce inflation in the Nash equilibrium to stimulate production when this monopolistic distortion measured by the markup becomes stronger.

We use cross-country panel data to estimate the effects of inflation on R&D and find that there is a statistically significant negative relationship between the inflation rate and the R&D share of GDP. Our preferred regression estimate shows that the semi-elasticity of R&D with respect to inflation is -0.374 (i.e., a 1% increase in the inflation rate is associated with a decrease in the R&D share of GDP by 0.374 percent). We also calibrate the two-country model to aggregate data in the Euro Area and the US to simulate the quantitative effects of inflation on R&D. We find that the simulated semi-elasticities of R&D with respect to inflation are -0.448 for the Euro Area and -0.266 for the US. These values are in line with the regression estimate.

In the numerical analysis of the Nash equilibrium, we consider the case in which final goods are produced by a CES aggregate of domestic and foreign intermediate goods, which introduces an international business-stealing effect across countries. In other words, when a country decreases inflation to improve domestic technology, domestic firms are able to capture a larger share of the global market due to the substitutability of domestic and foreign intermediate goods. This effect represents a negative externality of monetary policy. Together with the positive externality from technology spillovers, we find that the Nash equilibrium continues to feature an inflationary bias. Therefore, we proceed to quantify the welfare effects of decreasing the inflation rates from the Nash equilibrium to the optimal level. We find that the Friedman rule is optimal (i.e., a zero nominal interest rate maximizes welfare). In this case, decreasing the inflation rates from the Nash equilibrium to achieve a zero nominal interest rate in both economies would lead to nonnegligible welfare gains that are equivalent to a permanent increase in consumption of 1.038% in the US and 0.249% in the Euro Area. However, a unilateral deviation to decrease the inflation rate from the Nash equilibrium would hurt the domestic economy and only benefit the foreign economy. For example, we find that a unilateral decrease in the inflation rate in the Euro Area would reduce its welfare by 0.213% but increase welfare in the US by 1.079%.

This study relates to the literature of inflation and economic growth, which explores the long-run effects of inflation on capital investment. Stockman (1981) and Abel (1985) provide the seminal studies of the CIA constraint on capital investment in the Neoclassical

growth model. Instead of analyzing the effects of monetary policy in the Neoclassical growth model, we consider an R&D-based growth model in which economic growth is driven by R&D investment, which has potentially more important implications on growth and welfare than capital investment. The seminal study in this literature of inflation and *innovation-driven* growth is Marquis and Reffett (1994), who explore the effects of a CIA constraint on consumption in a Romer variety-expanding model.[2] In contrast, we consider a Schumpeterian quality-ladder model and analyze the effects of monetary policy via a CIA constraint on R&D investment as in Chu and Cozzi (2014).[3] Huang *et al.* (2013) also analyze the effects of monetary policy via CIA constraints on R&D investment but in a Schumpeterian model with endogenous market structure. The present study differs from the closed-economy analyses in Chu and Cozzi (2014) and Huang *et al.* (2013) by considering a two-country setting with trade in intermediate goods across countries. This open-economy model allows us to explore the interesting implications of policy competition and coordination across countries, which are absent in a closed economy. To our knowledge, this is the first study that analyzes the effects of monetary policy in a growth-theoretic framework featuring R&D-driven innovation in an open economy.

This study also relates to the new open economy macroeconomics literature that explores monetary policy coordination and competition across countries in the presence of nominal rigidity; see for example Obstfeld and Rogoff (2002), Benigno and Benigno (2003), Corsetti and Pesenti (2005) and Bergin and Corsetti (2013). These studies analyze interesting channels, such as output gap stabilization, terms of trade improvement and production reallocation externality, and their implications on welfare gains from monetary policy coordination. The present study complements these influential studies by exploring the internalization of technology spillovers as an additional source of welfare gains from monetary policy coordination given that R&D investment is an important component of corporate investment that central banks pay attention to when conducting monetary policy.

The rest of this study is organized as follows. Section 2 documents stylized facts. Section 3 presents the model. Section 4 analyzes the effects of inflation. Section 5 concludes.

2 Stylized facts

In this section, we use cross-country panel data to estimate the effects of inflation on R&D. Our data set covers 34 OECD countries for the period 1960-2012 at yearly frequency. We collect data on R&D from Eurostat/UNESCO and data on inflation, population, GDP, imports and exports from the World Development Indicators. We also use the Ginarte-Park index of patent rights from Park (2008) and the Fraser index of economic freedom.[4] We measure the level of income by real PPP-adjusted GDP per capita and the degree of

[2] Chu, Lai and Liao (2013) provide an analysis of the CIA constraint on consumption in a hybrid growth model in which economic growth in the long run is driven by both variety expansion and capital accumulation.

[3] See Chu and Lai (2013) for an analysis of the money-in-utility approach to model money demand in the quality-ladder growth model.

[4] The Ginarte-Park index is available once every 5 years for each country. We interpolate the data series by assuming that any missing year takes on the same value as the previously available year. We also apply the same procedure to the Fraser index.

openness to trade by the sum of exports and imports as a share of GDP. Table 1 reports the summary statistics of these variables.

Table 1: Descriptive statistics

	Mean	Standard deviation	Min	Max
R&D/GDP (%)	1.8	0.9	0.3	4.8
Inflation (%)	10.3	29.1	-30.2	665.4
Income	22591.5	10021.2	2431.7	74012.5
Patent rights	3.5	0.8	1.4	4.9
Economic freedom	6.9	1.2	3.4	8.8
Population (millions)	30.3	47.3	0.3	313.9
Trade/GDP (%)	34.5	21.8	0.0	166.7
Observations		648		

Our theoretical model predicts a negative relationship between inflation and R&D. Our regression results are consistent with this theoretical implication. Table 2 reports the results from our panel regressions and shows a negative relationship between inflation and R&D.

Table 2: Panel regression results

Dependent variable: 100*log(R&D/GDP)			
Method:	Pooled regression	Country FE	Country and year FE
Regressors			
Inflation	-1.0827***	-0.5637***	-0.3737***
	(0.000)	(0.000)	(0.000)
Income	0.0032***	0.0013***	0.0014***
	(0.000)	(0.001)	(0.003)
Patent rights	11.7772***	17.1994***	12.4010***
	(0.005)	(0.000)	(0.000)
Economic freedom	5.9472	6.5400***	6.9683***
	(0.101)	(0.001)	(0.003)
Population	-0.1110***	-0.3795***	-0.4614***
	(0.003)	(0.004)	(0.000)
Openness	0.8109***	0.0404	-0.1199
	(0.000)	(0.738)	(0.351)
Observations	648	648	648
Adj-R^2	0.4325	0.9254	0.9375

Notes: p-values in parentheses. FE denotes fixed effects.

The regression coefficients on inflation are all significantly different from zero at the 1 percent level. In our preferred regression specification with both country and year fixed effects, the estimated semi-elasticity of R&D with respect to inflation is -0.374. In other words, a 1% increase in the inflation rate is associated with a decrease in the R&D share of GDP by 0.374 percent. To identify whether it is the long-run or short-run component of inflation that is driving our results, we have also used the Hodrick-Prescott filter to extract

the trend and the cyclical component of inflation. After repeating the regressions in Table 2, we find that the negative relationship between R&D and inflation is all due to trend inflation; see Table 3 in which we report only the coefficient of trend inflation to conserve space.[5] Given that trend inflation is more likely to affect inflation expectations[6] and be reflected in the nominal interest rate that determines the opportunity cost associated with cash-in-advance constraints, we view these results as encouraging motivating evidence for our theory.

Table 3: Panel regressions using HP-trend

Dependent variable: 100*log(R&D/GDP)			
Method:	Pooled regression	Country FE	Country and year FE
Regressor			
Trend inflation	-1.2732***	-0.7065***	-0.4662***
p-values	(0.000)	(0.00)	(0.00)
Observations	648	648	648
Adj-R^2	0.4362	0.9214	0.9303
Notes: FE denotes fixed effects.			

3 An open-economy monetary Schumpeterian model

In this section, we develop an open-economy version of the monetary Schumpeterian growth model. The underlying quality-ladder model is based on the seminal work of Aghion and Howitt (1992), and we consider a version of the quality-ladder model in Grossman and Helpman (1991).[7] We remove scale effects in the Schumpeterian model by allowing for increasing complexity in innovation as in Segerstrom (1998).[8] Furthermore, we modify the Schumpeterian model by introducing money demand via CIA constraints on consumption and R&D investment as in Chu and Cozzi (2014) and extending the closed-economy model into a two-country setting with trade in intermediate goods. The home country is denoted with a superscript h, whereas the foreign country is denoted with a superscript f. Both countries invest in R&D, but we allow for asymmetry across the two countries in a number of structural parameters. Following a common treatment in this type of two-country models, we assume labor immobility across countries. Given that the quality-ladder model has been well-studied, we will describe the familiar components briefly but discuss new features in details. Furthermore, to conserve space, we will only present equations for the home country h, but readers are advised to keep in mind that for each equation we present, there is an analogous equation for the foreign country f.

[5] Regression results for cyclical inflation are available upon request; see Appendix B.

[6] We follow Orr *et al.* (1995), Ardagna *et al.* (2007) and Ardagna (2009) to use trend inflation from the Hodrick-Prescott filter as a proxy for inflation expectations.

[7] See also Segerstrom *et al.* (1990) for another seminal study of the quality-ladder model.

[8] See for example Jones (1999) for a discussion of scale effects in R&D-based growth models.

3.1 Household

In each country, there is a representative household. In country h, the population size is N_t^h, and its law of motion is $\dot{N}_t^h = nN_t^h$, where $n > 0$ is the exogenous population growth rate. Total population in the world is $N_t = N_t^h + N_t^f$, where N_t^f is the population size in country f, which is assumed to have the same population growth rate n. The lifetime utility function of the household in country h is given by[9]

$$U^h = \int_0^\infty e^{-\rho t} \left[\ln c_t^h + \theta^h \ln(1 - l_t^h)\right] dt, \tag{1}$$

where c_t^h denotes per capita consumption of final goods and l_t^h denotes the supply of labor per person in country h at time t. The parameters $\rho > 0$ and $\theta^h \geq 0$ determine respectively subjective discounting and leisure preference. We allow for asymmetry in θ^h across the two countries.

The asset-accumulation equation expressed in real terms (i.e., denominated in units of final goods) is given by

$$\dot{a}_t^h + \dot{m}_t^h = (r_t^h - n)a_t^h - (\pi_t^h + n)m_t^h + i_t^h b_t^h + w_t^h l_t^h + \tau_t^h - c_t^h. \tag{2}$$

a_t^h is the real value of financial assets (in the form of equity shares in monopolistic firms) owned by each member of the household in country h. r_t^h is the real interest rate in country h. According to the Fisher identity, it is equal to $r_t^h = i_t^h - \pi_t^h$, where i_t^h is the nominal interest rate and π_t^h is the inflation rate in country h. m_t^h is the real value of domestic currency held by each member of the household partly to facilitate the payment of consumption goods that are purchased domestically[10] and partly to facilitate money lending to R&D entrepreneurs subject to the following constraint: $b_t^h + \xi^h c_t^h \leq m_t^h$, where b_t^h is the real value of domestic currency borrowed by R&D entrepreneurs to finance their R&D investment and $\xi^h \geq 0$ parameterizes the strength of the CIA constraint on consumption. As the household accumulates more money m_t^h, its money lending b_t^h to R&D entrepreneurs also increases, and the rate of return on b_t^h is the nominal interest rate i_t^h.[11] w_t^h is the real wage rate in country h. Finally, τ_t^h is the real value of a lump-sum transfer (or tax if $\tau_t^h < 0$) from the government to each member of the household.

The household maximizes (1) subject to (2) and $b_t^h + \xi^h c_t^h \leq m_t^h$, which becomes a binding constraint in equilibrium. From standard dynamic optimization, the optimality condition for per capita consumption in country h is

$$c_t^h = \frac{1}{\eta_t^h(1 + \xi^h i_t^h)}, \tag{3}$$

[9] Here we assume that the utility function is based on per capita utility. Alternatively, one can assume that the utility function is based on aggregate utility in which case the effective discount rate simply becomes $\rho - n$.

[10] In reality, consumers do purchase consumption goods in other countries using foreign currencies; however, this represents a relatively small share of consumption expenditures of the average household. Therefore, in this model, we do not consider the use of foreign currency for the purchase of consumption goods in the other country.

[11] It can be shown as a no-arbitrage condition that the rate of return on b_t^h must be equal to i_t^h.

where η_t^h is the Hamiltonian co-state variable on (2). The optimality condition for labor supply is

$$l_t^h = 1 - \frac{\theta^h c_t^h (1 + \xi^h i_t^h)}{w_t^h}. \tag{4}$$

Finally, the intertemporal optimality condition is

$$-\frac{\dot{\eta}_t^h}{\eta_t^h} = r_t^h - \rho - n. \tag{5}$$

In the case of a constant nominal interest rate i^h, (3) and (5) simplify to the familiar Euler equation: $\dot{c}_t^h / c_t^h = r_t^h - \rho - n$.

We consider a global financial market. In this case, the real interest rates in the two countries must be equal such that $r_t^h = r_t^f = r_t$.[12] Given that the distribution of financial assets across the two countries is indeterminate, we follow Dinopoulos and Segerstrom (2010) to assume that monopolistic firms created by innovation of domestic entrepreneurs are owned by the domestic household.

3.2 Final goods

Final goods for consumption in the two countries are produced by competitive firms that aggregate two types of intermediate goods using a standard CES aggregator given by

$$C_t = \left[\alpha (Y_t^h)^{(\sigma-1)/\sigma} + (1-\alpha)(Y_t^f)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \tag{6}$$

where Y_t^h and Y_t^f denote intermediate goods produced by country h and country f, respectively. The parameter $\alpha \in (0,1)$ determines the importance of country h's intermediate goods in the production of final goods. The parameter $\sigma > 0$ measures the elasticity of substitution between intermediate goods produced by the two countries. From profit maximization, the conditional demand functions for Y_t^h and Y_t^f are respectively

$$Y_t^h = \left(\frac{\alpha}{p_{y,t}^h}\right)^{\sigma} C_t, \tag{7}$$

$$Y_t^f = \left(\frac{1-\alpha}{p_{y,t}^f}\right)^{\sigma} C_t, \tag{8}$$

where $p_{y,t}^h$ is the price of Y_t^h, and $p_{y,t}^f$ is the price of Y_t^f. Both of these prices are expressed in units of final goods.

Suppose the nominal price of final goods in country h is $p_{c,t}^h$, which is denominated in units of currency in country h. Then, because final goods can be freely traded across the two countries,[13] the law of one price holds such that the nominal price of final goods denominated in units of currency in country f is $p_{c,t}^f = \varepsilon_t p_{c,t}^h$, where ε_t is the nominal exchange rate.

[12] The nominal interest rates in the two countries would still be different if the inflation rates differ across countries.

[13] Even if final goods cannot be traded, the fact that intermediate goods are freely traded is sufficient to ensure $p_{c,t}^f = \varepsilon_t p_{c,t}^h$.

3.3 Intermediate goods

Intermediate goods are also produced by competitive firms. Competitive firms in country h produce Y_t^h by aggregating a unit continuum of differentiated domestic inputs $X_t^h(j)$ for $j \in [0,1]$. The standard Cobb-Douglas aggregator is given by[14]

$$Y_t^h = \exp\left(\int_0^1 \ln X_t^h(j) dj\right). \tag{9}$$

From profit maximization, the conditional demand functions for $X_t^h(j)$ is

$$X_t^h(j) = \frac{p_{y,t}^h}{p_{x,t}^h(j)} Y_t^h, \tag{10}$$

where $p_{x,t}^h(j)$ is the price (denominated in units of final goods) of $X_t^h(j)$. Finally, the standard price index of Y_t^h is $p_{y,t}^h = \exp\left(\int_0^1 \ln p_{x,t}^h(j) dj\right)$.[15]

3.4 Differentiated inputs

In country h, there is a unit continuum of differentiated inputs indexed by $j \in [0,1]$. In each industry $j \in [0,1]$, there is an industry leader who dominates the market temporarily until the arrival of the next innovation.[16] The industry leader employs domestic workers to produce $X_t^h(j)$.[17] Specifically, the production function is given by

$$X_t^h(j) = (z^h)^{q_t^h(j)} L_{x,t}^h(j), \tag{11}$$

where $L_{x,t}^h(j)$ denotes production labor in industry j of country h. $z^h > 1$ is the step size of innovation in country h, and we allow this parameter to differ across countries. $q_t^h(j)$ is the number of quality improvements that have occurred in industry j as of time t.[18]

Given $(z^h)^{q_t^h(j)}$ in industry j, the leader's marginal cost function for the production of $X_t^h(j)$ is

$$mc_t^h(j) = \frac{w_t^h}{(z^h)^{q_t^h(j)}}. \tag{12}$$

Standard Bertrand price competition leads to markup pricing. This markup ratio is assumed to equal the step size z^h of innovation in Grossman and Helpman (1991). Here we allow for

[14] Our results are robust to a more general CES aggregator, under which the monopolistic markup of differentiated inputs may be determined by the elasticity of substituition. For simplicity, we focus on the Cobb-Douglas aggregator.

[15] Derivations available upon request; see Appendix B.

[16] This is known as the Arrow replacement effect in the literature; see Cozzi (2007) for a discussion.

[17] In order to keep the analysis tractable, we do not consider production offshoring in this study; see Chu, Cozzi and Furukawa (2013) for a North-South analysis of monetary policy with production offshoring.

[18] It is useful to note that we here adopt a cost-reducing view of quality improvement as in Peretto (1998).

variable patent breadth similar to Li (2001) and Iwaisako and Futagami (2013) by assuming that the markup $\mu^h > 1$ is a policy instrument determined by the patent authority.[19] For simplicity, we focus on the case in which $\mu^h = \mu^f = \mu$, and this assumption can be partly justified by the harmonization of patent protection across countries as a result of the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPS) effective since 1996.[20] Furthermore, given that patent policy is not designed by the monetary authority in reality,[21] we treat μ as exogenous when deriving optimal monetary policy.

Given the markup ratio μ, the price of $X_t^h(j)$ is

$$p_{x,t}^h(j) = \mu \frac{w_t^h}{(z^h)^{q_t^h(j)}}. \tag{13}$$

Therefore, the real value of monopolistic profit earned by the industry leader j in country h is

$$\omega_t^h(j) = \frac{\mu - 1}{\mu} p_{x,t}^h(j) X_t^h(j) = \frac{\mu - 1}{\mu} p_{y,t}^h Y_t^h, \tag{14}$$

where the second equality follows from (10). Finally, wage income paid to industry j's workers in country h is

$$w_t^h L_{x,t}^h(j) = \frac{1}{\mu} p_{x,t}^h(j) X_t^h(j) = \frac{1}{\mu} p_{y,t}^h Y_t^h. \tag{15}$$

3.5 R&D

Denote $v_t^h(j)$ as the real value of the monopolistic firm $j \in [0,1]$ in country h. Because $\omega_t^h(j) = \omega_t^h$ for $j \in [0,1]$ from (14), $v_t^h(j) = v_t^h$ in a symmetric equilibrium that features an equal arrival rate of innovation across industries within a country.[22] In this case, the familiar no-arbitrage condition for v_t^h is

$$r_t = \frac{\omega_t^h + \dot{v}_t^h - \lambda_t^h v_t^h}{v_t^h}. \tag{16}$$

This condition equates the real interest rate r_t in the global financial market to the rate of return per unit of financial asset. The asset return is the sum of (a) monopolistic profit

[19] To model patent breadth, we first make a standard assumption in the literature, see for example Howitt (1999) and Segerstrom (2000), that once the incumbent leaves the market, she cannot threaten to reenter the market due to a reentry cost. As a result of the incumbent stopping production, the entrant is able to charge the unconstrained monopolistic markup, which is infinity due to the Cobb-Douglas specification in (9), under the case of complete patent breadth. However, with incomplete patent breadth, potential imitation limits the markup. Specifically, the presence of monopolistic profits attracts imitation; therefore, stronger patent protection allows monopolistic producers to charge a higher markup without the threat of imitation. This formulation of patent breadth captures Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".

[20] See Grossman and Lai (2004) for an analysis of the harmonization of patent protection under TRIPS.

[21] See Chu (2008) for a discussion of the political process in determining patent policy in the US.

[22] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi *et al.* (2007) for a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the Schumpeterian growth model.

ω_t^h, (b) any potential capital gain $\dot{v}_t^h$, and (c) expected capital loss $\lambda_t^h v_t^h$ due to creative destruction, where λ_t^h is the arrival rate of the next innovation in country h.

There is a unit continuum of R&D entrepreneurs indexed by $k \in [0,1]$ in each country, and they hire R&D labor for innovation. In country h, entrepreneur k's wage payment to R&D labor is $w_t^h L_{r,t}^h(k)$. However, to facilitate this wage payment, the entrepreneur needs to borrow domestic currency[23] from the domestic household.[24] The real value of money borrowed is $b_t^h(k) = \phi^h w_t^h L_{r,t}^h(k)$, where $\phi^h \in (0,1]$ is the fraction of wage payment that requires the use of currency. We follow the formulation in Chu and Cozzi (2014) to impose a CIA constraint on R&D such that the cost of borrowing is $i_t^h b_t^h(k)$. Therefore, the total cost of R&D is $(1+\phi^h i_t^h) w_t^h L_{r,t}^h(k)$. Free entry implies zero expected profit such that[25]

$$v_t^h \lambda_t^h(k) = (1+\phi^h i_t^h) w_t^h L_{r,t}^h(k), \tag{17}$$

where the *firm-level* arrival rate of innovation is $\lambda_t^h(k) = \overline{\varphi}_t^h L_{r,t}^h(k)$. To model two sources of R&D externality commonly discussed in the literature, we assume $\overline{\varphi}_t^h = \varphi/[(L_{r,t}^h)^\delta Z_t^h]$, where $L_{r,t}^h$ is aggregate R&D labor. Z_t^h denotes aggregate technology in country h capturing the effect of increasing innovation complexity.[26] This formulation of increasing R&D difficulty also removes scale effects in the innovation process as in Segerstrom (1998).[27] The parameter $\delta \in [0,1)$ measures the degree of R&D duplication externality as in Jones and Williams (2000).[28] The parameter $\varphi > 0$ determines R&D productivity. The *aggregate* arrival rate of innovation in country h is

$$\lambda_t^h = \int_0^1 \lambda_t^h(k) dk = \frac{\varphi (L_{r,t}^h)^{1-\delta}}{Z_t^h}. \tag{18}$$

3.6 Monetary authority

The nominal value of the aggregate money supply in country h is M_t^h. Then, the real value of the aggregate money balance in country h is $m_t^h N_t^h = M_t^h / p_{c,t}^h$, where $p_{c,t}^h$ is the price

[23] Given that this is wage payment to workers in the domestic economy, the wage payment is naturally paid in domestic currency. Furthermore, there is no incentive for the entrepreneurs to borrow foreign currency and convert it into domestic currency even when the nominal interest rates differ across countries. The reason is that the two countries pay the same real interest rate, so any difference in the nominal interest rates is due to the difference in the inflation rates. Given that the law of one price holds in our model, the difference in the inflation rates must be equal to the change in the nominal exchange rate.

[24] Due to the static nature of the R&D sector in this model, we cannot deal with the case in which R&D entrepreneurs accumulate cash reserves; however, inflation would have the same positive effect on the cost of R&D as in our current treatment in which entrepreneurs borrow cash from the household because the opportunity cost of using cash to finance R&D is determined by the nominal interest rate in both cases.

[25] In this study, we do not consider R&D subsidies; see for example Impullitti (2007, 2010) for an open-economy analysis of R&D subsidies in the Schumpeterian model.

[26] See Venturini (2012) for empirical evidence based on industry-level data that supports the presence of increasing R&D difficulty.

[27] Segerstrom (1998) considers an industry-specific index of R&D difficulty. Here we consider an aggregate index of R&D difficulty to simplify notation without altering the aggregate results of our analysis.

[28] We assume δ to be the same across countries in order to ensure that Z_t^h and Z_t^f grow at the same rate in the long run. Equation (27) shows that the balanced growth path would not exist (unless $\sigma \to 1$) if Z_t^h and Z_t^f grow at different rates in the long run.

of final goods denominated in units of currency in country h. Therefore, the growth rate of per capita real money balance is $\dot{m}_t^h/m_t^h = \dot{M}_t^h/M_t^h - n - \pi_t^h$, where $\pi_t^h \equiv \dot{p}_{c,t}^h/p_{c,t}^h$ is the inflation rate of the price of final goods in country h. The monetary policy instrument that we consider is the inflation rate π_t^h, which is exogenously chosen by the monetary authority in country h. Given π_t^h, the nominal interest rate in country h is endogenously determined according to the Fisher identity $i_t^h = \pi_t^h + r_t$, where r_t is the real interest rate in the global financial market. Then, the growth rate of the nominal money supply M_t^h in country h is endogenously determined according to $\dot{M}_t^h/M_t^h = \dot{m}_t^h/m_t^h + n + \pi_t^h$. Finally, the monetary authority in country h returns the seigniorage revenue as a real lump-sum transfer $\tau_t^h N_t^h = \dot{M}_t^h/p_{c,t}^h = [\dot{m}_t^h + (\pi_t^h + n)m_t^h]N_t^h$ to the domestic household.

3.7 Decentralized equilibrium

The equilibrium is a time path of allocations $\{l_t^h, l_t^f, c_t^h, c_t^f, C_t, Y_t^h, Y_t^f, X_t^h(j), X_t^f(j), L_{x,t}^h(j), L_{x,t}^f(j), L_{r,t}^h(k), L_{r,t}^f(k)\}_{t=0}^{\infty}$, a time path of prices $\{w_t^h, w_t^f, p_{c,t}^h, p_{c,t}^f, p_{y,t}^h, p_{y,t}^f, p_{x,t}^h(j), p_{x,t}^f(j), v_t^h, v_t^f, \varepsilon_t\}_{t=0}^{\infty}$ and a time path of policies $\{\pi_t^h, \pi_t^f, \tau_t^h, \tau_t^f\}_{t=0}^{\infty}$ such that the following conditions are satisfied:

- the representative household in country h chooses $\{l_t^h, c_t^h\}$ to maximize lifetime utility taking $\{w_t^h, p_{c,t}^h, \pi_t^h, \tau_t^h\}$ as given;
- the representative household in country f chooses $\{l_t^f, c_t^f\}$ to maximize lifetime utility taking $\{w_t^f, p_{c,t}^f, \pi_t^f, \tau_t^f\}$ as given;
- competitive final-good firms produce $\{C_t\}$ to maximize profit taking $\{p_{c,t}^h, p_{c,t}^f, p_{y,t}^h, p_{y,t}^f\}$ as given;
- competitive intermediate-good firms in country h produce $\{Y_t^h\}$ to maximize profit taking $\{p_{y,t}^h, p_{x,t}^h(j)\}$ as given;
- competitive intermediate-good firms in country f produce $\{Y_t^f\}$ to maximize profit taking $\{p_{y,t}^f, p_{x,t}^f(j)\}$ as given;
- monopolistic firms in country h produce $\{X_t^h(j)\}$ and choose $\{p_{x,t}^h(j)\}$ to maximize profit taking $\{w_t^h\}$ as given;
- monopolistic firms in country f produce $\{X_t^f(j)\}$ and choose $\{p_{x,t}^f(j)\}$ to maximize profit taking $\{w_t^f\}$ as given;
- competitive R&D entrepreneurs in country h employ $\{L_{r,t}^h(k)\}$ to maximize expected profit taking $\{w_t^h, v_t^h\}$ as given;
- competitive R&D entrepreneurs in country f employ $\{L_{r,t}^f(k)\}$ to maximize expected profit taking $\{w_t^f, v_t^f\}$ as given;
- the market-clearing condition for final goods holds such that $c_t^h N_t^h + c_t^f N_t^f = C_t$;

- the market-clearing conditions for labor in the two countries hold such that $l_t^h N_t^h = L_{x,t}^h + L_{r,t}^h$ and $l_t^f N_t^f = L_{x,t}^f + L_{r,t}^f$; and
- the value of assets equals the value of monopolistic firms in each country such that $a_t^h N_t^h = v_t^h$ and $a_t^f N_t^f = v_t^f$.

3.8 Aggregate economy

Substituting (11) into (9) yields the aggregate production function for Y_t^h given by

$$Y_t^h = Z_t^h L_{x,t}^h, \tag{19}$$

where aggregate technology Z_t^h in country h is defined as

$$Z_t^h \equiv \exp\left(\int_0^1 q_t^h(j)dj \ln z^h\right) = \exp\left(\int_0^t \lambda_\upsilon^h d\upsilon \ln z^h\right). \tag{20}$$

The second equality of (20) applies the law of large numbers. Differentiating the log of (20) with respect to t yields the growth rate of aggregate technology in country h given by

$$\frac{\dot{Z}_t^h}{Z_t^h} = \lambda_t^h \ln z^h = \frac{(L_{r,t}^h)^{1-\delta}}{Z_t^h}\varphi \ln z^h. \tag{21}$$

Similarly, the aggregate production function for Y_t^f is given by

$$Y_t^f = Z_t^f L_{x,t}^f, \tag{22}$$

where aggregate technology Z_t^f in country f is defined as

$$Z_t^f \equiv \exp\left(\int_0^1 q_t^f(j)dj \ln z^f\right) = \exp\left(\int_0^t \lambda_\upsilon^f d\upsilon \ln z^f\right). \tag{23}$$

Differentiating the log of (23) with respect to t yields the growth rate of aggregate technology in country f given by

$$\frac{\dot{Z}_t^f}{Z_t^f} = \lambda_t^f \ln z^f = \frac{(L_{r,t}^f)^{1-\delta}}{Z_t^f}\varphi \ln z^f. \tag{24}$$

Proposition 1 *Given constant nominal interest rates $\{i^h, i^f\}$ in the two countries, the aggregate economy gradually converges to a unique and stable balanced growth path along which each variable grows at a constant (possibly zero) rate.*

Proof. See Appendix A. ■

For the dynamics of the model, Proposition 1 shows that the aggregate economy gradually converges to a unique and stable balanced growth path (BGP). On the BGP, the share of labor allocated to each sector is stationary, and technologies $\{Z_t^h, Z_t^f\}$ grow at a constant rate. Consequently, (21) and (24) imply that $(L_{r,t}^h)^{1-\delta}/Z_t^h$ and $(L_{r,t}^f)^{1-\delta}/Z_t^f$ must be stationary in the long run. Given that the share of labor allocated to each sector is also stationary on the BGP, $L_{r,t}^h/N_t^h$ and $L_{r,t}^f/N_t^f$ are stationary in the long run. This analysis implies that the long-run growth rates of home and foreign technologies are

$$g^h \equiv \frac{\dot{Z}_t^h}{Z_t^h} = \lambda^h \ln z^h = (1-\delta)n, \tag{25}$$

$$g^f \equiv \frac{\dot{Z}_t^f}{Z_t^f} = \lambda^f \ln z^f = (1-\delta)n, \tag{26}$$

where the steady-state equilibrium arrival rates of innovation are also determined by exogenous parameters given by $\lambda^h = (1-\delta)n/\ln z^h$ and $\lambda^f = (1-\delta)n/\ln z^f$. Differentiating the log of (6) with respect to time yields the growth rate of aggregate consumption given by

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\alpha(Y_t^h)^{(\sigma-1)/\sigma} + (1-\alpha)(Y_t^f)^{(\sigma-1)/\sigma}} \left[\alpha(Y_t^h)^{(\sigma-1)/\sigma}\frac{\dot{Y}_t^h}{Y_t^h} + (1-\alpha)(Y_t^f)^{(\sigma-1)/\sigma}\frac{\dot{Y}_t^f}{Y_t^f}\right]. \tag{27}$$

On the BGP, the growth rates of final goods are

$$\frac{\dot{Y}_t^h}{Y_t^h} = \frac{\dot{Z}_t^h}{Z_t^h} + \frac{\dot{L}_{x,t}^h}{L_{x,t}^h} = g^h + n = (2-\delta)n, \tag{28}$$

$$\frac{\dot{Y}_t^f}{Y_t^f} = \frac{\dot{Z}_t^f}{Z_t^f} + \frac{\dot{L}_{x,t}^f}{L_{x,t}^f} = g^f + n = (2-\delta)n. \tag{29}$$

Therefore, the long-run growth rate of aggregate consumption is $g_C = (2-\delta)n$, and the long-run growth rate of per capita consumption in the two countries is $g_c^h = g_c^f = (1-\delta)n$.

3.9 Steady-state equilibrium labor allocations

Here we sketch out the derivations of the steady-state equilibrium labor allocations in country h. Integrating (17) over k yields the free-entry condition in the R&D sector given by $v_t^h \lambda_t^h = (1+\phi^h i_t^h) w_t^h L_{r,t}^h$. Equation (16) implies that the balanced-growth value of an innovation is $v_t^h = \omega_t^h/(r - g_v^h + \lambda^h)$, where g_v^h denotes the steady-state growth rate of v_t^h. It can be shown that $r - g_v^h = \rho$ on the BGP.[29] Substituting these conditions along with (14) and (15) into the R&D free-entry condition yields

$$\frac{l_r^h}{l_x^h} = \frac{\mu - 1}{1 + \phi^h i^h}\frac{\lambda^h}{\rho + \lambda^h}, \tag{30}$$

[29] Derivations available upon request; see Appendix B.

where $l_{r,t}^h \equiv L_{r,t}^h/N_t^h$ and $l_{x,t}^h \equiv L_{x,t}^h/N_t^h$ denote per capita labor allocations. The second condition for solving the steady-state equilibrium labor allocations is the resource constraint on labor given by

$$l^h = l_x^h + l_r^h. \tag{31}$$

To determine the steady-state equilibrium per capita labor supply l^h, we apply $a_t^h N_t^h = v_t^h$ (i.e., the assumption of domestic innovations being owned by the domestic household) on (2) such that

$$\dot{v}_t^h = r_t^h v_t^h + i_t^h b_t^h N_t^h + w_t^h L_{r,t}^h + w_t^h L_{x,t}^h - c_t^h N_t^h, \tag{32}$$

where we have also used $\tau_t^h = \dot{m}_t^h + (\pi_t^h + n)m_t^h$ and the resource constraint on labor in (31). Applying $r - g_v^h = \rho$ and (17) on (32) yields

$$c_t^h N_t^h = \rho v_t^h + \lambda_t^h v_t^h + w_t^h L_{x,t}^h = p_{y,t}^h Y_t^h, \tag{33}$$

where the second equality follows from $v_t^h = \omega_t^h/(\rho + \lambda^h)$, (14) and (15). Substituting (33) and (15) into (4) yields

$$l^h = 1 - \mu\theta^h(1 + \xi^h i^h) l_x^h. \tag{34}$$

Solving (30), (31) and (34) yields the steady-state equilibrium labor allocations.

Proposition 2 *The equilibrium labor allocations in country h are given by*

$$l_r^h = \frac{\frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}{1 + \mu\theta^h(1 + \xi^h i^h) + \frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}, \tag{35}$$

$$l_x^h = \frac{1}{1 + \mu\theta^h(1 + \xi^h i^h) + \frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}, \tag{36}$$

$$l^h = \frac{1 + \frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}{1 + \mu\theta^h(1 + \xi^h i^h) + \frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}, \tag{37}$$

where $i^h = \pi^h + r = \pi^h + \rho + n + g_c^h = \pi^h + \rho + (2 - \delta)n$, which is increasing in π^h.[30]

Proof. See Appendix A. ■

Equation (35) shows that R&D labor l_r^h is decreasing in i^h and π^h (given that $i^h = \pi^h + \rho + (2 - \delta)n$) via the CIA constraint on R&D (captured by ϕ^h) and the CIA constraint on consumption (captured by ξ^h). The intuition of the effect via ϕ^h is that a higher nominal interest rate increases the cost of R&D, which in turn causes R&D entrepreneurs to reduce their R&D spending. The intuition of the effect via ξ^h is that a higher nominal interest rate increases the cost of consumption relative to leisure; as a result, the household increases leisure and decreases labor supply, which also reduces R&D labor. Equation (36) shows that

[30] Empirical evidence supports a positive long-run relationship between inflation and the nominal interest rate; see for example Mishkin (1992) for US data and Booth and Ciner (2001) for European data.

i^h and π^h have a positive effect on production labor l_x^h via the CIA constraint on R&D but a negative effect on l_x^h via the CIA constraint on consumption. The positive effect of i^h and π^h on l_x^h via ϕ^h is due to the reallocation of labor from the R&D sector to the production sector. The negative effect of i^h and π^h on l_x^h via ξ^h is due to the reduced supply of labor. Equation (37) shows that labor supply l^h is decreasing in i^h and π^h via both CIA constraints.

3.10 Inflation and economic growth

We now explore the effects of inflation on the growth rate of technologies. To facilitate this analysis, we define a transformed variable $\varsigma_t^h \equiv Z_t^h/(N_t^h)^{1-\delta}$, and its growth rate is given by

$$\frac{\dot{\varsigma}_t^h}{\varsigma_t^h} \equiv \frac{\dot{Z}_t^h}{Z_t^h} - (1-\delta)\frac{\dot{N}_t^h}{N_t^h} = \frac{\dot{Z}_t^h}{Z_t^h} - \ (1-\delta)n. \tag{38}$$

Using the steady-state equilibrium condition $\dot{Z}_t^h/Z_t^h = (1-\delta)n$, we can rewrite (21) as

$$\varsigma^h = \frac{\varphi \ln z^h}{(1-\delta)n}(l_r^h)^{1-\delta}, \tag{39}$$

where the steady-state equilibrium R&D labor l_r^h is decreasing in the domestic nominal interest rate i^h and the domestic inflation rate π^h as shown in (35). Therefore, ς^h is also decreasing in i^h and π^h. In order for ς^h to decrease to a lower steady-state value in the long run, it must be the case that in the short run, $\dot{\varsigma}_t^h/\varsigma_t^h < 0$, which in turn implies that $\dot{Z}_t^h/Z_t^h < \ (1-\delta)n$. In other words, a permanent increase in the domestic inflation rate leads to a temporary decrease in the growth rate of domestic technology and a permanent decrease in the level of domestic technology ς^h. An analogous analysis would show that a permanent increase in the foreign inflation rate leads to a temporary decrease in the growth rate of foreign technology and a permanent decrease in the level of foreign technology ς^f.

4 Inflation and social welfare

In this section, we analyze the effects of domestic and foreign inflation on social welfare. On the BGP, the long-run welfare of the representative household in country h is given by

$$U^h = \frac{1}{\rho}\left[\ln c_0^h + \frac{g_c^h}{\rho} + \theta^h \ln(1-l^h)\right]. \tag{40}$$

For analytical tractability, we focus on the special case of $\sigma \to 1$ in (6) in this qualitative analysis.[31] Substituting (7) into (33) yields $c_t^h = \alpha C_t/N_t^h$. Substituting this condition along with (6) and $g_c^h = (1-\delta)n$ into (40) yields

$$\rho U^h = \ln C_0 + \theta^h \ln(1-l^h) = \alpha \ln Y_0^h + (1-\alpha)\ln Y_0^f + \theta^h \ln(1-l^h), \tag{41}$$

[31] We will consider the general case of $\sigma > 0$ in the subsequent quantitative analysis.

where we have dropped all the exogenous terms. The balanced-growth levels of final goods are given by

$$Y_0^h = Z_0^h l_x^h N_0^h, \tag{42}$$

$$Y_0^f = Z_0^f l_x^f N_0^f, \tag{43}$$

where the balanced-growth levels of technologies are given by

$$Z_0^h = \frac{(N_0^h)^{1-\delta}\varphi \ln z^h}{(1-\delta)n}(l_r^h)^{1-\delta}, \tag{44}$$

$$Z_0^f = \frac{(N_0^f)^{1-\delta}\varphi \ln z^f}{(1-\delta)n}(l_r^f)^{1-\delta}. \tag{45}$$

Substituting (42)-(45) into (41) yields

$$\rho U^h = \alpha[\ln l_x^h + (1-\delta)\ln l_r^h] + (1-\alpha)[\ln l_x^f + (1-\delta)\ln l_r^f] + \theta^h \ln(1-l^h), \tag{46}$$

where we have once again dropped the exogenous terms. In (46), $\{l_x^h, l_r^h, l^h\}$ depend on i^h and π^h and $\{l_x^f, l_r^f\}$ depend on i^f and π^f.

In the following subsections, we will derive (a) the inflation rate that is unilaterally chosen by each government to maximize domestic welfare and (b) the inflation rates that are chosen by cooperative governments who maximize the aggregate welfare of the two countries. Given that the results differ under the following three scenarios, we analyze them separately. In Section 4.1, we consider the case of inelastic labor supply. In Section 4.2, we consider elastic labor supply with only the CIA constraint on R&D investment. In Section 4.3, we consider elastic labor supply with only the CIA constraint on consumption. In Section 4.4, we calibrate the model to provide a quantitative analysis.

4.1 Inelastic labor supply

In this subsection, we consider the case of inelastic labor supply (i.e., $\theta^h = \theta^f = 0$). In this case, (35) and (36) simplify to

$$l_r^h = \frac{\frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}{1+\frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}, \tag{47}$$

$$l_x^h = \frac{1}{1+\frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}, \tag{48}$$

and $l^h = 1$. Due to inelastic labor supply, the effect of inflation operates solely through the CIA constraint on R&D investment. By analogous inference, the steady-state equilibrium allocations of R&D and production labors in country f are

$$l_r^f = \frac{\frac{\mu-1}{1+\phi^f i^f}\frac{\lambda^f}{\rho+\lambda^f}}{1+\frac{\mu-1}{1+\phi^f i^f}\frac{\lambda^f}{\rho+\lambda^f}}, \tag{49}$$

$$l_x^f = \frac{1}{1 + \frac{\mu-1}{1+\phi^f i^f}\frac{\lambda^f}{\rho+\lambda^f}}. \tag{50}$$

Substituting (47)-(50) into (46) and then differentiating U^h with respect to π^h, we obtain the following domestic inflation rate that is unilaterally chosen by the government in country h to maximize the domestic household's welfare:

$$\pi_{ne}^h = \frac{1}{\phi^h}\left[\frac{\mu-1}{1-\delta}\frac{\lambda^h}{\rho+\lambda^h} - 1\right] - r, \tag{51}$$

where $r = \rho + (2-\delta)n$ and $\lambda^h = (1-\delta)n/\ln z^h$ are determined by exogenous parameters. By analogous inference, the foreign inflation rate that is unilaterally chosen by country f's government to maximize the welfare of the household in country f is given by

$$\pi_{ne}^f = \frac{1}{\phi^f}\left[\frac{\mu-1}{1-\delta}\frac{\lambda^f}{\rho+\lambda^f} - 1\right] - r, \tag{52}$$

where $\lambda^f = (1-\delta)n/\ln z^f$.

We refer to the pair $\{\pi_{ne}^h, \pi_{ne}^f\}$ as the Nash-equilibrium inflation rates because each government pursues its own objective taking the other government's action as given. An interesting observation is that π_{ne}^f is also the foreign inflation rate that would be preferred by the government in country h. To see this result, we differentiate U^h with respect to π^f and find that the optimal foreign inflation rate for country h is also π_{ne}^f. Finally, we consider cooperative governments who choose $\{\pi^h, \pi^f\}$ to maximize aggregate welfare defined as $U^h + U^f$, and we refer to these inflation rates as the optimal inflation rates denoted as $\{\pi_*^h, \pi_*^f\}$. We find that $\{\pi_*^h, \pi_*^f\} = \{\pi_{ne}^h, \pi_{ne}^f\}$. In other words, the unilateral action of each government gives rise to an internationally optimal outcome; however, in the next subsection, we will show that this special result is due to the restriction of inelastic labor supply. We summarize the above results in the following proposition.

Proposition 3 *Under inelastic labor supply, the Nash-equilibrium inflation rate unilaterally chosen by each government coincides with the optimal inflation rate chosen by cooperative governments who maximize aggregate welfare of the two countries.*

Proof. See Appendix A. ■

The comparative statics of the optimal inflation rates can be summarized as follows. The optimal inflation rate in country h is decreasing in the domestic innovation step size z^h but increasing in the degree of duplication externality δ and the size of the markup μ. The intuition of these results can be easily understood if we compare the equilibrium allocation to the socially optimal allocation. It can be shown that the first-best optimal ratio of R&D to production labor is given by[32]

$$\frac{\widetilde{l}_r^h}{\widetilde{l}_x^h} = (1-\delta)\frac{g^h}{g^h+\rho}, \tag{53}$$

[32]Derivations available upon request; see Appendix B.

where $g^h = (1-\delta)n$. Then, we use $\lambda^h = g^h/\ln z^h$ to rewrite (30) and obtain the equilibrium ratio of R&D to production labor given by

$$\frac{l_r^h}{l_x^h} = \frac{\mu - 1}{1 + \phi^h i^h} \frac{g^h}{g^h + \rho \ln z^h}. \tag{54}$$

Comparing (53) and (54), we see that a larger z^h causes the equilibrium ratio l_r^h/l_x^h to decrease relative to the optimal ratio $\widetilde{l}_r^h/\widetilde{l}_x^h$ worsening the surplus-appropriability problem,[33] which is a positive externality. In this case, the optimal policy response is to reduce inflation to stimulate R&D. Second, a larger δ causes the equilibrium ratio l_r^h/l_x^h to increase relative to the optimal ratio $\widetilde{l}_r^h/\widetilde{l}_x^h$ capturing the negative duplication externality. In this case, the optimal policy response is to raise inflation to depress R&D. Finally, a larger μ also causes the equilibrium ratio l_r^h/l_x^h to increase relative to the optimal ratio $\widetilde{l}_r^h/\widetilde{l}_x^h$ due to a strengthening of the (domestic) business-stealing effect,[34] which is another source of negative R&D externality. In this case, the optimal policy response is also to raise inflation to depress equilibrium R&D.

4.2 Elastic labor supply with CIA on R&D only

In this subsection, we consider the case of elastic labor supply (i.e., $\theta^h > 0$) with the CIA constraint on R&D. However, we remove the CIA constraint on consumption by setting $\xi^h = \xi^f = 0$. In this case, (35), (36) and (37) simplify to

$$l_r^h = \frac{\frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}{1 + \mu\theta^h + \frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}, \tag{55}$$

$$l_x^h = \frac{1}{1 + \mu\theta^h + \frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}, \tag{56}$$

$$l^h = \frac{1 + \frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}{1 + \mu\theta^h + \frac{\mu-1}{1+\phi^h i^h}\frac{\lambda^h}{\rho+\lambda^h}}. \tag{57}$$

By analogous inference, the steady-state equilibrium allocations of R&D and production labors in country f are

$$l_r^f = \frac{\frac{\mu-1}{1+\phi^f i^f}\frac{\lambda^f}{\rho+\lambda^f}}{1 + \mu\theta^f + \frac{\mu-1}{1+\phi^f i^f}\frac{\lambda^f}{\rho+\lambda^f}}, \tag{58}$$

$$l_x^f = \frac{1}{1 + \mu\theta^f + \frac{\mu-1}{1+\phi^f i^f}\frac{\lambda^f}{\rho+\lambda^f}}. \tag{59}$$

[33] The surplus-appropriability problem refers to the case in which R&D entrepreneurs do not take into account the external benefits to consumers when new innovations occur.

[34] The business-stealing effect refers to the case in which R&D entrepreneurs do not take into account the external losses suffered by current industry leaders when new innovations occur.

Substituting (55)-(59) into (46) and then differentiating U^h with respect to π^h, we obtain the following domestic inflation that is unilaterally chosen by the government in country h to maximize the domestic household's welfare:

$$\pi^h_{ne} = \frac{1}{\phi^h}\left[\frac{1}{\alpha}\left(\frac{\alpha+\theta^h}{1+\mu\theta^h}\right)\frac{\mu-1}{1-\delta}\frac{\lambda^h}{\rho+\lambda^h}-1\right]-r, \tag{60}$$

where $r=\rho+(2-\delta)n$ and $\lambda^h=(1-\delta)n/\ln z^h$. The analogous inflation rate unilaterally chosen by country f's government to maximize the welfare of the household in country f is given by

$$\pi^f_{ne} = \frac{1}{\phi^f}\left[\frac{1}{1-\alpha}\left(\frac{1-\alpha+\theta^f}{1+\mu\theta^f}\right)\frac{\mu-1}{1-\delta}\frac{\lambda^f}{\rho+\lambda^f}-1\right]-r, \tag{61}$$

where $\lambda^f=(1-\delta)n/\ln z^f$. We next consider cooperative governments who choose $\{\pi^h,\pi^f\}$ to maximize aggregate welfare U^h+U^f, and the resulting optimal inflation rates are given by

$$\pi^h_{*} = \frac{1}{\phi^h}\left[\frac{1}{2\alpha}\left(\frac{2\alpha+\theta^h}{1+\mu\theta^h}\right)\frac{\mu-1}{1-\delta}\frac{\lambda^h}{\rho+\lambda^h}-1\right]-r, \tag{62}$$

$$\pi^f_{*} = \frac{1}{\phi^f}\left[\frac{1}{2(1-\alpha)}\left(\frac{2(1-\alpha)+\theta^f}{1+\mu\theta^f}\right)\frac{\mu-1}{1-\delta}\frac{\lambda^f}{\rho+\lambda^f}-1\right]-r. \tag{63}$$

We see that $\pi^h_{ne}>\pi^h_*$ and $\pi^f_{ne}>\pi^f_*$. In other words, the unilateral action of each government generally leads to excessively high inflation in the Nash equilibrium due to a cross-country spillover effect of monetary policy under elastic labor supply. This effect captures the inflationary bias due to monetary policy competition in Cooley and Quadrini (2003). However, the intuition of our model is different and can be explained as follows. When a country lowers its inflation rate, the welfare gain from a higher level of technology is shared by the other country, whereas the welfare cost of increasing labor supply (l^h in (57) is decreasing in π^h) falls entirely on the domestic household. As a result, the government does not lower the domestic inflation rate sufficiently in the Nash equilibrium. In contrast, cooperative governments would internalize the welfare gain from a higher level of technology in the other country.

Taking the difference of (60) and (62) yields the wedge between the Nash-equilibrium and optimal inflation rates in country h given by

$$\pi^h_{ne}-\pi^h_{*} = \frac{\mu-1}{\mu\theta^h+1}\frac{\theta^h}{2\alpha\phi^h(1-\delta)}\frac{\lambda^h}{\rho+\lambda^h}>0, \tag{64}$$

which is increasing in the markup μ. Intuitively, a larger markup strengthens the negative business-stealing externality as discussed before, and the resulting optimal policy response is to increase inflation to reduce R&D. However, in the Nash equilibrium, the cost of higher inflation that depresses the level of technology is shared by the other country. As a result, a noncooperative government would increase inflation more aggressively than a cooperative government would, and the wedge between the Nash-equilibrium and optimal inflation rates is monotonically increasing in the market power of firms. This result differs from the interesting result in Arseneau (2007), who shows that a larger market power of firms tends

to reduce the inflationary bias. The different implications between the two studies are due to the different CIA constraints. We have analyzed a CIA constraint on R&D, whereas Arseneau (2007) analyzes a CIA constraint on consumption. In the next subsection, we show that our model also delivers the insight of Arseneau (2007) under a CIA constraint on consumption.

Proposition 4 *Under elastic labor supply with only a CIA constraint on R&D, the Nash-equilibrium inflation rate unilaterally chosen by each government is higher than the optimal inflation rate chosen by cooperative governments who maximize aggregate welfare of the two countries. The degree of this inflationary bias is monotonically increasing in the market power of firms.*

Proof. See Appendix A. ■

4.3 Elastic labor supply with CIA on consumption only

In this subsection, we consider the case of elastic labor supply (i.e., $\theta^h > 0$) with the CIA constraint on consumption. However, we remove the CIA constraint on R&D by setting $\phi^h = \phi^f = 0$. In this case, (35), (36) and (37) simplify to

$$l_r^h = \frac{(\mu-1)\lambda^h/(\rho+\lambda^h)}{1+\mu\theta^h(1+\xi^h i^h)+(\mu-1)\lambda^h/(\rho+\lambda^h)}, \tag{65}$$

$$l_x^h = \frac{1}{1+\mu\theta^h(1+\xi^h i^h)+(\mu-1)\lambda^h/(\rho+\lambda^h)}, \tag{66}$$

$$l^h = \frac{1+(\mu-1)\lambda^h/(\rho+\lambda^h)}{1+\mu\theta^h(1+\xi^h i^h)+(\mu-1)\lambda^h/(\rho+\lambda^h)}. \tag{67}$$

By analogous inference, the steady-state equilibrium allocations of R&D and production labors in country f are

$$l_r^f = \frac{(\mu-1)\lambda^f/(\rho+\lambda^f)}{1+\mu\theta^f(1+\xi^f i^f)+(\mu-1)\lambda^f/(\rho+\lambda^f)}, \tag{68}$$

$$l_x^f = \frac{1}{1+\mu\theta^f(1+\xi^f i^f)+(\mu-1)\lambda^f/(\rho+\lambda^f)}. \tag{69}$$

Substituting (65)-(69) into (46) and then differentiating U^h with respect to π^h, we obtain the following domestic inflation that is unilaterally chosen by the government in country h to maximize the domestic household's welfare:

$$\pi_{ne}^h = \frac{1}{\xi^h}\left[\frac{1}{\alpha\mu(2-\delta)}\frac{\rho+\mu\lambda^h}{\rho+\lambda^h}-1\right]-r, \tag{70}$$

where $r = \rho + (2-\delta)n$ and $\lambda^h = (1-\delta)n/\ln z^h$. The analogous inflation rate unilaterally chosen by country f's government to maximize the welfare of the household in country f is given by

$$\pi^f_{ne} = \frac{1}{\xi^f}\left[\frac{1}{(1-\alpha)\mu(2-\delta)}\frac{\rho+\mu\lambda^f}{\rho+\lambda^f} - 1\right] - r, \tag{71}$$

where $\lambda^f = (1-\delta)n/\ln z^f$. We also consider cooperative governments who choose $\{\pi^h, \pi^f\}$ to maximize aggregate welfare $U^h + U^f$, and the resulting optimal inflation rates are given by

$$\pi^h_* = \frac{1}{\xi^h}\left[\frac{1}{2\alpha\mu(2-\delta)}\frac{\rho+\mu\lambda^h}{\rho+\lambda^h} - 1\right] - r, \tag{72}$$

$$\pi^f_* = \frac{1}{\xi^f}\left[\frac{1}{2(1-\alpha)\mu(2-\delta)}\frac{\rho+\mu\lambda^f}{\rho+\lambda^f} - 1\right] - r, \tag{73}$$

We see that $\pi^h_{ne} > \pi^h_*$ and $\pi^f_{ne} > \pi^f_*$. As in the previous case, the unilateral action of each government leads to excessively high inflation in the Nash equilibrium due to the cross-country spillover effect of monetary policy. However, the degree of this inflationary bias is now decreasing in the markup μ. To see this result, we take the difference of (70) and (72) and derive the following wedge between the Nash-equilibrium and optimal inflation rates in country h:

$$\pi^h_{ne} - \pi^h_* = \frac{\lambda^h + \rho/\mu}{\lambda^h + \rho}\frac{1}{2\alpha\xi^h(2-\delta)} > 0, \tag{74}$$

which shows that a larger markup μ would reduce the inflationary bias capturing the dampening effect of monopolistic distortion discussed in Arseneau (2007). It is useful to note from (65) and (66) that under the CIA constraint on consumption, increasing inflation does not lead to a reallocation of labor from R&D to production but decreases both R&D and production instead. Equation (66) also shows that when the markup μ increases, production labor decreases. In this case, the optimal policy response is to decrease inflation in order to stimulate production. Given that the inflation rate in the Nash equilibrium is higher to begin with, the government needs to reduce inflation more aggressively in order to achieve the same proportional increase in production l^h_x, which is a decreasing and convex function in i^h (and hence π^h).

Proposition 5 *Under elastic labor supply with only a CIA constraint on consumption, the Nash-equilibrium inflation rate unilaterally chosen by each government is higher than the optimal inflation rate chosen by cooperative governments who maximize aggregate welfare of the two countries. The degree of this inflationary bias is monotonically decreasing in the market power of firms.*

Proof. See Appendix A. ■

4.4 Quantitative analysis

In this section, we provide a numerical analysis of the growth and welfare effects of inflation across countries. We consider the general case with elastic labor supply and both CIA constraints on R&D and consumption. The two-country model features the following set of parameters $\{\sigma, n, \rho, \mu, z^h, z^f, \theta^h, \theta^f, \alpha, s, \delta, \xi^h, \xi^f, \phi^h, \phi^f, \pi^h, \pi^f\}$.[35] Given the calibrated parameter values, we then perform a quantitative analysis on the effects of inflation in the two economies.

To make this quantitative analysis more realistic, we allow for a non-unitary elasticity of substitution between home and foreign goods (i.e., $\sigma > 1$).[36] We consider a value of 2.46 for σ that is within the range of empirical estimates in Broda and Weinstein (2006). For the value of n, we set it to the average long-run growth rate of the number of R&D scientists and engineers[37] in the US[38] and the Euro Area[39]. As for the markup μ, we set it to 1.28, which corresponds to an intermediate value of the empirical estimates reported in Jones and Williams (2000). We follow Acemoglu and Akcigit (2012) to set the annual discount rate ρ to 0.05 and the time between innovation arrivals $\{1/\lambda^h, 1/\lambda^f\}$ to 3 years, which allows us to pin down the values of $\{z^h, z^f\} = \{\exp(g/\lambda^h), \exp(g/\lambda^f)\}$ given g. As for the leisure parameters $\{\theta^h, \theta^f\}$, we calibrate them by setting the per capita supply of labor $\{l^h, l^f\}$ to a standard value of 0.33. For the rest of the parameters, we calibrate the model using aggregate data from 1999 to 2007[40] in the US and the Euro Area. To fix notation, we consider the US as the home country h and the Euro Area as the foreign country f. We use data on the relative size of GDP in the US and the Euro Area to calibrate α by setting $(p_y^h Y^h + w^h L_r^h)/(p_y^h Y^h + w^h L_r^h + p_y^f Y^f + w^f L_r^f) = 0.58$.[41] As for the relative population size, we define $s \equiv N_t^h/N_t$ and calibrate it to data.[42] We also normalize N_0 to unity. The average growth rate of total factor productivity in the US and the Euro Area is 0.7%,[43] and we use this value to calibrate the duplication externality parameter $\delta = 1 - g/n$. We calibrate the consumption-CIA parameters $\{\xi^h, \xi^f\}$ to the ratios of M1 to consumption in the US and the Euro Area.[44] The average inflation rates in the US and the Euro Area are respectively 2.7% and 2.1%.[45] Given these empirical values of $\{\pi^h, \pi^f\}$, we calibrate $\{\phi^h, \phi^f\}$ by setting $\{\pi_{ne}^h, \pi_{ne}^f\} = \{\pi^h, \pi^f\}$. We report the parameter values in Table 4.

[35] It is useful to note that φ does not affect the other calibrated parameter values and the simulated welfare effects.

[36] We present the equations of the non-cooperative governments' best-response functions and their welfare functions in an unpublished appendix; see Appendix B.

[37] In the model, the long-run growth rate of technologies is driven by the growth rate of R&D labor as implied by (21) and (24); i.e., $\dot{Z}_t^h/Z_t^h = (1-\delta)\dot{L}_{r,t}^h/L_{r,t}^h$ and $\dot{Z}_t^f/Z_t^f = (1-\delta)\dot{L}_{r,t}^f/L_{r,t}^f$. Therefore, we set the value of n to the average long-run value of $\dot{L}_{r,t}^h/L_{r,t}^h$ and $\dot{L}_{r,t}^f/L_{r,t}^f$, instead of the population growth rate.

[38] Data source: National Center for Science and Engineering Statistics.

[39] Data source: Eurostat.

[40] We do not include data from 2008 onwards due to the international financial crises.

[41] Data source: Eurostat.

[42] Data sources: Eurostat, and OECD Labor Force Statistics.

[43] Data source: The Conference Board Total Economy Database.

[44] Data source: Federal Reserve Economic Data and ECB Statistical Data Warehouse.

[45] Data source: Eurostat.

Table 4: Calibrated parameter values														
σ	n	ρ	μ	z^h	z^f	θ^h	θ^f	α	s	δ	ξ^h	ξ^f	ϕ^h	ϕ^f
2.46	0.035	0.05	1.28	1.02	1.02	1.92	1.84	0.58	0.50	0.80	0.16	0.63	0.33	0.56

Under these calibrated parameter values, we can compute the effects of inflation on R&D in the two economies and compare these values to our regression estimate in Section 2. We find that when π^h increases by 1%, R&D/GDP in the US decreases by 0.266 percent (percent change). When π^f increases by 1%, R&D/GDP in the Euro Area decreases by 0.448 percent (percent change). These simulated values for the semi-elasticity of R&D with respect to inflation are in line with the panel regression estimate of -0.374 reported in Section 2.

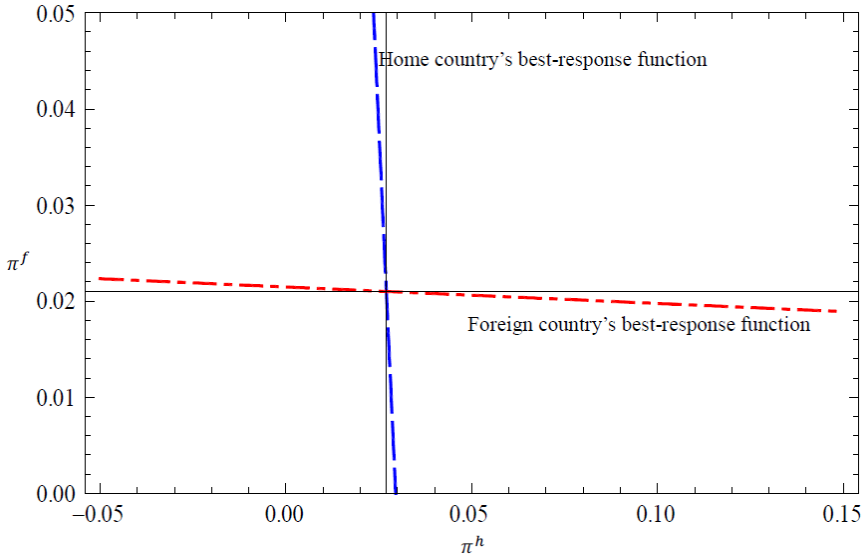

Figure 1: Non-cooperative governments' best-response functions

We can also numerically simulate the best-response functions of the two economies. Figure 2 shows that the best-response functions are downward-sloping implying that the monetary policy instruments $\{\pi^h, \pi^f\}$ are strategic substitutes. Under the CES aggregator in (6), one can show that given $\sigma > 1$, the market share of final goods (i.e., from (7), $p^h_{y,t}Y^h_t/C_t = \alpha^\sigma/(p^h_{y,t})^{\sigma-1}$) is decreasing in π^h and increasing in π^f due to an international business-stealing effect of technologies $\{Z^h_t, Z^f_t\}$ on market share.[46] Therefore, when the foreign government reduces π^f to increase foreign technology, the optimal response of the home government is also to reduce π^h in order to improve domestic technology and compete for market share. In this case, the best-response functions should be upward-sloping; however, there is also a technology-spillover effect across countries. From (33), the level of consumption in the home country is $c^h_t N^h_t = p^h_{y,t}Y^h_t = \alpha^\sigma C_t/(p^h_{y,t})^{\sigma-1}$, where the aggregate production of C_t is

$$C_t = \left[\alpha(Z^h_t L^h_{x,t})^{(\sigma-1)/\sigma} + (1-\alpha)(Z^f_t L^f_{x,t})^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \tag{75}$$

[46] Derivations available upon request; see Appendix B.

which uses (6), (19) and (22). We see that an increase in foreign technology Z_t^f increases aggregate consumption, which in turn increases home consumption (holding $p_{y,t}^h$ constant) capturing the technology-spillover effect. In other words, when the foreign government reduces π^f to increase foreign technology, the optimal response of the home government is to increase π^h to free-ride on the technology improvement in the foreign country. Equation (75) shows that an increase in Z_t^f is a closer substitute to an increase in Z_t^h as the substitution elasticity σ increases. The fact that the best-response functions are downward-sloping in Figure 2 implies that this technology-spillover effect dominates the international business-stealing effect under the calibrated parameter values.

Finally, our policy experiments are as follows. First, we lower the inflation rates in both economies from the Nash equilibrium to their globally optimal level and examine the effects on social welfare $\{U^h, U^f\}$. Second, we consider a unilateral deviation from the Nash equilibrium to the optimal inflation rate that maximizes aggregate welfare of the two economies and examine the asymmetric implications on the two economies. Under the current set of calibrated parameter values, the optimal nominal interest rates in both economies are zero (i.e., the Friedman rule is socially optimal) implying that the optimal inflation rates are $\{\pi_*^h, \pi_*^f\} = \{-r, -r\}$. We first consider the case in which the two governments are cooperative and agree to decrease the inflation rates from the Nash equilibrium to the globally optimal level of $-r$. In this case, the welfare gains are nonnegligible and equivalent to a permanent increase in consumption of 1.038% in the US and 0.249% in the Euro Area as reported in Table 5.[47] However, a unilateral deviation to decrease the inflation rate from the Nash equilibrium would hurt the domestic economy and only benefit the foreign economy, and the cross-country spillover effects are quantitatively significantly. For example, we find that a unilateral decrease in the inflation rate in the Euro Area would improve welfare in the US by 1.079% but reduce its own welfare by 0.213%. Intuitively, a decrease in inflation raises labor supply L^f via the CIA constraints, but the resulting expansion in production in the Euro Area increases consumption in both economies. It is useful to note that the welfare cost of decreasing leisure is borne by the Euro Area but by not the US. As a result, the US experiences a welfare gain whereas the Euro Area experiences a welfare loss. The opposite is true when the US unilaterally decreases inflation. We see in Table 5 that the Euro Area generally experiences a larger welfare loss (or a smaller welfare gain) than the US. The reason is that the money-consumption ratio is much higher in the Euro Area (0.63) than in the US (0.16), which in turn implies that the CIA parameters are larger in the Euro Area than in the US as reported in Table 4. In this case, when inflation decreases, leisure decreases by a larger amount in the Euro Area than in the US, generating the asymmetric welfare effects across the two countries.

Table 5: Welfare effects of monetary policy

	ΔU^h	ΔU^f
Cooperative policy (π^h=π^f= $-r$)	1.038%	0.249%
Unilateral policy (π^f= $-r$)	1.079%	−0.213%
Unilateral policy (π^h= $-r$)	−0.033%	0.470%

[47] Welfare gains are expressed as the usual equivalent variation in consumption.

5 Conclusion

In this study, we have analyzed the growth and welfare effects of inflation in an open-economy version of the Schumpeterian growth model with CIA constraints on consumption and R&D investment. We find that economic growth and social welfare are affected by domestic and foreign inflation. Furthermore, the cross-country welfare effects of inflation are quantitatively significant. These spillover effects give rise to an inflationary bias in the Nash equilibrium and prevent noncooperative governments from implementing optimal policies in the long run. Given that the optimal nominal interest rates in the two countries are zero in our simulation, a supranational authority choosing a uniform interest rate to maximize global welfare would improve welfare. Our analysis serves to provide a quantification of the potential welfare gains from a common monetary policy in monetary unions.[48]

Finally, our analysis is based on a semi-endogenous-growth version of the Schumpeterian model that removes scale effects. It may be a fruitful extension to explore the cross-country spillover effects of inflation in other vintages of the Schumpeterian growth model, such as the second-generation Schumpeterian growth model in Peretto (1998), Howitt (1999) and Segerstrom (2000). We leave this interesting extension to future research.

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Appendix A

Proof of Proposition 1. We assume that the monetary authority adjusts π_t^h to ensure a stationary i^h.[49] We define a transformed variable $\Phi_t \equiv p_{y,t}^h Y_t^h / v_t^h$. Then, differentiating Φ_t with respect to t yields

$$\frac{\dot{\Phi}_t}{\Phi_t} \equiv \frac{\dot{p}_{y,t}^h}{p_{y,t}^h} + \frac{\dot{Y}_t^h}{Y_t^h} - \frac{\dot{v}_t^h}{v_t^h} = \frac{\dot{c}_t^h}{c_t^h} + n - \frac{\dot{v}_t^h}{v_t^h}, \tag{A1}$$

where the second equality follows from (33). Combining (14), (16) and (18), the no-arbitrage condition for v_t^h can be expressed as

$$\frac{\dot{v}_t^h}{v_t^h} = r_t^h - \left(\frac{\mu - 1}{\mu}\right) \Phi_t^h + \frac{\varphi \left(l_{r,t}^h\right)^{1-\delta}}{\varsigma_t^h}, \tag{A2}$$

where $\varsigma_t^h \equiv Z_t^h / \left(N_t^h\right)^{1-\delta}$. Substituting the Euler equation $\dot{c}_t^h / c_t^h = r_t^h - \rho - n$ and (A2) into (A1) yields

$$\frac{\dot{\Phi}_t}{\Phi_t} = \left(\frac{\mu - 1}{\mu}\right) \Phi_t^h - \frac{\varphi \left(l_{r,t}^h\right)^{1-\delta}}{\varsigma_t^h} - \rho. \tag{A3}$$

To derive a relationship between $l_{r,t}^h$, Φ_t^h and ς_t^h, we first use $p_{y,t}^h = \exp\left(\int_0^1 \ln p_{x,t}^h(j) dj\right)$ and (13) to derive $p_{y,t}^h = \mu w_t^h / Z_t^h$. Substituting this condition, (19) and (33) into (4) yields

$$l_t^h = 1 - \mu\theta^h \left(1 + \xi^h i^h\right) l_{x,t}^h. \tag{A4}$$

Then, using (15) and (17) yields

$$\lambda_t^h = \left(\frac{1 + \phi^h i^h}{\mu}\right) \left(\frac{l_{r,t}^h}{l_{x,t}^h}\right) \Phi_t^h. \tag{A5}$$

Combining (A4), (A5) and $l_t^h = l_{r,t}^h + l_{x,t}^h$, we obtain

$$\lambda_t^h = \left\{ \frac{\left[1 + \mu\theta^h \left(1 + \xi^h i^h\right)\right] \left(1 + \phi^h i^h\right)}{\mu} \right\} \left(\frac{l_{r,t}^h}{1 - l_{r,t}^h}\right) \Phi_t^h. \tag{A6}$$

Combining (18) and (A6) yields the following relationship between $l_{r,t}^h$, Φ_t^h and ς_t^h:

$$l_{r,t}^h = J^h \left(\Phi_t^h, \varsigma_t^h\right), \tag{A7}$$

where

$$J_{\Phi^h}^h = -\left\{ \frac{\left[1 + \mu\theta^h \left(1 + \xi^h i^h\right)\right] \left(1 + \phi^h i^h\right)}{\mu\varphi \left[1 + \delta \left(1 - l_r^h\right) / l_r^h\right]} \right\} \left(l_r^h\right)^{\delta} \varsigma^h < 0, \tag{A8}$$

$$J_{\varsigma^h}^h = -\left\{ \frac{\left[1 + \mu\theta^h \left(1 + \xi^h i^h\right)\right] \left(1 + \phi^h i^h\right)}{\mu\varphi \left[1 + \delta \left(1 - l_r^h\right) / l_r^h\right]} \right\} \left(l_r^h\right)^{\delta} \Phi^h < 0. \tag{A9}$$

[49] In the steady state, a stationary π^h ensures a stationary $i^h = \pi^h + \rho + (2 - \delta)n$.

Based on (21), (38), (A3) and (A7), the following dynamic system in terms of Φ_t^h and ς_t^h can be described by

$$\frac{\dot{\Phi}_t}{\Phi_t} = \left(\frac{\mu-1}{\mu}\right)\Phi_t^h - \frac{\varphi\left[l_{r,t}^h\left(\Phi_t^h,\varsigma_t^h\right)\right]^{1-\delta}}{\varsigma_t^h} - \rho, \tag{A10}$$

$$\frac{\dot{\varsigma}_t^h}{\varsigma_t^h} = \frac{\left(\varphi \ln z^h\right)\left[l_{r,t}^h\left(\Phi_t^h,\varsigma_t^h\right)\right]^{1-\delta}}{\varsigma_t^h} - (1-\delta)\, n. \tag{A11}$$

Linearizing (A10) and (A11) around the steady-state equilibrium yields

$$\begin{bmatrix} \dot{\Phi}_t^h \\ \dot{\varsigma}_t^h \end{bmatrix} = \underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{Jacobian\ matrix} \begin{bmatrix} \Phi_t^h - \Phi^h \\ \varsigma_t^h - \varsigma^h \end{bmatrix}, \tag{A12}$$

where

$$a_{11} = \Phi^h\left[\left(\frac{\mu-1}{\mu}\right) - \frac{\varphi(1-\delta)}{\left(l_r^h\right)^{\delta}\varsigma^h} J_{\Phi^h}^h\right] > 0,\ a_{12} = -\frac{\varphi\left(l_r^h\right)^{1-\delta}\Phi^h}{\left(\varsigma^h\right)^2}\left\{\frac{(1-\delta)\varsigma^h}{l_r^h} J_{\varsigma^h}^h - 1\right\} > 0,$$

$$a_{21} = \frac{\left(\varphi \ln z^h\right)(1-\delta)}{\left(l_r^h\right)^{\delta}} J_{\Phi^h}^h < 0,\ a_{22} = \frac{\left(\varphi \ln z^h\right)\left(l_r^h\right)^{1-\delta}}{\varsigma^h}\left\{\frac{(1-\delta)\varsigma^h}{l_r^h} J_{\varsigma^h}^h - 1\right\} < 0.$$

Let κ_1 and κ_2 be the two characteristic roots of the dynamic system. The determinant of Jacobian is given by

$$\text{Det} = \kappa_1\kappa_2 = a_{11}a_{22} - a_{21}a_{12} = \left(\frac{\mu-1}{\mu}\right)\frac{\left(\varphi \ln z^h\right)\left(l_r^h\right)^{1-\delta}\Phi^h}{\varsigma^h}\left[\frac{(1-\delta)\varsigma^h}{l_r^h} J_{\varsigma^h}^h - 1\right] < 0. \tag{A13}$$

As indicated in (A13), the two characteristic roots have opposite signs. Together with the fact that Φ_t^h is a jump variable and ς_t^h is a state variable, these findings imply that the dynamic system displays saddle-path stability.

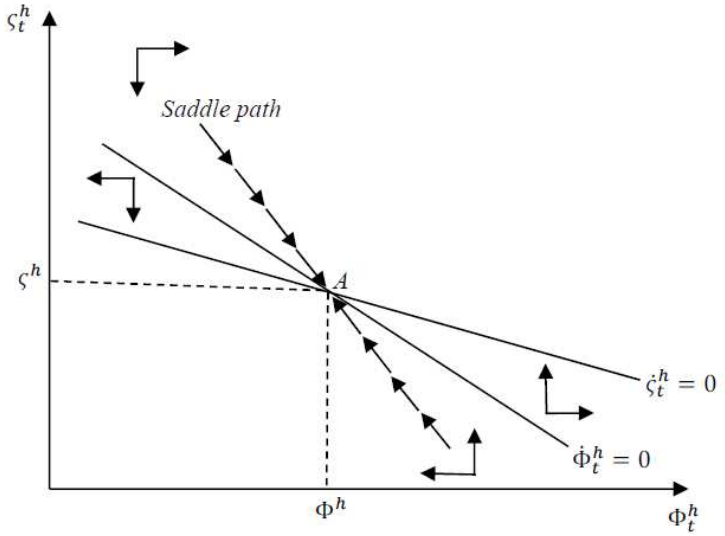

Figure 2: Phase diagram

The phase diagram is plotted in Figure 2, where the $\dot{\Phi}_t^h = 0$ locus is steeper than the $\dot{\varsigma}_t^h = 0$ locus. Figure 2 shows that Φ_t^h and ς_t^h gradually converge to a unique steady-state equilibrium in point A. An analogous proof would show that Φ_t^f and ς_t^f also gradually converge to their steady-state values. When $\{\Phi_t^h, \varsigma_t^h, \Phi_t^f, \varsigma_t^f\}$ are in the steady state, it can be shown that the economy is on a unique and stable balanced growth path. ■

Proof of Proposition 2. Setting $\dot{\Phi}_t^h = 0$ and $\dot{\varsigma}_t^h = 0$ in (A10) and (A11) yields the steady-state equilibrium values of Φ_t^h and ς_t^h given by

$$\Phi^h = \left(\frac{\mu}{\mu - 1}\right)\left\{\frac{(1-\delta)\,n}{\ln z^h} + \rho\right\}, \tag{A14}$$

$$\varsigma^h = \frac{\varphi \ln z^h}{(1-\delta)\,n}\left(l_r^h\right)^{1-\delta}, \tag{A15}$$

where l_r^h is still an endogenous variable. From (A15) and (18), the steady-state arrival rate of innovation in country h is exogenous and given by

$$\lambda^h = \frac{(1-\delta)\,n}{\ln z^h}. \tag{A16}$$

Substituting (A16) into (A14) yields $\Phi^h = \mu\left(\rho + \lambda^h\right)/(\mu - 1)$. We make use of this condition and (A5) to obtain (30). Solving (30), (A4) and $l^h = l_r^h + l_x^h$ yields the steady-state equilibrium labor allocations in (35), (36) and (37). Substituting (35) into (A15) yields the steady-state value of ς^h. ■

Proof of Proposition 3. The analogous expression of (46) for U^f is given by

$$\rho U^f = \alpha[\ln l_x^h + (1-\delta)\ln l_r^h] + (1-\alpha)[\ln l_x^f + (1-\delta)\ln l_r^f] + \theta^f \ln(1 - l^f). \tag{A17}$$

The analogous expressions of (35)-(37) in country f are

$$l_r^f = \frac{\frac{\mu-1}{1+\phi^f i^f}\frac{\lambda^f}{\rho+\lambda^f}}{1 + \mu\theta^f(1+\xi^f i^f) + \frac{\mu-1}{1+\phi^f i^f}\frac{\lambda^f}{\rho+\lambda^f}}, \tag{A18}$$

$$l_x^f = \frac{1}{1 + \mu\theta^f(1+\xi^f i^f) + \frac{\mu-1}{1+\phi^f i^f}\frac{\lambda^f}{\rho+\lambda^f}}, \tag{A19}$$

$$l^f = \frac{1 + \frac{\mu-1}{1+\phi^f i^f}\frac{\lambda^f}{\rho+\lambda^f}}{1 + \mu\theta^f(1+\xi^f i^f) + \frac{\mu-1}{1+\phi^f i^f}\frac{\lambda^f}{\rho+\lambda^f}}, \tag{A20}$$

where $i^f = \pi^f + r = \pi^f + \rho + n + g_c^f = \pi^f + \rho + (2-\delta)n$, which is increasing in π^f. Under inelastic labor supply, we set $\theta^h = \theta^f = 0$ in (35)-(37) and (A18)-(A20). Then, we substitute the resulting expressions into $U^h + U^f$ from (46) and (A17) and differentiate it with respect to $\{\pi^h, \pi^f\}$ to obtain the optimal inflation rates given by

$$\pi_*^h = \frac{1}{\phi^h}\left[\frac{\mu-1}{1-\delta}\frac{\lambda^h}{\rho+\lambda^h} - 1\right] - r, \tag{A21}$$

$$\pi_*^f = \frac{1}{\phi^f}\left[\frac{\mu-1}{1-\delta}\frac{\lambda^f}{\rho+\lambda^f}-1\right]-r. \tag{A22}$$

Therefore, $\{\pi_*^h, \pi_*^f\} = \{\pi_{ne}^h, \pi_{ne}^f\}$ in (51) and (52). ■

Proof of Proposition 4. In the absence of the CIA constraint on consumption, we set $\xi^h = \xi^f = 0$ in (35)-(37) and (A18)-(A20). The government in country h chooses π^h to maximize the welfare of the representative household in country h. We substitute (35)-(37) and (A18)-(A19)under $\xi^h = 0$ into U^h in (46) and then differentiate it with respect to π^h to obtain the Nash-equilibrium inflation rate π_{ne}^h in country h given by (60). Similarly, the government in country f chooses π^f to maximize the welfare of the representative household in country f. We substitute (35)-(36) and (A18)-(A20) under $\xi^f = 0$ into U^f in (A17) and then differentiate it with respect to π^f to obtain the Nash-equilibrium inflation rate π_{ne}^f in country f given by (61). The cooperative governments choose $\{\pi^h, \pi^f\}$ to maximize the welfare of both domestic and foreign households. We set $\xi^h = \xi^f = 0$ in (35)-(37) and (A18)-(A20) and substitute the resulting expressions into $U^h + U^f$ from (46) and (A17). Then, we differentiate $U^h + U^f$ with respect to $\{\pi^h, \pi^f\}$ to obtain the optimal inflation rates given by (62) and (63). Taking the difference between π_{ne}^h and π_*^h as shown in (64) and then differentiating it with respect to μ, we find that

$$\frac{\partial\left(\pi_{ne}^h - \pi_*^h\right)}{\partial\mu} = \frac{1+\theta^h}{\left(\mu\theta^h+1\right)^2}\frac{\theta^h}{2\alpha\phi^h\left(1-\delta\right)}\frac{\lambda^h}{\lambda^h+\rho} > 0. \tag{A23}$$

Equation (A23) shows that the wedge between the Nash-equilibrium and optimal inflation rates is monotonically increasing in the market power of firms. ■

Proof of Proposition 5. In the absence of the CIA constraint on R&D, we set $\phi^h = \phi^f = 0$ in (35)-(37) and (A18)-(A20). The government in country h chooses π^h to maximize the welfare of the representative household in country h. We substitute (35)-(37) and (A18)-(A19) under $\phi^h = 0$ into U^h in (46) and then differentiate it with respect to π^h to obtain the Nash-equilibrium inflation rate π_{ne}^h in country h given by (70). Similarly, the government in country f chooses π^f to maximize the welfare of the representative household in country f. We substitute (35)-(36) and (A18)-(A20) under $\phi^f = 0$ into U^f in (A17) and then differentiate it with respect to π^f to obtain the Nash-equilibrium inflation rate π_{ne}^f in country f given by (71). The cooperative governments choose $\{\pi^h, \pi^f\}$ to maximize the welfare of both domestic and foreign households. We set $\phi^h = \phi^f = 0$ in (35)-(37) and (A18)-(A20) and substitute the resulting expressions into $U^h + U^f$ from (46) and (A17). Then, we differentiate $U^h + U^f$ with respect to $\{\pi^h, \pi^f\}$ to obtain the optimal inflation rates given by (72) and (33). Taking the difference between π_{ne}^h and π_*^h as shown in (74) and then differentiating it with respect to μ, we find that

$$\frac{\partial\left(\pi_{ne}^h - \pi_*^h\right)}{\partial\mu} = -\frac{1}{\mu^2}\frac{\rho}{\lambda^h+\rho}\frac{1}{2\alpha\xi^h\left(2-\delta\right)} < 0. \tag{A24}$$

Equation (A24) shows that the wedge the Nash-equilibrium and optimal inflation rates is monotonically decreasing in the market power of firms. ■

Appendix B (not for publication)

Table 6: Panel regressions using HP-detrended

Dependent variable: 100*log(R&D/GDP)			
Method:	Pooled regression	Country FE	Country and year FE
Regressor			
Cyclical inflation	-0.1530	-0.2504	-0.1949
p-values	(0.797)	(0.261)	(0.361)
Observations	648	648	648
Adj-R^2	0.4036	0.9171	0.9286
Notes: FE denotes fixed effects.			

B.1 The price index $p_{y,t}^h$. Combining (9) and (10) yields

$$Y_t^h = \exp\left(\int_0^1 \ln\left[\left(p_{y,t}^h Y_t^h\right)/p_{x,t}^h(j)\right]dj\right). \tag{B1}$$

Then, manipulating (B1) yields the standard price index of Y_t^h given by $p_{y,t}^h = \exp\left\{\int_0^1 \ln[p_{x,t}^h(j)]dj\right\}$. ■

B.2 Proof of $r - g_v^h = \rho$ on the BGP. First, substituting (16) and (17) into (32), we obtain $c_t^h N_t^h = \omega_{x,t}^h + w_t^h L_{x,t}^h$. Combining this condition, (14) and (15) yields $c_t^h N_t^h = p_{y,t}^h Y_t^h$ as shown in (33). Then, substituting (18) into (17) and differentiating it with respect to time yields

$$\frac{\dot{v}_t^h}{v_t^h} = \frac{\dot{w}_t^h}{w_t^h} + n, \tag{B2}$$

where we have used (25). Using (15) and (33), (B2) can be rearranged as

$$g_v^h \equiv \frac{\dot{v}_t^h}{v_t^h} = \frac{\dot{c}_t^h}{c_t^h} + n. \tag{B3}$$

Finally, we make use of the familiar Euler equation $\dot{c}_t^h/c_t^h = r - \rho - n$ and (B3) to derive $g_v^h = r - \rho$ on the BGP. ■

B.3 The first-best optimal ratio of R&D to production labor. Using standard dynamic optimization, we maximize a lifetime utility function given by

$$U^h = \int_0^\infty e^{-\rho t}\left[\ln c_t^h + \ln c_t^f + \theta^h \ln(1 - l_t^h) + \theta^f \ln(1 - l_t^f)\right]dt, \tag{B4}$$

subject to (7), (8), (19), (21), (22), (24), $l_t^h = l_{x,t}^h + l_{r,t}^h$ and $l_t^f = l_{x,t}^f + l_{r,t}^f$. We obtain that the optimal ratio $l_{r,t}^h/l_{x,t}^h$ is

$$\frac{l_{r,t}^h}{l_{x,t}^h} = \frac{\eta_t^h\left(\varphi \ln z^h\right)\left(l_{r,t}^h N_t^h\right)^{1-\delta}}{2\alpha/(1-\delta)}. \tag{B5}$$

The intertemporal optimality condition is

$$-\frac{\dot{\eta}_t^h}{\eta_t^h} = \frac{2\alpha}{\eta_t^h Z_t^h} - \rho. \tag{B6}$$

Substituting (25) into (B5), we derive

$$\frac{l_{r,t}^h}{l_{x,t}^h} = \frac{(1-\delta)^2 n}{2\alpha}\eta_t^h Z_t^h \tag{B7}$$

Then, differentiating (B7) with respect to time yields $\dot{\eta}_t^h/\eta_t^h = -(1-\delta)n$. Combining this equation and (B6) and substituting it into (B7), we derive the optimal ratio $\tilde{l}_r^h/\tilde{l}_x^h$ as shown in (53). ■

B.4 The non-cooperative governments' best-response functions and their welfare functions. Substituting (7) into (33) yields $c_t^h N_t^h = \alpha\left(Y_t^h\right)^{(\sigma-1)/\sigma}(C_t)^{1/\sigma}$. Substituting this condition along with (6) and $g_c^h = (1-\delta)n$ into (40) yields

$$\rho U^h = \frac{\sigma-1}{\sigma}\ln Y_0^h + \frac{1}{\sigma-1}\ln\left[\alpha\left(Y_0^h\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(Y_0^f\right)^{\frac{\sigma-1}{\sigma}}\right] + \theta^h\ln\left(1-l^h\right), \tag{B8}$$

where we have dropped all the exogenous terms. Substituting (42)-(45) into (B8) yields

$$\begin{aligned}\rho U^h &= \frac{\sigma-1}{\sigma}\left[(1-\delta)\ln l_r^h + \ln l_x^h\right] \\ &+\frac{1}{\sigma-1}\ln\left\{\alpha\left[\frac{\left(\varphi\ln z^h\right)\left(l_r^h\right)^{1-\delta}l_x^h}{(1-\delta)n}\right]^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left[\frac{\left(\varphi\ln z^f\right)\left(l_r^f\right)^{1-\delta}l_x^f}{(1-\delta)n}\left(\frac{1-s}{s}\right)^{2-\delta}\right]^{\frac{\sigma-1}{\sigma}}\right\} \\ &+\theta^h\ln\left(1-l^h\right),\end{aligned} \tag{B9}$$

where we have once again dropped the exogenous terms. The government in country h chooses π^h to maximize the welfare of the representative household in country h. We substitute (35)-(37) into U^h in (B9) and then differentiate it with respect to π^h to obtain the best-response function in country h given by

$$(1-\delta)\left(\lambda^h+\rho\right)\left\{\mu\theta^h\xi^h\left(1+\phi^h i^h\right) + \phi^h\left[1+\mu\theta^h\left(1+\xi^h i^h\right)\right]\right\} - \Psi^h = \frac{\Delta^h}{\frac{\sigma-1}{\sigma} + \frac{\alpha/\sigma}{\alpha+(1-\alpha)\Omega^h}} \tag{B10}$$

where

$$\Psi^h \equiv \frac{(\mu-1)\lambda^h\phi^h - \mu\theta^h\xi^h\left(\lambda^h+\rho\right)\left(1+\phi^h i^h\right)^2}{1+\phi^h i^h}, \tag{B11}$$

$$\Delta^h \equiv \theta^h\frac{(\mu-1)\lambda^h\left[\xi^h\left(1+\phi^h i^h\right) + \phi^h\left(1+\xi^h i^h\right)\right] + \xi^h\left(\lambda^h+\rho\right)\left(1+\phi^h i^h\right)^2}{\left(1+\xi^h i^h\right)\left(1+\phi^h i^h\right)}, \tag{B12}$$

$$\Omega^h \equiv \left[\frac{Z_0^f l_x^f}{Z_0^h l_x^h}\left(\frac{1-s}{s}\right)\right]^{\frac{\sigma-1}{\sigma}}. \tag{B13}$$

Moreover, the analogous expression of (B9) for U^f is given by

$$\begin{aligned}\rho U^f &= \frac{\sigma-1}{\sigma}\left[(1-\delta)\ln l_r^f + \ln l_x^f\right] \\ &+\frac{1}{\sigma-1}\ln\left\{\alpha\left[\frac{\left(\varphi\ln z^h\right)\left(l_r^h\right)^{1-\delta}l_x^h}{(1-\delta)n}\left(\frac{s}{1-s}\right)^{2-\delta}\right]^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left[\frac{\left(\varphi\ln z^f\right)\left(l_r^f\right)^{1-\delta}l_x^f}{(1-\delta)n}\right]^{\frac{\sigma-1}{\sigma}}\right\} \\ &+\theta^f\ln\left(1-l^f\right).\end{aligned} \tag{B14}$$

The analogous expression of (B10) for the foreign government's best-response function is given by

$$(1-\delta)\left(\lambda^f+\rho\right)\left\{\mu\theta^f\xi^f\left(1+\phi^f i^f\right)+\phi^f\left[1+\mu\theta^f\left(1+\xi^f i^f\right)\right]\right\} - \Psi^f = \frac{\Delta^f}{\frac{\sigma-1}{\sigma}+\frac{(1-\alpha)/\sigma}{\alpha\Omega^f+(1-\alpha)}} \tag{B15}$$

where

$$\Psi^f \equiv \frac{(\mu-1)\lambda^f\phi^f - \mu\theta^f\xi^f\left(\lambda^f+\rho\right)\left(1+\phi^f i^f\right)^2}{1+\phi^f i^f}, \tag{B16}$$

$$\Delta^f \equiv \theta^f\frac{(\mu-1)\lambda^f\left[\xi^f\left(1+\phi^f i^f\right)+\phi^f\left(1+\xi^f i^f\right)\right]+\xi^f\left(\lambda^f+\rho\right)\left(1+\phi^f i^f\right)^2}{\left(1+\xi^f i^f\right)\left(1+\phi^f i^f\right)}, \tag{B17}$$

$$\Omega^f \equiv \left[\frac{Z_0^h l_x^h}{Z_0^f l_x^f}\left(\frac{s}{1-s}\right)\right]^{\frac{\sigma-1}{\sigma}}. \tag{B18}$$

We use (B10) and (B15) to numerically simulate the best-response functions of the two economies and use (B9) and (B14) to compute the welfare effect of monetary policy. Figure 1 and Table 5 present the results, respectively. ■

B.5 International business-stealing effect. Combining (7) and (33) yields $c_t^h N_t^h/C_t = \alpha\left(Y_t^h/C_t\right)^{(\sigma-1)/\sigma}$. Substituting (6) and (42) into this condition yields

$$\frac{c^h N^h}{C} = \frac{\alpha}{\alpha+(1-\alpha)\Omega^h}, \tag{B19}$$

where Ω^h is a function of the variables $\left\{Z_0^h, Z_0^f, l_x^h, l_x^f\right\}$ satisfying (B13). Substituting (35), (36), and (44) into (B19) and differentiating it with respect to π^h yields

$$\frac{\partial\left(c^h N^h/C\right)}{\partial\pi^h} = -\frac{\alpha(1-\alpha)(\sigma-1)\Omega^h/\sigma}{\left[\alpha+(1-\alpha)\Omega^h\right]^2}\left\{\frac{\Delta^h}{\frac{\sigma-1}{\sigma}+\frac{\alpha/\sigma}{\alpha+(1-\alpha)\Omega^h}}\right\} < 0, \tag{B20}$$

where we have used (B10). Substituting (45), (A18) and (A19) into (B19) and differentiating it with respect to π^f yields

$$\frac{\partial\left(c^h N^h/C\right)}{\partial \pi^f} = \frac{\alpha\left(1-\alpha\right)\left(\sigma-1\right)\Omega^h/\sigma}{\left[\alpha+\left(1-\alpha\right)\Omega^h\right]^2}\left\{\frac{\Delta^f}{\frac{\sigma-1}{\sigma}+\frac{(1-\alpha)/\sigma}{\alpha\Omega^f+(1-\alpha)}}\right\} > 0, \tag{B21}$$

where we have used (B15). Based on (B20) and (B21), the market share of final goods is decreasing in π^h and increasing in π^f due to the international business-stealing effect via technologies $\left\{Z_0^h, Z_0^f\right\}$. ■