Measuring Contagion Risk in High Volatility State between Major Banks in Taiwan by Threshold Copula GARCH Model

Su, EnDer

National Kaohsiung First University of Science and Technology

26 August 2014

Measuring Contagion Risk in High Volatility State between Major Banks in Taiwan by Threshold Copula GARCH Model

EnDer Su*

Abstract

This paper aims to study the structural tail dependences and risk magnitude of contagion risk during high risk state between domestic and foreign banks. Empirically, volatility of stock returns has the properties of persistence, clustering, heteroscedasticity, and regime switchs. Thus, the threshold regression model having piecewise regression capability is used to classify the volatility index of influential foreign banks as "high" and "low" of two volatility states to further estimate Kendall taus i.e. structural tail dependences between banks using three models: Gaussian, t, and Clay copula GARCH. Using fuzzy c-means method, both domestic and foreign banks are categorized into 10 groups. Through the groups, 5 domestic and 7 foreign representative banks are identified as the research objects. Then, the in-sample data of daily banks' stock prices covering 01/03/2003 ~06/30/2006 are used to estimate the parameters of threshold copula GARCH model and Kendall taus. The out-of-sample data covering 07/01/2006~03/25/2014 are used to estimate the Kendall taus gradually using rolling window technique. Several research findings are described as follows. In high state, the tail dependences are two times much larger than in low state and most of them have up-trend property after sub-prime crisis and reach the peak during Greek debt. It implies that the volatility is high in risk event and the contagion is high after risk event. In high state, HNC has the highest tail dependences with foreign banks but its value at risk is the lowest. It can be considered as an international attribute bank with lower risk. On the contrary, YCB and FCB have the lower tail dependences with foreign banks but their value at risks are quite high. They are viewed as a local attribute bank with higher risk. The Bank of American, Citigroup, and UBS AG have the relatively higher value at risk. Citigroup has been tested to Granger cause ANZ and all domestic banks. It is necessary to beware the contagion risk from Citigroup. Among three models, in low state, Gaussian and t copula models have the better significance of estimation than Clay copula model. However in high state, Clay copula model has the same acceptable estimation and more capability to uncover the instant nonlinear jumps of tail dependences while Gaussian and t copula models reveal the linear changes of tail dependences as a curve.

Keywords: Contagion Risk, Threshold GARCH, Copula, Tail Dependences

---

*Corresponding author,
Risk Management and Insurance
National Kaohsiung First University of Science and Technology
No.2, Jhuoyue Rd., Nansih District, Kaohsiung City 811, Taiwan, R.O.C.
Phone: 886-7-6011000-3022
E-mail: suender@ccms.nkfust.edu.tw

1. Research Objectives and Background

The financial liberalization and internationalization have made the financial markets of Taiwan susceptible to different global contagion risks. To date and looking forward to the future, with the promotion of Basel II, the studies of nonlinear extreme risk or fat tail effect in major underlyings such as stock prices, interest rates, and foreign exchanges have become more and more important. In reality, there are strong fat tail dependences and volatility spillover because of financial risk disasters such as Asian financial crisis, U.S. sub-prime crisis, Greek debt crisis, etc. Acting like the butterfly effect or domino effect, the financial risks swiftly spread globally and as a result, the financial institutions suffer substantially everywhere.

Nevertheless, the major domestic and foreign banks in Taiwan such as Mega International Commercial Bank, Chinatrust Commercial Bank, Cathay United Bank, Taiwan Business Bank, Yunata Commercial Bank, Fubon Commercial Bank etc. and Australia and New Zealand Bank, Mizuho Corporate Bank, JPMorgan, Citigroup, Deutsche Bank, ING Bank, Standard Chartered Bank, etc. have owned a number of international financial portfolios constructed by stock indices, bonds, currency, CMOs, and CDOs etc and definitively are not immune from the effect of global financial risks. To measure the extent to which the financial risks between the major banks spread, a new concept of contagion risk therefore has used as well as value at risk(VaR) to describe how the financial disasters affect the median and small size banks in Taiwan compared to global larger banks such as JPMorgan, Bank of American, HSBC, and Citigroup.

However, the contagious risk disperses differently as the bank's volatility stay in different states. For example, in high volatility state, the volatility spillover effect, leverage effect, fat tail effect, or asymmetric effect become stronger and larger. Hence, the contagious risk influences much more widely and deeply among major banks in high volatility state while it is much weaker in low volatility state. Therefore, it is necessary to apply the threshold regression technique beforehand to distinguish the state of contagion risk and then evidence the contagion risk spreads among banks.

In addition, the multivariate distribution with nonlinear properties for dynamic process is too complicated to be estimated. The traditional measure of correlation lacks to describe the multivariate distribution between underlying assets and cannot specify the multivariate tail dependences either. Fortunately, a new structural dependence called copula is used recently to describe the multivariate distribution and structural tail dependences in many financial aspects including credit risk of bond portfolios, default risk of mortgages, and contagion risk of financial markets. The multi-dimensional distribution is able to be constructed by simply combing uni-variate marginal distributions and one proper copula according to Sklar's theorem (1959). As a result, the structural tail dependences and risk measurement are estimated by various copulas to describe the contagious risk accordingly.

To discrete the risk states and model the multivariate distribution, this paper develops a threshold GARCH model with copulas to study the contagious risk among the major banks in Taiwan. The downside structural tail dependences and risk measurement such as value at risk or expected shortfall are used to describe the extent to which a large shock of one bank impacts the other banks in certain context particularly when the global financial markets crash together i.e. occur in a high volatility state.

2. Literature Review

The contagion risk is studied by not only the structural tail dependences but the tail jumps among multivariate random processes. As shown by Embrechts et al. (2001), the Pearson correlation is too restricted to describe the linear co-movements of two random processes. However, the copulas (e.g. Joe, 1997; Nelsen, 1999) have the advantages to measure the conditional time-varying concordance and tail dependences and thus have been widely and successfully used to study the contagion risk.

It is noticed that the skewness Student-t but not the linear Gaussian copula can measure tail dependences. Evidences show that the stock returns drop more than rise in the size of movements (Ang and Chen, 2002) while the correlation of stock returns is generally higher in a high volatility than in a low volatility regime (Ang and Bekaert, 1999). This phenomenon is called asymmetric effect that cannot be caught by symmetric elliptical copulas such as Gaussian and Student-t copulas. Thus, the Archimedean copulas including Gumbel, Clayton, and Frank copulas are considered to be more plausible to model the asymmetric tail dependences. The empirical evidences reported that the properties of time-varying volatility of stock returns including volatility asymmetry, clustering, persistence, and leptokurtosis exist in stock returns. To catch the conditional heteroskedasticity volatility, the ARCH model was developed by Engle (1982) and extended by Bollerslev (1986) to create the GARCH model. To date, several GARCH type models were proposed to capture the volatility asymmetry such as the exponential GARCH (EGARCH) model by Nelson (1991), the asymmetric GARCH (AGARCH) by Engle and Ng (1993), the GJR-GARCH by Glosten et al. (1993), the power ARCH by Ding et al. (1993) etc.

Hamilton (1990, 1991) has found that the stock returns exhibits high- and low-state volatilities and the volatility state tends to persist for a while. In this regard, Hamilton and Susmel (1994) proposed an unobserved Markov chain in ARCH to model the property of volatility regime switch. They however recognized that the regime switch in GARCH is not feasible because the condition variances have the issue of path dependence. Thus, the alternative method to deal with different regimes such as the state of returns or volatilities in regression is to use the self-exciting threshold autoregressive (SETAR) first developed by Tong and Lim, 1980. The SETAR model creates and tests threshold value to distinguish different regimes in regression without considering the matrix of transition probability. Later, to model higher nonlinear data, Chan and Tong (1986) considered a smooth transition function of variables such as a logistic or exponential function with higher degree flexibility of parameter in SETAR which is then called smooth threshold autoregression (STAR). Tsay (1989) has provided a reliable procedure to test threshold autoregression and similarly Teräsvirta (1994) has tested STAR. To date, the threshold regression has been considered more plausible (Tong, 1990) and applied by authors such as Tse and Tsui (2002) and Jamaleh and Venezia (2001) who have applied GARCH model with threshold regression.

For the conditional variances and co-variances model of multivariate assets, multivariate GARCH (MGARCH) has been used in Bollerslev, Engle, and Wooldridge (1988), Ng (1991), and Hansson and Hordahl (1998). It was applied to explain the spillover effects of contagion in Tse and Tsui (2002) and Bae, Karolyi and Stulz (2003). Further, Ramchand and Susmel (1998), Ang and Bekaert (2002), Honda (2003) and Haas et al. (2004) have estimated a Markov-switching MGARCH model.

An alternative of MGARCH is the use of copula GARCH proposed by Patton (2001) and Jondeau and Rockinger (2002). Later, Jondeau and Rockinger (2006), Patton (2006), and Hu (2006) applied different copulas in GARCH model to study the tail dependences between financial markets. Rodriguez (2007) without considering GARCH term to avoid estimating the unknown process of regimes has added regime switch to the copula ARCH model. Whereas, Chollete et al. (2009) and Garcia and Tsafack (2011) added regime switch to copula GARCH using the iteratedly conditional expectation of unknown regime process.

3. Data and Methodology

Because the number of unobservable regimes and co-variances grow exponentially and multiplicatively respectively with times, to specify multivariate GARCH (MGARCH) with several underlying assets using regime switch governed by unrealized regime process is not appropriate to study the structural dependence. Thus, the regimes of stock returns are suggested to be classified by fewer threshold parameters in space of previous process as first done by Tong and Lim (1980) who have developed the threshold piecewise autoregression to approximate the nonlinear random process of returns. Since then, the threshold approach was applied by Cao and Tsay (1992) to model the time-varying conditional volatility in different regimes and later the threshold GARCH model was created by Zakoïan (1994). In addition, since the copulas can describe the structural dependence between marginal distributions, they are much better advantageous to be used in multivariate model.

In this paper, the both advantages of the threshold autoregression and copula are used to study the multivariate distribution, tail dependences, and tail jumps in different regimes for the contagion risk between banks. Expectedly, the threshold copula GARCH can reveal valuable information about contagion risk when banks crash together.

3.1 Bank Data

According to statistical data of Financial Supervisory Commission, 38 domestic banks and 28 foreign banks have business operations in Taiwan. The stock prices of domestic banks can be collected from Taiwan Economic Journal (TEJ) while those of foreign banks can be retrieved from the market data of Yahoo Finance that covers the fundamental data of most exchange-traded stocks. The sample period is starting from 2003 up to present 2014 at a daily frequency to cover the durations of major risk events such as sub-prime disaster in early 2008 and Greece debt crisis in April 2010.

It is interesting to reveal the contagion risk between banks through tail dependences and jumps in different states of risk. Thus, the daily bank stock data is distinguished into high or low volatility state according to the threshold value of volatility index. Then, to explain the tail dependences and jumps, the Kendall taus are estimated using threshold value and GARCH model.

3.2 Threshold GARCH and threshold value

The self-exciting threshold autoregressive (SETAR, Tong and Lim, 1980) studies how to use the self-threshold variable u_{t-d} (d is threshold lag) to classify the nonlinear financial process into several regression regimes and explain the behavior of nonlinear process such as volatility heteroscedasticity or volatility switch. Specifically, it intends to create regimes and uses several piecewise autoregression to approximate the nonlinear process. That is why it is called switching linear regression as well. The threshold GARCH would take advantage of the SETAR to model the volatility in different states.

3.2.1 Threshold regression of actual volatility index

The trends and volatility of stock returns are the most important factors to study the properties of stock. In fact, they have the elements of both positive and negative stock returns. Thus, it is not plausible to use threshold values to separate the stock returns without regard to the intact of trends and volatility. One way to solve this issue is to transform stock returns into actual stock volatility which is then further classified into different states by threshold values. It is considered appropriately to divide volatility into “low” and “high” of two states because the volatility switches often between either in high or low state. The actual stock volatility is formulated as

$$v_{i,t} = \sqrt{\frac{\pi}{2}}\left|r_{i,t}\right|, \tag{1}$$

where $r_{i,t}$ the ith stock returns at time t. The stock returns is the natural logarithm of the relative stock prices.

The number of states would increase doubly as one series of bank data is added to study. To keep the number of states concise, the weight average of actual volatilities of several major banks selected by fuzzy c-means is used to estimate the threshold value. It is called actual volatility index and is written as

$$V_t = \sum_{i=1}^{n} w_i v_{it}, \tag{2}$$

where w_i is the weight of the ith bank's capital amount.

The threshold variable is $V_{t-d} \in R_j$ for $j = 1, 2, ..., l-1$ (R_j is the set of jth regime for the actual volatility index of banks and l is the number of regimes). Then, the jth threshold autoregression equation is expressed as

$$v_t = b_0^{(j)} + \sum_{h=1}^{k} b_h^{(j)} v_{t\text{-}h} + \zeta_t^{(j)} \ as \ L_{j\text{-}1} < v_{t\text{-}d} < L_j, \tag{3}$$

where $\zeta_t^{(j)}$ is the autoregressive error of jth regime, k is the order of autoregression, d is the lag of threshold ($d \leqq k$). In other words, $L_1, ..., L_{l-1}$ are the threshold values that can divide v_{t-d} into l regimes of equations as Equation (3).

3.2.2 Threshold GARCH

Suppose that the stock returns of jth regime $\xi_t^{(j)}$ for a specific bank is considered as a time-varying volatility process. According to Engle (1983), the multiplicative conditional heteroscedastic model of $\xi_t^{(j)}$ is

$$\xi_t^{(j)} = \varepsilon_t^{(j)} \sqrt{h_t^{(j)}}, \tag{4}$$

where F_{t-1} is filtration at time t-1 and $\varepsilon_t^{(j)} \mid F_{t-1} \sim N(0,1)$. Thus, the general autoregression conditional heteroscedastic (GARCH) model is written as

$$h_t^{(j)} = c + \alpha_1 \xi_{t-1}^{2,(j)} + ... + \alpha_q \xi_{t-q}^{2,(j)} + \beta_1 h_{t-1}^{(j)} + ... + \beta_p h_{t-p}^{(j)}. \tag{5}$$

However, the empirical h_t might have leverage effect or volatility asymmetry (i.e. bad news has a higher impact on bank stock prices than good news). Thus, the types of asymmetric effect adjusted models are given as follows:

(1) Student-t GARCH

The residual $\varepsilon_t^{(j)}$ in Equation (4) is considered to follow a Student-t distribution as

$$f(\varepsilon_t^{(j)}) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})} \frac{1}{\sqrt{\pi\nu}} (1 + \frac{\varepsilon_t^{(j)2}}{\nu})^{-\frac{\nu+1}{2}}, \tag{6}$$

where ν is the degree of freedom. The Student-t distribution is used to catch the skewness effect and build a likelihood function.

(2) GJR GARCH

To capture the leverage effect between banks' stock returns and volatility, the GJR model (Glosten, Jagannathan, and Runkle, 1993) is applied as

$$h_t^{(j)} = \alpha\varepsilon_{t-1}^{2,(j)} + \beta h_{t-1}^{(j)} + \gamma\varepsilon_{t-1}^{2,(j)} I_{t-1}, \tag{7}$$

where $I_{t-1} = 1$ if $\varepsilon_{t-1}^{(j)} < 0$ and $I_{t-1} = 0$ otherwise.The leverage effect exists if $\gamma > 0$ for GJR.

(3) EGARCH

Similarly, the EGARCH model (Nelson, 1991) can be used to catch the volatility asymmetry and it is represented as

$$\ln(h_t^{(j)}) = \beta\ln(h_{t-1}^{(j)}) + \gamma\frac{\varepsilon_{t-1}^{(j)}}{\sqrt{h_{t-1}^{(j)}}} + \alpha\left[\frac{|\varepsilon_{t-1}^{(j)}|}{\sqrt{h_{t-1}^{(j)}}} - E\left(\frac{|\varepsilon_{t-1}^{(j)}|}{\sqrt{h_{t-1}^{(j)}}}\right)\right], \tag{8}$$

where $\varepsilon_{t-1}^{(j)} \sim N(0, h_{t-1}^{(j)})$ and so that $E\left(\frac{|\varepsilon_{t-1}^{(j)}|}{\sqrt{h_{t-1}^{(j)}}}\right) = \sqrt{\frac{2}{\pi}}$. The impact of $\varepsilon_t^{(j)}$ for good news as $\varepsilon_t^{(j)} > 0$ is $(\gamma+\alpha)\frac{\varepsilon_t^{(j)}}{\sqrt{h_t^{(j)}}}$ and for bad news as $\varepsilon_t^{(j)} < 0$ is $(\gamma-\alpha)\frac{\varepsilon_{t-1}^{(j)}}{\sqrt{h_{t-1}^{(j)}}}$. Thus, for EGARCH, the volatility of price changes is symmetric if $\gamma = 0$ and asymmetric otherwise. Noticeably, if leverage effect exists, γ must be negative.

3.2.3 Test and estimation of the threshold value *L*

If the volatility heteroscedasticity exists, the threshold effect is significant. Hansen (1996) has proposed an asymptotic statistic of Lagrange multiplier, LM, to test the difference between autoregression and threshold autoregression. The error terms ε_t can be obtained from both autoregression and threshold autoregression to create the statistics: SupLM, ExpLM, and AveLM that stand for supremum, exponential and average LM respectively as formulated by Hansen to test threshold effect. Thus, the LM is maximized to find the optimal threshold value *L* under certain threshold parameter *d* which in turn is found by the maximum of maximum LM in different *d*. Regarding stock returns of a bank, *d* is small and less than 7 days of a week.

The volatility of bank stock returns is either in a high or low state. Hence, the number of regimes in threshold autoregression is set to two, i.e. the number of *L* is one. Undoubtedly, the most important of structural tail dependences for contagion risk is when the volatilities of bank stock returns are in a high state. At that time, the structural tail dependences between banks are interesting for the analysis of contagion risk.

3.3 Classification of Banks by Fuzzy C-means Method

To reduce the mass of the analysis, all banks are required to be categorized into a few groups and some representative banks are selected from the groups. The task is performed by the classification method of fuzzy c-means using banks' stock returns, volatility, skewness, and kurtosis.

The fuzzy c-means method is described as follows. With X_i represents the vector of *i*th bank's capital size, stock return and high-state volatility for *i*th bank and *i*=1..*n*, where *n*=the number of the banks and c_j denotes the center of cluster j, i.e. centroid for j=1..k, where k=the number of groups. To find the c-means clusters and centroids is to solve the optimal problem of minimizing sum of square error (SSE) written as

$$\begin{aligned} &\underset{p}{Min}\ \text{SSE} = \sum_{i=1}^{n}\sum_{j=1}^{k} w_{i,j}^{p} dist(X_i, \mathbf{c}_j)^2 \\ &s.t. \sum_{i=1}^{n} w_{i,j}^{p} = 1, \end{aligned} \tag{9}$$

where p is parameter that determines the influence of weights $w_{i,j}$, $\mathbf{c}_j$ is the centroid of cluster C_j, $dist(X_i, \mathbf{c}_j)$ is the distance between vector X_i and centroid $\mathbf{c}_j$, and $w_{i,j}$ is the degree of membership of X_i belonging to cluster C_j. The detail formula for above symbols are as follows:

$$dist(X_i, \mathbf{c}_j)^2 = (X_i - \mathbf{c}_j)'(X_i - \mathbf{c}_j),\ \ w_{i,j} = \frac{(1/dist(X_i - \mathbf{c}_q)^2)^{\frac{1}{p-1}}}{\sum_{q=1}^{k}(1/dist(X_i - \mathbf{c}_j)^2)^{\frac{1}{p-1}}},\ \ \mathbf{c}_j = \frac{\sum_{i=1}^{n} w_{i,j}^{p} X_i}{\sum_{i=1}^{n} w_{i,j}^{p}}. \tag{10}$$

3.4 Threshold copula GARCH framework

It is described previously that the threshold regression could distinguish volatility index into different volatility states and the fuzzy c-means method could classify out several representative banks. However, to study structural tail dependences between multivariate stock returns of banks, it is more appropriate to use the approach of copulas because it can handle the structural dependence using marginal distributions of banks' stock returns combined with just one suitable copula.

3.4.1 The vector autoregression model

Since the correlation exists between the banks' stock returns, the vector autoregression (VAR) is used to catch the first order effect of return process. The bank stock returns at time t is written as $r_t = \ln(P_t / P_{t-1})$ and the vector autoregression of $\mathbf{r}_t$ which denotes the vector of returns of bank stocks at time t is written with p lags in standard form as

$$\mathbf{r}_t = A_0 + \sum_{i=1}^{p-1} A_i \mathbf{r}_{t-i} + \boldsymbol{\upsilon}_t, \tag{11}$$

where $\boldsymbol{\upsilon}_t$ is the error term at time t which has the properties of conditional time-varying volatility and regime switch.

Using VAR model, the Granger causality can be tested to discover if causality exists between banks. In fact, the smaller-size domestic banks such as Cathay United Bank, Chinatrust Commercial Bank and Fubon Commercial Bank etc. are susceptible to larger-size foreign banks such as Australia and New Zealand Bank, JPMorgan, and Citigroup etc.

3.4.2 Bivariate distribution and copulas

The copulas introduced by Nelsen (1999) and applied by Patton (2006) can decompose a multi-dimension distribution into a few marginal distributions and structural dependence, i.e. copula. According to (Sklar, 1959), suppose that F is a multivariate distribution function in the unit hypercube [0,1] with marginal uniform function $F_i(x_i)$ for i=1,…,m (m is the number of variable).

Then there exists an m-dimensional copula $C(F_1(x_1),...,F_m(x_m))$ such that for x ∈ Rn, $C(F_1(x_1),...,F_m(x_m)) = F(x_1,...,x_m)$ and the copula function $C:[0,1]^m \to [0,1]$.

In this context, the residual $\varepsilon_{i,t}^{(j)}$ of ith bank returns in Equation (4) is equivalent to x_i. However, the copula is much easier to deal with bivariate distribution. Thus, for $\varepsilon_{i,t}^{(j)}$ between any two banks with i=1...2 in regime j at time t, the bivariate copula can be written as

$$\begin{aligned} C(F_1(\varepsilon_{1,t}^{(j)}),F_2(\varepsilon_{2,t}^{(j)})) &= \Pr(U_1 \le F_1(\varepsilon_{1,t}^{(j)}),U_2 \le F_2(\varepsilon_{2,t}^{(j)})) \\ &= \Pr(F^{-1}(U_1) \le \varepsilon_{1,t}^{(j)},F^{-1}(U_2) \le \varepsilon_{2,t}^{(j)}) = F(\varepsilon_{1,t}^{(j)},\varepsilon_{2,t}^{(j)}), \end{aligned} \tag{12}$$

where U is a standard uniform random variable. If $F_1(\varepsilon_{1,t}^{(j)})$ and $F_2(\varepsilon_{2,t}^{(j)})$ are all continuous, C is uniquely determined on $F_1(\varepsilon_{1,t}^{(j)}) \times F_2(\varepsilon_{2,t}^{(j)})$. Conversely, if C is copula with marginal $F_1(\varepsilon_{1,t}^{(j)})$ and $F_2(\varepsilon_{2,t}^{(j)})$, then F is a bivariate distribution. To obtain the density of F, i.e. $f(\varepsilon_{1,t}^{(j)},\varepsilon_{2,t}^{(j)})$, just take the derivative of F as

$$\begin{aligned} f(\varepsilon_{1,t}^{(j)},\varepsilon_{2,t}^{(j)}) &= \frac{\partial^2 F(\varepsilon_{1,t}^{(j)},\varepsilon_{2,t}^{(j)})}{\partial\varepsilon_{1,t}^{(j)}\partial\varepsilon_{2,t}^{(j)}} = \frac{\partial^2 C(F_1(\varepsilon_{1,t}^{(j)}),F_2(\varepsilon_{2,t}^{(j)}))}{\partial F_1(\varepsilon_{1,t}^{(j)})\partial F_2(\varepsilon_{2,t}^{(j)})} \times \prod_i^2 \frac{\partial F_i(\varepsilon_{i,t}^{(j)})}{\partial \varepsilon_{i,t}^{(j)}} \\ &= c(u_{1,t}^{(j)},u_{2,t}^{(j)}) \times \prod_i^2 f_i(\varepsilon_{i,t}^{(j)}), \end{aligned} \tag{13}$$

where u is a random number in U, $c(u_{1,t}^{(j)},u_{2,t}^{(j)})$ is the copula density function, and $f_i(\varepsilon_{i,t}^{(j)})$ is the marginal density function of x_i. To catch the leptokurtosis effect, $f_i(\varepsilon_{i,t}^{(j)})$ can be considered as skewed t-distribution, GED, or mixture normal distribution.

Therefore, it is apparent that the joint probability function of bi-variable can be separated into the product of structural dependence i.e. copula and two marginal probability functions. Since the marginal probability functions bare no information at all about dependence between variables, the structural dependence between two variables definitely exists in the copula. That's why the copula is described as the structural dependence.

3.4.3 Elliptical and Archimedean copulas

There are several candidate copulas common used in modeling. The elliptical copulas including Gaussian and Student-t copulas have embedded linear correlation and symmetric shape in copula function. Using the Sklar's theorem, the distribution function of Gaussian denoted by C_N can be constructed from the Gaussian bivariate distribution as

$$C_N(u_1,u_2;\rho) = \int_\infty^{\Phi^{-1}(u_1)}\int_\infty^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{(1-\rho^2)}} \exp\left[-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}\right]dxdy, \tag{14}$$

where ρ is the Pearson correlation that is linear correlation between random variable x and y, and Φ is the cumulative univariate standard normal distribution. Similarly, the distribution function of Student-t copula denoted by C_{St} is given by

$$C_{St}(u_1,u_2;\nu,\rho) = \int_\infty^{t_\nu^{-1}(u_1)}\int_\infty^{t_\nu^{-1}(u_2)} \frac{1}{2\pi\sqrt{(1-\rho^2)}} \left\{1+\frac{x^2-2\rho xy+y^2}{\nu(1-\rho^2)}\right\}^{-\frac{\nu+2}{2}} dxdy, \tag{15}$$

where t_v^{-1} is the inverse univariate Student-t distribution and v is its degree of freedom.

The Archimedean copulas including Clayton (1978), Frank (1979) and Gumbel (1960) are expressed in Appendix A. Those copula functions reveal that Clayton has intensive density to the left tail (dropping together), Gumbel has intensive density to the right tail (rising together), and Frank is symmetric without skewness.

One useful copula combining both Clayton and Gumbel copulas is the Clayton-Gumbel copula given by

$$C_{CG}(u_1,u_2;\theta,\delta)=\left\{1+\left[(u_1^{-\theta}-1)^{\delta}+(u_2^{-\theta}-1)^{\delta}\right]^{\frac{1}{\delta}}\right\}^{-\frac{1}{\theta}},\delta\geq 1,\theta\geq 0. \tag{16}$$

(1) as δ=1, $C_{CG}(u_1,u_2;\theta,\delta)$ implies Clayton copula that is lower tail dependent and upper tail independent.

$$C_{CG}(u_1,u_2;\theta,1)=\left\{u_1^{-\theta}+u_2^{-\theta}-1\right\}^{-\frac{1}{\theta}}. \tag{17}$$

(2) as $\theta\to 0$, $C_{CG}(u_1,u_2;\theta,\delta)$ implies Gumbel copula that is lower tail independent and upper tail dependent.

$$C_{CG}(u_1,u_2;0\downarrow,\delta)=\exp\left\{-\left[(-\ln u_1)^{\delta}+(-\ln u_2)^{\delta}\right]^{\frac{1}{\delta}}\right\}. \tag{18}$$

3.4.4 Maximum likelihood estimation

The maximum likelihood estimation (MLE) is used to estimate the parameters embedded in the estimated copula function $\tilde{c}(\cdot)$ and each marginal function $f_i(\cdot)$. Suppose that ϕ_i is a constant parameter vector in the ith marginal density function $f_i(\cdot)$, e.g. Equation (13). For a structural tail dependences between two series of banks' stock returns with regime j representing the high volatility state at time t, the conditional log-likelihood function for $\varepsilon_{i,t}^{(j)}$ with i=1,2 is given by

$$\text{lnL}(\phi^{(j)})=\sum_{t=1}^{n_j}\sum_{i=1}^{2}\ln f_i^{(j)}(\varepsilon_{i,t}^{(j)},\phi_i^{(j)}\mid\varepsilon_{t-1}), \tag{19}$$

where $\phi^{(j)}$ is equal to $[\phi_1^{(j)},\phi_2^{(j)}]'$ and n_j denotes the number of observations in regime j. Given the MLE estimation, it follows that the tail dependences and risk measure can be implemented in the following.

3.6 Measure the tail dependences

Several measures of asymmetric dependence can be used for analyzing contagion risk such as tail dependences and exceedance correlation described as follows. The advanced studies can be found in Longin and Solnik (2001) and Ang and Chen (2002).

3.5.1 Kendall's ρ_τ and Spearman's ρ_s

Unlike the simple correlation estimating the linear co-moment of two random variables, the Kendall's ρ_τ measures the various dependences between two random variables as

$$\rho_\tau=E[sign\{(X_1-X_2)(Y_1-Y_2)\}], \tag{20}$$

where (X_1, Y_1) and (X_2, Y_2) are two pairs of independent and equally distributed random variables and *sign* is a sign function.

In this context, suppose that the *i*th $\xi_{i,t}^{(j)}$ of bank returns for *i*=1,2 in Equation (4) is equivalent to above *X* and *Y* random variables. Then, the Kendall's ρ_τ for $\xi_{1,t}^{(j)}$ and $\xi_{2,t}^{(j)}$ in terms of copula is given by Schweizer and Wolff (1981) as

$$\rho_\tau^{(j)} = 4\int\int_{[0,1]^2} \tilde{C}^{(j)}(u_1^{(j)}, u_2^{(j)})\tilde{c}^{(j)}(u_1^{(j)}, u_2^{(j)})du_1^{(j)}du_2^{(j)} - 1. \tag{21}$$

Note that $\rho_\tau^{(j)}$ depends only on copula function but not the multivariate distribution. The Spearman's correlation† ρ_s i.e. the correlation coefficient of copula is given by

$$\rho_s^{(j)}=12\int\int_{[0,1]^2} u_1^{(j)}u_2^{(j)}d\tilde{C}^{(j)}(u_1^{(j)}, u_2^{(j)}) - 3. \tag{22}$$

Note that ρ_s depends only on the marginal distributions.

The Kendall's ρ_τ and Spearman's ρ_s for elliptical and Archimedean copulas are displayed in Table 1.

Table 1 Kendall's ρ_τ and Spearman's ρ_s

Copulas	ρ_τ	ρ_s
Gaussian	$\frac{2}{\pi}\arcsin(\rho)$	$\frac{6}{\pi}\arcsin(\frac{\rho}{2}) \approx \rho$
Student-t	$\frac{2}{\pi}\arcsin(\rho)$	-
Clayton	$\theta/(\theta+2)$	-
Gumbel	$1-1/\theta$	-
Frank	$1-4\theta^{-1}(1-D_1(\theta))$	$1-12\theta^{-1}(D_1(\theta)-D_2(\theta))$
Clayton-Gumbel	$\frac{(2+\theta)\delta-2}{(2+\theta)\delta}$	-

Note: $D_1(\theta)$ and $D_2(\theta)$ are the Debye function that is $D_m(x) = \frac{m}{x^m}\int_0^x \frac{t_m}{e^t-1}dt$.

3.5.2 Tail dependences for copulas

One way to reveal the financial contagion risk is to describe the tail behaviors of financial markets while crashing or booming together. The lower or upper tail dependences of a copula is considered as a most proper method to describe the tail behaviors between markets.

The coefficient of lower tail dependences is a function of standard uniform distribution and defined as

† See Embrechts et al. (2002) for relation between ρ, ρ_τ, and ρ_s

$$\lambda_L(u)=\Pr\left\{Y<G^{-1}(u)\,|\,X<F^{-1}(u)\right\}=\Pr\left\{G(Y)<u\,|\,F(X)<u\right\}$$

$$=\frac{\Pr\left\{G(Y)<u,F(X)<u\right\}}{\Pr\left\{F(X)<u\right\}}=\frac{C(u,u)}{u}. \tag{23}$$

As previous for $\xi_{i,t}^{(j)}$ and i=1,2, a bivariate copula has the lower tail dependences such that

$$\lim_{u\to 0^+}\frac{\tilde{C}^{(j)}(u,u)}{u}=\lambda_L\,. \tag{24}$$

If $\lambda_L\in(0,1]$, $\tilde{C}^{(j)}$ has the lower tail dependences and if $\lambda_L=0$, it has the lower tail independence.

The upper tail dependences is defined as

$$\lambda_U(u)=\Pr\left\{Y>G^{-1}(u)\,|\,X>F^{-1}(u)\right\}$$
$$=\frac{1-2u+C(u,u)}{1-u}. \tag{25}$$

As for $\xi_{i,t}^{(j)}$, a bivariate copula has the upper tail dependences such that

$$\lim_{u\to 0^-}\frac{1-2u+\tilde{C}^{(j)}(u,u)}{1-u}=\lambda_U\,. \tag{26}$$

If $\lambda_L\in(0,1]$, $\tilde{C}^{(j)}$ has the upper tail dependences and if $\lambda_U=0$, it has the upper tail independence.

The Gaussian copula has zero lower and upper tail dependences $\lambda_L=\lambda_U=0$, supposed that correlation $\rho\neq 0$. The Student-t copula has the same lower and upper tail dependences $\lambda_L=\lambda_U=2t_{v+1}(-\sqrt{v+1}\sqrt{\frac{1-\rho}{1+\rho}})$ in which t_{v+1} is a univariate Student-t distribution with $v+1$ degree of freedom. The tail dependences of Archimedean copulas are shown in Table 2.

3.5.3 Risk measures of a bank

(1) Value at risk and expected shortfall

Value at risk (VaR) measures the worst case loss (i.e. a threshold loss) at a given confidence level and investment horizon whereas expected shortfall (ES) measures the expected value of losses over the threshold loss under the similar condition. Clearly, VaR answers how bad the loss is and ES answers what the expected loss is if loss does happen. Both measures are related and complementary to each other.

Thus, given a $(1-\alpha)$ 100% confidence level, the VaR is the α 100% quantile for a continuous distribution and is written as

$$\mathrm{VaR}_\alpha(X)=-\inf[x\,|\,P(X\leq x)>\alpha], \tag{27}$$

where X is a random variable denoting the profit and loss. The ES is the mean of the α 100% worst outcomes and can be written as

$$\mathrm{ES}_\alpha(X)=E[X\,|\,X\leq-\mathrm{VaR}_\alpha]=\frac{1}{\alpha}\int_{-\infty}^{-VaR_\alpha}\left[Xf(X)dX\right], \tag{28}$$

where $f(X)$ is the probability density function of X.

Suppose that $\xi_{1,t}^{(j)}$ in Equation (4) is equivalent to X. Then, its risk measures can be computed using above Equations (27) and (28).

Table 2 Tail dependences of Copulas

Copulas	λ_L	λ_U
Gaussian	0 (as $\rho \neq 0$)	0 (as $\rho \neq 0$)
Student-t	$2t_{v+1}(-\sqrt{v+1}\sqrt{\frac{1-\rho}{1+\rho}})$	$2t_{v+1}(-\sqrt{v+1}\sqrt{\frac{1-\rho}{1+\rho}})$
Clayton	$2^{-\frac{1}{\theta}}$	-
Gumbel	0	$2-2^{-\frac{1}{\theta}}$
Frank	0	0
Clayton-Gumbel	$2^{-\frac{1}{\delta\theta}}$	$2-2^{-\frac{1}{\delta}}$

4. Empirical Result

The aim of this paper is to study how the domestic banks are affected by the the large and dominant foreign banks when the risk events occurred such as sub-prime risk and Greek debt crisis. In fact, the tail (downside jumps) distributions, dependences, and correlations between banks' stock returns would uncover the changes of contagion risk.

4.1 Bank Data

The period of bank data covers from 01/03/2003 to 03/25/2014 daily as the full sample data. Actually, the important periods are the sub-prime in 2008 and the Greek debt crisis in 2011 for the study of the contagion risk between the larger-size foreign banks and the smaller-size domestic banks.

The stock prices of bank data are collected based on 26 domestic and 24 foreign totaling 50 banks who have issue of stock shares in Taiwan. It is necessary to category the stock data of banks into several groups to succinct the research. To perform the task, the stock prices of bank data are transformed into stock returns of bank data using the logarithm of relative daily stock price. Then, the mean, standard deviation, skewness, and kurtosis of stock returns of each bank are calculated for classification. According to the method of fuzzy c-means described previously, our bank data are categorized into 10 groups with respect to domestic and foreign banks as shown in Table 3.

However, some groups such as Groups 1, 3, and 10 are unique and some groups such as Groups 2 and 9 are minor. Hence, those groups are not considered in study and Groups 4, 5, 7, and 8 comprised of more banks are selected to study.

For the proper domestic bank selections, Cathay United Bank (CUB) in Group 4 , Chinatrust Commercial Bank(CCB) in Group5, Yuanta Commercial Bank(YCB) in Group 7, Fubon Commercial Bank Co., Ltd (FCB) in Group 8, and Hua Nan Commercial Ltd (HNC) in Group 9 totaling 5 banks are selected as the representatives of domestic banks for studying contagion risk.

Table 3 The Classification of Domestic and Foreign Bank Data

Group no.	Domestic Banks	Group no.	Foreign Banks
Group1	Taiwan Cooperative Bank	Group1	Australia and New Zealand Bank (ANZ), Deutsche Bank AG, BNP Paribas, The Bank of East Asia Ltd.
Group2	Taiwan Business Bank, Jih Sun International Bank	Group2	Mizuho Corporate Bank Ltd. (MFG), Sumitomo Mitsui Banking Corporation, Societe Generale,
Group3	Waterland Financial Holdings	Group3	JPMorgan Chase Bank, N.A. (JPM), UBS AG, ING BANK, N. V.,
Group4	Ta Chong Bank, Cathay United Bank, E.Sun Commercial Bank, Ltd., Mega International Commercial Bank	Group4	The Bank Of Tokyo-Mitsubishi UFJ, Ltd. (MTU), Calyon Corporate and Investment Bank, The Bank of Nova Scotia, DBS Bank Ltd, Banco Bilbao Vizcaya Argentaria S.A.
Group5	China trust Commercial Bank, Taishin International Bank, Taiwan Shin Kong Commercial Bank, Bank Sinopac Company Limited,	Group5	Bank of America Corporation, United Overseas Bank, The Bank of New York Mellon, Wells Fargo Bank, National Association
Group6	Union Bank Of Taiwan	Group6	Citibank N. A.
Group7	Entie Commercial Bank, Cosmos Bank, Taiwan, Yuanta Commercial Bank	Group7	Standard Chartered Bank
Group8	Chang Hwa Commercial Bank, Kings Town Bank, Far Eastern International Bank, Fubon Commercial Bank Co., Ltd., China Development Industrial Bank, First Commercial Bank,	Group8	The Hongkong and Shanghai Banking Corp.Ltd
Group9	Taichung Commercial Bank, Bank Of Kaohsiung, Hua Nan Commercial Ltd.,	Group9	State Street Bank and Trust Company
Group10	Bank of Taipei	Group10	Barclays Bank PLC

Note: Banks without the issue of stock shares in Taiwan stock market are excluded here such as Bank of Taiwan, Land Bank of Taiwan, The Export-Import Bank of the Republic of China, Sunny Bank Ltd., Bank Of Panhsin, Hwatai Bank, and Cota Bank and so do the affiliations of foreign banks in Taiwan.

On the other hand, the foreign banks selected should be more influential to domestic banks. Hence, the larger and well-known foreign banks would be the selection priority. As a result, Australia and New Zealand Bank (ANZ), JPMorgan Chase Bank(JMB), UBS AG(UBS) Bank of American (BAC), Citi Group (C), Standard Chartered Plc (STAN), The Hong Kong and Shanghai Banking Corp. Ltd. (HSBC) totaling 7 banks are chose to analyze the contagion risk for the 5 domestic banks. Hereafter, the short names of banks as above in parentheses are used for the rest of the paper.

Figure 1 shows the stock price changes from 01/03/2003 to 03/25/2014 for 5 domestic banks and 7 foreign banks. It is apparent during the period of sub-prime crisis around Aug. 2008 that all the banks exhibit the larger stock price drops especially for UBS, BAC, and C. which have dropped respectively from \$64.98, \$47.01, and \$508.35 to \$7.11, \$3.08, and \$10.17. In fact, C i.e. Citi Group has favorable reputation and popularity in U.S. banking industry. It is unbelievable that the stock prices of C i.e. Citi Group tumbled so tremendously in just the period of sub-prime crisis.

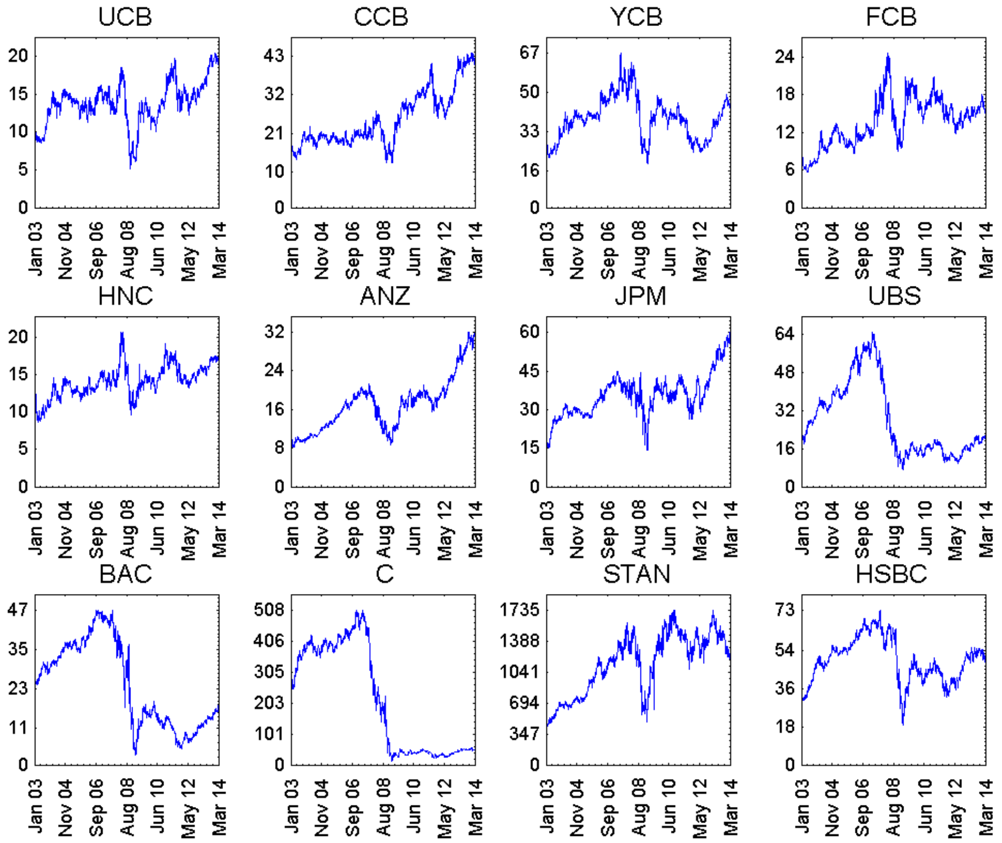


Figure 1 The Stock Price Changes of Domestic and Foreign Banks for Full Data Sample

4.1 Statistical Description of Bank Data

As aforementioned, the number of banks has been reduced to 5 domestic and 7 foreign banks. Table 4 panel A reports the statistical description of the stock returns for 12 banks. Except for CB, the means of stock returns of the banks are all positive. However, the test of the mean of returns equal to zero is not significant except CCB, FUB, and UBS. The foreign banks C and BAC have larger standard deviations of 0.0334 and 0.0301 respectively which imply higher financial risk than other banks. On the other hand, among domestic banks, FCB and CUB have slighter large standard deviations of 0.0232 and 0.0207 respectively. In terms of Sharpe ratio, CCB and BAC representing domestic and foreign banks respectively have better performance in stock price. Table 4 panel B shows that overall the domestic banks have higher correlations with ANZ, HSBC, and STAN while having the least correlations with JPM.

Table 5 Panel A reports the Granger causality test between each of the 5 domestic banks and 7 foreign banks with three lags of returns. The tests reveal several important causality information. First, the changes of the stock returns of JPM, C, and STAN foreign banks could most affect the changes of stock returns of all domestic banks while UBS and HSBC could roughly do so. On the other hand, BAC could not influence any domestic bank.

Table 5 Panel B reports the Granger causally test between foreign banks. The tests reveal that JPM, C, and BAC appear to influent the other foreign banks most. It is notable that ANZ could not influence the change of stock returns of any foreign bank.

Table 4 Statistical Description of Bank Data

Panel A. Basic statistics

	CUB	CCB	YCB	FCB	HNC	ANZ	JPM	UBS	BAC	C	STAN	HSBC
Min	-0.0884	-0.0766	-0.0902	-0.0957	-0.0842	-0.1330	-0.2325	-0.1977	-0.3430	-0.4948	-0.6848	-0.2083
Max	0.0722	0.0836	0.0919	0.1028	0.0985	0.1370	0.2241	0.2728	0.3017	0.4564	0.6378	0.1348
Mean	0.0003	0.0003	0.0003	0.0003	0.0001	0.0005	0.0004	-0.0001	-0.0002	-0.0006	0.0003	0.0002
Stdev	0.0217	0.0202	0.0212	0.0245	0.0187	0.0167	0.0267	0.0289	0.0347	0.0369	0.0305	0.0181
Sharpe Ratio	0.0119	0.0172	0.0126	0.0134	0.0071	0.0283	0.0166	-0.0018	-0.0044	-0.0176	0.0110	0.0087
Skewness	-0.0635	-0.1012	-0.0057	0.0633	0.0153	0.0089	0.3492	0.1461	-0.3114	-0.4874	-0.8022	-1.1120
Kurtosis	2.117	2.319	2.068	1.309	3.242	8.666	14.798	11.763	23.057	35.728	171.051	17.563
JK test	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
Test of Mean	0.6155	0.8916	0.6511	0.6942	0.3686	1.4626	0.8587	-0.0907	-0.2283	-0.9091	0.5675	0.4520

Note: The full sample data of stock returns covering from 01/03/2003~03/25/2014 are used to calculate statistics.

Panel B. Correlations

	CUB	CCB	YCB	FCB	HNC	ANZ	JPM	UBS	BAC	C	STAN	HSBC
CUB	1	0.6466	0.6622	0.6184	0.6535	0.3147	0.0831	0.1193	0.1342	0.1431	0.1220	0.1690
	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(1.68E-05)	(6.23E-10)	(3.26E-12)	(1.05E-13)	(2.50E-10)	(0.00E+00)
CCB	0.6466	1	0.7032	0.6299	0.6347	0.3060	0.0827	0.1370	0.1402	0.1525	0.1862	0.1841
	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(1.88E-05)	(1.13E-12)	(3.36E-13)	(2.22E-15)	(0.00E+00)	(0.00E+00)
YCB	0.6622	0.7032	1	0.6643	0.7041	0.3393	0.0491	0.1024	0.0988	0.1108	0.1915	0.1647
	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(1.11E-02)	(1.13E-07)	(3.12E-07)	(9.23E-09)	(0.00E+00)	(0.00E+00)
FCB	0.6184	0.6299	0.6643	1	0.6691	0.2921	0.0141	0.0763	0.0791	0.0868	0.1430	0.1365
	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(4.65E-01)	(7.94E-05)	(4.25E-05)	(7.03E-06)	(1.13E-13)	(1.38E-12)
HNC	0.6535	0.6347	0.7041	0.6691	1	0.3317	0.0819	0.1250	0.1142	0.1370	0.1882	0.2024
	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(2.27E-05)	(8.81E-11)	(3.27E-09)	(1.15E-12)	(0.00E+00)	(0.00E+00)
ANZ	0.3147	0.3060	0.3393	0.2921	0.3317	1	0.1070	0.1490	0.1387	0.1306	0.1958	0.1750
	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(2.91E-08)	(9.77E-15)	(5.87E-13)	(1.22E-11)	(0.00E+00)	(0.00E+00)
JPM	0.0831	0.0827	0.0491	0.0141	0.0819	0.1070	1	0.6804	0.7895	0.7272	0.2668	0.6433
	(1.68E-05)	(1.88E-05)	(1.11E-02)	(4.65E-01)	(2.27E-05)	(2.91E-08)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)
UBS	0.1193	0.1370	0.1024	0.0763	0.1250	0.1490	0.6804	1	0.6313	0.6308	0.3467	0.6623
	(6.23E-10)	(1.13E-12)	(1.13E-07)	(7.94E-05)	(8.81E-11)	(9.77E-15)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)
BAC	0.1342	0.1402	0.0988	0.0791	0.1142	0.1387	0.7895	0.6313	1	0.7945	0.2774	0.6109
	(3.26E-12)	(3.36E-13)	(3.12E-07)	(4.25E-05)	(3.27E-09)	(5.87E-13)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)
C	0.1431	0.1525	0.1108	0.0868	0.1370	0.1306	0.7272	0.6308	0.7945	1	0.2855	0.6040
	(1.05E-13)	(2.22E-15)	(9.23E-09)	(7.03E-06)	(1.15E-12)	(1.22E-11)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)
STAN	0.1220	0.1862	0.1915	0.1430	0.1882	0.1958	0.2668	0.3467	0.2774	0.2855	1	0.4119
	(2.50E-10)	(0.00E+00)	(0.00E+00)	(1.13E-13)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)
HSBC	0.1690	0.1841	0.1647	0.1365	0.2024	0.1750	0.6433	0.6623	0.6109	0.6040	0.4119	1
	(0.00E+00)	(0.00E+00)	(0.00E+00)	(1.38E-12)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)

Note: The number in parentheses refers to the *p*-value of the correlation test.

4.2 Multivariate Copula GARCH Estimations for Two States

Since the stock markets generally have two kinds of volatility states. One is the normal or low volatility state that has occurs more frequent and usually. The other is the high volatility state that occurs less frequent and unusually. The estimation of threshold value is proposed to suit this phenomenon and classify our data into two kinds of states: “low” and “high” volatilities. With one threshold value, the data can be classified into low and high states. Because our bank data are tested nonlinear, the t, and Clay models are used here to adjust the nonlinear effect beside the Gaussian model. As a result, the two state multivariate copula GARCH are proposed to estimate Kendall taus as well as value at risks between domestic and foreign banks.

To cover the events of market risk such as the sub-prime and Greek debt crises, the period of out-of-sample data is set from 07/03/2006 to 03/25/2014 having 1836 data samples and the in-sample data period is set from 01/03/2003 to 06/30/2006 yielding 836 data samples.

Table 5 Granger Causality Test between Banks

Panel A. One of domestic banks vs. 7 foreign banks

	Dependent Variable									
	CUB		CCB		YCB		FCB		HNC	
Indep.Variables	Chi-sq	Prob.	Chi-sq	Prob.	Chi-sq	Prob.	Chi-sq	Prob.	Chi-sq	Prob.
ANZ	4.46	0.216	10.96	0.012	9.17	0.027	12.41	0.006	9.58	0.023
JPM	15.74	0.001	14.27	0.003	15.43	0.002	24.92	0.000	17.12	0.001
UBS	10.56	0.014	5.53	0.137	26.47	0.000	12.32	0.006	22.38	0.000
BAC	2.46	0.482	4.01	0.261	6.88	0.076	4.77	0.190	3.70	0.295
C	25.65	0.000	8.20	0.042	21.32	0.000	18.09	0.000	23.49	0.000
STAN	31.26	0.000	20.17	0.000	8.60	0.035	38.99	0.000	20.00	0.000
HSBC	15.14	0.002	18.72	0.000	13.75	0.003	8.58	0.035	0.69	0.875
All	392.05	0.000	317.50	0.000	442.70	0.000	402.76	0.000	336.88	0.000

Note: Dependent variable means one of domestic banks and independent variables means 7 foreign banks. The Granger causality test which 7 independent variables with 3 lags could cause the change of one dependent variable.

Panel B. One of foreign banks vs. the other 6 foreign banks

	Dependent Variable													
	ANZ		JPM		UBS		BAC		CB		STAN		HSBC	
Indep. Variables	Chi-sq	Prob.	Chi-sq	Prob.	Chi-sq	Prob.	Chi-sq	Prob.	Chi-sq	Prob.	Chi-sq	Prob.	Chi-sq	Prob.
ANZ	-	-	3.39	0.335	3.81	0.283	4.59	0.204	2.80	0.423	6.96	0.073	7.15	0.067
JPM	14.72	0.002	-	-	10.10	0.018	18.74	0.000	15.62	0.001	16.28	0.001	12.92	0.005
UBS	34.06	0.000	14.54	0.002	-	-	14.05	0.003	4.37	0.224	2.15	0.543	8.90	0.031
BAC	12.79	0.005	13.25	0.004	25.30	0.000	-	-	17.99	0.000	6.95	0.073	8.23	0.042
C	37.42	0.000	37.62	0.000	27.18	0.000	27.63	0.000	-	-	30.58	0.000	43.34	0.000
STAN	8.80	0.032	11.78	0.008	7.27	0.064	12.12	0.007	5.24	0.155	-	-	3.63	0.304
HSBC	1.64	0.650	20.75	0.000	7.91	0.048	11.86	0.008	2.83	0.419	4.63	0.201	-	-
All	520.11	0.000	84.00	0.000	80.61	0.000	86.68	0.000	51.82	0.000	220.00	0.000	87.49	0.000

4.2.1 Threshold value estimation

The threshold value is supposed to be estimated according to Equation (3). Table 6 reports the result of the threshold value estimation using in-sample data. According to the supreme LM (SupLM) test by Hansen(1996), the threshold value is significant at 10% level. The number of usable observations is 830 for full sample, 651 for low state, and 178 for high state. It is clear that the low state numbers are far larger than the high state numbers.

4.2.2 Multivariate copula GARCH estimations for two states

To optimize the log likelihood of copula GARCH effectively, it is better to use two step estimations. The first step is to estimate the marginal i.e. univariate GARCH model for each of banks and the second step is to estimate the multivariate copula. The estimations of two-state multivariate copula GARCH using in-sample data are reported in Tables 7 and 8. Note that at present, the bivaratie but not three-variate above Clayton copula GARCH can be estimated. Thus, there are 15 combinations of bivariate copula GARCH for six indices. Four important combination only are excerpted here.

Apparently, for low state situation, the marginal GARCH is acceptable for its GARCH terms i.e. γ. According to the the number of significant parameters, the Gaussian copula and t copula are favorable while Clay copula is less favorable but acceptable. Compared to the low state estimations, the high state estimations for marginal GARCH, Gaussian copula, and t copula appears to be less acceptable according to the significant numbers, AIC, SBC, and LogL (i.e. log likelihood). Though, the t copula has strong significance of degree of freedom for either state. On the contrary, the Clay copula appear to have better estimation in the high state. Its AIC and SBC are lower and LogL is larger compared to those in the low state. Besides, its significant parameters in high sate is still as numerous as in low state.

Table 6 Estimation and Test of the Threshold Value of the Actual Volatility Index

	C	Vol(1)	Vol(2)	Vol(3)	Vol(4)	Vol(5)	Vol(6)	Obs	SSE
Full Sample	0.0039***	0.1181***	0.1153***	0.0840***	0.1065***	0.1121***	0.0828***	830	3.53E-05
	(0.0006)	(0.0347)	(0.0349)	(0.0350)	(0.0350)	(0.0346)	(0.0346)		
Vol<=Threshold	0.0040***	0.1392***	0.1580***	-0.0044	0.0839**	0.1071***	0.1173*	651	0.000029
	(0.0008)	(0.0395)	(0.0386)	(0.0393)	(0.0397)	(0.0373)	(0.0746)		
Vol>Threshold	0.0078***	0.0631	-0.0096	0.2476***	0.1648***	0.1449*	-0.1269	178	5.53E-05
	(0.0022)	(0.0725)	(0.0774)	(0.0743)	(0.0731)	(0.0828)	(0.0940)		
Threshold	0.01341	Lags	6						
SupLM	26.083	*p*-value	0.057						
ExpLM	7.456	*p*-value	0.100						
AveLM	5.569	*p*-value	0.860						

Notes: 1. The daily in-sample data are from 01/03/2003 to 06/30/2006. 2. The number of simulation is 2,500 for Lagrange multiplier (LM) tests. With total number of 836 in-sample data except 6 lag numbers, the usable observations of sample data in low state is 651 and in high state is 178.

4.3 Calculating Out-of-Sample Kendall Taus

To analyze the changes of contagion risk from foreign banks to domestic banks, the in-sample data are used to estimate the parameters of models and the Kendall taus between our banks are callused using the estimated parameters. Whereas, the out-of-sample data are used to compute the Kendall taus for each sample of out-of-sample data. As a result, the rolling windows technique is used here to move forward out-of-sample data one by one with the size of in-sample data. Thus, the Kendall taus of the entire out-of-sample data are obtained and the contagion risk between banks in different periods could be studied and assessed subsequently.

Overall, the periods of contagion risk in out-of-sample data interesting to us are the pre risk, sub-prime, Greek debt, and post-risk periods which cover respectively from 07/03/2006, 03/01/2007, 01/04/2010, and 07/02/2012 to 02/27/2007, 02/27/2009, 12/30/2011, and 03/25/2014. The next sections would study and compare contagion risk i.e. Kendall taus and risk measures such as value at risk or expected shortfall for domestic banks between the four periods.

4.3.1 Calculating out-of-sample threshold values of volatility index

Using Equation (3), the threshold value of the in-sample volatility index is estimated as shown in Table 6. Based on rolling window technique, the threshold values for entire out-of-sample volatility index are obtained. Then, each out-of-sample bank data is classified into either low or high volatility state according to the threshold value of volatility index. Figure 2 exhibits the volatility index and the threshold value changes. It is indeed that the volatility indices are much higher between the late of 2008 to the early of 2009 (around sub-prime crisis) and during the late of 2011 (around Greek debt crisis). However, the threshold values of the latter is higher than those of the former period.

Table 7 Low State Multivariate Copula GARCH Estimations

Panel A. Stage 1: marginal univariate GARCH

	CUB	CCB	YCB	FCB	HNC	ANZ	JPM	UBS	BAC	C	STAN	HSBC
c	6.01e-06	-7.55e-05	0.0002	0.0008	0.0002	0.0011***	0.0004	0.0005	0.0007***	0.0003	0.0004	0.0002
	(1.59e-05)	(0.0002)	(0.0004)	(0.0007)	(0.0005)	(0.0004)	(0.0003)	(0.0004)	(0.0003)	(0.0003)	(0.0004)	(0.0002)
α	6.71e-06	3.24e-06*	1.51e-06	2.96e-05	5.62e-06	7.93e-05***	1.19e-05	4.27e-05	2.09e-05***	6.52e-07	1.38e-06	8.84e-07
	(1.24e-05)	(1.93e-06)	(1.31e-06)	(3.01e-05)	(5.26e-06)	(1.01e-05)	(7.46e-06)	(3.04e-05)	(7.91e-06)	(6.40e-07)	(1.12e-06)	(8.30e-07)
β	0.0692	0.0273***	0.0492***	0.0780*	0.0481*	0.1193	0.0647	0.1089*	0.0318	0.0247*	0.0265**	0.0377*
	(0.0630)	(0.0093)	(0.0149)	(0.0457)	(0.0285)	(0.0755)	(0.0421)	(0.0644)	(0.0436)	(0.0141)	(0.0124)	(0.0209)
γ	0.9084***	0.9610***	0.9471***	0.8484***	0.9296***	3.27e-05***	0.7524***	0.4849	0.5429***	0.9624***	0.9645***	0.9412***
	(0.1055)	(0.0100)	(0.0155)	(0.1171)	(0.0436)	(3.84e-06)	(0.1356)	(0.3221)	(0.1787)	(0.0228)	(0.0156)	(0.0366)
AIC	-3575.75	-3551.04	-3581.25	-3289.15	-3574.46	-4248.80	-4454.67	-4142.60	-4634.30	-4565.63	-3898.62	-4752.80
SBC	-3557.81	-3533.10	-3563.31	-3271.21	-3556.52	-4230.86	-4436.73	-4124.66	-4616.36	-4547.69	-3880.68	-4734.86
LogL	-1791.87	-1779.52	-1794.63	-1648.57	-1791.23	-2128.40	-2231.34	-2075.30	-2321.15	-2286.82	-1953.31	-2380.40

Panel B. Stage 2: multivariate Gaussian copula

	CUB vs All FB	CCB vs All FB	YCB vs All FB	FCB vs All FB	HNC vs All FB
$w1$	0.0217***	0.0201***	0.0207***	0.0162**	0.0196***
	(0.0078)	(0.0078)	(0.0079)	(0.0072)	(0.0073)
$w2$	0.5693***	0.5679***	0.6411***	0.5500***	0.5328***
	(0.1260)	(0.1653)	(0.1624)	(0.1804)	(0.1289)
AIC	-861.77	-872.37	-872.82	-866.27	-886.17
SBC	-852.80	-863.40	-863.85	-857.30	-877.21
LogL	432.89	438.18	438.41	435.14	445.09

Notes: 1.The two-stage procedure of estimation is used to estimate elliptical copula GARCH model for 6 stock price indices together. 2.The numbers in parenthesis are standard errors. 3. All FB stands for all foreign banks.

Panel C. Stage 2: multivariate Student-t copula

	CUB vs All FB	CCB vs All FB	YCB vs All FB	FCB vs All FB	HNC vs All FB
v_c	37.28***	43.29***	39.99***	40.37***	34.72***
	(11.68)	(13.64)	(12.65)	(15.36)	(8.631)
$w1$	0.0214***	0.0205***	0.0205***	0.0159**	0.0195***
	(0.0079)	(0.0080)	(0.0083)	(0.0074)	(0.0074)
$w2$	0.5628***	0.5426***	0.6316***	0.5332***	0.5155***
	(0.1291)	(0.1815)	(0.1841)	(0.1954)	(0.1345)
AIC	-870.78	-878.67	-881.39	-874.11	-897.98
SBC	-857.33	-865.22	-867.93	-860.66	-884.53
LogL	438.39	442.34	443.69	440.06	451.99

Note: The degree of freedom of t distribution v and v_c are estimated in stage 1 and stage 2 respectively.

Panel D. Stage 2: bivariate Clayton copula

	CUB vs One of 7 Foreign Banks						
	ANZ	JPM	UBS	BAC	C	STAN	BB
$w0$	-11.29***	-2.126	-6.915***	-15***	-1.014	-2.566	-14.99***
	(0.7179)	(2.1)	(2.553)	(4.131)	(0.8675)	(3.052)	(4.122)
$w1$	-5.079***	-7.178	1.151	5.34*	14.97	1.088	4.397***
	(0.1114)	(9.503)	(1.044)	(3.188)	(21.26)	(2.196)	(1.481)
$w2$	12.93***	-0.1154	-0.8855***	-0.8147***	15**	0.0619	-0.9305***
	(0.1702)	(0.3627)	(0.1197)	(0.1043)	(7.231)	(1.002)	(0.0158)
AIC	5.5613	2.0383	1.7912	5.2041	4.8855	-5.1691	3.2620
SBC	19.0152	15.4922	15.2451	18.6580	18.3394	8.2848	16.7159
LogL	0.2193	1.9808	2.1044	0.3979	0.5573	5.5846	1.3690

Table 7 Continued

			CCB vs One of 7 Foreign Banks				
	ANZ	JPM	UBS	BAC	C	STAN	HSBC
w0	-4.628***	0.1034	-4.755***	0.0638	-15	-3.64***	-8.335
	(0.7614)	(0.4409)	(1.095)	(0.0772)	(15.72)	(0.6730)	(12.79)
*w*1	0.6176	-2.185	0.8508	-0.7421	-3.362***	0.3944	-3.865
	(0.7097)	(1.449)	(0.7401)	(0.7931)	(0.5946)	(0.6253)	(6.369)
*w*2	-0.9024***	0.8436***	-0.5598***	2.175	-0.9546***	-0.7559***	-0.9294***
	(0.0745)	(0.0687)	(0.2344)	(1.361)	(0.1202)	(0.1393)	(0.0166)
AIC	-5.9305	2.8964	2.7036	2.0918	4.8257	-14.0284	-8.5124
SBC	7.5234	16.3503	16.1575	15.5458	18.2796	-0.5745	4.9415
LogL	5.9653	1.5518	1.6482	1.9541	0.5871	10.0142	7.2562
			YCB vs One of 7 Foreign Banks				
	ANZ	JPM	UBS	BAC	C	STAN	HSBC
w0	-4.862***	-11.03	-12.13*	-12.32***	-8.76***	-2.932***	-1.564***
	(0.8657)	(9.889)	(6.615)	(0.7377)	(3.082)	(0.5523)	(0.5436)
*w*1	0.7926	-15	1.264***	0.6861***	15***	-0.7976	0.9129
	(0.6804)	(58.81)	(0.4915)	(0.0172)	(5.772)	(1.508)	(1.745)
*w*2	-0.7825***	-0.5446	-0.9664***	2.09***	1.748***	-0.6295**	0.5313**
AIC	(0.1644)	(1.499)	(0.0202)	(0.0203)	(0.2083)	(0.2886)	(0.2680)
SBC	-0.4123	5.5536	4.3161	6.0875	3.5604	-12.7746	2.2283
LogL	13.0416	19.0075	17.7700	19.5414	17.0143	0.6793	15.6822
	3.2061	0.2232	0.8419	-0.0437	1.2198	9.3873	1.8858
			FCB vs One of 7 Foreign Banks				
	ANZ	JPM	UBS	BAC	C	STAN	HSBC
w0	-10.94***	-13.25***	-15	-15***	-0.5674***	-4.586***	-1.524
	(0.6912)	(0.9211)	(22.03)	(3.951)	(0.0093)	(1.102)	(1.799)
*w*1	3.516***	0.2463***	-13.47	-3.383	-12.51***	1.501*	14.99
	(0.0502)	(0.0115)	(26.32)	(2.996)	(3.034)	(0.8507)	(17.66)
*w*2	13.88***	1.895***	-0.7123***	-0.9139***	12.42***	-0.7762***	14.92**
	(0.1596)	(0.0227)	(0.0872)	(0.0670)	(0.5412)	(0.0480)	(7.103)
AIC	5.2275	5.9670	1.5963	4.4313	5.8902	-13.4779	4.2545
SBC	18.6814	19.4209	15.0502	17.8852	19.3441	-0.0240	17.7084
LogL	0.3863	0.0165	2.2018	0.7844	0.0549	9.7390	0.8727
			HNC vs One of 7 Foreign Banks				
	ANZ	JPM	UBS	BAC	C	STAN	HSBC
w0	-10.42***	-12.89***	-7.111***	-13.98***	0.4478	-14.01***	-2.979***
	(0.7033)	(0.7455)	(2.229)	(0.9732)	(0.7737)	(5.39e-07)	(1.194)
*w*1	12.49***	1.553***	-3.629*	-2.326***	-14.99	0.1976***	0.7888
	(0.5564)	(0.0228)	(2.183)	(0.2098)	(14.15)	(5.25e-07)	(1.864)
*w*2	12.96***	1.014***	-0.7749***	0.2594	14.98	1.409***	-0.2495
	(1.253)	(0.0096)	(0.0841)	(0.3076)	(15.25)	(1.69e-10)	(0.3153)
AIC	5.0039	5.8616	2.4233	5.9957	5.5904	5.9314	-6.6965
SBC	18.4578	19.3155	15.8772	19.4496	19.0443	19.3853	6.7574
LogL	0.4980	0.0692	1.7884	0.0021	0.2048	0.0343	6.3483

Notes: Clayton Kendall ρ_τ is estimated using Patton(2006) as $\rho_\tau = \Lambda(w_0 + w_1\rho_{\tau-1} + w_2 \mid u_{1,t-i} - u_{2,t-i} \mid)$ where Λ is the logistic probability transformation function.

Table 8 Multivariate Copula GARCH Estimations for High State

Panel A. Stage 1: marginal univariate GARCH

	CUB	CCB	YCB	FCB	HNC	ANZ	JPM	UBS	BAC	C	STAN	HSBC
c	0.0016	0.0012	0.0024	0.0006	-0.0003	-0.0008	0.0018	0.0029**	0.0001	0.0008	0.0024	0.0025**
	(0.0016)	(0.0014)	(0.0016)	(0.0021)	(0.0022)	(0.0010)	(0.0016)	(0.0014)	(0.1703)	(0.0015)	(0.0015)	(0.0012)
α	4.59e-05	7.43e-05*	0.0001	0.0003***	0.0002	5.56e-06	8.15e-06	2.09e-05*	6.51e-07***	9.75e-06	0.0002	7.00e-05
	(2.87e-05)	(4.38e-05)	(6.98e-05)	(0.0001)	(0.0001)	(4.93e-06)	(6.25e-06)	(1.24e-05)	(2.06e-08)	(9.61e-06)	(0.0003)	(5.40e-05)
β	0.0730	0.1238	0.1893**	0.2394	0.1181	0.1219	0.0546**	0.0805***	2.13e-09	0.0272	0.0481	0.1272
	(0.0447)	(0.0787)	(0.0903)	(0.1675)	(0.1207)	(0.1003)	(0.0265)	(0.0312)	(1.87e-09)	(0.0218)	(0.0739)	(0.0820)
γ	0.8410***	0.6880***	0.6313***	0.3634**	0.5698***	0.8544***	0.9266***	0.8732***	0.9981***	0.9466***	0.4073	0.5639***
	(0.0498)	(0.1172)	(0.1282)	(0.1840)	(0.2235)	(0.0846)	(0.0294)	(0.0345)	(0.1556)	(0.0328)	(0.9269)	(0.2388)
AIC	-854.41	-910.19	-843.69	-802.63	-844.99	-1049.10	-833.58	-880.60	-952.69	-891.09	-917.14	-1002.30
SBC	-841.61	-897.40	-830.90	-789.84	-832.20	-1036.30	-820.79	-867.80	-939.90	-878.30	-904.35	-989.51
LogL	-431.20	-459.10	-425.85	-405.32	-426.50	-528.55	-420.79	-444.30	-480.35	-449.55	-462.57	-505.15

Panel B. Stage 2: multivariate Gaussian copula

	CUB vs All FB	CCB vs All FB	YCB vs All FB	FCB vs All FB	HNC vs All FB
$w1$	3.67e-06***	2.29e-08	3.70e-08	4.19e-08	1.59e-07
	(9.32e-07)	(0.1507)	(0.0133)	(0.0502)	(0.0101)
$w2$	0.2572***	0.2399	0.2577***	0.2512	0.2853***
	(0.0227)	(7.89)	(0.0195)	(4.246)	(0.0208)
AIC	-707.63	-701.58	-698.68	-705.27	-708.25
SBC	-701.24	-695.19	-692.28	-698.87	-701.85
LogL	355.82	352.79	351.34	354.63	356.13

Notes: 1.The two-stage procedure of estimation is used to estimate elliptical copula GARCH model for 6 stock price indices together. 2.The numbers in parenthesis are standard errors. 3. All FB stands for all foreign banks.

Panel C. Stage 2: multivariate Student-t copula

	CUB vs All FB	CCB vs All FB	YCB vs All FB	FCB vs All FB	HNC vs All FB
v_c	92.34***	196.1***	188.7***	147.7***	171.3***
	(2.01)	(3.159)	(20.98)	(1.316)	(1.728)
$w1$	4.34e-08	2.62e-08	4.32e-07***	8.89e-07	2.83e-08
	(0.0141)	(0.0247)	(4.63e-09)	(0.0106)	(0.0120)
$w2$	0.2162***	0.1176	0.9614***	0.1944***	0.4143***
	(0.0282)	(0.2893)	(0.0078)	(0.0095)	(0.0426)
AIC	-706.94	-699.83	-696.57	-703.72	-706.41
SBC	-697.34	-690.23	-686.97	-694.12	-696.82
LogL	356.47	352.91	351.28	354.86	356.21

Note: The degree of freedom of t distribution v and v_c are estimated in stage 1 and stage 2 respectively.

Panel D. Stage 2: bivariate Clayton copula

	CUB vs One of 7 Foreign Banks						
	ANZ	JPM	UBS	BAC	C	STAN	HSBC
$w0$	-5.091	-1.779***	-1.94	-5.966***	-3.298***	-1.21***	-2.378***
	(4.492)	(0.6504)	(1.233)	(1.828)	(0.9195)	(0.4929)	(0.6626)
$w1$	15	0.4911	1.478	1.343*	0.6239	1.452	-0.0352
	(13.76)	(3.35)	(2.474)	(0.6930)	(1.222)	(1.213)	(0.8109)
$w2$	0.9919***	0.1760	0.1760	-0.9101***	-0.7759***	0.3157	-0.8258***
	(0.0077)	(0.7550)	(0.1557)	(0.0349)	(0.2566)	(0.2121)	(0.1012)
AIC	-1.393	1.258	-0.036	1.755	-1.356	-11.853	-7.315
SBC	8.203	10.853	9.559	11.351	8.240	-2.258	2.280
LogL	3.696	2.371	3.018	2.122	3.678	8.927	6.658

Table 8 Continued

	ANZ	JPM	UBS	BAC	C	STAN	HSBC
			CCB vs One of 7 Foreign Banks				
	ANZ	JPM	UBS	BAC	C	STAN	HSBC
w0	-3.711***	-12.11	-11.42***	-9.035	-5.641	0.1988	-2.945***
	(1.274)	(26.07)	(4.223)	(7.306)	(6.788)	(0.4702)	(0.7321)
*w*1	1.656**	13.91	15***	8.696	-1.454	-8.539***	-0.3808
	(0.8426)	(28.85)	(5.604)	(9.863)	(2.284)	(3.079)	(0.5922)
*w*2	-0.3938	-0.0235	0.2340***	-0.1165	-0.9978***	-0.1010	-0.9476***
	(0.3697)	(0.2411)	(0.0775)	(0.3759)	(0.0138)	(0.1474)	(0.0312)
AIC	-0.537	0.374	-2.293	3.784	1.117	-13.639	-4.343
SBC	9.059	9.969	7.302	13.380	10.712	-4.043	5.252
LogL	3.268	2.813	4.147	1.108	2.442	9.819	5.172
			YCB vs One of 7 Foreign Banks				
	ANZ	JPM	UBS	BAC	C	STAN	HSBC
w0	-0.2426	-1.936	-2.205***	-7.278**	-3.829*	-2.353***	-2.083***
	(0.4640)	(1.634)	(0.8479)	(3.246)	(2.026)	(0.7404)	(0.7851)
*w*1	0.9688	-4.382	-1.216	2.26*	-1.079	2.239*	2.673*
	(1.964)	(5.316)	(2.359)	(1.338)	(2.082)	(1.171)	(1.373)
*w*2	1.028***	-0.2726	-0.3509*	-0.9154***	-0.8481***	-0.2131	0.1812
	(0.0166)	(0.3980)	(0.2107)	(0.0276)	(0.2253)	(0.2152)	(0.1931)
AIC	-2.958	1.020	0.106	3.198	1.675	-10.429	-8.429
SBC	6.637	10.616	9.701	12.793	11.270	-0.833	1.167
LogL	4.479	2.490	2.947	1.401	2.163	8.214	7.214
			FCB vs One of 7 Foreign Banks				
	ANZ	JPM	UBS	BAC	C	STAN	HSBC
w0	-5.494***	-1.766*	-15***	-4.937***	-3.799***	-1.039**	-3.511***
	(1.704)	(0.9028)	(3.97)	(1.306)	(1.122)	(0.4555)	(1.473)
*w*1	2.159*	-0.3157	7.384***	1.912*	0.8182	0.9423	0.8380
	(1.167)	(4.78)	(2.471)	(1.132)	(0.7593)	(1.038)	(0.9139)
*w*2	-0.8516***	0.1232	-0.8173***	-0.8815***	-0.9245***	0.4009*	-0.9325***
	(0.0631)	(1.165)	(0.0288)	(0.0422)	(0.0425)	(0.2372)	(0.0524)
AIC	-0.192	2.899	-1.389	-4.340	-2.718	-9.630	-4.167
SBC	9.404	12.495	8.206	5.256	6.877	-0.034	5.428
LogL	3.096	1.550	3.695	5.170	4.359	7.815	5.084
			HNC vs One of 7 Foreign Banks				
	ANZ	JPM	UBS	BAC	C	STAN	HSBC
w0	-11.13***	-0.6653	-6.479	-6.364**	-3.426**	-1.155	-14.91
	(3.137)	(1.787)	(4.529)	(2.955)	(1.538)	(1.727)	(22.07)
*w*1	15***	-14.16	10.2	2.105	1.018	-1.813*	-0.8605**
	(4.659)	(28.6)	(6.426)	(1.285)	(1.136)	(1.005)	(0.4169)
*w*2	0.4224***	-0.1739	0.3144**	-0.9205***	-0.9132***	-1.016***	-0.9930***
	(0.0441)	(0.3008)	(0.1553)	(0.0190)	(0.0350)	(0.0466)	(0.0113)
AIC	2.108	-0.856	-3.391	0.528	-4.176	-19.586	-1.834
SBC	11.704	8.740	6.204	10.124	5.420	-9.991	7.762
LogL	1.946	3.428	4.696	2.736	5.088	12.793	3.917

Note: Clayton Kendall ρ_τ is estimated using Patton(2006) as $\rho_\tau = \Lambda(w_0 + w_1\rho_{\tau-1} + w_2 \,|u_{1,t-i} - u_{2,t-i}|)$ where Λ is the logistic probability transformation function.

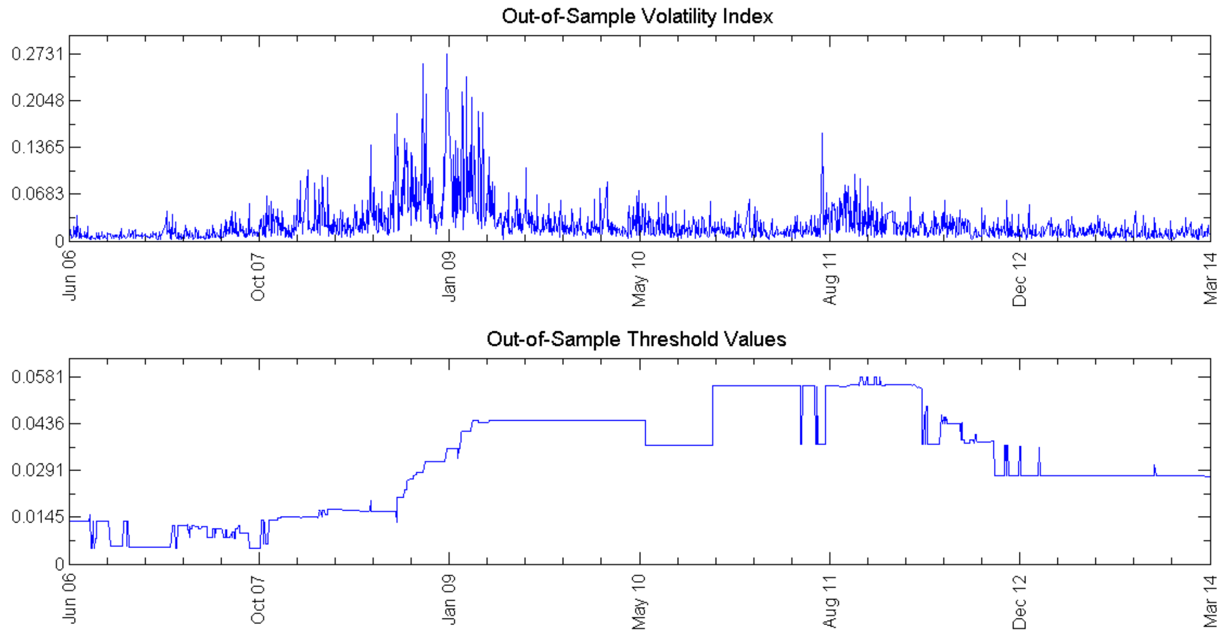


Figure 2 Out-of-sample Volatility index and Threshold Values

4.3.2 Estimating out-of-sample copula models

Similarly using rolling window method, the parameters of the threshold GARCH are estimated and Kendall taus are obtained for the whole out-of-sample data. It is appealing to capture and study the Kendall taus for 5 domestic banks against 12 foreign banks while the Kendall taus within either domestic banks or foreign banks are not considered.

To analyze Kendall taus between banks thoroughly, four dimension comparisons are proposed: one is for 4 sub-periods: pre risk, sub-prime, Greek debt, and post-risk, the second is for 3 models: Gaussian, t, and Clay, the third is for different stats: “low” and “high”, and the fourth is for different domestic against foreign banks, namely total 35 combinations.

Tables 9, 10, and 11 reports respectively the Gaussian, t, and Clay copula Kendall taus. As examining the total mean of Kendall taus for each domestic bank, all three copula GARCH models point out that the trend of Kendall taus is increasing for all domestic banks in either “low” or “high” state. The high state has over two times larger Kendall taus than the lower state. According to the function of threshold value, the volatility index composed by the stock return volatilities of all foreign banks is higher in high state compared to in low state. In other words, it implies that higher volatility has higher Kendall taus i.e. contagion risk.

The number of observations in low state for 4 sub-periods is 82 of 156 for pre-risk, 147 of 476 for sub-prime, 434 of 475 for Greek debt, and 386 of 408 for post-risk. It is obvious that the sub-prime has the largest ratio of high state sample and Greek debt has the smallest ratio of low state sample. However, it has to note that the threshold values increase gradually during the full sample period. This phenomenon means that most high state volatilities are found regularly over low threshold value in sub-prime but just a few high state volatilities are found regularly over higher threshold value. In other words, the tail or downside jumps are likely larger in Greek debt than in sub-prime period. Thus, in high state, the Greek debt is supposed to have much higher tail dependences or jumps as shown in Tables 9~11.

Table 9 Gaussian Copula Kendall Taus in Four Sub-Periods

Panel A. Low state

		CUB				CCB				YCB				FCB				HNC			
		PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR
ANZ	M	0.1135	0.1922	0.3019	0.3082	0.1290	0.1843	0.2904	0.3166	0.1069	0.1985	0.3833	0.3601	0.1235	0.2176	0.3269	0.3485	0.1194	0.2052	0.3703	0.3559
	S	0.0517	0.0521	0.0731	0.0439	0.0337	0.0466	0.0696	0.0283	0.0255	0.0585	0.0580	0.0315	0.0350	0.0510	0.0814	0.0228	0.0608	0.0593	0.0720	0.0281
JPM	M	0.0603	0.0487	0.0155	0.0572	0.0578	0.0135	0.0168	0.0437	-0.0158	-0.0605	0.0115	0.0244	-0.0005	-0.0084	-0.0287	0.0124	0.0588	0.0155	0.0041	0.0081
	S	0.0344	0.0329	0.0499	0.0270	0.0433	0.0332	0.0539	0.0216	0.0320	0.0262	0.0468	0.0248	0.0410	0.0406	0.0629	0.0243	0.0333	0.0463	0.0596	0.0317
UBS	M	0.0367	0.0573	0.0799	0.1185	0.0691	0.0800	0.0740	0.1030	0.0422	0.0255	0.0487	0.0589	0.0804	0.0501	0.0538	0.0865	0.0474	0.0738	0.0843	0.0782
	S	0.0473	0.0266	0.0453	0.0295	0.0481	0.0246	0.0341	0.0211	0.0329	0.0303	0.0434	0.0263	0.0395	0.0291	0.0390	0.0340	0.0419	0.0376	0.0448	0.0264
BAC	M	-0.0104	0.0208	0.0835	0.0918	-0.0073	0.0014	0.0678	0.0863	-0.0400	-0.0505	0.0611	0.0754	-0.0338	-0.0286	0.0443	0.0714	-0.0058	0.0076	0.0513	0.0448
	S	0.0205	0.0257	0.0400	0.0258	0.0356	0.0226	0.0440	0.0266	0.0410	0.0282	0.0505	0.0264	0.0472	0.0335	0.0627	0.0389	0.0303	0.0274	0.0553	0.0443
C	M	0.0147	0.0348	0.0815	0.0871	0.0566	0.0677	0.0861	0.1107	0.0258	-0.0115	0.0626	0.0651	-0.0210	0.0215	0.0727	0.0820	-0.0129	0.0190	0.0910	0.0698
	S	0.0555	0.0337	0.0390	0.0382	0.0679	0.0297	0.0390	0.0258	0.0616	0.0334	0.0431	0.0280	0.0306	0.0254	0.0434	0.0377	0.0272	0.0317	0.0416	0.0458
STAN	M	0.0973	0.1229	0.1798	0.0918	0.1731	0.2166	0.1766	0.1103	0.1574	0.1944	0.1934	0.0894	0.1576	0.1889	0.1862	0.1093	0.1696	0.2040	0.2110	0.1079
	S	0.0491	0.0312	0.0308	0.0309	0.0361	0.0234	0.0313	0.0265	0.0277	0.0299	0.0354	0.0305	0.0342	0.0353	0.0343	0.0324	0.0443	0.0346	0.0324	0.0300
HSBC	M	0.0374	0.0735	0.1678	0.1501	0.0960	0.1225	0.1729	0.1829	0.0746	0.0763	0.1661	0.1838	0.1149	0.1311	0.1471	0.1721	0.1252	0.1732	0.2026	0.1851
	S	0.1042	0.0516	0.0492	0.0337	0.1080	0.0466	0.0430	0.0263	0.0922	0.0349	0.0658	0.0277	0.0670	0.0340	0.0760	0.0246	0.1444	0.0457	0.0629	0.0342
Total M		0.0499	0.0786	0.1300	0.1293	0.0820	0.0980	0.1264	0.1362	0.0502	0.0532	0.1324	0.1224	0.0602	0.0818	0.1146	0.1260	0.0717	0.0998	0.1449	0.1214
Total S		0.0700	0.0670	0.1000	0.0846	0.0793	0.0826	0.0975	0.0867	0.0809	0.1064	0.1292	0.1108	0.0839	0.0975	0.1236	0.1057	0.0921	0.0943	0.1277	0.1141

Notes: 1. M and S represent respectively mean and standard deviation of Kendall taus. 2. PreR, SP, GD, and PosR represent respectively the four sub-periods: pre-risk, sub-prime, Greek debt, and post-risk covering respectively from 07/03/2006, 03/01/2007, 01/04/2010, and 07/02/2012 to 02/27/2007, 02/27/2009, 12/30/2011, and 03/25/2014. 3. Total M is the mean of Kendall taus for one of domestic bank vs. 7 foreign banks during one of sub-periods while M is the mean of Kendall taus for one of domestic banks vs one of the foreign banks. 4.The number of low state data estimated in PreR, SP, GD, and PosR is 82 of 156, 147 of 476, 434 of 475, and 386 of 408 respectively.

Panel B. High state

		CUB				CCB				YCB				FCB				HNC			
		PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR
ANZ	M	0.0940	0.2710	0.4579	0.4593	0.0946	0.2927	0.4678	0.4670	0.0789	0.3315	0.4738	0.4811	0.0986	0.2498	0.4543	0.4887	0.1304	0.3103	0.5070	0.5492
	S	0.0251	0.0958	0.0267	0.0406	0.0275	0.1096	0.0178	0.0407	0.0312	0.1335	0.0153	0.0466	0.0305	0.0936	0.0382	0.0417	0.0341	0.1033	0.0245	0.0331
JPM	M	0.1172	0.1113	0.1647	0.0689	0.0798	0.0211	0.2076	0.0822	0.0635	0.0397	0.1192	0.1195	0.0892	-0.0367	0.0615	0.1276	0.0858	0.0382	0.1790	0.1505
	S	0.0275	0.0341	0.0157	0.0393	0.0206	0.0737	0.0397	0.0330	0.0406	0.0379	0.0376	0.0313	0.0330	0.0648	0.0288	0.0290	0.0313	0.0440	0.0259	0.0413
UBS	M	0.1478	0.1556	0.1966	0.1402	0.1156	0.0982	0.2890	0.1490	0.0843	0.0754	0.2019	0.1315	0.1014	-0.0025	0.1333	0.1948	0.1130	0.0930	0.2228	0.1860
	S	0.0396	0.0554	0.0163	0.0332	0.0260	0.0907	0.0297	0.0258	0.0329	0.0656	0.0136	0.0286	0.0216	0.0849	0.0117	0.0214	0.0306	0.0662	0.0116	0.0266
BAC	M	0.0483	0.1314	0.2474	0.1100	0.0175	0.0632	0.3137	0.1499	-0.0082	0.0206	0.1918	0.1340	-0.0198	-0.0435	0.1605	0.1897	0.0240	0.0688	0.2508	0.1726
	S	0.0240	0.0619	0.0182	0.0397	0.0220	0.1030	0.0281	0.0458	0.0245	0.0577	0.0197	0.0360	0.0366	0.0806	0.0179	0.0229	0.0232	0.0899	0.0123	0.0378
C	M	0.0807	0.1259	0.2207	0.1143	0.0776	0.0694	0.2935	0.1618	0.0361	0.0376	0.1940	0.1430	0.0707	0.0136	0.1531	0.1554	0.0631	0.0809	0.2308	0.1724
	S	0.0532	0.0443	0.0191	0.0477	0.0228	0.0802	0.0442	0.0518	0.0435	0.0476	0.0229	0.0470	0.0425	0.0568	0.0217	0.0426	0.0503	0.0602	0.0194	0.0484
STAN	M	0.1924	0.2341	0.1962	0.1920	0.2290	0.2365	0.3170	0.2482	0.2132	0.2763	0.3218	0.2818	0.2108	0.1846	0.2703	0.2604	0.2597	0.2844	0.3221	0.2635
	S	0.0459	0.0413	0.0124	0.0325	0.0146	0.0553	0.0312	0.0440	0.0337	0.0596	0.0088	0.0517	0.0239	0.0516	0.0156	0.0367	0.0348	0.0393	0.0154	0.0358
HSBC	M	0.2062	0.1517	0.2217	0.1705	0.2158	0.1160	0.2758	0.2070	0.1803	0.0844	0.2057	0.2241	0.1835	0.0189	0.1839	0.2529	0.2683	0.1559	0.2667	0.2542
	S	0.0392	0.0644	0.0200	0.0262	0.0189	0.0746	0.0248	0.0433	0.0243	0.0552	0.0175	0.0305	0.0191	0.0920	0.0210	0.0171	0.0201	0.0643	0.0157	0.0351
Total M		0.1267	0.1687	0.2436	0.1793	0.1186	0.1282	0.3092	0.2093	0.0926	0.1236	0.2441	0.2164	0.1049	0.0549	0.2024	0.2385	0.1349	0.1473	0.2828	0.2498
Total S		0.0659	0.0817	0.0928	0.1262	0.0747	0.1255	0.0796	0.1229	0.0798	0.1369	0.1110	0.1279	0.0765	0.1310	0.1203	0.1159	0.0937	0.1226	0.1020	0.1341

Note: The number of high state data found in PreR, SP, GD, and PosR is 74 in 156, 369 in 476, 41 in 475, and 22 in 408 respectively.

Table 10 Student-t Copula Kendall Taus in Four Sub-periods

Panel A. Low state

		CUB				CCB				YCB				FCB				HNC			
		PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR
ANZ	M	0.1138	0.1932	0.3028	0.3059	0.1290	0.1849	0.2918	0.3170	0.1072	0.1995	0.3824	0.3587	0.1226	0.2163	0.3262	0.3483	0.1191	0.2050	0.3688	0.3552
	S	0.0516	0.0527	0.0758	0.0428	0.0332	0.0473	0.0726	0.0280	0.0251	0.0592	0.0591	0.0307	0.0345	0.0500	0.0853	0.0221	0.0604	0.0591	0.0751	0.0273
JPM	M	0.0597	0.0478	0.0167	0.0554	0.0574	0.0143	0.0178	0.0422	-0.0159	-0.0613	0.0154	0.0247	-0.0008	-0.0089	-0.0243	0.0131	0.0569	0.0137	0.0077	0.0080
	S	0.0341	0.0332	0.0499	0.0258	0.0434	0.0304	0.0546	0.0206	0.0317	0.0266	0.0462	0.0236	0.0408	0.0399	0.0629	0.0232	0.0334	0.0471	0.0600	0.0307
UBS	M	0.0365	0.0569	0.0815	0.1164	0.0684	0.0778	0.0765	0.1017	0.0418	0.0264	0.0526	0.0595	0.0798	0.0496	0.0568	0.0866	0.0463	0.0732	0.0872	0.0783
	S	0.0470	0.0261	0.0473	0.0280	0.0477	0.0225	0.0368	0.0195	0.0325	0.0323	0.0447	0.0252	0.0389	0.0287	0.0395	0.0320	0.0411	0.0372	0.0469	0.0250
BAC	M	-0.0091	0.0216	0.0845	0.0906	-0.0074	0.0025	0.0691	0.0839	-0.0399	-0.0490	0.0643	0.0757	-0.0336	-0.0278	0.0473	0.0727	-0.0068	0.0086	0.0547	0.0452
	S	0.0200	0.0246	0.0402	0.0253	0.0353	0.0227	0.0457	0.0273	0.0405	0.0286	0.0515	0.0255	0.0469	0.0331	0.0647	0.0387	0.0305	0.0272	0.0572	0.0439
C	M	0.0143	0.0342	0.0799	0.0854	0.0561	0.0684	0.0844	0.1099	0.0256	-0.0109	0.0632	0.0653	-0.0209	0.0210	0.0712	0.0819	-0.0136	0.0190	0.0904	0.0697
	S	0.0554	0.0337	0.0398	0.0364	0.0678	0.0279	0.0402	0.0244	0.0615	0.0338	0.0446	0.0272	0.0303	0.0252	0.0446	0.0353	0.0270	0.0314	0.0430	0.0443
STAN	M	0.0977	0.1227	0.1837	0.0921	0.1743	0.2181	0.1783	0.1108	0.1577	0.1938	0.1956	0.0897	0.1579	0.1892	0.1870	0.1108	0.1692	0.2021	0.2129	0.1076
	S	0.0491	0.0292	0.0319	0.0308	0.0362	0.0233	0.0329	0.0264	0.0273	0.0316	0.0371	0.0296	0.0339	0.0354	0.0361	0.0321	0.0439	0.0351	0.0335	0.0296
HSBC	M	0.0382	0.0746	0.1675	0.1483	0.0959	0.1205	0.1705	0.1821	0.0747	0.0792	0.1666	0.1833	0.1151	0.1316	0.1463	0.1725	0.1250	0.1728	0.2041	0.1826
	S	0.1044	0.0507	0.0502	0.0330	0.1080	0.0438	0.0445	0.0250	0.0920	0.0366	0.0667	0.0261	0.0669	0.0343	0.0778	0.0223	0.1440	0.0456	0.0639	0.0328
Total M		0.0502	0.0787	0.1309	0.1277	0.0820	0.0981	0.1269	0.1354	0.0502	0.0540	0.1343	0.1224	0.0600	0.0816	0.1158	0.1266	0.0709	0.0992	0.1465	0.1209
Total S		0.0698	0.0670	0.1008	0.0839	0.0795	0.0822	0.0983	0.0871	0.0808	0.1068	0.1283	0.1100	0.0836	0.0972	0.1233	0.1050	0.0921	0.0941	0.1272	0.1134

Table 10 Continued

Panel B. High state

		CUB				CCB				YCB				FCB				HNC			
		PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR
ANZ	M	0.0941	0.2733	0.4580	0.4624	0.0951	0.2931	0.4660	0.4685	0.0802	0.3318	0.4718	0.4864	0.0971	0.2465	0.4546	0.4876	0.1301	0.3129	0.5122	0.5507
	S	0.0250	0.0964	0.0262	0.0433	0.0264	0.1103	0.0180	0.0403	0.0300	0.1333	0.0164	0.0520	0.0299	0.0959	0.0365	0.0427	0.0340	0.1068	0.0230	0.0335
JPM	M	0.1172	0.1102	0.1637	0.0668	0.0803	0.0208	0.2072	0.0800	0.0651	0.0399	0.1231	0.1192	0.0901	-0.0363	0.0635	0.1243	0.0847	0.0367	0.1786	0.1494
	S	0.0275	0.0349	0.0161	0.0393	0.0182	0.0751	0.0394	0.0337	0.0392	0.0383	0.0362	0.0326	0.0326	0.0660	0.0283	0.0285	0.0315	0.0448	0.0257	0.0426
UBS	M	0.1480	0.1528	0.1954	0.1445	0.1143	0.0971	0.2851	0.1491	0.0845	0.0753	0.1999	0.1314	0.0988	-0.0022	0.1323	0.1974	0.1124	0.0916	0.2211	0.1879
	S	0.0394	0.0576	0.0178	0.0344	0.0250	0.0906	0.0304	0.0250	0.0321	0.0659	0.0153	0.0304	0.0212	0.0844	0.0114	0.0222	0.0308	0.0653	0.0145	0.0274
BAC	M	0.0479	0.1300	0.2493	0.1133	0.0167	0.0645	0.3164	0.1513	-0.0076	0.0209	0.1957	0.1369	-0.0182	-0.0445	0.1627	0.1907	0.0230	0.0672	0.2532	0.1754
	S	0.0240	0.0656	0.0190	0.0416	0.0215	0.1054	0.0267	0.0464	0.0234	0.0589	0.0211	0.0375	0.0360	0.0810	0.0187	0.0236	0.0234	0.0892	0.0127	0.0389
C	M	0.0812	0.1221	0.2185	0.1155	0.0766	0.0690	0.2909	0.1620	0.0375	0.0351	0.1928	0.1446	0.0716	0.0125	0.1517	0.1552	0.0629	0.0771	0.2275	0.1735
	S	0.0529	0.0455	0.0208	0.0478	0.0231	0.0795	0.0443	0.0514	0.0437	0.0468	0.0244	0.0475	0.0422	0.0572	0.0220	0.0420	0.0505	0.0585	0.0205	0.0482
STAN	M	0.1917	0.2327	0.1946	0.1911	0.2290	0.2370	0.3177	0.2476	0.2133	0.2784	0.3220	0.2820	0.2096	0.1847	0.2681	0.2619	0.2593	0.2859	0.3222	0.2642
	S	0.0457	0.0421	0.0125	0.0327	0.0168	0.0564	0.0307	0.0449	0.0331	0.0622	0.0094	0.0563	0.0242	0.0518	0.0151	0.0407	0.0352	0.0409	0.0160	0.0377
HSBC	M	0.2068	0.1469	0.2210	0.1701	0.2143	0.1128	0.2761	0.2044	0.1807	0.0809	0.2086	0.2226	0.1831	0.0168	0.1868	0.2517	0.2679	0.1534	0.2692	0.2509
	S	0.0390	0.0666	0.0205	0.0279	0.0195	0.0753	0.0248	0.0434	0.0231	0.0583	0.0180	0.0316	0.0192	0.0929	0.0213	0.0180	0.0206	0.0652	0.0170	0.0364
Total	M	0.1267	0.1669	0.2429	0.1805	0.1181	0.1278	0.3085	0.2090	0.0934	0.1232	0.2449	0.2176	0.1046	0.0539	0.2028	0.2384	0.1343	0.1464	0.2834	0.2503
Total	S	0.0659	0.0838	0.0933	0.1273	0.0745	0.1262	0.0794	0.1235	0.0792	0.1380	0.1097	0.1299	0.0757	0.1309	0.1198	0.1161	0.0939	0.1244	0.1040	0.1345

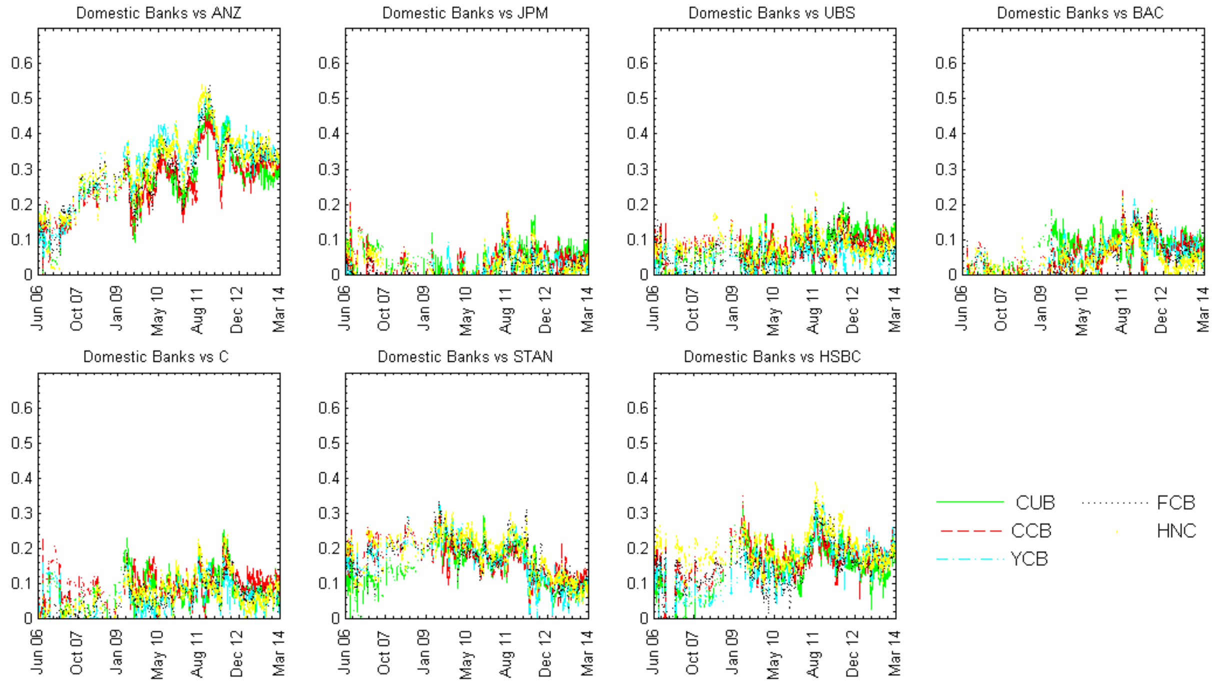
Table 11 Clay Copula Kendall Taus in Four Sub-periods

Panel A. Low state

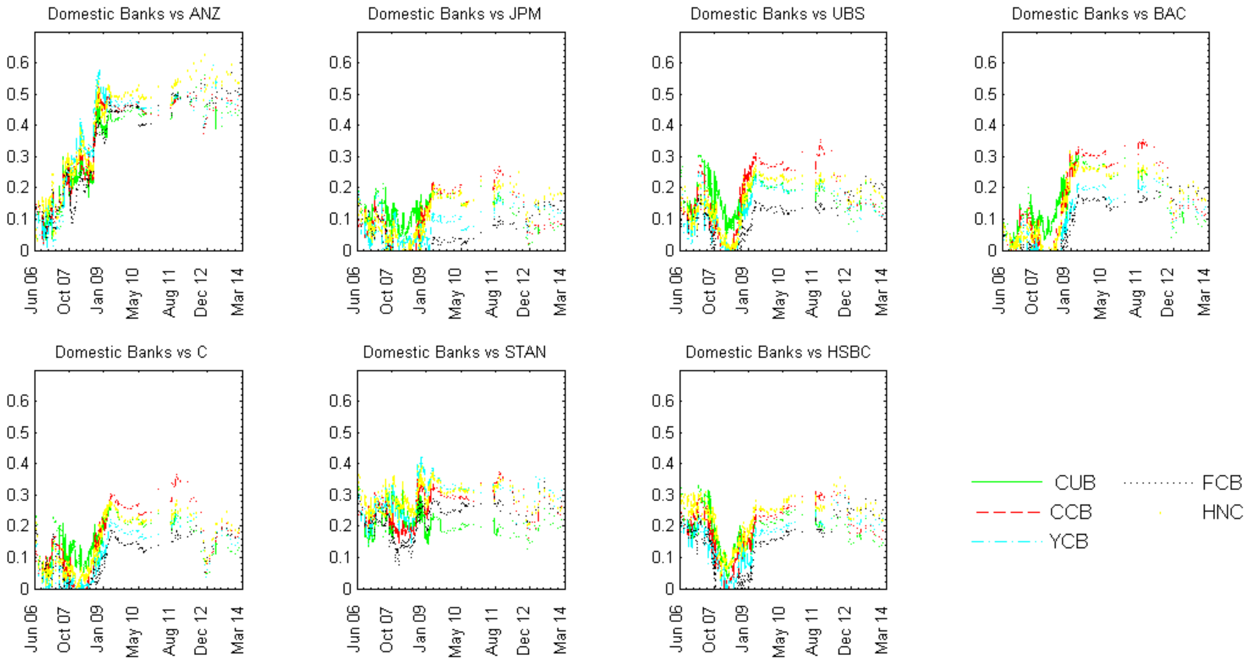
		CUB				CCB				YCB				FCB				HNC			
		PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR
ANZ	M	0.052	0.101	0.181	0.181	0.056	0.089	0.162	0.185	0.046	0.113	0.215	0.219	0.078	0.130	0.188	0.202	0.091	0.116	0.208	0.210
	S	0.046	0.050	0.040	0.061	0.036	0.038	0.042	0.039	0.032	0.041	0.062	0.037	0.046	0.060	0.069	0.040	0.056	0.038	0.055	0.044
JPM	M	0.032	0.015	0.007	0.031	0.019	0.009	0.016	0.017	0.001	0.001	0.007	0.018	0.003	0.006	0.001	0.010	0.023	0.003	0.009	0.009
	S	0.060	0.031	0.014	0.038	0.031	0.019	0.021	0.017	0.003	0.011	0.020	0.015	0.008	0.012	0.008	0.011	0.040	0.014	0.012	0.011
UBS	M	0.015	0.033	0.023	0.031	0.023	0.029	0.031	0.028	0.021	0.010	0.011	0.001	0.030	0.017	0.007	0.005	0.007	0.019	0.025	0.016
	S	0.026	0.066	0.022	0.073	0.025	0.028	0.037	0.070	0.089	0.047	0.027	0.005	0.026	0.040	0.023	0.007	0.013	0.036	0.035	0.019
BAC	M	0.001	0.008	0.043	0.025	0.001	0.008	0.035	0.027	0.001	0.000	0.031	0.028	0.000	0.007	0.028	0.013	0.000	0.010	0.027	0.019
	S	0.004	0.016	0.030	0.026	0.003	0.011	0.034	0.056	0.004	0.000	0.027	0.046	0.000	0.032	0.028	0.020	0.000	0.037	0.034	0.040
C	M	0.005	0.008	0.043	0.044	0.024	0.040	0.029	0.034	0.010	0.002	0.023	0.037	0.005	0.014	0.037	0.027	0.001	0.010	0.037	0.031
	S	0.018	0.031	0.038	0.024	0.059	0.054	0.031	0.036	0.030	0.008	0.027	0.033	0.016	0.020	0.034	0.030	0.004	0.019	0.029	0.035
STAN	M	0.055	0.086	0.120	0.052	0.086	0.119	0.118	0.061	0.076	0.103	0.111	0.038	0.070	0.101	0.120	0.062	0.077	0.112	0.129	0.060
	S	0.038	0.054	0.031	0.034	0.035	0.041	0.046	0.038	0.053	0.044	0.058	0.039	0.048	0.057	0.037	0.026	0.046	0.051	0.038	0.033
HSBC	M	0.010	0.032	0.092	0.080	0.031	0.061	0.080	0.085	0.035	0.035	0.069	0.089	0.044	0.054	0.051	0.085	0.060	0.085	0.098	0.098
	S	0.034	0.034	0.039	0.022	0.045	0.032	0.023	0.029	0.050	0.045	0.053	0.052	0.059	0.049	0.054	0.019	0.038	0.033	0.041	0.022
Total	M	0.024	0.040	0.073	0.063	0.034	0.051	0.067	0.062	0.027	0.038	0.067	0.061	0.033	0.047	0.062	0.058	0.037	0.051	0.076	0.063
Total	S	0.042	0.056	0.065	0.067	0.045	0.052	0.062	0.070	0.053	0.057	0.082	0.078	0.046	0.063	0.075	0.070	0.049	0.059	0.077	0.074

Panel B. High state

		CUB				CCB				YCB				FCB				HNC			
		PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR	PreR	SP	GD	PosR
ANZ	M	0.071	0.152	0.231	0.280	0.046	0.150	0.283	0.306	0.039	0.212	0.284	0.304	0.057	0.142	0.215	0.302	0.075	0.178	0.261	0.313
	S	0.066	0.117	0.050	0.105	0.019	0.116	0.076	0.120	0.020	0.120	0.089	0.110	0.036	0.097	0.081	0.092	0.031	0.090	0.086	0.092
JPM	M	0.041	0.032	0.036	0.017	0.040	0.019	0.087	0.077	0.023	0.009	0.030	0.071	0.031	0.009	0.049	0.085	0.034	0.022	0.079	0.103
	S	0.043	0.074	0.039	0.030	0.082	0.066	0.094	0.079	0.039	0.041	0.078	0.077	0.025	0.040	0.094	0.063	0.039	0.063	0.067	0.157
UBS	M	0.063	0.069	0.084	0.062	0.046	0.048	0.103	0.045	0.051	0.030	0.099	0.091	0.064	0.019	0.061	0.051	0.053	0.035	0.090	0.100
	S	0.030	0.075	0.082	0.035	0.034	0.077	0.075	0.039	0.068	0.053	0.063	0.080	0.031	0.050	0.063	0.041	0.033	0.057	0.076	0.038
BAC	M	0.015	0.060	0.102	0.032	0.003	0.043	0.128	0.080	0.009	0.012	0.077	0.062	0.023	0.013	0.071	0.108	0.010	0.040	0.110	0.074
	S	0.016	0.090	0.066	0.042	0.009	0.071	0.086	0.047	0.042	0.052	0.103	0.067	0.051	0.046	0.075	0.084	0.028	0.068	0.082	0.080
C	M	0.012	0.044	0.084	0.070	0.010	0.033	0.141	0.095	0.025	0.014	0.106	0.115	0.029	0.018	0.070	0.086	0.026	0.041	0.121	0.093
	S	0.028	0.074	0.059	0.084	0.022	0.067	0.106	0.056	0.057	0.057	0.153	0.094	0.029	0.063	0.055	0.067	0.043	0.074	0.065	0.082
STAN	M	0.117	0.147	0.084	0.115	0.115	0.141	0.141	0.157	0.111	0.153	0.185	0.174	0.120	0.113	0.138	0.149	0.132	0.157	0.159	0.149
	S	0.054	0.095	0.029	0.052	0.043	0.071	0.052	0.086	0.034	0.074	0.100	0.042	0.061	0.099	0.032	0.044	0.033	0.074	0.042	0.084
HSBC	M	0.078	0.064	0.100	0.072	0.105	0.059	0.120	0.080	0.080	0.041	0.093	0.124	0.094	0.029	0.089	0.147	0.125	0.068	0.136	0.132
	S	0.041	0.092	0.064	0.036	0.043	0.083	0.056	0.041	0.059	0.066	0.098	0.069	0.045	0.066	0.077	0.103	0.032	0.066	0.081	0.034
Total	M	0.057	0.081	0.103	0.092	0.052	0.070	0.143	0.120	0.048	0.067	0.125	0.135	0.060	0.049	0.099	0.133	0.065	0.077	0.137	0.138
Total	S	0.055	0.100	0.080	0.102	0.058	0.094	0.099	0.109	0.058	0.103	0.127	0.110	0.053	0.086	0.089	0.105	0.056	0.092	0.092	0.116

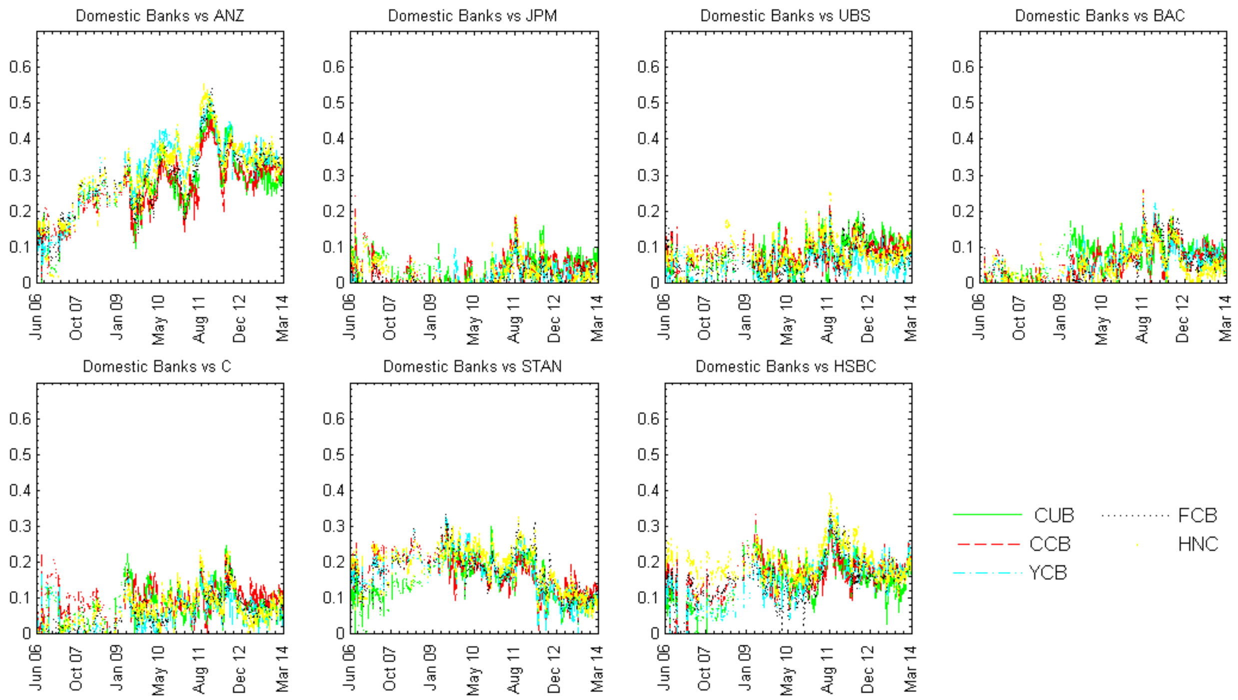


(a) Low state Kendall taus

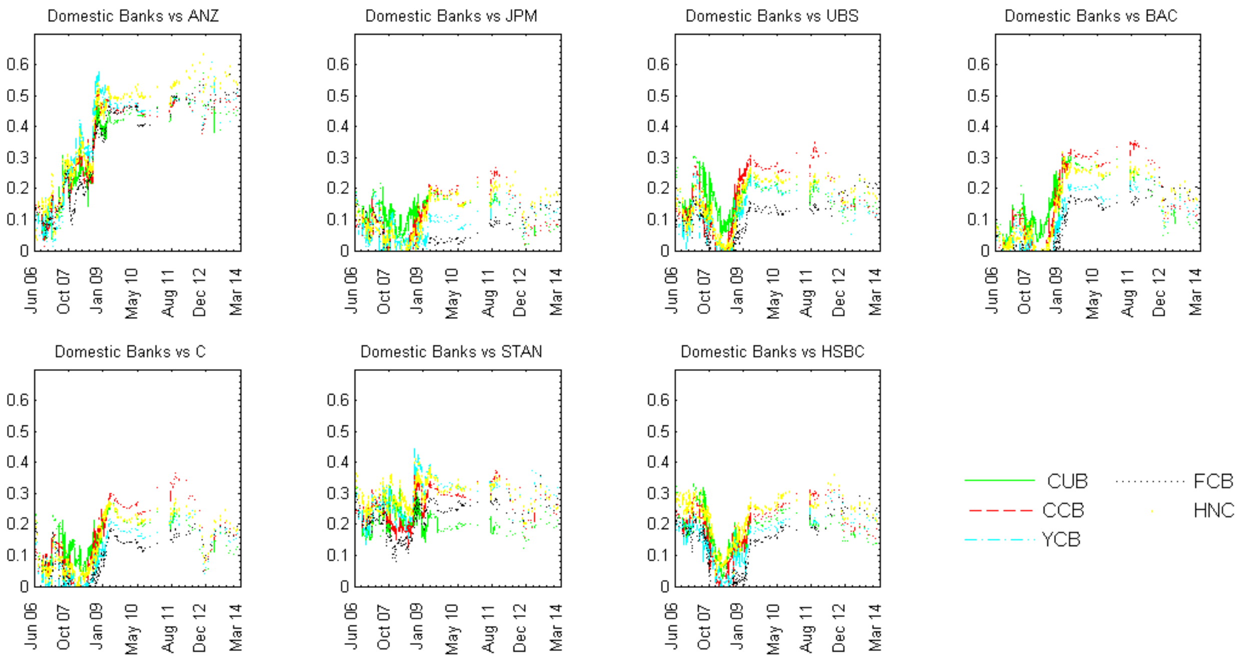


(b) High state Kendall taus

Figure 3 Out-of-Sample Gaussian Copula Kendall Taus

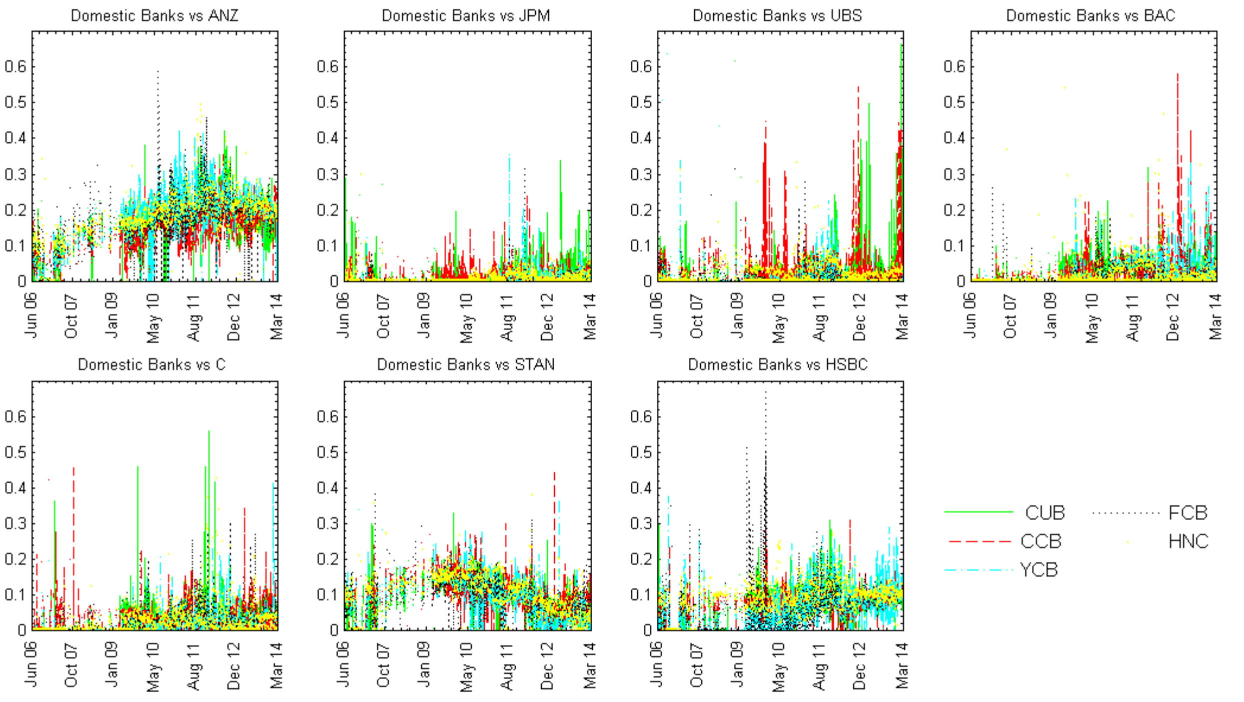


(a) Low state Kendall taus

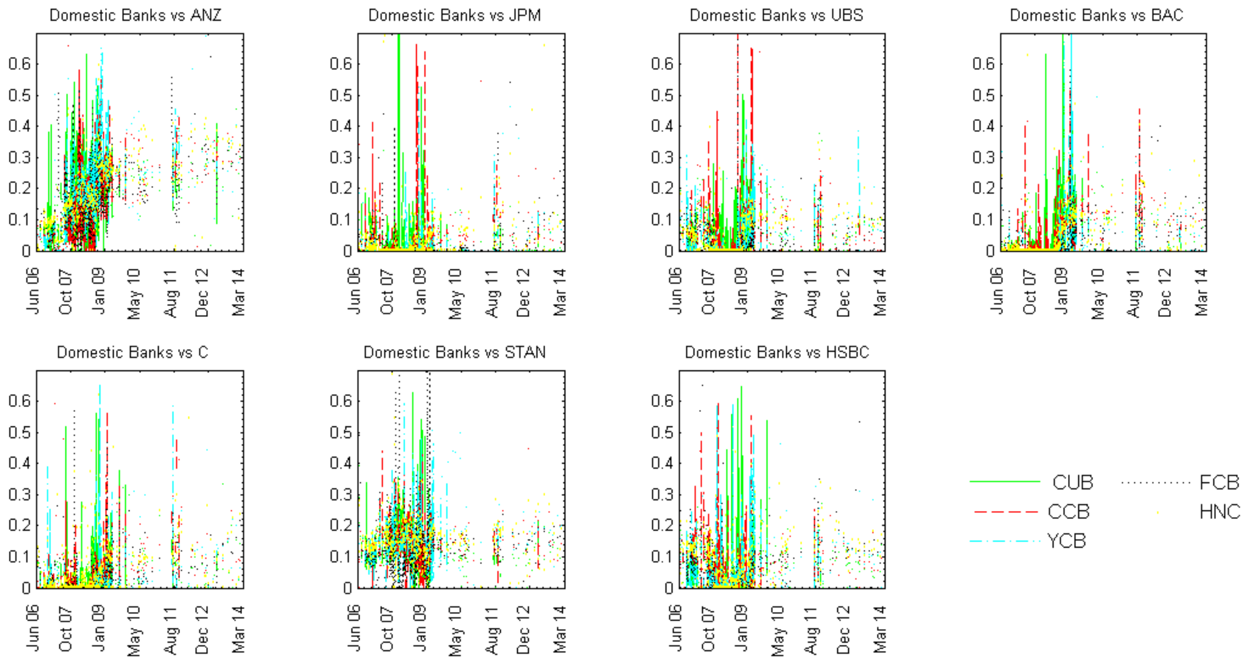


(b) High state Kendall taus

Figure 4 Out-of-Sample Student-t Copula Kendall Taus



(a) Low state Kendall taus



(b) High state Kendall taus

Figure 5 Out-of-Sample Clay Copula Kendall Taus

As a result, the three two-state copula GARCH models have all shown that the high state with higher Kendall taus is much more important to study the contagion risk. Thus, to analyze contagion risk, it is necessary to check the Kendall taus in high state situation. Compared between 5 domestic banks in high state, all three models reveal that on average HNC has highest Kendall taus while CUB has lowest Kendall taus against 7 foreign banks. It implies that HNC is more liable to the contagion risk while CUB is not so. Compared to the foreign banks, ANZ has the most tail correlation with the domestic banks. The second and third, are STAN and HSBC. Though UBS. C, JPM, and BAC have shown smaller tail correlation with domestic banks. They all have larger tail correlation with domestic banks in high state.

Figure 3 to 5 display the three models' Kendall taus respectively. Those figures illustrate that the ANZ, STAN, and HSBC indeed have higher tail correlation with domestic banks. Although JPM, BAC, C, and UBS have smaller tail correlations with domestic banks, their tail correlations all jump much higher in high state than in low state.

Note that the Kendall taus computed by Clay copula are more like occasional jumps that could reach more than 0.3 and upto 0.7. This situation is rare seen for t and Gaussian Copula estimation results except for ANZ who has regularly higher tail correlations with domestic banks. It appears that although Clay copula estimates the lower total mean of Kendall taus, it reflects tail correlations by jumps but not the smooth curve of tail correlations. This explains why it has lower total mean of Kendall taus. Specifically, Clay copula estimates tail correlations as the tail jumps occur. As a result, the Clay copula is used to expose the instant contagion risk.

Hence, it is not appropriate to use the total mean to measure long term Clay copula estimated Kendall taus. This fact could be explained by examining various Kendall tau equations. The Gaussian and t copula estimated Kendall taus are both comprised by ρ which is dependent on linear copula. That's why their estimations exhibit as smooth curve. The Clay Kendall taus is comprised by θ only which is the parameter of the nonlinear copula. That's why its estimation displays jumps.

4.4 Calculating Value at Risk and Expected Shortfall

As aforementioned, HNC has been tail correlated (i.e. contagious) with foreign banks most especially with ANZ and STAN. However, the aftermath of contagion risk is measure by the seriousness of tail jump but not the tail correlation. Thus, value at risk (VaR) and expected shortfall (ES) are proposed to further expose the consequence of contagion risk.

To analyze the seriousness of contagion risk, it is most appropriate to emphasize the risk in high state for the tail jump i.e. VaR as well as ES measures. As shown in Table 12, in high state YBC as well as FCB has the much higher VaR and ES compared to domestic banks while C and BAC have the much higher VaR and ES after sub-prime crisis.

Although HNC has the highest tail correlation with the foreign banks, its tail jumps measured by VaR and ES are not larger than domestic banks. ANZ encounters the same situation as HNC. In other words, HNC and ANZ are sensible but robust to the spread of contagion risk.

5. Conclusions

When the bad news emerges, it often causes the stock markets tumble deeply. It encore the tail jumps and volatility. The tail jumps and volatility also spread out risk everywhere which is called the contagion risk. The study of contaign risk aims to reveal the true tail correlation i.e. Kendall tau in state of high volatility but not low volatility for domestic banks. Thus, the two state threshold copula GARCH model is proposed to perform the task. Compared to two states, three models, four

Table 12 Value at Risk and Expected Shortfall in Four Sub-periods

	Full sample								Low State Sample								High State Sample							
	PreR		SP		GD		PosR		PreR		SP		GD		PosR		PreR		SP		GD		PostR	
	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES
CUB	0.0246	0.0305	0.0595	0.0698	0.0324	0.0487	0.0187	0.0236	0.0241	0.0291	0.0334	0.0491	0.0316	0.0466	0.0173	0.0221	0.0245	0.0308	0.0651	0.0719	0.0494	0.0579	0.0245	0.0356
CCB	0.0240	0.0331	0.0509	0.0662	0.0292	0.0472	0.0199	0.0299	0.0212	0.0296	0.0326	0.0476	0.0267	0.0435	0.0195	0.0289	0.0250	0.0346	0.0599	0.0691	0.0462	0.0591	0.0284	0.0322
YCB	0.0219	0.0305	0.0572	0.0690	0.0311	0.0470	0.0184	0.0255	0.0188	0.0210	0.0320	0.0455	0.0281	0.0422	0.0182	0.0239	0.0267	0.0357	0.0654	0.0722	0.0549	0.0662	0.0303	0.0324
FCB	0.0277	0.0396	0.0665	0.0703	0.0345	0.0474	0.0230	0.0318	0.0237	0.0389	0.0449	0.0570	0.0341	0.0456	0.0226	0.0304	0.0283	0.0364	0.0691	0.0708	0.0457	0.0559	0.0357	0.0413
HNC	0.0139	0.0203	0.0467	0.0644	0.0263	0.0390	0.0146	0.0182	0.0126	0.0181	0.0349	0.0545	0.0256	0.0358	0.0142	0.0173	0.0147	0.0212	0.0498	0.0661	0.0400	0.0520	0.0204	0.0236
ANZ	0.0133	0.0181	0.0442	0.0663	0.0233	0.0297	0.0168	0.0243	0.0128	0.0195	0.0275	0.0487	0.0226	0.0274	0.0163	0.0222	0.0133	0.0151	0.0475	0.0715	0.0321	0.0393	0.0246	0.0399
JPM	0.0137	0.0181	0.0650	0.1082	0.0384	0.0519	0.0194	0.0264	0.0106	0.0128	0.0132	0.0174	0.0292	0.0374	0.0182	0.0220	0.0158	0.0212	0.0744	0.1243	0.0677	0.0802	0.0371	0.0474
UBS	0.0174	0.0307	0.0859	0.1228	0.0449	0.0701	0.0262	0.0382	0.0116	0.0161	0.0299	0.0353	0.0326	0.0485	0.0250	0.0358	0.0269	0.0381	0.0941	0.1369	0.0897	0.0911	0.0428	0.0445
BAC	0.0117	0.0182	0.0876	0.1611	0.0520	0.0773	0.0229	0.0356	0.0059	0.0082	0.0176	0.0231	0.0370	0.0487	0.0202	0.0296	0.0154	0.0236	0.1158	0.1867	0.1006	0.1479	0.0477	0.0615
C	0.0122	0.0222	0.0920	0.1888	0.0531	0.0717	0.0241	0.0343	0.0089	0.0131	0.0194	0.0336	0.0376	0.0482	0.0222	0.0281	0.0174	0.0269	0.1347	0.2203	0.1025	0.1310	0.0506	0.0585
STAN	0.0210	0.0265	0.0676	0.0973	0.0331	0.0540	0.0253	0.0438	0.0193	0.0231	0.0351	0.0481	0.0271	0.0472	0.0236	0.0412	0.0216	0.0285	0.0774	0.1078	0.0534	0.0582	0.0415	0.0462
HSBC	0.0124	0.0185	0.0414	0.0709	0.0314	0.0483	0.0177	0.0260	0.0080	0.0110	0.0101	0.0145	0.0224	0.0329	0.0162	0.0230	0.0139	0.0228	0.0483	0.0811	0.0726	0.0810	0.0313	0.0337

Note: Refer to Table 9 for the data size and date period of four sub-periods: PreR, SP, GD, and PosR.

subperiods, and out-sample Kendall taus, several findings are demonstrated for 5 domestic and 7 foreign banks as follows:

(1) In high state, larger contagion risk indeed exists between 12 banks than in low state.

(2) The uptrend of contagion risk after prerisk period indeed is found between most banks. In general, the Greek debt period has the highest contagion risk.

(3) The t and Gaussian copula model are estimated better in low state than in high state while the Clay copula model is estimated in high state as well as in low state. The result of the Kendatll taus shows that t and Gaussian is suitable for longer tem Kendall tau estimation but the Clay copula is suitable for instant Kendall tau estimation as revealed by the Kendall tau Equations.

(4) ANZ and HNC have the higher Kendall tau which is the tail correlation. However, they have lower VaR and ES which is the tail jumps. In other words, though they are liable to the contagion risk, but they are robust to expose lighter symptom of disease.

(5) Though YBC and FBC do not have the larger Kendall tau, they still have higher VaR and ES as the bad news occurs. BAC, C, and UBS are found so similar as YBC and FBC.

Actually, the Conation risk spreads from larger foreign banks to smaller domestic banks through the tail jumps and tail correlation. It ought to study both of the tail affects. The bad news often incurs extremal events or tail jumps in instant time. It appears right to use Clay copula to study the nonlinear instant extremal events which are depicted by jumps. On the contrary, the t or Gaussian copula is used to study the linear longer extremal events which are displayed by smooth curve. To further analyze the correlation of tail jumps, it seems valuable to use extreme value theory to find the probability distribution of the tail correlation to compare and control the contagion risk.

Appendix A. Archimedean Copulas

(1) Clayton copula

$$C_{Cl}(u_1,u_2;\theta)=\left\{u_1^{-\theta}+u_2^{-\theta}-1\right\}^{-\frac{1}{\theta}},\theta\in(0,\infty). \tag{B.1}$$

This copula is characterized by lower tail dependences and upper tail independence.

(2) Gumbel copula

$$C_G(u_1,u_2;\theta)=\exp\left\{-\left[(-\ln u_1)^{\theta}+(-\ln u_2)^{\theta}\right]^{\frac{1}{\theta}}\right\}\ ,\theta\in[1,\infty). \tag{B.2}$$

This copula is characterized by lower tail independence and upper tail dependences.

(3) Frank Copula

$$C_F(u_1,u_2;\theta)=-\frac{1}{\theta}\ln\left\{1+\frac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)}{e^{-\theta}-1}\right\}\ ,\theta\in R\setminus\{0\}. \tag{B.3}$$

This copula is characterized by upper and lower tail independence.

References

Ang, A. and G. Bekaert (1999). International asset allocation with time-varying correlations. NBER Working Paper 7056.

Ang, A. and G. Bekaert (2002), International asset allocation with regime shifts. *Review of Financial Studies*, 11, 1137-1187.

Ang, A. and J. Chen (2002). Asymmetric correlations of equity portfolios, *Journal of Financial Economics*, 63(3), 443-494.

Bae, K.H., G. A. Karolyi, and R. M. Stulz (2003). A new approach to measuring financial contagion. *The Review of Financial Studies*, 16, 717–763.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31, 307-327.

Bollerslev, T., R. F. Engle, and J. M. Wooldridge (1988). A capital asset pricing model with time-varying covariances. *The Journal of Political Economy*, 96, 116–131.

Cao, C.Q. and R.S. Tsay (1992). Nonlinear time series analysis of stock volatilities. *Journal of Applied Econometrics*, 7, 165-185.

Chan, K. S. and H. Tong (1986). On estimating thresholds in autoregressive models. *Journal of Time Series Analysis*, 7, 178-190.

Chollete, L., A. Heinen, and A. Valdesogo (2009). Modeling international financial returns with a multivariate regime switching copula. *Journal of Financial Econometrics*, 7(4), 437-480.

Clayton, D.G. (1978). A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence. *Biometrika*, 65, 141-151.

Ding, Z., C.W.J. Granger, and R. F. Engle, 1993, "A Long Memory Property of Stock Market Returns and a New Model" *Journal of Empirical Finance*, 1, 83-106.

Embrechts, P., F. Lindskog, and A. McNeil (2001). Modelling dependence with copulas and applications to risk management. ETHZ, Working Paper.

Embrechts, P., A. McNeil, and D. Strauman (2002). Correlation and dependence properties in risk management: properties and pitfalls, in M. Dempster, ed., Risk management: Value at Risk and beyond, Cambridge University Press.

Engle, R. F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation. *Econometrica*, 50, 987-1008.

Engle, R. F. (1983). Estimates of the variance of U.S. inflation based upon the ARCH model. *Journal of Money, Credit, and Banking*, 15, 286-301.

Engle, R. F. and V. K. Ng (1993). Measuring and testing the impact of news on volatility. *The Journal of Finance*, 48, 1749-1777.

Frank, M.J. (1979). On the simultaneous associativity of F(x,y) and x+y−F(x,y). *Aequationes Mathamatics*, 19, 194-226.

Garcia, R. and G. Tsafack (2011). Dependence structure and extreme comovements in international equity and bond markets, *Journal of Banking and Finance*, 35, 1954-1970.

Glosten, L., R. Jagannathan, and D. Runkle (1993). Relationship between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*, 48, 1779-1801.

Gumbel, E.J. (1960). Bivariate exponential distributions. *Journal of the American Statistical Association*, 55, 698-707.

Haas, M., S. Mittnik, and M.S. Paolella (2004). A new approach to Markov switching GARCH models. *Journal of Financial Econometrics*, 2, 493-530.

Hamilton, J.D. (1990). Analysis of time series subject to changes in regime. *Journal of Econometrics*, 45, 39-70.

Hamilton, J.D. (1991). A quasi-Bayesian approach to estimating parameters for mixtures of normal distributions. *Journal of Business and Economic Statistics*, 9, 27-39.

Hamilton, J.D. and R. Susmel (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics*, 64, 307-333.

Hansen, B.E. (1996). Inference when a nuisance parameter is not identified under the null hypothesis. *Econometrica*, 64, 413-430.

Hansson, B. and P. Hordahl (1998). Testing the conditional CAPM using multivariate GARCH. *Applied Financial Economics*, 8, 377-388.

Honda, T. (2003). Optimal portfolio choice for unobservable and regime-switching mean returns. *Journal of Economic Dynamics and Control*, 28, 45-78.

Hu, L. (2006). Dependence patterns across financial markets: a mixed copula approach. *Applied Financial Economics*, 16, 717-729.

Jamaleh, A. and G. Venezia (2001). Threshold model for Italian stock market volatility. *Revista Politica Economia*, 91(2), 79-132.

Joe, H. (1997). Multivariate models and dependence concepts. Chapman and Hall.

Jondeau, E. and M. Rockinger (2002). Conditional dependency of financial series: The copula-GARCH Model. FAME Research Paper Series rp69.

Jondeau, E. and M. Rockinger (2006). The copula-Garch model of conditional dependencies: an international stock market application. *Journal of International Money and Finance*, 25, 827-853.

Longin, F. and B. Solnik (2001). Extreme correlations in international equity markets. *Journal of Finance*, 56, 649-676.

Nelsen, R. B. (1999). An introduction to copulas. Lectures notes in statistics, 139, Springer Verlag, New York.

Nelson, D.B. (1991). Conditional heteroskedasticity in asset returns: a new approach. *Econometrica*, 59, 347-370.

Ng, L. (1991). Tests of the CAPM with time-varying covariances: a multivariate GARCH approach. *The Journal of Finance*, 46, 1507-1521.

Patton, A. J. (2001). Modelling time-varying exchange rate dependence using the conditional copula, Working Paper, U.C. San Diego.

Patton, A.L. (2006). Modelling asymmetric exchange rate dependence, *International Economic Review*, 47(2), 527-556.

Ramchand, L. and R. Susmel (1998). Cross correlations across major international markets. *Journal of Empirical Finance*, 5, 397-416.

Rodriguez, J. (2007), Measuring financial contagion: a copula approach, *Journal of Empirical Finance*, 14, 401-423.

Schweizer, B. and E. Wolff (1981). On nonparametric measures of dependence forrandom variables. *Annals of Statistics*, 9, 879-885.

Sklar, A.W. (1959). Fonctions de répartitionà n-dimension et leurs marges. *Publications de lSInstitut de Statistique de lSUniversité* de Paris, 8, 229-231.

Teräsvirta, T. (1994). Specification, estimation, and evaluation of smooth transition autoregressive models, *Journal of the American Statistical Association*, 89, 208-218.

Tong, H. (1990). Non-linear time series: a dynamical system approach, Oxford, Oxford University Press.

Tong, H. and K. Lim (1980). Threshold Autoregression, Limit Cycles, and Cyclical Data. *Journal of the Royal Statistical Society*, 42, 245-292.

Tsay, R. S. (1989). Testing and modeling threshold autoregressive processes, *Journal of the American Statistical Association*, 84, 231-240.

Tse, Y. K. and K. C. Tsui (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business and Economic Statistics*, 20, 351-362.

Zakoïan, J.M. (1994). Threshold heteroskedastic models, *Journal of Economic Dynamics and Control*, 18, 931-955.