On the Emergence of Private Insurance in Presence of Mutual Agreements

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Abstract

The aim of this paper is to analyze the impact of the existence of mutual firms on the behavior of an insurance company and more precisely to study in which situations a private insurance firm may emerge in presence of an incumbent mutual firm. Our approach differs from the existing literature as we integrate the investment choices of the company and the fact that, because it commits on a fix contract, it can become insolvent. In such a situation we are able to characterize the unique optimal choices of an entrant company and the conditions favoring or preventing its appearance.

Key words: Insurance market, Mutual firms, Commitment, Insolvency

JEL Classification No. : L1, D8, G22

1 Introduction

Historically, mutual agreements have been the first mean used to cope with risk. Starting from benevolent societies in the ancient Greece or guilds in the Middle-Age, reciprocal help and mutual assistance has been first used by people to be insured against various risks as fire, robbery or floods. The emergence of private insurance companies is very posterior. Starting in medieval Genoa during the 14th century with third-party insurance in shipping industry, private insurance definitively arises in Great Britain with fire insurance in the 17th century and the use of external capital. Since, both organizational forms experienced various success and none of them really dominates insurance markets. For example, as stated in Hansmann (1985), many changes in organizational forms happened in the United States during the 20st century. First organized as private organizations, many of the largest insurance companies choose to mutualize in the earlier part of the century. The share of mutual firms in life insurance even hit 69 percent in 1947. However, during the second part of the 20st century and the beginning of the 21st, the reverse effect arisen and many mutual firms have converted to the stock form. These phenomenons called mutualization and demutualization waves, rise the issue of the parameters driving the preference for one form or the other. Similarly, as mutual forms control health insurance in most European countries and as risk is largely insured with mutual agreements in developing countries it seems worthwhile to study why insurance companies have very low market shares in some countries or sectors.

The main objective of this paper is to analyze the influence of the pre-existence of mutual firms on the choices of an entrant insurance company when optimal behavior consists in decisions about both offered coverage and capital stock. We analyze more precisely the impact of the existence of mutual firms as individuals outside option on the optimal profit of an unique entrant insurance company. We focus on the effects of parameters as the cost of capital, the distribution of income, the degree of risk aversion and the size of the population. In this way we are able to analyze how and when an insurance company may attract mutual firms policyholders and to determine which variable make or not an insurance company enter the market. However in this paper we do not consider the entry of further insurance companies and are thus unable to study the impact of openness to competition in a regulated market. Still, by studying when a company can not rule out mutual agreements our paper analyzes when mutual agreements may be sustainable and why some market remain reserved to such arrangements even without any regulation.

To do so, we build a model that captures the main features distinguishing mutual and stock insurers, namely : (i) a difference in the ownership structure : while insurance companies are owned by their shareholders, mutual firms belong to their policyholders, (ii) a difference in the objective of the organization : whereas insurance companies aim to maximize return on invested capital, mutual firms theoretically maximize its members satisfaction, and (iii) a difference in the definition of risk : stock firms have to precisely define at stake risks to contract on a fixed premia when mutual ones can define risk ex-post as they systematically adjust offered premia a posteriori. These three differences imply several trade-offs between the two organizational forms. Firstly, as shareholders are assumed to be risk-neutral, while policyholders are risk adverse, the insurance company appears to have a comparative advantage in bearing risk. However, raising capital externally is costly and as shareholders are profit seeker, conflicts may arise between shareholders and policyholders. As this paper introduces in the discussion the investment choice of the insurance company, the last difference will have an important role in our setting. Indeed, although external capital is highly useful when aggregate loss is high, it may become insufficient to honor the specific contract the firm commits on and the company may become insolvent. Like most of the papers on this topic we then assume that agents are perfectly rational and thus take into account the probability of insolvency when making their choices. Therefore an individual may not ever wish an increase in coverage as it also increases the insolvency probability of the insurance company.

Under such considerations, we characterize in this paper the optimal choice of coverage and capital investment of a single entrant company facing an incumbent mutual firm, and show it is unique. In doing so we are able to determine the conditions under which this equilibrium gives a positive expected profit, that is to state when an insurance firm can enter the market. Analyzing these conditions, this paper provides interesting comparative statics, either based on analytical results or simulations. This way we show that a decrease in the cost of capital raises the optimal capital stock, lowers optimal proposed coverage and thus rises the likelihood for a stock firm to be set up. We also prove in this paper that when a distribution of aggregate income dominates another one in the sense of first order stochastic dominance, the optimal offered coverage increases. Another result of interest is the fact that a higher individual degree of risk aversion increases optimal capital reserves, decreases optimal coverage and that those two forces result in an increase in the optimal profit. Simulations on the influence of the insured population size then allows to state that, when risks are independent, an increase in the number of policyholders raises the optimal offered coverage and lowers the possibility for a stock company to emerge. Lastly, we prove in this paper that the opportunity for an insurance firm to enter the market is higher when individual risk is high, as an increase in the variance of income increases optimal capital reserves and decreases optimal coverage. Those two last results are consistent with the findings of previous empirical works either on demutualization or on the difference between the two organizational forms.

We briefly discuss the relationship of the paper with the most closely related literature. This paper fits into the literature on organizational form in insurance that first tries to explain the co-existence of mutual firms with insurance companies. Focusing mainly on the difference in the ownership structure, Mayers and Smith (1988) argue that the two organizational forms coexist because each ownership structure has a comparative advantage in preventing different types of agency problems (mutual firms prevent for conflict between shareholders and policyholders but have less incentive to control their managers). Alternatively, Smith and Stutzer (1990) and Doherty and Dionne (1993) take into account the additional feature that policyholders of mutual firms bear the aggregate risk (that is that mutual firms – contrary to stock companies – offer participating policies) to explain this coexistence arguing that stock and mutual firms insure different kind of individuals or different kind of risks [1].

Ours is not the first paper to includes the possibility of insolvency. Focusing on the solvability regulation, Rees, Gravelle and Wambach (1999) show that, in absence of mutual firms, it is optimal for the insurance companies to hold enough capital to avoid insolvency when total losses are bounded. However, if it is not always feasible to escape bankruptcy or if the market is not frictionless, this result does not hold. Laux and Muermann (2006) study optimal choices of mutual and stock insurers when there are frictions and more precisely when there exist conflicts between managers and owners. They first show that, without any competition between stock and mutual firms, it is optimal for policyholders to transfer wealth between solvency and insolvency states. Making capital choice endogenous, they show that capital stock and premia are both decreasing with governance problems and increasing with competition. Finally they prove that the incentive to increase the number of policyholders is higher for mutual firms. They however consider stock and mutual firms as independent entities that do not compete to attract policyholders. On the contrary when analyzing the impact of mutual firms on the insurance market, Fagart, Fombaron and Jeleva (2002) model the interactions between insurance companies and mutual firms but do not study optimal capital choice. They show that the expected utility of the consumers depends on the size of the organization they belong to and thus that the existence of mutual firms modifies optimal behavior of insurance companies, when it only consists of offered premia. Here, we endogenize in addition the choice of capital and define the optimal choice of the company in a way allowing for more comparative statics. We are then able to better analyze the emergence of

[1] Doherty and Dionne (1993) prove that, if covered risks are decomposable into diversifiable (idiosyncratic) and non-diversifiable parts, it is optimal, given participating nature of mutual firms policies, to combine insurance firms (non-participating) coverage with mutual risk sharing arrangements. On the same direction, Smith and Stutzer (1990) show, using a variant of adverse selection model of Rothschild and Stiglitz (1976), that because of their participating nature, mutual firms attract low risk individuals who want to signal their type.

private insurance. Compared to Fagart, Fombaron and Jeleva (2002) this is done at the price of considering a unique stock firm.

Our paper contributes to the literature on insurance forms by studying both the interactions between the two organizational forms and the investment choices of insurance companies.

The rest of the paper is structured in the following way. We present the model in Section 2 before characterizing the optimum and its implication on firms participation (in Section 3). Comparative statics either based on analytical results or simulations are provided in Section 4 and compared to previous empirical findings in Section 5. Our conclusion and directions for future research are outlined in Section 6.

2 The Model

2.1 General Assumptions and Notations

We consider n identical risk averse individuals with increasing and concave utility function $u(.)$ that satisfies the Inada conditions. Each agents receive random revenue $\widetilde{x}_i$, $i = 1, ..., n$, the $\widetilde{x}_i$s being independent[2] and identically distributed. We assume then that aggregate revenue in the economy called $\widetilde{\omega} \equiv \sum_{i=1}^{n} \widetilde{x}_i$ is distributed according to some cumulative distribution function $F(.)$ with density $f(.)$. This random variable may be interpreted as total crop or a sum of revenues adjusted for uncetain health spending for example.

2.2 The Insurance Process

As agents are risk averse, they want to be insured against risks of changes in revenue and we consider that they face two kinds of organizations to do so.

[2]The assumption of independence is not necessary for our main results to hold. However, to relax it we need to model a specific form of correlation. For example, as shown in Henry (1981), our analysis is likely to remain accurate for a f-modulated stochastic dependence

- They originally share risk thanks to a mutual agreement. In our static framework, such an agreement corresponds to a sharing rule of the aggregate revenue. Indeed, whatever the coverage specified, a mutual firm being collectively owned by its policyholders, they receive any extra profit at the end of the period, such that the whole revenue is shared. Following Borch (1962), Eeckhoudt and Gollier (1995) and Fagart, Fombaron and Jeleva (2002) we then have that the optimal sharing rule is characterized by following proposition.

Proposition 1 *When individual risks are independent and identically distributed, a mutual aggreement optimally provides equal sharing of resource such that each policyholder of a mutual firm with n members get $u\left(\frac{\omega}{n}\right)$ whatever the state of the world. Moreover, as shown in Fagart, Fombaron and Jeleva (2002), the expected utility of mutual policyholders is increasing with the number of people in such an agreement.*

Proof*: See Appendix*

- They can choose to subscribe to a policy in an entrant insurance company that proposes to provide a fixed income y to each of its policyholders. This firm is owned by shareholders that invest in the insurance market a capital stock K at the beginning of the process (i.e. before the realization of the $\widetilde{x}_i$s is known) and gets the profit of the company Π at the end (that is after having indemnified the policyholders). They however face a discount factor δ that represents the opportunity cost of capital[3]. Thus, if this company insures the entire population, it goes bankrupt when $\omega < n.y - K$ and its probability of insolvency is equal to $F(n.y - K)$. In such cases, as policyholders have priority, the firm equally shares its whole resources (premia plus capital) among its policyholders and each policyholder of a company that insures the n agents gets $u\left(\frac{\omega + K}{n}\right)$ (which is less than $u(y)$ when

[3]If capital has no opportunity cost or if there is no discount factor, the company has an incentive to accumulate an infinite amount of capital and thereby avoid bankruptcy.

$\omega < n.y - K)$ [4]. As we assume that policyholders fully anticipate this probability of insolvency, the expected utility of an individual insured with all the others in the entrant insurance company is :

$$U(y, K) \equiv [1 - F(n.y - K)].u(y) + \int_{-\infty}^{n.y-K} u\left(\frac{\omega + K}{n}\right) f(\omega)\, d\omega \tag{1}$$

Remark 1 *The expected utility of an individual insured in the entrant stock firm is increasing in both the offered coverage and the company capital stock.*

The reader should note that this contract can easily be related to usual insurance contracts that implies an indemnity and a premium. Considering $\widetilde{x}_i$ as composed by a certain revenue $\overline{x}$ minus a random positive loss $\widetilde{l}_i$, y can then be defined as $y = \overline{x} - \widetilde{l}_i - \pi + i\left(\widetilde{l}_i\right)$, where π represents the premium and $i(l)$ the indemnity paid when the loss l occurs. As we focus here on complete insurance, we also need y to be certain that is $i\left(\widetilde{l}_i\right)$ to be equal to $\widetilde{l}_i + c$, where c is a constant. Now, to avoid for usual problems of moral hazard, we also need policyholders not to have an incentive to declare a loss but when it occurs. We thus need $i\left(\widetilde{l}_i\right) \leqslant \widetilde{l}_i$, that is $c \leqslant 0$. So, $y = \overline{x} - \pi + c \leqslant \overline{x} - \pi$ and y is upward bounded by $\overline{x}$.

2.3 The Incentive Constraint and the Profit of the Insurance Company

Let us suppose a two-stage game where

- at $t = 1$: the insurance company raises capital K and offers a contract (y, K)
- at $t = 2$: the policyholders has to choose whether to stay in the mutual or to go to the entrant insurance company .

[4] The main results and properties of our analysis remain unchanged even if policyholders only have priority on total premia and a portion $0 \leqslant \lambda \leqslant 1$ of the capital stock. In this case it can be shown that it is all the more difficult for a insurance company to enter the market as λ is low, that is as shareholders have priority on a large part of the capital stock

As it insures the same risk as the pre-existing mutual agreement the entrant firm has to provide its policyholders with at least as much utility as under the equal-sharing rule among n individuals. As all agents are homogenous and supposing that insurance contracts are anonymous, since the utility in the mutual firm is increasing with its number of members (see Proposition 1), an insurance company managing to attract one individual insures the entire population. Thus, to enter the market, the insurance company only need to offer a contract (y, K) providing – when every individuals sign it (the stock firm then fully take advantage of risk pooling among its policyholders) – at least as much utility as the mutual firm. The equal-sharing rule among n individual then defines the lower bond of what the insurance company need to provide its policyholders with. To enter the market it thus has to offer a contract (y, K) satisfying the following incentive constraint:

$$[1 - F(n.y - K)].u(y) + \int_{-\infty}^{n.y-K} u\left(\frac{\omega + K}{n}\right) f(\omega)\, d\omega \geq E\left(u\left(\frac{\omega}{n}\right)\right) \tag{2}$$

This constraint implies that only one organizational form exists at the equilibrium. Indeed, even if this incentive constraint binds, our model does not bring out coexistence of mutual with stock firms at the equilibrium, as then

- either every policyholders stay in the mutual firm,
- or one policyholder moves to the stock company making it more attractive to all the others (because of Proposition 1) and the insurance company is the only form to perform.

Remark 2 *If it does not hold any capital, the insurance company is unable to sell any policy as agents are then better off in the mutual firm whatever the coverage proposed by the company.*

If $K = 0$, the company can never do better than the mutual firm when it is solvent as then $y < \frac{\omega}{n}$. The only way for it to satisfy the constraint is then to always go bankrupt, that is to set

$y = \overline{x} \equiv \frac{\overline{\omega}}{n}$ (where $\overline{\omega}$ represent the upper bar of the distribution of $\widetilde{\omega}$). However, in this case its behavior exactly amounts to the one of a mutual firm as, when it goes bankrupt, the company equally shares its whole resource that then only consist in $\widetilde{\omega}$. So, without capital, an insurance company can not actually exist as its optimal choice is then to act just like a mutual firm. This moreover implies that the existence of a mutual firm by itself forces the company to hold capital as, in the absence of a mutual firm, an insurance firm can still make positive profits even if it does not hold capital. In this case, the outside option is for the individual to be uninsured what can be overstepped by the insurance company even when it goes bankrupt, thanks to risk pooling effects between policyholders.

So, in order to attract policyholders, an insurance company facing an incumbent mutual agreement has to invest in capital stock before the realization of the risk variable. Its expected profit can then be written as:

$$\Pi(y, K) = \delta. \int_{n.y-K}^{+\infty} (\omega + K - n.y)\, f(\omega)\, d\omega - K \tag{3}$$

As this expected profit is decreasing in both y and K, the only incentive for the company to increase y and K is to attract policyholders. The fact that the insolvency probability influence policyholders' behavior leads us to study the stock of capital as a choice variable of the insurance company.

3 Optimal Behavior of a Single Entrant Insurance Company Facing an Incumbent Mutual Firm

The program of the entrant insurance company consists in the maximization of $\Pi(y,K)$ under the constraint that individuals subscribe its policy, that can be rewritten as:

$$C(y,K) \equiv \int_{-\infty}^{n.y-K} \left[u\left(\frac{\omega+K}{n}\right) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega + \int_{n.y-K}^{+\infty} \left[u(y) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega \geqslant 0 \quad (4)$$

As this constraint is increasing in both y and K (see Remark 1) when profit is decreasing with those two variables, it is satisfied with equality.

The problem thus become

$$\max_{y,K} \left\{\delta.\int_{n.y-K}^{+\infty} (\omega + K - n.y)\, f(\omega)\, d\omega - K\right\} \tag{5}$$
$$\text{s.t.} \quad \left\{ \begin{array}{l} C(y,K) \equiv \displaystyle\int_{-\infty}^{n.y-K} \left[u\left(\frac{\omega+K}{n}\right) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega \\ \qquad + \displaystyle\int_{n.y-K}^{+\infty} \left[u(y) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega = 0 \end{array} \right.$$

Proposition 2 *Suppose that either the support of $\widetilde{\omega}$ is unbounded or the upper bound of the support $\overline{\omega}$ satisfies*

$$\frac{1-\delta}{\delta} < \frac{\displaystyle\int_{-\infty}^{\overline{\omega}} \left(u'\left(\frac{\omega}{n}\right) - u'\left(\frac{\overline{\omega}}{n}\right)\right) f(\omega) d\omega}{u'\left(\dfrac{\overline{\omega}}{n}\right)} \tag{6}$$

Then there exists a unique optimal solution for program (5) that yields a positive profit, fully *characterized by the two following equations:*

$$\Phi(y,K) \equiv \int_{-\infty}^{n.y-K} \left[\frac{u'\left(\frac{\omega+K}{n}\right) - u'(y)}{u'(y)} \right] f(\omega)\, d\omega - \frac{1-\delta}{\delta} = 0 \tag{7}$$

$$\begin{aligned} C(y,K) \equiv & \int_{-\infty}^{n.y-K} \left[u\left(\frac{\omega+K}{n}\right) - u\left(\frac{\omega}{n}\right) \right] f(\omega)\, d\omega \\ & + \int_{n.y-K}^{+\infty} \left[u(y) - u\left(\frac{\omega}{n}\right) \right] f(\omega)\, d\omega = 0 \end{aligned} \tag{8}$$

Proof*: See Appendix*

Proposition 2 states that, if aggregate wealth can be infinite, an insurance company can always enter a market controlled by mutual firms. However, when aggregate wealth is bounded, an insurance company may not be able to make a positive profit by entering the market.

When an insurance company can enter the market, Proposition 2 moreover characterizes its optimal behavior, that consists of investing capital stock K^* and proposing a coverage y^* that satisfies the first order condition $\Phi(y,K) = 0$ and the constraint $C(y,K) = 0$. One can also show that this optimal behavior is unique as the first order condition expresses an increasing relationship between the offered premium and the stock of capital whereas the constraint defines a decreasing mapping between those two variables. The direction of those two relationships characterizing the equilibrium may be intuitively explained. The fact that the profit maximization gives rise to an increasing relationship between the coverage and the stock of capital may be explained by the effect of those two variables on the insolvency probability. As an increase in coverage increases this probability, the company has to also rise the stock of capital to restore a reasonable insolvency probability. Concerning the interactions with the incumbent mutual insurer, an increase in y increases the attractiveness of the company. Thus, everything else being equal, the company can decrease its capital stock without losing any policyholders. Figure 1 illustrates the first order condition and the constraint in the plan (K, y).

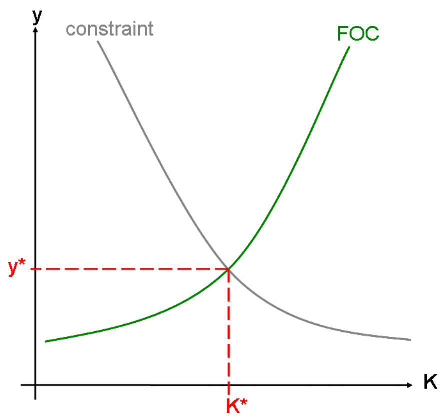


Figure 1: The Optimal Behavior of the Insurance Company

4 Comparative Statics

In this section, we analyze the effect of different variables on the firm's optimal choices and profit. By studying how parameters of the model affect the entrant firm's profit, we are able to characterize conditions under which formal insurance companies are likely to emerge.

4.1 Analytical results

We first derive three analytical results, relating the insurance company's choices and profit to the cost of capital, distribution of aggregate income and degree of risk aversion.

Proposition 3 *:*

(i) A decrease in the cost of capital (i.e. an increase in the discount factor δ) increases the optimal capital stock (K^*) and decreases optimal proposed coverage (y^*)

(ii) Let $F_1(.)$ and $F_2(.)$ be two distributions of aggregate income. Then, if $F_1(.)$ stochastically dominates $F_2(.)$, optimal offered coverage (y^*) is higher under $F_2(.)$ than under $F_1(.)$

(iii) The profit of an entrant insurance company that wants to rule out a mutual firm is always increasing with the insured's degree of risk aversion. Moreover, when $\delta < 1$ and individuals are risk neutral, an insurance firm can not enter a market in which a mutual insurer performs.

Proof*: See Appendix.*

In providing comparative statics on δ, Proposition 3 first states the effect of changes in the cost of capital on the optimal choice of the insurance company: as it increases the return on invested capital (by decreasing the cost of capital), an increase in δ increases optimal capital (K^*) and then allows the company to lower y^* without increasing its insolvency probability. This is illustrated in Figure 2. By equation (6), it follows that it is then easier for an insurance company to emerge as δ is high, that is as capital is cheap.

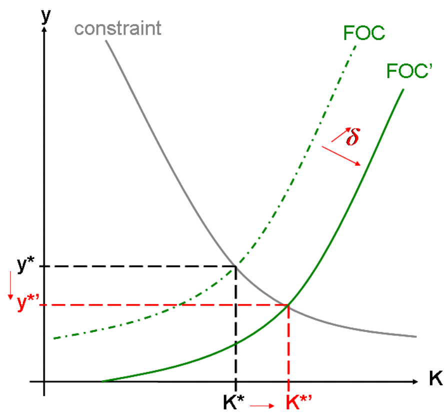


Figure 2: The Effect of Changes in δ

The second part of Proposition 3 shows that if a distribution of aggregate income dominates (in the sense of first order stochastic dominance) another one, the optimal offered coverage increases. The effect of such a change on optimal capital stock is however ambiguous as it results from two effects:

- as it lowers the probability of low aggregate revenue and thus of bankruptcy, this change leads to a decrease in optimal invested capital
- but because of the increase in offered premia, the insurance company has to raise K^* not to increase its insolvency probability.

One can see from the proof that these effects come from the fact that the change in the distribution we study here shifts the first order condition to the North-West and the constraint to the North-Est resulting, as shown in figure 3, in an increase in y^* and an ambiguous effect on K^*.

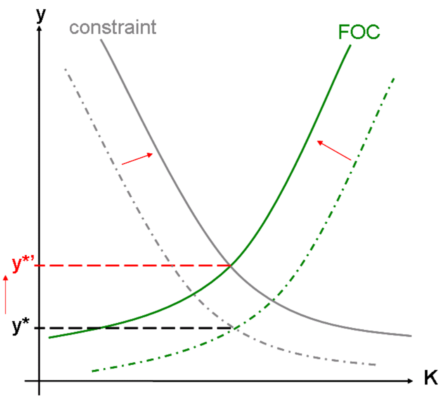


Figure 3: The Effect of Changes in the Distribution of Risks

As it impacts individual insurance decision, risk aversion also has an important part in the determination of the equilibrium and optimal profit. If the effect of the degree of risk aversion on choice variables is hard to grasp in the general case (see next section), Proposition 3 states that, as it increases expected profit, an increase in the degree of risk aversion increases the opportunity for an insurance firm to emerge. Thus, an insurance company seems to be more likely to be set up when individuals are highly risk averse. This important result can be intuitively explained by the fact that an increase in individual degree of risk aversion increases the attractiveness of the insurance company that, contrary to a mutual firm, bears aggregate risk when it is solvent.

Moreover, when policyholders are risk neutral, the insurance company losses its comparative advantage in bearing risk arising from the neutrality of its shareholders and can not emerge.

4.2 Simulations

If the effects of the variables mentioned in the previous subsection can be analytically analyzed, the effect of other variables requires the use of simulations. For exemple, the effect of the size of the population (n) is complex because it affects the distribution of aggregate income $\left(\sum_{i=1}^{n} \widetilde{x}_i\right)$ and we need to specify completely the relationship between the distribution of individual and aggregate income to analyze changes in n. In order to study the impact of individual risk aversion on the choice variables (Proposition 3 only states the effect of risk aversion on optimal profit) we also need to specify an utility function and a cumulative distribution function. For all these complex comparative statics, we use simulations with specific utility function and distribution, that also allows for the study of the impact of changes in risk aversion on the choices of insurance company.

4.2.1 The Retained Specification

We focus on individuals facing independent and normally distributed risks. So, we assume that $\widetilde{x_i}$ follows a $N(m, \sigma^2)$ distribution and thus that $\widetilde{\omega}$ is also distributed according to a normal distribution: $N(n.m, n.\sigma^2)$ [5]. Except in the study of changes in the variance of individual income, we analyze the case of agents' revenues with zero mean ($m = 0$) and a variance equal to one ($\sigma = 1$).

We suppose that agents have a CARA (Constant Absolute Risk Aversion) utility function: $u(c) = -\frac{1}{\rho}.\exp(-\rho.c)$ where ρ represents the coefficient of risk aversion. When necessary, we

[5] The specification of normally distributed individual risks is not necessary for our simulations to be relevant. The only requirement is the aggregate risk to be distributed though a $N(n.m, n.\sigma^2)$. Given the Central Limit Theorem this can be achieved with any individual distribution for n high enough

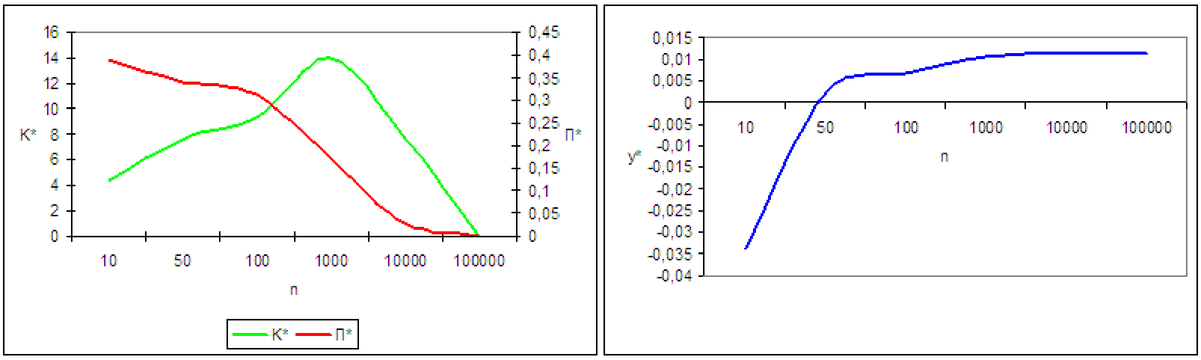
specify a coefficient of risk aversion, ρ, equal to 0.9 [6].

As we want to study the effects of changes in different variables on the optimal behavior of the insurance company, we focus on cases where it has a high incentive to be set up, that is on situations where the cost of capital is low. We thus specify here $\delta = 0.99$.

Lastly, we have to give a specific value to n when the size of the population is not the studied variable. In those cases we specify $n = 100$ [7].

4.2.2 The Effect of Changes in the Size of the Population

As we already pointed out, the analyzis of the effect of changes in the size of the population (n) is complex because it also implies changes in the distribution of aggregate income. To study the implications of such variations on the optimal behavior of the insurance company we thus need to resort to simulations (c.f. Figure 4 drawn on a logarithmic scale).



$$(\delta = 0.99, u(c) = -\frac{1}{0.9}.\exp(-0.9.c), \widetilde{\omega} \sim N(0, n))$$

Figure 4: The Effect of Changes in n

According to those simulations it appears that: *when risks are independent, an increase in the number of policyholders increases the optimal coverage offered by the insurance company and decreases its optimal profit.*

[6] As recommended in most of recent papers (see for exemple Chetty, 2006 or Bombardini and Trebbi, 2005) we use here a coefficient of risk aversion around one

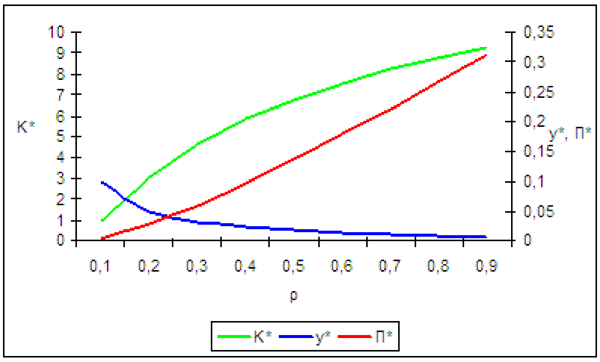
[7] It is usually agreed to be enough for the Central Limit Theorem to hold

The positive effect of an increase in the number of policyholders on the offered coverage and its negative effect on optimal profit are intuitive as, by increasing risk pooling, an increase in n improves the performances of the mutual firm. The effect on the optimal stock of capital is then ambiguous as it is driven by two conflicting forces. First, as the sum of due coverage increases with n and y, the firm has to raise capital not to increase its insolvency probability. However, because it increases risk pooling, the increase in n lowers the risk of bankruptcy and the need for capital. It seems from our simulations that this last effect dominates for high values of n. Anyway, as an increase in n decreases optimal profit, it seems that it is all the more difficult for an insurance company to be set up as risk is initially shared among a lot of individuals.

4.2.3 The Effect of Changes in Risk Aversion

Proposition 3 states the effect of changes in risk aversion on profit in the general case, but is silent on its effect on the insurance company's choice. We simulate the effect of changes in the degree of risk aversion ρ on optimal capital and coverage.

Figure 5 outlines the outcomes of those simulations.



$(\delta = 0.99, \widetilde{\omega} \sim N(0, n), n = 100, u(c) = -\frac{1}{\rho}.\exp(-\rho.c))$

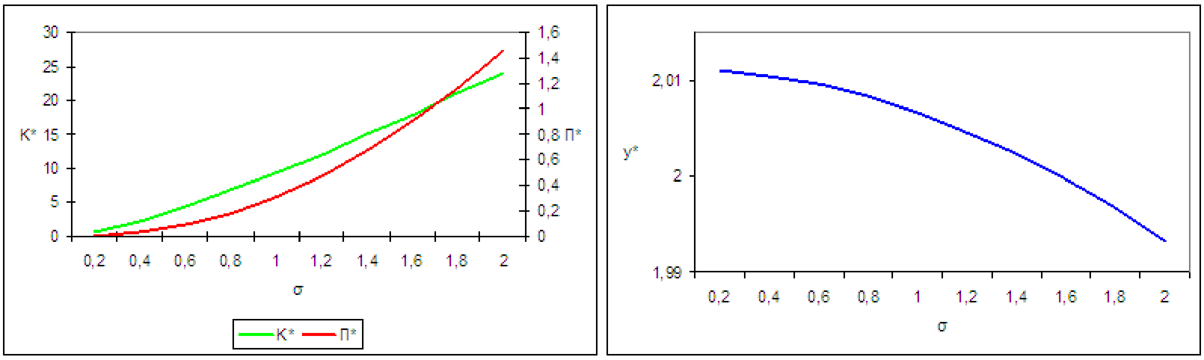
Figure 5: The Effect of Changes in ρ

Based on those results we learn that: *an increase in individual risk aversion increases optimal capital reserves and decreases optimal coverage*.

As already pointed out, because of the increase of their risk aversion, individuals are less demanding, and the company can propose a lower coverage. However, the result of such an increase is also to make policyholders even more reluctant to insolvency of the insurance firm that thus has to increase its capital stock. Still, as the decrease in y also has a negative effect on K, the first effect dominates. As stated in Proposition 3, the total influence on optimal profit is then positive. Those simulations confirm that the higher risk aversion, the larger the opportunity for an insurance firm to enter the market.

4.2.4 The Effect of Changes in the Variance of Individual Income

The last interesting analysis allowed by the specification of a particular cumulative distribution function concerns the effect of changes in the variance of individual income, σ.



$(\delta = 0.99, u(c) = -\frac{1}{0.9}.\exp(-0.9.c), n = 100, \widetilde{\omega} \sim N(2.n, n.\sigma^2))$

Figure 6: The Effect of Changes in σ

Those simulations give rise to the next finding: *an increase in risk (that is in the variance of each individual income distribution) increases optimal capital reserves, decreases optimal coverage and increases optimal profit.*

These effects can be intuitively explained by the fact that for higher σ, the insurance company becomes more attractive with respect to the mutual firm. As it automatically raises the variance of aggregate income ($\widetilde{\omega}$) when income are independent, an increase in the variance of individual income favors the insurance company that enables policyholders to avoid aggregate risk when it is solvent. The firm can then lower offered coverage without losing policyholders. However, the effect this decrease in y has on the capital stock, seems to be offset by the need for capital induced by the increase in aggregate risk. Still, maybe because of this effect of y^* on K^*, optimal profit appears to be positively affected by this increase in risk. So, the likelihood for a company to be set up and to rule out the mutual firm is higher when risks are high.

5 Evidences from the Empirical Literature

To explain the co-existence of mutual with stock firms, many authors have empirically analyzed the difference between the two forms and the reasons making an entity changing from one form to another. In this section, we try to relate our results of comparative statics with the outcome of this empirical literature. Even if this is not precisely a test of our model (as we analyzed previously the entry of the first insurance company) this allows us to check if the main effects highlighted in this paper are accurate. Let us first sum up our main results of comparative statics in the following table:

Effect of...	optimal coverage	optimal capital stock	optimal profit
...a decrease in the cost of capital on	$-$	$+$	$+$
...an increase in individual degree of risk aversion on	$-$	$+$	$+$
...an increase in the number of policyholders on	$+$	?	$-$
...an increase in risk (variance of individual income) on	$-$	$+$	$+$

Table 1: Results of comparative statics

To our knowledge, no empirical study have analyzed the effect of our different parameters neither on capital stock nor on the coverage offered by insurance companies. However, it seems possible to check our results on optimal profit through the effect it has on the choice of organizational form. In our paper higher profit also mean higher probability for the emergence of an insurance company. Our model thus predict that insurance company perform in situation where capital is cheap, individual are risk adverse, risk is high and when few individual need to be insured. If information about the cost of capital and the individual degree of risk aversion are hard to grasp, several empirical papers have analyzed the effect of risk and the size of the insured population on the composition of insurance market.

Concerning the number of policyholders, the outline of our simulation seems first to be consistent with finding of Nekby (2004). Focusing on Swedish historical (1902-1910) data on health insurance, when insurance market was unregulated, this paper establishes that mutual firms were significantly larger than stock companies. This confirm our result stating that it is all the more difficult for an insurance company to emerge when risk is initially shared among a lot of policyholders. This statement is also compatible with the findings of the empirical literature on recent demutualization waves (see for example Viswanathan and Cummins, 2003 or Erhemjamtsa and Leverty, 2006) that asserts that an increase in competition favors demutualization. Higher competition lowering the number of policyholders in mutual firms, it may – consistently with our findings – give the mutual an incentive change its organizational form [8].

Our result on the effect of an increase in risk also seems to be supported by empirical researchs. Lamm-Tenant and Starks (1993) for example show that, compared to mutual insurers, insurance company insures higher-risk activities (when underwriting risk is measured by the variance of loss ratio). This is moreover confirmed by Mayers and Smith (2002) that analyzes 98 conversions of mutual insurers to stock forms in the US between 1920 and 1990. Using alternatively means and medians analysis and a probit regression, they assert that the probability to convert from a mutual to a stock company is higher when risk is high.

Even if those empirical works do not really form a test of a model, they corroborate some of

[8] Our model also fits with the issue of a mutual firm that wants to demutualize. As the mutual firm is owned by its policyholders, they have to agree on the change in status. With homogeneous agents this will only be the case if the contract proposed by the company gives all the policyholders at least as much as what they had in the mutual form. We thus end up with the same problem as in equation (5).

our theoretical findings namely that insurance companies are more likely to emerge in presence of mutual agreements when risk is high and initially shared among few individuals.

6 Conclusion

In studying both the interaction between organizational forms in insurance and the investment choice of the stock firm, this paper highlights the relationship between insurance contracts and capital stock, through the probability of insolvency. Given this interdependence, we specify the optimal choice of coverage and capital investment of an entrant insurance company facing an incumbent mutual agreement, and show they are unique. This paper moreover establishes that the possibility for a stock firm to rule out a mutual firm (or mutual risk-sharing arrangements) is higher as the size of the insured population and the capital cost are low, and as risk and individual risk aversion are high.

This model explains how and why some risks (or some areas) are exclusively insured through mutual or stock firms but, as it considers homogeneous agents, does not explain another feature of insurance market: the coexistence of the two organizational forms. As agents are here homogeneous, the insurance company just needs to give the same utility as the mutual firm to insure the entire population. A possible way to model the coexistence of stock and mutual insurers in our framework would be to introduce heterogeneity. Insurance companies may then attract some kinds of individuals the others staying in the mutual company. Because the agents remaining in the mutual firm are not indifferent to the emergence of an insurance company, such an extension may allow a welfare analysis that would lead to policy advices concerning the regulation of insurance markets.

Our paper also emphasizes situations where a mutual agreement is sustainable, that is when no stock company can enter the market. It moreover endogenizes the choice of capital of a stock firm when it can enter the market. However we are not able to analyze, in our setting, the entry of further companies because of difficulties in aggregating risk distributions. This issue prevent us to analyze the effect of potential shift of a policyholders from an insurance company to one other, and thus to define an equilibrium. Such a study would yet be meaningful as it would allow us to study the impact of openness to insurance companies competition of markets reserved to mutual firms. This way we would be able to study the impact of deregulations making an insurance line reserved to mutual insurers (as health insurance in France) contestable. With an exogenous capital stock, Fagart, Fombaron and Jeleva (2002) already studied a similar situation. However, with an endogenous choice of capital stock, there results are likely to change mainly because of the ambiguous effect (highlighted is our paper) the number of policyholders has on optimal capital stock.

It also seems interesting to take into account the dynamic implications of the possibility of insolvency. In future work it would indeed be worthwhile to analyze the long term effects of bankruptcy of insurance companies on their policyholders' utility. It might be that this expected utility is no longer strictly increasing with the coverage offered by the company. The long-run effect of a bankruptcy might then make the negative influence of an increase in coverage on insolvency probability exceed the positive one it has on monetary gains in case of solvency. Moreover, the introduction of a dynamic framework might also change the optimal agreement offered by the mutual firm since, as shown by Génicot and Ray (2003), it is not always optimal for mutual insurance agreements to provide equal sharing.

Appendix

Proof of Proposition 1

Let us consider individual revenues $\widetilde{x_i}$ with the same expectation and a combination α such that $\sum\limits_{i=1}^{n}\alpha_i = 1$. Then, as

$$\sum_{i=1}^{n} \alpha_i.\widetilde{x}_i = \frac{1}{n}.\sum_{i=1}^{n} \widetilde{x}_i + \sum_{i=1}^{n} \left(\alpha_i - \frac{1}{n}\right) \widetilde{x}_i \tag{9}$$

we have that $\sum\limits_{i=1}^{n} \alpha_i.\widetilde{x}_i$ is a mean-preserving spread of $\frac{1}{n}.\sum\limits_{i=1}^{n} \widetilde{x}_i$ if:

$$E\left(\sum_{i=1}^{n} \left(\alpha_i - \frac{1}{n}\right) \widetilde{x}_i \middle| \sum_{k=1}^{n} \widetilde{x}_k\right) = 0 \tag{10}$$

that is if the variable $\sum\limits_{i=1}^{n} \alpha_i.\widetilde{x}_i$ is equal to $\frac{1}{n}.\sum\limits_{i=1}^{n} \widetilde{x}_i$ augmented by a noise with null conditional expectation.

Then, $\sum\limits_{i=1}^{n} \alpha_i.\widetilde{x}_i$ is more risky than $\frac{1}{n}.\sum\limits_{i=1}^{n} \widetilde{x}_i$ if:

$$\sum_{i=1}^{n} \left(\alpha_i - \frac{1}{n}\right) E\left(\widetilde{x}_i \middle| \sum_{k=1}^{n} \widetilde{x}_k\right) = 0 \tag{11}$$

for which a sufficient condition is:

$$E\left(\widetilde{x}_i \middle| \sum_{k=1}^{n} \widetilde{x}_k\right) = E\left(\widetilde{x}_j \middle| \sum_{k=1}^{n} \widetilde{x}_k\right) \forall i, j \tag{12}$$

Thus, if for each given macroscopic state $\sum\limits_{k=1}^{n} \widetilde{x}_k$, individual risks have a common expectation (which is the case for independent risks), an equal sharing agreement makes everybody better off. Moreover, equal sharing is obviously Pareto efficient since it maximizes (for instance) the utilitarian criterion:

$$\widetilde{\chi}_1^* = \widetilde{\chi}_2^* = ... = \widetilde{\chi}_n^* = \frac{\sum\limits_{k=1}^{n} \widetilde{x}_k}{n} = \frac{\widetilde{\omega}}{n} = \arg\max \left\{ \frac{1}{n} \sum_{i=1}^{n} \mathrm{E}\left(u\left(\widetilde{\chi}_i\right)\right), \sum_{i=1}^{n} \widetilde{\chi}_i = \widetilde{\omega} \right\} \tag{13}$$

Indeed for all $(\widetilde{\chi}_i)_{i=1...n}$ such that $\sum_{i=1}^{n} \widetilde{\chi}_i = \widetilde{\omega}$, we have:

$$\begin{aligned}
\frac{1}{n}\sum_{i=1}^{n} E\left(u\left(\widetilde{\chi}_i\right)\right) &= \mathrm{E}\left(\sum_{i=1}^{n} \frac{1}{n} u\left(\widetilde{\chi}_i\right)\right) \\
&\leq \mathrm{E}\left(u\left(\frac{1}{n}\sum_{i=1}^{n} \widetilde{\chi}_i\right)\right) = \mathrm{E}\left(u\left(\frac{\widetilde{\omega}}{n}\right)\right) \\
\text{that is} &: \\
\frac{1}{n}\sum_{i=1}^{n} E\left(u\left(\widetilde{\chi}_i\right)\right) &\leq \frac{1}{n}\sum_{i=1}^{n} E\left(u\left(\frac{\widetilde{\omega}}{n}\right)\right)
\end{aligned}$$

As individuals are the owner of the mutual firm, equal sharing of resources is then optimal.

The proof of the second part of Proposition 1 is provided in Fagart, Fombaron and Jeleva (2002).

Proof of Proposition 2

- First Order Conditions

The program of the insurance company being:

$$\begin{aligned}
&\max_{y,K} \quad \Pi(y,K) \equiv \delta. \int_{n.y-K}^{+\infty} (\omega + K - n.y)\, f(\omega)\, d\omega - K \qquad (14)\\
&\text{s.t.} \quad C(y,K) \equiv \int_{-\infty}^{n.y-K} \left[u\left(\frac{\omega+K}{n}\right) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega + \int_{n.y-K}^{+\infty} \left[u(y) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega = 0
\end{aligned}$$

its first order condition can be written as:

$$
\begin{aligned}
& -\frac{\partial \Pi(y,K)/\partial K}{\partial C(y,K)/\partial K} = -\frac{\partial \Pi(y,K)/\partial y}{\partial C(y,K)/\partial y} \\
\Leftrightarrow \quad & \frac{1-\delta[1-F(n.y-K)]}{\frac{1}{n}\int_{-\infty}^{n.y-K} u'\left(\frac{\omega+K}{n}\right) f(\omega)\, d\omega} = \frac{n.\delta}{u'(y)} \\
\Leftrightarrow \quad & \frac{1-\delta[1-F(n.y-K)]}{\delta F(n.y-K)} = \frac{E\left(u'\left(\frac{\omega+K}{n}\right) | \frac{\omega+K}{n} \leq y\right)}{u'(y)} \\
\Leftrightarrow \quad & \Phi(y,K) \equiv \int_{-\infty}^{n.y-K} \left[\frac{u'\left(\frac{\omega+K}{n}\right) - u'(y)}{u'(y)}\right] f(\omega)\, d\omega - \frac{1-\delta}{\delta} = 0
\end{aligned}
$$

- Second Order Conditions

It can then be proved that this first order condition along with the constraint correspond to necessary and sufficient conditions that describe a maximum.

To do so let us study the following larger problem:

$$
\begin{aligned}
& \max_{y,K,z(.)} \quad \int_{-\infty}^{+\infty} \omega f(\omega) d\omega - n \int_{-\infty}^{+\infty} z(\omega) f(\omega) d\omega - \frac{1-\delta}{\delta} K \\
& \text{s.t.} \quad \begin{cases} \int_{-\infty}^{+\infty} u(z(\omega)) f(\omega) d\omega \geq \int_{-\infty}^{+\infty} u\left(\frac{\omega}{n}\right) f(\omega) d\omega \\ \min\left(\frac{\omega+K}{n}, y\right) \geq z(\omega)\ \forall \omega \end{cases}
\end{aligned}
\tag{15}
$$

The objective function of (15) $\left(V(K,y,z(.)) = \int_{-\infty}^{+\infty} \omega f(\omega) d\omega - n \int_{-\infty}^{+\infty} z(\omega) f(\omega) d\omega - \frac{1-\delta}{\delta} K\right)$ is linear in K, y and $z(.)$. Thus if the constraints define a convex set, then problem (15) is regular and its first order conditions are both necessary and sufficient for a maximum.

To verify whether it is true let us consider two triplets $K_1, y_1, z_1(.)$ et $K_2, y_2, z_2(.)$ that satisfy the constraints:

$$\begin{cases} \int_{-\infty}^{+\infty} u(z_1(\omega))f(\omega)d\omega \geq \int_{-\infty}^{+\infty} u\left(\frac{\omega}{n}\right) f(\omega)d\omega \\ \min\left(\frac{\omega + K_1}{n}, y_1\right) \geq z_1(\omega)\ \forall\omega \end{cases} \tag{16}$$

$$\begin{cases} \int_{-\infty}^{+\infty} u(z_2(\omega))f(\omega)d\omega \geq \int_{-\infty}^{+\infty} u\left(\frac{\omega}{n}\right) f(\omega)d\omega \\ \min\left(\frac{\omega + K_2}{n}, y_2\right) \geq z_2(\omega)\ \forall\omega \end{cases} \tag{17}$$

Then:

1. $\forall\alpha \in [0, 1]$
   $\alpha \int_{-\infty}^{+\infty} u(z_1(\omega))f(\omega)d\omega + (1-\alpha)\int_{-\infty}^{+\infty} u(z_2(\omega))f(\omega)d\omega \geq \int_{-\infty}^{+\infty} u\left(\frac{\omega}{n}\right) f(\omega)d\omega,$
   $\alpha \int_{-\infty}^{+\infty} u(z_1(\omega))f(\omega)d\omega + (1-\alpha)\int_{-\infty}^{+\infty} u(z_2(\omega))f(\omega)d\omega \leq \int_{-\infty}^{+\infty} u(\alpha z_1(\omega) + (1-\alpha)z_2(\omega))f(\omega)d\omega$
   and thus: $\int_{-\infty}^{+\infty} u(\alpha z_1(\omega) + (1-\alpha)z_2(\omega))f(\omega)d\omega \geq \int_{-\infty}^{+\infty} u\left(\frac{\omega}{n}\right) f(\omega)d\omega$

2. Similarly, $\forall\alpha \in [0, 1]$, we have
   $\alpha \min\left(\frac{\omega + K_1}{n}, y_1\right) + (1-\alpha)\min\left(\frac{\omega + K_2}{n}, y_2\right) \geq \alpha z_1(\omega) + (1-\alpha)z_2(\omega)$
   Then, the two variables function $\min(a, b)$ being concave from $\mathbb{R}^2$ into $\mathbb{R}$, it follows that:
   $\alpha \min\left(\frac{\omega + K_1}{n}, y_1\right) + (1-\alpha)\min\left(\frac{\omega + K_2}{n}, y_2\right) \leq \min\left(\frac{\omega + \alpha K_1 + (1-\alpha)K_2}{n}, \alpha y_1 + (1-\alpha)y_2\right)$
   Which leads to: $\min\left(\frac{\omega + \alpha K_1 + (1-\alpha)K_2}{n}, \alpha y_1 + (1-\alpha)y_2\right) \geq \alpha z_1(\omega) + (1-\alpha)z_2(\omega)$

Thus, the constraints of program (15) define a convex set and, as already pointed out, its first order conditions are both necessary and sufficient for a maximum.

Now, to prove that the equations $\Phi(y, K) = 0$ and $C(y, K) = 0$ define the optimum choice of the insurance company we have to prove that they satisfy the first order conditions of the program (15). Indeed, if those equations define the maximum of program (15) that is larger than (14) in imposing $\min\left(\frac{\omega + K}{n}, y\right) \geq z(\omega)\ \forall\omega$ instead of $\min\left(\frac{\omega + K}{n}, y\right) \equiv z(\omega)\ \forall\omega$, they also define the maximum of (14).

Calling λ and $\mu(\omega)f(\omega)$ the multipliers associated with the constraints, the first order conditions of (15) can be written as:

$$\begin{aligned} \forall\omega, \qquad -nf(\omega)+\lambda u'(z(\omega)f(\omega) &= \mu(\omega)f(\omega) && (18)\\ \int_{ny-K}^{+\infty}\mu(\omega)f(\omega)d\omega &= 0 && (19)\\ -\frac{1-\delta}{\delta}+\frac{1}{n}\int_{-\infty}^{ny-K}\mu(\omega)f(\omega)d\omega &= 0 && (20) \end{aligned}$$

the complementarity conditions being:

$$\begin{aligned} \lambda &\geq 0 && (21)\\ \mu(\omega)f(\omega) &\geq 0 && (22)\\ \lambda\int_{-\infty}^{+\infty}\left(z(\omega)-u\left(\frac{\omega}{n}\right)\right)f(\omega)d\omega &= 0 && (23)\\ \mu(\omega)f(\omega)\left(\min\left(\frac{\omega+K}{n},y\right)-z(\omega)\right) &= 0 && (24) \end{aligned}$$

(18) together with $\mu(\omega)\geq 0$ then leads to $\omega \geq ny-K \Longrightarrow \mu(\omega)f(\omega)=0$
which gives, using (19): $\omega\geq ny-K \Longrightarrow z(\omega)=cte=z$

We then have:

$$\begin{aligned} \lambda u'(z) &= n && (25)\\ \frac{1}{n}\int_{-\infty}^{ny-K}[-n+\lambda u'(z(\omega))]f(\omega)d\omega &= \frac{1-\delta}{\delta} && (26)\\ \mu(\omega)f(\omega)\left(\frac{\omega+K}{n}-z(\omega)\right) &= 0 \text{ for } \omega\leq ny-K && (27)\\ z(\omega) &= z \text{ for } \omega \geq ny-K && (28)\\ \lambda(\int_{-\infty}^{+\infty}\left(u(z(\omega))-u\left(\frac{\omega}{n}\right)\right)f(\omega)d\omega &= 0 && (29) \end{aligned}$$

And one then can see that $\lambda, z(\omega), y, K$ verifying:

$$\begin{aligned} \lambda u'(y) &= n & (30) \\ \frac{1}{n}\int_{-\infty}^{ny-K}\left[-n+\lambda u'\left(\frac{\omega+K}{n}\right)\right]f(\omega)d\omega &= \frac{1-\delta}{\delta} & (31) \\ z(\omega) &= \min\left(\frac{\omega+K}{n}, y\right) & (32) \\ \int_{-\infty}^{+\infty}\left(u(z(\omega))-u\left(\frac{\omega}{n}\right)\right)f(\omega)d\omega &= 0 & (33) \end{aligned}$$

(that are the solutions of (14)) are solutions of previous equations (that is of (15)) with $y = z$.

Thus, the solutions of the program of the insurance company being the maximum of a larger program containing the one of the firm, it is also the maximum our this first program.

- The Existence of An Optimum

Once the optimum characterized we need to focus on the conditions under which an optimum giving a positive expected profit exists. As the company can always make null profit by mimicking a mutual firm in setting $K = 0$ and $y = \frac{\overline{\omega}}{n}$ (as explained in section 2.2, to avoid for moral hazard, the company has to propose $y \leqslant \overline{x} \equiv \frac{\overline{\omega}}{n}$), this will be the case when there exists an optimum different from $K = 0$ and $y = \frac{\overline{\omega}}{n}$.

Rewriting the first order condition as: $\int_{-\infty}^{ny-K}\left(u'\left(\frac{\omega+K}{n}\right)-u'(y)\right)f(\omega)d\omega = \frac{1-\delta}{\delta}u'(y)$ we can see that the left hand side is increasing in y and decreasing in K, when the right hand side is decreasing in y and independent on K. Thus, a necessary condition for a solution not to exist is that for $K = 0$ and $y = \frac{\overline{\omega}}{n}$ the right hand side to be strictly lower than the left hand side, that is $\int_{-\infty}^{\overline{\omega}}\left(u'\left(\frac{\omega}{n}\right)-u'\left(\frac{\overline{\omega}}{n}\right)\right)f(\omega)d\omega < \frac{1-\delta}{\delta}u'\left(\frac{\overline{\omega}}{n}\right)$.

So,

- if $\tilde{\omega}$ is not bounded ($\overline{\omega} = +\infty$), as the utility function satisfies the Inada condition, $u'(+\infty) = 0$ and an optimum that gives positive profit always exists

- if $\widetilde{\omega}$ is upward bounded ($\overline{\omega} < +\infty$) equilibrium exists only if $\dfrac{1-\delta}{\delta} < \dfrac{\int_{-\infty}^{\overline{\omega}} \left(u'\left(\frac{\omega}{n}\right) - u'\left(\frac{\overline{\omega}}{n}\right)\right) f(\omega) d\omega}{u'\left(\frac{\overline{\omega}}{n}\right)}$

- Uniqueness of The Optimum

Rewriting the first order condition of the firm's program as:

$$\frac{E\left(u'\left(\frac{\widetilde{\omega}+K}{n}\right) \Big| \frac{\widetilde{\omega}+K}{n} \leq y\right)}{u'(y)} = \frac{1-\delta.[1-F(n.y-K)]}{\delta.F(n.y-K)} \tag{34}$$

one can then show that when it holds, this condition corresponds to a unique mapping between the offered premia (y) and the capital stock (K). Indeed, keeping K constant, the left hand side is increasing in y from 1 to $+\infty$, when the right hand side is decreasing from $+\infty$ to 1. So, for each value of stock of capital the first order condition of studied programm gives a unique optimal premium.

Moreover,

- As:
  $\dfrac{\partial \Phi(y,K)}{\partial K} = \dfrac{1}{n.u'(y)} \displaystyle\int_{-\infty}^{n.y-K} u''\left(\frac{\omega+K}{n}\right) f(\omega)\, d\omega < 0$, and
  $\dfrac{\partial \Phi(y,K)}{\partial y} = -\dfrac{u''(y)}{u'(y)^2} \displaystyle\int_{-\infty}^{n.y-K} u'\left(\frac{\omega+K}{n}\right) f(\omega)\, d\omega > 0$,
  the first order condition of our problem gives rise to an increasing relationship between y and K $\left(\dfrac{\partial y}{\partial K} \equiv -\dfrac{\partial \Phi(y,K)/\partial K}{\partial \Phi(y,K)/\partial y} > 0\right)$.

- Likewise, as
  $\dfrac{\partial C(y,K)}{\partial K} = \dfrac{1}{n}. \displaystyle\int_{-\infty}^{n.y-K} u'\left(\frac{\omega+K}{n}\right) f(\omega)\, d\omega > 0$
  $\dfrac{\partial C(y,K)}{\partial y} = [1-F(n.y-K)].u'(y) > 0$
  the constraint corresponds to an decreasing relationship between y and K $\left(\dfrac{\partial y}{\partial K} \equiv -\dfrac{\partial C(y,K)/\partial K}{\partial C(y,K)/\partial y} < 0\right.$

So, the optimal behavior of an entrant insurance company that competes with an incumbent mutual firm, characterized by the two equations $\Phi(y,K)=0$ and $C(y,K)=0$, is unique (when it exists).

Proof of Proposition 3

(i) Effect of Changes in δ

As the optimal choice of the company is fully characterized by the two equations $\Phi(y,K)=0$ and $C(y,K)=0$, and as $C(y,K)$ is independent on δ, the effect of the capital cost (δ) on the equilibrium has to verify:

$$\left\{\begin{array}{l} \frac{\partial C(y,K)}{\partial y}.\frac{\partial y}{\partial \delta}+\frac{\partial C(y,K)}{\partial K}.\frac{\partial K}{\partial \delta}=0 \\ \frac{\partial \Phi(y,K,\delta)}{\partial y}.\frac{\partial y}{\partial \delta}+\frac{\partial \Phi(y,K,\delta)}{\partial K}.\frac{\partial K}{\partial \delta}+\frac{\partial \Phi(y,K,\delta)}{\partial \delta}=0 \end{array}\right. \tag{35}$$

That is:

$$\frac{\partial y}{\partial \delta} = \frac{\frac{\partial C(y,K)}{\partial K}.\frac{\partial \Phi(y,K,\delta)}{\partial \delta}}{\frac{\partial C(y,K)}{\partial y}.\frac{\partial \Phi(y,K,\delta)}{\partial K}-\frac{\partial C(y,K)}{\partial K}.\frac{\partial \Phi(y,K,\delta)}{\partial y}} \tag{36}$$

and,

$$\frac{\partial K}{\partial \delta} = \frac{-\frac{\partial C(y,K)}{\partial y}.\frac{\partial \Phi(y,K,\delta)}{\partial \delta}}{\frac{\partial C(y,K)}{\partial y}.\frac{\partial \Phi(y,K,\delta)}{\partial K}-\frac{\partial C(y,K)}{\partial K}.\frac{\partial \Phi(y,K,\delta)}{\partial y}} \tag{37}$$

Now as, $\frac{\partial C(y,K)}{\partial y}>0$, $\frac{\partial C(y,K)}{\partial K}>0$, $\frac{\partial \Phi(y,K,\delta)}{\partial y}>0$, $\frac{\partial \Phi(y,K)}{\partial K}<0$ and $\frac{\partial \Phi(y,K,\delta)}{\partial \delta}=\frac{1}{\delta^2}>0$ one ends up with $\frac{\partial y}{\partial \delta}<0$ and $\frac{\partial K}{\partial \delta}>0$.

(ii) Effect of Changes in The Distribution of $\widetilde{\omega}$

If $F_2(\widetilde{\omega})$ first-order stochastically dominates $F_1(\widetilde{\omega})$ $\Big(F_1(\widetilde{\omega})\geq F_2(\widetilde{\omega})\,\forall\widetilde{\omega}\in]-\infty,+\infty[\Big)$ then:

- The first order condition: $\Phi(y,K)=0$ can be written as:

$$\int_{-\infty}^{n.y-K} u'\left(\frac{\omega+K}{n}\right) f(\omega)\,d\omega - \frac{1-\delta\left[1-F(n.y-K)\right]}{\delta}u'(y)=0 \tag{38}$$

that is, after integrating by parts: $\Phi_p(y,K,F)\equiv -\int_{-\infty}^{ny-K} u''\left(\frac{\omega+K}{n}\right) F(\omega)\,d\omega - \frac{1-\delta}{\delta}u'(y)=0$

Thus,

$$\Phi_p(y,K,F_1)-\Phi_p(y,K,F_2)=\int_{-\infty}^{ny-K} \underbrace{\left[F_2(\omega)-F_1(\omega)\right]}_{<0 \text{ by assumption}}\underbrace{u''\left(\frac{\omega+K}{n}\right)}_{<0} d\omega \geq 0 \tag{39}$$

- Similarly, integrating by parts the incentive constraint $C(y,K)=0$, one gets:

$$\begin{aligned} C_p(y,K,F) \;\equiv\; & u(y)-\lim_{a\to+\infty}u(a)+\int_{-\infty}^{n.y-K}\left[u'\left(\frac{\omega}{n}\right)-u'\left(\frac{\omega+K}{n}\right)\right]F(\omega)\,d\omega \\ & +\int_{ny-K}^{+\infty} u'\left(\frac{\omega}{n}\right)F(\omega)\,d\omega=0 \end{aligned} \tag{40}$$

And,

$$\begin{aligned} C_p(y,K,F_1)-C_p(y,K,F_2) \;=\; & \int_{-\infty}^{n.y-K}\left[u'\left(\frac{\omega}{n}\right)-u'\left(\frac{\omega+K}{n}\right)\right]\left[F_1(\omega)-F_2(\omega)\right]d\omega \\ & +\int_{ny-K}^{+\infty} u'\left(\frac{\omega}{n}\right)\left[F_1(\omega)-F_2(\omega)\right]d\omega \geq 0 \end{aligned} \tag{41}$$

Using (39) and (41) one can then prove by contradiction that changing the distribution from $F_1(.)$ to $F_2(.)$ leads to a increase in the optimal offered coverage.

Indeed, if $y_1^* > y_2^*$ and F_2 first-order stochastically dominates F_1 then, according to the first order condition, we necessarily have $SP(y_1^*, K_2^*, F_1) = SP(y_2^*, K_2^*, F_2) = 0$ which means, from

(39) and as $\dfrac{\partial \Phi_p(y,K,F)}{\partial y} > 0$ and $\dfrac{\partial \Phi_p(y,K,F)}{\partial K} > 0$, that the company will optimally choose $K_1^* < K_2^*$.

However, under the same assumption, according to the incentive constraint, we also necessarily have that $C_p(y_1^*, K_2^*, F_1) = C_p(y_2^*, K_2^*, F_2) = 0$. Now, with (41) and as $\dfrac{\partial C_p(y,K,F)}{\partial y} > 0$ and $\dfrac{\partial C_p(y,K,F)}{\partial K} > 0$ this would mean that $K_1^* > K_2^*$ which enters in contradiction with the previous result.

Thus, if F_2 first-order stochastically dominates F_1, then the company will necessarily choose $y_1^* < y_2^*$.

(iii) Effect of Changes in individuals degree of risk aversion

In the program of the insurance company:

$$\begin{aligned}
&\max_{y,K} \quad \left\{\delta.\int_{n.y-K}^{+\infty} (\omega + K - n.y)\, f(\omega)\, d\omega - K\right\} \\
&\text{s.t.} \quad C(y,K,u(.)) \equiv \int_{-\infty}^{n.y-K} \left[u\left(\frac{\omega+K}{n}\right) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega + \int_{n.y-K}^{+\infty} \left[u(y) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega \geq 0
\end{aligned} \tag{42}$$

the objective function being independent of insureds utility function, the effect of risk aversion on optimal profit only goes through the constraint.

Let us take a strictly increasing and concave function g and set $w = g \circ v$. We then have that w is a Von Neumann Morgenstern utility function of a more risk averse individual than v.

We now can prove that because $C(y,K,w(.)) \geq C(y,K,v(.))$, that is because it enlarge the set of possible choices, an increase in individual risk aversion increase optimal profit.

Indeed as g is increasing and concave:

$$\begin{aligned}
w\left(\frac{\omega+K}{n}\right)-w\left(\frac{\omega}{n}\right) &= g\left(v\left(\frac{\omega+K}{n}\right)\right)-g\left(v\left(\frac{\omega}{n}\right)\right)\\
&\geq g'\left(v\left(\frac{\omega+K}{n}\right)\right).\left(v\left(\frac{\omega+K}{n}\right)-v\left(\frac{\omega}{n}\right)\right)\\
&\geq g'(v(y)).\left(v\left(\frac{\omega+K}{n}\right)-v\left(\frac{\omega}{n}\right)\right) \quad \forall\omega<n.y-K\\
\Rightarrow \int_{-\infty}^{n.y-K}\left[w\left(\frac{\omega+K}{n}\right)-w\left(\frac{\omega}{n}\right)\right]f(\omega)d\omega &\geq \int_{-\infty}^{n.y-K}\left[v\left(\frac{\omega+K}{n}\right)-v\left(\frac{\omega}{n}\right)\right]f(\omega)d\omega \qquad (43)\\
&\text{and}\\
w(y)-w\left(\frac{\omega}{n}\right) &= g\left(v(y)\right)-g\left(v\left(\frac{\omega}{n}\right)\right)\\
&\geq g'(u(y)).\left(v(y)-v\left(\frac{\omega}{n}\right)\right)\\
\Rightarrow \int_{n.y-K}^{+\infty}\left[w(y)-w\left(\frac{\omega}{n}\right)\right] &\geq \int_{n.y-K}^{+\infty}\left[v(y)-v\left(\frac{\omega}{n}\right)\right] \qquad (44)
\end{aligned}$$

which together leads to $C(y,K,w(.))\geq C(y,K,v(.))$.

It follows that if (y,K) is acceptable for a given individual, it is also acceptable for a more risk averse one. Thus, the profit of an insurance company increases with individuals risk aversion.

The second part of the proposition is obvious. Indeed, if individuals are risk neutral, the constraint becomes

$$\int_{-\infty}^{n.y-K}\left[\frac{\omega+K}{n}-\frac{\omega}{n}\right]f(\omega)\,d\omega+\int_{n.y-K}^{+\infty}\left[y-\frac{\omega}{n}\right]f(\omega)\,d\omega\geq 0 \qquad (45)$$

As the constraint is binding, it then leads to $\int_{n.y-K}^{+\infty}(\omega-n.y)f(\omega)d\omega=K.F(n.y-K)$ and the profit becomes $\Pi=-(1-\delta).K$

The optimal choice is thus to set $K=0$, and $y=\frac{\overline{\omega}}{n}$, which exactly amount to the behavior of a mutual firm.

References

Bombardini, M. and Trebbi, F., "Risk aversion and expected utility theory: A field experiment with large and small stakes", mimeo University of British Columbia and Harvard University, 2005

Borch, K., "Equilibrium in a reinsurance market", Econometrica 30, pp. 424–444, 1962

Chetty, R., "A bound on risk aversion using labor supply elasticities", forthcoming American Economic Review, 2006.

Doherty, N. and Dionne, D., "Insurance with undiversifiable risk: Contract structure and organizational form of insurance firms", Journal of Risk and Uncertainty (6), pp. 187–203, 1993.

Eeckhoudt, L. and Gollier, C., Risk-Evaluation, Management and Sharing, London: Harvester Wheatsheaf, 1995.

Erhemjamts O. and J. T. Leverty, "Efficiency changes around US life insurer demutualizations" working paper, 2006.

Fagart, M., Fombaron, N. and Jeleva, M., "Risk mutualization and competition", The Geneva Papers on Risk and Insurance Theory (27), pp. 115–141, 2002.

Génicot, G. and Ray, D., "Group formation in risk-sharing arrangements", Review of Economic Studies (70), pp. 87–113, 2003

Hansmann, H., "The Organization of Insurance Companies: Mutual versus Stock Henry", Journal of Law, Economics, & Organization, (1), pp. 125-153, 1985.

Henry, C., "Criteres Simples pour la Prise en Compte du Risque", Econometrica (49), pp. 153-170, 1981.

Lamm-Tennant, J. and L. T. Starks, "Stock Versus Mutual Ownership Structures: The Risk Implications", The Journal of Business (66), 29-46, 1993.

Laux, C. and Muermann, A., "Mutual versus stock insurers: Fair Premium, Capital and Solvency", working paper, 2006.

Mayers, D. Smith, C., "Ownership structure across lines of property-casualty insurance", Journal of Law and Economics (31), pp. 351–378, 1988.

Mayers, D. Smith, C., "Ownership Structure and Control: Property-Casualty Insurer Conversion to Stock Charter", Journal of Financial Services Research (21), pp. 117–144, 2002.

Nekby, L., "Pure Versus Mutual Health Insurance: Evidence From Swedish Historical Data", The Journal of Risk and Insurance (71), pp. 115-134, 2004

Rees, R., Gravelle, H. and Wambach, A., "Regulation of insurance markets", The Geneva Papers on Risk and Insurance Theory (24), pp. 55–68, 1999.

Rothschild, M. and Stiglitz, J., "Equilibrium in competitive insurance markets: An essay on the economics of imperfect information", Quarterly Journal of Economics (90), pp. 629–650, 1976.

Smith, B. and Stutzer, M., "Adverse selection, aggregate uncertainty, and the role for mutual insurance contracts", The Journal of Business 63(4), 1990.

Viswanathan, K. S. and J. D. Cummins, "Ownership Structure Changes in the Insurance Industry: An Analysis of Demutualization", Journal of Risk and Insurance (70), 401-437, 2003.