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Abstract

This paper explores the relationship between factor substitution in production and the steady-sate level of capital stock in a growing economy. Unlike the foregoing studies on this topic that have exclusively utilized one-sector growth models, we consider a two-sector economy where investment and consumption goods are produced by different technologies. We show that the relation between the elasticity of substation and the long-run capital formation critically depends on the factor-intensity ranking between the two sectors.

Keywords: CES production function, two-sector model, capital accumulation, factor-intensity ranking

JLE classification: O11, O33, O46

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1 Introduction

Recently, there is a renewed interest in the role of capital-labor substitution in growing economies. Following Klump and de La Grandville (2000) and Klump and Pressiler (2000), a number of authors have investigated how the magnitude of elasticity of substitution between capital and labor affects capital accumulation. Those studies have revealed that the elasticity of factor substitution would be a relevant determinant for economic growth.[1]

In this literature the existing contributions have exclusively investigated one-sector growth models under alternative assumptions on saving behavior of households. The present paper examines the role of factor substitution in a two-sector neoclassical growth model where investment goods and consumption goods are produced by use of different technologies. We show that in this generalized setting the relation between elasticity of substitution in production and capital formation critically depends on the factor-intensity ranking between the two production sectors.

2 Production in a Two-Sector Economy

Consider a two-sector economy where the first sector produces pure investment goods and the second sector produces pure consumption goods. The production technology of each sector is specified as

$$Y_i = T_i \left(\gamma_i K_i^{\frac{\sigma_i - 1}{\sigma_i}} + (1-\gamma_i) L_i^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}}, \quad T_i > 0, \quad 0 < \gamma_i < 1, \quad \sigma_i > 0, \quad i = 1, 2, \quad (1)$$

where Y_i, K_i and L_i respectively denote output, capital and labor of the i-th sector. In addition, σ_i denotes the elasticity of substitution between capital and labor, T_i stands for an efficiency parameter and γ_i expresses a distribution parameter. For notational simplicity, we set $T_i = 1$.[2]

[1] Xue and Yip (2009) present a through summary of the main results obtained so far.

[2] Klump and de La Grandville (2000) emphasize that if we fix the magnitude of σ_i, then T_i and γ_i in (1) may depend on σ_i. To specify $T_i(\sigma_i)$ and $\gamma_i(\sigma_i)$, we should select baseline values of k_i and Y_i. The foraging studies have revealed that such a normalization procedure also affects the relation between σ_i and capital formation: see Klump et al. (2011) for a detailed discussion on the normalized CES functions. This paper uses CES functions without normalization in order to highlight the role of factor-intensity ranking in our two-sector economy.

Markets are competitive so that the gross rate of return to capital, r, and the real wage rate, w, fulfill the following conditions:

$$r=\frac{\partial Y_1}{\partial K_1}=p\frac{\partial Y_2}{\partial K_2},\qquad w=\frac{\partial Y_1}{\partial L_1}=p\frac{\partial Y_2}{\partial L_2}, \tag{2}$$

where p denotes the price of consumption good in terms of the investment good. Letting $k_i\equiv K_i/L_i$ and $w/r\equiv\omega$, we see that conditions in (2) yield

$$\frac{1-\gamma_i}{\gamma_i}k_i^{1/\sigma_i}=\omega,$$

implying that the factor intensity in each sector, k_i, is written as

$$k_i=\beta_i\omega^{\sigma_i},\quad \text{where } \beta_i\equiv\gamma_i/\left(1-\gamma_i\right)>0,\quad i=1,2. \tag{3}$$

We assume that capital and labor are fully employed:

$$K_1+K_2=K,\quad L_1+L_2=1.$$

Here, the labor supply is assumed to be fixed and it is normalized to one.[3] The full-employment conditions give

$$L_1=\frac{K-k_2}{k_1-k_2},\qquad L_2=\frac{k_1-K}{k_1-k_2}. \tag{4}$$

We focus on an interior equilibrium where both goods are produced. In order to have a feasible, interior equilibrium where $0<L_1<1$, the factor price ratio satisfies $\omega\in[\underline{\omega},\ \bar{\omega}]$, where $[\underline{\omega}$ and $\bar{\omega}$ depend on the level of K. We also assume that there is no factor-intensity reversal in the sense that

$$\beta_1\omega^{\sigma_1}\neq\beta_2\omega^{\sigma_2}\ \text{ for all } \omega\in[\underline{\omega},\ \bar{\omega}].$$

Using (4), we write the production function of each sector as

$$Y_1=\frac{K-k_2}{k_1-k_2}\left(\gamma_1 k_1^{\frac{\sigma_1-1}{\sigma_1}}+1-\gamma_1\right)^{\frac{\sigma_1}{\sigma_1-1}}, \tag{5}$$

$$Y_2=\frac{k_1-K}{k_1-k_2}\left(\gamma_2 k_2^{\frac{\sigma_2-1}{\sigma_2}}+1-\gamma_2\right)^{\frac{\sigma_2}{\sigma_2-1}}. \tag{6}$$

Substituting $k_1=\beta_1\omega^{\sigma_1}$ into (5) and (6) presents the supply function of each product:

$$Y_1\left(K,\omega\right)=\frac{K-\beta_2\omega^{\sigma_2}}{\beta_1\omega^{\sigma_1}-\beta_2\omega^{\sigma_1}}\left(\gamma_1\left(\beta_1\omega^{\sigma_1}\right)^{\frac{\sigma_1-1}{\sigma_1}}+1-\gamma_1\right)^{\frac{\sigma_1}{\sigma_1-1}}, \tag{7}$$

$$Y_2\left(K,\omega\right)=\frac{\beta_1\omega^{\sigma_1}-K}{\beta_1\omega^{\sigma_1}-\beta_2\omega^{\sigma_1}}\left(\gamma_2\left(\beta_{21}\omega^{\sigma_1}\right)^{\frac{\sigma_2-1}{\sigma_2}}+1-\gamma_1\right)^{\frac{\sigma_2}{\sigma_2-1}}. \tag{8}$$

[3]Introducing population growth and labor-augmenting technical progress will not alter our main conclusion.

3 Savings and Capital Formation

3.1 A Representative-Agent Economy

We now specify the saving behavior of households. As an example, consider the representative agent economy. The representative family maximizes

$$U = \int_0^\infty \frac{C^{1-1/\eta}}{1-(1/\eta)} e^{-\rho t} dt, \quad \eta > 0, \quad \rho > 0$$

subject to

$$\dot{K} = rK + w - pC - \delta K, \quad 0 < \delta < 1$$

and the initial holding of capital, K_0. In the above, C is consumption, ρ the time-discount rate, η the intertemporal elasticities of substitution in consumption, and δ is the rate of capital depreciation.

The optimization conditions give the Euler equation for consumption such that

$$\frac{\dot{C}}{C} = \frac{1}{\eta}(\rho + \delta - r) + \frac{\dot{p}}{p}, \tag{9}$$

together with the transversailty condition: $\lim_{t\to\infty} e^{-\rho t} C^{-\eta} K = 0$.

Since capital goods are produced by the first sector, the capital stock changes according to

$$\dot{K} = Y_1(K, \omega) - \delta K. \tag{10}$$

We restrict our attention on the steady-state equilibrium. Since consumption and the relative price stay constant in the steady state, from (9) it holds that

$$r = \rho + \delta \tag{11}$$

in the steady state. This means that the steady-state level of factor intensity of the first sector is uniquely determined by the following modified golden-rule condition:

$$k_1^{*-\frac{1}{\sigma_1}} \left(\gamma_1 k_1^{*\frac{\sigma_1 - 1}{\sigma_1}} + 1 - \gamma_1\right)^{\frac{\sigma_1}{\sigma_1 - 1}} = \rho + \delta, \tag{12}$$

where k_1^* represents the steady-state level of k_1. Therefore, the steady-state value of factor price ratio, ω^*, is given by

$$\beta_1 \omega^{*\sigma_1} = k_1^*. \tag{13}$$

Equation (10) means that the capital stock stays constant when $Y_1(K,\omega) = \delta K$, and thus from (7) the steady-state level of capital determined by

$$\frac{K - \beta_2 \omega^{*\sigma_2}}{k_1^* - \beta_2 \omega^{*\sigma_1}} \left(\gamma_1 k_1^{*\frac{\sigma_1 - 1}{\sigma_1}} + 1 - \gamma_1 \right)^{\frac{\sigma_1}{\sigma_1 - 1}} = \delta K, \tag{14}$$

where k_1^* and ω^* respectively given by (12) and (13). Figures 1-a and 1-b depict the graphs of left and right hands of (14). As the figures show, the graph of the left-hand side (LHS) of (14) changes its position depending on the factcor-intensity ranking, i.e. sign $(k_1 - k_2)$. In both cases, there is a unique level of capital stock, K^*, in the steady-state.[4]

Now suppose that σ_1 increases. Since k_1^* is fixed by (12), a higher σ_1 depresses ω^*. The resulting shift of the graph of LHS of (14) is shown by Figures 2-and 2-b. We see that if the investment good sector uses a more capital intensive technology than the consumption good sector, then a rise in σ_1 lowers K^*. Otherwise, a higher σ_1 raises K^*.

Similarly, Figures 3-a and 3-b depict the case where σ_2 increases. In this case, ω^* does not change so that an increase in σ_2 only raises $\omega^{*\sigma}$. As Figures 3-a and 3-b demonstrate, we obtain the opposite results to the case of a rise in σ_1.

3.2 Non-Optimizing Saving Behavior

Uzawa (1962 and 1963) present non-optimizing models of two-sector economies. Uzawa (1962) assumes that the entire rental revenue is saved, while all the wage income is spent for consumption. Capital formation is thus determined by

$$\dot{K} = Y_1(K,\omega) - \delta K = (r - \delta) K.$$

In the steady state we obtain $r = \delta$. Hence, the steady-state characterization is essentially the same as that of the representative agent model. Uzawa (1963) examines an alternative formulation of savings that is close to the Solow tradition: the aggregate saving is proportional to the real income. In this case we obtain

$$\dot{K} = Y_1(K,\omega) - \delta K = s[Y_1(K,\omega) + pY_2(K,\omega)] - \delta K,$$

[4] We assume that the parameter values involved in our model satisfy the conditions that ensure the presence of a feasible steady-state equilibrium.

where $s \in (0,1)$ is a fixed saving rate. In the steady state, it holds that $Y_1(K,\omega) = \frac{s}{1-s} pY_2(K,\omega) = \delta K$. Therefore, from (7) and (8) we obtain the following two conditions in the steady state:

$$\frac{K - \beta_2 \omega^{\sigma_2}}{\beta_1 \omega^{\sigma_1} - \beta_2 \omega^{\sigma_1}} \left(\gamma_1 (\beta_1 \omega^{\sigma_1})^{\frac{\sigma_1 - 1}{\sigma_1}} + 1 - \gamma_1 \right)^{\frac{\sigma_1}{\sigma_1 - 1}} = \delta K,$$

$$p \frac{\beta_1 \omega^{\sigma_1} - K}{\beta_1 \omega^{\sigma_1} - \beta_2 \omega^{\sigma_1}} \left(\gamma_2 (\beta_{21} \omega^{\sigma_1})^{\frac{\sigma_2 - 1}{\sigma_2}} + 1 - \gamma_1 \right)^{\frac{\sigma_2}{\sigma_2 - 1}} = \frac{\delta}{s} K,$$

where the relative price satisfies

$$p = \frac{\partial Y_1 / \partial K_1}{\partial Y_2 / \partial K_2} = \frac{k_1^{-1/\sigma_2} \left(\gamma_1 k_1^{\frac{\sigma_1 - 1}{\sigma_1}} + 1 - \gamma_1 \right)^{\frac{\sigma_1}{\sigma_1 - 1}}}{k_2^{-1/\sigma_2} \left(\gamma_2 k_2^{\frac{\sigma_2 - 1}{\sigma_2}} + 1 - \gamma_2 \right)^{\frac{\sigma_2}{\sigma_2 - 1}}} = p(\omega).$$

The two equations shown above may determine the steady-state values of ω and K. It is seen that the relation between the level of σ_i and and the steady-state capital stock is much more complex than that established in the representative-agent economy.

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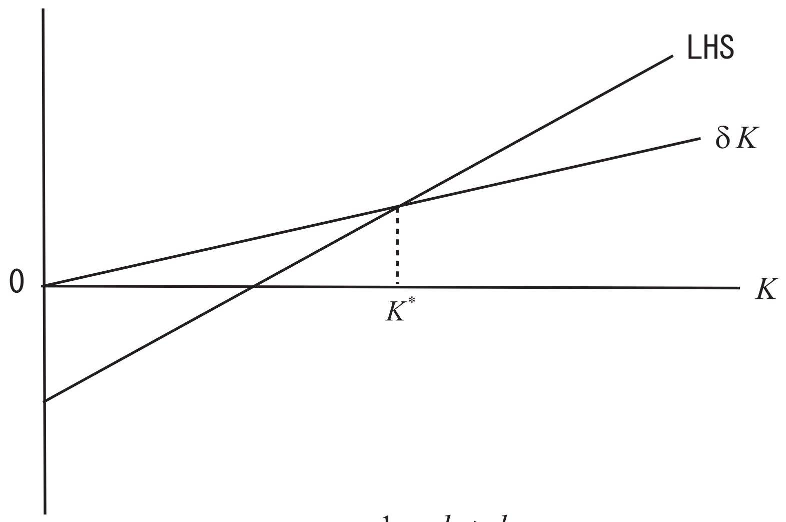
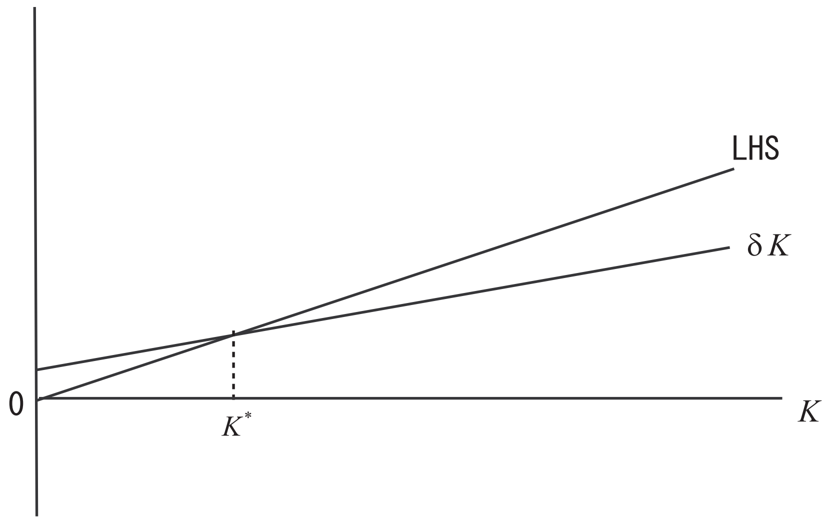
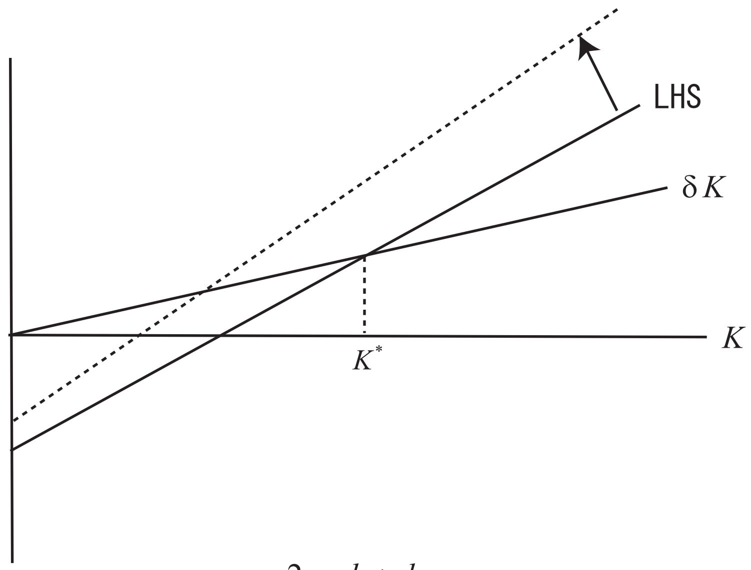


1-a: $k_1 > k_2$

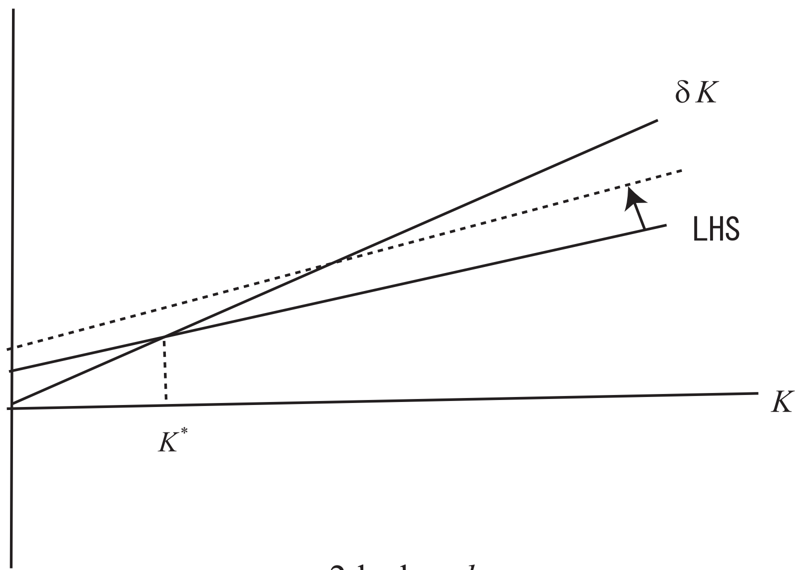


1-b: $k_1 < k_2$

Figure 1

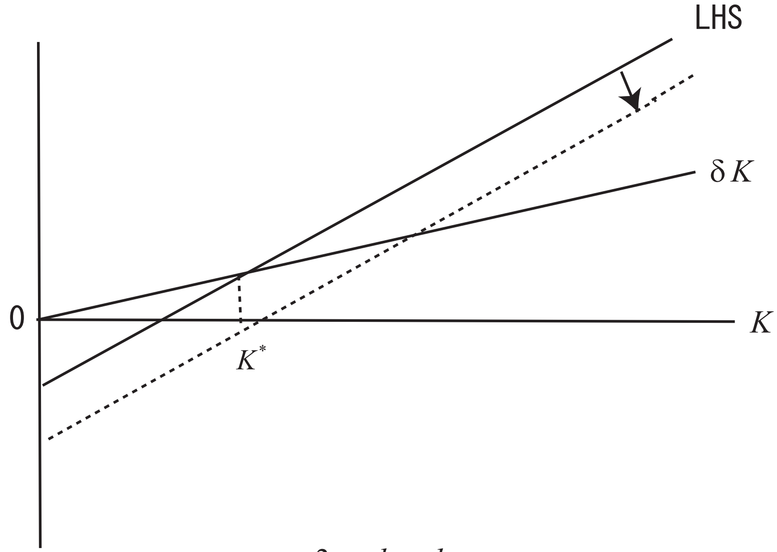


2-a: $k_1 > k_2$

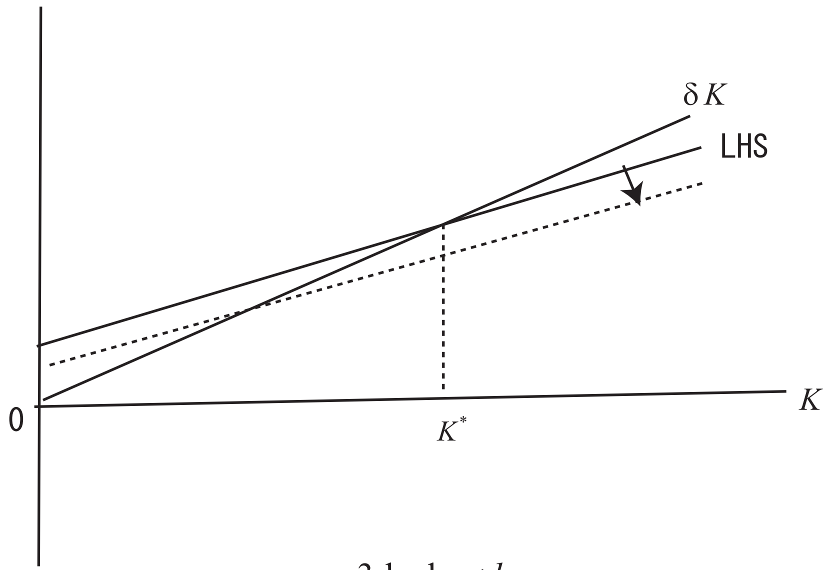


2-b: $k_1 < k_2$

Figure 2



3a: $k_1 > k_2$



3-b: $k_1 < k_2$

Figure 3