Unionised labour market, efficiency wage and endogenous growth

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Abstract:
In this paper, we analyse the effect of unionisation on the growth of the economy in the presence of ‘Efficiency Wage Hypothesis’. We use both ‘Efficient Bargaining’ model and ‘Right to Manage’ model to solve the negotiation problem. Unionisation raises negotiated wage rate and the effort (efficiency) level of the worker. In the case of ‘efficient bargaining model’, unionisation reduces the negotiated number of workers but improves the effort level when the union is neutral in its orientation. As a result, effective employment is increased; and this leads to a rise in the growth rate and welfare level of the economy. However, in the ‘Right to manage model’ of bargaining, unionisation in the labour market raises the effort level of worker but lowers the number of workers irrespective of the orientation of the labour union; and raises effective employment, balanced growth rate and welfare level if the wage elasticity of efficiency is greater than the unemployment rate.

JEL classification: J51; O41; J31

Keywords: Labour union; Efficiency Wage Hypothesis; Endogenous growth; Efficient bargaining; Right to manage model

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1. **Introduction:**

Unionisation exists in labour markets of almost every country with a varying degree. Though some works\(^1\) show that labour unions have become weaker over time in some countries like United States, Australia, New Zealand, Japan etc., they still play a crucial role to determine wage rate and employment in many other countries, for example in European countries\(^2\). In recent years, many European countries are suffering from high unemployment rate as well as from low growth rate; and there is a view that reducing labour market frictions are necessary to promote economic growth. So it is important to make a theoretical analysis of the effect of unionisation on economic growth.

There already exists a set of literature\(^3\) dealing with the effect of unionisation on the long run growth rate of the economy. However, these works mainly emphasise on the effects of unionisation on wage, intersectoral labour allocation and unemployment and thereby on economic growth but does not focus on the role of ‘Efficiency Wage Hypothesis’\(^4,5\). Since unionisation raises wage rate of employed workers, so, according to ‘Efficiency Wage Hypothesis’, effort (efficiency) level per worker rises\(^6\); and this may cause an overall positive effect on the production level. A few works have focused on the role of efficiency wage on union firm bargaining\(^7\). However, no one has analysed the effect of unionisation on economic growth in the presence of ‘Efficiency Wage Hypothesis’.

The present paper attempts to develop a model to analyse effects of unionisation in the labour market on the employment level, growth rate and welfare level of an economy in the presence of ‘Efficiency Wage Hypothesis’. The model developed here is an AK model with an unionised labour market and with an unemployment benefit scheme. In this model, we use two alternative versions of bargaining models – the ‘Efficient Bargaining Model’ of McDonald and Solow (1981) and the ‘Right to Manage Model’ of Nickell and Andrews (1983).

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\(^1\) See for example Freeman and Ichniowski (1988), Dinardo et al. (1996) and Visser (2006).
\(^2\) See for example Page 6 of Lingens (2004).
\(^5\) An earlier version of Palokangas (2004), i.e., Palokangas (2003) incorporates ‘Efficiency Wage Hypothesis’ in his model, but does not emphasise on its role while determining the effect of unionisation. In fact, in a footnote in that paper, he states that “However, the results in this paper hold even if the effort per worker is wholly inflexible…….” The published version of the paper, i.e., Palokangas (2004) does not incorporate ‘Efficiency Wage Hypothesis’.
\(^6\) See sections 9.2 and 9.3 of Romer (2006).
\(^7\) See, for example, Garino and Martin (2000), Marti (1997), Mauleon and Vannetelbosch (2003), Pereau and Sanz (2006) etc.
We derive interesting results from this model. In the ‘Efficient Bargaining model’, unionisation in the labour market reduces the negotiated number of workers but raises the effort (efficiency) level per worker when the labour union is neutral in its orientation, i.e., when it is neither wage oriented nor membership oriented. Effective employment measured in efficiency unit is increased; and this leads to a rise in the growth rate and welfare level of the economy. However, in the ‘Right to Manage Model’, unionisation in the labour market raises the effort level per worker but lowers the number of workers irrespective of the orientation of the labour union; and raises effective employment, balanced growth rate and welfare level if and only if the wage elasticity of effort is greater than the unemployment rate. This sufficient condition is valid when the income tax rate charged by the government to finance unemployment benefit expenditure is very low.

The paper is organized in the following way. In section 2, we describe the basic model and analyse the effect of unionisation with ‘Efficient Bargaining’. In section 3, we do the same with a ‘Right to Manage’ model. Section 4 concludes.

2. The model:

2.1 Firms:

The representative competitive firm produces the final good, \( Y \), using private capital, \( K \), and effective labour, \( eL \); and its production function\(^8\) is given by

\[
Y = AK^\alpha (eL)^\beta \bar{K}^{1-\alpha}
\]  

(1)

satisfying \( \alpha, \beta, \alpha + \beta \in (0,1) \).

Here \( A > 0 \) is a time independent technology parameter. \( \bar{K} \) represents average amount of capital stock of all firms available in the economy; and \( 0 < \alpha < 1 \) ensures that external effect of capital is positive. Here \( L \) denotes the number of workers and \( e \) represents the work effort given by each worker.\(^9\) Existence of decreasing returns to private inputs leads to super normal profit in equilibrium; and this acts as the rent in the bargaining process to be negotiated between the employers’ association and the labour union. Following Chang et al. (2007), we assume that a

\(^8\) This production function is identical to that in Chang et al. (2007) except for the fact that Chang et al. (2007) does not consider effort of workers, \( e \).

\(^9\) We assume that all workers have identical effort levels.
fixed quantity of land is necessary for a firm to operate; and thus the number of firms is fixed even in the presence of positive profit due to fixed availability of land.\textsuperscript{10}

We introduce the efficiency wage hypothesis\textsuperscript{11} which states that the effort level of a worker, $e$, varies positively with the premium of wage over his alternative reservation income. For simplicity, we assume that a worker's reservation income is the unemployment benefit per unemployed worker, $b$. So the worker's effort function is given by \textsuperscript{12}

$$e = h \left( \frac{w}{b} \right)^{\delta}.$$  \hspace{1cm} (2)

Here $h$ is a positive parameter and it denotes worker's effort level when effort level is independent of wage premium, i.e. when $\delta = 0$. Here $\delta$ represents the elasticity of effort with respect to the wage rate; and it is assumed to be a positive constant satisfying $0 < \delta < 1$. Chang et al. (2007) does not consider efficiency wage hypothesis. In Chang et al. (2007), $e \equiv 1$, i.e., $\delta = 0$ and $h = 1$.

The firm maximises profit, $\pi$, defined as

$$\pi = Y - wL - rK.$$

Here $w$ and $r$ represent wage rate and rental rate on private capital respectively.

Private capital market is perfectly competitive. The equilibrium value of rental rate on private capital is determined by the supply-demand equality. The demand function for private capital is derived from firms’ profit maximization exercise; and it is given by

$$r = \alpha AK^{\alpha-1}(eL)^{\beta}K^{1-\alpha} = \frac{\alpha Y}{K}.$$  \hspace{1cm} (4)

\subsection*{2.2 Government:}

\textsuperscript{10} Number of firms is normalized to unity. The equilibrium in the product market is always a short run competitive equilibrium with positive profit. Lai and Wang (2010) and Chang et al. (2007) also consider that union bargaining takes place in competitive production sector. However, Adjemian et al. (2010), Bräuninger (2000) and Lingens (2003b) assume a monopolistically competitive sector; and Lingens (2003a) assumes a monopoly product market. Ramos-Parreño and Sánchez-Losada (2002) and Irmen and Wigger (2002/2003) consider monopoly labour union model.

\textsuperscript{11} See footnote 4.

\textsuperscript{12} Danthine and Kurmann (2006) has also used almost similar functional form.
The government finances unemployment benefit scheme by charging an exogenously
given rate of income tax, \( \tau \); and balances its budget at each point of time. The budget balancing
equation is given by
\[
\tau Y = b(1 - L). \quad (5)
\]
Here \( (1 - L) \) is the unemployment level.

2.3 **Labour union and Efficient bargaining:**

The labour union in this model derives utility from the hike in the wage rate over the
unemployment benefit rate\(^{13} \) as well as from the size of membership. All employed workers
are assumed to be members of the union. The utility function of the labour union is given by
\[
u_T = (w - b)^m L^n. \quad (6)
\]
Here \( u_T \) stands for the utility of the labour union. \( m \) and \( n \) are elasticities of labour union’s
utility with respect to wage premium and with respect to number of members respectively. The
labour union is said to be ‘wage oriented’ (‘employment oriented’) (‘neutral’) if \( m > (<) (=) n \).
Chang et al. (2007) contains a brief discussion about these parameters. Others works including
Lingens (2003a, 2003b) and Adjemian et al. (2010) assume \( m = n = 1 \), i.e., the labour union to be neutral.

We now consider the ‘Efficient bargaining model’ where wage rate and number of
employed workers are determined jointly by the labour union and the employer’s association;
and they maximize the ‘generalised Nash product’ function given by
\[
\psi = (u_T - \bar{u}_T)^\theta (\pi - \bar{\pi})^{(1-\theta)}. \quad (7)
\]
Here \( \bar{u}_T \) and \( \bar{\pi} \) stand for the reservation utility level of the labour union and the reservation
profit level of the firm respectively. Bargaining disagreement discontinues production process
and this implies \( \bar{u}_T = \bar{\pi} = 0 \). The relative bargaining power of the labour union is represented
by \( \theta \in (0,1) \). Unionisation is defined as an exogenous increase in the relative bargaining power
of the labour union, i.e. in the value of \( \theta \).

\(^{13}\) Irmen and Wigger (2003), Lingens (2003a) and Lai and Wang (2010) assume that the difference between the
bargained wage rate and the competitive wage rate is an argument in the labour union’s utility function. Contrary
to this, Adjemian et al. (2010) and Chang et al. (2007) assume that the difference between the after tax bargained
wage rate and the unemployment benefit is an argument in the labour union’s utility function. So, this paper
follows the second kind of modelling.
Finally, using equations (3), (6) and (7), we obtain

\[
\psi = \{(w - b)^m L^n\}^\theta \{Y - wL - rK\}^{(1-\theta)} .
\]  

(8)

Here \(\psi\) is to be maximised with respect to \(w\) and \(L\). Using equations (1), (2), (4) and (5), and two first order conditions of optimisation, we solve for optimal \(w\) and \(L\).\(^{14}\) These are given by

\[
L^* = \frac{\theta_2 \theta_4}{\theta_2 \theta_4 + \tau \theta_1 \theta_3} ;
\]

(9)

and

\[
w^* = \frac{AK^\alpha h^\beta K^{1-\alpha}}{(\theta_2 \theta_4 + \tau \theta_1 \theta_3)^{\beta-1}} \left( \frac{\theta_2^\beta \theta_3^{\beta \delta}}{\theta_1 \theta_4^{1-\beta (1-\delta)}} \right) .
\]

(10)

Here,

\[
\theta_1 = (1 - \theta + \theta n) > 0 ,
\]

(11)

\[
\theta_2 = [\theta n (1 - \alpha) + \beta (1 - \theta)] > 0 ,
\]

(12)

\[
\theta_3 = [\theta n (1 - \alpha - \beta) + \beta (1 - \beta)(1 - \theta + \theta n)] > 0 ,
\]

(13)

and

\[
\theta_4 = [\theta (n - m)(1 - \alpha - \beta) + \beta (1 - \delta)(1 - \theta + \theta n)] .
\]

(14)

We assume \(\theta_4\) to be positive to ensure \(0 < L^* < 1\). This assumption implies that elasticity of union’s utility with respect to wage hike, \(m\), cannot be far greater than the corresponding elasticity with respect to membership, \(n\). If the union is neutral or employment oriented, i.e., \(m \leq n\), then \(\theta_4\) is always positive. From equation (9), we obtain

\[
\frac{\partial L^*}{\partial \tau} = -\frac{\theta_2 \theta_4 \theta_1 \theta_3}{[\theta_2 \theta_4 + \tau \theta_1 \theta_3]^2} < 0 .
\]

(9.a)

Equation (9.a) shows that employment level varies inversely with the rate of income tax used to finance unemployment benefit. This is so because, a rise in the tax rate raises unemployment benefit per worker; and this lowers both the efficiency of a worker and union’s utility from wage hike. As a result, wage rate rises and employment level falls.

\(^{14}\) Derivation of optimal \(w\) and \(L\) is provided in Appendix A.
Now, from equations (2), (5), (9) and (10), we obtain the effort level per worker as given by

$$e^* = h\left(\frac{\theta_3}{\theta_4}\right)^{\delta}.$$  \hspace{1cm} (15)

From equations (9) and (15), we obtain effective level of employment i.e., the level of employment in efficiency unit, as given by

$$e^*L^* = h\frac{\theta_2\theta_4^{1-\delta}\theta_3^{\delta}}{\theta_2\theta_4 + \tau\theta_1\theta_3}.$$  \hspace{1cm} (16)

### 2.4 Households:

The representative household obtains instantaneous utility only from consumption of the final good. She maximises her discounted present value of instantaneous utility over the infinite time horizon subject to the intertemporal budget constraint. The household’s problem is given by the following.

$$\max_{c, k} \int_0^\infty \frac{c^{\sigma} - 1}{1 - \sigma} e^{-\rho t} \, dt$$

subject to

$$\dot{K} = (1 - \tau)Y - c$$

$$K(0) = K_0 \quad (K_0 \text{ is historically given})$$

and $$c \in [0, (1 - \tau)Y].$$

Here $c$ denotes consumption level of the representative household; and $\sigma$ and $\rho$ are the two parameters representing elasticity of marginal utility of consumption and the rate of discount respectively. Savings is always invested and there is no depreciation of private capital.

Solving this dynamic optimisation problem, we obtain the growth rate of consumption as given by

$$g = \frac{k}{c} = \frac{(1 - \tau)\alpha AK^{\alpha-1}(eL)\beta K^{1-\alpha} - \rho}{\sigma}.$$  \hspace{1cm} (19)

### 2.5 Equilibrium:

15 Derivation is provided in Appendix B.
We assume a symmetric equilibrium where $\tilde{K} = K$, i.e., all firms have equal amount of capital; and hence, from equation (19), we obtain a time independent growth rate of consumption given by

$$g = \frac{\dot{c}}{c} = \frac{(1 - \tau)A(eL)^\beta - \rho}{\sigma}.$$  \hspace{1cm} (20)

The economy is always in the steady state equilibrium. It does not have transitional dynamic properties because this is an AK model. In equilibrium, all variables like number of workers, $L$, income tax rate, $\tau$, rental rate on capital, $r$, effort level of worker, $e$, and effective employment, $eL$, are time-independent. Capital stock, $K$, final output, $Y$, negotiated wage rate, $w^*$, firm’s profit, $\pi$, and unemployment benefit, $b$, grow at the same rate in the steady-state equilibrium.

### 2.6 Effect of unionisation:

From equations (9), (11), (12), (13) and (14), we obtain

$$\frac{\partial L^*}{\partial \theta} = \frac{\tau(1 - \alpha - \beta)\left[\frac{(n - m)\{\beta(1 - \delta)\theta_1 \theta_2 + n(1 - \alpha - \beta)\theta_3\}}{-\beta^2 \delta n(1 - \delta)\theta_1^2}\right]}{[\theta_2 \theta_4 + \tau \theta_1 \theta_3]^2}.$$  \hspace{1cm} (21)

Equation (21) shows that the effect of unionisation on the employment of workers consists of two components. First component is the union’s membership effect. It is ambiguous in sign and depends on the nature of orientation of labour union. Second component is the substitution effect on employment. An increase in worker’s efficiency lowers the employer’s demand for workers. So the second component is negative. We find that employment orientation of labour union is necessary but not sufficient condition to have a positive relationship between unionisation and the number of workers (members). When labour union is wage oriented or even neutral, unionisation must result into a reduction in the number of workers (members). In Chang et al. (2007), effect of unionisation on employment solely depends on the membership effect and thus on the nature of orientation of the labour union.

When the labour union is neutral, i.e., $m = n$, then

$$\frac{\partial L^*}{\partial \theta} = -\frac{\tau(1 - \alpha - \beta)\beta^2 \delta n(1 - \delta)\theta_1^2}{[\theta_2 \theta_4 + \tau \theta_1 \theta_3]^2} < 0 \text{ for } 0 < \delta < 1.$$  \hspace{1cm} (21.a)
When the labour union is neutral, employment effect is nil in Chang et al. (2007) because \( \delta = 0 \) in that model. However, the effect on employment of workers is negative in our model because \( 0 < \delta < 1 \).

Now, from equations (13), (14) and (15), we obtain

\[
\frac{\partial e^*}{\partial \theta} = \delta h \left( \frac{\theta_3}{\theta_4} \right)^{-1} m(1 - \alpha - \beta) \frac{\beta(1 - \delta)}{\theta_4^2} > 0 .
\]

Equation (22) shows that the effort level of a worker varies positively with the degree of unionisation in the labour market. Negotiated wage rate rises with the rise in the relative bargaining power of the labour union; and this induces the worker to put greater effort. This positive relationship between unionisation and effort level is valid only in the presence of ‘Efficiency Wage Hypothesis’.

Again, from equations (11), (12), (13), (14) and (16), we obtain

\[
\frac{\partial e^L}{\partial \theta} = \frac{(n - m)(1 - \alpha - \beta) \tau h \theta_3 \theta_4}{[\theta_2 \theta_4 + \tau \theta_1 \theta_3]^2 \theta_4^2} \left\{ \theta n \theta_3 (1 - \alpha - \beta) + \beta(1 - \delta) \theta_1 \theta_3 \right\} \\
+ \frac{h \beta(1 - \delta)(1 - \alpha - \beta)[\theta_2^2 m \theta_4 + \tau \theta_1 \theta_3 n m (1 - \alpha - \beta)]}{[\theta_2 \theta_4 + \tau \theta_1 \theta_3]^2 \theta_3^{1-\delta} \theta_4^\delta} .
\] 

Equation (23) shows that unionisation affects effective employment through two channels – changing the number of workers (members) and changing effort level of workers. The membership effect depends partially on the orientation of labour union. However, the other effect is originated from the rise in effort level of workers and hence this effect is always positive. So employment orientation or neutrality of the labour union is sufficient but not necessary to have a positive relationship between effective employment and unionisation in the presence of ‘Efficiency Wage Hypothesis’. This implies that, in the presence of ‘Efficiency Wage Hypothesis’, unionisation may raise effective employment through a rise in effort even if the number of workers (members) is reduced. However, in the absence of this hypothesis, i.e., when \( \delta = 0 \), unionisation does not raise workers’ effort level; and its effect on employment (number of workers) depends solely on the orientation of the labour union.

When the labour union is neutral, i.e., when \( m = n \), then
\[
\frac{\partial e^*L^*}{\partial \theta} = \frac{h\beta \delta(1-\delta)(1-\alpha-\beta)\theta\theta_1\theta_2\theta_3\theta n(1-\alpha-\beta)}{[\theta_2\theta_4 + \tau\theta_1\theta_3]^2\theta_3\theta_4^{1-\delta}} > 0 \quad \text{(23.a)}
\]

So when the labour union is neutral, unionisation raises the effective employment in our model. In Chang et al. (2007), \( \delta = 0 \); and hence the employment effect is nil in that model. Lingens (2003a, 2003b) and Adjemian et al. (2010) assume labour union to be neutral but show that a rise in its relative bargaining power reduces employment due to rise in the wage rate. Lai and Wang (2010) shows that unionisation raises (lowers) the employment level if and only if the balanced growth equilibrium is locally determinate (indeterminate).

Now, equation (20) shows that the balanced growth rate, \( g \), varies positively with the level of effective employment. So effect of unionisation on growth rate is qualitatively similar to that on effective employment. This effect is given by

\[
\frac{\partial g}{\partial \theta} = \left( \frac{(1-\tau)}{\sigma} \right) \frac{\partial e^*L^*}{\partial \theta} \quad \text{(24)}
\]

Sign of \( \frac{\partial g}{\partial \theta} \) depends on the sign of \( \frac{\partial e^*L^*}{\partial \theta} \). In Chang et al. (2007), the nature of growth effect of unionisation depends totally on orientation of the labour union because \( e \equiv 1 \) there. However, our model incorporates ‘Efficiency Wage Hypothesis’; and so effective employment is crucial rather than the employment of workers.

The welfare level of the representative household, \( \omega \), is obtained from equations (1), (17), (18) and (20); and is given by

\[
\omega = K_0^{1-\sigma} \left[ \frac{\rho + \sigma g - \alpha g}{\alpha} \right]^{1-\sigma} \left[ \frac{1}{\rho - g(1-\sigma)} \right] + \text{constant} \quad \text{(25)}
\]

Equation (25) shows that welfare level varies positively with growth rate as we assume \( 1 > \sigma \) > \( \alpha \) and \( \rho > g(1-\sigma) \). Hence welfare effect of unionisation is qualitatively similar to the growth effect of unionisation.

Chang et al. (2007) shows that the employment effect, growth effect and welfare effect of unionisation are nil when labour union is neutral. However, our model shows that these effects of unionisation do consist not only of membership effect but also of substitution effect; and, as a result, they are not nil in the presence of ‘Efficiency Wage Hypothesis’ even if the union is neutral.
So we can establish the following proposition.

**Proposition 1:** In the presence of ‘Efficiency Wage Hypothesis’, unionisation in the labour market reduces the negotiated number of workers but raises the effort level when the labour union is neutral. As a result, effective employment is increased; and this leads to a rise in the growth rate and welfare level of the economy.

We now compare our result to the findings of existing literature. In Palokangas (1996), unionisation lowers employment of both unskilled labour and skilled labour in the final good sector due to their complementary relationship; and this results an increase in the employment of skilled labour in the R&D sector and therefore a rise in the growth rate. Sorensen (1997) shows that unionization may raise the growth rate because it raises the skill of the workers but lowers the profit and, in turn, the marginal return from skill accumulation. The growth rate is reduced (increased) in the ‘Efficient bargaining model’ (‘Right to manage model’). In Bräuninger (2000), unionisation, in general, lowers capital accumulation and the growth rate. However, in the case of heterogeneous individuals, it may raise the growth rate through increase in skill of workers. Lingens (2003a) uses a creative destruction growth model where unionisation lowers the expected profit of the innovators and employment of skilled labour in the manufacturing sector. This surplus labour is absorbed in the R&D sector and rate of innovation is raised. The net effect on growth is ambiguous and depends on the elasticity of substitution between the two types of labour in the manufacturing sector. Irmen and Wigger (2002/2003) uses an OLG model where unionisation causes a transfer of income from the dissaving old to the saving young; and this raises capital accumulation and growth rate. Lingens (2003b) develops a model of endogenous skill formation where unionisation in the unskilled labour market lowers the skilled unskilled relative wage and thus lowers the supply of skilled labour. If the long-run equilibrium level of skilled workers is low (high), then unionisation lowers (raises) the growth rate. Lai and Wang (2010) shows that unionisation raises (lowers) the growth rate if and only if the balanced growth equilibrium is locally determinate (indeterminate). In Adjemian et al. (2010), unionization reduces profit and thereby expected value of innovation; and this discourages R&D and economic growth. However, none of these works considers the role of efficiency wage hypothesis.

3. **The ‘Right to manage model’:**

In this section, we use the ‘Right to manage model’ of bargaining where the two parties bargain over the wage rate only. The firm unilaterally decides the level of employment from
its labour demand function resulting from the profit maximising behaviour. The inverted labour demand function of the representative firm is given by

\[ w = \left[ \beta AK^{aR^{1-a}L^{\beta-1}h^{\theta} b^{-\beta\delta}} \right]^{\frac{1}{1-\beta\delta}}. \]  

(26)

So firms’ association and labour union jointly maximises the ‘generalised Nash product’ function given by equation (8), with respect to \( w \) only, subject to the firm’s labour demand function given by equation (26). Using the first order condition and equations (1), (2), (4), (5) and (26), optimum values of \( L \) and \( w \) are obtained as\(^{16}\)

\[ L^* = \frac{(\beta)\{\theta n(1 - \alpha - \beta)(1 - \beta\delta) + \beta(1 - \delta)(1 - \theta)(1 - \beta)\}}{(\beta + \tau)\{\theta n(1 - \alpha - \beta)(1 - \beta\delta) + \beta(1 - \delta)(1 - \theta)(1 - \beta)\}} < 1. \]  

(27)

and

\[ w^* = AK^{aR^{1-a}h^{\theta} \beta^{1+\beta\delta} L^{\beta-1-\beta\delta} \tau^{-\beta\delta} (1 - L^*)^{\beta\delta}} ; \]  

(28)

We assume \( \{\theta n(1 - \alpha - \beta)(1 - \beta\delta) + \beta(1 - \delta)(1 - \theta)(1 - \beta)\} > \theta m(1 - \beta)(1 - \alpha - \beta) \) to ensure that \( L^* > 0 \).

From equations (1), (2), (5) and (28), we obtain the effort level per worker as given by

\[ e^* = h \left[ \frac{\beta (1 - L^*)}{L^*} \right]^{\delta}. \]  

(29)

The government’s budget balance equation as well as the representative household’s behaviour in this model is identical to that in the ‘Efficient Bargaining’ model. So equations and solutions derived here are same as those obtained in section 2 except that \( L^* \) is replaced by \( L^{**} \) and \( e^* \) is replaced by \( e^{**} \).

Now, from equation (27), we have

\[ \frac{\partial L^{**}}{\partial \theta} = \frac{-\theta m \beta^2 (1 - \delta)(1 - \alpha - \beta)(1 - \beta)^2}{\{\beta + \tau\{\theta n(1 - \alpha - \beta)(1 - \beta\delta) + \beta(1 - \delta)(1 - \theta)(1 - \beta)\}\}} < 0. \]  

(30)

---

\(^{16}\) We assume that second order condition of maximisation is satisfied.
So in this model, unionisation in the labour market lowers the employment of workers irrespective of the orientation of the labour union.

Again, from equation (29), we obtain

\[
\frac{\partial e^*}{\partial \theta} = -\delta h \left[ \frac{\beta}{\tau} \right] \left[ \frac{(1 - L^*)}{L^*} \right]^{\delta - 1} \frac{1}{L^{**2}} \frac{\partial L^*}{\partial \theta} > 0 .
\]

and

\[
\frac{\partial (e^*L^*)}{\partial \theta} = \frac{e^*(1 - \delta - L^*)}{(1 - L^*)} \frac{\partial L^*}{\partial \theta} .
\]

Equation (31) shows that in the presence (absence) of ‘Efficiency Wage Hypothesis’, effort level of a worker goes up (does not change) with unionisation in the labour market. This is similar to that in ‘Efficient Bargaining Model’. However, contrary to the ‘Efficient Bargaining Model’, equation (32) shows that the effect of unionisation on effective employment depends not on the orientation of the labour union but on the mathematical sign of \((1 - \delta - L^*)\). If \((1 - \delta - L^*)\) is negative (positive), then effective employment varies positively (inversely) with unionisation in the labour market because the number of workers varies inversely with unionisation. This implies that, unionisation raises (lowers) effective employment if the rate of unemployment, \((1 - L^*)\), is less (greater) than the value of wage elasticity of effort parameter, \(\delta\).

The intuition behind this result is as follows. Unionisation affects effective employment by changing both effort level and number of workers. As the first effect is positive and the second effect is negative, so the aggregate effect depends on the relative strength of these two effects. When unionisation raises wage rate and thereby the effort level of worker, the wage elasticity of effort parameter, \(\delta\), captures the strength of this effect. However, when unionisation raises the number of unemployed workers, the strength of this effect is captured by \((1 - L^*)\). Hence \((1 - L^*) < \delta\) implies that the first effect dominates the second effect.

This condition \((1 - L^*) < \delta\) has important implication for policy prescription. Using equation (27), we find that \((1 - L^*) < \delta\)

\[
\tau < \bar{\tau} = \frac{\beta \delta}{(1 - \delta)} \left[ 1 - \frac{\theta m(1 - \beta)(1 - \alpha - \beta)}{\theta n(1 - \alpha - \beta)(1 - \beta \delta) + \beta (1 - \delta)(1 - \theta)(1 - \beta)} \right] .
\]
Here $\bar{\tau} > 0$ if

$$\theta (1 - \alpha - \beta) [m(1 - \beta) - n(1 - \beta \delta)] < \beta (1 - \delta)(1 - \theta)(1 - \beta) .$$

If $m \leq n$, then this inequality is always valid.

So if the tax rate is very low, then unionisation raises effective employment. The level of employment varies inversely with $\tau$. So a low value of $\tau$ leads to a low rate of unemployment such that a rise in the effort level of each worker compensates the fall in employment due to unionisation. So, in the presence of ‘Efficiency Wage Hypothesis’, if the income tax rate charged by the government to finance unemployment benefit expenditure is very low, then unionisation may have a positive effect on effective employment. However, in the absence of ‘Efficiency Wage Hypothesis’, i.e., when $\delta = 0$, unionisation always lowers effective employment level, which is identical to the number of workers.

Growth rate and welfare level of the economy in this model are identical to those given by equations (20) and (25) in efficient bargaining model except that $L^*$ and $e^*$ are replaced by $L^{**}$ and $e^{**}$. So the effect of unionisation on growth rate and welfare level are qualitatively similar to its effect on effective employment. So we can conclude that unionisation raises the growth rate and the welfare level if $\delta > (1 - L^{**})$. This result is different from that obtained in the case of the ‘Efficient bargaining model’ where effect of unionisation on growth and welfare depends on the nature of orientation of the labour union.

Important results derived in this section are summarized in the following proposition.

**Proposition 2:** In the ‘Right to manage model’ of bargaining, unionisation in the labour market raises the effort level of worker but lowers the number of workers irrespective of the orientation of the labour union; and raises (lowers) effective employment, balanced growth rate and welfare level if the wage elasticity of effort is greater (less) than the unemployment rate.

**4. Conclusion:**

This paper develops a model to investigate the effect of unionisation in the labour market on the long run growth rate of an economy in the presence of ‘Efficiency Wage Hypothesis’. Here we use two alternative versions of bargaining models – the ‘Efficient bargaining model’ of McDonald and Solow (1981) and the ‘Right to manage model’ of Nickell.
and Andrews (1983). The existing literature that analyses the role of unionisation on economic growth does not consider ‘Efficiency Wage Hypothesis’.

We derive different results from these two versions of bargaining models. In the ‘Efficient Bargaining model’, unionisation in the labour market lowers the negotiated number of workers but raises the effort level of a worker when the labour union is neutral. Effective employment is increased; and this leads to an increase in the growth rate and welfare level of the economy. However, in the ‘Right to Manage Model’, unionisation raises the effort level of the worker but reduces the number of workers irrespective of the orientation of the labour union. This raises effective employment, balanced growth rate and welfare level of the economy if the wage elasticity of effort (efficiency) exceeds the unemployment rate; and this sufficient condition is likely to be valid when the income tax rate is very low.

However, our model is abstract and fails to consider many aspects of reality. We do not consider the possibility of human capital accumulation, population growth, technological progress, positive externality of public capital etc. Hence the allocation of government’s budget and of household’s income to education, R&D etc. is ignored in this work. For simplicity, we assume ‘closed shop labour union’, which is rare in reality than the more common ‘open shop labour union’; and thus ignore the role of membership dynamics. We plan to do further research in future attempting to get rid of these limitations.

References


Appendix

Appendix A

Derivation of optimal $w$ and $L$

From equations (1) and (8), we obtain two first order conditions given by

$$\frac{\theta m}{(w - b)} + \frac{(1 - \theta) \left[ \beta \delta \frac{Y}{w} - L \right]}{Y - wL - rK} = 0 \quad . (A. 1)$$

$$\frac{\theta n}{L} + \frac{(1 - \theta) \left[ \beta \frac{Y}{L} - w \right]}{Y - wL - rK} = 0 \quad . (A. 2)$$

From equations (A.2) and (4), we obtain
\[
\frac{Y}{wL} = \frac{(1 - \theta + \theta n)}{[\theta n(1 - \alpha) + \beta(1 - \theta)]} .
\] (A.3)

From equations (A.1), (4) and (5), we obtain

\[
\frac{\theta m}{1 - \left(\frac{\tau Y}{w[1 - L]}\right)} = \frac{(1 - \theta) \left[1 - \beta \delta \frac{Y}{wL}\right]}{(1 - \alpha) \left[\frac{Y}{wL} - 1\right]} .
\] (A.4)

Incorporating equation (A.3) in equation (A.4), we obtain

\[
\frac{\theta m}{1 - \left(\frac{\tau L}{[1 - L] \left[\theta n(1 - \alpha) + \beta(1 - \theta)\right]}\right)} = \frac{(1 - \theta) \left[1 - \frac{\beta \delta(1 - \theta + \theta n)}{[\theta n(1 - \alpha) + \beta(1 - \theta)]}\right]}{(1 - \alpha) \left[\frac{1 - \beta \delta(1 - \theta + \theta n)}{[\theta n(1 - \alpha) + \beta(1 - \theta)]} - 1\right]} .
\] (A.4a)

Solving equation (A.4a), we obtain the optimal value of \(L\) as given in equation (9) in the body of the paper.

Now, using equations (1) and (5), we obtain

\[
Y = [AK^\alpha R^{1 - \alpha} h^\beta L^\beta \tau^{-\beta \delta} (1 - L)^{\beta \delta}]^{\frac{1}{1 + \beta \delta}} .
\] (A.5)

Using equations (A.3) and (A.5), we obtain

\[
\frac{[AK^\alpha R^{1 - \alpha} h^\beta L^\beta \tau^{-\beta \delta} (1 - L)^{\beta \delta}]^{\frac{1}{1 + \beta \delta}}}{wL} = \frac{(1 - \theta + \theta n)}{[\theta n(1 - \alpha) + \beta(1 - \theta)]} .
\]

\[
\Rightarrow w = [AK^\alpha R^{1 - \alpha} h^\beta L^{\beta - (1 + \beta \delta) \tau^{-\beta \delta} (1 - L)^{\beta \delta}}]^{\frac{1}{1 + \beta \delta}} \left\{\frac{[\theta n(1 - \alpha) + \beta(1 - \theta)]}{(1 - \theta + \theta n)}\right\}^{1 + \beta \delta} .
\] (A.6)

Using equations (A.6) and (9), we obtain the optimal value of \(w\) as given in equation (10) in the body of the paper.

**Second order conditions**:

From equations (A.1) and (A.2), we obtain
\[
\frac{\partial^2 \psi}{\partial w^2}\psi - \left(\frac{\partial \psi}{\partial w}\right)^2 = -\frac{\theta m}{(w-b)^2} + \frac{(1-\theta) \left[ \beta \delta \delta - 1\right] \frac{Y}{w^2} (Y - wL - rK) - \left[ \beta \delta \frac{Y}{w} - L \right]^2}{(Y - wL - rK)^2} ; \quad (A.7)
\]

\[
\frac{\partial^2 \psi}{\partial L^2}\psi - \left(\frac{\partial \psi}{\partial L}\right)^2 = -\frac{\theta n}{L^2} + \frac{(1-\theta) \left[ \beta (\beta - 1) \frac{Y}{L^2} (Y - wL - rK) - \beta \frac{Y}{L} - w \right]^2}{(Y - wL - rK)^2} ; \quad (A.8)
\]

and

\[
\frac{\partial^2 \psi}{\partial \xi \partial w}\psi - \frac{\partial \psi}{\partial \xi}\frac{\partial \psi}{\partial w} = \frac{\left[ \beta \delta \delta \delta - \frac{Y}{wL} \left( Y - wL - rK \right)^2 \right]}{(1-\theta)^{-1}(Y - wL - rK)^2} . \quad (A.9)
\]

Using equations (1), (4), (5), (9), (11), (12), (13), (14), (A.3), (A.7), (A.8), (A.9) and \(\frac{\partial \psi}{\partial \xi} = 0\), we obtain respectively

\[
\frac{\partial^2 \psi}{\partial w^2} = -\frac{(1-\theta + \theta m)\theta_3^2 + (1-\alpha-\beta)(1-\theta)\beta \delta \theta_1 (1-\beta) \theta m}{w^2 (1-\alpha-\beta)^2 \theta m (1-\theta)} < 0 ; \quad (A.10)
\]

\[
\frac{\partial^2 \psi}{\partial L^2} = -\frac{\theta n (1-\alpha-\beta) \theta_1 + \theta_1 \beta (1-\theta) (1-\beta)}{(1-\alpha-\beta) (1-\theta)L^2} < 0 ; \quad (A.11)
\]

and

\[
\frac{\partial^2 \psi}{\partial \xi \partial w} = \frac{[\theta n + \beta (1-\theta) - \theta_1 \theta_2]}{(1-\alpha-\beta) (1-\theta) wL} . \quad (A.12)
\]

Now using equations (A.10), (A.11) and (A.12), we have

\[
\frac{\partial^2 \psi}{\partial w^2} \frac{\partial^2 \psi}{\partial L^2} - \left(\frac{\partial^2 \psi}{\partial \xi \partial w}\right)^2
\]
\[
\left\{(1 - \theta + \theta m) \theta_3^2 + (1 - \alpha - \beta)(1 - \theta)\beta \delta \theta_1 (1 - \beta \delta) \theta m\right\} \\
\frac{\{\theta n (1 - \alpha - \beta) \theta_1 + \theta_1 \beta (1 - \theta) (1 - \beta)\}}{-\{\theta n + \beta (1 - \theta) - \theta_1 \theta_2\}^2 (1 - \alpha - \beta) \theta m} \\
= \frac{w^2 (1 - \alpha - \beta)^3 \theta m (1 - \theta)^2 L^2}{(A.13)}.
\]

We assume that the R.H.S. of equation (A.13) is positive in order to satisfy the second order conditions.

**Appendix B**

**Derivation of equation (15):**

From equations (2) and (5), we obtain

\[
e = h \left(\frac{w (1 - L)}{\tau Y}\right)^\delta.
\]

Using equations (A.3) and (9), we obtain equation (15) in the body of the paper.

Derivations of section 3 are similar to that of section 2.