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**Abstract**

Demographic transition theory is developed highlighting cultural transmission pattern as key driver. Individuals maximize cultural fitness, i.e. rate of own cultural type absorbtion by future generations. With low population density, one's culture can be picked up only by own children, thus cultural fitness equals genetic fitness, individuals allocate all energy surplus to reproduction, and Malthusian regime occurs. With rising population density, cultural transmission between non-relatives accelerates; knowledge production by an individual makes her culture more attractive. Individuals reallocate some of energy surplus from reproduction to knowledge production, causing technological growth. The model fits observed demographic transition patterns.

*Keywords:* Endogenous growth, Cultural transmission, Demographic transition

*JEL codes*: J11, O44, Z19

## 1. Introduction

The life history theory, a branch of theoretical biology, makes very specific predictions about such elements of an organism's lifecycle as the age of reproduction, fertility, and life expectancy. All of these are shaped to maximize the population growth rate.

While for most of their history humans were seemingly no different from other species in their effort to maximize the population rate of growth, in the last two centuries a dramatic transformation of human life histories has occurred. Humans deter reproduction until later

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age, have fewer children, and live longer. They no longer allocate all spare resources towards reproduction. What caused such demographic transition? Why only among humans, and why only in the last two centuries? Clearly, an explanation must include some features of the *Homo sapiens* that are non-existent in other species, and that became especially pronounced in the last two centuries.

The abundant Economics literature on the demographic transition has virtually ignored this logic. The reason is in the fundamental assumptions of the Economic theory: instead of building on the Darwinian *Homo Sapiens*, whose goal is to maximize own trace in history, theories are typically built on the Adam Smith's *Homo Economicus*, whose primary goal is to enjoy himself. Several economists have pointed out that, according to the Darwinian logic, private consumption is an intermediate rather than the final good (e.g. Robson (2001), Bergstrom (2007)). Nevertheless, this logic did not find its way into Economic Demography, and virtually all theories of the demographic transition remain transfixed on the human as the consumption maximizer. To explain why such a human would have any children at all, additional bells and whistles have been attached to the consumption-centered objective function. Becker (1960) proposes to view children as something that enters parents' utility along with consumption goods. Becker et al. (1990) assume that parent's utility includes personal consumption, as long as (with a smaller weight) that of each of their children. Galor and Weil (2000), Kalemli-Ozcan (2002) assume that parents are concerned about own consumption as well as about aggregate income of all of their (surviving) children. Baudin (2010) assumes that parents derive utility from own labor income, and from the fact that their children have adopted their own (as opposed to someone else's) views of life. Galor and Moav (2002) assume that parents derive utility from own consumption, as well as the number and "quality"/education of children. In their model, personal consumption enters the utility with an exogenous weight, while the importance of the other two components is subject to evolutionary selection. While the authors acknowledge that the same evolutionary pressure should drive the weight of personal consumption down to zero, they “navigate around” this problem by (verbally) asserting that such consumption is needed to increase individual reproductive success; this idea is never formulated rigorously within their model.

This paper makes a case for abandoning the traditional consumption-based utility, and modeling a human whose objectives follow logically from the Darwinian genetic-fitness-maximizing paradigm. As mentioned above, to explain demographic transition within this paradigm one has to highlight some unique features of the *Homo sapiens*. This paper emphasizes the role of one such suspect, namely the human ability for *cultural transmission*. Culture is a set of behavioral recipes that are passed from one generation of humans to the next, and which may differ across societies. Boyd and Richerson (1985) convincingly argue that the human ability for cultural transmission is unprecedented; such ability strongly affects the course of the evolution of mankind. Importantly, culture is not always transmitted from parents to children: a child may pick a cultural trait from outside the family. Newson et al. (2007) links the declining fertility to the increasing exposure of people to the “outside” culture: when culture is picked up primarily from parents or other close relatives, the evolution of cultural traits parallels the evolution of genes, and cultural traits emphasizing high fertility are favored. When the chance of picking up culture from outside is increased, another cultural traits, prescribing lower fertility, dominate. Zakharenko (2013a) formalizes the idea in a model.

On the empirical side, several studies indicate the link between cultural transmission patterns and fertility. La Ferrara et al. (2012) finds that arrival of TV shows portraying low-fertility lifestyles reduces actual fertility in Brazilian communities. Axinn and Barber (2001); Axinn and Yabiku (2001) use the purpose-collected dataset from rural Nepal to show that living near social places such as school (even controlling for actual school attendance), a bus stop, or a market, in the early childhood, increases the chance of contraception use during adulthood. The countries of Eastern Europe had a sharp decline in fertility beginning from around 1987, at the time of sharply increased openness to culture of Western Europe.[1] The Amish community, perhaps the most culturally isolated in the United States, also has some of the highest fertility with 7.7 children per woman (Greksa, 2002).

Zakharenko (2013a) also shows that, under mild sufficient conditions, the most successful cultural traits are the ones that encourage their hosts to spread them around as much as possible, i.e. maximize the "cultural fitness". Other cultural traits (e.g. those that prescribe the concern for personal consumption) are copied less frequently and become extinct in the process of cultural evolution. This result is reminiscent of the "selfish meme" hypothesis (e.g. Blackmore (2000)), but does not require the existence of memes as discrete physical entities. This idea is employed in the theory we develop: we assume that individuals behave in a way that maximizes their *cultural fitness*, i.e. the extent to which their cultural type has been picked up by the young. The traditional for Economists concern for consumption (own or that of others) is eliminated entirely from the objective function; personal consumption is merely an intermediate good that reduces the rate of mortality.

While the theoretical link between fertility and cultural transmission patterns has already been discussed in Newson et al. (2007) and Zakharenko (2013a), this paper goes beyond and also explains the trend in the other element of demographic transition, namely declining mortality. Additionally, the pattern of cultural transmission is endogenized, which helps to explain why the demographic transition has happened only recently in human history. We now proceed to explain the key ideas of the model in greater detail.

The cultural transmission process is elaborated from Zakharenko (2013a). Every individual is born to parent(s) of a specific cultural type, chooses her own cultural type immediately after birth, and belongs to that cultural type throughout lifetime. Such assumption may not

[1] The socioeconomic conditions such as women's wage or old age security, which are typically cited by Economists as key drivers of fertility, began to change several years *after* the Eastern European fertility began to decline. Moreover, the socioeconomic change was in opposite direction from what was needed to reduce fertility according to typical Economic theories of demographic transition.

be the most accurate representation of actual cultural transmission, but it indeed improves the tractability of the model. With some probability, the individual simply inherits the cultural type of parents. Such probability is greater when a community is isolated so there are few or no alternative role models; this probability decreases as the global population density becomes higher. With the remaining probability, the newly born samples a large number of potential role models from at large,[2] and adopts the cultural type of the most *culturally attractive* role model. Cultural attractiveness is stochastic, such that more *knowledgeable* role models are stochastically more attractive. The knowledge produced by an individual is a matter of choice, and can be viewed as a costly effort that enables individuals of a certain cultural type to attract followers born to parents of other cultural types. This motive for production of knowledge, to increase popularity of own cultural type, is non-standard in the Economics literature (which, again, assumes that selfish individuals create knowledge only to increase own consumption), but does have some empirical support. For example, (Mokyr, 2010, p. 90), emphasizes that "Scientists who discovered matters of significant insight to industry, such as Count Rumford, Joseph Priestley, or Humphry Davy, usually wanted credit, not profit". In the interpretation of the current paper, "credit" is own cultural fitness, i.e. popularity of own cultural type.

The above assumptions about cultural transmission differ from a growing body of Economics literature on cultural transmission commenced by Bisin and Verdier (2001). In that line of research, individuals undertake costly effort to prevent own children from being "lost" to other cultural types, rather than to attract followers from other cultural types. The two approaches, by Zakharenko (2013a) and by Bisin and Verdier (2001), as well as a combination of both, yield similar qualitative results in a model that follows. We use the first approach due to its better mathematical tractability.

[2]There is no geography in the model, hence all individuals are equally likely to be sampled as potential role models.

Individuals require *energy* to live and to reproduce. Energy is produced off the *land*, the only factor of production. The productivity of land depends on the level of technology; naturally, there is less land available per capita as population grows. Therefore, the level of technology checks the maximal population size on the globe. The level of technology, in turn, is a byproduct of knowledge produced by individuals in order to attract cultural followers. A similar assumption was made in Zakharenko (2013b).

Part of the produced energy is utilized by individuals for own mortality prevention. The remaining energy is divided between new births and the production of knowledge.

The model predicts that, with low population density, cultures are largely isolated, cultural transmission between non-relatives is unlikely, and therefore the incentives for knowledge production are low. Individuals allocate the entire energy surplus towards reproduction, and population is limited only by the productivity of land. With little knowledge production, land productivity remains at a low level and grows at a slow rate, which prevents the population size and density from rising too fast.

When the population density finally becomes high enough, individuals begin to deliberately invest in knowledge in order to improve their cultural success. With greater knowledge production, technological progress and land productivity accelerate. A greater rate of growth of energy production results in both increased population growth rate and increased individual productivity. As the total population continues to grow, cultural fitness becomes increasingly dependent on knowledge production, and individuals increasingly reallocate their energy surplus away from physical reproduction to knowledge production. As individual productivity continues to grow, is becomes optimal for individuals to invest more into mortality prevention and thus to live longer.

In contrast with conventional growth models which admit an infinite exponential growth in productivity, we assume that technology eventually levels out at some maximal level. Such assumption is motivated by the fact that we model energy production rather than

GDP; while such production continues to grow, there is little ground to assume that global energy production may grow indefinitely and exponentially. As the level of technology levels out, the model predicts that both population and energy production per capita stabilize at some (high) level. In that terminal steady state, individuals are characterized by high life expectancy, they also limit fertility and allocate a significant proportion of their energy surplus to the production of knowledge.

To the best of the author's knowledge, this is the first paper that links endogenous technology growth, cultural transmission, fertility, and mortality in one model. The above mentioned Becker et al. (1990), Galor and Weil (2000), Kalemli-Ozcan (2002) link fertility to endogenous growth; the latter also features child mortality as an element of her model. Baudin (2010) links fertility to cultural transmission patterns; fertility decline is triggered by exogenously changing productivity. Ponthiere (2011) studies the culturally transmitted lifestyles resulting in heterogenous mortality. Chakraborty (2004) models an endogenous interaction between mortality and growth.

## 2. Model

Consider a dynamic world with continuous time, indexed by $t$. The world is populated by a continuum of individuals, each belonging to one of cultural types. The number of cultural types is countable and is equal to $N$. Note that the distribution of individuals across cultural types need not be perfectly correlated with the distribution across genetic types: individuals from the same cultural type may differ genetically (e.g. belong to different races); individuals from the same biological family may belong to different cultures (e.g. practice different religions). Throughout the model, we keep track only of distribution of individuals across cultural types. The set of all individuals belonging to a cultural type $n$ at time $t$ is denoted $\mathcal{L}_n(t)$; the measure of that set (i.e. the population size) is denoted $L_n(t)$. By $\mathcal{L}(t) = \cup_{n=1\ldots N}\mathcal{L}_n(t)$ we denote the set of all individuals alive at $t$; $L(t) = \sum_n L_n(t)$ is the total population at time $t$.

### 2.1. Objective function

In the world of competition among cultural types, survival necessitates maximization of the cultural fitness, i.e. maximization of the share of population of a given cultural type in the global population, which we assume to be the objective of every member of every cultural type. This implies, in particular, that members of the same cultural types should redistribute energy between one another if that improves the fitness of their culture. Thus, in the model we keep track of the total energy balance for each cultural type, but disregard individual energy balances.

Every evolutionarily successful cultural type should be adaptive, i.e. respond to permanent changes in the environment such that the *steady state* cultural fitness is maximized. At the same time, there is no requirement for a cultural type to be forward-looking along a transition path from one steady-state to another: since transition lasts only for a limited period of time, non-forward-looking behavior may survive the evolutionary selection. This reasoning allows us to assume that decision makers treat some aggregate model parameters such as population growth rate and personal productivity as time-invariant even when they are not, which considerably simplifies calculation of their optimal choices.

### 2.2. Cultural transmission process

With some probability $1 - q$, a newborn simply inherits the cultural type of her parents. The parameter $q \in [0, 1)$ presents the degree of cultural openness of an individual to alternative cultures, and is assumed to be a continuous and increasing function of the global population size: $q = q(L(t))$. A smaller $q$ implies more cultural isolation. The fact that $q$ rises with $L$ implies that cultural isolation decreases as the global population grows.

With the remaining probability $q$, the individual randomly samples a large number $M$ of *potential role models* from $\mathcal{L}(t)$, compares their *cultural attractiveness*, and adopts the type of the most culturally attractive individual. We now describe the sampling process and the process that determines cultural attractiveness.

Every random draw of a potential role model is independent from another draws. The probability of being drawn is the same for every member of $\mathcal{L}(t)$. The drawn individual is a potential role model. Thus, the probability of drawing a potential role model from a cultural type $n$ is equal to the share of that cultural type in the global population, $z_n(t) = \frac{L_n(t)}{L(t)}$.

The cultural attractiveness $\psi_{ij} \geq 0$ of every potential role model $j$ drawn by individual $i$ is a realization of random variable from Frechet distribution with parameters $h^j \geq 0$ and $\theta > 1$, such that $\text{Prob}(\psi_{ij} < x) = e^{-h^j x^{-\theta}}$. The realizations of $\psi_{ij}$ are independent across $i, j$. The parameter $h^j$ is referred to as the knowledge of $j$, and is endogenously determined by individual $j$, prior to the role model sampling, as detailed below. The attractiveness of an individual with a higher knowledge stochastically dominates that of an individual with a lower knowledge. Note that, to simplify mathematical tractability, we have assumed that knowledge is a flow rather than a stock variable, i.e. it is not accumulated over time but depends only on instantaneous effort to produce it.

From the properties of Frechet distribution, for a given sample $\{j_1 \ldots j_M\}$ of potential role models drawn by $i$, the probability that $j_k$ is the most culturally attractive is merely $\frac{h^{j_k}}{\sum_l h^{j_l}}$. Then, the probability that $i$ becomes a member of cultural type $n$, conditional on *not* picking up parents' culture and on the sample $\{j_1 \ldots j_M\}$ of potential role models, is

$$\frac{\sum_k h^{j_k} I(j_k \in L_n(t))}{\sum_k h^{j_k}}, \tag{1}$$

where $I(\cdot)$ is the indicator function. As $M$ grows infinitely large, by law of large numbers, (1) converges in probability to

$$\frac{h_n(t) z_n(t)}{\sum_{m \in 1 \ldots N} h_m(t) z_m(t)} = \frac{h_n(t) L_n(t)}{\sum_{m \in 1 \ldots N} h_m(t) L_m(t)}, \tag{2}$$

where

$$h_n(t) \equiv \frac{\int_{j \in L_n(t)} h^j(t) \mathrm{d}j}{L_n(t)} \tag{3}$$

is the average level of knowledge of type $n$ at time $t$. Throughout the paper, we assume that $M$ is large enough so the probability of choosing type $n$ can be approximated by (2). According to (2), only the average level of knowledge, but not its distribution across members of a cultural type, matters for cultural transmission. In the analysis that follows, we keep track only of the average level $h_n(t)$ for a given cultural type.

Assuming that the number of births to type $n$ at time $t$ is $b_n(t)L_n(t)$ where $b_n(t)$ is referred to as the birth *rate*, we can also calculate the number of individuals adopting cultural type $n$ at time $t$, $g_n(t)L_n(t)$, where $g_n(t)$ is referred to as the rate of adoption:

$$\begin{aligned} g_n(t) &= (1-q)b_n(t) + q \sum_{m=1\ldots N} \frac{h_n(t)L_n(t)}{\sum_{l=1\ldots N} h_l(t)L_l(t)} b_m(t) \frac{L_m(t)}{L_n(t)} \\ &= (1-q)b_n(t) + q h_n(t) \frac{\sum_{m=1\ldots N} b_m(t)L_m(t)}{\sum_{m=1\ldots N} h_m(t)L_m(t)} \\ &= (1-q)b_n(t) + q h_n(t) \frac{b(t)}{h(t)}, \end{aligned} \tag{4}$$

where $h(t) \equiv \frac{\sum_{m=1\ldots N} h_m(t)L_m(t)}{L(t)}$ and $b(t) \equiv \frac{\sum_{m=1\ldots N} b_m(t)L_m(t)}{\sum_{m=1\ldots N} L(t)}$ are population averages of knowledge and fertility, respectively. Throughout the paper, we assume that each cultural type constitutes a small fraction of the global population, and decisions made by members of a given cultural type have a negligible effect on $h(t)$ and $b(t)$.

Observe also that $q = 0$ implies $g_n(t) = b_n(t)$, hence, for isolated cultures which have no contact with others, knowledge production does not affect the cultural transmission process. Observe also that, since cultural transmission by itself does not increase or decrease the number of people, the average rate of adoption $g(t) \equiv \frac{\sum_{m=1\ldots N} g_m(t)L_m(t)}{\sum_{m=1\ldots N} L(t)}$ is always equal to $b(t)$.

### 2.3. *Individual lifecycle*

The model of human lifecycle is simplified from Robson and Kaplan (2003a), who analyze the unique features of a (pre-industrial) human, compared to other species, by drawing on the lifecycle theory.

#### 2.3.1. *Production of energy*

Each individual produces energy using the land, the only factor of production, which is available in fixed stock $X$ but with endogenously evolving productivity $B(t)$. As the land is divided equally between all living individuals, the per-capita energy production is equal to

$$y(t) = \frac{B(t)X}{L(t)}. \tag{5}$$

The land productivity evolves with the production of knowledge as follows:

$$\frac{\partial B(t)}{\partial t} = h(t)L(t)\left(1 - \frac{B(t)}{B_m}\right), \tag{6}$$

where $B_m$ is the maximum possible productivity of a unit of land, also referred to as the limit of productivity. Knowledge creation reduces the gap between the current productivity and its limit.

#### 2.3.2. *Use of energy*

As mentioned in previous discussion, all members of a typical cultural type $n$ share a common goal hence can pass energy from one to another as necessary. For this reason, we only keep track of the aggregate energy balance for $n$ at each moment of time $t$. Part of (individual) produced energy $y(t)$ is allocated towards (individual) prevention of mortality $s(t)$. The remaining (aggregate) energy surplus $E_n^s(t) \equiv (y(t) - s(t))\, L_n(t)$ is divided between new births $E_n^b(t)$ and knowledge production $E_n^h(t)$. We now characterize each of the three uses in more detail.

Each individual faces the instantaneous mortality hazard $\mu$, a function $\mu : R_+ \to R_+$ of mortality prevention effort $s$, with properties similar to that of Robson and Kaplan (2003b). Specifically, $\mu(s) > 0, \mu'(s) < 0, \mu''(s) > 0, \forall s \geq 0$. We also assume $\mu'(0) = -\infty$, so there is never a corner solution. Denote by $p(t', t)$ the probability of survival from time $t$ to time $t'$, we then have that $p(t,t) = 1, \frac{\partial p(t',t)}{\partial t'} \frac{1}{p(t',t)} = -\mu(s(t'))$.

It takes $e_0 > 0$ units of energy to give birth to an individual. Thus, the birth rate at time $t$ is given by

$$b_n(t) = \frac{1}{L_n(t)} \frac{E_n^b(t)}{e_0}. \tag{7}$$

A byproduct of the reproduction process is knowledge creation: every unit of energy allocated towards reproduction results in creation of $c_0 > 0$ units of knowledge. Thus, there is (slow) technological progress even if the entire energy surplus is allocated towards reproduction. Such knowledge creation can be viewed as the "learning by doing" type of knowledge.

The production of knowledge however is much more effective when resources are allocated specifically towards that goal: every unit of energy dedicated to it results in $c$ units of knowledge, with $c > c_0$. Thus, the rate of knowledge production by cultural type $n$ is given by

$$h_n(t) = \frac{1}{L_n(t)} \left( c_0 E_n^b(t) + c E_n^h(t) \right). \tag{8}$$

## 3. Analysis

### *3.1. Allocation of energy surplus*

Suppose the members of a cultural type $n$ at time $t$ have made their decisions about mortality prevention $s(t)$, and face the dilemma of dividing their energy surplus $E_n^s(t)$ between new births $E_n^b(t)$ and knowledge production $E_n^h(t)$. The division is made in a way that maximizes the population growth rate of the cultural type, (4), subject to (7) and to (8).

Such maximization yields one of the three following solutions (dropping the time argument): (i) if $L_n \frac{\partial g_n}{\partial E_n^b} = (1-q)\frac{1}{e_0} - q(c - c_0)\frac{b}{h} > 0$, then no energy is allocated to knowledge production, and the entire energy surplus is allocated towards new births, thus $E_n^b = E_n^s$ and $b_n = \frac{E_n^s}{e_0}$; (ii) if $\frac{\partial g_n}{\partial E_n^b} = 0$, the returns to both types of energy allocation are the same, and any allocation is growth-maximizing; (iii) if $\frac{\partial g_n}{\partial E_n^b} < 0$, all of energy surplus is allocated to knowledge production, and no new births are made.

Since the above optimal solution is the same for all cultural types, we can draw inference on the aggregate birth rate $b$ and knowledge production $h$ for each of the three above outcomes. The outcome (i) implies

$$b = \frac{1}{e_0}\frac{\sum_n E_n^s L_n}{L}, \tag{9}$$

$$h = c_0 \frac{\sum_n E_n^s L_n}{L}. \tag{10}$$

Such outcome will take place iff $q \leq \frac{c_0}{c}$. In other words, when cultures are sufficiently isolated from each other and cultural transmission is mostly vertical (from parents to children), a Malthusian economy occurs where the entire energy surplus is allocated towards reproduction. The outcome (ii) occurs when $q > \frac{c_0}{c}$, i.e. the rate of "oblique" (i.e. between non-relatives) cultural transmission is high enough. Equality of $\frac{\partial g_n}{\partial E_n^b}$ to zero implies that

$$b = (1-q)\frac{c}{c-c_0}\frac{1}{e_0}\frac{\sum_n E_n^s L_n}{L}, \tag{11}$$

$$h = qc\frac{\sum_n E_n^s L_n}{L}. \tag{12}$$

In other words, only a fraction of the energy surplus is allocated towards reproduction; the rest is spent on deliberate knowledge creation. Finally, outcome (iii) implies that $b = 0$, which makes it impossible for $\frac{\partial g_n}{\partial E_n^b}$ to be negative for any $q < 1$. Thus, we conclude that such outcome can be ruled out.

*3.2. Lifecycle decisions*

We assume that individuals make their decisions regarding mortality prevention $s(t)$ under the assumptions that (i) there is no population growth, and that (ii) the individual energy production $y(t)$ is time-invariant, as well. As shown below, during the pre-industrial ("Malthusian") era, population growth is negligible so (i) is close to the truth, while (ii) is exactly true. In the time limit, as the technology $B(t)$ reaches its maximum, both (i) and (ii) are true. Thus, such assumptions are irrelevant only along the transition path from the former to the latter; suboptimal decisions along a transition path do not necessarily lead to extinction of cultural types and thus are empirically plausible.

The objective of every individual is to contribute to the growth rate of own cultural type, which in turn requires to produce as much lifetime energy surplus as possible.[3] The energy surplus produced by all living members of a cultural type is pooled and is allocated toward creation of cultural followers, either by birth or by attraction from other cultural types.

The expected energy surplus produced during remaining lifetime by an individual at time $t$, who faces (perceived constant) individual output $y$ is then

$$V(y, s^E(\cdot, t)) \equiv \int_t^\infty p^E(t', t) \left[y - s^E(t', t)\right] \mathrm{d}t', \tag{13}$$

where $p^E(t', t)$ $(s^E(t', t))$ is the expected survival probability (mortality prevention effort) at time $t'$ for an individual making decisions at time $t$. The $V(\cdot)$ is being maximized subject to constraints $p^E(t, t) = 1$, $\frac{\partial p^E(t',t)}{\partial t'} \frac{1}{p^E(t',t)} = -\mu(s^E(t', t))$. As the problem is (perceived) time-invariant, individuals expect that their mortality prevention effort will not change over time: $s^E(t', t) = s, \forall t' \geq t$, for some $s$. Then, $p^E(t', t) = \exp(-\mu(s)(t' - t))$, and the expected lifetime energy surplus can then be shown to be $V(y, s) = \frac{y-s}{\mu(s)}$. Maximization of the latter

[3] With population growth, a unit of energy produced earlier is more valuable than the one produced later, because the former results in a greater impact on the share of one's cultural type in the global population. We disregard this phenomenon due to perceived zero population growth rate.

with respect to $s$ yields the optimal mortality prevention effort $s(y)$,

$$\frac{\partial V(y,s(y))}{\partial s} = -\frac{1}{\mu(s(y))}\left[1+\mu'(s(y))V(y,s(y))\right] = 0. \tag{14}$$

With $\mu'(\cdot) < 0$, it must be that a positive energy surplus is produced, $V(y,s(y)) > 0$, thus $s(y) < y$. We can also show that the second derivative is negative: $\frac{\partial^2 V(y,s)}{\partial s^2} = -\frac{\mu''(s(y))}{\mu(s(y))}V(y,s(y)) < 0$, so the second-order condition of optimality is indeed met. Additionally, we calculate the marginal response of optimal $s$ to changes in $y$, $\frac{\partial s(y)}{\partial y} = -\frac{\frac{\partial^2 V(y,s)}{\partial y \partial s}}{\frac{\partial^2 V(y,s)}{\partial s^2}} = -\frac{\mu'(s)}{\mu''(s)(y-s)} > 0$. Not surprisingly, a greater individual productivity results in a greater mortality prevention effort.

*3.3. Determining the aggregates*

Next, we proceed to the analysis of the path of the economy through time. The model allows to track such parameters as population size, technology level, fertility, and mortality. We divide the time dimension into three periods: (i) the Malthusian epoch, when the cultural transmission parameter $q$ is below the threshold ${}^{c_0}/_{c}$, (ii) the demographic transition period, and (iii) the time limit, when the economy stabilizes in a steady state.

*3.3.1. The Malthusian economy*

The Malthusian economy occurs when global population is small enough so that $q(L(t)) \leq \frac{c_0}{c}$ and the entire energy surplus is allocated to reproduction. Because of that, population is checked only by the available resources, and individual productivity $y(t)$ remains at a lowest possible level needed for reproduction. The mortality prevention effort is low, hence life expectancy is low, too.

The evolution of the technology level is characterized by (cf. (6),(5))

$$r_B(t) = \frac{\partial B(t)}{\partial t}\frac{1}{B(t)} = h(t)\frac{L(t)}{B(t)}\left(1-\frac{B(t)}{B_m}\right) = c_0(y(t)-s(y(t)))\frac{L(t)}{B(t)}\left(1-\frac{B(t)}{B_m}\right) \tag{15}$$

In the Malthusian regime, the level of technology is very far from its limit so we assume the term $\left(1-\frac{B(t)}{B_m}\right)$ is close enough to unity so we can approximate (15) by $c_0(y(t)-s(y(t)))\frac{L(t)}{B(t)} = c_0(y(t)-s(y(t)))\frac{X}{y(t)}$.

The population growth rate is, naturally, equal to the fertility rate minus the mortality rate:

$$r_L(t) = \frac{\partial L(t)}{\partial t}\frac{1}{L(t)} = b(t) - \mu(s(y(t))) = \frac{1}{e_0}(y(t)-s(t)) - \mu(s(t)). \tag{16}$$

From (14), we have that $1+\frac{\mu'(s)}{\mu(s)}(y-s)=0$ and therefore $\mu(s) = -\mu'(s)(y-s)$ which enables us to rewrite (16) as follows: $r_L(t) = (y(t)-s(y(t)))\left(\frac{1}{e_0}+\mu'(s(y(t)))\right)$.

The evolution of income is then characterized by (cf.(5))

$$r_y(t) = \frac{\partial y(t)}{\partial t}\frac{1}{y(t)} = r_B(t) - r_L(t) = (y(t)-s(y(t)))\left(c_0\frac{X}{y(t)} - \frac{1}{e_0} - \mu'(s(y(t)))\right) \tag{17}$$

The term $(y(t)-s(t))$ in (17) is always positive; the remaining term is strictly decreasing with $y$ from infinity to $-\frac{1}{e_0}$, and is equal to zero at some unique value of $y(t)=y_0$:

$$c_0\frac{X}{y_0} - \frac{1}{e_0} - \mu'(s(y_0)) = 0. \tag{18}$$

Therefore, $y_0$ is the unique long-run time-invariant income level in the Malthusian regime. Life expectancy in this regime is equal to $\frac{1}{\mu(s(y_0))}$; the population grows at the same rate as the technology does, $r_L(t) = r_B(t) = c_0X\left(1-\frac{s(y_0)}{y_0}\right) \equiv r_0$.

*3.3.2. The demographic transition*

As the population size $L(t)$ hits the threshold $q(L(t)) = \frac{c_0}{c}$, the demographic transition begins. The evolution of productivity is now characterized by

$$r_B(t) = q(L(t))cX\left(1-\frac{s(y(t))}{y(t)}\right)\left(1-\frac{B(t)}{B_m}\right), \tag{19}$$

while the evolution of population size is

$$r_L(t) = (y(t) - s(y(t)))\left((1 - q(L(t)))\frac{c}{c - c_0}\frac{1}{e_0} + \mu'(s(y(t)))\right) \tag{20}$$

As $qc$ climbs above $c_0$, the productivity growth accelerates while the population growth (shortly) slows down, which results in increase in per-capita income $y(t)$. Further developments depend on how the energy surplus $y - s(y)$ responds to increases in income $y$; we make the following

**Assumption 1.** $\frac{\partial s(y)}{\partial y} < 1$.

This assumption is true if the function $\mu(s)$ is sufficiently convex. For example, $\mu(s) = As^{-\alpha} + \delta, A > 0, \alpha > 0, \delta \geq 0$ results in $\frac{\partial s}{\partial y} = \frac{\alpha A}{(\alpha+1)(A+\delta s^{\alpha})} < 1$.

With Assumption 1, an income rise causes both technology and population growth to accelerate. Thus, the overall effect of changing $q$ and $y$ is unambiguously positive for the technological growth. Rising income also reduces mortality $\mu$.

The effect on the birth rate $b$ is ambiguous: on the one hand, greater $q$ implies that a smaller fraction of energy surplus is allocated towards fertility; on the other hand, the energy surplus itself is rising. At the initial stages of demographic transition, we argue, the second (positive) effect is likely to dominate, because a change in the *level* of $q$ results in an increase in the rate of *growth* of $y$, leading to an increase in the birth rate.

As the population growth accelerates, the process starts to feed on itself: a faster growing $L$ implies a faster growing $q$, which further implies a faster growing technology level and increasing per-capita energy production. The latter contributes to further population growth, but at a decreasing rate: with rising $q$, an ever-smaller fraction of energy surplus is allocated towards new births. As a result, the effects of technological growth gradually shift from predominantly increasing the population size to predominantly increasing the per-capita productivity.

Finally, as the level of technology rises high and starts to approach its limit, the technological growth slows down, causing both the population size and energy production per capita to stabilize at some (high) level.

*3.3.3. Time limit*

In the time limit, the economy is characterized by the level of technology being equal to its maximum $B_m$, and the population level being stabilized at some $L_m$ so that the fertility rate equals the mortality rate:

$$(y_m - s(y_m))\left((1-q(L_m))\frac{c}{c-c_0}\frac{1}{e_0} + \mu'(s(y_m))\right) = 0. \tag{21}$$

This, together with the definition of $y$, (cf.(5)) $y_m = \frac{B_m X}{L_m}$, allow to find the unique values of $L_m$ and $y_m$.

**Proposition 1.** *If the Malthusian rate of population growth $r_0$ is slow enough, the time limit income $y_m$ is greater than the Malthusian income $y_0$.*

Note that the Assumption 1 is not required for this result to be true.

**Proof.** From (21) and (18), it follows that

$$\mu'(s(y_m)) - \mu'(s(y_0)) = \frac{1}{e_0}\left[1 - \frac{c}{c-c_0}(1-q(L_m))\right] - c_0\frac{X}{y_0}. \tag{22}$$

By the definition of post-Malthusian regime, the term in square brackets in (22) is positive. The Malthusian rate of growth is determined by the parameter $c_0$; if it is small enough, the right-hand side of (22) is positive, making the left-hand side to be positive, as well. The fact that $\mu(\cdot)$ is concave then implies that $s(y_m) > s(y_0)$, which further implies $y_m > y_0$. ■

## 4. Calibration

To illustrate how the model fits the observed pattern of the demographic transition, we calibrate the model using the known history of population growth, birth and death rates.

We take the land stock $X$ equal to the earth surface, roughly 510 million square kilometers.[4] The data on population levels is from the HYDE dataset (Klein Goldewijk et al., 2010, 2011). The dataset provides an estimate on the global population size from year 10000BC until 2000AD, with time intervals decreasing from 1000 to 10 years.

We assume that the beginning of the demographic transition is year 1800, with population $L_0 = 990$ million people. To normalize the Malthusian per-capita energy production at $y_0 = 1$, we assume the technology level in that year is equal to $B(1800) = B_0 = \frac{y_0 L_0}{X} = 1 \times 990/510 = 1.94$.

We assume the mortality function takes the form $\mu(s) = As^{-\alpha} + \delta, A > 0, \alpha > 0$, which results in some optimal mortality prevention effort $s(y)$. Clark (2008) estimates the Malthusian death rate at $\mu_0 = 0.03$, thus we assume $\mu(s(y_0)) = \mu_0$ which implies $\delta = \mu_0 - As(y_0)^{-\alpha}$.

The cultural openness function takes the form $q(L) = \frac{L^\gamma}{L^\gamma + \theta^\gamma}, \gamma > 0, \theta > 0$. To ensure that the demographic transition indeed begins in year 1800, it must be that $q(L_0) = \frac{c_0}{c}$.

Since the technology and population grow at the same rate during the Malthusian era, we can match both to the known data on the population growth rate during that period. According to the HYDE dataset, global population has risen from 461 to 990 million people between years 1500 and 1800, which is consistent with the annual growth rate of $b_0 - \mu_0 = 0.0025$, where $b_0$ is the Malthusian birth rate. Therefore, $\frac{\partial B(1800)}{\partial t} \frac{1}{B_0} = c_0 \left(1 - \frac{s(y_0)}{y_0}\right) X =$

[4] We include the water surface, as the world oceans can surely contribute to the production of energy in various ways.

$b_0 - \mu_0$. We can then express the technological growth rate at any later date as follows:

$$\frac{\partial B(t)}{\partial t}\frac{1}{B(t)} = (b_0 - \mu_0)\frac{q(L(t))}{q(L_0)}\frac{1-\frac{s(y(t))}{y(t)}}{1-\frac{s(y_0)}{y_0}}\left(1-\frac{B(t)}{B_m}\right). \tag{23}$$

The Malthusian birth rate must then be equal to $b_0 = \frac{1}{e_0}(y_0 - s(y_0))$, from which a later birth rate $b(t)$ can be derived: $b(t) = (1-q(L(t)))\frac{c}{c-c_0}\frac{1}{e_0}(y(t)-s(y(t))) = b_0\frac{1-q(L(t))}{1-q(L_0)}\frac{y(t)-s(y(t))}{y_0-s(y_0)}$.

The remaining unknown parameters $A, \alpha, \gamma, \theta, B_m$ are calibrated by matching the predicted demographic transition path to the following observations: global population size in years 1960 and 2000 (from the HYDE dataset), global birth rate in 1960, global death rates in years 1960 and 2000 (all from World Development Indicators). The resultant values are $A = 0.0153, \alpha = 0.39, \gamma = 4.84, \theta = 2095, B_m = 3669$.

The results are illustrated in figure 1. Despite the fact that the model was calibrated using only five bits of post-Malthusian data, it fits the observed demographic patterns fairly well. The model predicts that the global population size will eventually level off at slightly under 9 billion people; the per-capita energy production will be more than 200 times that of the Malthusian era. At the same time, the fraction of energy allocated to own mortality prevention will drop from 25% to 16%; the rest will be spent on new births and (largely) on knowledge creation.

## 5. Discussion

The main goal of this paper was not to make accurate numerical predictions about the demographic transition process, but to highlight an alternative (relative to the existing literature) mechanism at work. To emphasize the novel elements of the mechanism, some other elements have been deliberately simplified. For example, the model completely ignores the population age structure and its effects on the birth and death rates, and all living individuals at a point in time are assumed to be the same. Such simplification indeed

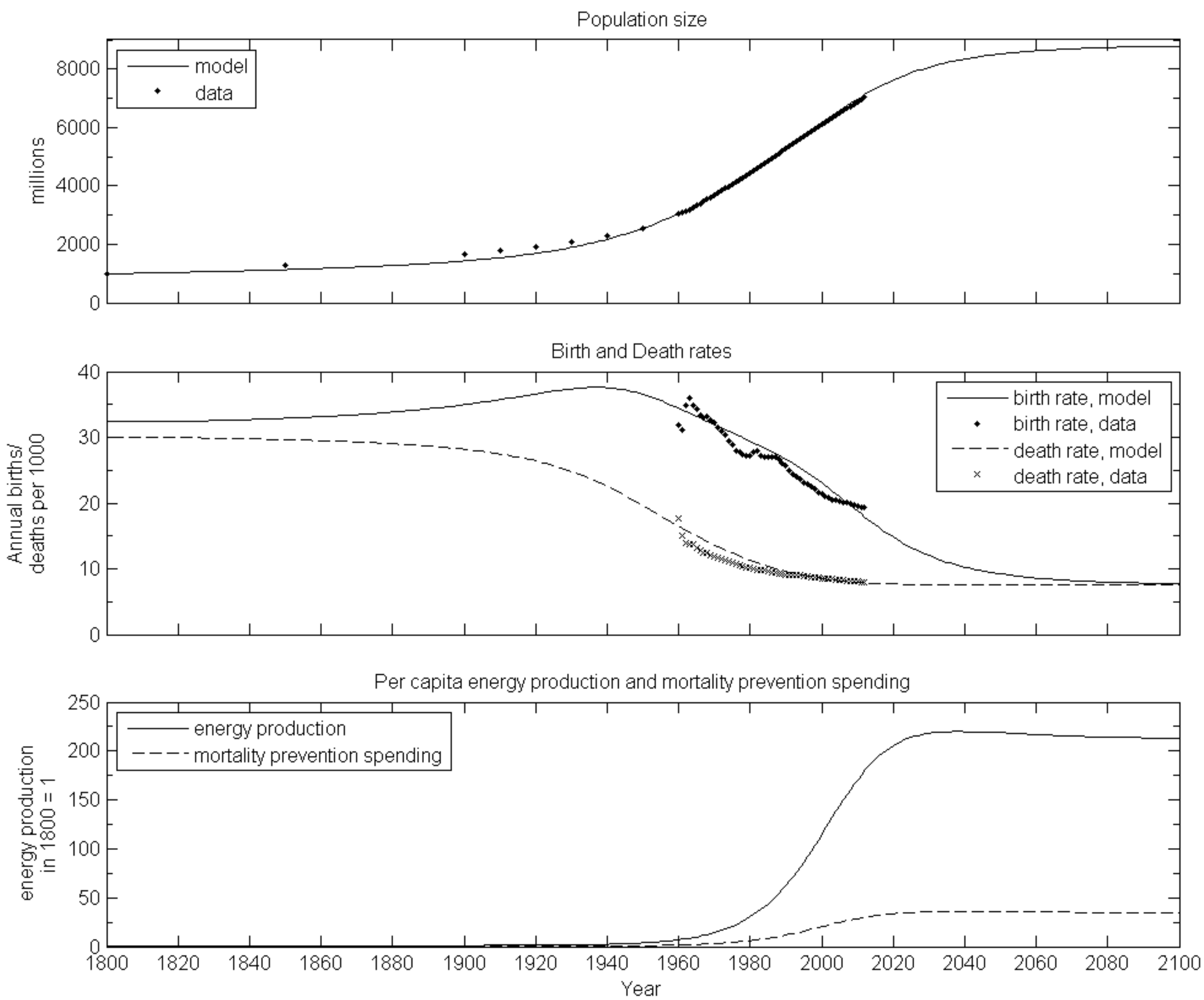


Figure 1: Predicted vs. observed elements of the demographic transition

results in unrealistic predictions about some demographic elements. For example, the World Development Indicators dataset estimates the death rate in 2012 at about 8 per 1000 people. If all living individuals indeed faced the same risk of mortality, that would imply a life expectancy of 125 years; the real figure (from the same dataset) is only 70.8 years. Unless life expectancy indeed increases to 125 years, the death rate should increase in the future, which is something our simple model cannot predict.

To address the issue, the endogenous population age structure can be introduced into the model, along the lines of Robson and Kaplan (2003a). Individual energy production per unit of land depends not only on land productivity, but also on personal productivity which exogenously depends on age. Personal productivity is low in the young age, peaks during adulthood, and declines with older age. As individuals maximize the expected energy surplus, the individual mortality prevention effort should then rise during early childhood and decline in the old age; the mortality rate should follow the opposite pattern. Since members of the same cultural type share the same goal of increasing their type's fitness, energy transfers between more productive and less productive individuals are possible.

A further extension of the model could include endogenous human capital, whose increase is often associated with the demographic transition. Suppose individual energy production per unit of land depends, besides land productivity and personal productivity, on the amount of endogenously accumulated human/somatic capital. Accumulation of human capital requires the use of energy. Individuals then optimally accumulate some amount of such capital in the young age. As the land productivity increases with technological progress, the expected energy surplus rises which induces individuals to accumulate more human capital.

## References

Axinn, W. G., Barber, J. S., 2001. Mass education and fertility transition. American Sociological Review 66 (4), 481–505.

Axinn, W. G., Yabiku, S. T., 2001. Social change, the social organization of families, and fertility limitation1. American Journal of Sociology 106 (5), 1219–1261.

Baudin, T., 2010. A role for cultural transmission in fertility transitions. Macroeconomic Dynamics 14 (04), 454–481.

Becker, G. S., 1960. An economic analysis of fertility. In: Demographic and Economic Change in Developed Countries. UMI, pp. 225 – 256.

Becker, G. S., Murphy, K. M., Tamura, R., 1990. Human capital, fertility, and economic growth. The Journal of Political Economy 98 (5), S12–S37.

Bergstrom, T. C., 2007. Some evolutionary economics of family partnerships. The American economic review 97 (2), 482–486.

Bisin, A., Verdier, T., 2001. The economics of cultural transmission and the dynamics of preferences. Journal of Economic Theory 97 (2), 298 – 319.

Blackmore, S., 2000. The meme machine. Oxford University Press.

Boyd, R., Richerson, P. J., 1985. Culture and the Evolutionary Process. The University of Chicago Press.

Chakraborty, S., 2004. Endogenous lifetime and economic growth. Journal of Economic Theory 116 (1), 119 – 137.

Clark, G., 2008. A farewell to alms: a brief economic history of the world. Princeton University Press.

Galor, O., Moav, O., 2002. Natural selection and the origin of economic growth. Quarterly Journal of Economics 117 (4), 1133 – 1191.

Galor, O., Weil, D. N., 2000. Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond. The American Economic Review 90 (4), 806–828.

Greksa, L. P., 2002. Population growth and fertility patterns in an Old Order Amish settlement. Annals of Human Biology 29 (2), 192 – 201.

Kalemli-Ozcan, S., 2002. Does the mortality decline promote economic growth? Journal of Economic Growth 7 (4), 411–439.

Klein Goldewijk, K., Beusen, A., de Vos, M., van Drecht, G., 2011. The hyde 3.1 spatially explicit database of human induced land use change over the past 12,000 years. Global Ecology and Biogeography 20, 73–86.

Klein Goldewijk, K., Beusen, A., Janssen, P., 2010. Long term dynamic modeling of global population and built-up area in a spatially explicit way, hyde 3 .1. The Holocene 20, 565–573.

La Ferrara, E., Chong, A., Duryea, S., 2012. Soap operas and fertility: Evidence from brazil. American Economic Journal: Applied Economics 4 (4), 1–31.

Mokyr, J., 2010. The Enlightened Economy. Yale University Press.

Newson, L., Postmes, T., Lea, S., Webley, P., Richerson, P. J., McElreath, R., 2007. Influences on communication about reproduction: the cultural evolution of low fertility. Evolution and Human Behavior 28 (3), 199 – 210.

Ponthiere, G., 2011. Mortality, family and lifestyles. Journal of Family and Economic Issues 32 (2), 175–190.

Robson, A. J., 2001. The biological basis of economic behavior. Journal of Economic Literature 39 (1), 11–33.

Robson, A. J., Kaplan, H. S., 2003a. The evolution of human life expectancy and intelligence in hunter-gatherer economies. The American Economic Review 93 (1), 150–169.

Robson, A. J., Kaplan, H. S., 2003b. The evolution of human life expectancy and intelligence in hunter-gatherer economies. American Economic Review, 150–169.

Zakharenko, R., 2013a. Nothing Else Matters: Evolution of Preference for Social Prestige. mimeo.

Zakharenko, R., 2013b. The Role of Passionate Individuals in Economic Development. mimeo.